The meanfield laboratory to study five-dimensional gauge theories

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- Introduction
- Meanfield results for torus geometry
- Monte Carlo results for torus geometry
- The orbifold case
- Outlook

Introduction

Gauge-Higgs unification

$$\underbrace{A_M}_{\text{5d gauge field}} \xrightarrow{\mathcal{M}_5 = E_4 \times S^1} \{ \underbrace{A_\mu}_{W:\ \text{4d gauge field}} , \underbrace{A_5}_{H:\ \text{4d Higgs}} \}$$

- Higgs potential is generated by quantum corrections and can trigger spontaneous symmetry breaking (Hosotani mechanism)
- 5d gauge symmetry keeps the potential finite
- Triviality requires a cut-off $\longrightarrow$ lattice
- Does a continuum limit exist non-perturbatively? $\longrightarrow$ meanfield, Monte Carlo
- Dimensional reduction?

Meanfield results for torus geometry

Meanfield expansion [Drouffe and Zuber, 1983]
$SU(N)$ gauge links U are replaced by $N \times N$ complex matrices V and Lagrange multipliers H

$$\langle \mathcal{O}[U] \rangle = \frac{1}{Z} \int \mathrm{D}V \int \mathrm{D}H \, \mathcal{O}[V] \mathrm{e}^{-S_{\mathrm{eff}}[V,H]}$$

$$S_{\mathrm{eff}} = S_G[V] + u(H) + (1/N)\mathrm{Re}\,\mathrm{tr}\{HV\}\,, \quad \mathrm{e}^{-u(H)} = \int \mathrm{D}U \, \mathrm{e}^{(1/N)\mathrm{Re}\,\mathrm{tr}\{UH\}}$$

Saddle point solution (background)

$$H \longrightarrow \bar{H}\mathbf{1}\,, \quad V \longrightarrow \bar{V}\mathbf{1}\,, \quad S_{\mathrm{eff}}[\bar{V},\bar{H}] = \text{minimal}$$

Corrections calculated from Gaussian fluctuations

$$H = \bar{H} + h \quad \text{and} \quad V = \bar{V} + v$$

Covariant gauge fixing is imposed on v [Rühl, 1982]

Meanfield results for torus geometry

Our setup: periodic boundary conditions

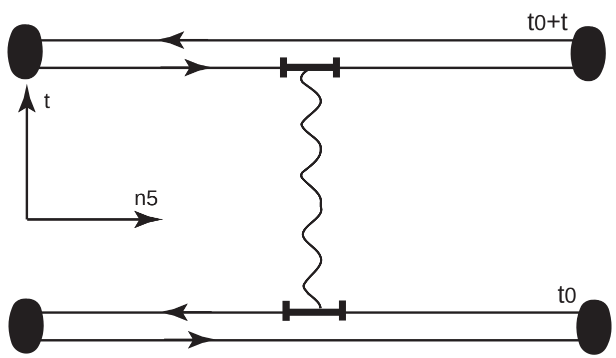
$L_T \times L^3 \times L_5$ lattice, $SU(2)$ gauge theory with anisotropic plaquette action

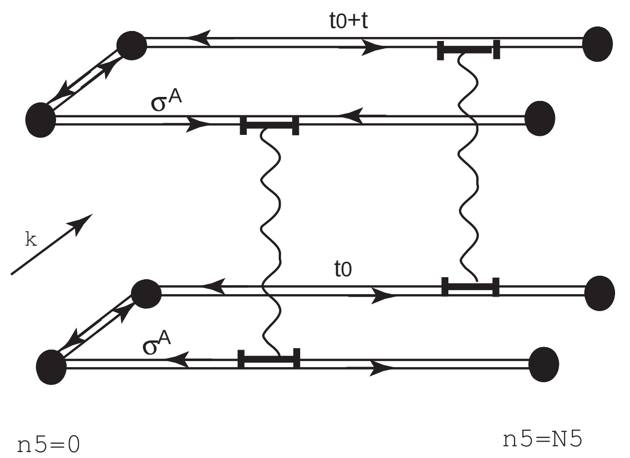
$$S_W = \frac{\beta}{4}\Big[\frac{1}{\gamma}\sum_{\rm 4d-p} {\rm tr}\Big(1 - U_p\Big) + \gamma \sum_{\rm 5d-p} {\rm tr}\Big(1 - U_p\Big)\Big]\,, \quad \gamma = \frac{a_4}{a_5} \text{ (tree level)}$$

The background is $\overline{v}_0$ along directions $\mu = 0,1,2,3$ and $\overline{v}_5$ along the extra dimension

Observables

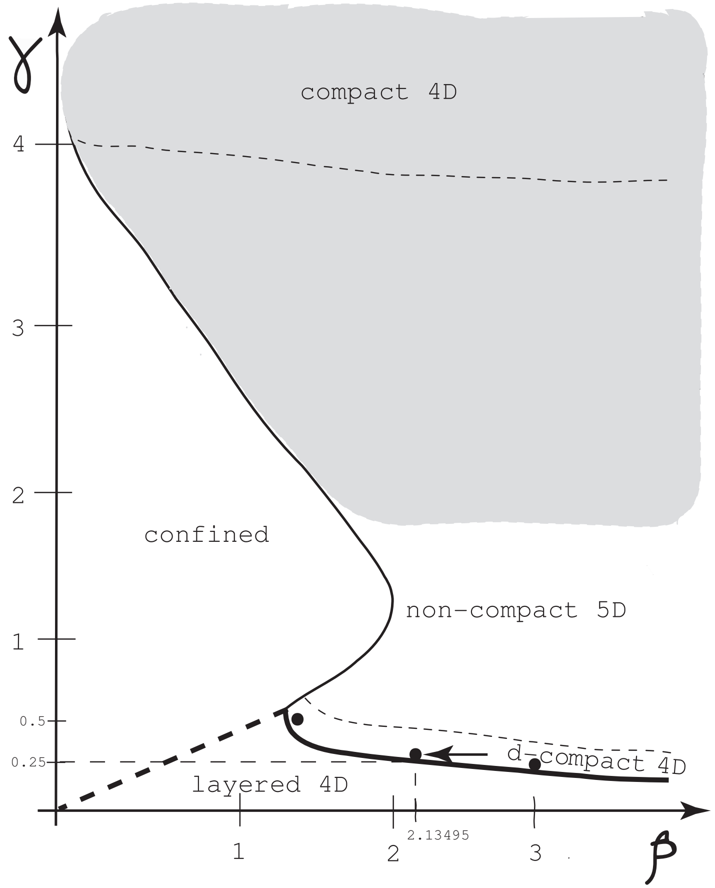
- Static potential V_4 along the 4d hyperplanes and V_5 along the extra dimension
- Higgs (1st order) m_H and gauge boson (2nd order) m_W masses





Meanfield results for torus geometry

Phase diagram



[Irges and Knechtli, 2009]

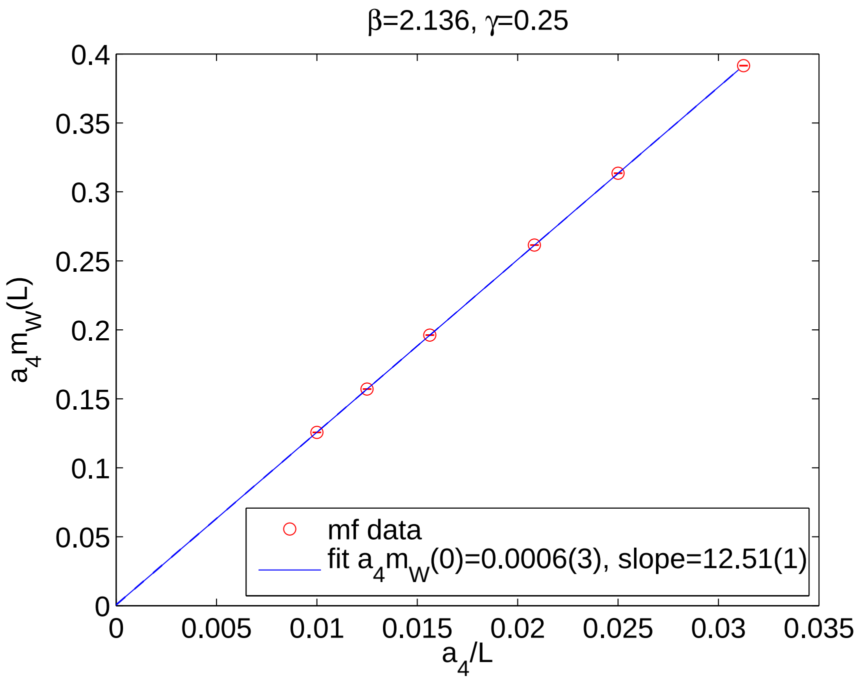
The deconfined phase ($\overline{v}_0 \neq 0$, $\overline{v}_5 \neq 0$) has a rich structure:

- at $\gamma >> 1$ (compact phase) V_4 at short distances is 4d Coulomb
- at $\gamma < 1$ there is a line of 2nd order phase transitions; close to the layered phase V_4 is 4d Coulomb again, we call it the "d-compact" phase

Meanfield results for torus geometry

Spectrum

$a_4 m_H(\beta, \gamma)$ does not depend at 1st order on the geometry



The gauge boson mass at 2nd order depends significantly only on L

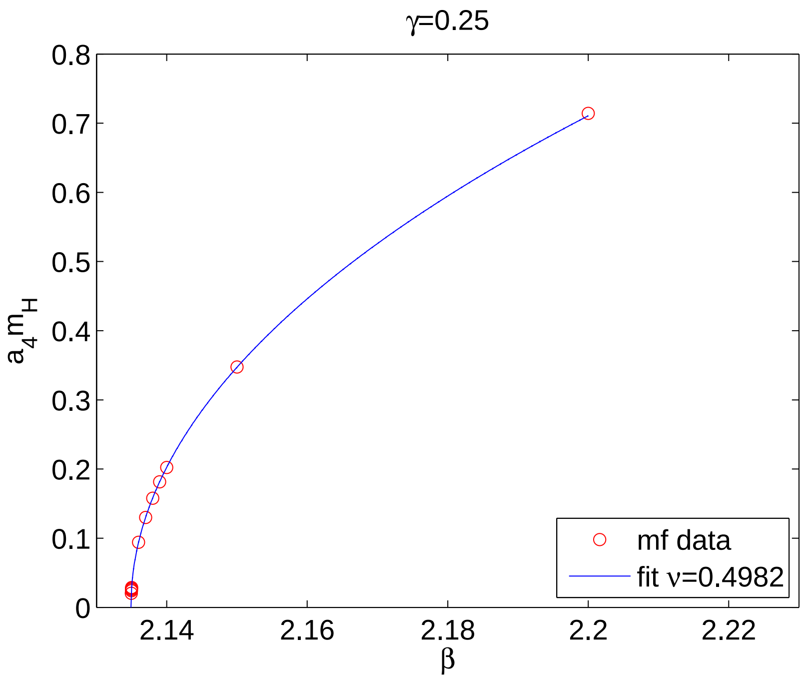
$$a_4 m_W = c_L / L \,, \quad c_L = 12.5$$

Extrapolation $L \to \infty$ is consistent with zero (we cannot exclude a exponentially small mass)

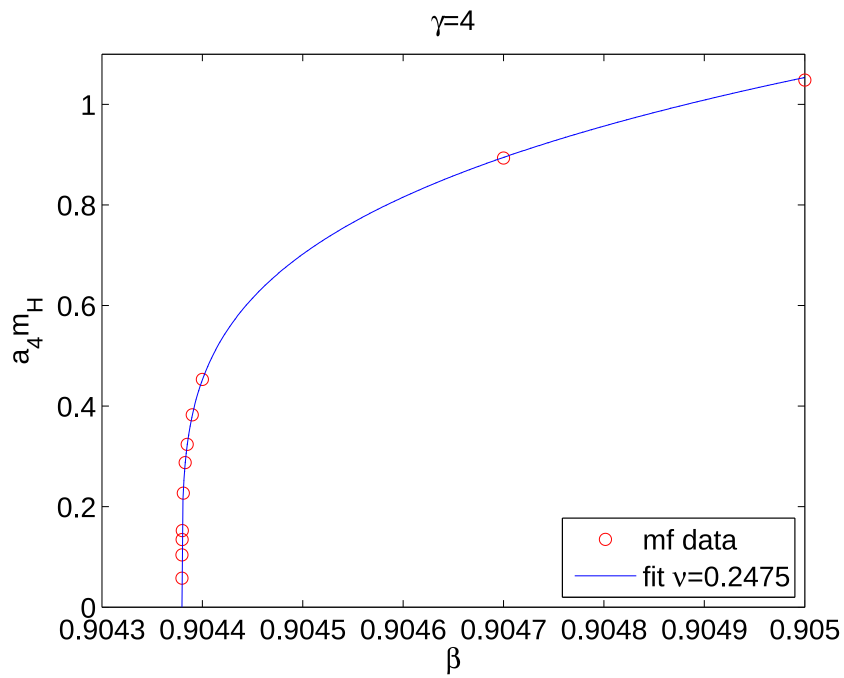
Meanfield results for torus geometry

The second order phase transition separating the d-compact phase from the layered phase:

$$a_4 m_H \sim (1 - \beta_c/\beta)^{\nu}$$



$\nu = 1/2$: 4d Ising model, confirms [Svetitsky and Yaffe, 1982]



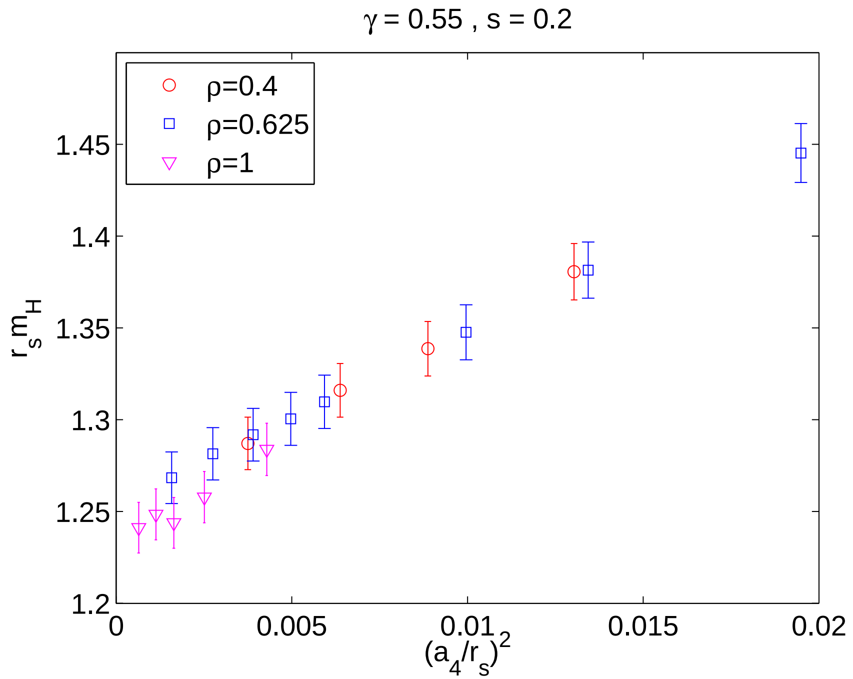
$\nu = 1/4$, the mass does never go to zero

Meanfield results for torus geometry

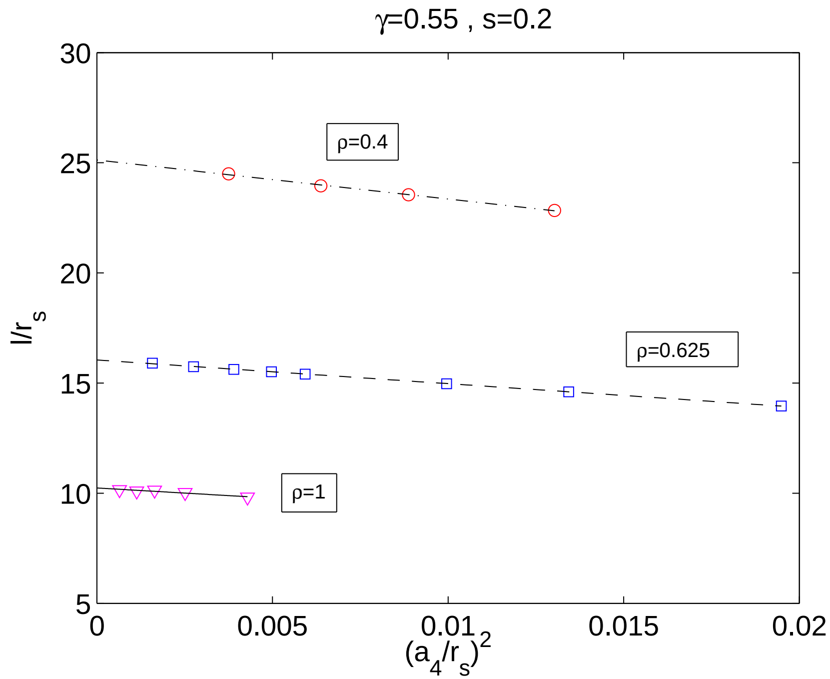
Lines of constant physics [Irges and Knechtli, 2010]

$$(L = L_T = L_5 \longrightarrow \infty, \quad \beta \longrightarrow \beta_c)|_{\gamma,\rho=m_W/m_H} \quad \Longleftrightarrow \quad \text{continuum limit}$$

A physical scale r_s is defined through $r^2 F(r)|_{r=r_s} = s = 0.2$ with $F = V_4'$.
Fixing $\gamma = 0.55$:



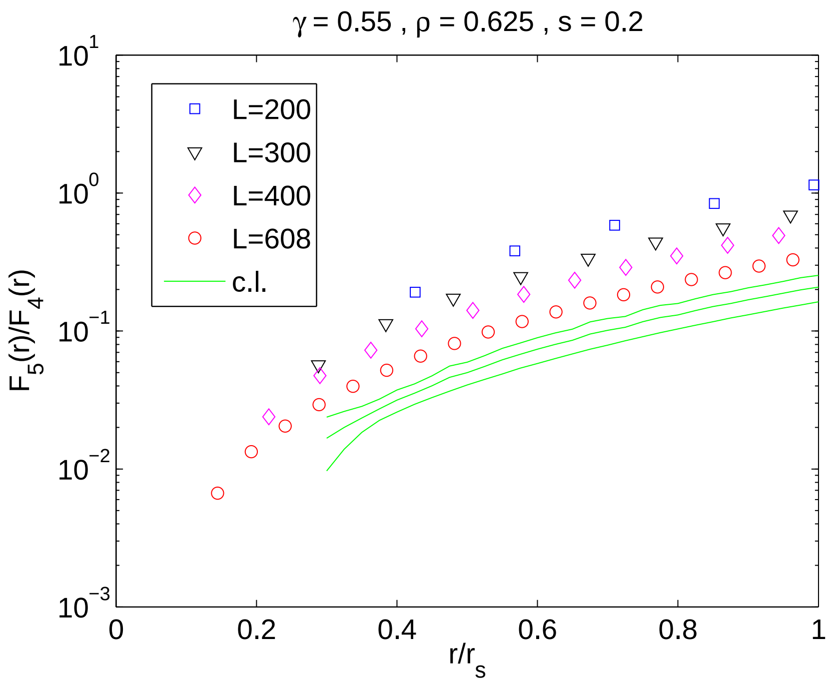
m_H is independent on ρ



ρ determines the physical box size l

Meanfield results for torus geometry

Line of constant physics $\gamma = 0.55$, $\rho = 0.625$: dimensional reduction



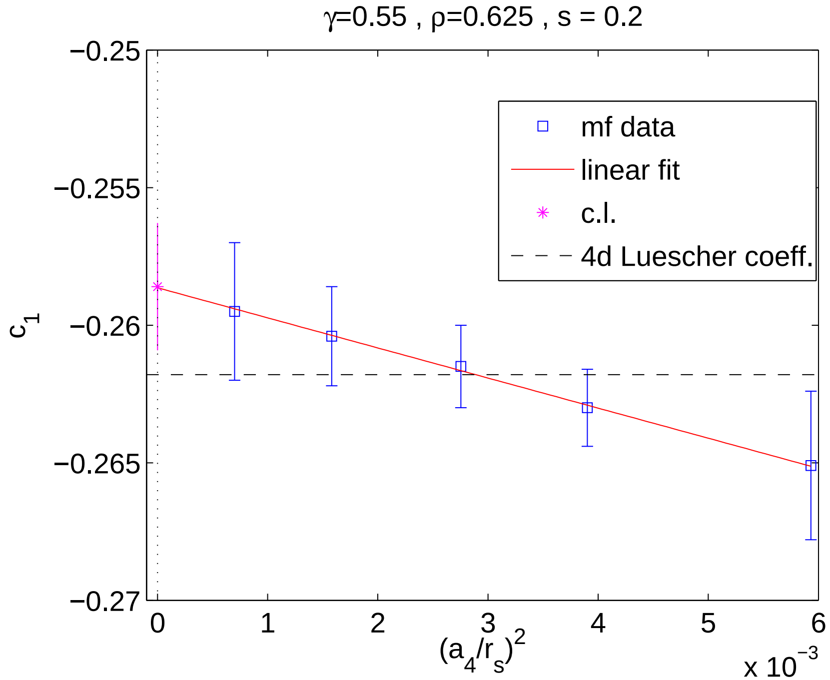
- The force F_4 has a physical nonzero continuum limit
- F_5/F_4 tends to zero in the continuum limit $\Rightarrow$ localization
- Dimensional reduction to 4d Georgi–Glashow model. It must be in the confined phase

Meanfield results for torus geometry

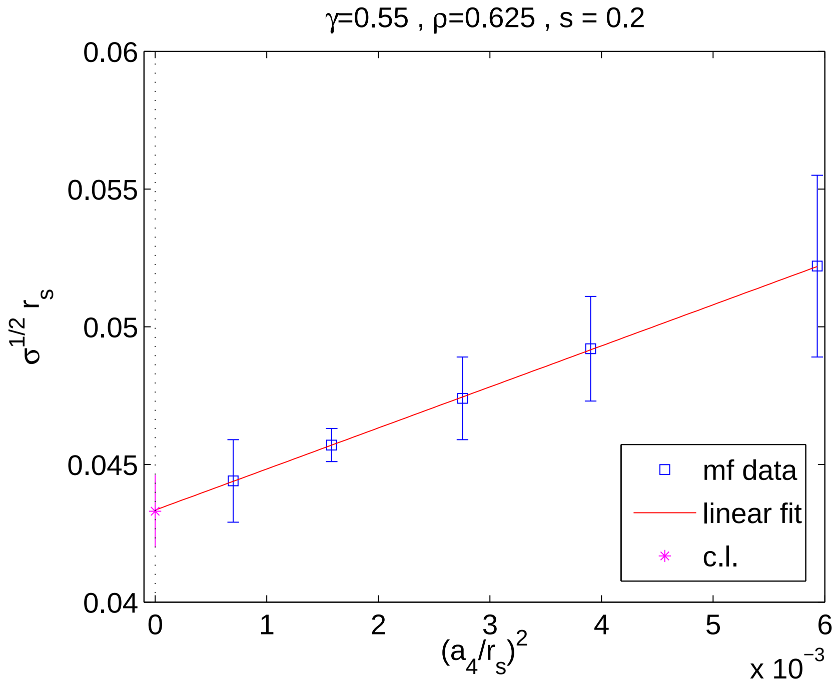
Line of constant physics $\gamma = 0.55$, $\rho = 0.625$: confinement

$$V_4(r) = \mu + \sigma r + c_0 \log(r) + \frac{c_1}{r} + \frac{c_2}{r^2}\,, \quad r/r_s > 1$$

Perform local fits, there are *simultaneous* plateaus for all four coefficients
Continuum limit of plateau values in the range $r/r_s \in [2.15\,,\,2.80]$:



We get the universal 4d value $-\pi/12$!



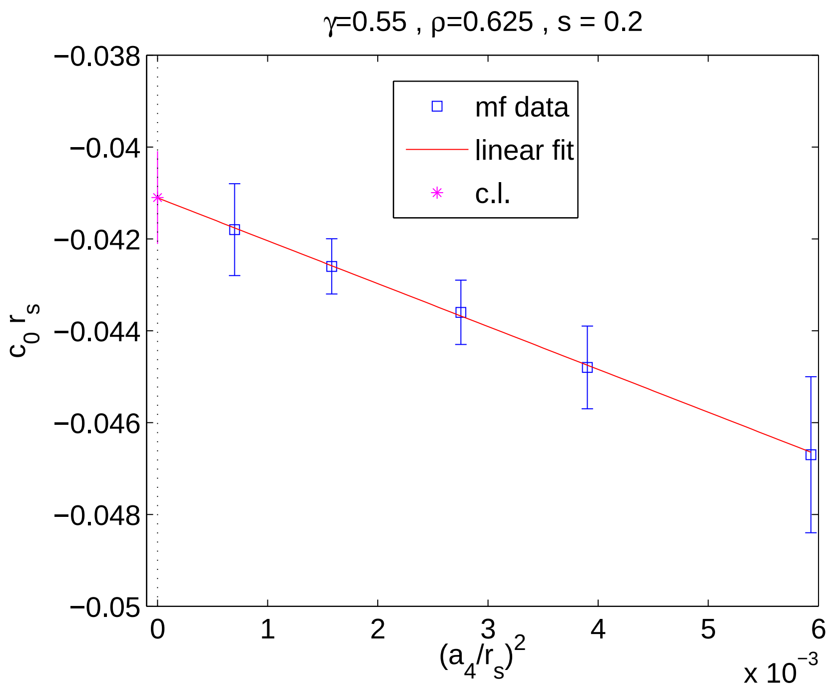
There is a positive string tension

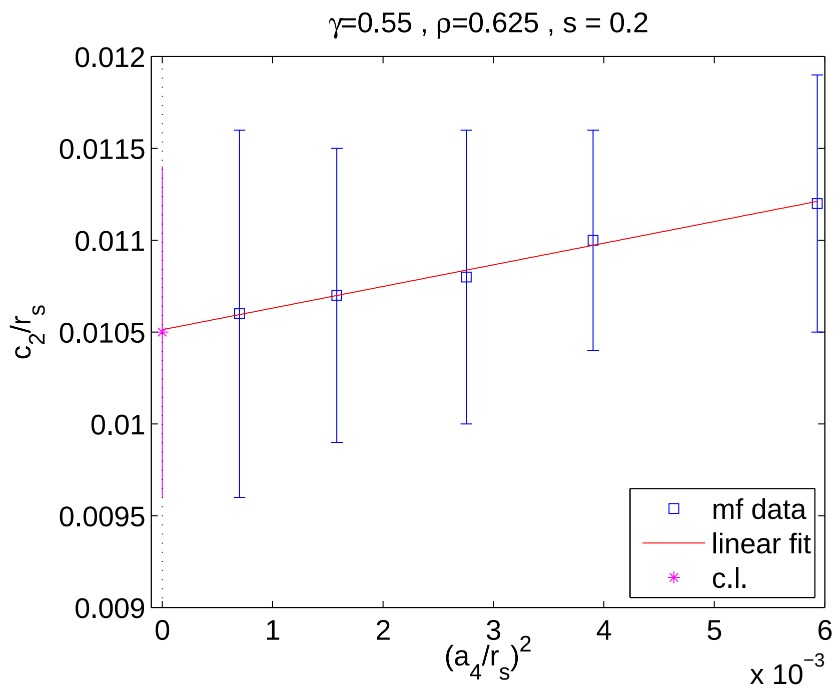
Meanfield results for torus geometry

Line of constant physics $\gamma = 0.55$, $\rho = 0.625$: confinement

$$V_4(r) = \mu + \sigma r + c_0 \log(r) + \frac{c_1}{r} + \frac{c_2}{r^2}\,, \quad r/r_s > 1$$

Continuum limit of plateau values in the range $r/r_s \in [2.15\,,\,2.80]$:

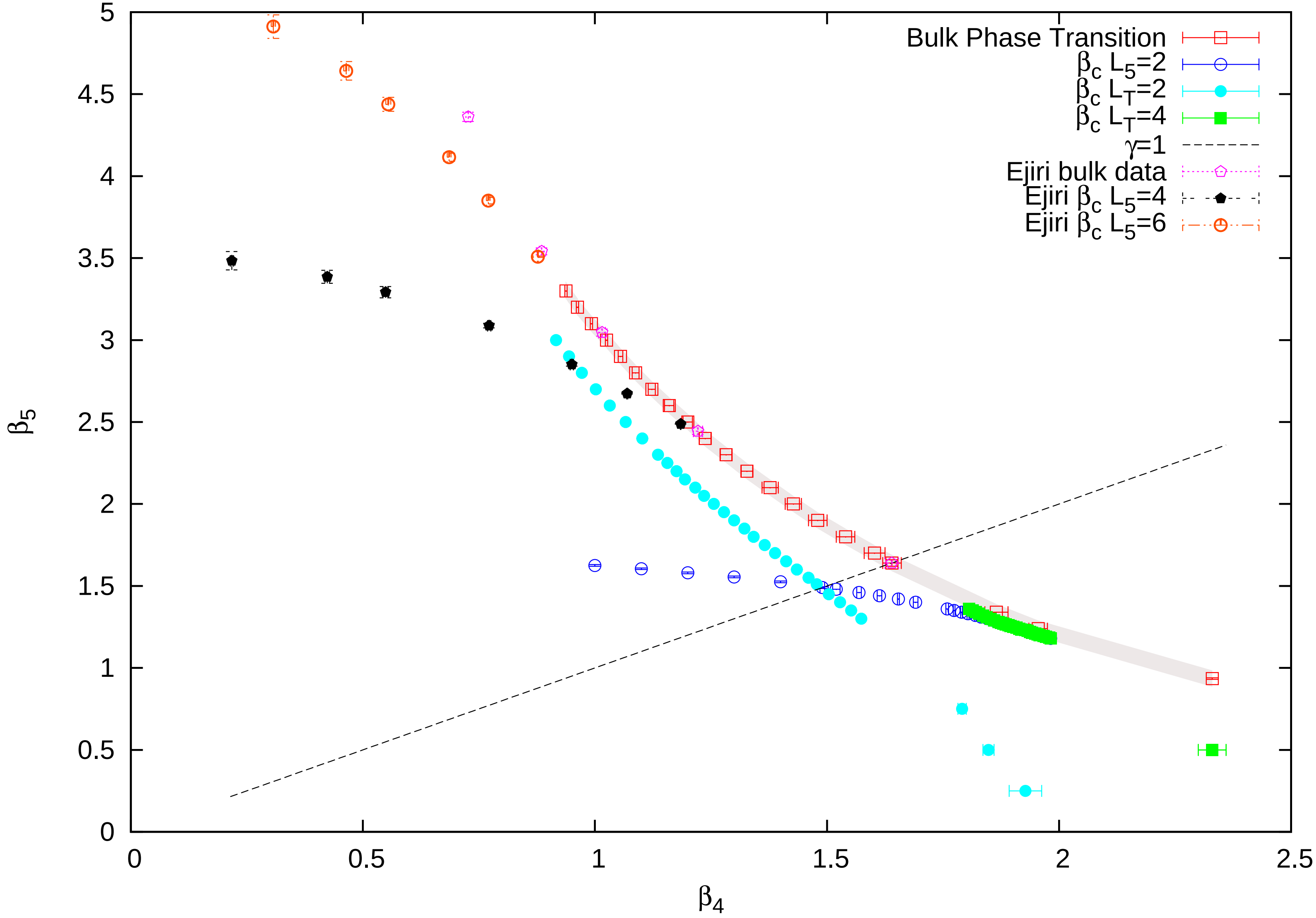




A large negative log term is present (origin?)

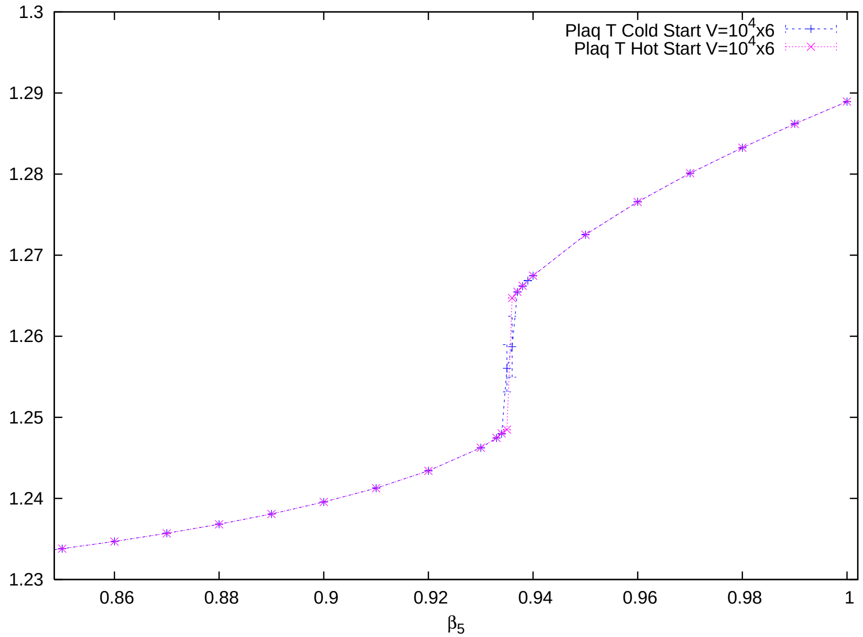
There are also higher order corrections

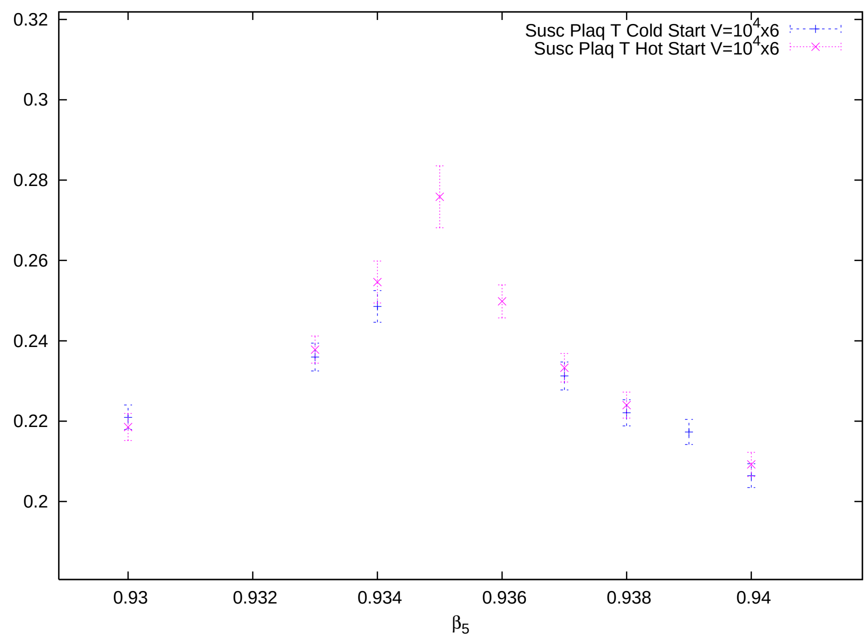
Monte Carlo results for torus geometry: $\beta_4 = \beta/\gamma$, $\beta_5 = \beta\,\gamma$



Monte Carlo results for torus geometry

Bulk phase transition at $\gamma < 1$: $10^4 \times 6$ at $\beta_4 = 2.33$:



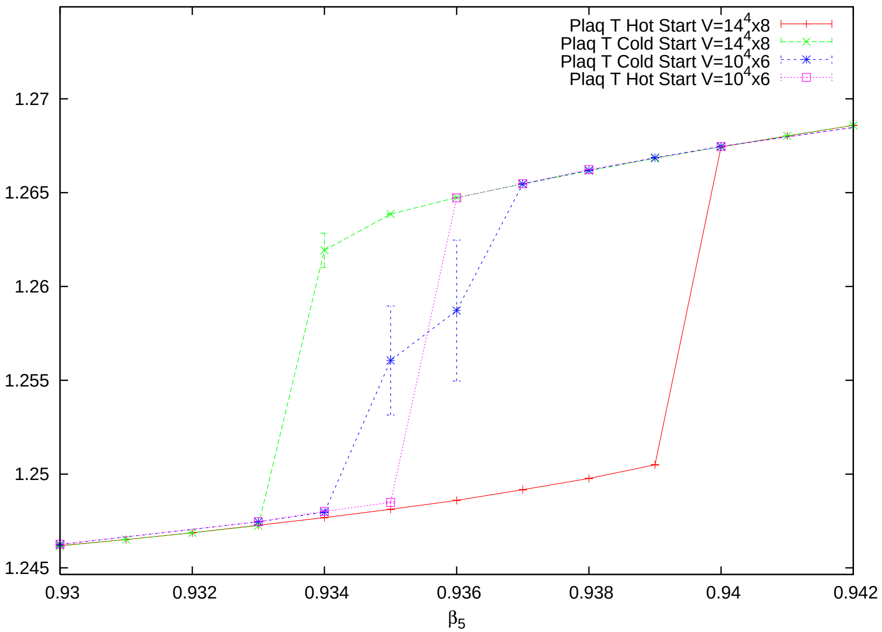


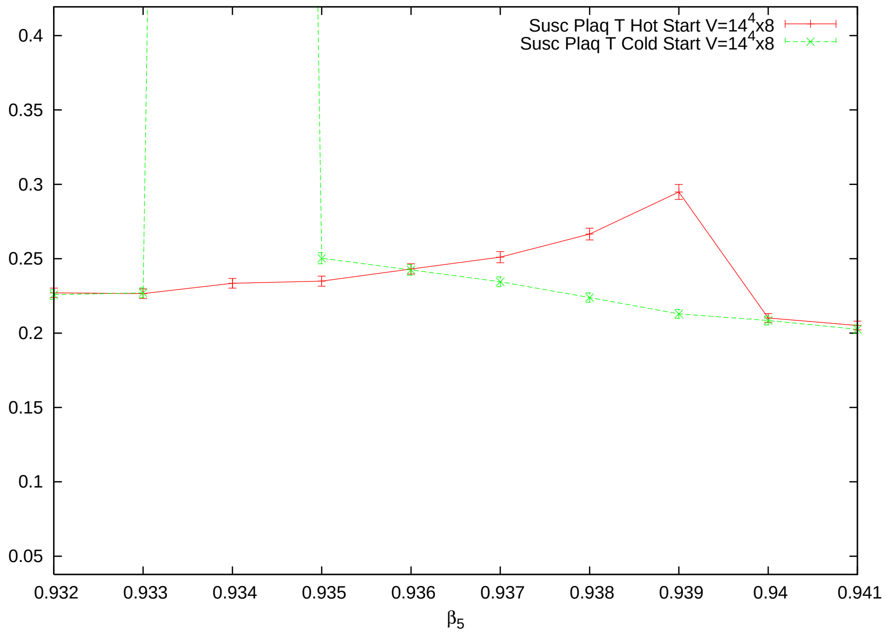
$P = (t\,\mu)$–plaquette: looks like there is no hysteresis . . .

and its susceptibility $\chi_P = L_T L^3 L_5 \langle (P - \langle P \rangle)^2 \rangle$ has one peak
(similar results for the 5μ plaquette)

Monte Carlo results for torus geometry

But: larger volume $14^4 \times 8$ at $\beta_4 = 2.33$:





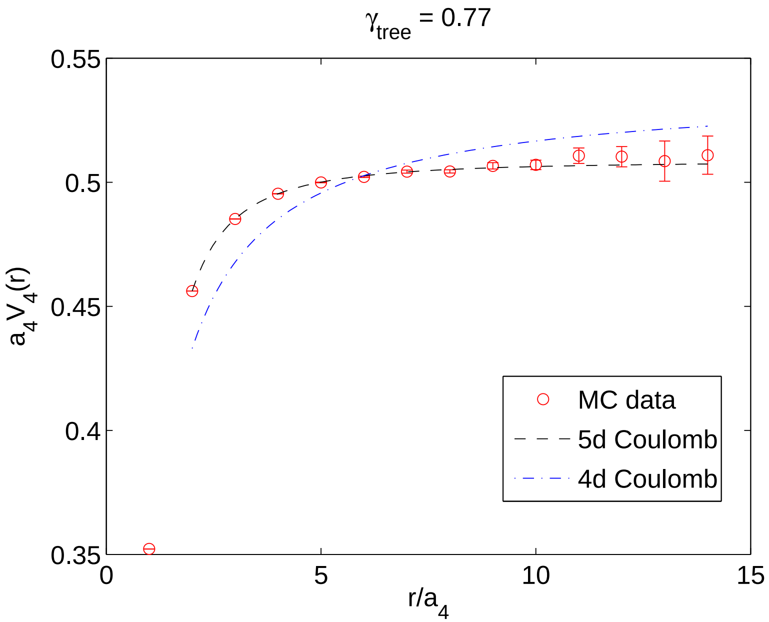
$(t\ \mu)$–plaquette: there is a strong hysteresis . . .

and the susceptibility has a double peak
$\Rightarrow$ first order phase transition
(similar results for the 5μ plaquette)

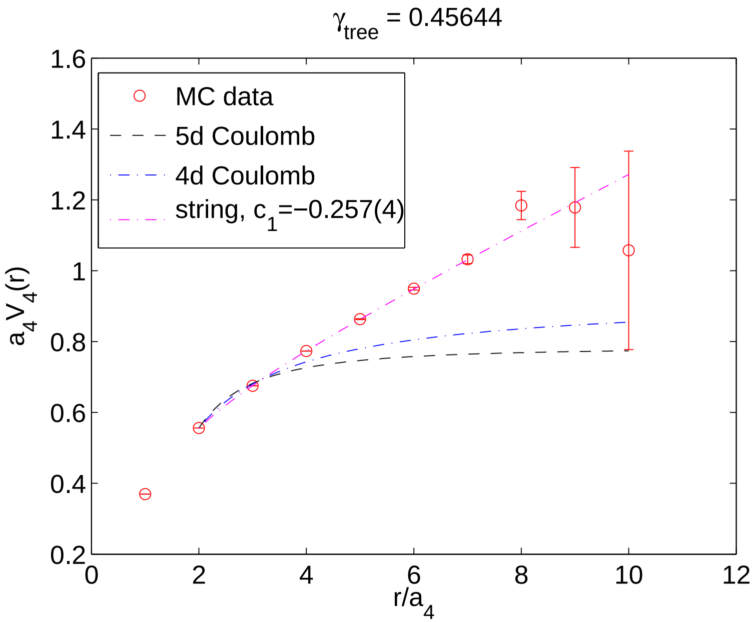
Monte Carlo results for torus geometry

- We confirm [Ejiri, Kubo and Murata, 2000; de Forcrand, Kurkela and Panero, 2010]:
  – in infinite volume there is only a first order bulk phase transition (shaded line);
  – at $\gamma > 1$ there are second order phase transitions due to compactification
- We located second order phase transitions when
  $\gamma < 1$ and $L_T \ll L, L_5$
- We can accurately compute the static potential (using 2 levels of 4d spatial HYP smearing), examples at $\gamma < 1$:
  – $32^4 \times L_5 = 16$ lattice in the deconfined phase at $\beta_5 = 1.24$, $\beta_4 = 2.10$
  – $32^3 \times L_z = 4 \times L_5 = 16$ lattice at $\beta_5 = 0.5$, $\beta_4 = 2.4$ close to second order phase transition

Monte Carlo results for torus geometry



Five-dimensional Coulomb phase

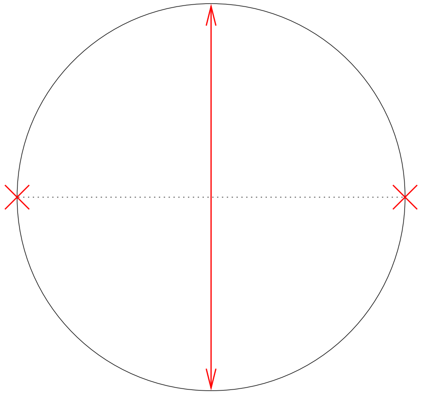


Four-dimensional confined phase

The fitted coefficient c_1 of the $1/r$ term agrees with $-\pi/12 = -0.2618$ of the 4d universal Lüscher term

The orbifold case

Orbifold $S^1/\mathbb{Z}_2$



S^1 : $x_5 \in (-\pi R, \pi R]$; Reflection

$$\mathcal{R}: \; z = (x_\mu, x_5) \;\; \longrightarrow \;\; \bar{z} = (x_\mu, -x_5)$$
$$A_M(z) \;\; \longrightarrow \;\; \alpha_M A_M(\bar{z}) \, , \quad \alpha_\mu = 1 \, , \; \alpha_5 = -1$$

Fixed points $z = \bar{z} \Leftrightarrow x_5 = 0$ and $x_5 = \pi R$ define 4d boundaries

$\mathbb{Z}_2$ projection for gauge fields

$$\begin{aligned} \mathcal{R}\, A_M &= g\, A_M\, g^{-1} \, , \qquad g^2 \in \text{centre of } SU(N) \\ \mathcal{R}\, \partial_5 A_M &= g\, \partial_5 A_M\, g^{-1} \\ &\vdots \end{aligned}$$

Parities of $SU(N)$ generators

$$g\, T^a\, g^{-1} \; = \; T^a \text{ (unbroken)} \, , \quad g\, T^{\hat{a}}\, g^{-1} \; = \; -T^{\hat{a}} \text{ (broken)}$$

The orbifold case

Dirichlet boundary conditions at $z = \bar{z}$

$$A_\mu = g\, A_\mu\, g^{-1} \qquad \text{and} \qquad A_5 = -g\, A_5\, g^{-1}$$

$\Rightarrow$ Only *even* components A_μ^a and $A_5^{\hat{a}}$ are $\neq 0$: breaking of the gauge symmetry

$$G = SU(p+q) \xrightarrow{\mathbb{Z}_2} \mathcal{H} = SU(p) \times SU(q) \times U(1)$$

depending on the choice of g

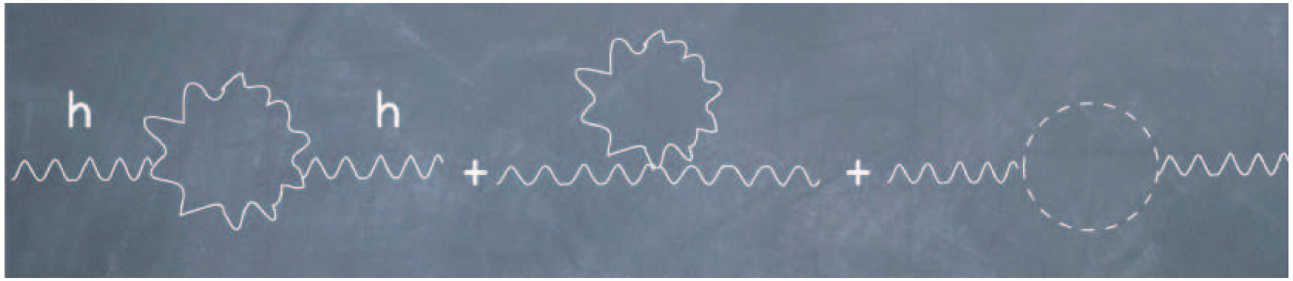
❍ $SU(2) \xrightarrow{\mathbb{Z}_2} U(1)$ with $g = \mathrm{diag}(-i, i)$: even fields
A_μ^3: "photon/Z"
$A_5^{1,2}$: complex "Higgs"

❍ $SU(3) \xrightarrow{\mathbb{Z}_2} SU(2) \times U(1)$ with $g = \mathrm{diag}(-1, -1, 1)$: even fields
$A_\mu^{1,2,3,8}$: "photon,Z and $W^\pm$"
$A_5^{4,5,6,7}$: doublet of complex "Higgs" in the fundamental representation of $SU(2)$

The orbifold case

Mass of the Higgs zero mode $h = A_5^{\hat{a},(0)}$

- zero at tree level (5d gauge invariance)
- 1-loop vacuum polarization
  [von Gersdorff, Irges and Quiros, 2002 and 2003; Cheng, Matchev and Schmaltz, 2002; Del Debbio, Kerrane and Russo, 2009 (S^1)]



$$(m_H R)^2 = \frac{9N\zeta(3)}{32\pi^4} g_4^2 \,, \quad g_4^2 = \frac{g_5^2}{2\pi R}$$

- finite (bulk) mass!
- logarithmic (bulk-boundary) corrections from 2-loops
  [von Gersdorff and Hebecker, 2005]

The orbifold case

Hosotani mechanism [Hosotani, 1983; 1989]

$$\alpha \;=\; g_5 \langle A_5^1 \rangle R$$

α is determined by dynamics:

$$SU(N) \stackrel{\mathbb{Z}_2}{\longrightarrow} \mathcal{H} \stackrel{SSB}{\longrightarrow} ?$$

- KK masses for $SU(2)$ [Kubo, Lim and Yamashita, 2002]

$$\begin{aligned} A_\mu^{3,(0)}\ (Z): &\qquad (m_Z R)^2 = \alpha^2 \\ A_5^{1,2(0)}\ (\mathrm{Higgs}): &\qquad (m_{A_5} R)^2 = \alpha^2\,,\ 0 \\ \text{higher KK modes}: &\qquad (m_n R)^2 = n^2\,,\ (n \pm \alpha)^2 \end{aligned}$$

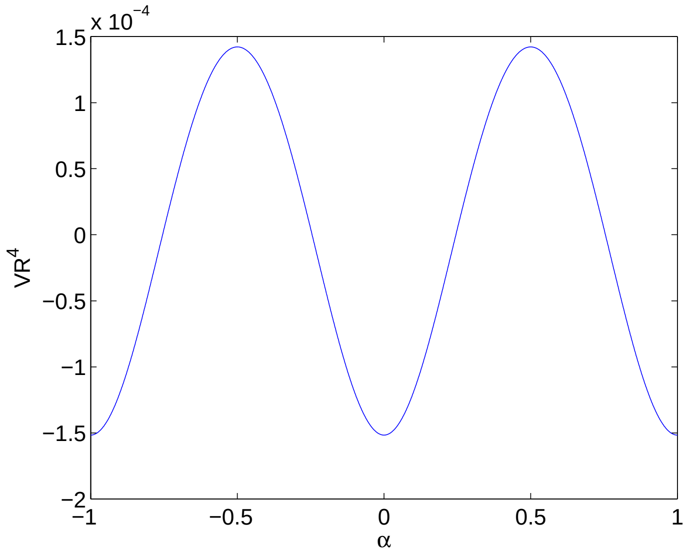
- 1-loop Coleman–Weinberg (CW) scalar potential V

$$\int [\mathrm{D}\phi] \mathrm{e}^{-S_\mathrm{E}} \sim \mathrm{e}^{-V} \;\equiv\; \frac{1}{\sqrt{\det[-\partial_\mu \partial_\mu + M^2]}}$$

- Take $D = 4$, use KK masses m_n and Poisson resummation

The orbifold case

$$V = -\frac{9}{64\pi^6 R^4} \sum_{m=1}^{\infty} \frac{\cos(2\pi m\alpha)}{m^5}$$



Minima at $\alpha = \alpha_{\min} = 0 \bmod \mathbb{Z} \Rightarrow \mathcal{H} = U(1)$ unbroken

Next step in perturbation theory: introduce fermions to get SSB. We go on the lattice . . .

The orbifold case

Lattice action: $SU(2)$, $g = -i\sigma^3$, $L_T \times L^3 \times (L_5 + 1)$ lattice

$$S_W^{\mathrm{orb.}} = \frac{\beta}{4}\Big[\frac{1}{\gamma}\sum_{\mathrm{4d-p}} w(p)\mathrm{tr}\Big(1 - U_p\Big) + \gamma \sum_{\mathrm{5d-p}} \mathrm{tr}\Big(1 - U_p\Big)\Big]$$

$$w(p) = \begin{cases} \frac{1}{2} & p \text{ in the boundary} \\ 1 & \text{in all other cases.} \end{cases}$$

- Periodic boundary conditions (b.c.) in 4d. In the 5th dimension, only Dirichlet b.c.

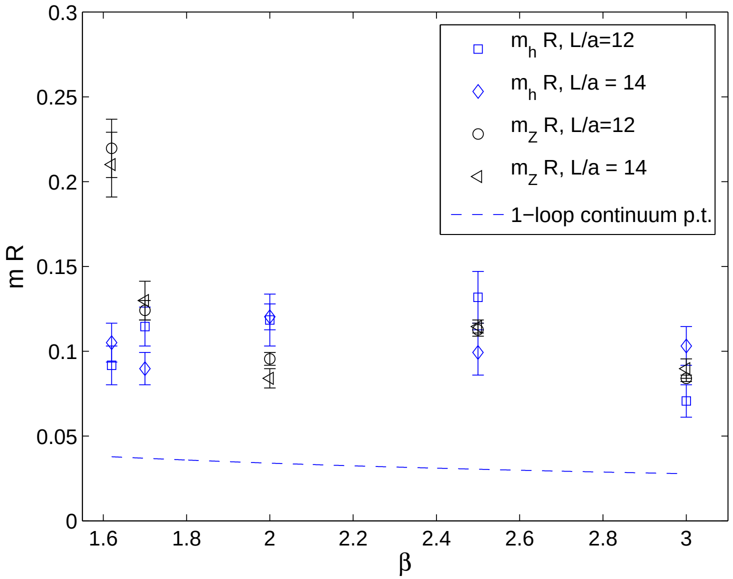
$$A_\mu = g\, A_\mu\, g^{-1} \quad \longrightarrow \quad U(z,\mu) = g\, U(z,\mu)\, g^{-1} \qquad \text{at } n_5 = 0\,,\ L_5$$

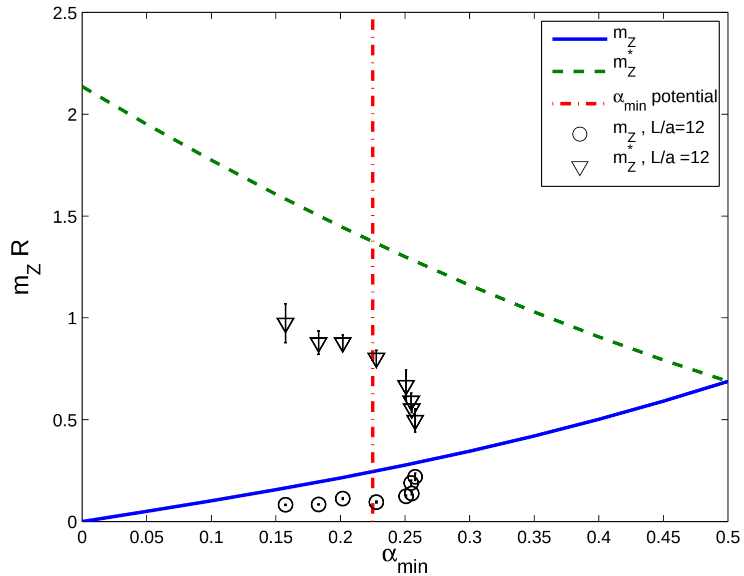
- No boundary counterterm $\mathrm{tr}\{[A_5, g]^2\}$ [von Gersdorff, Irges and Quiros, 2003; Irges and F.K., 2005]
- Meanfield: background

$$v(n,\mu) = \overline{v}_0(n_5)\mathbf{1}\,, \qquad v(n,5) = \overline{v}_0(n_5 + 1/2)\mathbf{1}$$

Twisted orbifold: vev for $v^1(n,5)$ is equivalent to $S^1/(\mathbf{Z}_2 \times \mathbf{Z}_2')$ orbifold on circle of radius $2R$ [Scrucca, Serone and Silvestrini, 2003]

The orbifold case



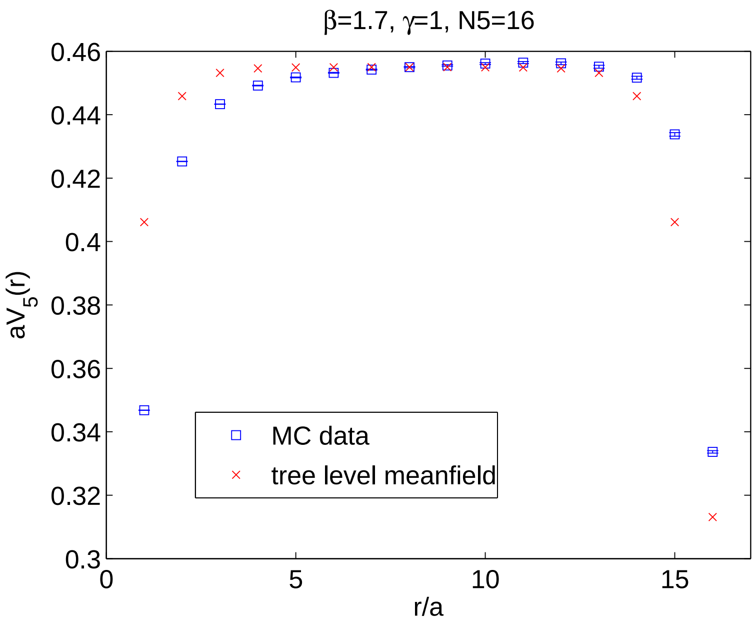


– Higgs mass is finite close to 1-loop value
– Z-boson is massive: there is spontaneous symmetry breaking (SSB)
[Irges and F.K., 2007]

Contradiction with perturbation theory (no SSB) is resolved if cut-off effects are included in the Coleman–Weinberg potential calculation
[Irges, F.K. and Luz, 2007]

The orbifold case

Meanfield computation of the potential along the extra dimension



$$a_4 V_5(r) = -\ln\left[\overline{v}_0(0)\overline{v}_0(n_5)\right]$$
$$n_5 = r/a_5$$

- potential barrier at tree level
- good agreement with Monte Carlo
- compute corrections in the meanfield, study behavior of the barrier as L_5 grows, localization?

Outlook

Meanfield:

- Convergence of the meanfield expansion: second order correction to the Higgs mass (ongoing)
- Spontaneous symmetry breaking in the meanfield laboratory (ongoing)

Monte Carlo:

- Map of the phase diagram on the torus, order of the phase transition and dimensional reduction for $\gamma < 1$ (ongoing)
- Spectrum, orbifold boundary conditions, $SU(3)$

... in order to be ready with predictions when first LHC results will come!

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