Center clusters in QCD and their percolation at the deconfinement transition

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Motivation and basic facts

Center symmetry of gluodynamics

- In pure gauge theory we can identify the deconfinement transition using the Polyakov loop:

$$\text{Local loop:} \qquad L(\mathbf{x}) \;=\; \mathrm{Tr} \prod_t U_4(\mathbf{x}, t)$$

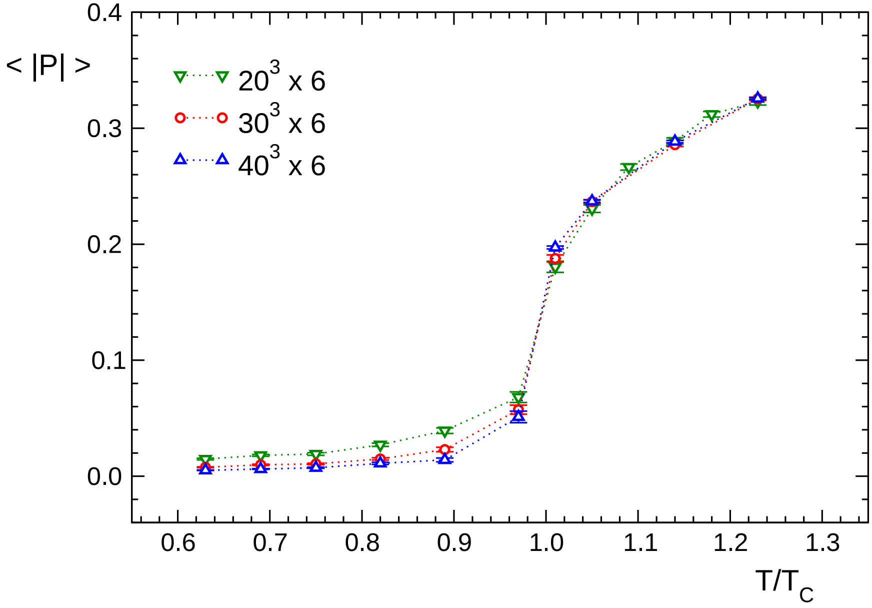
$$\text{Averaged loop:} \qquad P \;=\; \frac{1}{V} \sum_{\mathbf{x}} L(\mathbf{x})$$

- The Polyakov loop tests for center symmetry ($z \in \mathbb{Z}_3$):

$$L(\mathbf{x}) \;\longrightarrow\; z\, L(\mathbf{x}) \quad , \quad P \;\longrightarrow\; z\, P \;.$$

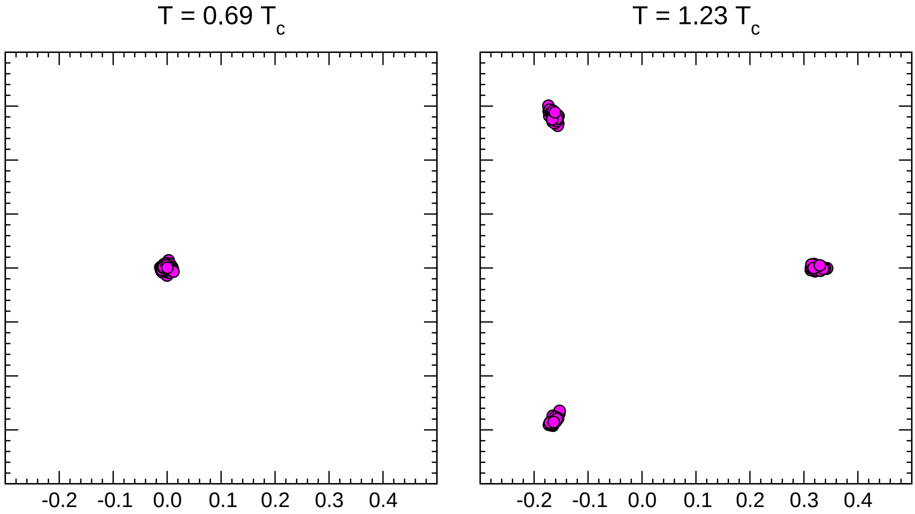
- A non-vanishing expectation value $\langle P \rangle \;=\; \langle L(\mathbf{x}) \rangle \neq 0$ signals that the center symmetry is broken spontaneously.

Polyakov loop as function of temperature for SU(3) gauge theory



Spontaneous breaking of center symmetry

- The spontaneous breaking of center symmetry may lead to any of the three center sectors, as seen in scatter plots of P:



Svetitsky-Yaffe conjecture

An influential idea for understanding the phase transition of pure gluodynamics is the Svetistky-Yaffe conjecture (1981):

- At T_c the critical behavior of SU(N) gauge theory in $d+1$ dimensions can be decribed by a d - dimensional spin system with a $\mathbb{Z}_N$ - invariant effective action. The spins are related to the local loops $L(\mathbf{x})$.

- Leading term of the effective action ($+$ term for determinant):

$$S[L] \; = \; -\,\tau \sum_{\langle \mathbf{x},\mathbf{y} \rangle} \Big[\, L(\mathbf{x}) L(\mathbf{y})^\star \, + \, L(\mathbf{y}) L(\mathbf{x})^\star \, \Big] \; - \; \kappa \sum_{\mathbf{x}} \Big[\, L(\mathbf{x}) \, + \, L(\mathbf{x})^\star \, \Big]$$

Properties of spin systems in QCD?

- The critical behavior of spins systems is well understood.
- At T_c a spin system describes the critical behavior of QCD.
- Can we identify characteristic properties of spin systems directly in QCD?
- Are these properties important only at T_c, or is there a range of temperatures where they play a role?
- Here we focus on clusters with coherent spin values and their percolation properties near T_c.
- Previous studies for SU(2): S. Fortunato, H. Satz, ...
- First results for SU(3): C. Gattringer, Phys. Lett. B 690 (2010) 179
  S. Borsanyi et al, arXiv:1007.5403

Clusters and percolation in spin systems

- For many spin systems clusters of coherent spins can be defined which percolate at the temperature of the magnetic transition.
- These clusters do not simply bind parallel spins, but have a more complicated structure (Fortuin-Kasteleyn clusters).
- For the example of the Ising model two parallel spins are linked only with probability

$$p_{FK} = 1 - e^{-2\beta}$$

- As an alternative strategy the linking probability p has been considered as a free parameter and properties of the corresponding clusters were studied.

Local properties of the Polyakov loop

Setting of our analysis

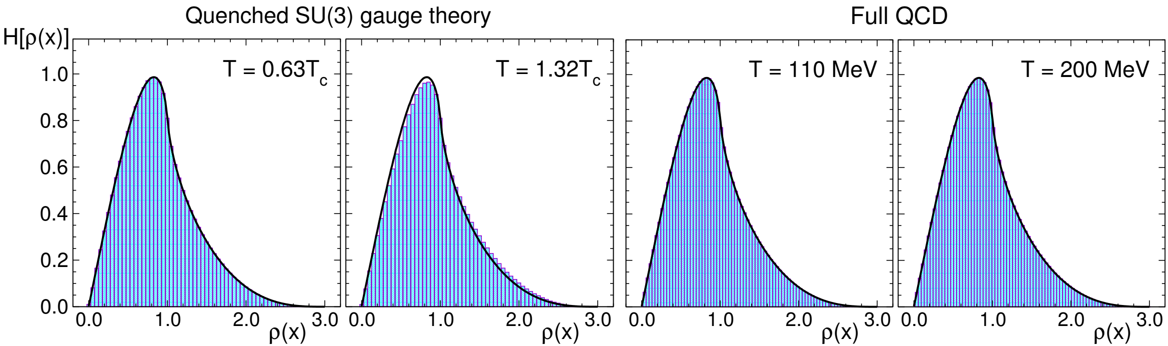
- We study clusters and critical percolation directly in SU(2) and SU(3) lattice gauge theory, and in full QCD with dynamical fermions.

- Technicalities:
  - SU(2), SU(3): Lüscher-Weisz gauge action with lattice sizes $20^3 \times 6$ ... $40^3 \times 10$ and temperatures $T \in [\,0.63\,T_c\,,\,1.32\,T_c\,]$.
  - Full QCD: 2+1 flavors of staggered fermions with lattice sizes $18^3 \times 6$, $36^3 \times 6$, $24^3 \times 8$ and temperatures from 110 - 320 MeV.

- We write the local loops $L(\mathbf{x})$ as

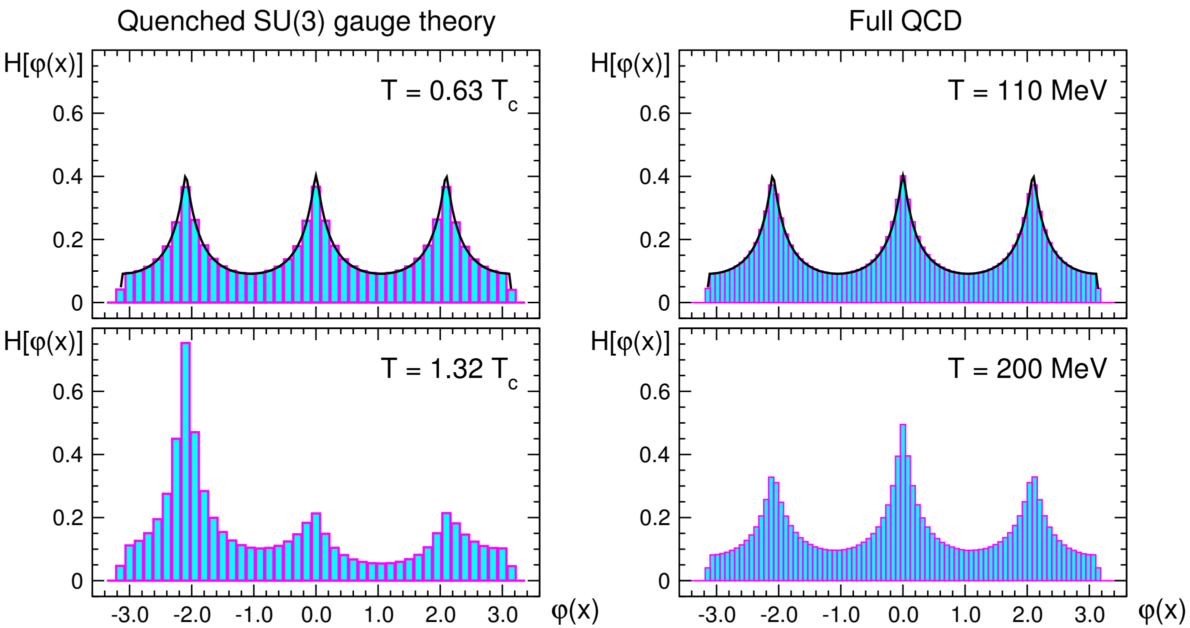
$$L(\mathbf{x}) \;=\; \rho(\mathbf{x})\, \mathbf{e}^{\,\mathbf{i}\,\varphi(\mathbf{x})}$$

Histograms for the modulus of the local loops



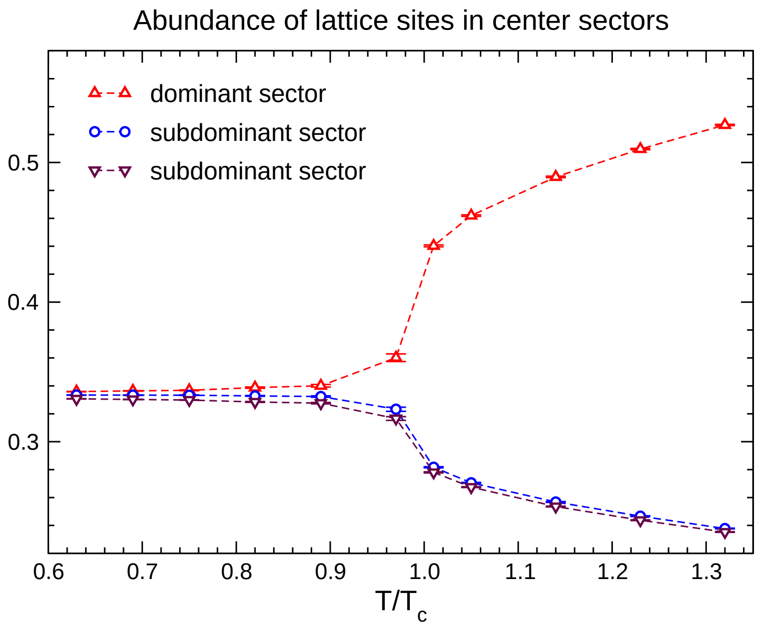
The values $\rho(\mathbf{x}) \equiv |\, L(\mathbf{x}) \,|$ are distributed according to Haar measure. The distribution is insensitive to temperature, lattice spacing and quenching.

Histograms for the phase $\varphi(\mathbf{x})$ of the local loops



Haar measure distribution below T_c. Enhancement of one sector above T_c.

Center sectors across the phase transition, SU(3) gauge theory

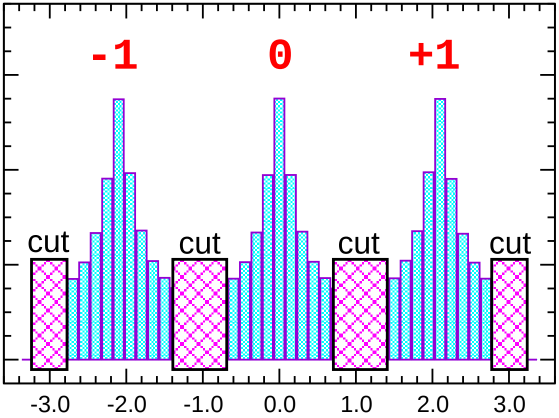


Non-vanishing $\langle P \rangle$ is driven by increasing population in one of the sectors.

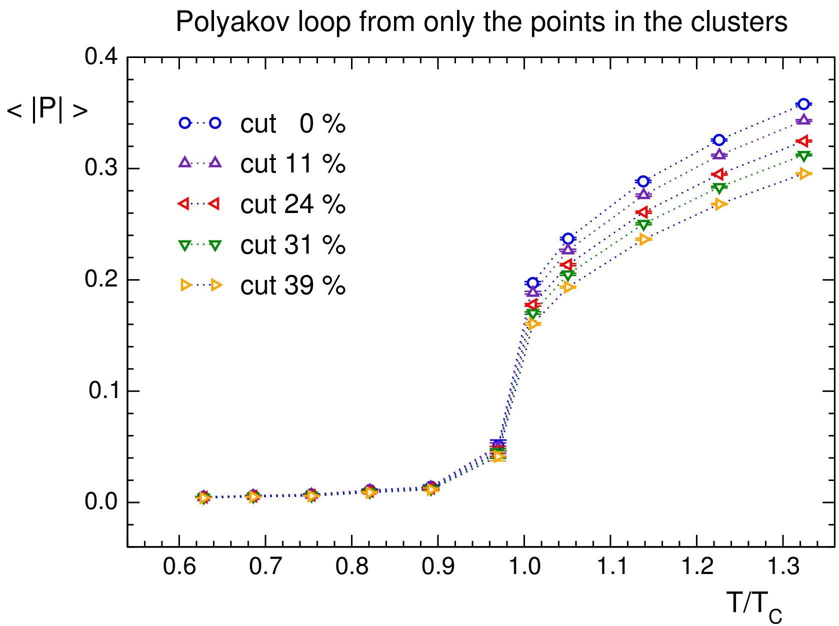
Cluster- and percolation properties of center domains

Center domains of the local Polyakov loop

- The behavior of the Polyakov loop P across the phase transition is driven by the dynamics of the phases of the local loops $L(\mathbf{x})$.
- We assign sector numbers `-1, 0, +1` to the three sectors and study properties of the corresponding clusters.
- We study the effect of a cut on the fluctuations.

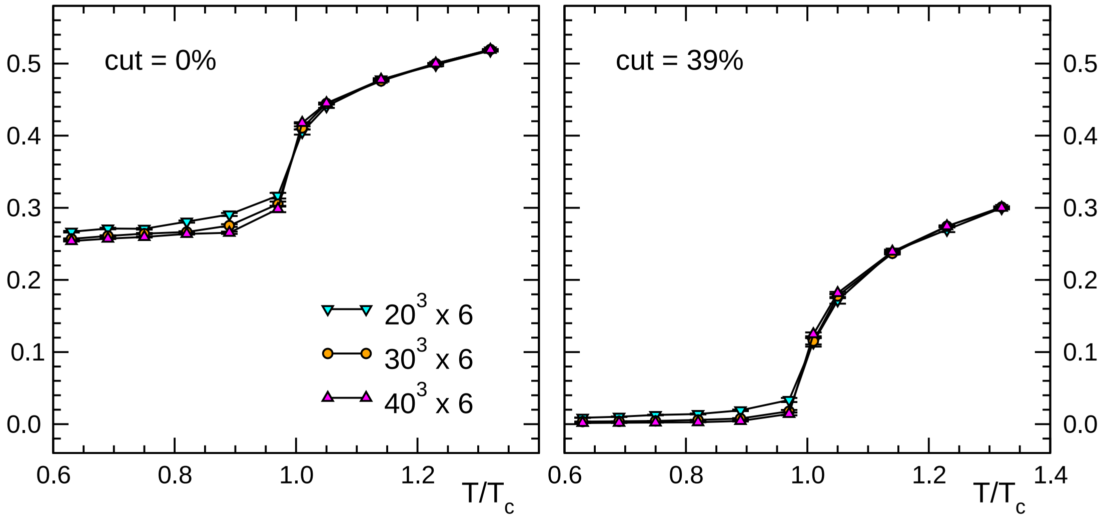


Cut leaves behavior of Polyakov loop essentially unchanged



Behavior of the largest cluster

Size of largest cluster normalized with the volume



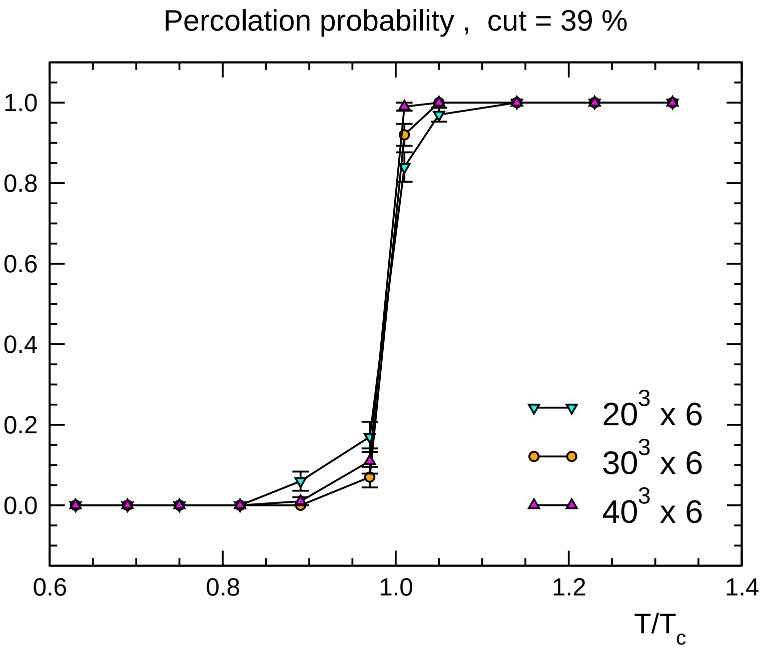
Aspects of the percolation phenomenon

- In 3 dimensions the critical occupation probability for site percolation is $p_c = 0.3116$.

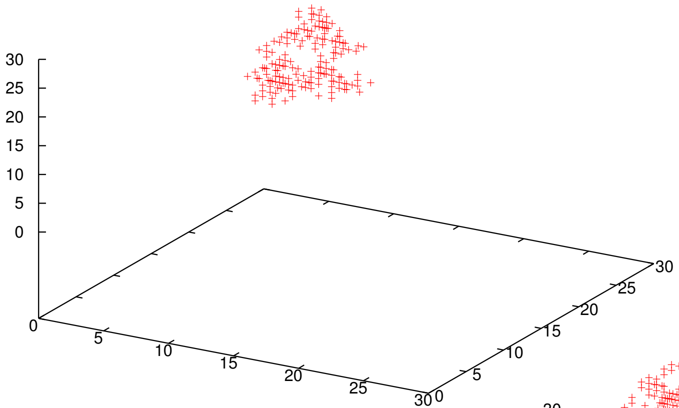
- With a naive cluster definition we always encounter a percolating cluster.

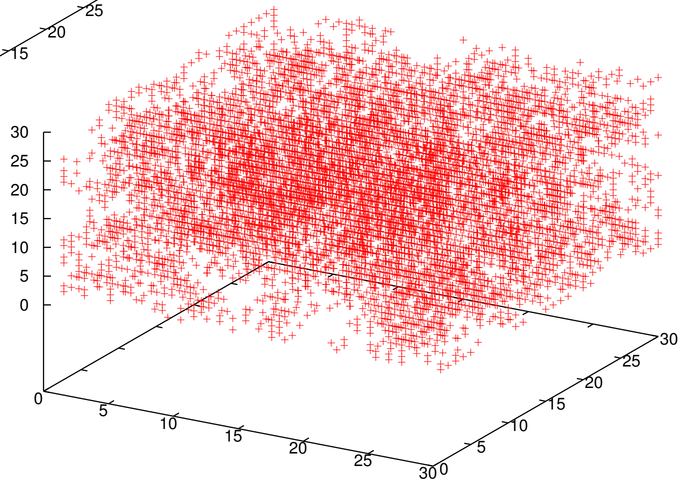
- For suitably constructed clusters the deconfinement transition may be characterized by the onset of percolation.

$Percolation\ at\ T_c$



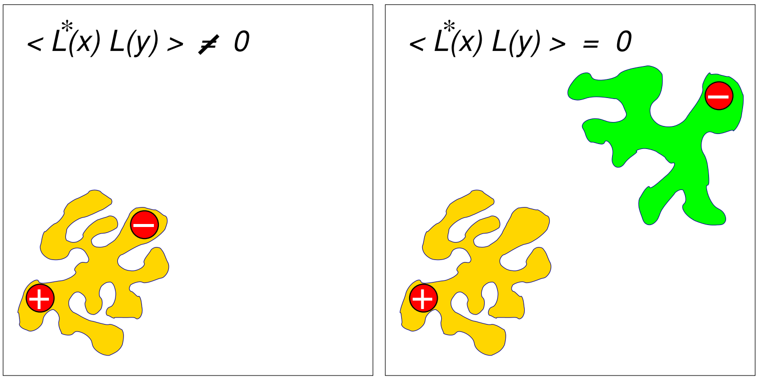
Clusters below and above T_c



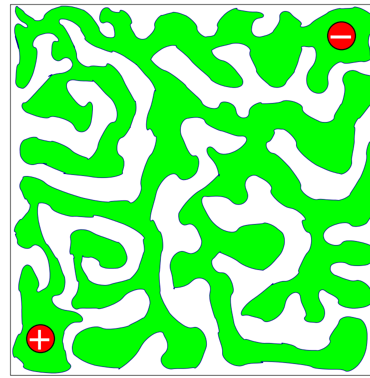


A geometrical picture for confinement and the deconfinement transition

$T < T_c$



$T > T_c$



Below T_c two static sources (= local loops) have a non–vanishing expectation value only if they fit into the same cluster, such that the phases cancel.

When the clusters percolate the sources can be put at arbitrary distances.

Physical size and scaling of the clusters

Diameter of the clusters in physical units

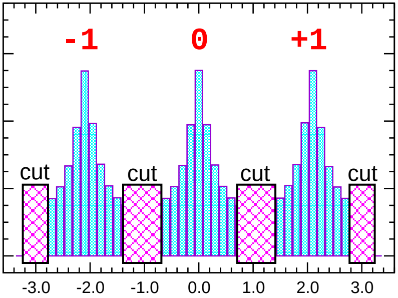
- We consider 2-point functions $C(r)$ within the clusters and define a diameter d through the exponential decay of $C(r)$

$$C(r) \sim \exp(-2r/d)$$

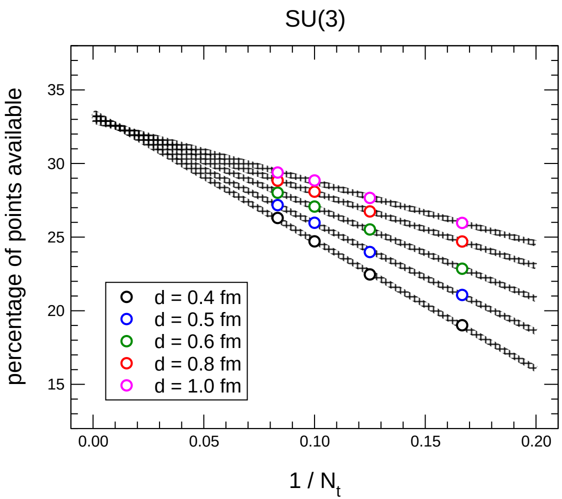
- The average diameter of the clusters depends on the cut we impose.

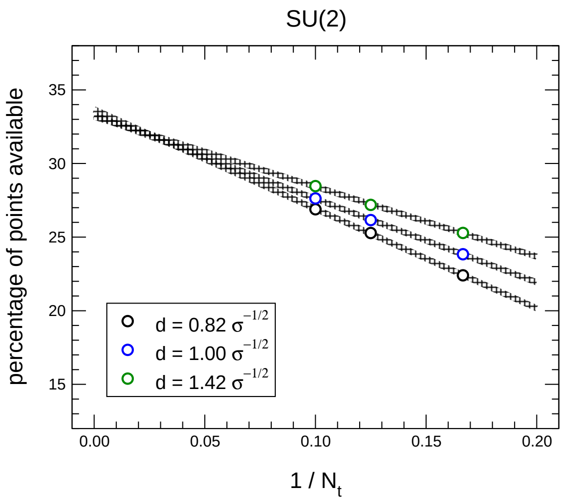
- We adjust the cut such that at our lowest temperature, $T = 0.63T_c$, the diameter is a fixed number in physical units, e.g., $d = 0.5 fm$.

- The procedure is implemented on lattices with different lattice constant a and we compare the flow of the cut.



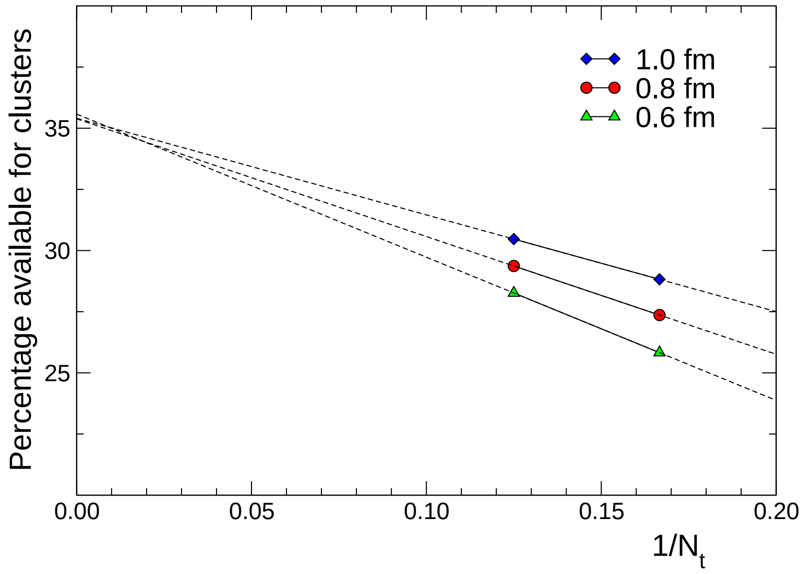
Fraction of points available for clusters per sector at $0.63T_c$





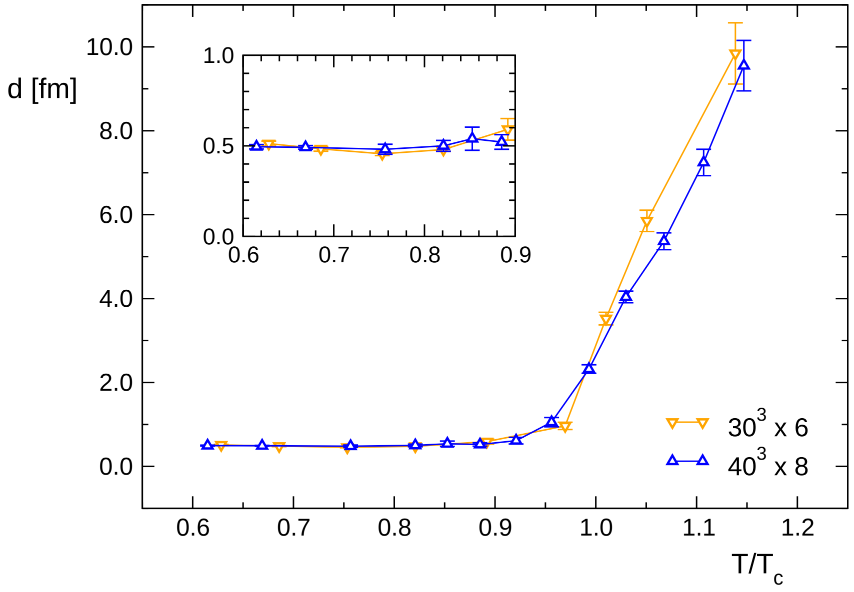
In the continuum limit the number of available lattice points is above the percolation threshold. Thus for the clusters a continuum limit seems possible.

Fraction of points available for clusters per sector for full QCD

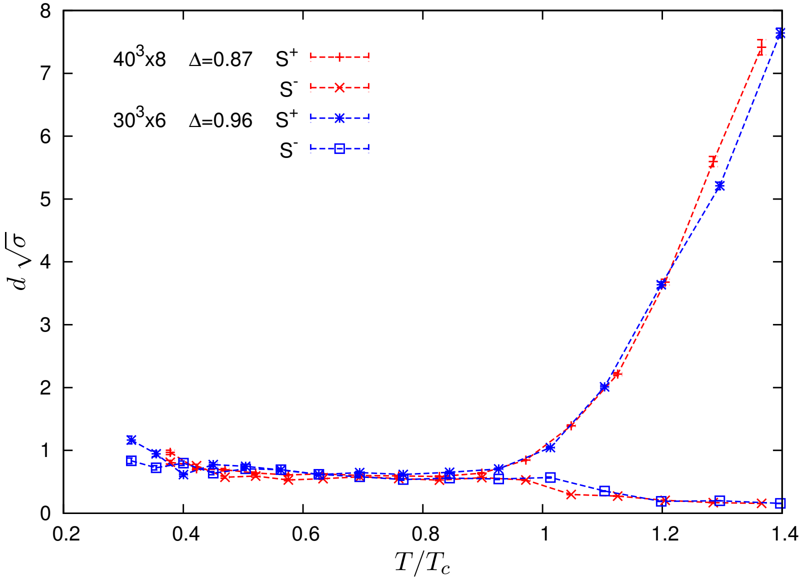


Preliminary !

Cluster size in physical units, SU(3) gauge theory



Cluster size in physical units, SU(2) gauge theory



Summary

- We study the role of center symmetry in the deconfinement transition by analyzing local Polyakov loops.

- The phases of the local loops have preferred values near all center angles $0, \pm i2\pi/3$.

- The phases form spatially localized clusters.

- For pure gauge theory one of the sectors starts to dominate at the phase transition and the corresponding clusters show percolation.

- Our cluster definition allows for a continuum limit and we find universal curves for the physical cluster diameter (pure gauge theory so far).