(A proposal for) B-physics on current lattices

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Future directions in lattice gauge theories - LGT10
CERN - July 26^{th}, 2010

Outline of the talk

- Main results
- Generalities
  - Flavour physics and the SM
  - Where we are in parameter (a, μ_q, V) space
  - Mtm-LQCD
- The ratio method for heavy-light ($h\ell$) meson physics
  - B-physics
    - m_b
    - f_B
    - f_{B_s}
  - D-physics
    - m_c
    - f_D
    - f_{D_s}
- Conclusions and outlook

References - I

HQET

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H. Georgi, *An effective field theory for heavy quarks at low-energies*, *Phys. Lett.* **B 240** (1990) 447 [SPIRES].

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References - II

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Finite size scaling

M. Guagnelli, F. Palombi, R. Petronzio and N. Tantalo, *f_B and two scales problems in lattice QCD*, *Phys. Lett.* **B 546** (2002) 237 [hep-lat/0206023] [SPIRES].

G.M. de Divitiis, M. Guagnelli, R. Petronzio, N. Tantalo and F. Palombi, *Heavy quark masses in the continuum limit of quenched Lattice QCD*, *Nucl. Phys.* **B 675** (2003) 309 [hep-lat/0305018] [SPIRES].

G.M. de Divitiis, M. Guagnelli, F. Palombi, R. Petronzio and N. Tantalo, *Heavy-light decay constants in the continuum limit of lattice QCD*, *Nucl. Phys.* **B 672** (2003) 372 [hep-lat/0307005] [SPIRES].

D. Guazzini, R. Sommer and N. Tantalo, *Precision for B-meson matrix elements*, *JHEP* **01** (2008) 076 [arXiv:0710.2229] [SPIRES].

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Improved fermion action

A.X. El-Khadra, A.S. Kronfeld and P.B. Mackenzie, *Massive fermions in lattice gauge theory*, *Phys. Rev.* **D 55** (1997) 3933 [hep-lat/9604004] [SPIRES].

S. Aoki, Y. Kuramashi and S.-i. Tominaga, *Relativistic heavy quarks on the lattice*, *Prog. Theor. Phys.* **109** (2003) 383 [hep-lat/0107009] [SPIRES].

N.H. Christ, M. Li and H.-W. Lin, *Relativistic heavy quark effective action*, *Phys. Rev.* **D 76** (2007) 074505 [hep-lat/0608006] [SPIRES].

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Based on ...

JEHP 04(2010)049 (arXiv:0909.3187)

B. Blossier, P. Dimopoulos, R. Frezzotti, G. Herdoiza, K. Jansen,

V. Lubicz, G. Martinelli, C. Michael, G.C. Rossi, A. Shindler,

S. Simula, C. Tarantino, C. Urbach

European Twisted Mass Collaboration

Main results

From ETMC maximally twisted fermion data (arXiv:0909.3187)

- *B*-physics
  - *b*-mass

$$\hat{\mu}_b^{\overline{MS},N_f=2}(2\text{ GeV}) = 5.35(32)\text{ GeV}$$

  - decay constants

$$f_B = 194(16)\text{ MeV} \qquad f_{B_s} = 235(11)\text{ MeV}$$

- *D*-physics
  - *c*-mass

$$\hat{\mu}_c^{\overline{MS},N_f=2}(2\text{ GeV}) = 1.23(06)\text{ GeV}$$

  - decay constants

$$f_D = 211(9)\text{ MeV} \qquad f_{D_s} = 252(7)\text{ MeV}$$

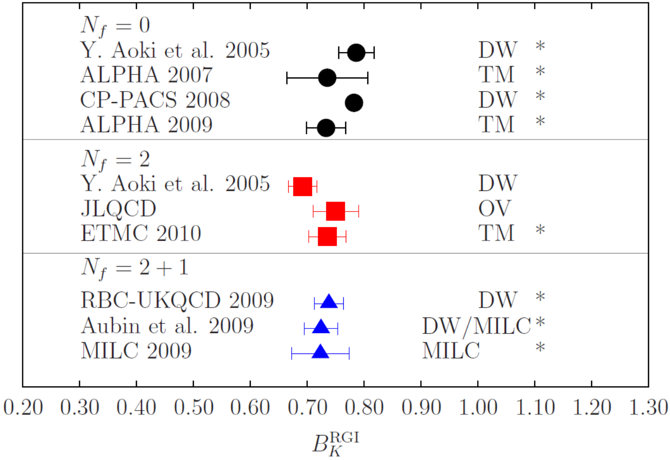
Flavour physics & SM

	$B_d - \bar{B}_d$	
oscillations		$K_0 - \bar{K}_0$
	$B_s - \bar{B}_s$	
	$f_B \; f_{B_s}$	f_K
parameters		
	$B_B \; B_{B_s}$	B_K

- $\Delta M_{B_d} = \frac{G_F^2}{6\pi^2} M_W^2 \eta_b S(x_t) M_{B_d} f_{B_d}^2 B_{B_d} |V_{tb}|^2 |V_{td}|^2$
- $\frac{\Delta M_{B_d}}{\Delta M_{B_s}} = \frac{M_{B_d}}{M_{B_s}} \frac{f_{B_d}^2}{f_{B_s}^2} \frac{B_{B_d}}{B_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2}$
- $BR(B \to \tau\nu)|_{SM} = (0.79 \pm 0.07) \times 10^{-4}$ with V_{ub} and f_B as inputs
- $BR(B \to \tau\nu)|_{exp} = (1.74 \pm 0.34) \times 10^{-4}$

See M. Bona for UTfit at FPCP 2010

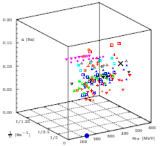
B_K compilation

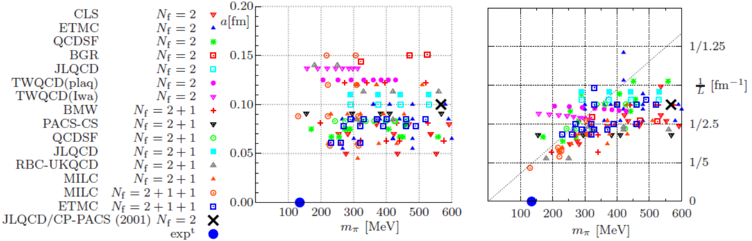


Simulation landscape - I

dynamical simulations : parameters landscape

- number of flavours : N_f
- lattice spacing : a
- lattice size : L
- pion masses : m_{PS}





oblique dotted line : $m_\pi L = 3.5$

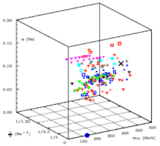
Caveat in plots : no information on systematic effects (scale setting, cut-off effects, ...), m_s, m_c, ...

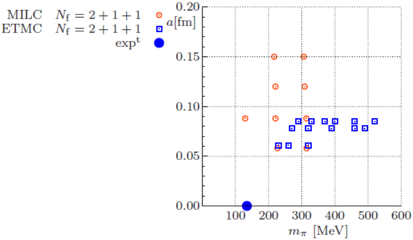
review talk by Christian Hoelbling

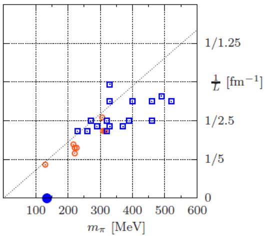
Simulation landscape - II

dynamical simulations : parameters landscape

- number of flavours : $N_f = 2 + 1 + 1$
- lattice spacing : a
- lattice size : L
- pion masses : m_{PS}







oblique dotted line : $m_\pi L = 3.5$

Caveat in plots : no information on systematic effects (scale setting, cut-off effects, ...), m_s, m_c, ...

review talk by Christian Hoelbling

Twisted-mass formulation

- The Mtm lattice regularization of ($N_f = 2$) QCD action reads ($r = 1$)

$$S^{\rm ph}_{N_f=2} = S_L^{\rm YM} + a^4 \sum_x \bar\psi(x) \left[\gamma\cdot\widetilde\nabla - i\gamma_5\tau^3 \left(-\frac{a}{2} r\nabla^*\nabla + M_{\rm cr}(r)\right) + \mu_q\right] \psi(x)$$

- ψ is a flavour doublet, $M_{\rm cr}(r)$ is the critical mass and τ^3 acts on flavour indices
- From the "physical" basis (where the quark mass is real), the non-anomalous

$$\psi = \exp(i\pi\gamma_5\tau^3/4)\chi\,, \quad \bar\psi = \bar\chi\exp(i\pi\gamma_5\tau^3/4)$$

transformation brings the lattice action in the so-called "twisted" basis

$$S^{\rm tw}_{N_f=2} = S_L^{\rm YM} + a^4 \sum_x \bar\chi(x) \left[\gamma\cdot\widetilde\nabla - \frac{a}{2} r\nabla^*\nabla + M_{\rm cr}(r) + i\mu_q\gamma_5\tau^3\right] \chi(x)$$

- Unlike the standard Wilson regularization, here the subtracted Wilson operator $-\frac{a}{2}r\nabla^*\nabla + M_{\rm cr}(r)$ is "chirally rotated" w.r.t. the quark mass.

Virtues and drawbacks

- Virtues
  - Automatic $O(a)$ improvement
  - Positivity of the Dirac-Wilson matrix determinant, with (lowest eigenvalue)^2 bound from below by μ_q^2
  - Simplified operator renormalization (e.g. quark mass, $f_\pi, \ldots$)
  - Mass non-degenerate quark pairs can be introduced, loosing none of the above properties (from $N_f = 2$ to $N_f = 2 + 1 + 1$)
- Drawbacks
  - $O(a^2)$ breaking of parity and isospin (e.g. $m_{\pi^\pm}$ *vs.* m_{π^3})
  - $O(a^2)$ contaminations from mixing of different parity/isospin states
- Further "useful" features
  - Valence quarks (*à la* OS) can be introduced on any N_f sea
    - Isospin restoration ...
    - No wrong chirality mixing in CP-conserving $\mathcal{H}_{eff}^{w}$ ...
  - ... up to only $O(a^2)$ unitarity violations

Proof of automatic O(a) improvement

- Mtm action invariant under [sym] $\equiv \mathcal{P} \times \mathcal{D}_d \times (\mu_q \to -\mu_q)$

  [S.E.] = Symanzik Expansion (no reference to r)

$$\mathcal{P}: \begin{cases} \psi(x) \to \gamma_0 \psi(x_P)\,, \quad \bar\psi(x) \to \bar\psi(x_P)\gamma_0 \\ U_0(x) \to U_0(x_P)\,, \\ U_k(x) \to U_k^\dagger(x_P - a\hat{k})\,, \quad k = 1,2,3 \end{cases} \qquad \mathcal{D}_d: \begin{cases} U_\mu(x) \to U_\mu^\dagger(-x - a\hat\mu)) \\ (\psi(x)\,,\bar\psi(x)) \to e^{3i\pi/2}(\psi(-x)\,,\bar\psi(-x)) \end{cases}$$

- $\langle O(x)\rangle\Big|_{(r,\mu_q)} \stackrel{[\mathrm{S.E.}]}{=} \Big[\zeta_O^O(r)\langle O(x)\rangle + a\sum_\ell \eta_{O_\ell}^O(r)\langle O_\ell(x)\rangle\Big]_{(\mu_q)}^{\mathrm{cont}} + \mathrm{O}(a^2) =$

$$\stackrel{[\mathrm{sym}]^L}{=} (-1)^{P[O]+d}\langle O(-x_P)\rangle\Big|_{(r,-\mu_q)} \stackrel{[\mathrm{S.E.}]}{=}$$

$$= (-1)^{P[O]+d}\Big[\zeta_O^O(r)\langle O(-x_P)\rangle + a\sum_\ell \eta_{O_\ell}^O(r)\langle O_\ell(-x_P)\rangle\Big]_{(-\mu_q)}^{\mathrm{cont}} + \mathrm{O}(a^2) =$$

$$\stackrel{[\mathrm{sym}]^{\mathrm{cont}}}{=} \Big[\zeta_O^O(r)\langle O(x)\rangle - a\sum_\ell \eta_{O_\ell}^O(r)(-1)^{P[O_\ell]+P[O]}\langle O_\ell(x)\rangle\Big]_{(\mu_q)}^{\mathrm{cont}} + \mathrm{O}(a^2)$$

- $-(-1)^{P[O_\ell]+P[O]} = 1 \Longrightarrow (P[O] = 0 \to O_\ell$ parity-odd) thus $\langle O_\ell(x)\rangle\Big|_{(\mu_q)}^{\mathrm{cont}} = 0$
- true for all odd powers of a

B-physics on current lattices

Aim extract μ_b, f_B, f_{B_s}, ... from available data where $\mu_b \gg a^{-1}$

$$\text{HQET} \to \int d^3x \langle A_{h\ell}(\vec{x},t) A_{h\ell}(0)\rangle = \frac{e^{-M_{h\ell}t}}{2M_{h\ell}} f^2_{h\ell} M^2_{h\ell} \overset{\mu_h\to\infty}{\longrightarrow} O(\Lambda^3_{QCD}) e^{-\mu_h t}$$

- Computational strategy for μ_b (ignoring for a while renormalization)
  - Consider the $h\ell$-meson mass ratios, $M^L_{h\ell}(\mu_\ell,\mu_h)/M^L_{h\ell}(\mu_\ell,\mu'_h)$
    - ⋆ at pairs of nearby h-quark masses $\sim \mu_c < \mu_h < \mu'_h <\sim 2\mu_c$
    - ⋆ scaled in units of some fixed number, $\lambda = \mu_h/\mu'_h$
  - Ratios expected to have smooth χ-al/continuum extrapolation
    - ⋆ have an exact infinite h-quark mass limit, namely 1
    - ⋆ fit them with a 2^{nd} order polynomial in $1/\mu_h$, passing through 1
  - Intersecting the fitted curve at the B-meson mass yields μ_b
- Computational strategy for f_B, f_{B_s} (ignoring for a while renormalization)
  - Same, for $\frac{f_{h\ell}=\langle\Omega|A_0^{h\ell}|M_{h\ell}(\mu_\ell,\mu_h)\rangle}{f'_{h\ell}=\langle\Omega|A_0^{h\ell}|M_{h\ell}(\mu_\ell,\mu'_h)\rangle}\left(\frac{\mu'_h}{\mu_h}\right)^{1/2}$ ratios
  - Hit the value of $f_B(\mu_\ell=\mu_{u/d})$ (or $f_B(\mu_\ell=\mu_s)$) at $\mu_h=\mu_b$

General strategy - I

- Consider the lattice y-ratios

$$y^L(x^{(n)}, \lambda; \hat\mu_\ell, a) = \frac{M^L_{h\ell}(\hat\mu_h^{(n)}; \hat\mu_\ell, a)}{M^L_{h\ell}(\hat\mu_h^{(n-1)}; \hat\mu_\ell, a)} \cdot \frac{\rho(\log \hat\mu_h^{(n-1)})\hat\mu_h^{(n-1)}}{\rho(\log \hat\mu_h^{(n)})\hat\mu_h^{(n)}}\,, \quad n = 2, \cdots, N$$

[where $\rho(\log \hat\mu_h)\hat\mu_h = \mu_h^{\rm pole}$ and $\hat\mu = Z_P^{-1}\mu$]

- inspired by the large h-quark mass limit

$$\lim_{\mu_h^{\rm pole}\to\infty} \frac{M_{h\ell}(\mu_h^{\rm pole})}{\mu_h^{\rm pole}} = 1$$

- Keeping fix

$$\lambda = \frac{\hat\mu_h^{(n)}}{\hat\mu_h^{(n-1)}} = \frac{\mu_h^{(n)}}{\mu_h^{(n-1)}} = \frac{x^{(n-1)}}{x^{(n)}}\,, \qquad x^{(n)} = \frac{1}{\hat\mu_h^{(n)}}$$

- construct the chiral and continuum limit of the lattice y-ratios

$$y(x^{(n)}, \lambda; \hat\mu_{u/d}) \equiv \lim_{\hat\mu_\ell\to\hat\mu_{u/d}} \lim_{a\to 0} y^L(x^{(n)}, \lambda; \hat\mu_\ell, a) = \lambda^{-1} \frac{M_{hu/d}(1/x^{(n)})}{M_{hu/d}(1/\lambda x^{(n)})} \frac{\rho(\log \lambda x^{(n)})}{\rho(\log x^{(n)})}$$

[with the short-hand definition $M_{hu/d}(1/x) \equiv M_{hu/d}(1/x, \hat\mu_{u/d})$]

- then

$$\lim_{x\to 0^+} y(x, \lambda; \hat\mu_{u/d}) = 1$$

General strategy - II

- Notice that, by truncating ρ in PT at N^{p}LL order, one gets

$$y(x,\lambda;\hat\mu_{u/d})\Big|_p - 1 \overset{x\sim 0^+}{=} \mathrm{O}\left(\frac{1}{(\log x)^{p+1}}\right)$$

- Taking $\lambda = 1.278$ (see below), we successively consider the $N=4$ h-quark masses

$$\hat\mu_h^{(1)} = 1.230\ \mathrm{GeV} \qquad \hat\mu_h^{(2)} = \lambda\hat\mu_h^{(1)} = 1.572\ \mathrm{GeV}$$
$$\hat\mu_h^{(3)} = \lambda^2\hat\mu_h^{(1)} = 2.009\ \mathrm{GeV} \qquad \hat\mu_h^{(4)} = \lambda^3\hat\mu_h^{(1)} = 2.568\ \mathrm{GeV}$$

and compute the numbers

$$y_p^{(n)} = y(x^{(n)}, 1.278; \hat\mu_{u/d})\Big|_p, \quad n=2,3,4 \quad p=0,1,2$$

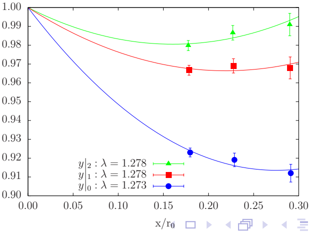
- Using the simple ansatz

$$y(x,\lambda;\hat\mu_{u/d})\Big|_p = 1 + \eta_p^{(1)}(\log x,\lambda;\hat\mu_{u/d})\,x + \eta_p^{(2)}(\log x,\lambda;\hat\mu_{u/d})\,x^2$$

one gets the fitting curves ($x = 1/\mu_h$) flatter and flatter as p increases

$p=0$ TL $\longrightarrow$ blue dots
$p=1$ LL $\longrightarrow$ red squares
$p=2$ NLL $\longrightarrow$ green triangles

How do we fix λ?

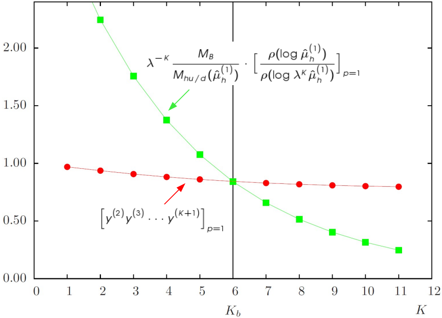


General strategy - III

- Interpolation to $\mu_h = \mu_b$ is done through the formula ($\hat{\mu}_h^{(K+1)} = \lambda^K \hat{\mu}_h^{(1)}$)

$$\left[y^{(2)} y^{(3)} \cdots y^{(K+1)}\right]_p = \lambda^{-K} \frac{M_{hu/d}(\hat{\mu}_h^{(K+1)})}{M_{hu/d}(\hat{\mu}_h^{(1)})} \cdot \left[\frac{\rho(\log \hat{\mu}_h^{(1)})}{\rho(\log \hat{\mu}_h^{(K+1)})}\right]_p$$

- $M_{hu/d}(\hat{\mu}_h^{(1)})$ is an input $h\ell$-mass around charm
- $M_{hu/d}(\hat{\mu}_h^{(K+1)})$ is set equal to M_B
- Solving for the integer K gives (λ is slightly p dependent)



$\hat{\mu}_b = \lambda^{K_b} \hat{\mu}_h^{(1)}$, $K_b = 6$

$\hat{\mu}_b^{N_f=2}(\hat{\mu}_b^{N_f=2}) = 4.63(27)$ GeV

Error budget

- 5% $\leftarrow$ input $M_{hu/d}(\hat{\mu}_h^{(1)})$
- 1% $\leftarrow$ perturbative ρ evaluation
- 1% $\leftarrow$ $\prod y$ (and K_b determination)
- 1% $\leftarrow$ constancy of η

Determining f_B and f_{B_s} - I

- Take

$$z(x,\lambda;\hat\mu_\ell) = \lambda^{1/2}\,\frac{f_{h\ell}(1/x)}{f_{h\ell}(1/x\lambda)}\cdot\frac{C_A^{stat}(\log(x\lambda))}{C_A^{stat}(\log x)}\,\frac{[\rho(\log x)]^{1/2}}{[\rho(\log \lambda x)]^{1/2}}$$

with the short-hand notation

$$f_{h\ell}(1/x) \equiv f_{h\ell}(1/x,\hat\mu_\ell)$$

- inspired by the large h-quark mass limit (in HQET, currents need to be renormalized)

$$\lim_{x\to 0^+}\left[\frac{\rho(\log x)}{x}\right]^{1/2}\frac{f_{h\ell}(1/x)}{C_A^{stat}(\log x)} = \text{constant} \neq 0$$

- Then

$$\lim_{x\to 0^+} z(x,\lambda;\hat\mu_\ell) = 1$$

- At N^{p}LL order

$$z(x,\lambda;\hat\mu_\ell)\Big|_p - 1 \overset{x\sim 0^+}{=} \mathrm{O}\Big(\frac{1}{(\log x)^{p+1}}\Big)$$

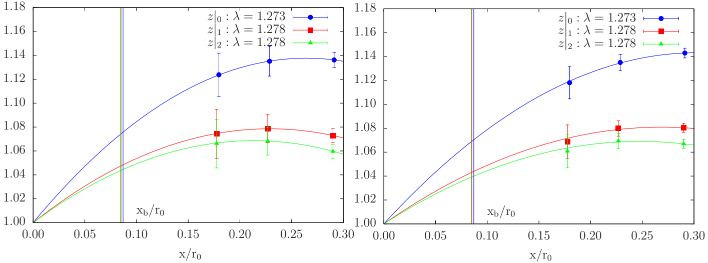
- Similarly as before, try a fit to lattice data of the kind

$$z(x,\lambda,\hat\mu_\ell) = 1 + \zeta_1(\log x,\lambda;\hat\mu_\ell)\,x + \zeta_2(\log x,\lambda;\hat\mu_\ell)\,x^2$$

Determining f_B and f_{B_s} - II

- We get for

$$\mu_\ell = \mu_{u/d} \qquad \text{and} \qquad \mu_\ell = \mu_s$$



- From the iterative equation ($\mu_\ell = \mu_{u/d}$ or $\mu_\ell = \mu_s$)

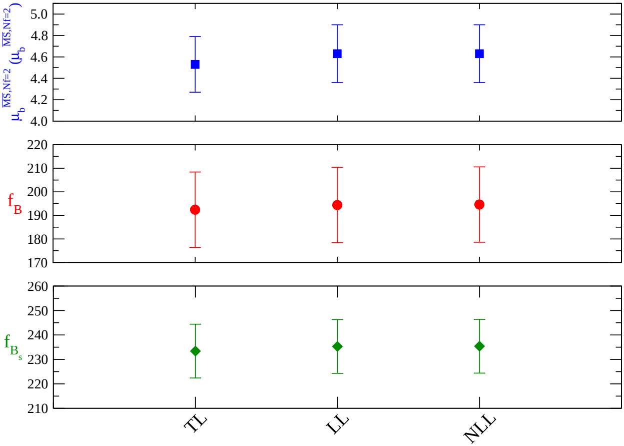
$$\left[z^{(2)} z^{(3)} \cdots z^{(K+1)}\right]_p = \lambda^{K/2} \frac{f_{h\ell}(\hat{\mu}_h^{(K+1)})}{f_{h\ell}(\hat{\mu}_h^{(1)})} \cdot \left[\frac{C_A^{stat}(\log \hat{\mu}_h^{(1)})}{C_A^{stat}(\log \hat{\mu}_h^{(K+1)})} \left(\frac{\rho(\log \hat{\mu}_h^{(K+1)})}{\rho(\log \hat{\mu}_h^{(1)})}\right)^{1/2}\right]_p$$

- inserting the measured values of $f_{h\ell}(\hat{\mu}_h^{(1)})$ ($\ell = u/d$ or $\ell = s$), one finds at μ_b

$$f_B = 194(16)\ \text{MeV} \qquad f_{B_s} = 235(11)\ \text{MeV}$$

p-dependence

- Very mild p dependence

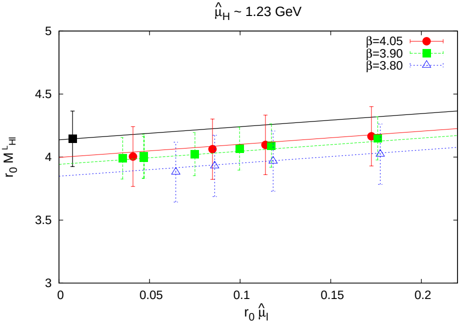


Extrapolation - I

- Chiral and continuum extrapolations of $M_{h\ell}(\hat{\mu}_h^{(1)})$

  (SU(2) χPT @ NLO) & continuum fit (Roessl '99, RBC '08)

$$M_{h\ell} r_0 = C_0 + [C_1 + 0 \cdot \log(\frac{2B_0 \hat{\mu}_\ell}{16\pi^2 f_0^2})]\hat{\mu}_\ell r_0 + D_0 \frac{a^2}{r_0^2}$$

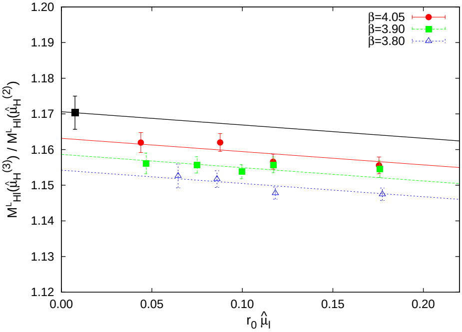


Extrapolation - II

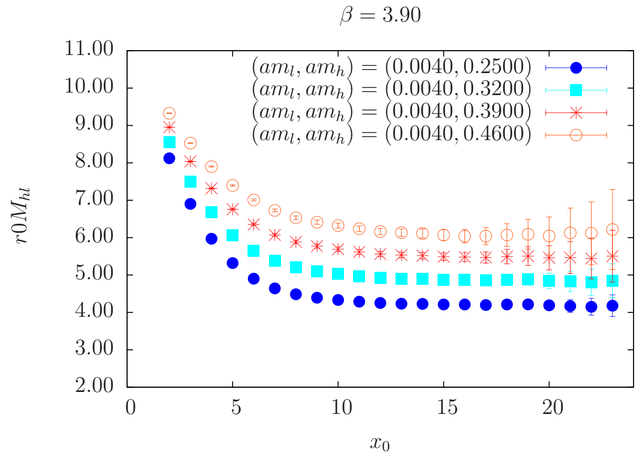
- Chiral and continuum extrapolations of $M_{h\ell}$-ratios

(SU(2) χPT @ NLO) & continuum fit (Roessl '99, RBC '08)

$$M_{h\ell}r_0 = C_0 + [C_1 + 0 \cdot \log(\frac{2B_0\hat{\mu}_\ell}{16\pi^2 f_0^2})]\hat{\mu}_\ell r_0 + D_0 \frac{a^2}{r_0^2}$$



Effective-mass plateau for $M_{h\ell}$

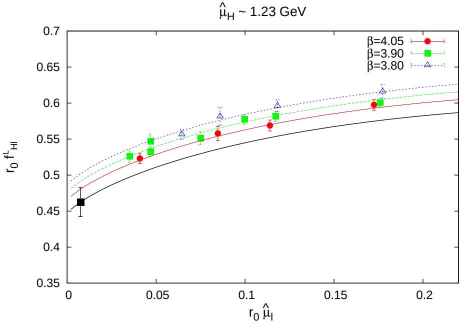


Extrapolation - III

- Chiral and continuum extrapolations of $f_{h\ell}(\hat{\mu}_h^{(1)})$

(SU(2) χPT @ NLO) & continuum fit (Roessl '99, RBC '08)

$$f_{h\ell} r_0 = C_0 + [C_1 + \gamma_1 \cdot \log(\frac{2B_0 \hat{\mu}_\ell}{16\pi^2 f_0^2})]\hat{\mu}_\ell r_0 + D_0 \frac{a^2}{r_0^2}$$

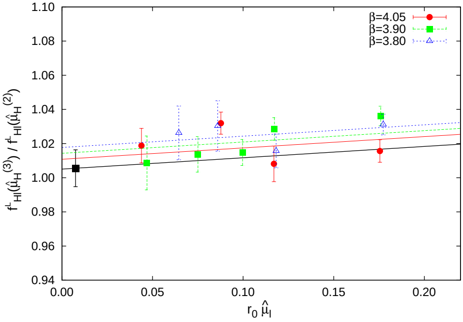


Extrapolation - IV

- Chiral and continuum extrapolations of $f_{h\ell}$-ratios

  (SU(2) χPT @ NLO) & continuum fit (Roessl '99, RBC '08)

$$f_{h\ell} r_0 = C_0 + [C_1 + \gamma_1 \cdot \log(\frac{2B_0 \hat{\mu}_\ell}{16\pi^2 f_0^2})]\hat{\mu}_\ell r_0 + D_0 \frac{a^2}{r_0^2}$$



Fitting coefficients

p	$\eta_1 r_0$	$\eta_2 r_0^2$	$\zeta_1 r_0$	$\zeta_2 r_0^2$
0	-0.63(4)	1.14(19)	1.04(26)	-1.97(90)
1	-0.30(5)	0.70(21)	0.69(26)	-1.50(87)
2	-0.24(5)	0.75(22)	0.64(25)	-1.50(87)

Table: Best-fit $\eta_{1,2}$ and $\zeta_{1,2}$ coefficients (with statistical errors in parenthesis) in r_0-units as obtained from the $y|_p(x)$ and $z|_p(x)$ ratios defined above

Summary and Comparison

- cheap method
- works with any kind of fermions
- present ETMC results can be easily improved
  - more points in the charm region
  - better determination of inputs
    - more lattice spacings
    - more light masses
    - better NP-determination of Z_P (in Mtm-LQCD $\hat{\mu} = Z_P^{-1} \mu$)
  - $N_f = 2 \to N_f = 2 + 1 + 1$
- can be employed in other interesting cases ($M_{B_s}, B_B, B_{B_s}, \ldots$)
- good consistency with other determinations

Comparison

Rolf & Sint, hep-ph/0209255, Della Morte *et al.*, 0710.2201 & 0710.1553, Chetyrkin *et al.*, 0907.2110, Blossier *et al.*, 0810.3145, 0909.3187, 0911.3757

- Charm

  $\hat{\mu}_c^{N_f=5}(\hat{\mu}_c^{N_f=5}) = 1.28(7)$ GeV vs. $\hat{\mu}_c^{N_f=5}(\hat{\mu}_c^{N_f=5}) = 1.28(1)$ GeV

  $f_D = 211(9)$ MeV vs. $197(9)$ vs. $\underline{206.7 \pm 8.5 \pm 2.5|_{exp}}$ [Rosner & Stone, 1002.1655]

  $f_{D_s} = 252(7)$ MeV vs. $244(8)$ vs. $\underline{254.6 \pm 5.9|_{exp}}$ [HFAG 2010]

- Bottom

  $\hat{\mu}_b^{N_f=5}(\hat{\mu}_b^{N_f=5}) = 4.04(25)$ GeV vs. $\hat{\mu}_b^{N_f=5}(\hat{\mu}_b^{N_f=5}) = 4.16(02)$ GeV

  $f_B = 194(16)$ MeV vs. $f_B = 191(14)$

  $f_{B_s} = 235(11)$ MeV vs. $f_{B_s} = 243(14)$ vs $\underline{f_{B_s}\sqrt{B_{B_s}} = 265(4)|_{exp}}$ [www.utfit.org]

- Mass ratios
  - $\mu_c^{RGI,N_f=2}/\mu_b^{RGI,N_f=2} = 0.232(13)$

    $[\mu_c^{RGI,N_f=0} = 1.654(45)\ GeV]/[\mu_b^{RGI,N_f=0} = 6.758(86)\ GeV] = 0.245(7)$
  - $\mu_c^{\overline{MS},N_f=2}(3\ \text{GeV})/\mu_b^{\overline{MS},N_f=2}(10\ \text{GeV}) = 0.274(15)$

    $\mu_c^{\overline{MS},N_f=4}(3\ \text{GeV})/\mu_b^{\overline{MS},N_f=5}(10\ \text{GeV}) = 0.273(3)$

Acknowledgments

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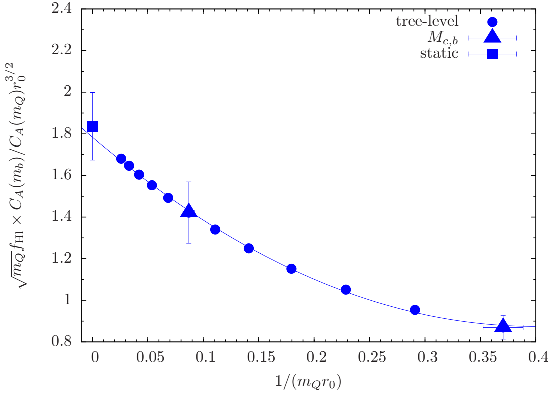
- All people of the ETM Collaboration for discussions
- in particular P. Dimopoulos, R. Frezzotti, G. Hedoiza, K. Jansen, V. Lubicz, S. Simula for their help in preparing this seminar
- . . . and . . .
- all of you for listening

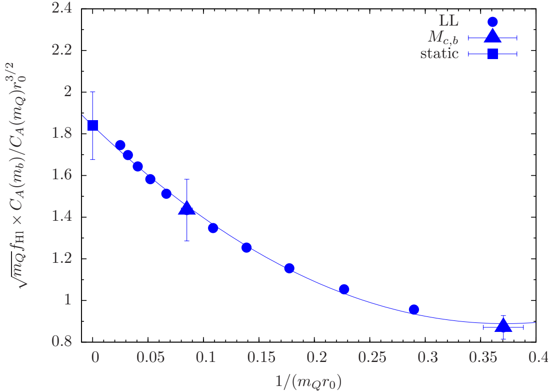
Comparison with the static point - TL

- Static point of $\sqrt{m_Q} f_{Hl}(m_Q)$ from
  B. Blossier *et al.* (ETM Collaboration) PoS LATTICE2009:151, 2009 (arXiv:0911.3757)
- $m_Q = \hat{\mu}_h^{\overline{MS}}$



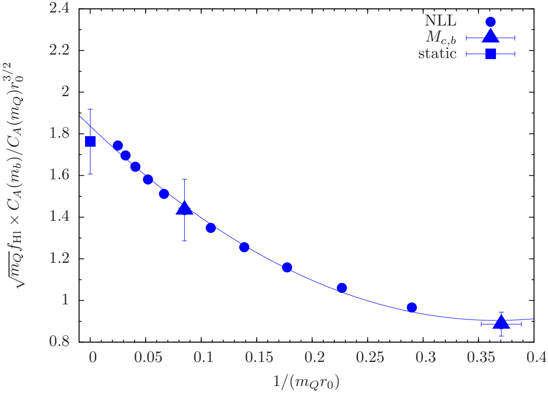
Comparison with the static point - LL

- $m_Q = \hat{\mu}_h^{\overline{MS}}$



Comparison with the static point - NLL

- $m_Q = \hat{\mu}_h^{\overline{MS}}$



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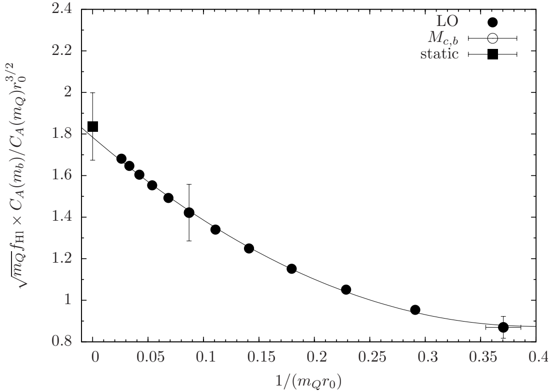

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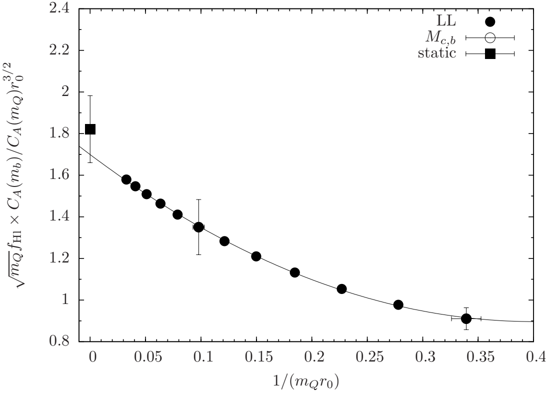
- $m_Q = \hat{\mu}_h^{pole}$



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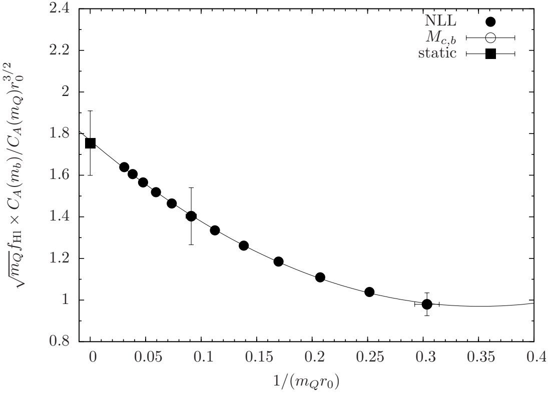
- $m_Q = \hat{\mu}_h^{pole}$



On higher twist operators

$$\hat{R}(Q^2)_{fi} \sim C_1^{(k_1)}(\alpha_s(Q^2/\mu^2))\langle f|O_1(\mu)|i\rangle + \frac{C_2^{(k_2)}(\alpha_s(Q^2/\mu^2))}{(Q^2)^\Delta}\langle f|O_2(\mu)|i\rangle$$

$$2\Delta = \dim O_2 - \dim O_1$$

The following facts are related (Martinelli & Sachrajda, Nucl. Phys. B476 (1996) 660, hep-ph 9605336)

- Perturbative series are only asymptotic, and terms $O(\alpha_s(Q^2/\mu^2)^{k+1} \sim \log(Q^2/\mu^2)^{-(k+1)}$ are neglected
- Higher twisted terms are kept which are exponentially small compared to neglected perturbative contributions. Indeed

$$e^{-4\pi\Delta/\beta_0\alpha_s(Q^2/\mu^2)} \sim e^{-\Delta\log(Q^2/\mu^2)} \sim \left(\frac{\mu^2}{Q^2}\right)^\Delta$$

- Perturbative series (are supposed to) develop renormalon ambiguities
- O_2 mixes with O_1 with a power divergent coefficient