Lattice QCD+QED simulation

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[T.Blum, T.Doi, M.Hayakawa, T.Izubuchi, S.Uno, N.Yamada and R.Zhou] ,
"Electromagnetic mass splittings of the low lying hadrons and quark masses from 2+1 flavor lattice QCD+QED", `arXiv:1006.1311`, (83 pages).

[R. Zhou, S. Uno] ,
"Isospin breaking in 2+1 flavor QCD+QED", PoS(LATTICE 2009) 182 (7pages).

[R. Zhou, T.Blum, T.Doi, M.Hayakawa, T.Izubuchi, and N.Yamada] ,
"Isospin symmetry breaking effects in the pion and nucleon masses" PoS(LATTICE 2008) 131.

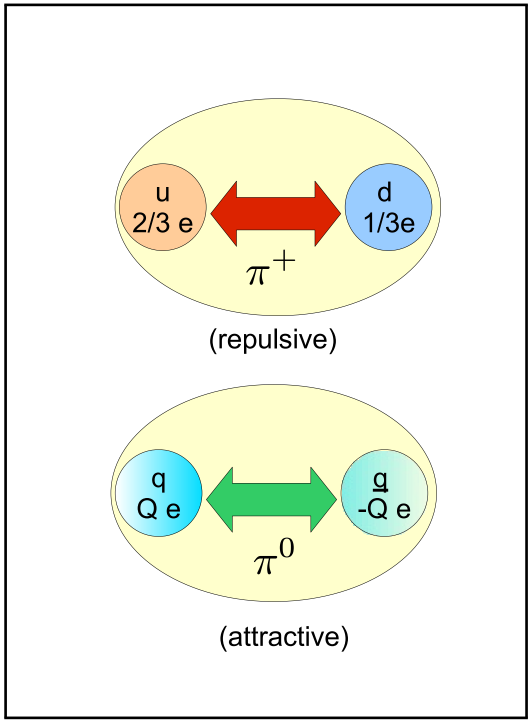
[T. Blum, T. Doi, M. Hayakawa, TI, N. Yamada] ,
"Determination of light quark masses from the electromagnetic splitting of psedoscalar meson masses computed with two flavors of domain wall fermions"
Phys. Rev.D76 (2007) 114508 (38 pages).

"The isospin breaking effect on baryons with Nf=2 domain wall fermions"
PoS(LAT2006) 174 (7 pages).

"Electromagnetic properties of hadrons with two flavors of dynamical domain wall fermions".
PoS(LAT2005) 092 (7 pages).

"Hadronic light-by light scattering contribution to the muon g-2 from lattice QCD: Methodology"
PoS(LAT2005) 353 (7 pages).

Physical Impact of Isospin Breakings



- The effect of isospin breaking due to electromagnetic (EM) and the up, down quark mass difference has phenomenological impacts for accurate hadron spectrum, quark mass determination.

- Isospin breaking's are measured very accurately :

$$m_{\pi^\pm} - m_{\pi^0} = 4.5936(5)\text{MeV},$$

$$m_N - m_P = 1.2933321(4)\text{MeV}$$

- The positive mass difference between Neutron (udd) and Proton (uud) stabilizes proton thus make our world as it is.

Physical Impact of Isospin Breakings (contd.)

[discussion with A.Juttner, C.Sachrajda, G. Colangelo, L. Lellouch]

- f_K/f_π is getting very precise:

  $f_K/f_\pi = 1.193(6)$ [0.5%] [WA by FlaviaNet Kaon WG 2010]

- CKM matrix elements ratio from charged π and K leptonic decay widths:

$$\frac{\Gamma(K^+ \to l^+\nu(\gamma))}{\Gamma(\pi^+ \to l^+\nu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2} \times \frac{f_K^2}{f_\pi^2} \times \frac{m_K(1-m_l^2/m_K^2)^2}{m_\pi(1-m_l^2/m_\pi^2)^2} \times (1+\delta_{\text{EM}})$$

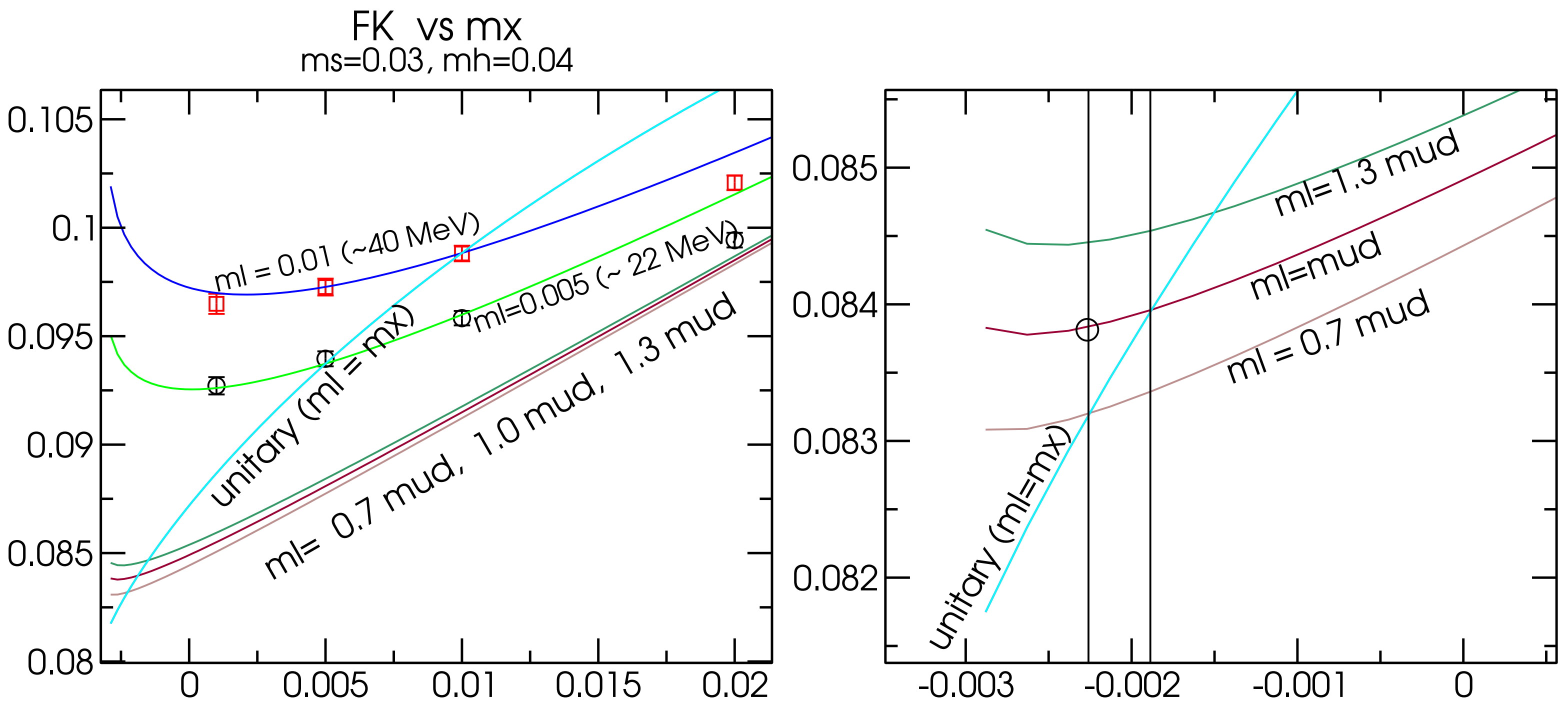
- At which quark masses, f_π and f_K should be computed ?

  - f_K: Should light quark mass be m_u or $m_{ud} = (m_u + m_d)/2$?
    $m_u/m_{ud} \sim 0.6 - 0.8$
  - f_π: Is the π mass shift from EM effect totally removed by δ_{EM} ?
    $m_\pi^0 = 135$ MeV vs $m_\pi^\pm = 139$ MeV ?

- Which is the best way to correct isospin breakings in the $|V_{us}/V_{ud}|$ extraction ?

F_K

[RBC/UKQCD PRD78:114509(08)& in prep.]

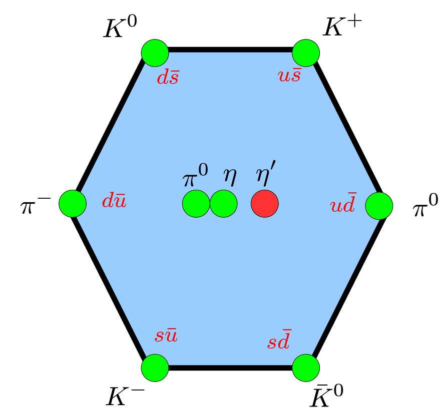


- $K^+ = \bar{s}u$ (light sea quark mass: m_l, light valence quark mass : m_x)
- f_K @ $m_l = m_x = m_{ud}$: 149.6(7) MeV
- f_K @ $m_x = 0.7m_{ud}, \ \ m_l = m_{ud}$: [-0.15%] vs the total error of current WA, 0.5%
- f_K @ $m_l = m_x = 0.7m_{ud}$: [-0.904%]

Isospin Breaking Effects

- PS meson spectrum and quark masses.

  - Asymmetry due to Quark mass differences :
    $m_u \neq m_d \neq m_s$
  - Asymmetry due to QED interactions :
    $Q_u = 2/3e, Q_d = Q_s = -1/3e$
  - QCD axial anomaly makes m_η' heavy.



- One needs to understand Isospin breaking effects on Hadrons to determine each individual, up, down (and strange) quark masses.

- Could $m_u \simeq 0$, which would explain the very small Neutron EDM ? (Strong CP problem) [D.Nelson,G.Fleming, G.Kilcup,PRL90:021601,2003.]

- Although isospin breakings are small, non-perturbative analysis are needed, via either QCD matrix elements, e.g. $\Pi_{V-A}(q^2)$ for $\pi^\pm - \pi^0$ splittings, or the direct QCD+QED simulation.

QCD+QED lattice simulation

- In 1996, Duncan, Eichten, Thacker carried out SU(3)×U(1) simulation to do the EM splittings for the hadron spectroscopy using quenched Wilson fermion on $a^{-1} \sim 1.15$ GeV, $12^3 \times 24$ lattice. [Duncan, Eichten, Thacker PRL76(96) 3894, PLB409(97) 387]

- Using $N_F = 2+1$ Dynamical DWF ensemble (RBC/UKQCD) would have benefits of chiral symmetry, such as better scaling and smaller quenching errors.

- Especially smaller systematic errors due to the the quark massless limits, $m_f \longrightarrow -m_{res}(Q_i)$, has smaller Q_i dependence than that of Wilson fermions, $\kappa \longrightarrow \kappa_c(Q_i)$.

- Generate Feynman gauge fixed, quenched non-compact U(1) gauge action with $\beta_{QED} = 1$. $U_\mu^{EM} = \exp[-iA_{\text{em}\,\mu}(x)]$.

- Quark propagator, $S_{q_i}(x)$ with EM charge $Q_i = q_i e$ with Coulomb gauge fixed wall source

$$D\left[(U_\mu^{EM})^{Q_i} \times U_\mu^{SU(3)}\right] S_{q_i}(x) = b_{src}, \quad (i = \text{up,down})$$

$$q_{\text{up}} = 2/3, \quad q_{\text{down}} = -1/3$$

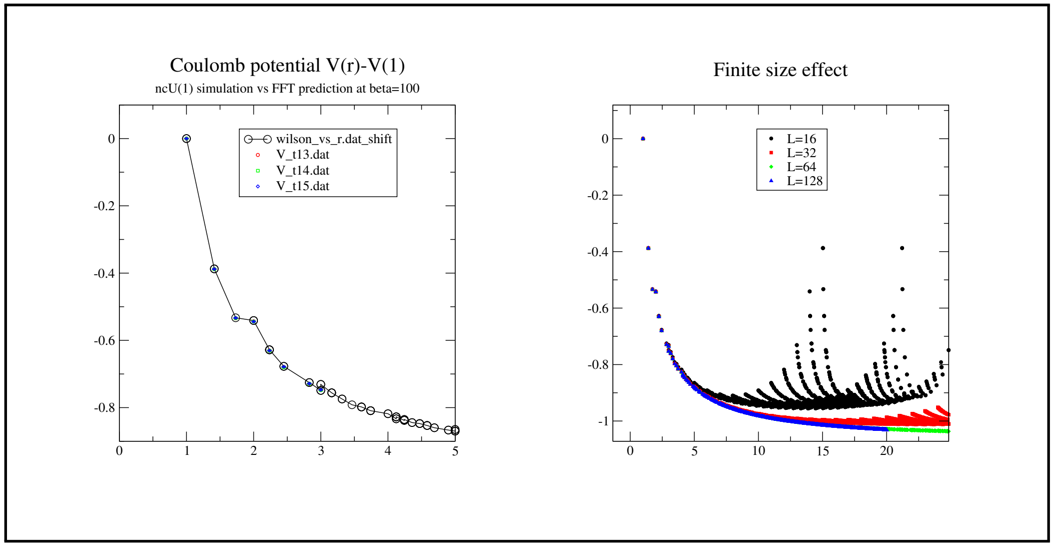
photon field on lattice

- non-compact $U(1)$ gauge is generated by using Fast Fourier Transformation (FFT). Feynman gauge with eliminating zero modes.

- static lepton potential on $16^3 \times 32$ lattice ($\beta_{QED} = 100$, 4,000 confs) vs lattice Coulomb potential.

- In our quenched QED simulation, QED coupling e is set by the static Coulomb potential in infinite volume limit to be,

$$V(r) = \frac{e^2}{4\pi}\frac{1}{r} = 1/137, \quad e = 0.30286$$



Measurements

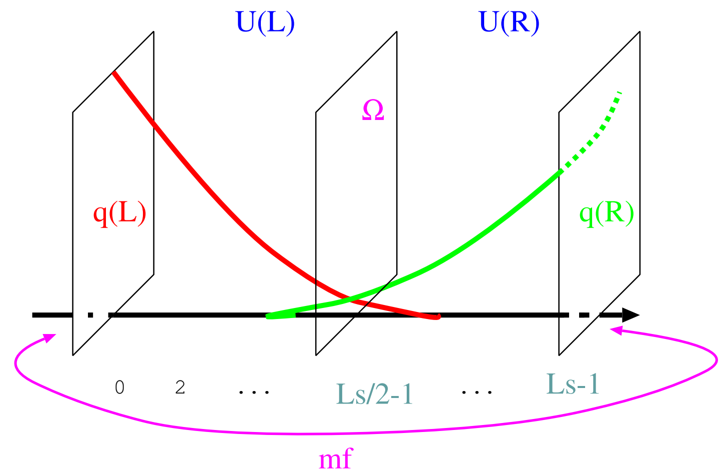
lat	m_{sea}	m_{val}	Trajectories	Δ	N_{meas}	t_{src}
16^3	0.01	0.01, 0.02, 0.03	500-4000	20	352	4,20
16^3	0.02	0.01, 0.02, 0.03	500-4000	20	352	4,20
16^3	0.02	0.01, 0.02, 0.03	500-4000	20	352	4,20
24^3	0.005	0.00{1,5}, 0.0{1,2,3}	900-8660	40	195	0
24^3	0.01	0.001, 0.0{1,2,3}	1460-5040	20	180	0
24^3	0.02	0.02	1800-3580	20	360	0,16,32,48
24^3	0.03	0.03	1260-3040	20	360	0,16,32,48

- $N_F = 2 + 1$ DWF QCD ensemble generated by [RBC/UKQCD, PRD78:114509(08), in prep.]
- a^{-1} =1.784 (44) GeV, $V = (16a = 1.76$ fm$)^3$ and $(24a = 2.65$ fm$)^3$
- $m_v = 0.0001$ ($\sim$ 9 MeV), 0.005 ($\sim$ 22 MeV) , 0.01 ($\sim$ 40 MeV), 0.02 ($\sim$ 70 MeV), , 0.03 ($\sim$ 100 MeV)
- $m_{res} = 0.003148(46)$ ($\sim$ 8.9 MeV)
- In total, $\sim$ 200 charge/mass combinations are measured.

The residual chiral symmetry breaking in QCD+QED

- Using DWF's PCAC relation, in terms of the mid-point correlator $J_{5q}(L_s/2)$, for the flavor off-diagonal current with same EM charge quarks, q_i. Parametrize the EM charge dependence in terms of C_2:

$$m_{\text{res}}(q_i, q_i) = \frac{\left\langle \sum_x J^a_{5q}(\vec{x}, t)\pi^a(0) \right\rangle}{\left\langle \sum_x J^a_5(\vec{x}, t)\pi^a(0) \right\rangle},$$



$$m_{\text{res},i}(q_i, q_i) - m_{\text{res}}(0, 0) = e^2 C_2\, q_i^2,$$

	16^3	24^3
m_{sea}	m_{res}	m_{res}
chiral limit	0.003148(46)	0.003203(15)
0.005	N/A	0.003222(16)
0.01	0.003177(31)	0.003230(15)
0.02	0.003262(29)	0.003261(16)
0.03	0.003267(28)	0.003297(15)

L_s	$C_2 u\bar{u}$	$C_2 d\bar{d}$
	16^3 lattice size	
16	2.597(23)	2.532(22)
32	0.309(16)	0.301(16)
	24^3 lattice size	
16	2.585(7)	2.519(7)

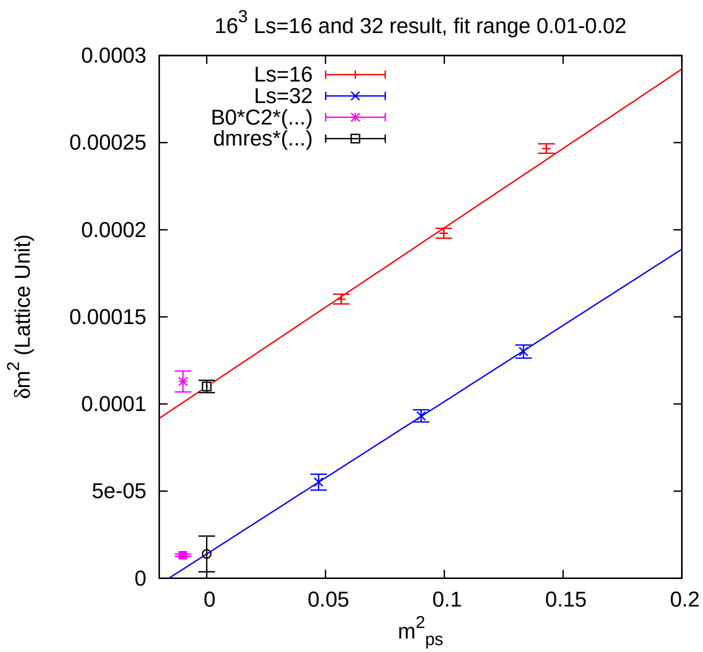
- In the massless quark limit of QCD, $m_f = -m_{res}(0,0)$, $Neutral$ PS meson (should still be a NG boson upto α^2), has additive mass shift due to the additional chiral symmetry breaking from photon field, $m_{\text{res},i}(q_i, q_i) - m_{\text{res}}(0,0)$.

- This effect is expressed in the DWF-ChPT as

$$\Delta m^2 = M^2_{\text{PS}^0}(e \neq 0) - M^2_{\text{PS}^0}(e = 0) = BC_2 e^2 (q_1^2 + q_3^2),$$

where $\chi = 2Bm_q$ is the LO PS mass squared.

- $L_s = 16$ and 32 (partially quenched) consistent with DWF-PCAC.



$\mathcal{O}(e)$ error reduction

- On the infinitely large statistical ensemble, term proportional to odd powers of e vanishes. But for finite statistics,

$$\langle O \rangle_e = \langle C_0 \rangle + \langle C_1 \rangle\, e + \langle C_2 \rangle\, e^2 + \cdots$$

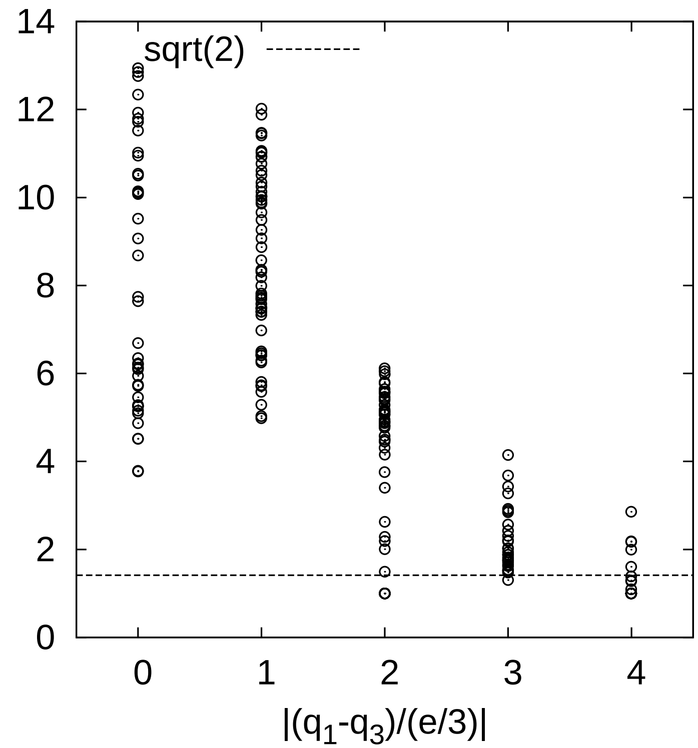
$\langle C_{2n-1} \rangle$ could be finite and source of large statistical error as e^{2n-1} vs e^{2n}.

- By averaging $+e$ and $-e$ measurements on the same set of QCD+QED configuration,

$$\frac{1}{2}[\langle O \rangle_e + \langle O \rangle_{-e}] = \langle C_0 \rangle + \langle C_2 \rangle\, e^2 + \cdots$$

$\mathcal{O}(e)$ is exactly canceled.

- More than a factor of 10 error reduction, corresponding to $\times 100$ measurements by only twice computational cost (vs naive reduction factor $\sqrt{2}$).



EM splittings

- Axial WT identity with EM for massless quarks $(N_F = 3)$,

$$\mathcal{L}_{\text{em}} = eA_{\text{em}\,\mu}(x)\bar{q}Q_{\text{em}}\gamma_\mu q(x), \; Q_{\text{em}} = \text{diag}(2/3, -1/3, -1/3)$$

$$\partial^\mu \mathcal{A}^a_\mu = ieA_{\text{em}\,\mu}\,\bar{q}\,[T^a, Q_{\text{em}}]\,\gamma^\mu\gamma_5 q - \frac{\alpha}{2\pi} tr\left(Q^2_{\text{em}}T^a\right) F^{\mu\nu}_{\text{em}}\widetilde{F}_{\text{em}\,\mu\nu},$$

neutral currents, four $\mathcal{A}^a_\mu(x)$, are conserved (ignoring $\mathcal{O}(\alpha^2)$ effects): π^0, K^0, $\overline{K^0}$, η_8 are still a NG bosons.

- ChPT with EM at $\mathcal{O}(p^4, p^2e^2)$:

$$M^2_{\pi^\pm} = 2mB_0 + 2e^2\frac{C}{f_0^2} \qquad +\mathcal{O}(m^2\log m, m^2) + I_0e^2m\log m + K_0e^2m$$

$$M^2_{\pi^0} = 2mB_0 \qquad +\mathcal{O}(m^2\log m, m^2) + I_\pm e^2m\log m + K_\pm e^2m$$

Dashen's theorem :
The difference of squared pion mass is independent of quark mass up to $\mathcal{O}(e^2m)$,

$$\Delta M^2_\pi \equiv M^2_{\pi^\pm} - M^2_{\pi^0} = 2e^2\frac{C}{f_0^2} + (I_\pm - I_0)e^2m\log m + (K_\pm - K_0)e^2m$$

C, $K_\pm$, K_0 is a new low energy constant. $I_\pm$, I_0 is known in terms of them.

ChPT+EM at NLO

- Double expansion of $M^2_{\mathrm{PS}}(m_1, q_1; m_3, q_3)$ in $\mathcal{O}(\alpha), \mathcal{O}(m_q)$.
  QCD LO:
  $$M^2_{\mathrm{PS}} = \chi_{13} = B_0(m_1 + m_3)$$
  QCD NLO: $(1/F_0^2 \times)$
  $$(2L_6 - L_4)\chi_{13}^2 + (2L_5 - L_8)\chi_{13}\bar{\chi}_1 + \chi_{13} \sum_{I=1,3,\pi,\eta} R_I \chi_I \log(\chi_I/\Lambda_\chi^2),$$
  QED LO: (Dashen's term)
  $$\frac{2C}{F_0^2}(q_1 - q_3)^2$$
  QED NLO: $(\bar{Q}_2 = \sum q^2_{\mathrm{sea}-i}$, no $\bar{Q}_1$ in $\mathrm{SU(3)}_{N_F})$
  $$-Y_1\bar{Q}_2\chi_{13} + Y_2(q_1^2\chi_1 + q_3^2\chi_3) + Y_3 q_{13}^2\chi_{13} - Y_4 q_1 q_3 \chi_{13} + Y_5 q_{13}^2\bar{\chi}_1$$
  $$+\chi_{13}\log(\chi_{13}/\Lambda_\chi^2)q_{13}^2 + \bar{B}(\chi_\gamma, \chi_{13}, \chi_{13})q_{13}^2\chi_{13} - \bar{B}_1(\chi_\gamma, \chi_{13}, \chi_{13})q_{13}^2\chi_{13} + \cdots$$

- QED LO adds mass to $\pi^\pm$ at $m_q = 0$, QED NLO changes slope, B_0, in m_q.

- Partially quenched formula ($m_{\mathrm{sea}} \neq m_{\mathrm{val}}$) $\mathrm{SU(3)}_{N_F}$ [Bijnens Danielsson, PRD75 (07)] $\mathrm{SU(2)}_{N_F}$+Kaon+FiniteV [Hayakawa Uno, PTP 120(08) 413] [RBC/UKQCD] (also [C. Haefeli, M. A. Ivanov and M. Schmid, EPJ C53(08)549])

SU(3)+EM ChPT LEC

[R. Zhou] [Bijnens Danielsson, PRD75 (07)]

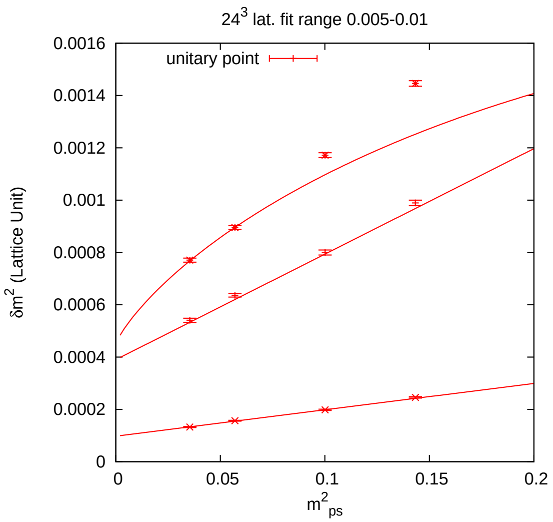
- By fitting charge splitting

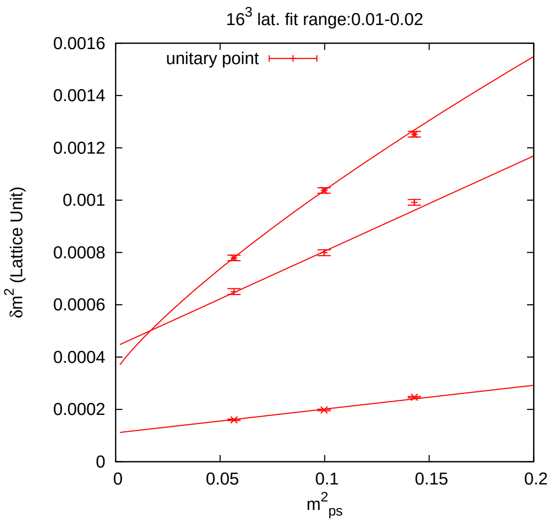
$$\delta M^2 = M^2_{\text{PS}}(m_1, q_1; m_2, q_2; m_l) - M^2_{\text{PS}}(m_1, 0; m_2, 0; m_l)$$

  by SU(3) ChPT+EM formula at NLO, 3 QCD LECs (1 LO + 2 NLO), 5 QED LECs (1 LO + 4 NLO) are determined.

- Requiring $m_1, m_3, m_l \leq$ 0.01 (0.02), 58 (124) partially quenched data for $M_{\text{PS}}(m_1, q_1; m_2, q_2; m_l)$ are used in the fit (to see NNLO effects).

- Finite volume effects are observed by repeating the fit on (1.8 fm)^3 and and (2.7 fm)^3.



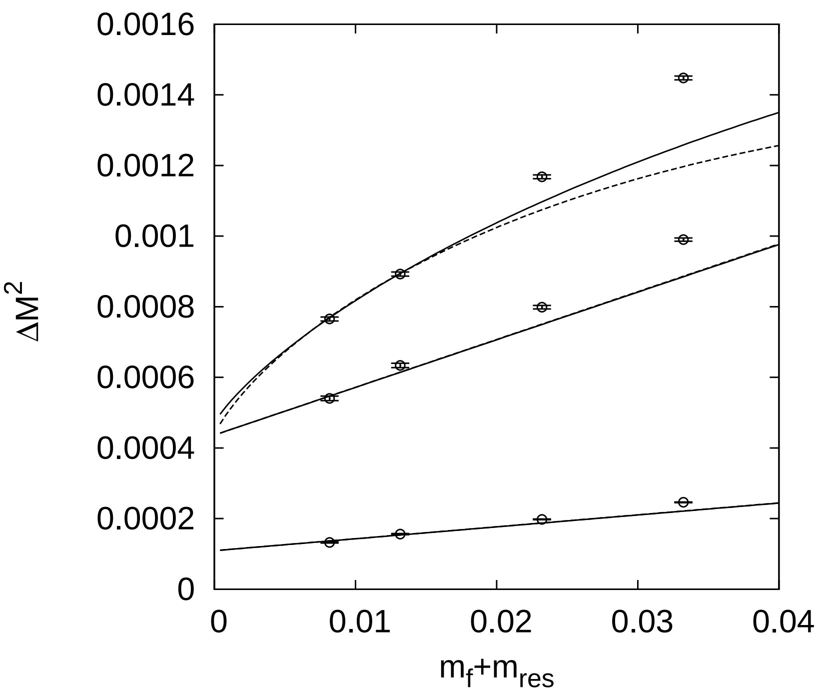


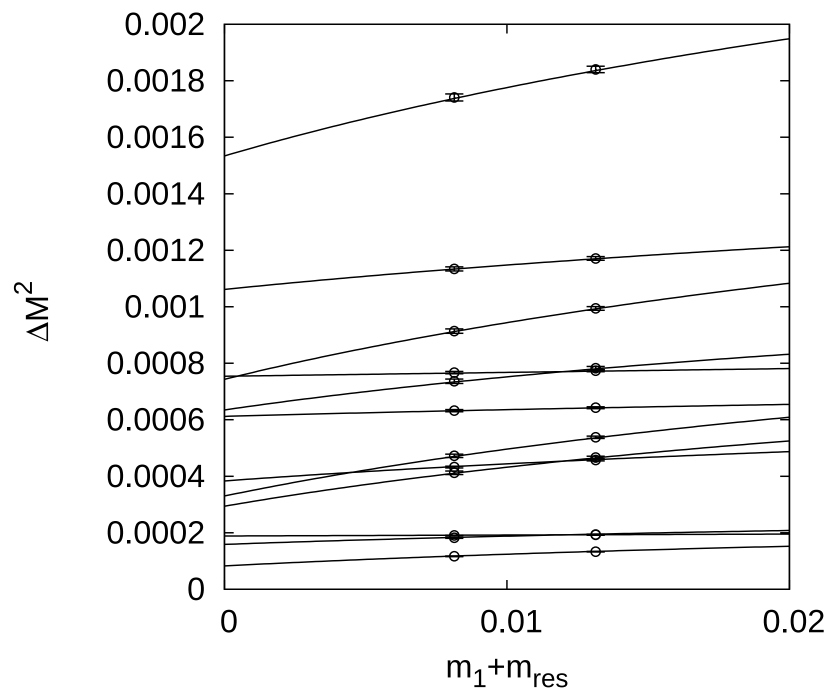
SU(2)+ Kaon+EM ChPT Fit

-
$$
\begin{aligned}
M_K^2 \;=\;& M^2 - 4B(A_3 m_1 + A_4(m_4+m_5)) \\
&+e^2\left(2\left(A_5^{(1,1)}+A_5^{(2,1)}\right)q_1^2 + A_5^{(s,1,1)}q_3^2 + 2A_5^{(s,2)}q_1q_3\right) \\
&-\frac{e^2}{(4\pi)^2F^2}\left((A_5^{(1,1)}+3A_5^{(2,1)})q_1^2 + A_5^{(s,2)}q_1q_3\right)\sum_{s=4,5}\chi_{1s}\log\frac{\chi_{1s}}{\mu^2} \\
&+e^2 m_1\left(x_3^{(K)}(q_1+q_3)^2 + x_4^{(K)}(q_1-q_3)^2 + x_5^{(K)}(q_1^2-q_3^2)\right) \\
&+e^2\frac{m_4+m_5}{2}\left(x_6^{(K)}(q_1+q_3)^2 + x_7^{(K)}(q_1-q_3)^2 + x_8^{(K)}(q_1^2-q_3^2)\right) \\
&+e^2\delta_{mres}(q_1^2+q_3^2),
\end{aligned}
$$

- EM splitting NLO/LO is still large ($\sim$ 50% at $m_q = 40$ MeV) for Pion but small ($\sim$ 10% at $m_q = 70$ MeV) for Kaon. But quark mass determination is stable under NLO correction.

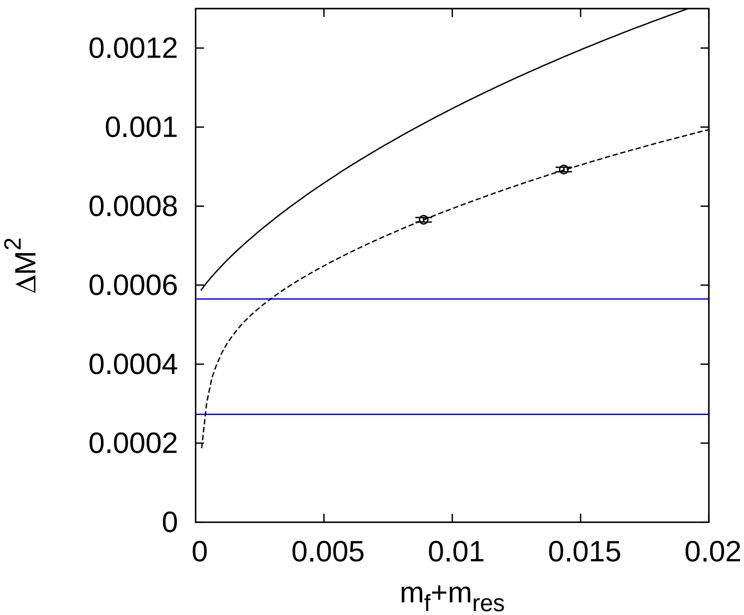
- An accidental flat direction of χ^2 function in our data set (degenerate light quark) : increase light mass range ($ml \le 0.02$) or fix QED NLO LEC to zero to see the effects on quark mass (included in systematic error).

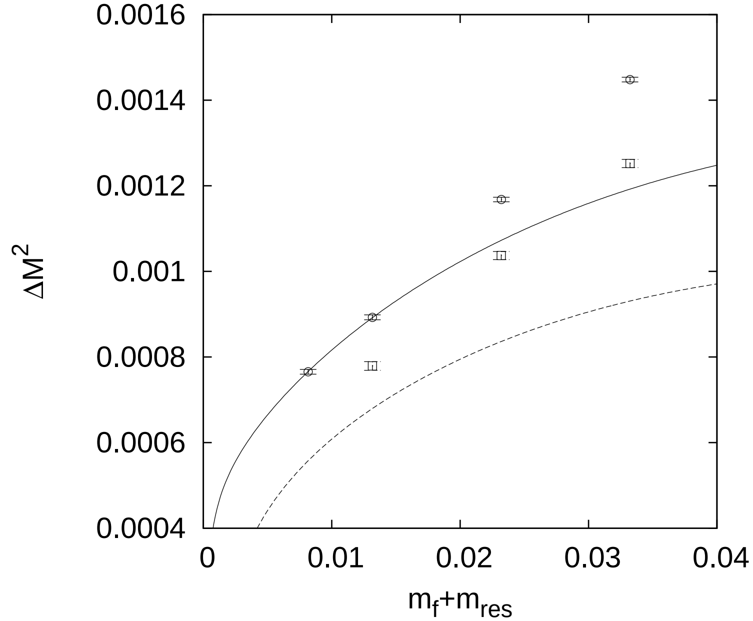




- Left: Pion fit, $\bar{u}d, \bar{u}u, \bar{d}d$ from top. SU(2) fit is in solid curve and dashed curve is SU(3) fit.
- Right: Kaon fit for various charge combinations.
- Infinite volume fit formula are shown.

Finite Volume effect on ChPT fits





- We use finite volume (FV) ChPT formula to fit data.
- Left: Pion unitary points. lower line: δm_{res}, upper line: LO (Dashen's) term
- NLO contributions at simulation points are 50-100% $\times$ LO. But only +2% contribution to $m_d - m_u$ from NLO.
- Left: Using FV fit on (2.7 fm)^3, dotted curve are predicted for (1.8 fm)^3, which overshoots the data by a factor of 2.

Quark mass determination

- Using the LECs, B_0, F_0, L_i, C_0, Y_i, from the fit, we could determine the quark masses $m_{\mathrm{up}}, m_{\mathrm{dwn}}, m_{\mathrm{str}}$ by the solving equations [PDG08] :

$$M_{\mathrm{PS}}(m_{\mathrm{up}}, 2/3, m_{\mathrm{dwn}}, -1/3) = 139.57018(35)\mathrm{MeV}$$
$$M_{\mathrm{PS}}(m_{\mathrm{up}}, 2/3, m_{\mathrm{str}}, -1/3) = 493.673(14)\mathrm{MeV}$$
$$M_{\mathrm{PS}}(m_{\mathrm{dwn}}, -1/3, m_{\mathrm{str}}, -1/3) = 497.614(24)\mathrm{MeV}$$

- $(m_{\mathrm{up}} - m_{\mathrm{dwn}})$ is mainly determined by Kaon charge splittings,

$$M_{K^\pm}^2 - M_{K^0}^2 = B_0(m_{\mathrm{up}} - m_{\mathrm{dwn}}) + \frac{2C}{F_0^2}(q_1 - q_3)^2 + \mathrm{NLO}$$

- π^0 mass is not used for now (disconnected quark loops).

- The term proportional to sea quark charge, $-Y_1\bar{Q}_2\chi_{13}$, is omitted. We will estimate the systematics by varying Y_1.

Quark mass results

- $\overline{MS}$ at 2 GeV, using NPR, RI-SMOM$_{\gamma\mu}$ scheme [C.Sturm et.al PRD (09) 014501, Y.Aoki, PoS LAT2009 012, L. Almeida C.Sturm arXiv:1004.4613, P.Boyle et. al. arXiv:1006.0422, RBC/UKQCD in prep.] as a intermediate scheme. (10% $\rightarrow$ 5% $\rightarrow$ 2,3% error)
- $m_1, m_3 \leq 0.01(\sim 40\text{MeV})$, $M_{ps} \leq$ 250 MeV
- SU(3)$_{N_F}$/SU(2)$_{N_F}$ in infinite/finite volume.
- Uncertainties in QED LEC have small effect to quark mass.

	SU(3)		SU(2)	
	inf.v	f.v	inf.v.	f.v.
m_u [MeV]	2.606(89)	2.318(91)	2.54(10)	2.37(10)
m_d [MeV]	4.50(16)	4.60(16)	4.53(15)	4.52(15)
m_s [MeV]	89.1(3.6)	89.1(3.6)	97.7(2.9)	97.7(2.9)
$m_d - m_u$ [MeV]	1.900(99)	2.28(11)	1.993(67)	2.155(63)
m_{ud} [MeV]	3.55(12)	3.46(12)	3.54(12)	3.44(12)
m_u/m_d	0.578(11)	0.503(12)	0.5608(87)	0.5238(93)
m_s/m_{ud}	25.07(36)	25.73(36)	27.58(27)	28.34(29)

- Only statistical error shown above.

Error budget

- Statistic error is small, especially for ratios.
- Chiral fit error: $m_q \leq$ 40 or 70 MeV ($M_{ps} \leq$ 250 or 420 MeV).
- Finite Volume Effect by comparing (1.9 fm)^3 and (2.7 fm)^3.

$$\frac{\Delta^{\text{EM}} M_{PS}^2(\infty)\Big|_{V.S.M}}{\Delta^{\text{EM}} M_{PS}^2(L \approx 1.9\ fm)\big|_{V.S.M}} = 1.10\,.$$

FV ChPT overestimate the FV effect. Generally quark masses are stable against $\Delta M_{PS} \sim$ O(10) %. ($M_{\pi^\pm}, M_{K^\pm}, M_{K^0}$ inputs)

	stat. err (%)	fit(%)	fv(%)	$\mathcal{O}(a^2)$ (%)	QED qnch(%)	renorm(%)
m_u	4.22	5.51	6.86	4	2	2.8+
m_d	3.31	2.05	~5	4	2	2.8+
m_s	3.00	0.12	0.04	4	2	2.8+
$m_d - m_u$	2.92	7.46	8.12	4	2	2.8+
m_{ud}	3.49	4.27	2.65	4	2	2.8+
m_u/m_d	1.78	6.32	6.60	4	2	-
m_s/m_{ud}	1.02	2.15	2.76	4	2	-

- QED Z_m $\mathcal{O}(\alpha) \sim$ 1%. Error of $m_s^{\text{sea}} \sim$ 2 %.

QED reweighting for sea quark charge

[Tomomi Ishikawa] [C. Jung LAT09, LAT10]

- Quenched QED error estimation via reweighting [Duncan, Eichten, Sedgewick PRD71 (05) 094509] .

$$\langle \mathcal{O} \rangle_{\text{full QED}} = \langle w \mathcal{O} \rangle \;/\; \langle w \rangle$$

- Need careful treatment for the reweighting factor [A. Hasenfratz et.al. PRD78 (08) 014515, M.Luscher F.Palombi PoS(LATTICE 2008)049, PACS-CS PRD81(10) 074503, T.Ishikawa et. al PoS(LATTICE 2009)035,]

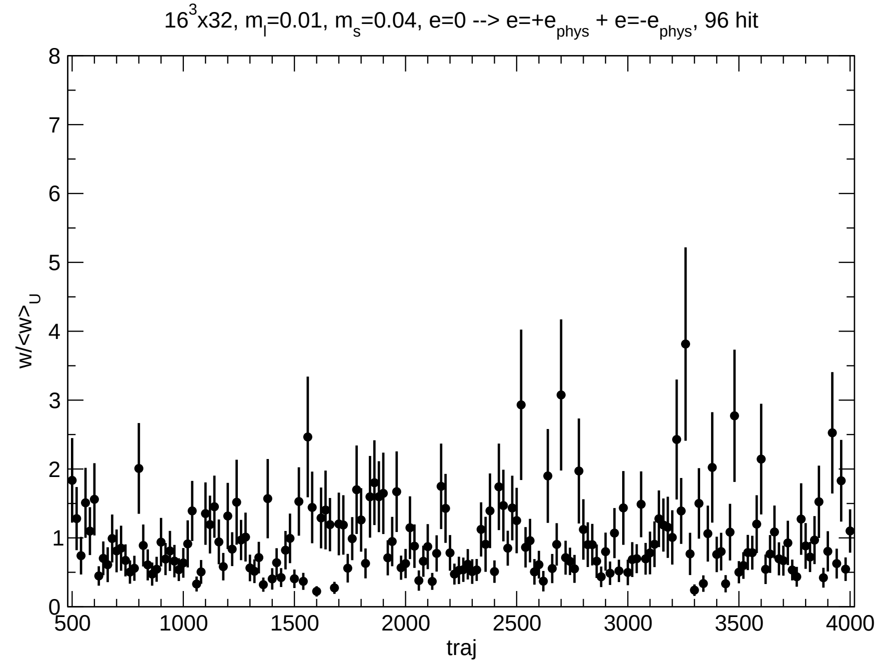
$$w = \det \mathcal{D}(e = e_{phys}) / \det \mathcal{D}(e = 0)$$

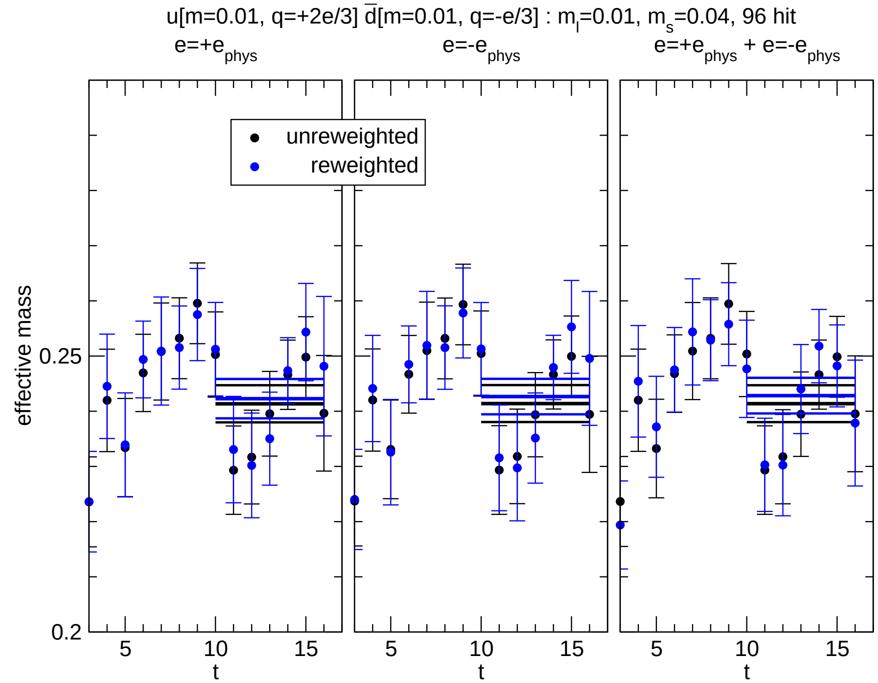
- Use rational approximation to take Nth root [T.Ishikawa et. al PoS(LATTICE 2009)035]

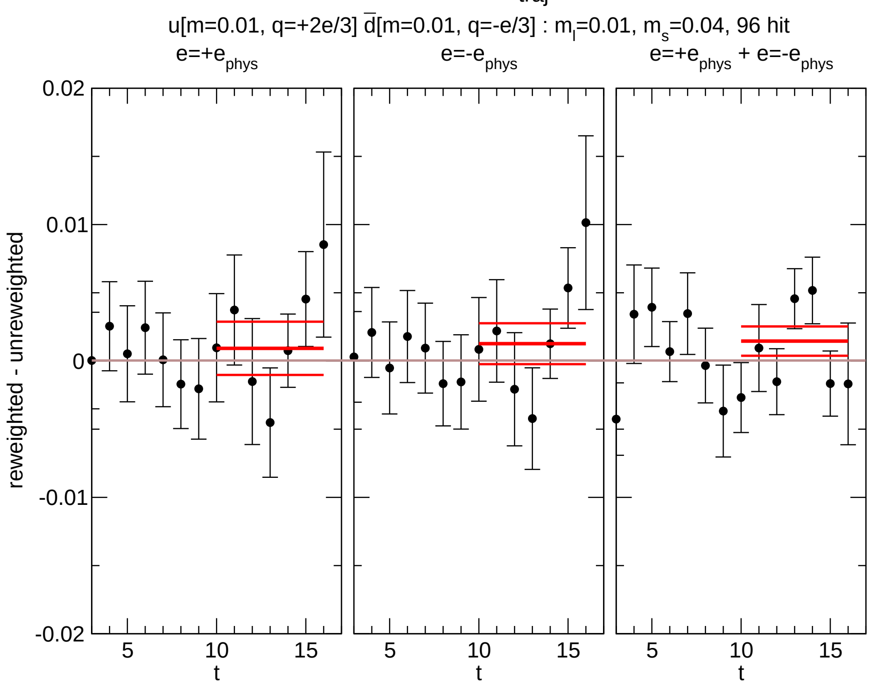
$$w = \left\langle e^{-\xi^\dagger (\Omega - 1)\xi} \right\rangle_\xi = \prod_i^N \left\langle e^{-\xi_i^\dagger (\Omega_N - 1)\xi_i} \right\rangle_{\xi_i},$$

$$\det \Omega_N = [\det \mathcal{D}(e = e_{phys}) / \det \mathcal{D}(e = 0)]^{1/N}$$

so that Ω_N is close to unity.







- 24-th root$\times$ 4 hits
- sea charges $q_u = 2/3, q_d = q_s = -1/3$ for $m_u = m_d$
- Size of the sea charge LEC, Y_1, is roughly a ball park of other LEC, consistent with systematic error estimate.

Quark masses and ratios

Final quotes from 24^3 run:

$$
\begin{aligned}
m_u &= 2.37 \pm 0.10 \pm 0.24 \;\; MeV \\
m_d &= 4.52 \pm 0.15 \pm 0.26 \;\; MeV \\
m_s &= 97.7 \pm 2.9 \pm 5.2 \;\; MeV \\
m_{ud} &= 3.44 \pm 0.12 \pm 0.24 \; MeV \\
m_d - m_u &= 2.155 \pm 0.063 \pm 0.263 \;\; MeV \\
m_u/m_d &= 0.5238 \pm 0.0093 \pm 0.0533 \\
m_s/m_{ud} &= 28.34 \pm 0.28 \pm 1.61,
\end{aligned}
$$

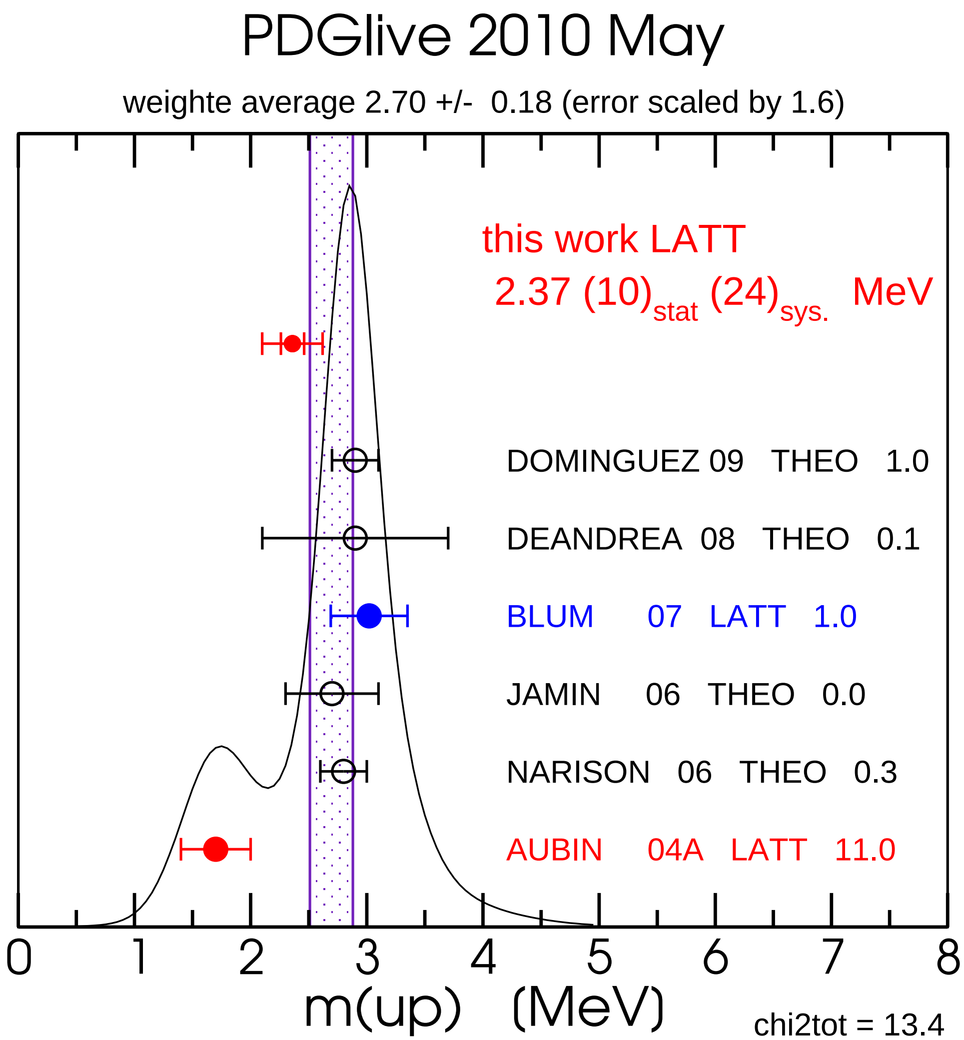
Belows are still preliminary values (statistical error only) [R.Zhou, S. Uno] :

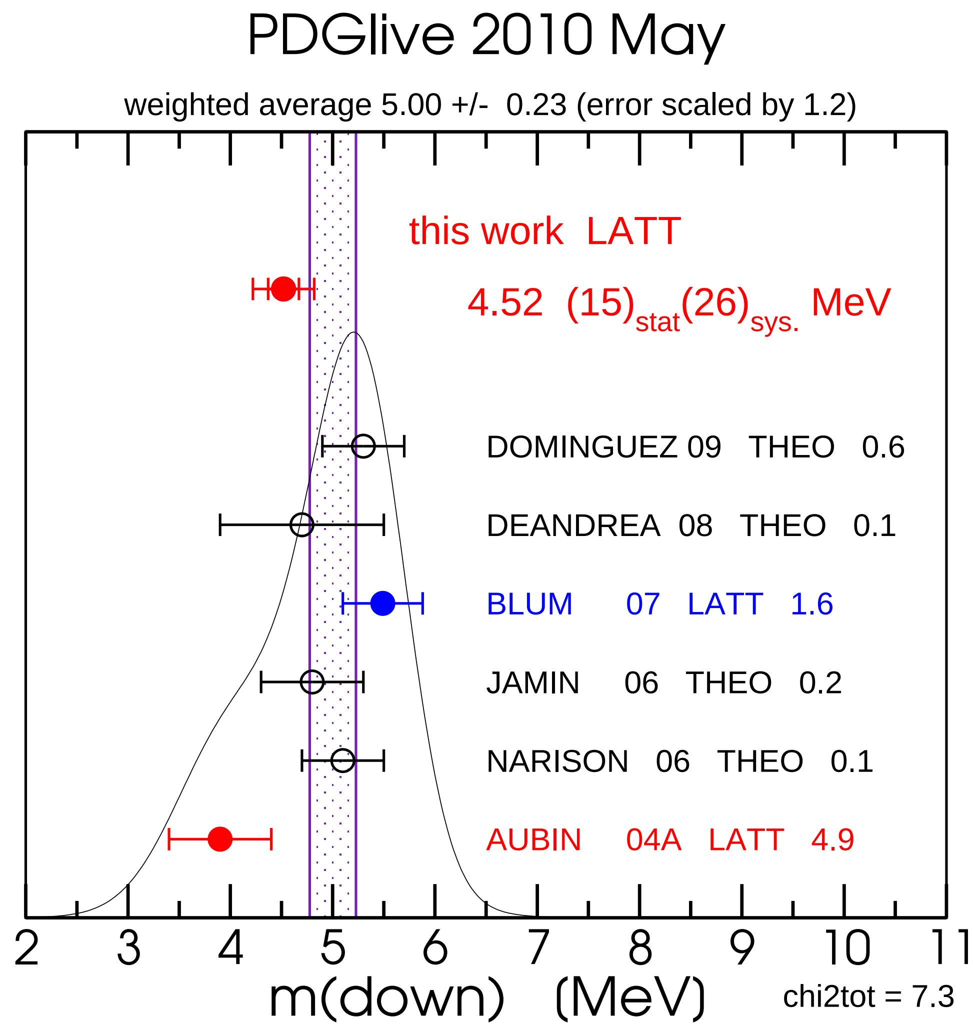
$(m_d - m_u)/(m_d + m_u) = 0.3124(80)$

$M_K(m_{ud}, 0, m_s, 0) = 494.7(2)$MeV (su2 formula)
$M_\pi(m_{ud}, 0, m_{ud}, 0) = 134.97(23)$MeV (su2 formula)

Comparison with u,d masses in PDG

red $N_F = 2+1$ staggered, DWF blue N_F=2 DWF (2.7 fm)^3





Components of Kaon masses splittings

- Reason why the iso doublet, (K^+, K^0), has the mass splitting

$$M_{K^\pm} - M_{K^0} = -3.937(29) \quad \text{MeV}, \qquad \text{[PDG]}$$

  - ▷ $(m_{\text{dwn}} - m_{\text{up}})$ *: makes* $M_{K^+} - M_{K^0}$ *negative.*
  - ▷ $(q_u - q_d)$ *: makes* $M_{K^+} - M_{K^0}$ *positive.*

- Using the determined quark masses and SU(3) LEC, we could isolate (to $\mathcal{O}((m_{\text{up}} - m_{\text{dwn}})\alpha)$) each of contributions,

$$M^2_{\text{PS}}(m_{\text{up}}, 2/3, m_{\text{str}}, -1/3) - M^2_{\text{PS}}(m_{\text{dwn}}, -1/3, m_{\text{str}}, -1/3)$$

$$\simeq M^2_{\text{PS}}(m_{\text{up}}, 0, m_{\text{str}}, 0) - M^2_{\text{PS}}(m_{\text{dwn}}, 0, m_{\text{str}}, 0) \qquad [\Delta M(m_{\text{up}} - m_{\text{dwn}})]$$

$$+M^2_{\text{PS}}(\bar{m}_{ud}, 2/3, \bar{m}_{ud}, -1/3) - M^2_{\text{PS}}(\bar{m}_{ud}, -1/3, m_{\text{str}}, -1/3) \qquad [\Delta M(q_u - q_d)]$$

- ▷ $\Delta M(m_{\text{up}} - m_{\text{dwn}})$ *= -5.23 (14) MeV* *[133(4)% in $\Delta M^2(m_{\text{up}} - m_{\text{dwn}})$]*
  ▷ $\Delta M(q_u - q_d)$ *= 1.327(37) MeV* *[-34(1)% in $\Delta M^2(q_u - q_d)$]*

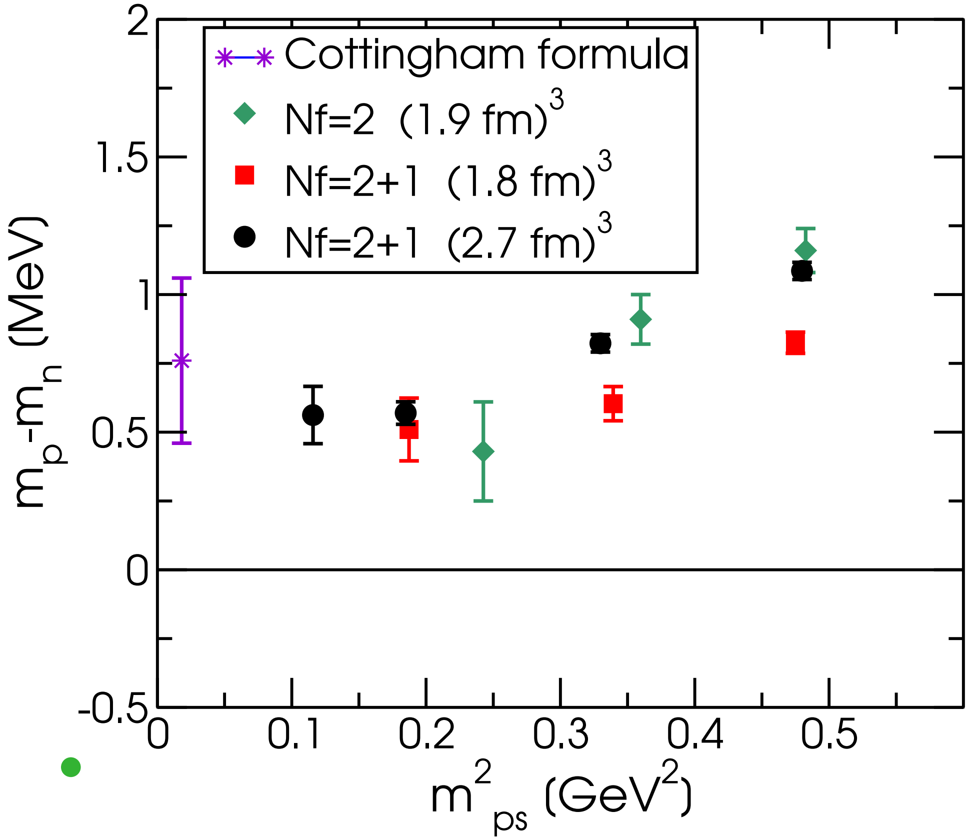
  Also SU(3) ChPT, $\Delta M(m_{\text{up}} - m_{\text{dwn}})$=-5.7(1) MeV and $\Delta M(q_u - q_d)$=1.8(1) MeV.

- Similar analysis for π is possible, but facing a difficulty of isolating sea strange quark terms. $m_{\pi^\pm} - m_{\text{“}\pi^0\text{”}} = 4.50(23)$ MeV

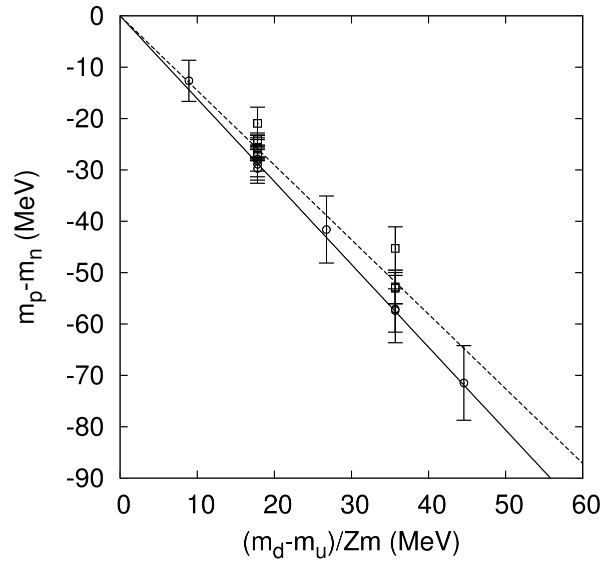
Nucleon mass splitting in $N_F = 2, 2+1$

[R.Zhou, T.Doi]

$(q_u - q_d)$ effect



$(m_{\text{up}} - m_{\text{dwn}})$ effect



$$M_N - M_p|_{\text{QED}} = 0.383(68) \text{ MeV}$$

$$M_N - M_p|_{\text{quark mass}} = -2.24(12) \text{ MeV}$$

$$\Longrightarrow M_N - M_p| = -1.86(14)(47)_{\text{FV,fit}} \text{ MeV}$$

Other recent works of isospin breaking on lattice

- [A.Portelli, LAT10] EM correction to hadron masses
- [A. Torok, LAT10] [E. Freeland, LAT10]
  [MILC Collaboration (S. Basak et al.) PoS LAT2008 127]
  EM splitting using MILC ensembles. The breaking of Dashen's theorem

$$\Delta M_D^2 = (M_{K^\pm}^2 - M_{K^0}^2)_{\text{em}} - (M_{\pi^\pm}^2 - M_{\pi^0}^2)_{\text{em}}$$

- [A. Wallker-Loud, LAT10]
  Using anisotropic clover, $m_u - m_d$ from $m_{\Xi^-} - m_{\Xi^0}$, derive $(m_p - m_n)_{m_d - m_u}$.
- [I.Baum, LAT10]
- [McNeile, Michael, Urbach (ETMC) PLB674(09) 286] $\rho - \omega$ mass splitting using twisted Wilson fermion. Discussed $\rho - \omega$ mixing from $m_u - m_d$. Measure disconnected quark loop correlation.
- [JLQCD (E.Shintani et. al.)PRL 101(08) 242001, PRD79(09)] Calculate $\Pi_V - \Pi_A$, derive the EM contribution to the pion's charge splittings in quark massless limit and the S-parameter using overlap fermion.
- [NPLQCD NPB 768 (07) 38] Calculate $(m_p - m_n)_{m_d - m_u}$. PQChPT for nucleon mass.

Summary and Future perspective

- Using $a^{-1} \sim$ 1.7 GeV DWF N_F =2+1 ensemble, two volumes, we determined each individual masses of up, down and strange quarks, and their ratios.
- We are currently improving the error estimates by a better treatment of FV for Kaon, QED reweighting. (quark mass central values may change, but within error quoted in this talk)
- We also derive { quark mass, QED} origins of $m_{K^\pm} - m_{K^0}$, $m_p - m_n$, (and Pion).
- Isospin breaking effects are interesting and inevitable for precise understanding of hadron physics, which could now be addressed by QCD+QED simulations from the first principle.

Future plans

- Analysis on the finer lattice, $a \sim 0.08$ fm and lighter/larger volume [Iwasaki+DSDR Lattice, Tom Blum's talk] ,
- Decay constants ($f_{\pi^\pm}/f_{\pi^0}$: QED dominate)

$$f_{\pi^\pm} - f_{\pi^0} = \left(0.3 + 0.03 \ln \frac{m_\gamma^2}{1\text{MeV}^2}\right) \text{MeV}$$

- $m_\rho^+ - m_\rho^0$, $\Gamma_{\rho^+}, \Gamma_{\rho^0}$ are related to the conversion of $\Gamma(\tau \to$ Hadrons$)$ to $\Gamma(e^+e^- \to$ Hadrons$)$ to determine leading QCD correction to muon $g-2$.
- π^0-η-η' mixings : $(m_u - m_d)(+\mathcal{O}(e^2p^2))$ [Q.Liu et. al. (RBC) arXiv:1002.2999]
- $\Gamma(\pi^0 \to \gamma\gamma)$: $(m_u - m_d)$ [E.Shintani; LAT10]
- $\mathcal{O}(\alpha)$ contribution to $g_\mu - 2$ (pure QED). $\mathcal{O}(\alpha^3)$ contribution (light-by-light) to $g_\mu - 2$. Chiral magnetic effect in QGP. [T.Blum]

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