Using volume reduction to study QCD-like theories at large N_c

Steve Sharpe
University of Washington

Based on work with Barak Bringoltz & Mateusz Koren:
arXiv:0805.2146, 0906.3538, and in progress

CERN workshop on Future Directions in LGT, July 27 2010

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Can we use simulations on 1^4 lattices to study QCD-like theories at large N_c ?

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Outline

* Motivation
* A short history of "volume reduction"
* Application to QCD with single Dirac adjoint fermion: mapping the phase diagram of 1^4 model
* Into the guts of reduction: eigenvalue distributions for the adjoint Dirac operator
* Ongoing work with two adjoint fermions
* Future prospects

Motivation

- Intriguing idea! ..... but does it work?
- May provide an alternative method for studying gauge theories at large N
  - replace V & N extrapolation with single N extrap.
- Theories simplify at large N, yet share key non-perturbative properties with low-N versions
  - QCD retains confinement and CSB at large N, but mesons do not interact
- Connect to analytic approaches (e.g. gauge-gravity duality)

Reduction already in use:

* Partial volume independence [Narayan & Neuberger]
  * If $L > L_c \approx 1$ fm then results independent of L
* Single-site SUSY models (reduced from SUGRA) using non-compact gauge variables [see e.g. Nishimura, LAT09]

History of large-N volume reduction

First example

VOLUME 48, NUMBER 16 PHYSICAL REVIEW LETTERS 19 APRIL 1982

Reduction of Dynamical Degrees of Freedom in the Large-*N* Gauge Theory

Tohru Eguchi and Hikaru Kawai
Department of Physics, Faculty of Science, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan
(Received 19 January 1982)

$$\text{Lattice } SU(N) \text{ on } L^d \overset{N=\infty}{\equiv} \text{Lattice } SU(N) \text{ on } 1^d$$

Now usually called "large-N volume independence"

Large-N volume independence

Eguchi Kawai `82

$$\text{Lattice } SU(N) \text{ on } L^d \overset{N=\infty}{\equiv} \text{Lattice } SU(N) \text{ on } 1^d$$

gauge theory	"reduced" or "matrix" model
$U_{n,\mu} \in SU(N)$	$U_\mu \in SU(N)$
$S_{\text{gauge}} = Nb \sum_{n, \mu<\nu} 2\text{Re}\,\text{Tr}\left(U_{n,\mu} U_{n+\mu,\nu} U^\dagger_{n+\nu,\mu} U^\dagger_{n,\nu}\right)$ $b = (g^2 N)^{-1}$	$S_{EK} = Nb \sum_{\mu<\nu} 2\text{Re}\,\text{Tr}\,(U_\mu U_\nu U^\dagger_\mu U^\dagger_\nu)$ $b = (g^2 N)^{-1}$
$W_C = \frac{1}{N} \text{tr}\; U_{x,\hat\mu} U_{x+\hat\mu,\hat\nu} \cdots U_{x-\hat\nu-\hat\rho,\hat\rho} U_{x-\hat\nu,\hat\nu}\,,$	$W_C^{\text{reduced}} = \frac{1}{N} \text{tr}\; U_\mu U_\nu \cdots U_\rho U_\nu\,.$
$U_{n\mu} \rightarrow \Omega_n\, U_{n\mu}\, \Omega^\dagger_{n+\mu} \quad ; \quad \Omega_n \in SU(N)$ $U_{[(\vec{n},\tau),\mu]} \rightarrow U_{[(\vec{n},\tau),\mu]}\, z_\mu \quad ; \quad z_\mu \in Z_N$	$U_\mu \rightarrow \Omega\, U_\mu\, \Omega^\dagger \quad ; \quad \Omega \in SU(N)$ $U_\mu \rightarrow U_\mu z_\mu \quad ; \quad z_\mu \in Z_N$

$$\langle W_C \rangle_{\text{gauge theory}} = \langle W_C^{\text{reduced}} \rangle_{\text{reduced}} + O(1/N^2).$$

EK's demonstration of vol. indep.

- Show equivalence of Dyson-Schwinger eqs for Wilson loops

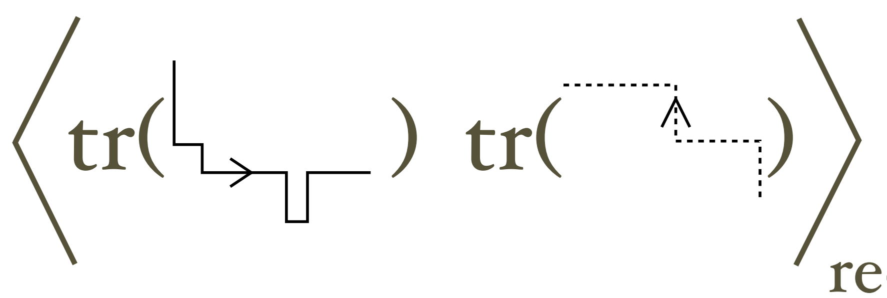
gauge	reduced
$U_{n,\mu} \to U_{n\mu}\left(1 + i\epsilon t^a\right)$	$U_\mu \to U_\mu\left(1 + i\epsilon t^a\right)$

- Crucial difference

gauge	reduced
$\mathrm{tr}\left(\cdots \underline{U_{n,\mu}} U_{n+\mu,\nu} \cdots \underline{U^\dagger_{m,\mu}} U_{m-\mu,\rho} \cdots\right)$	$\mathrm{tr}\left(\cdots \underline{U_\mu} U_\nu \cdots \underline{U^\dagger_\mu} U_\rho \cdots\right)$

- Get extra terms on the reduced side: must vanish for reduction to hold

e.g.

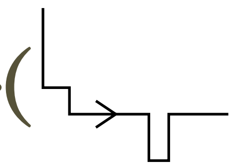
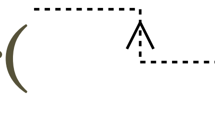


= 0

- Extra terms correspond to "open loops" in gauge theory

e.g. $\left\langle \mathrm{tr}\left(U_\mu U_\nu^\dagger\right)\ \mathrm{tr}\left(U_\mu^\dagger U_\nu\right)\right\rangle_{\mathrm{reduced}} = 0$

EK's demonstration (continued)

Reduction holds if $\Big\langle$ tr() tr() $\Big\rangle_{\text{reduced}}$ = 0

- Valid if have large-N factorization

$$\langle W_{C_1} W_{C_2} \rangle_{\text{reduced}} = \langle W_{C_1} \rangle_{\text{reduced}} \langle W_{C_2} \rangle_{\text{reduced}} + O(1/N^2),$$

- ... and if center symmetry is unbroken $(Z_N^4 : \; U_\mu \to U_\mu z_\mu)$

$$\langle W_{\text{open}} \rangle_{\text{reduced}} = 0.$$

CONCLUSION: $\text{tr}U_\mu, \;\; \text{tr}U_\mu U_\nu, \;\; \text{etc.}$

must all vanish in the reduced model for reduction to hold

Reduction fails! [Bhanot, Heller & Neuberger '82]

- Qualitatively: Small L ⇔ High T ⇒ deconfinement ⇒ $\mathrm{tr}(U_\mu) \neq 0$
- Can understand in weak coupling limit as due to clustering of eigenvalues of U_μ [BHN '82, Kazakov & Migdal '82]

$$Z_{EK} = \int DU\, exp\left[Nb \sum_{\mu<\nu} 2\mathrm{Re}\,\mathrm{tr}(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger)\right]$$

$$U_\mu = V_\mu^\dagger \Lambda_\mu V_\mu \qquad\qquad \Lambda_\mu = \mathrm{diag}\left[e^{i\theta_\mu^1}, \ldots, e^{i\theta_\mu^N}\right]$$

$$Z_{EK} = \int \prod_{\mu,a} \frac{d\theta_\mu^a}{2\pi} \Delta^2(\theta) \int DV\ \exp S_{EK} \equiv \int \prod_{\mu,a} \exp - F_{EK}(\theta)$$

$$F_{EK} \stackrel{b\to\infty}{\longrightarrow} \sum_{a<b} \log\left[\sum_\mu \sin^2\left(\frac{\theta_\mu^a - \theta_\mu^b}{2}\right)\right]$$

➡ Eigenvalues attract for d>2 ⇒ $\theta_\mu^a = \theta_\mu^b$ and so: $\mathrm{tr}\ U_\mu \neq 0$

- Note that θ_μ appear as momenta in gluon propagator

Alternative view of reduction

- Volume independence is an example of a large-N orbifold equivalence [Kovtun, Unsal & Yaffe]

Restrict to zero-momentum fields

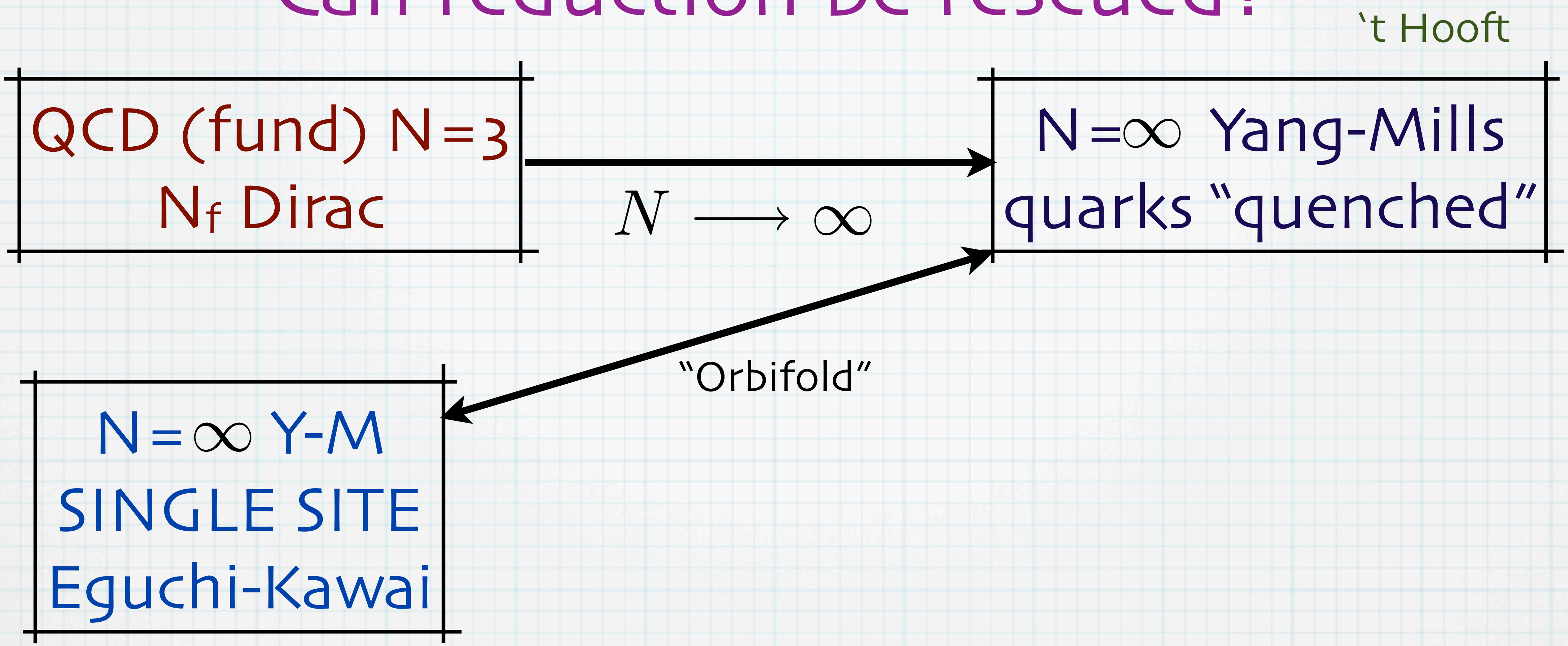
SU(N) gauge theory → SU(N) on single site

SU(N) on single site → SU(N) gauge theory

Orbifold w.r.t. combined gauge and center transfomation

- Orbifold equivalence holds if "orbifolding symmetries" (translation invariance and center symmetry) are unbroken

Can reduction be rescued?



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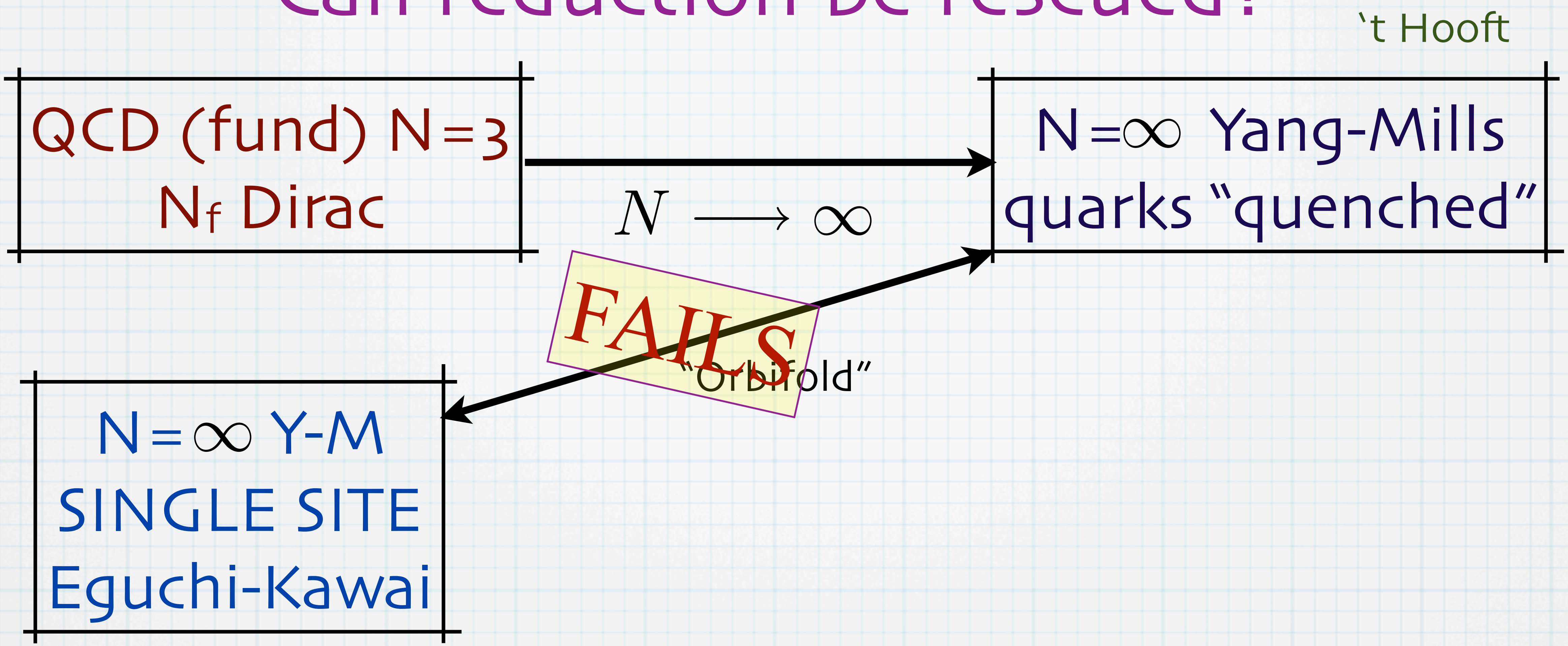

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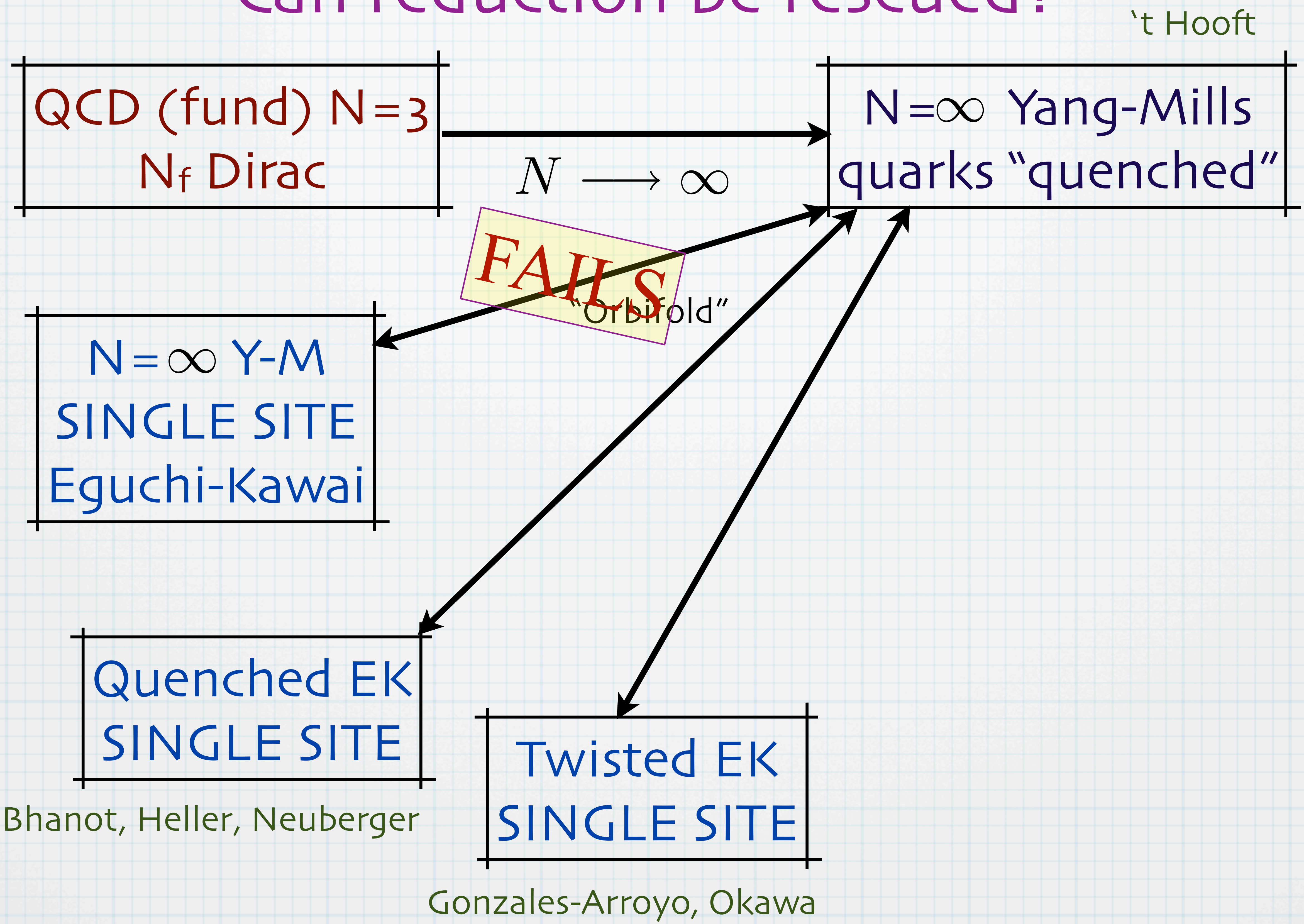

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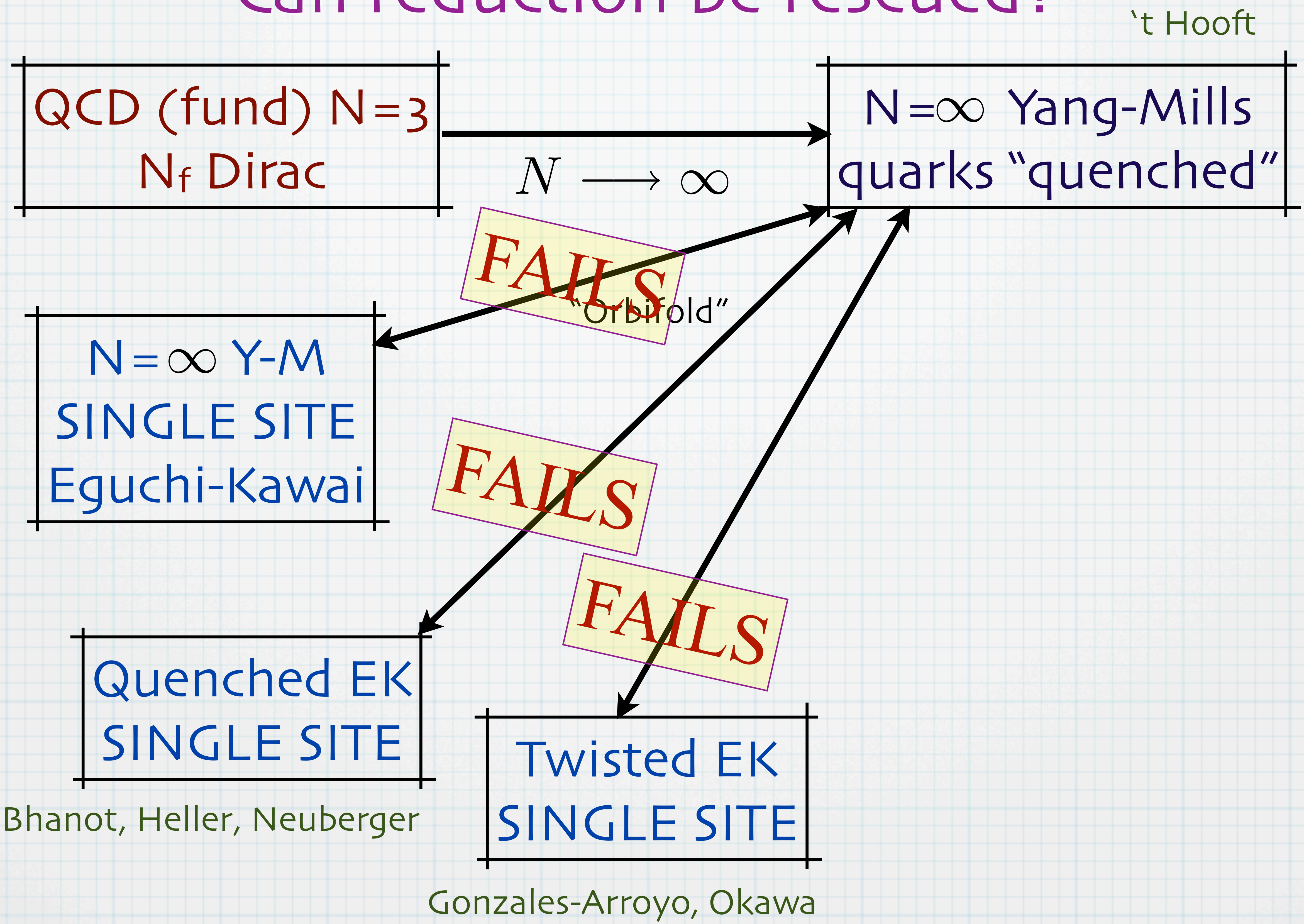

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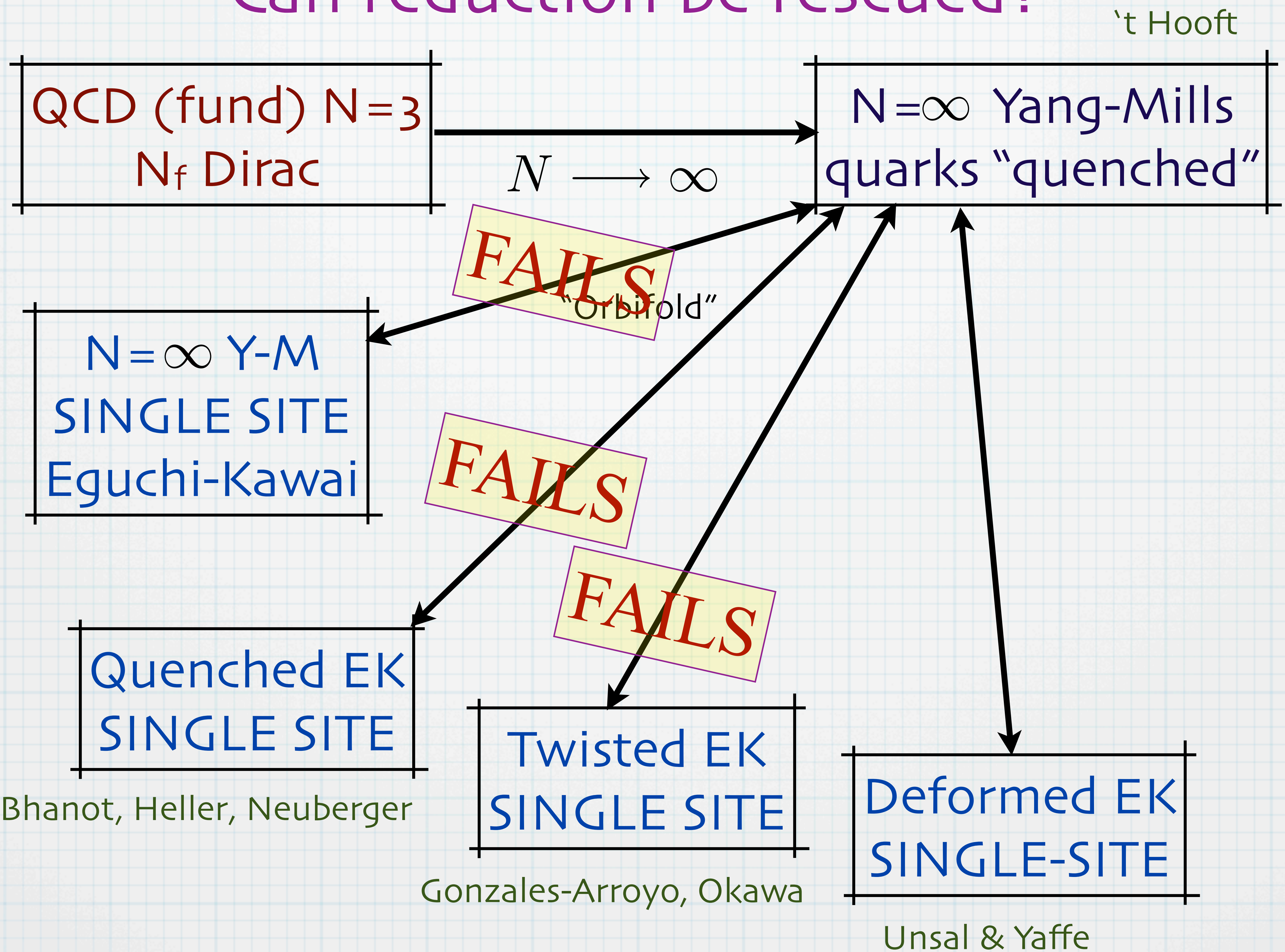

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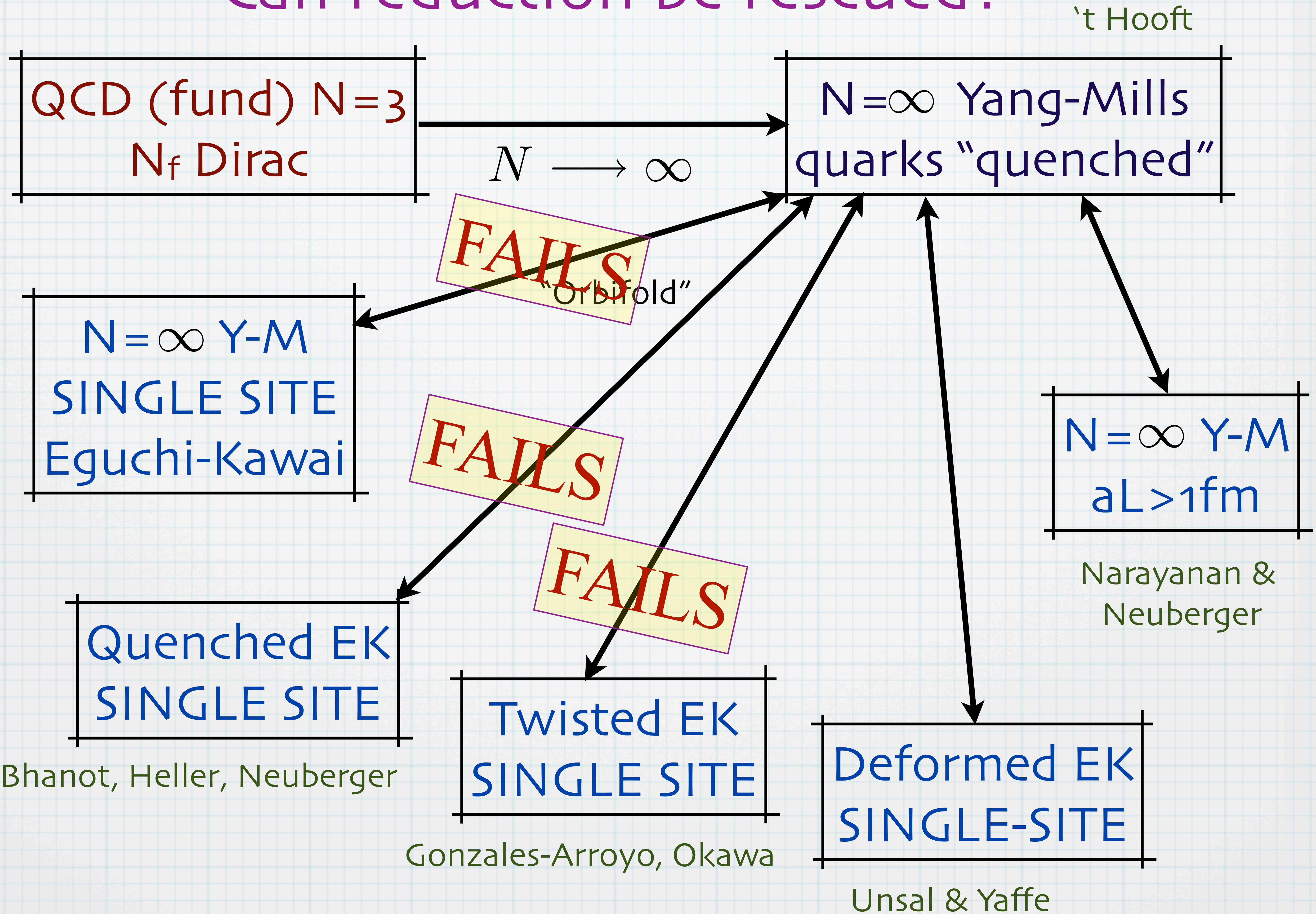

An alternative approach

QCD ($N=3$)
$2N_f$ Dirac fermions
in AS irrep (q^{ab})
infinite volume

An alternative approach

[Corrigan-Ramond
Armoni-Shifman-Veneziano]

QCD (N=3) 2N_f Dirac fermions in AS irrep (q^{ab}) infinite volume	$N \longrightarrow \infty$ →	QCD (N=infinity) 2N_f Dirac fermions in AS irrep (q^{ab}) infinite volume

An alternative approach

[Corrigan-Ramond
Armoni-Shifman-Veneziano]

QCD (N=3)
$2N_f$ Dirac fermions
in AS irrep (q^{ab})
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$N \longrightarrow \infty$

QCD (N=infinity)
$2N_f$ Dirac fermions
in AS irrep (q^{ab})
infinite volume

"Orientifold" equivalence
(in even C sectors)
[ASV]

QCD (N=infinity)
N_f Dirac fermions
in Adjoint irrep
infinite volume

An alternative approach

[Corrigan-Ramond
Armoni-Shifman-Veneziano]

QCD (N=3)
$2N_f$ Dirac fermions
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$N \longrightarrow \infty$

QCD (N=infinity)
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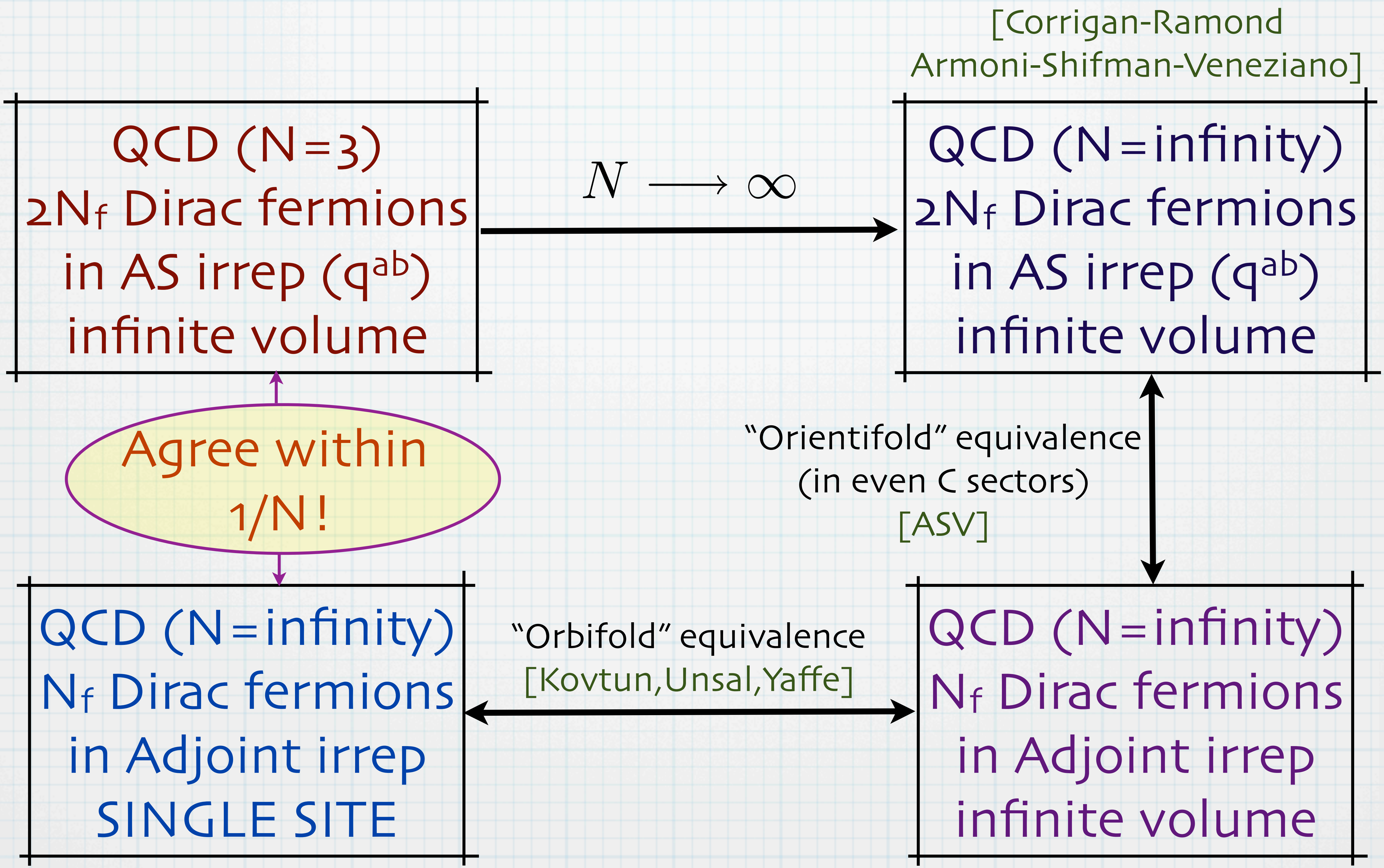
“Orientifold” equivalence
(in even C sectors)
[ASV]

QCD (N=infinity)
N_f Dirac fermions
in Adjoint irrep
SINGLE SITE

“Orbifold” equivalence
[Kovtun,Unsal,Yaffe]

QCD (N=infinity)
N_f Dirac fermions
in Adjoint irrep
infinite volume

An alternative approach



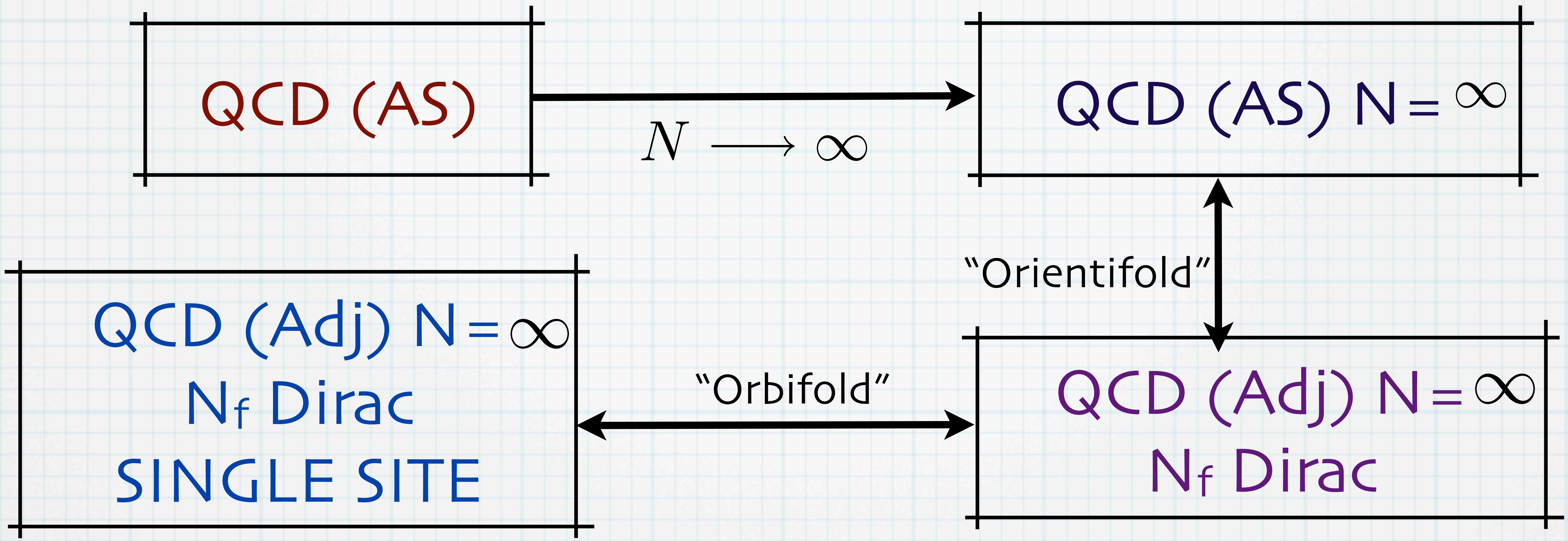
Why do adjoint fermions help?

- Adjoint fermions survive in large N limit (unlike fundamentals)
- With PBC, lead to repulsion between link eigenvalues ($\theta_\mu^{ab}=\theta_\mu^a - \theta_\mu^b$)

$$F_{EK}(b\to\infty) = 2\sum_{a<b}\log\left[\frac{4}{a^2}\sum_{\mu=1}^{4}\sin^2\left(\frac{\theta_\mu^{ab}}{2}\right)\right] - 4N_f^D\sum_{a<b}\log\left(\frac{1}{a^2}\sum_{\mu=1}^{4}\sin^2\theta_\mu^{ab} + \left(m_0+\frac{2}{a}\sum_{\mu=1}^{4}\sin^2\left(\frac{\theta_\mu^{ab}}{2}\right)\right)^2\right)$$

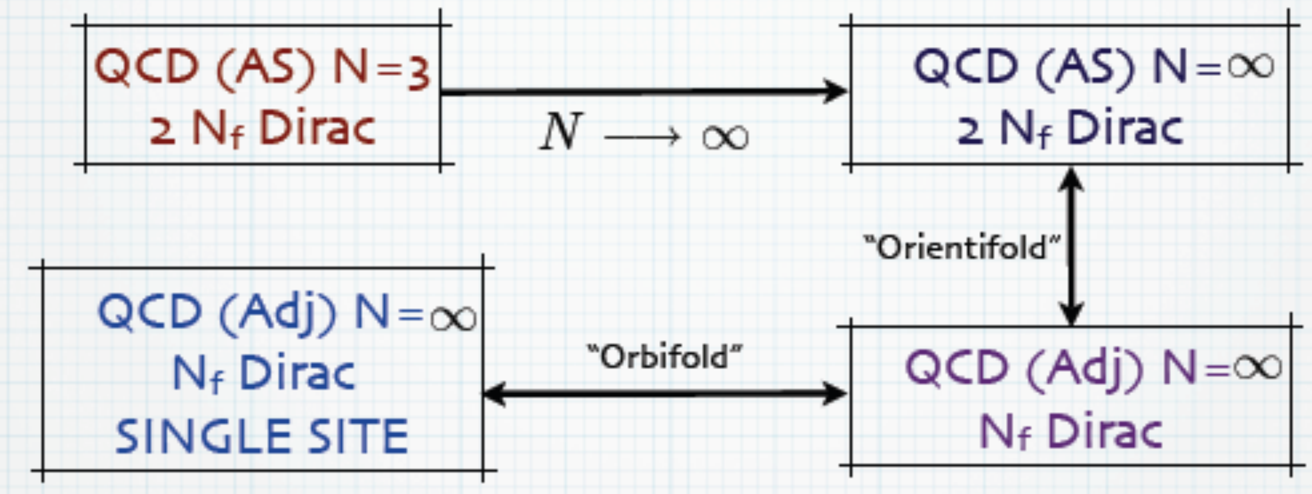
- Repulsion wins for $N_f > 1$ massless fermions
- PT suggests need $m_{phys} < 1/(aN)$ to avoid center symmetry breaking [Ogilvie & Myers, Hollowood & Myers, Bringoltz]
- However, one-loop unreliable for $\theta_\mu^{ab} \to 0$
- Furthermore, couplings of interest are in non-perturbative domain
- Need to study non-perturbatively!

Overall aims of our calculations



- Use single-site QCD(Adj) for N large to learn about 3 theories of great interest
  - $N_f=1$: learn about QCD with 2 flavors in Corrigan-Ramond large-N limit
  - $N_f=2$: alternative window on "minimal" walking technicolor theory
  - [$N_f=1/2$: equivalent to SYM, for which exact results are known]
- Even though "matrix model" lives on a single site, one can calculate many physical quantities (string tension, pion mass, ...)

Conditions for equivalences to hold



1. Large-N factorization holds
2. Orientifold: C not broken in QCD(AS,Adj)
3. Orbifold: Translation invariance unbroken in QCD (Adj.) in infinite volume
4. Orbifold: $(Z_N)^4$ center symmetry unbroken in QCD (Adj.) on a single site

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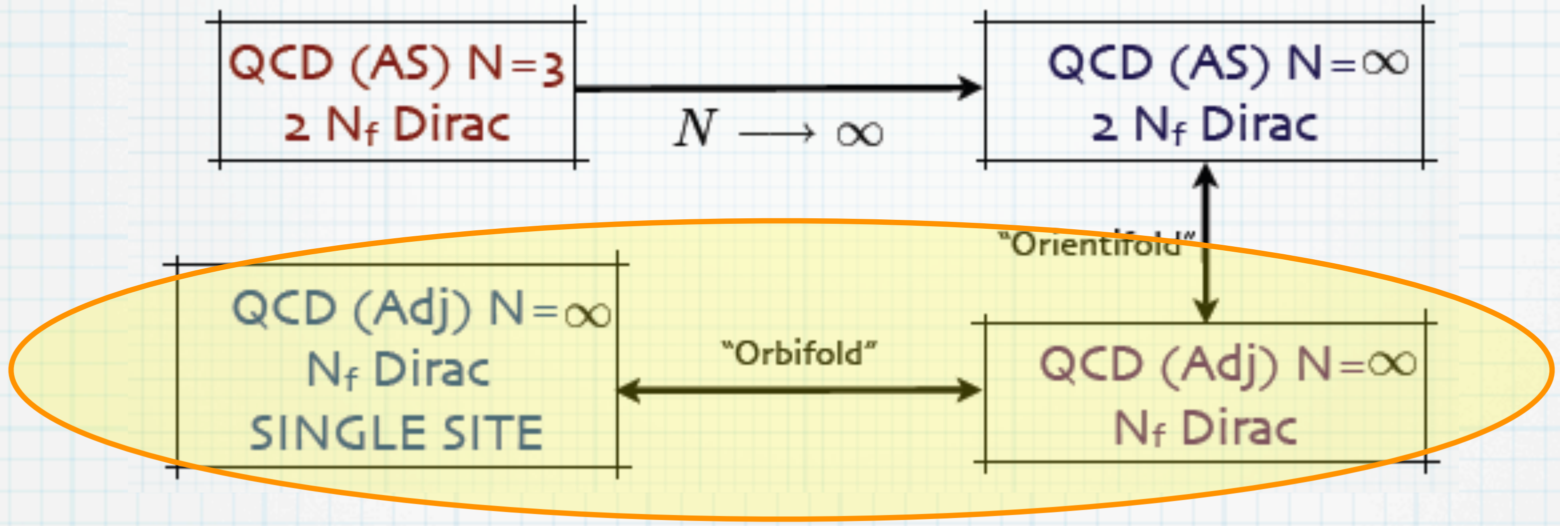

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IN THIS TALK:

We assume the first three hold and study the last

Application to N_f=1 adjoint QCD

Main aims of initial study

Bringoltz & SS, arXiv:0906.3528 (PRD)

* Determine region in phase diagram of single-site model for which center symmetry is unbroken
* Study some basic observables
* Understand how L_{eff} scales with N
* To get started, need conjecture for phase diagrams
* Keep in mind that for N=3, $\beta_{SU(3)}=6/g^2=18\, b$

The (possibly) equivalent theories (I)

(1) Infinite volume QCD(adj), N_f=1 Dirac

$$S_{\text{gauge}} = 2Nb \sum_{\text{P}} \text{ReTr} U_P \,, \qquad b = 1/(g^2 N)$$

$$S_F = \bar{\psi} D_W \psi$$

$$(D_W)_{xy} = \delta_{xy} - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U^{\text{adj}}_{x,\mu} \delta_{y,x+\mu} + (1+\gamma_\mu) U^{\dagger\,\text{adj}}_{x,\mu} \delta_{y,x-\mu} \right]$$

- Use Wilson fermions for computational simplicity
- PBC in all directions
- Symmetries: gauge, center $(Z_N)^4$ and flavor U(1) (SO(2) if write as two Majorana fields)
- This theory not simulated previously--though lots of work for N_f=1/2 (SUSY) and N_f=2 (nearly conformal)

The (possibly) equivalent theories (II)

(2) Single-site theory, PBC in all directions:

$$S_{\text{gauge}} = 2Nb \sum_{\mu<\nu} \text{ReTr} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \,, \qquad b = 1/(g^2 N)$$

$$S_F = \bar{\psi} D_W \psi$$

$$(D_W) = \mathbf{1} - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu) U_\mu^{\text{adj}} + (1+\gamma_\mu) U_\mu^{\text{adj}\,\dagger} \right]$$

Symmetries:

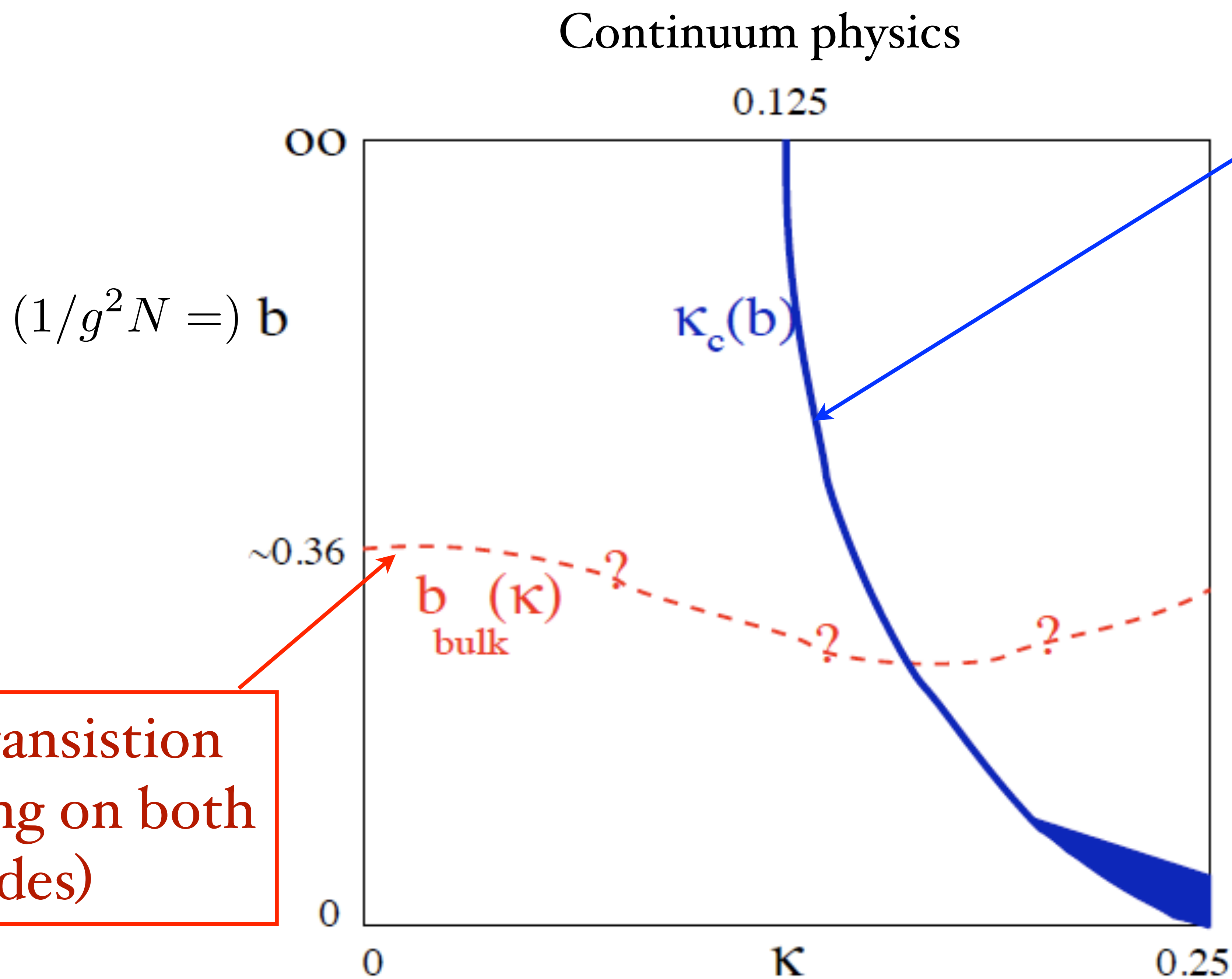
gauge: $U_\mu \longrightarrow \Omega U_\mu \Omega^\dagger \quad (\text{all } \mu) \quad \Omega \in SU(N)$

center $(Z_N)^4$: $U_\mu \longrightarrow U_\mu e^{2\pi i n_\mu / N} \quad n_\mu \in Z_N$

* Equivalence relates theories having same b, kappa
* Requires $(Z_N)^4$ to be unbroken

Expected phase diagrams (I)

1. Infinite volume QCD(adj) (large N, N_f=1)



Critical line (or Aoki phase): quarks are light. Similar to QCD although symmetries differ

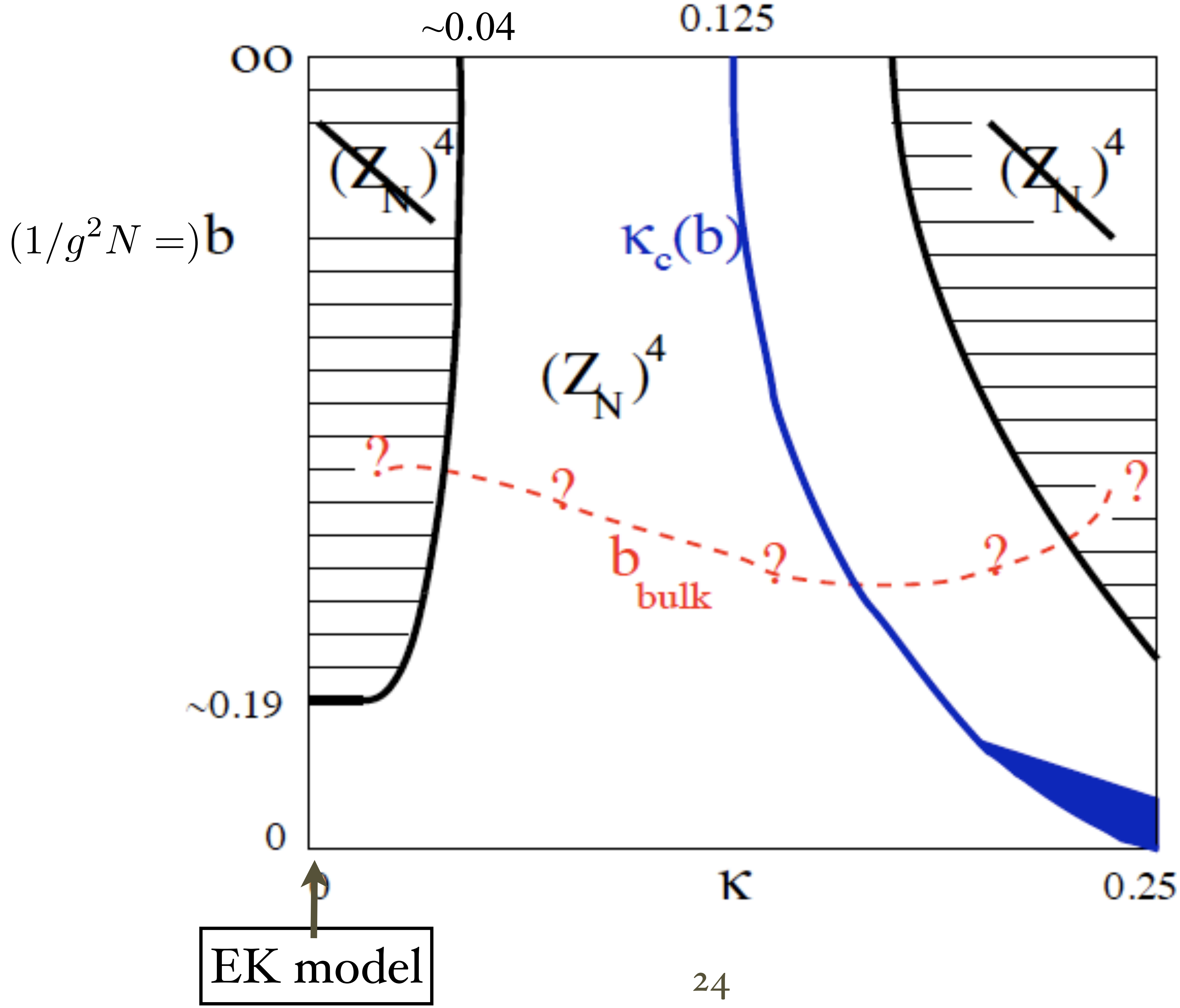
$$m_0 = \frac{1}{2\kappa} - \frac{1}{2\kappa_c}$$

Bulk transistion (confining on both sides)

pure-gauge theory

Expected phase diagram (2): single-site theory

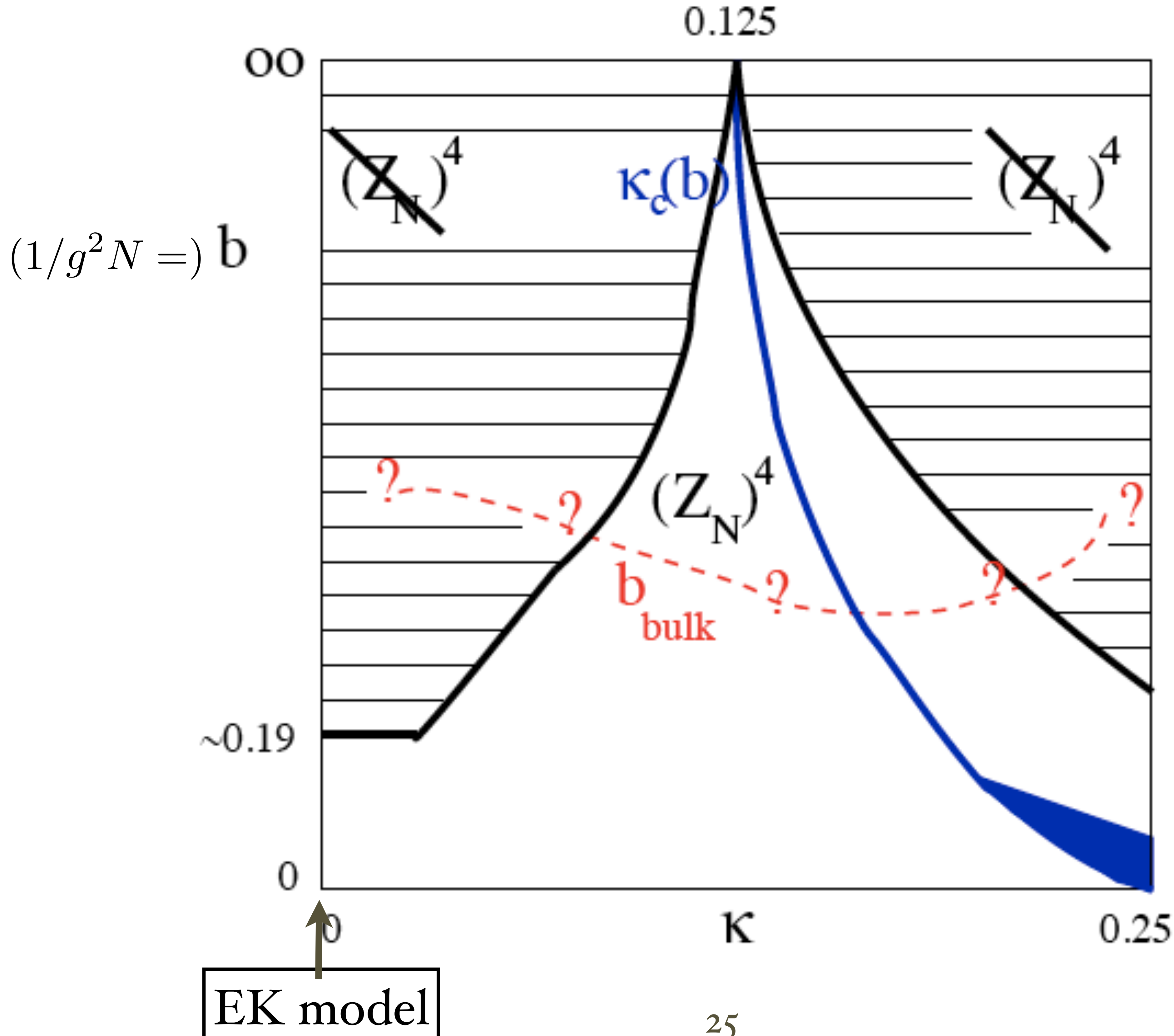
- Based on knowledge of EK model and PT (2009)



Identical to infinite volume theory (at large N) within "funnel"

Expected phase diagram (2): single-site theory

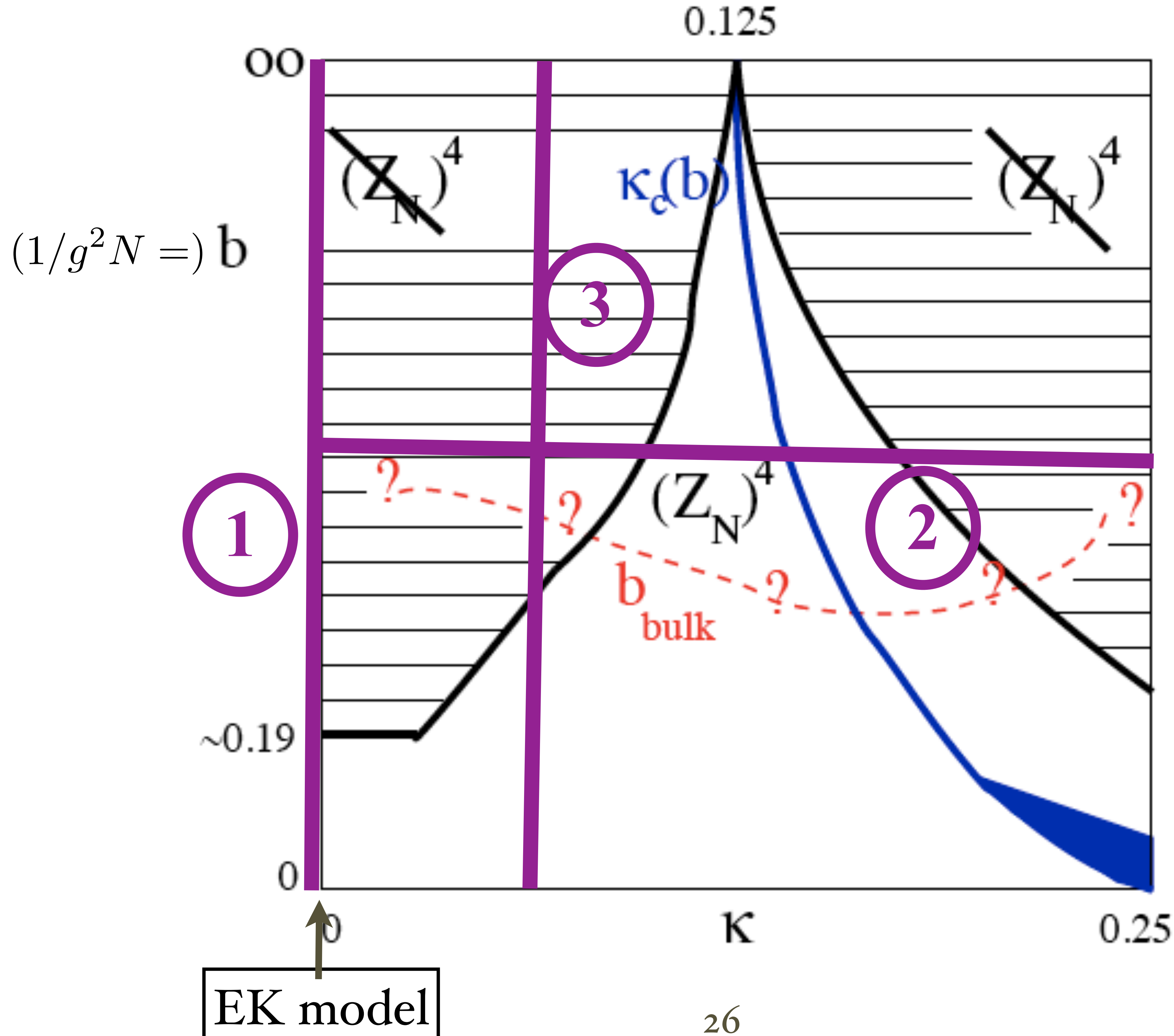
- Based on knowledge of EK model and PT (early '10)



Identical to infinite volume theory (at large N) within "peak"

Studying phase diagram of single-site theory

- Do scans along lines in phase plane



Numerical lattice study

Details of initial simulation

* Use Metropolis algorithm with weight

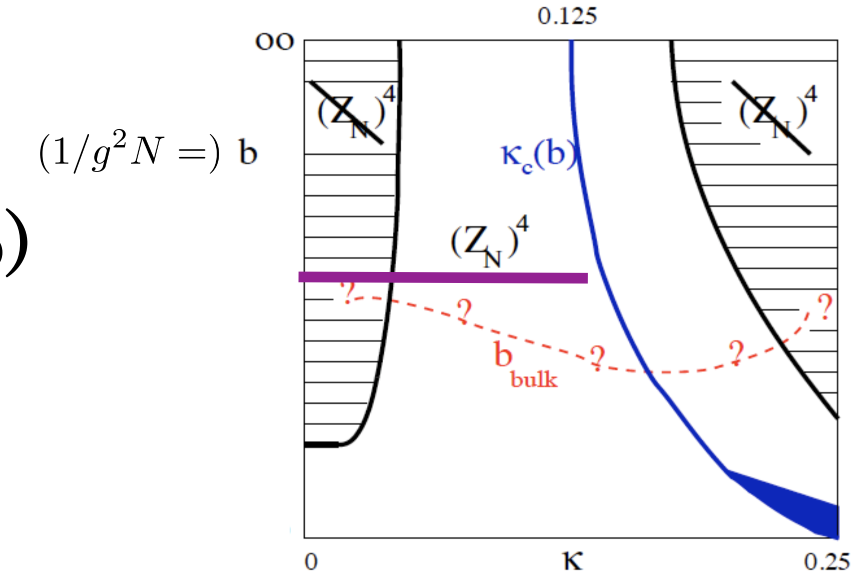
$$P(U) = e^{S_{\text{EK}}(U)} \det D_W^{\text{red}}(U)$$

* Determinant is real & positive (for integer N_f)
* Update N(N-1)/2 SU(2) subgroups in turn on each link, then move to next link (4 in all!)
* Evaluate determinant explicitly: 50-60% accept.
* Scaling is $(N^2)^3 \times N^2$---can reach N=15 on PCs
* Measure every 5 sweeps after ~50 sweeps therm.
* 100-3700 measurements

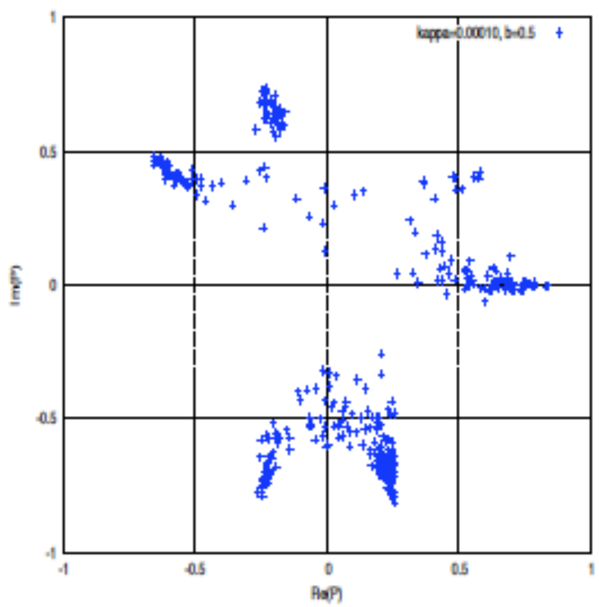
Scans 2: decreasing m_q at fixed b

Z_N symmetry restored for $\kappa \gtrsim 0.6$ at b=0.5

,SU(10)



$\kappa \approx 0$



Scatter plots of Polyakov loops

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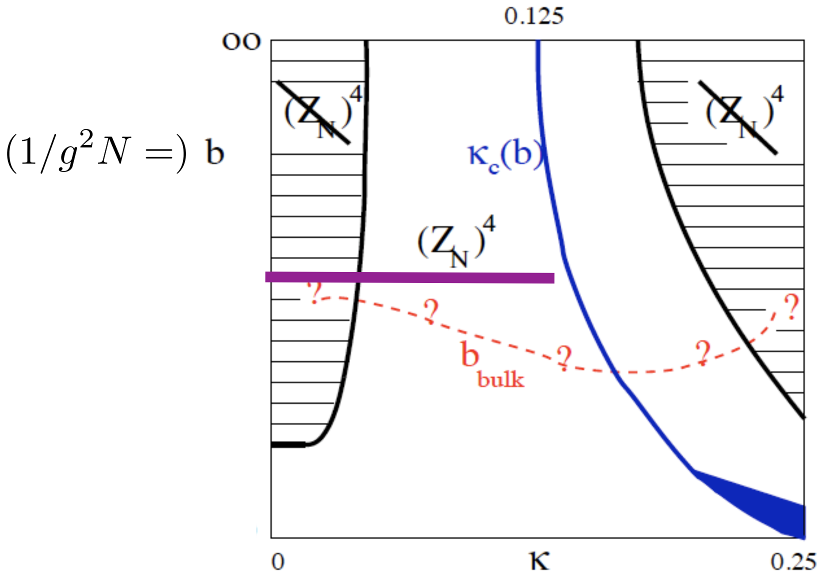

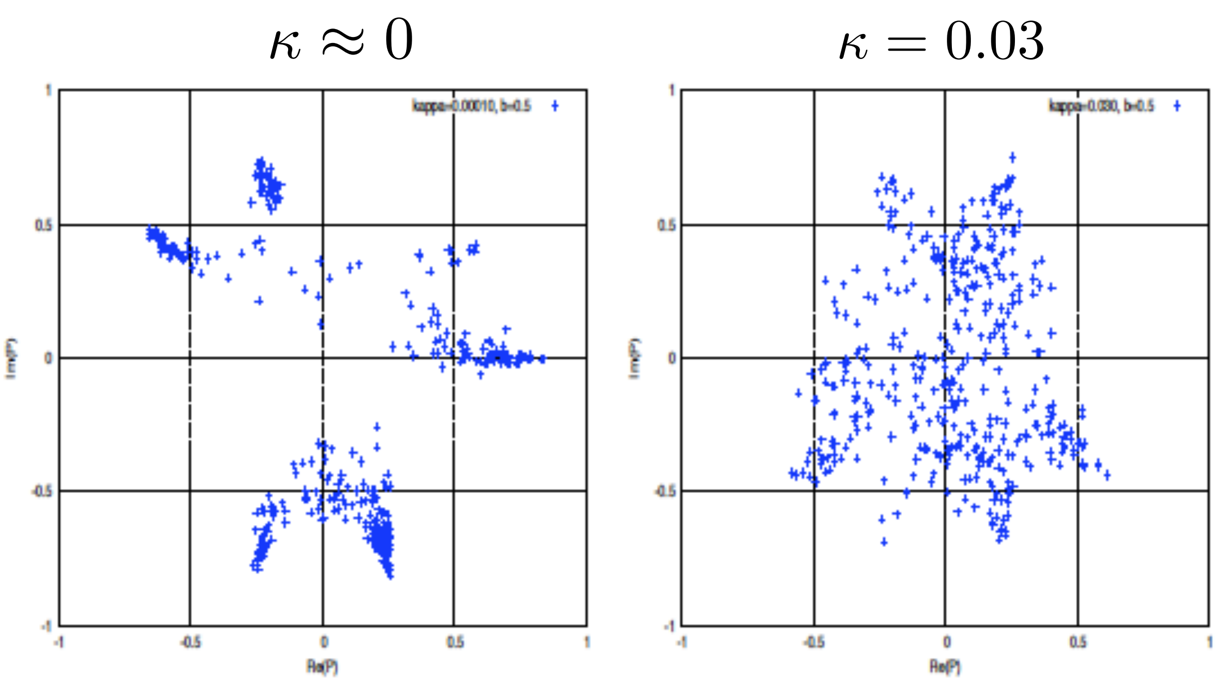

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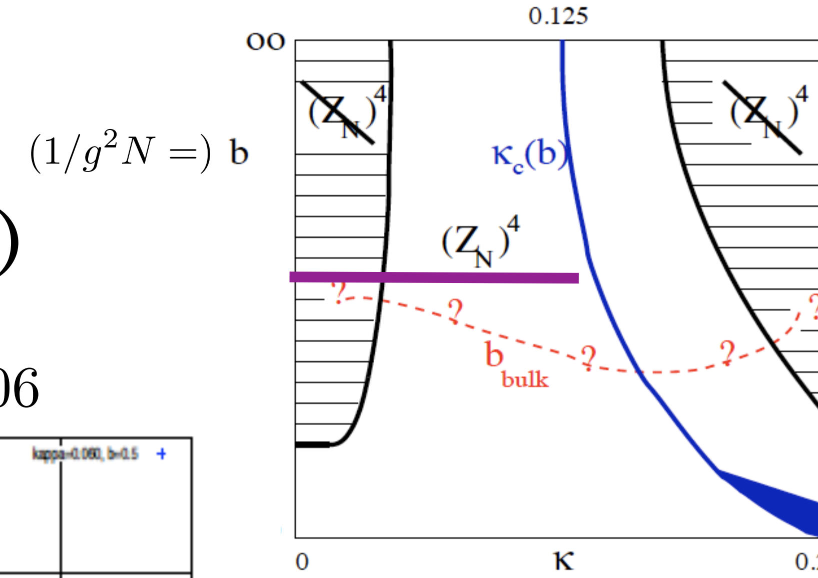
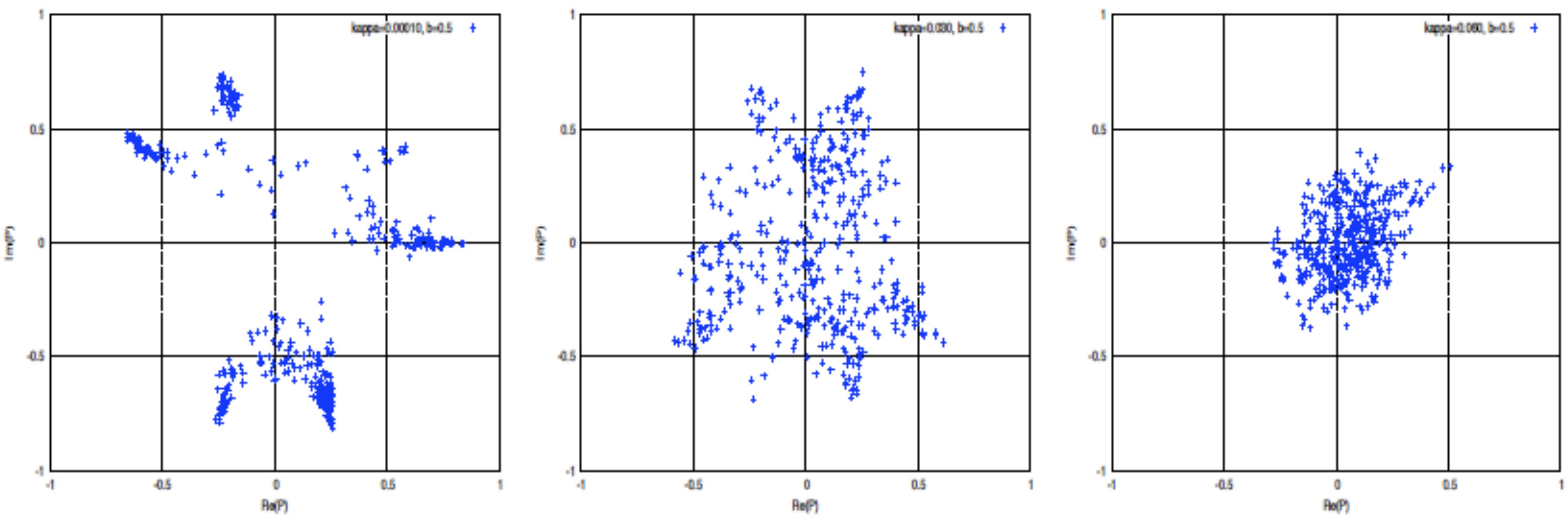


$\kappa \approx 0$ $\kappa = 0.03$ $\kappa = 0.06$



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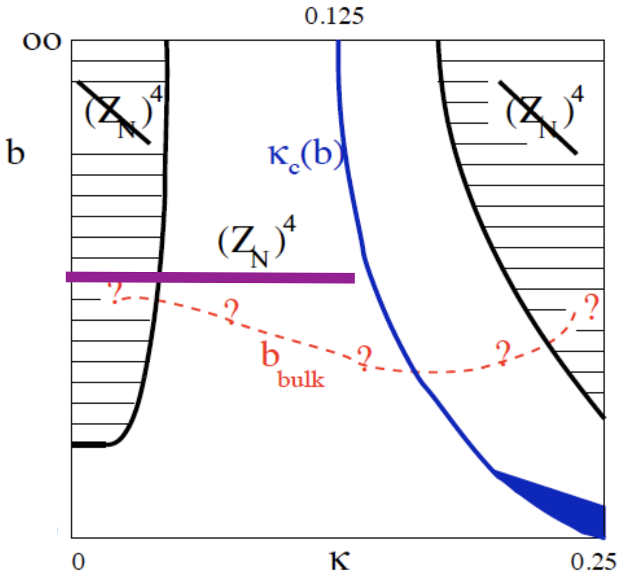
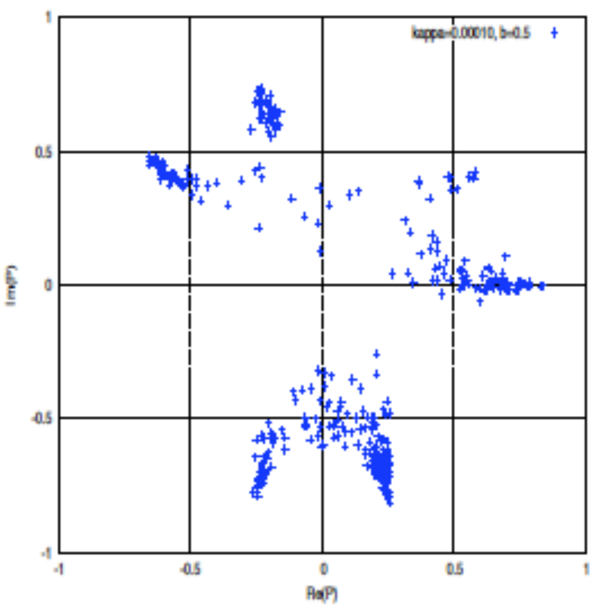
,SU(10)

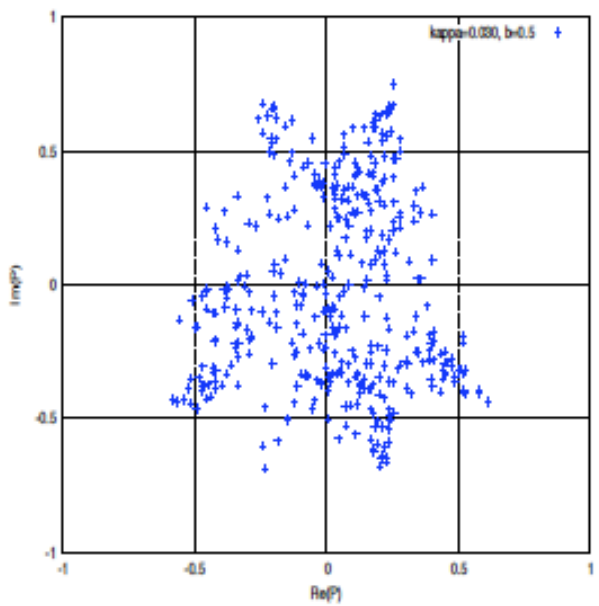
$(1/g^2N =)$ b



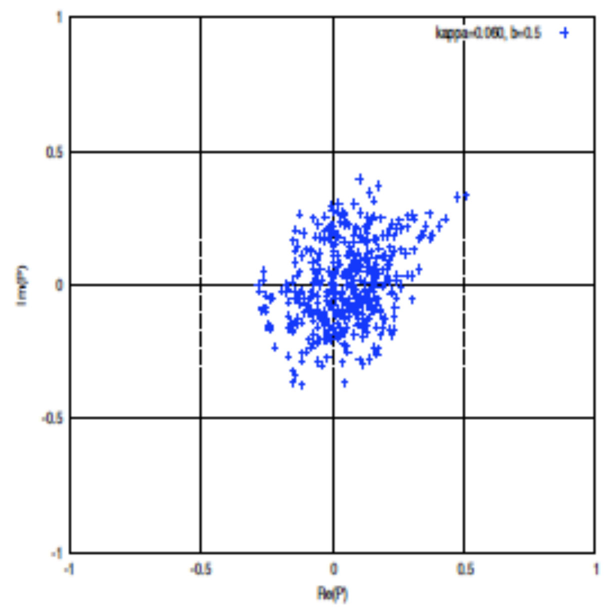
$\kappa \approx 0$

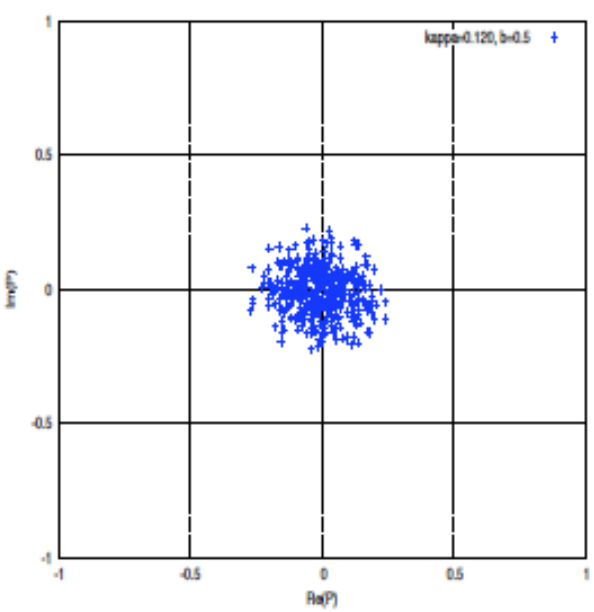


$\kappa = 0.03$



$\kappa = 0.06$





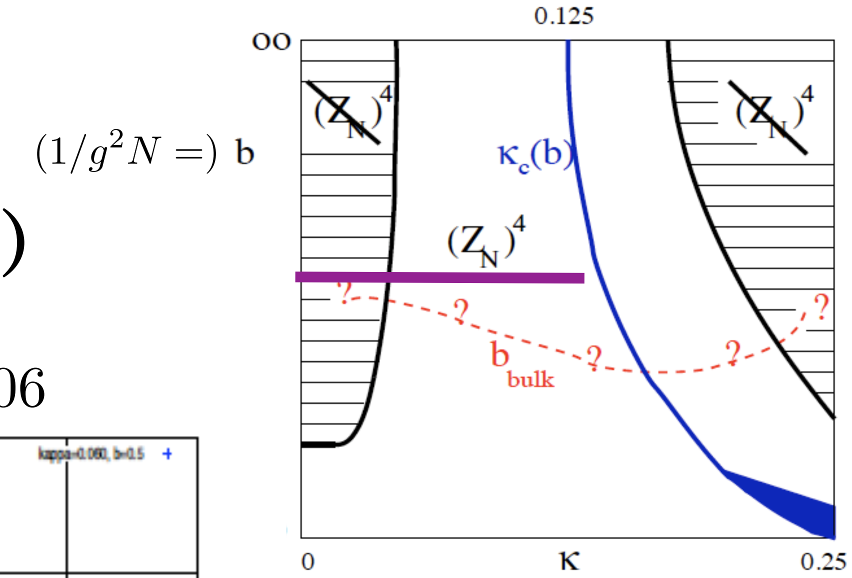
$\kappa = 0.12$

Scatter plots of Polyakov loops

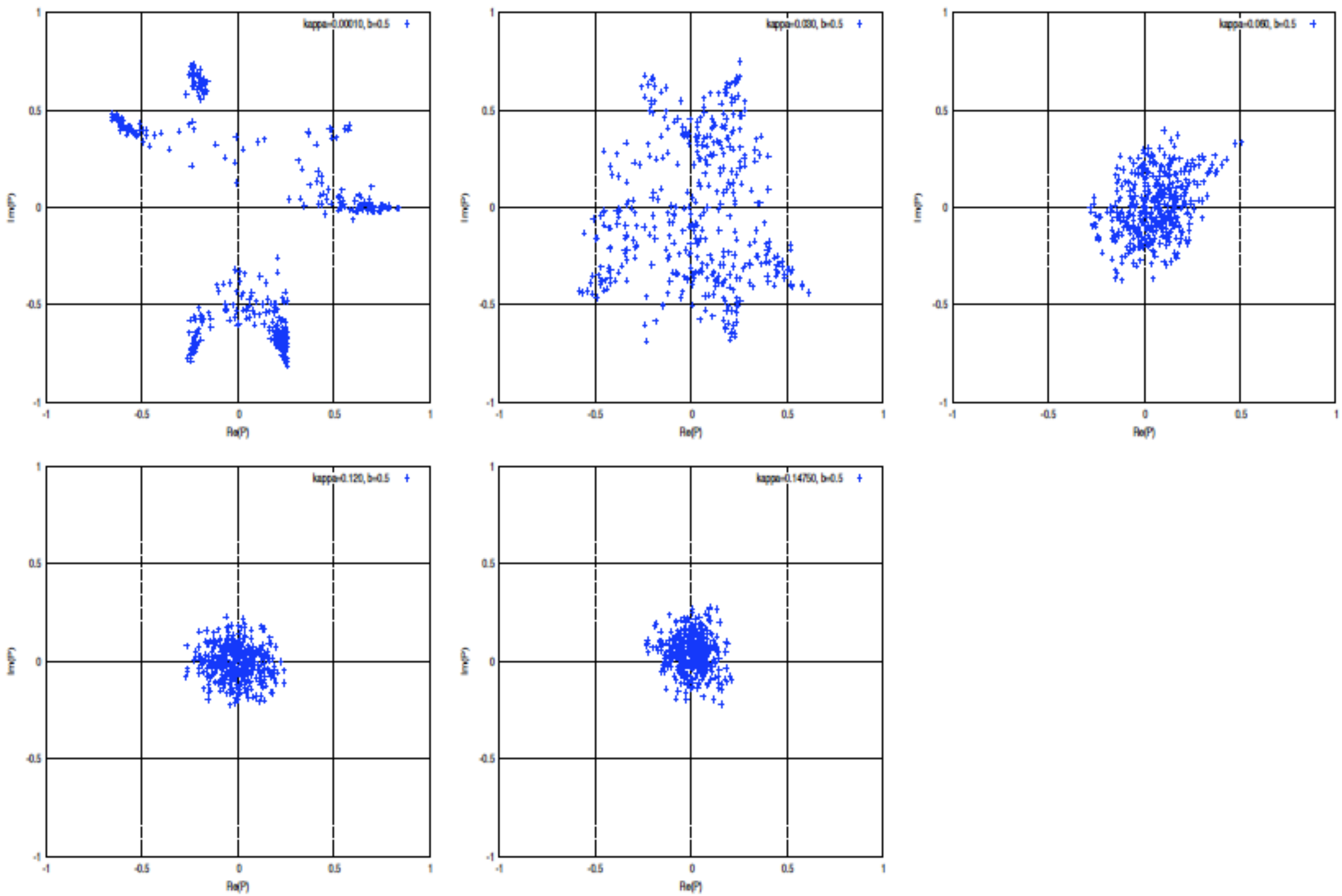
Scans 2: decreasing m_q at fixed b

Z_N symmetry restored for $\kappa \gtrsim 0.6$ at b=0.5

,SU(10)



$\kappa \approx 0$ $\kappa = 0.03$ $\kappa = 0.06$



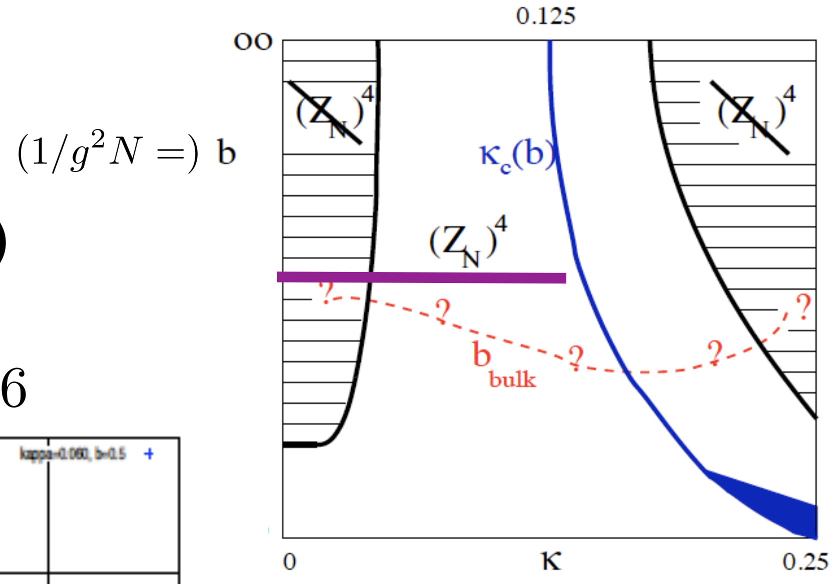
$\kappa = 0.12$ $\kappa = 0.1475$

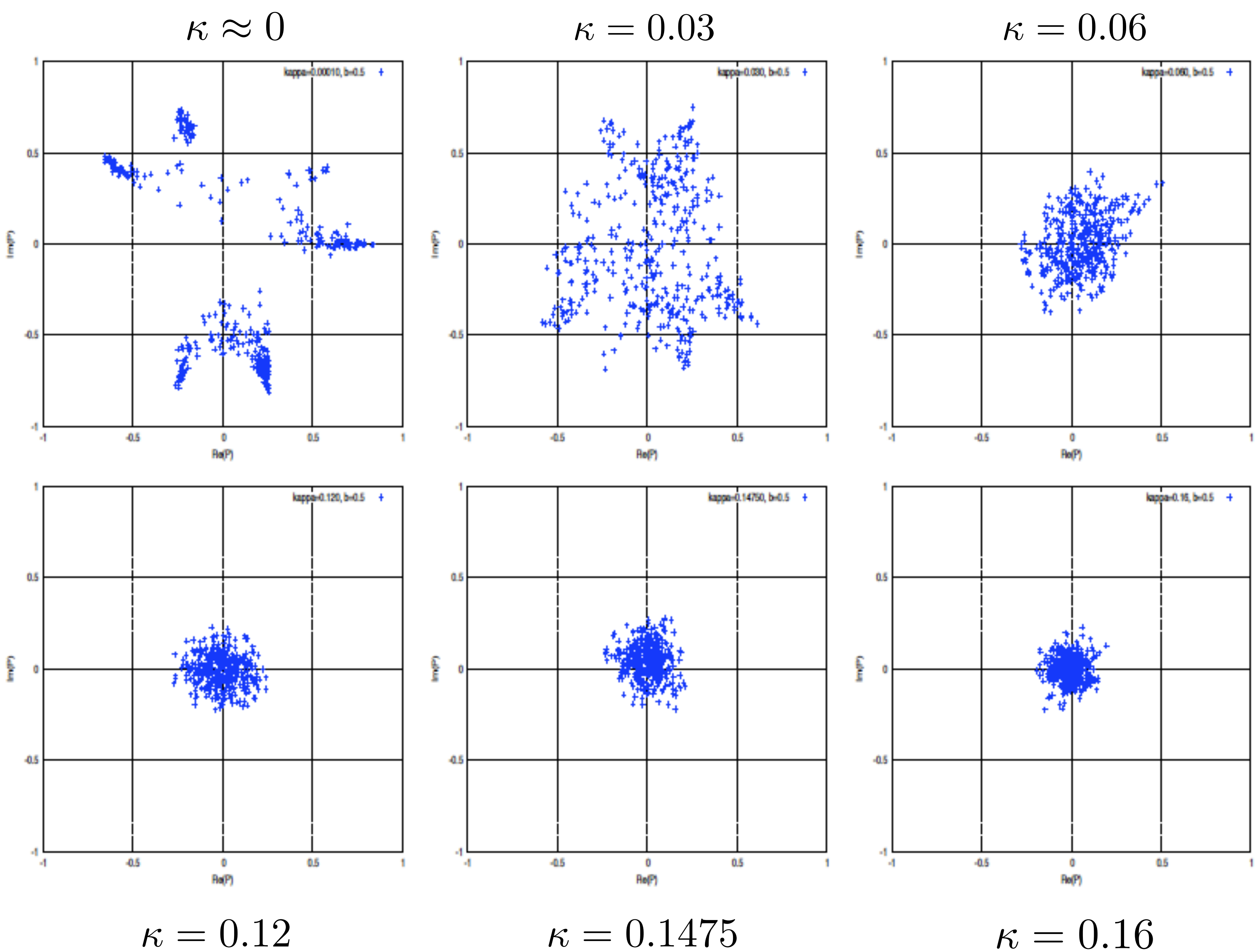
Scatter plots of Polyakov loops

Scans 2: decreasing m_q at fixed b

Z_N symmetry restored for $\kappa \gtrsim 0.6$ at b=0.5

,SU(10)



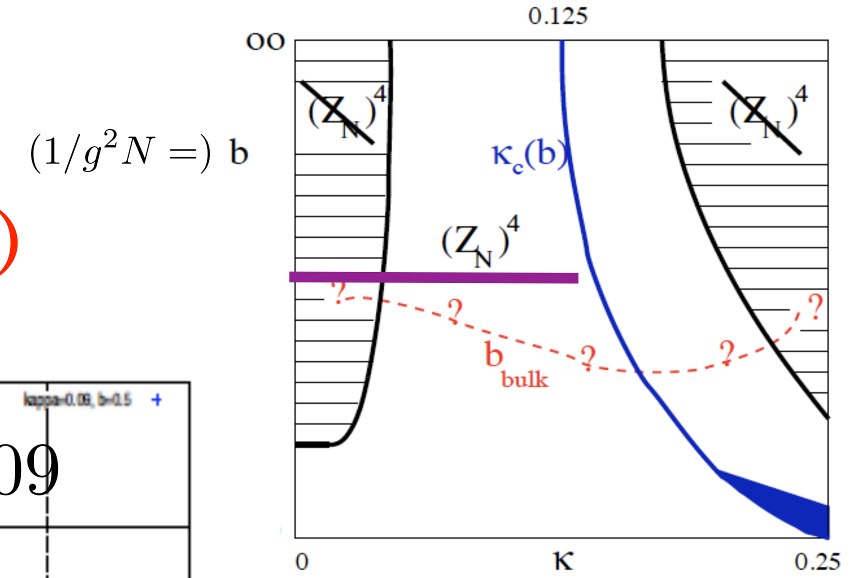


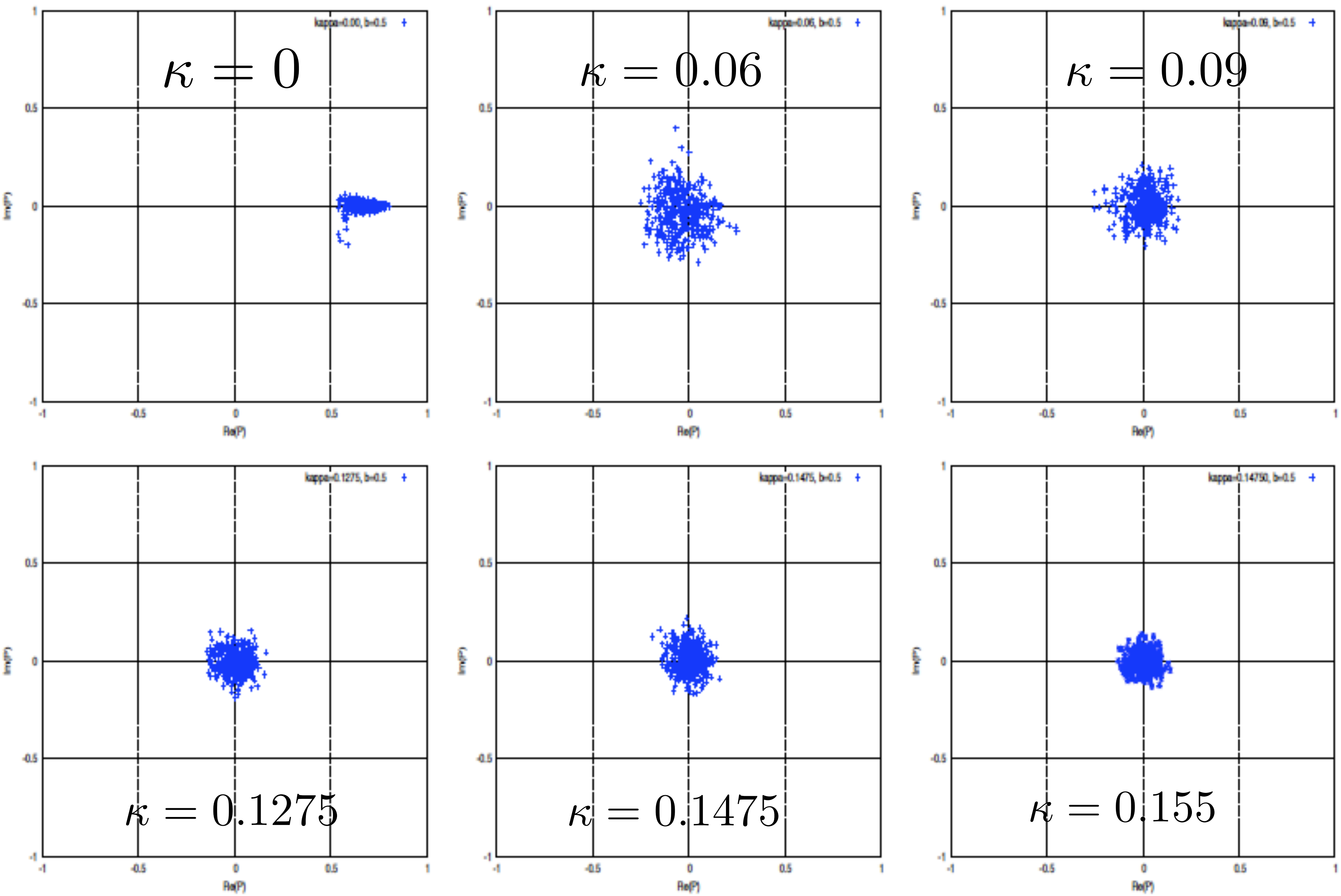
Scatter plots of Polyakov loops

Scans 2: decreasing m_q at fixed b

Z_N symmetry restored for $\kappa \gtrsim 0.6$ at b=0.5

,SU(15)

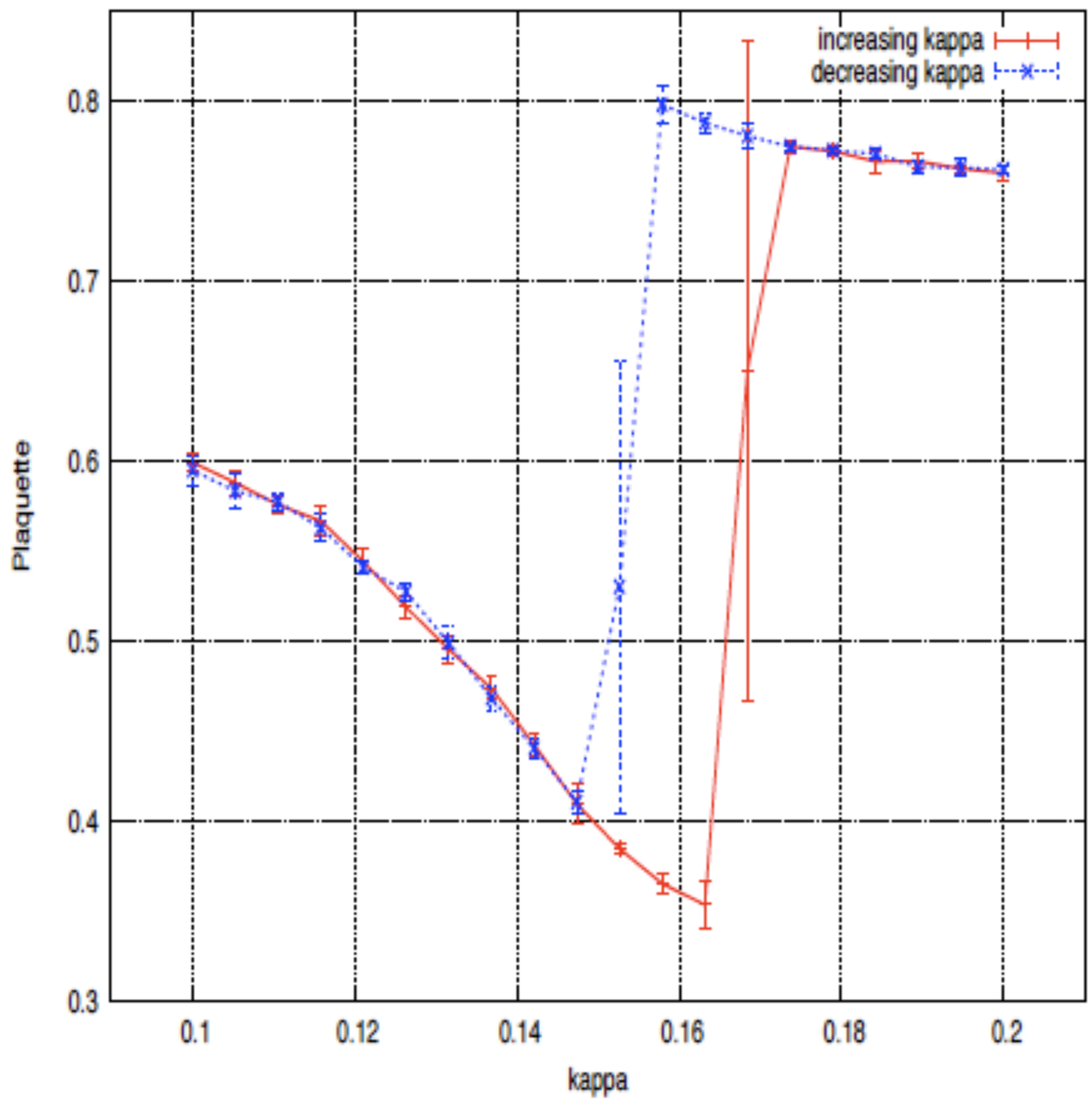




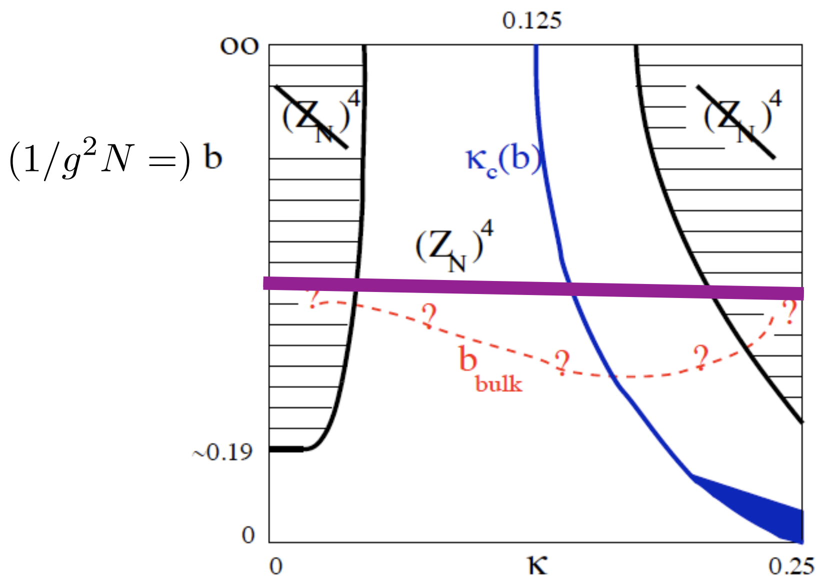
Scatter plots of Polyakov loops

Scans 2: looking for critical line

<Plaquette>

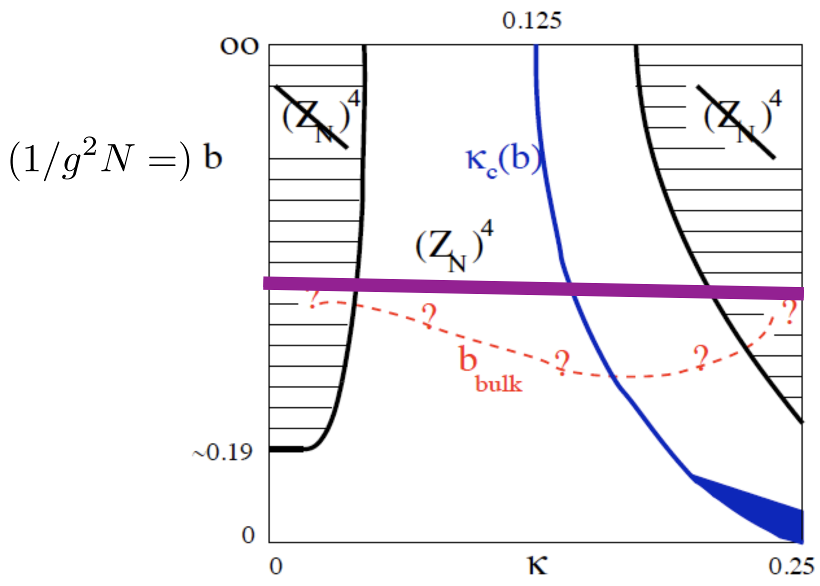


$\varkappa$

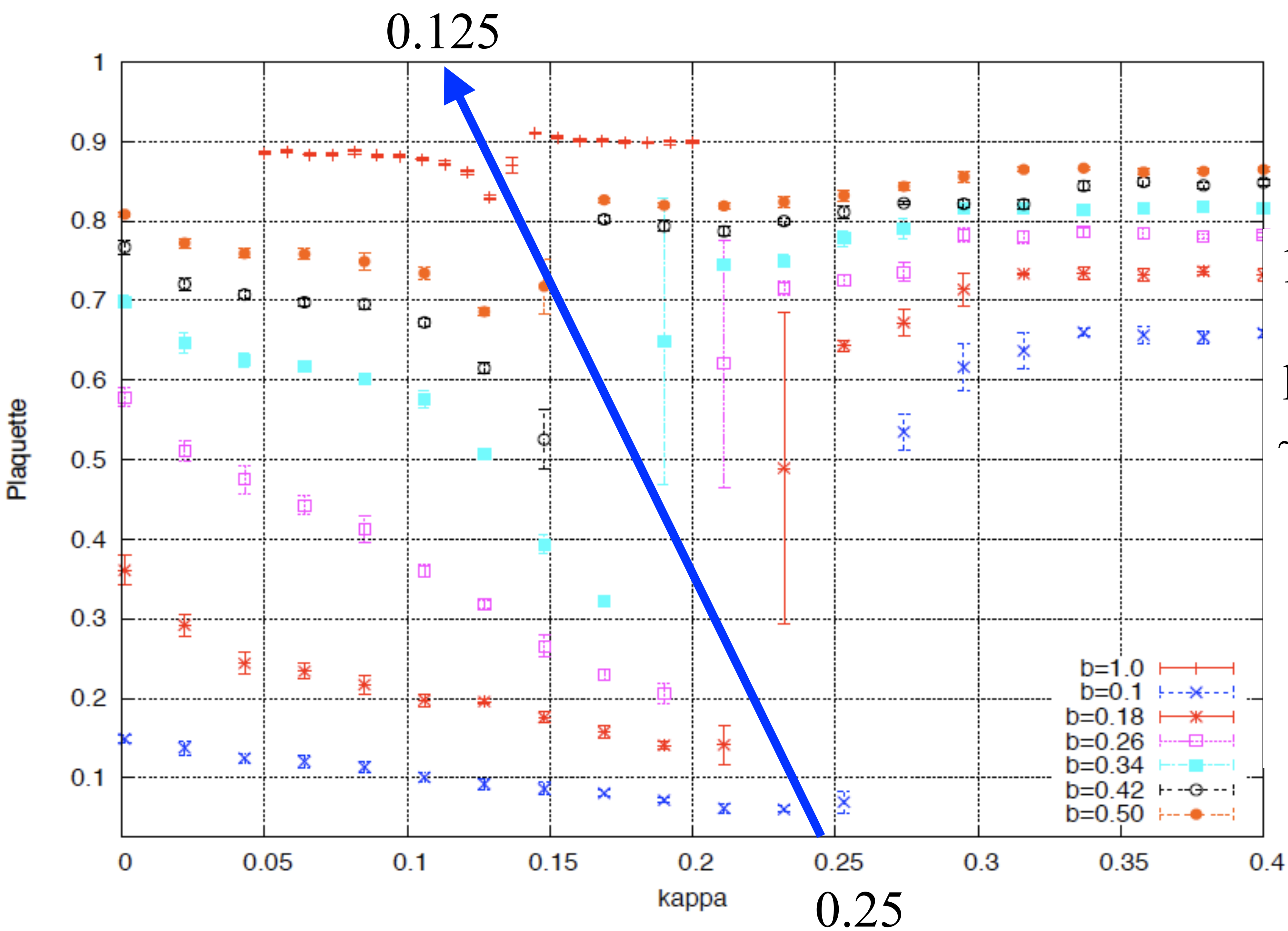


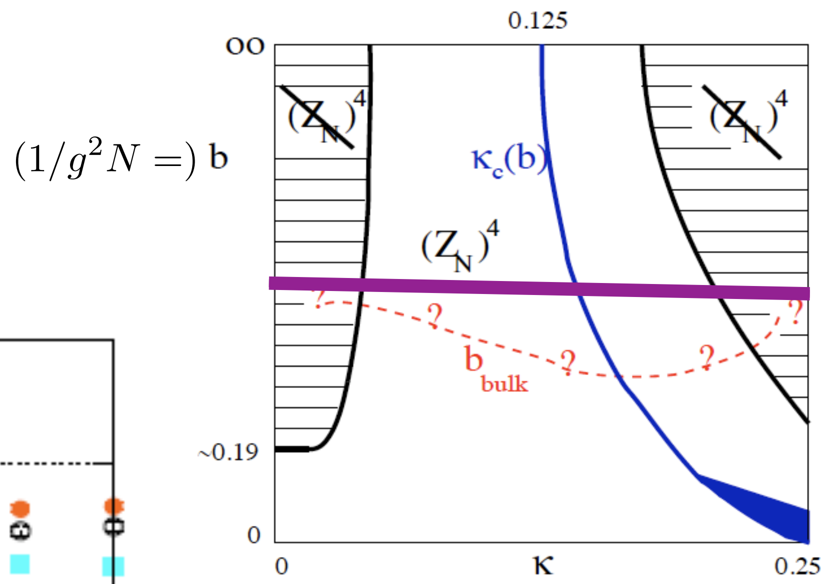
b=0.35:
1st order transition
at kappa~0.15
with Z_N unbroken on
both sides

Scans 2: looking for critical line



Scans 2: looking for critical line



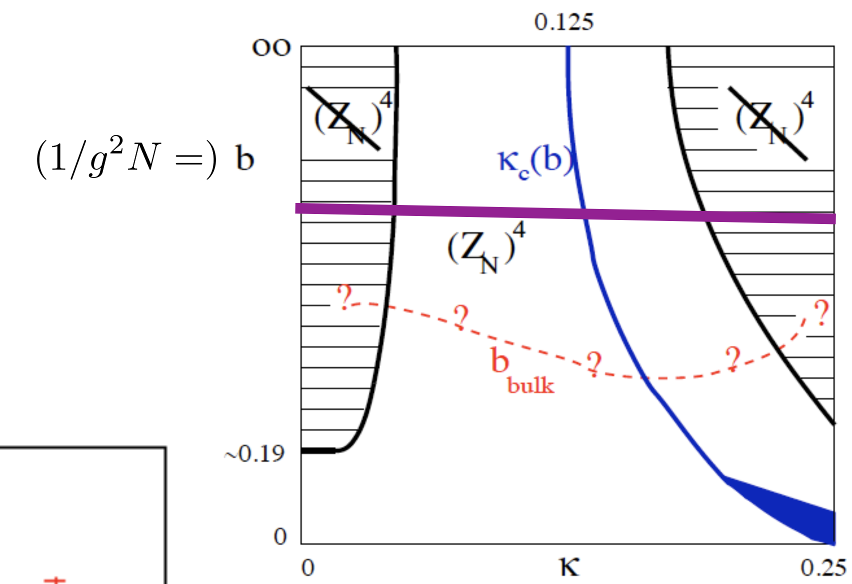


1st order transition at all b moving from kappa ~0.25 towards ~0.125 as b increases

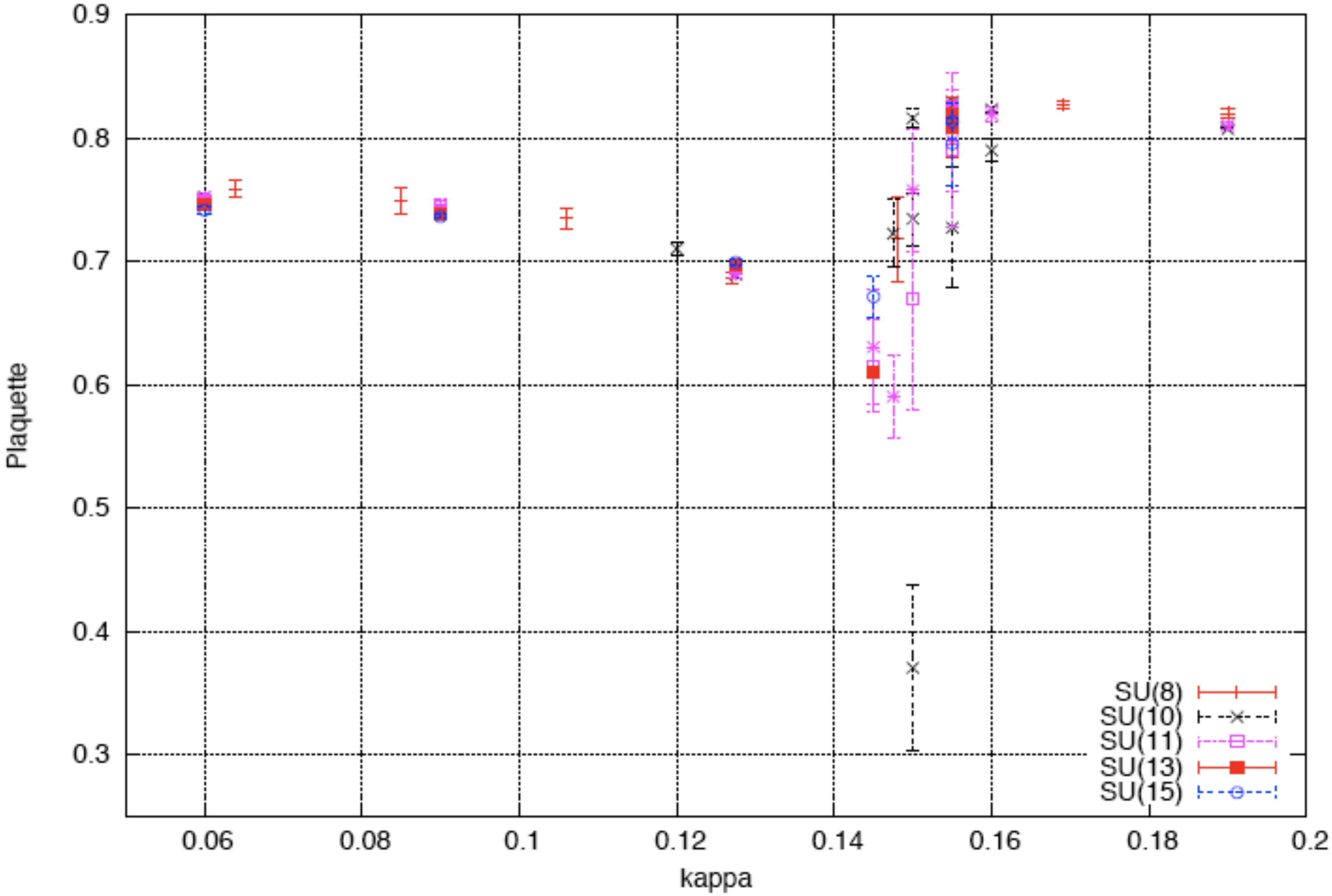
Our interpretation: critical line in "first-order scenario" [SS+Singleton]

SU(8)

Scans 2: N dependence?



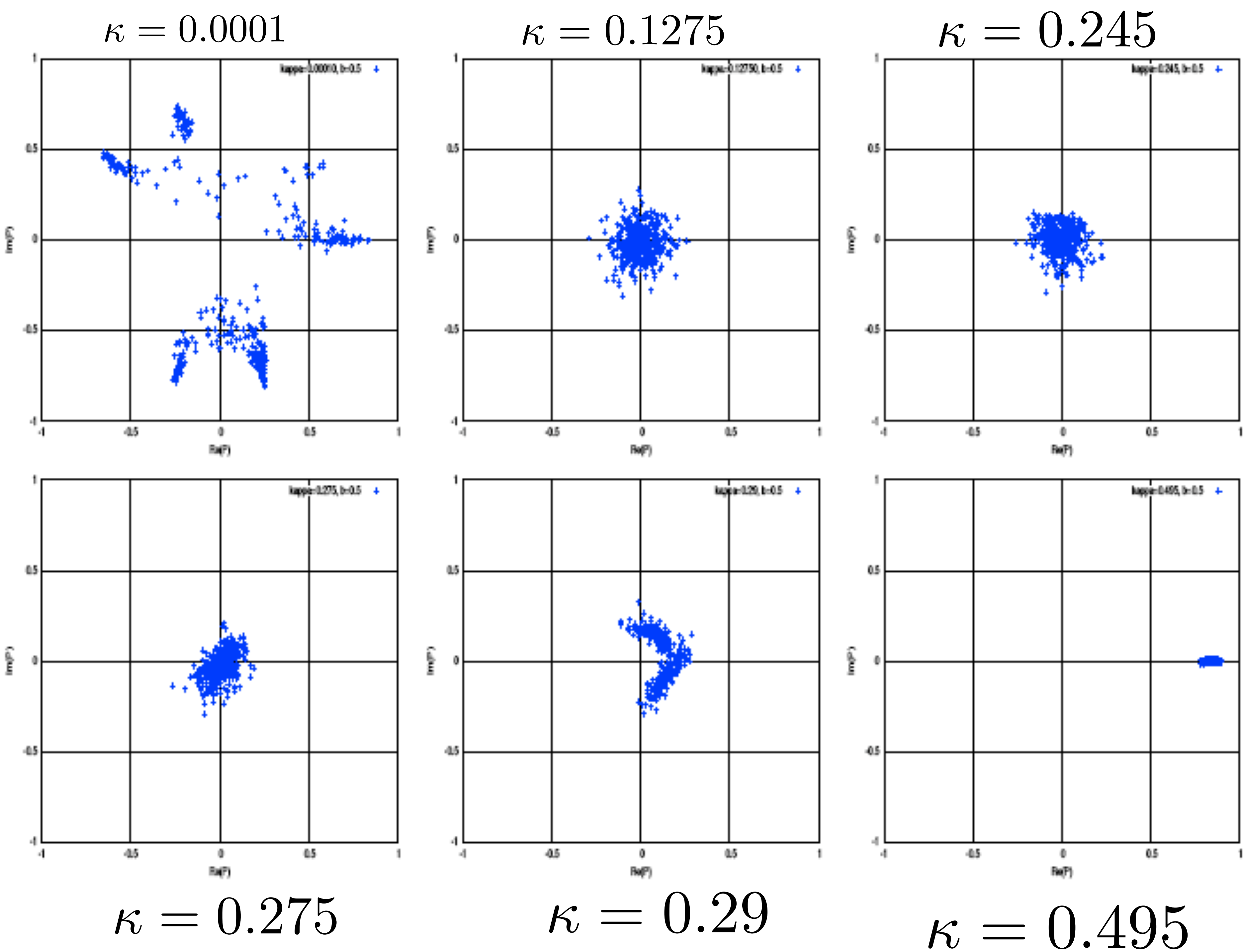
"Transition" present for all N studied
e.g. b=0.5, N=8, 10, 11, 13, 15:

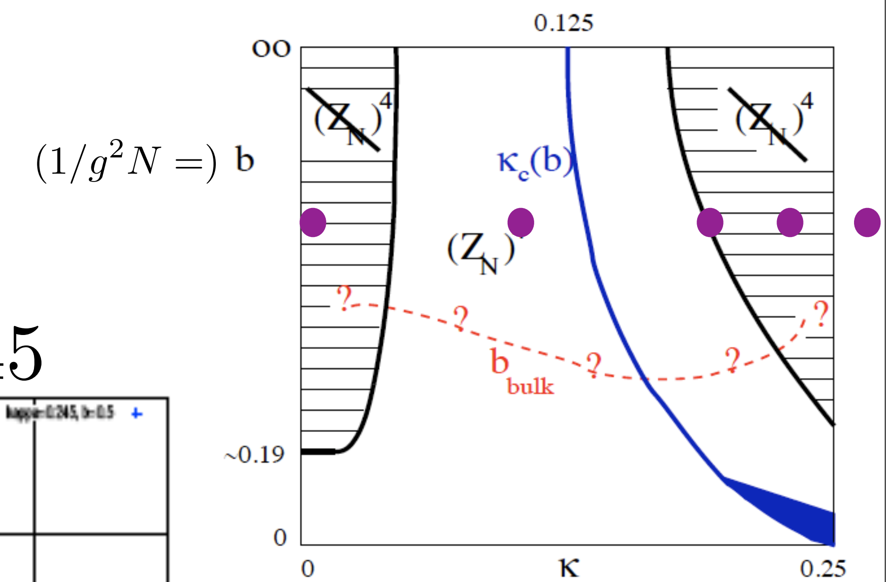


Only true transition when N infinite

Scans 2: larger kappa

b=0.5, SU(10)



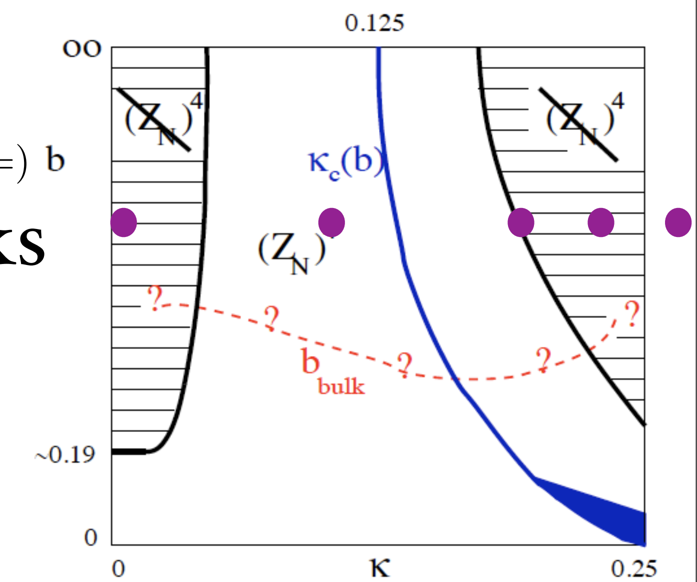


Polyakov loops indicate Z_N breaking for $\kappa \gtrsim 0.28$

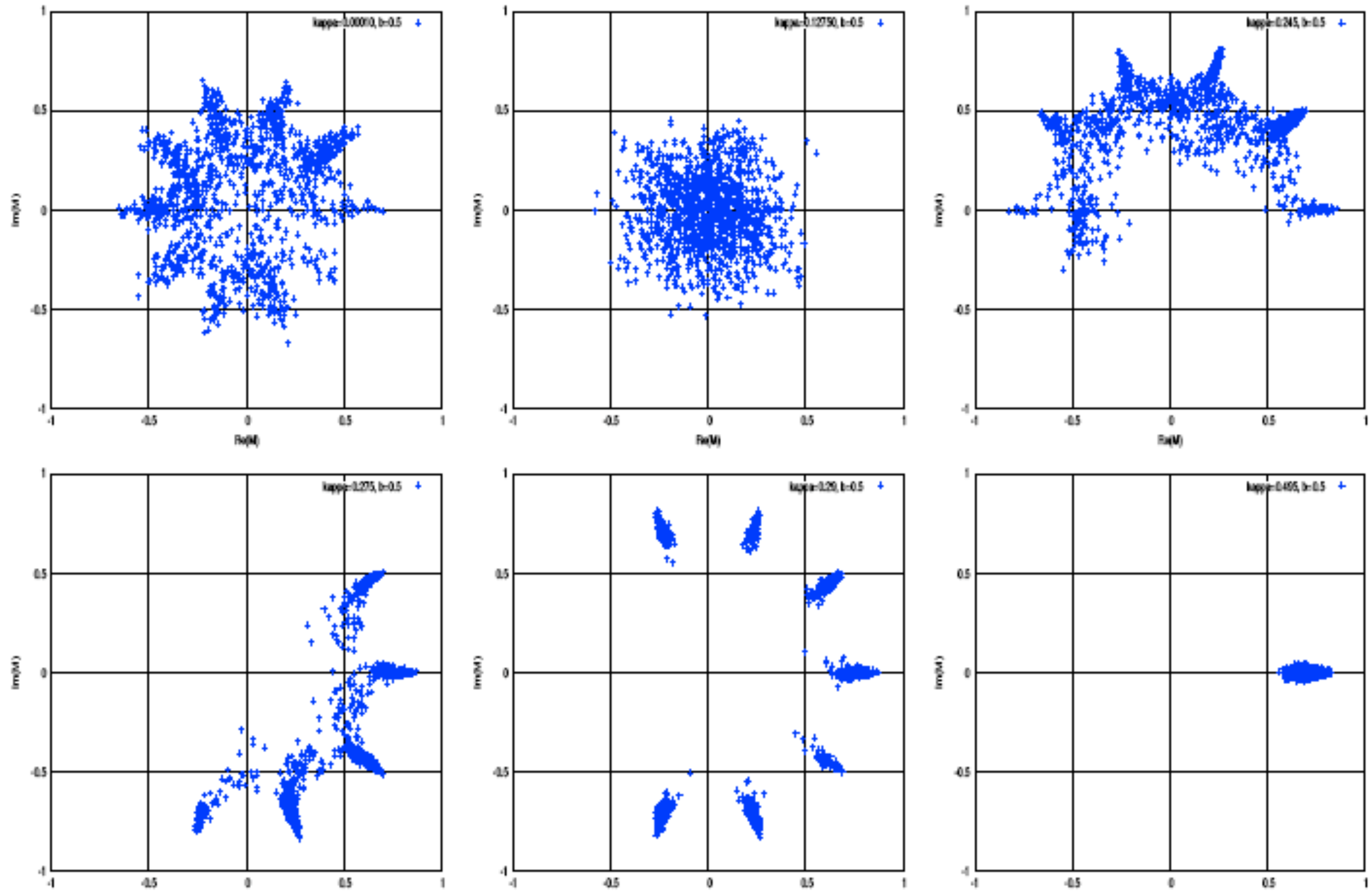
Scans 2: larger kappa

New observables: $M_{\mu,\pm\nu} = \frac{1}{N}\mathrm{tr}U_\mu U_{\pm\nu}$

Monitor Z_N breaking involving correlations between links



$\kappa = 0.0001$ $\kappa = 0.1275$ $\kappa = 0.245$



$\kappa = 0.275$ $\kappa = 0.29$ $\kappa = 0.495$

Indicate Z_N breaking for $\kappa \gtrsim 0.24$

Tentative conclusions

* For b<1 ($\beta_{SU(3)}$<18) there is a range of κ's on both sides of the putative κ_c for which reduction holds
* Surprise: range goes up to $|m_{phys}| \sim 1/a$
* Possible caveat (from our experience with QEK model): center-symmetry breaking may show up only in more complicated expectation values

Approaching the continuum: high statistics at b=1

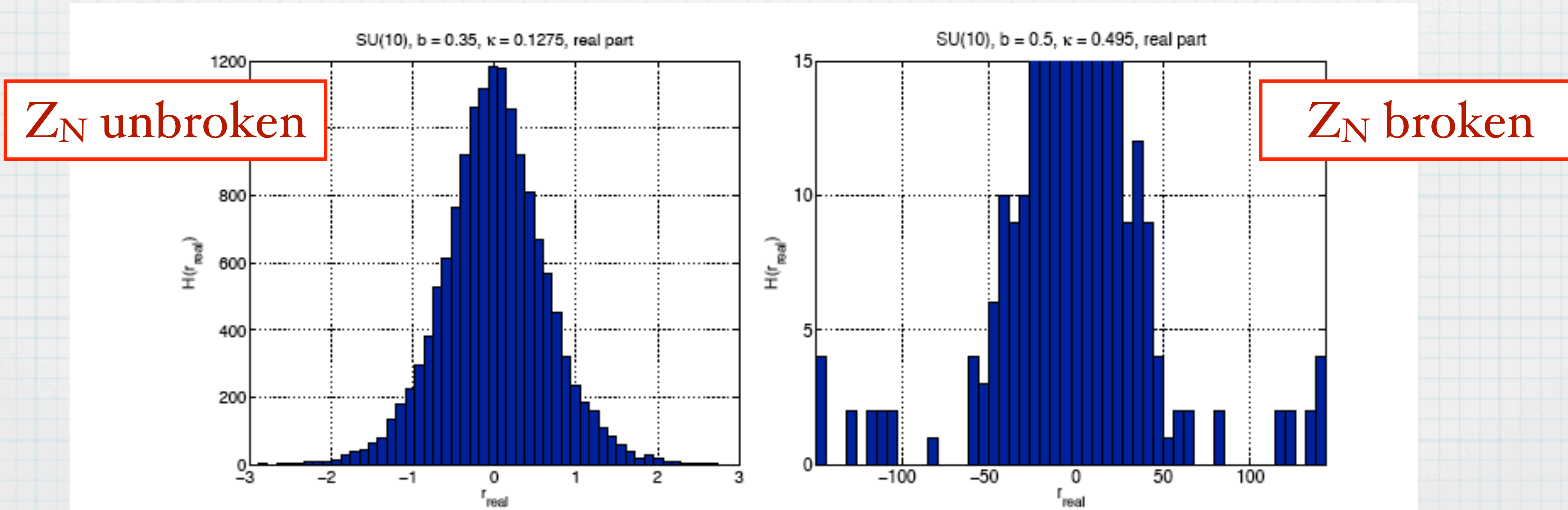
New observables

- To be sensitive to many patterns of sym. breaking we calculated 14641 different traces:

$$K_{\vec{n}} \equiv \frac{1}{N} \mathrm{tr}\; U_1^{n_1} U_2^{n_2} U_3^{n_3} U_4^{n_4}, \quad \text{with } n_\mu = 0, \pm 1, \pm 2, \ldots, \pm\, 5$$

- For each we calculated the signal-to-noise for the real and imag. part and then formed a histogram

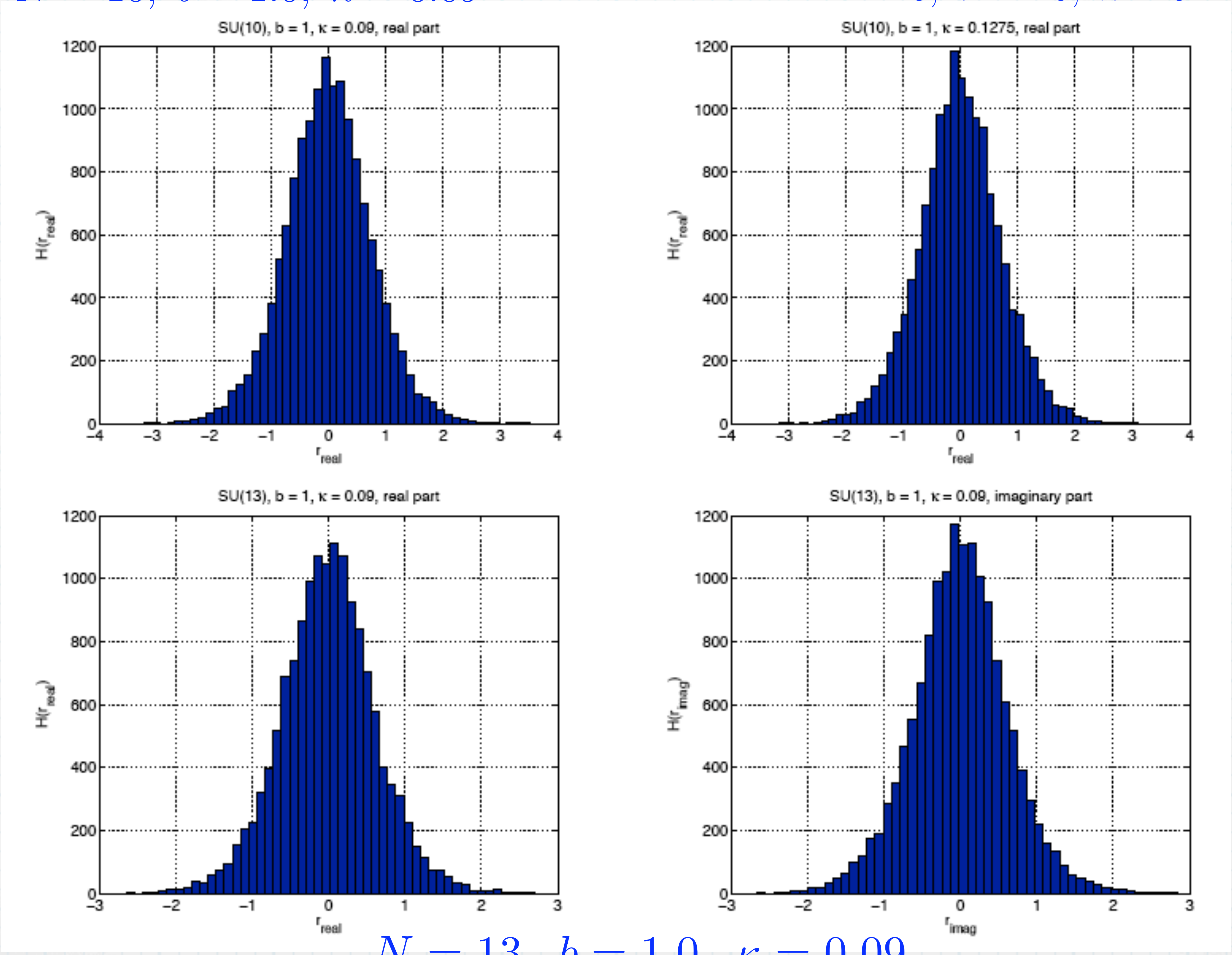
- Expectations exemplified by:



$N = 10,\ b = 0.35,\ \kappa = 0.1275$ $\quad$ $N = 10,\ b = 0.5,\ \kappa = 0.495$

Results for K_n at b=1

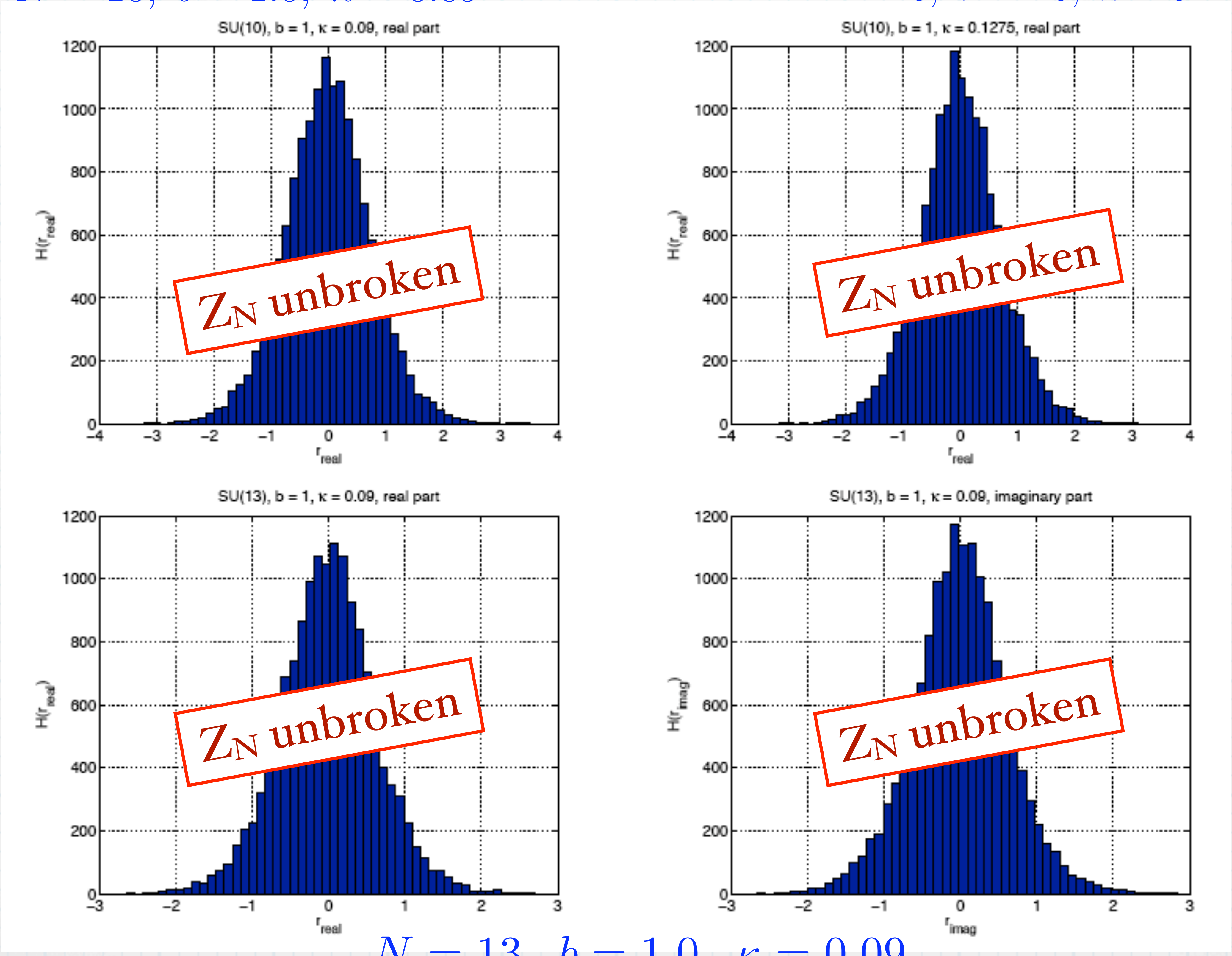
$N = 10,\ b = 1.0,\ \kappa = 0.09$ $N = 10,\ b = 1.0,\ \kappa = 0.1275$



$N = 13,\ b = 1.0,\ \kappa = 0.09$

Results for K_n at b=1

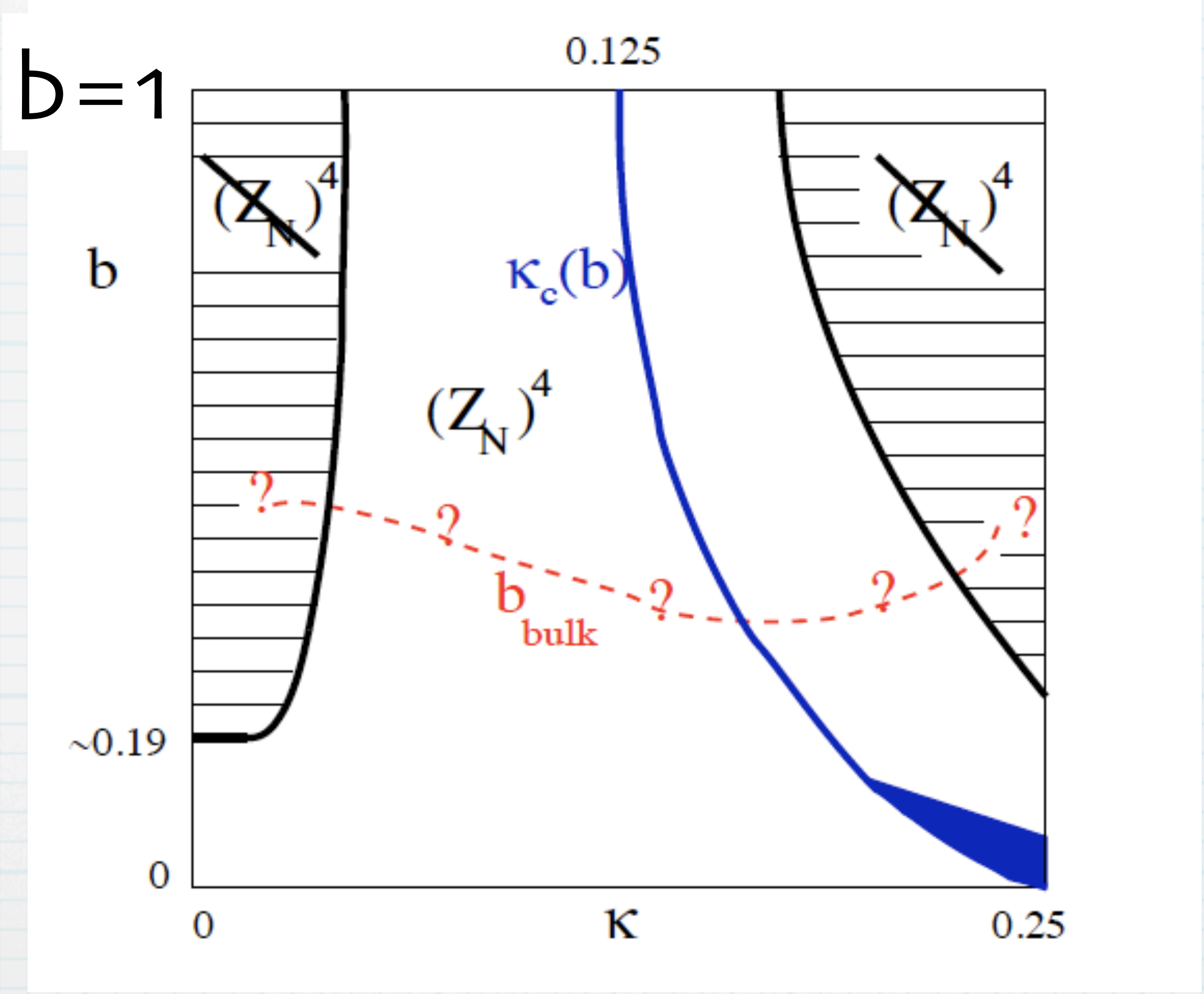
$N = 10,\ b = 1.0,\ \kappa = 0.09$ $N = 10,\ b = 1.0,\ \kappa = 0.1275$



$N = 13,\ b = 1.0,\ \kappa = 0.09$

Conclusion on N_f=1 phase diagram

* Our results are consistent with volume independence for interesting range of couplings



Identical to infinite volume theory (at large N) within "funnel"

* Far from critical line, but inside funnel, long distance theory is pure-gauge theory ⇒ realization of EK idea!

Updates on $N_f=1$

* [Heitanen & Narayan] find center-symmetry unbroken with massless overlap fermions at b=5

* [Azenayagi, Hanada, Unsal & Yakoby] check the existence of the funnel with Wilson fermions for m_{phys}~1/a using rHMC algorithm, and extend calculation to 1 uncompactified direction (allowing study of finite temperature transition)

* [AHUY] conjecture that "funnel" closes in continuum limit as

$$|am_{\mathrm{phys}}| < \frac{1}{b^{1/4}}$$

➡ can take continuum limit for any fixed m_{phys} within funnel

Update on N_f=1: Spectrum of adjoint Dirac operator

Reduction in Perturbation Theory

* $U_\mu \approx \mathrm{diag}\left(e^{i\theta_{\mu,1}}, e^{i\theta_{\mu,2}}, \ldots, e^{i\theta_{\mu,N}}\right)$
* $U_\mu^{\mathrm{Adj}} = U_\mu \otimes U_\mu^\dagger \approx \mathrm{diag}\left(\ldots, e^{i(\theta_{\mu,j}-\theta_{\mu,k})}, \ldots\right)$
* $D_W^{\mathrm{adj}}(m_0 = 0) \approx \mathrm{diag}\left(\ldots, \left\{(4 - \sum_\mu \cos\theta_\mu^{jk}) + i\sum_\mu \sin\theta_\mu^{jk}\gamma_\mu\right\}, \ldots\right)$
* For SU(N) have 4(N-1) zero modes---irrelevant in PT
* Remaining $4(N^2-N)$ modes have infinite volume form with $p_\mu \longrightarrow \theta_\mu^{jk} = \theta_{\mu,j} - \theta_{\mu,k}$
* If eigenvalues repel and are uncorrelated in different directions [$(Z_N)^4$ unbroken] $\theta_\mu^{jk} \approx \frac{2\pi}{N}(\mathrm{perm}_\mu^j - \mathrm{perm}_\mu^k)$
* Build up spectrum of D_W on N^4 lattice from $\sim N^2$ random samples, and have $L_{eff}=N$

Reduction in Perturbation Theory

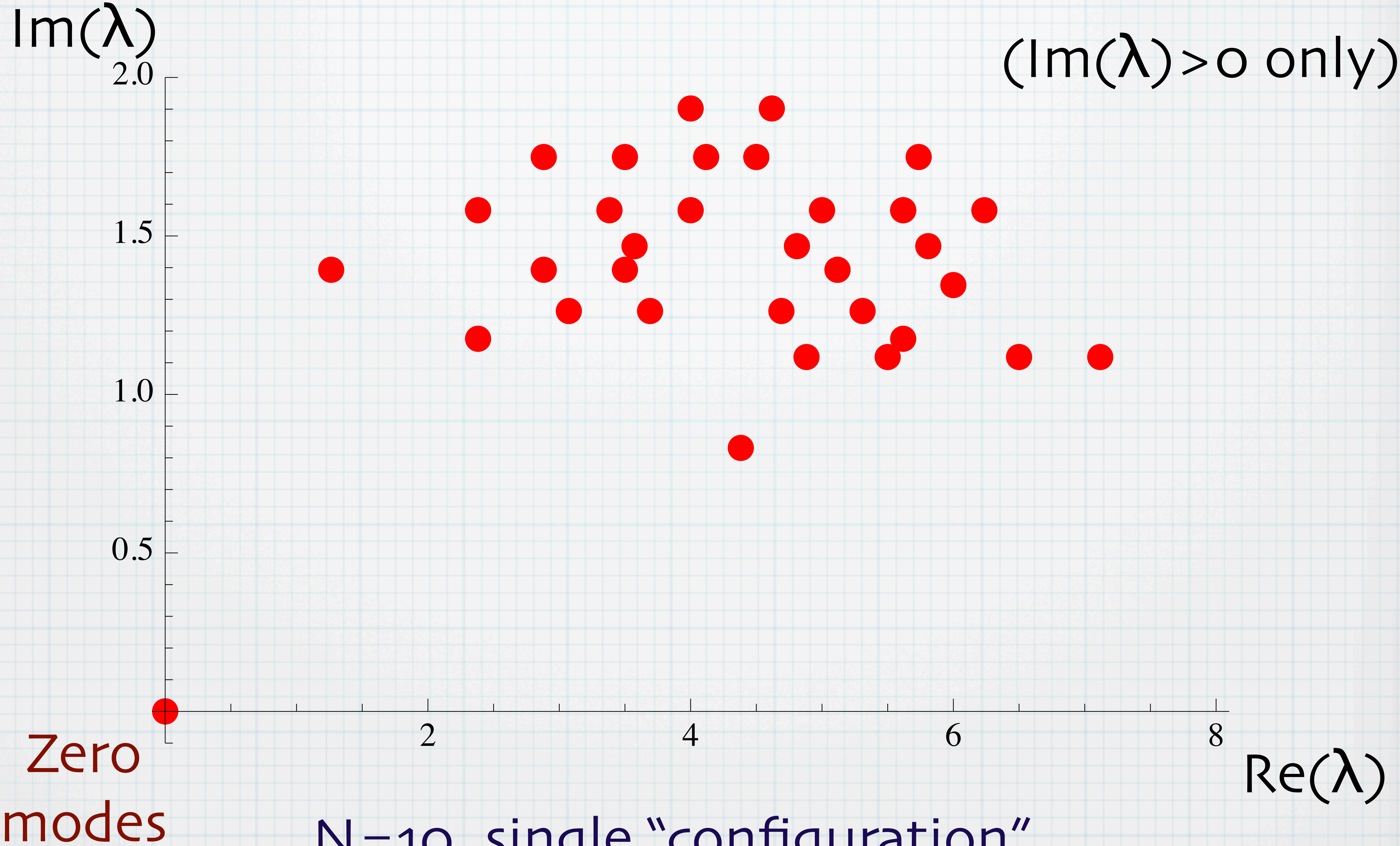
* Alternatively, eigenvalues lie close to a regular crystal within 4-d Brillouin zone [Bars, Unsal & Yaffe]

* Build up spectrum of D_W on $L_{eff} = N^{1/4}$ with 1 configuration

* For our values of N (N_{max}=15) would have $L_{eff} < 2$!

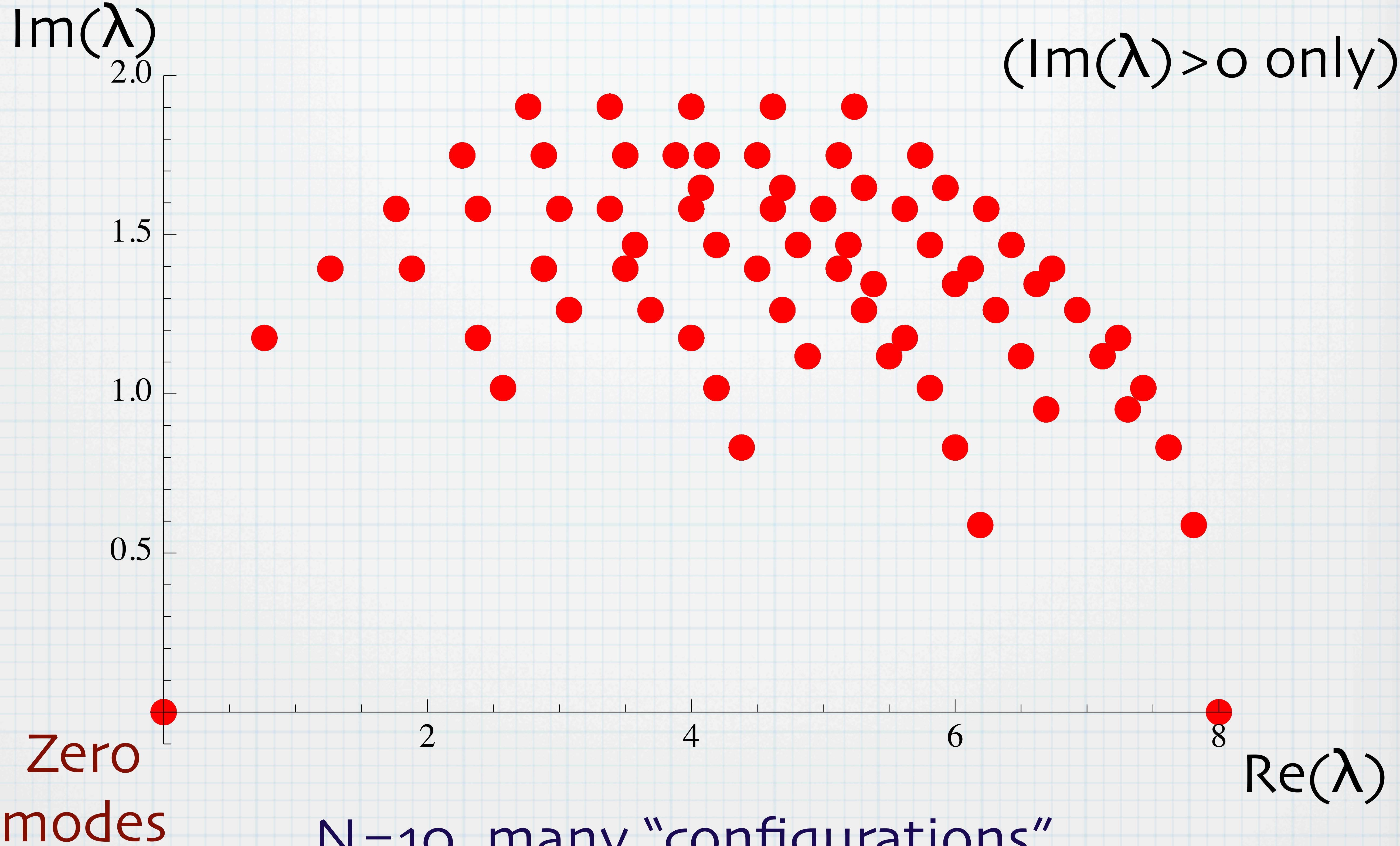
* Numerical data can distinguish these possibilities

Spectrum of $D_W(adj, m_0=0)$ in PT



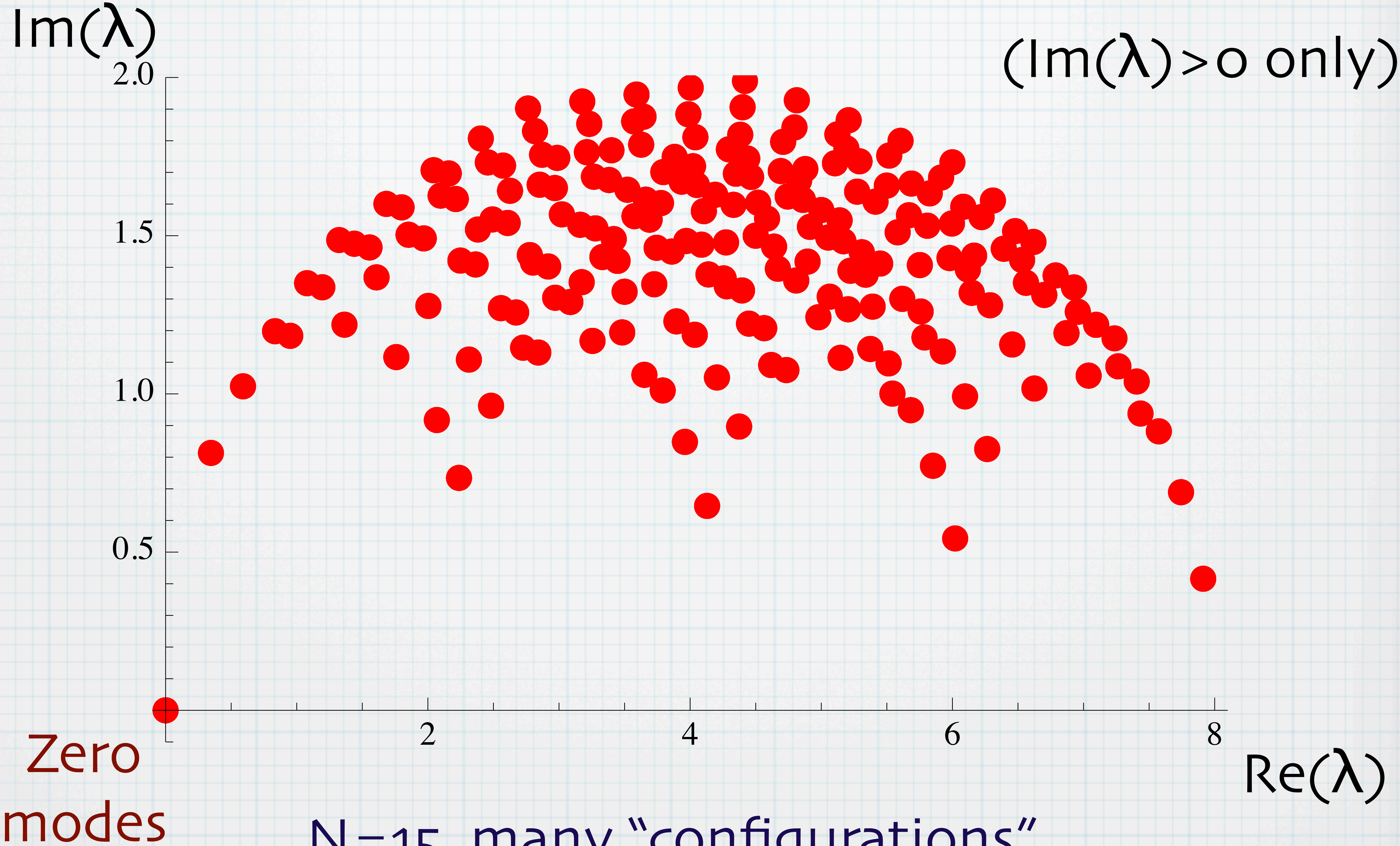
N=10, single "configuration"

Spectrum of $D_W(adj, m_0=0)$ in PT



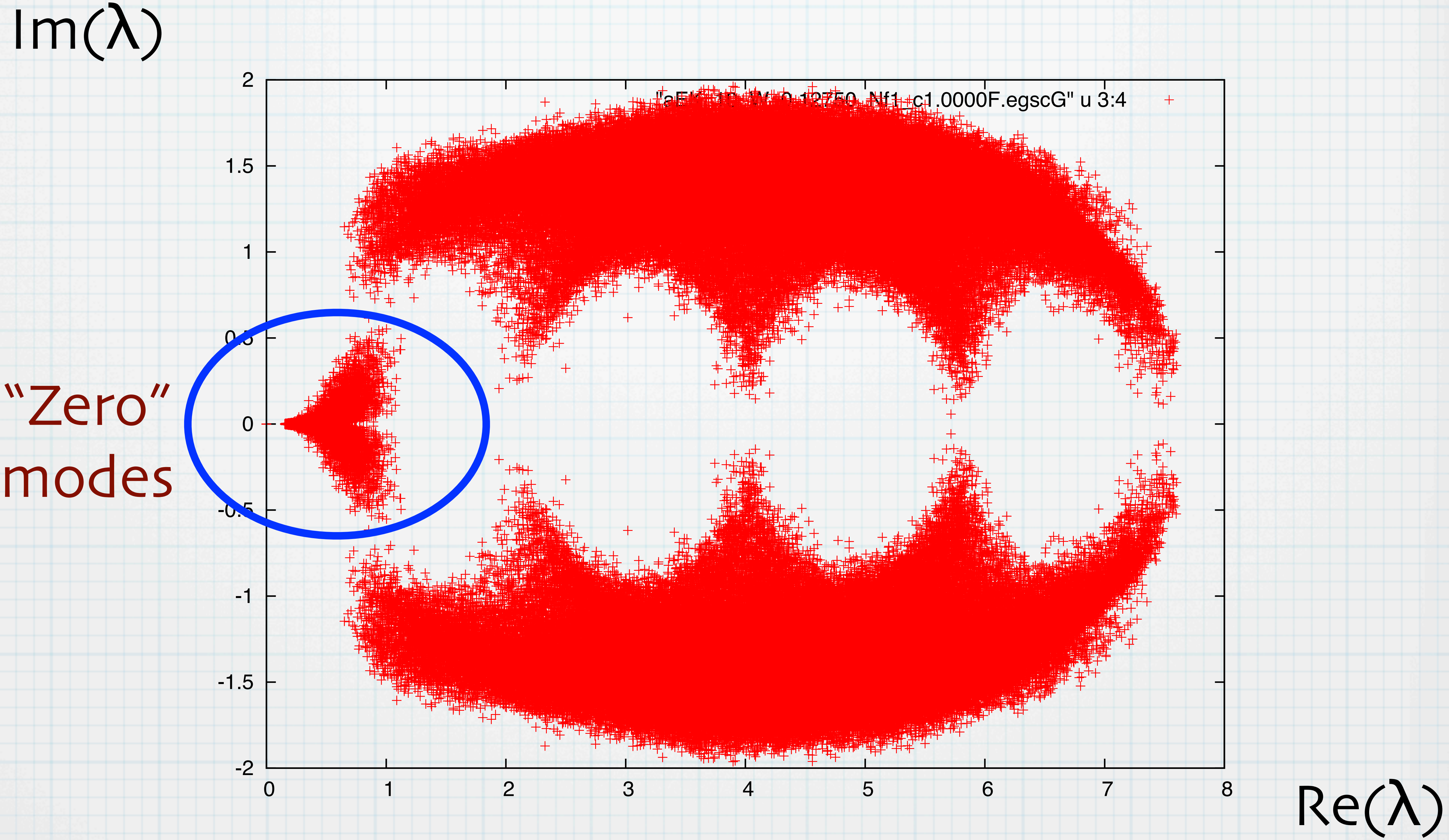
N=10, many "configurations"

Spectrum of $D_W(\mathrm{adj}, m_0 = 0)$ in PT



N=15, many "configurations"

Spectrum of $D_W(adj, m_0=0)$ in AEK model



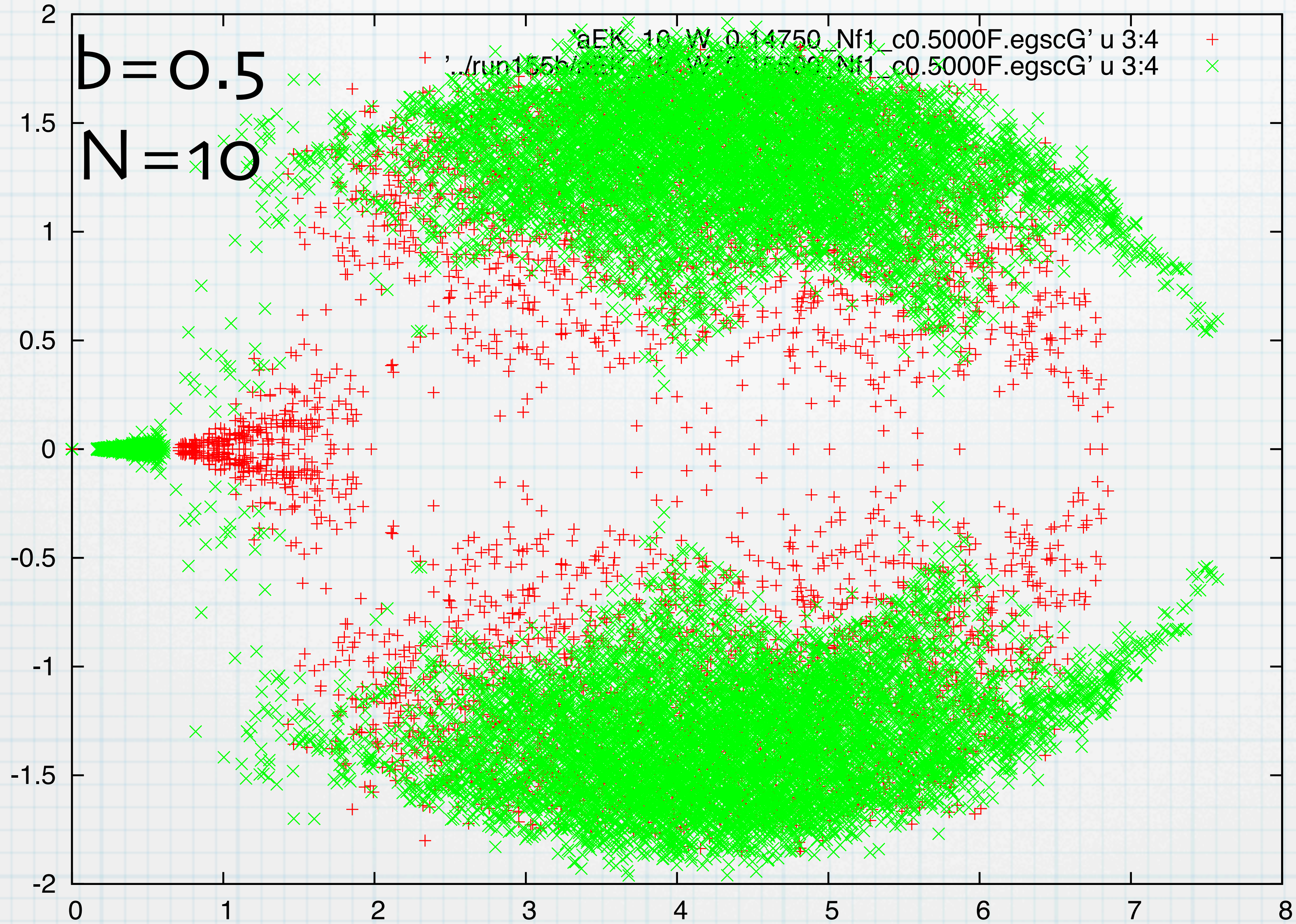
$N=10$, $b=1$, $\kappa=0.1275$ ($m_0=-0.08$)

(just below transition)

What do we learn from spectrum of D_W(adj)?

* "Zero-modes" can influence dynamics (for finite N)
* "Non-zero modes" have desired 4-d "fingers" --- which reach close to real axis
* Provides a nontrivial test that eigenvalue distribution does not break center symmetry
* Suggests that induced $L_{eff} = N$

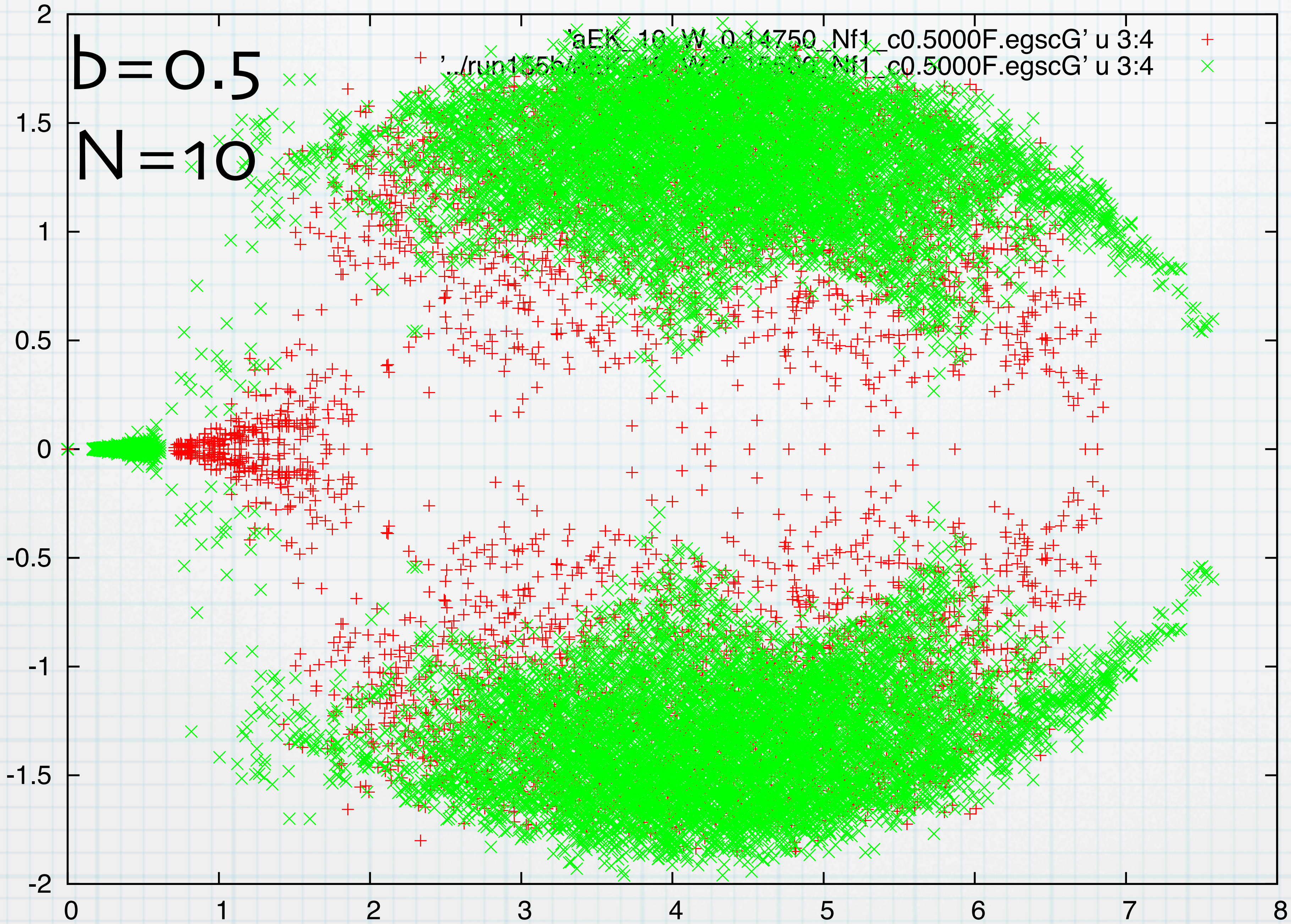
How important are “zero-modes” for phase structure?



κ=0.1475 (m_0=-0.6)
(below transition)

κ=0.155 (m_0=-0.77)
(above transition)

How important are “zero-modes” for phase structure?



Need larger N to answer this question

κ=0.1475 (m_0=-0.6)
(below transition)

κ=0.155 (m_0=-0.77)
(above transition)

Onwards to N_f=2

Present Project [w/ Bringoltz & Koren]

* Need to work at larger N, both to study 1/N effects and to calculate physical quantities
  - (r)HMC algorithm
  - Scaling is $N^3 \times (N^2)^{1/4}$? [Catterall, Galvez & Unsal]
* Have working HMC for $N_f=2$
  - Timings for $N_f=1$ suggest that we can reach N=30 (running on ~10 processors)
  - rHMC for $N_f=1$ in progress

Status for $N_f=2$

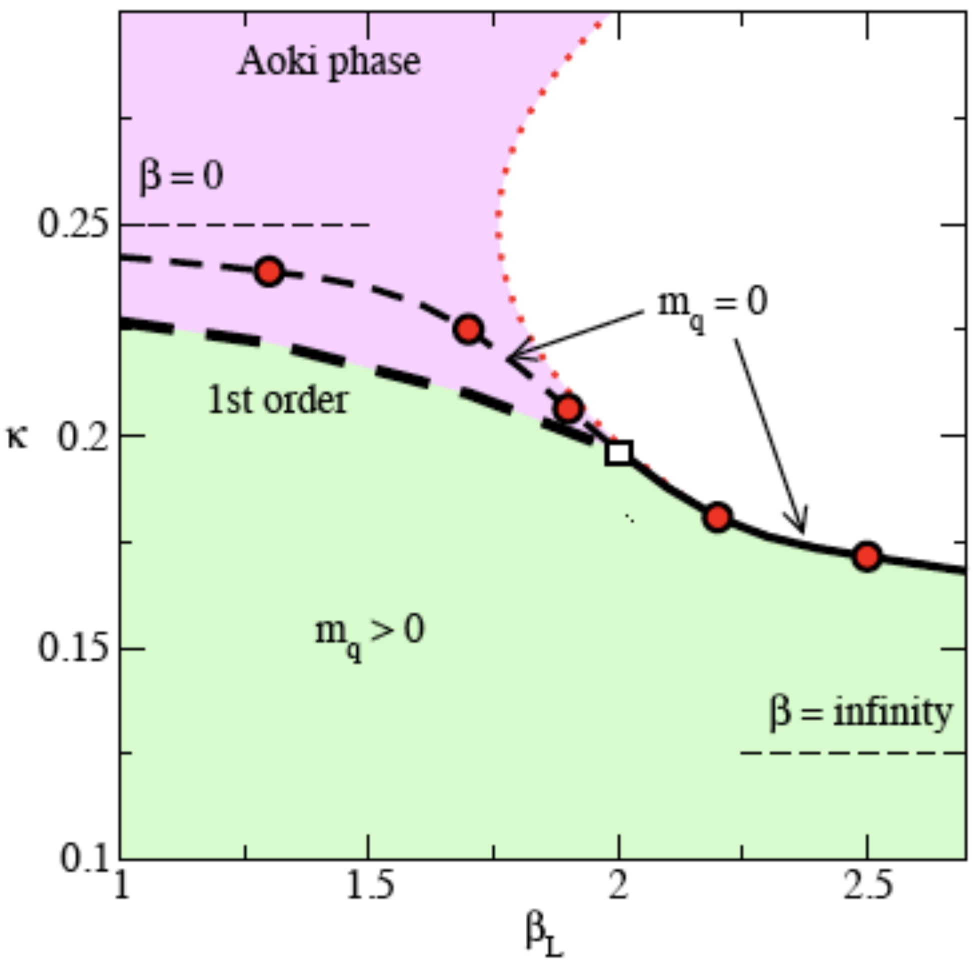
* N=2 gauge theory ("minimal walking technicolor") subject of many recent studies

  - Expect mild N dependence

➡ Have good idea of phase diagram of gauge theory

e.g. [Heitanen et al]

Differs from $N_f=1$



Status for $N_f=2$

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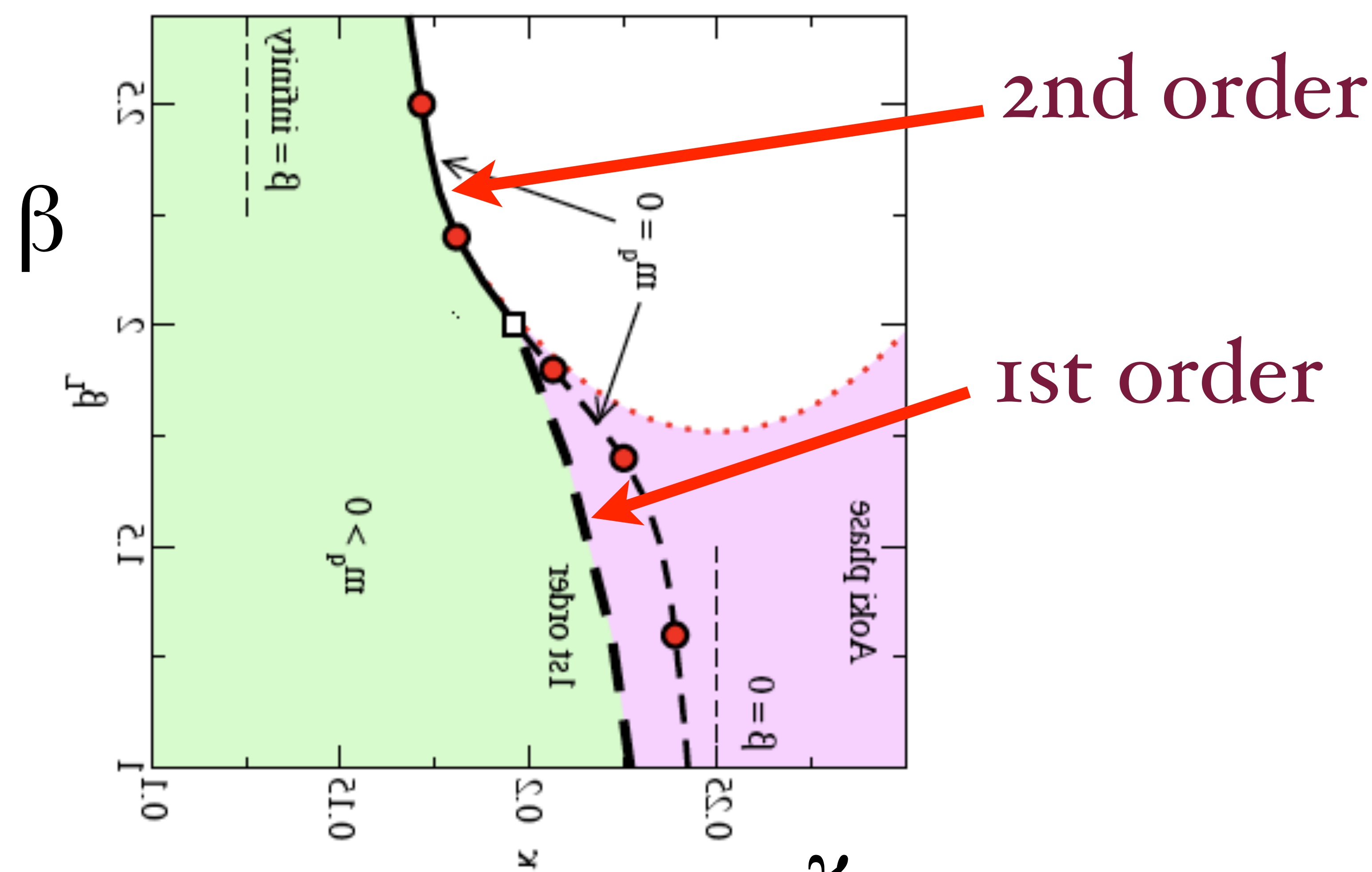
Differs from $N_f=1$



Status for $N_f=2$

* Scans with $N=10$ find results similar to $N_f=1$
  - Large region where center-symmetry unbroken
  - Only 1st order transition, no sign of end-point
* Find exotic (metastable?) phases for $b<0.5$, large κ
  - C spontaneously broken (complex plaquette)
  - $Z_{10} \to Z_3$ ("skewed" phase) [Myers & Ogilvie]
* Calculating eigenvalue distributions
* 2^4 model recently studied by [Catterall, Galvez & Unsal]

Future prospects

Future Plans & Prospects

* Crucial to check N_f=1,2 results at larger N
* Need to check interpretation by calculating m_π, m_{PCAC}, string tension, ε-regime e'values, ...
  - Need larger L_{eff}
  - Key issue is scaling of L_{eff}: $N^{1/4}$, $N^{1/2}$ or N ?
* Can extend to glueball masses and glueball-$q\bar{q}$ mixing by having one long direction
* So far, only used few CPUs, so lots of room for growth!

Any questions?