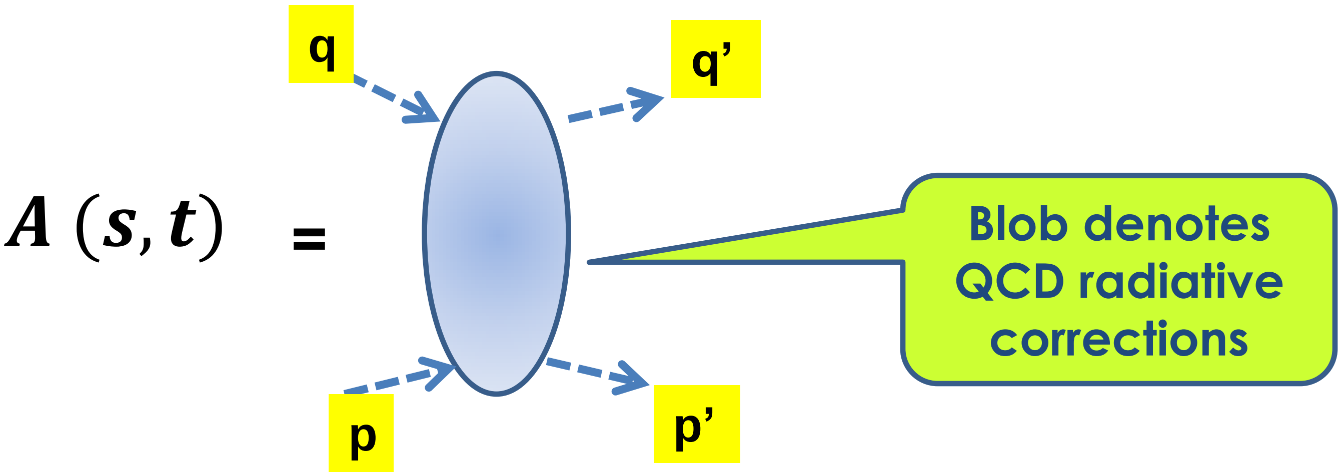
Krakow 6 Sept 2017

B. I. Ermolaev

Amplitudes of light-by-light scattering in Double-Logarithmic Approximation

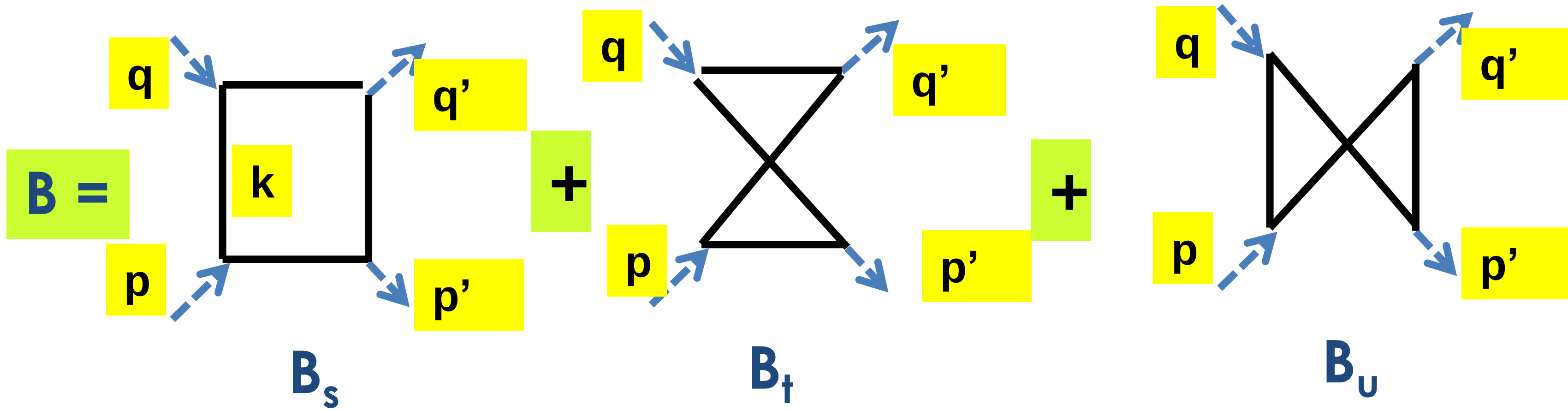
talk based on results obtained in collaboration with
D.I. Ivanov and S.I. Troyan

We consider amplitudes of the elastic scattering of virtual photons $\gamma^*(p)\gamma^*(q) \rightarrow \gamma^*(p')\gamma^*(q')$ in the forward kinematics and presume that all the photons being non-polarized



This process, apart of its experimental importance, is interesting from the theoretical point of view because, in contrast to hadronic reactions, it is free of non-perturbative contributions, so it can be regarded as a test-field for various theoretical approaches

Born (lowest order) approximation:



standard notations:

$$s = (p+q)^2 \approx -u = (p - q')^2 \gg -t = (p - q')^2 \,,$$

$$|p^2| \equiv Q_1{}^2 \approx |p'^2|, \; |q^2| \approx |q'^2| \equiv Q_2{}^2 \gg \mu^2$$

IR cut-off/mass scale

Forward kinematics: $s \approx -u \gg -t$

In the Born approximation there are two kinematic regions:

Moderately virtual photons: $s\,\mu^2 \gg {Q_1}^2 {Q_2}^2$

$$B_s = -\left(e^4/16\pi^2\right)\left[ln^2\left(-s/\mu^2\right) - ln^2\left({Q_2}^2/\mu^2\right) - ln^2\left({Q_1}^2/\mu^2\right)\right]$$

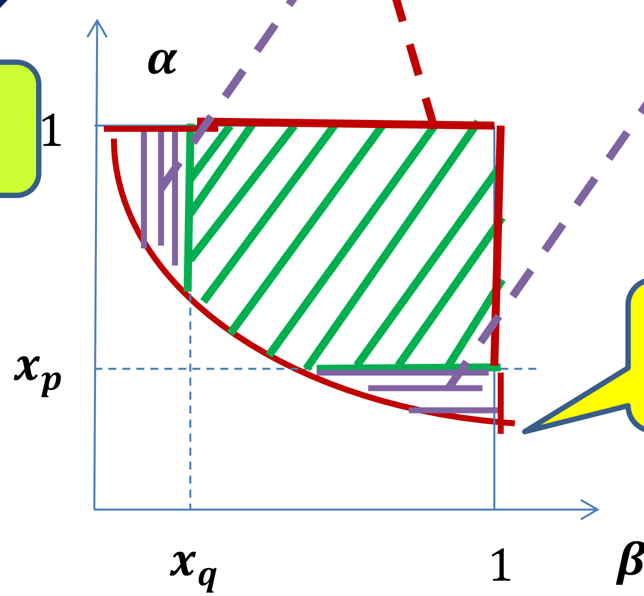
Sudakov parametrization

$$k = -\,\alpha\, q' + \beta\, p' + k_\perp$$

light-cone vectors

$$q' = q - x_q\, p,\quad p' = p - x_p\, q$$

$$x_q = {Q_2}^2/s,\quad x_p = {Q_1}^2/s$$

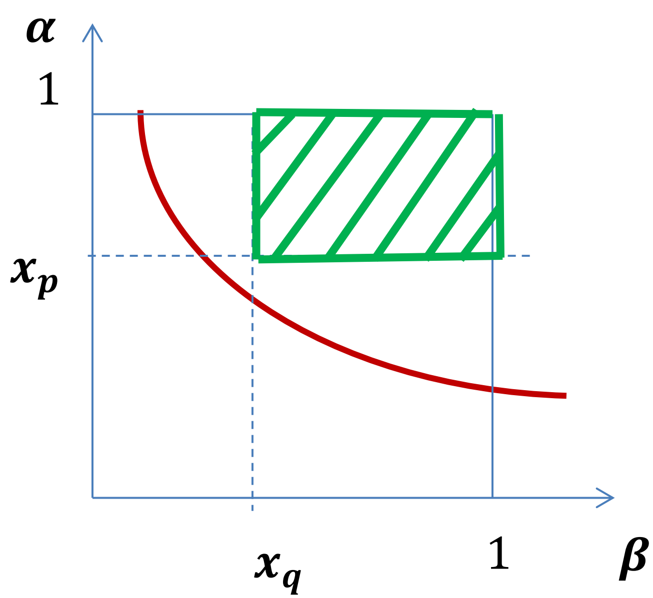


$\alpha\beta = \mu^2/s$

Deeply virtual photons $s\,\mu^2 \ll {Q_1}^2{Q_2}^2$

$$B_s = -\left(e^4/16\pi^2\right)\left[\,2\,ln\left(-s/{Q_1}^2\right)ln\left(-s/{Q_2}^2\right)\right]$$

does not depend on μ^2

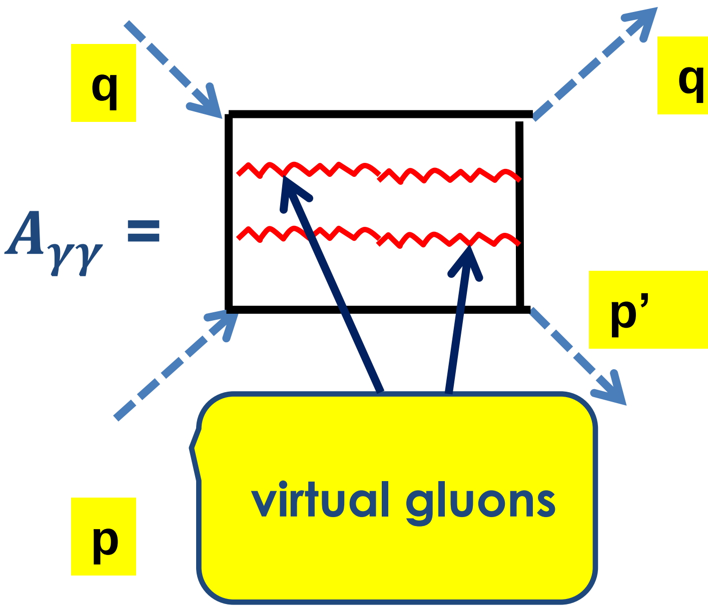


$$B_u\,(u) \approx B_s\,(s)$$

B_t can be neglected in the forward kinematics because it does not contain logs of s and u

Accounting for DL corrections to the Born amplitude can be done in two steps

STEP 1: photons scatter via a single quark loop



Bartels-Lublinsky

Calculated A_S at t=0, QCD coupling is fixed

Ermolaev-Ivanov-Troyan

Calculated A_S and A_U at non-zero t. QCD coupling is running

We start with considering Collinear kinematics where t=0.
To calculate $A_{\gamma\gamma}$ we compose and solve InfraRed Evolution Equations for it.

Essence of the approach is to introduce an IR cut-off in the transverse space and factorize DL contributions of parton with minimal transverse momentum.

L.N. Lipatov

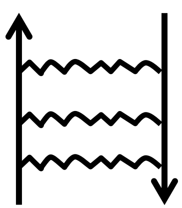
It is convenient to use the Mellin transform

$$A_{\gamma\gamma}(s, {Q_1}^2, {Q_2}^2) = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\pi i} \left(\frac{s}{\mu^2}\right)^{\omega} \xi^{(+)}(\omega)\, F_{\gamma\gamma}(\omega, {Q_1}^2, {Q_2}^2)$$

signature factor

$$\xi^{(+)} = \left(1 + e^{-i\pi\omega}\right)/2$$

and present the result in terms of the quark-quark scattering amplitude f_0

f_0 ⇔ 

$$f_0(\omega) = 4\pi^2\omega\sqrt{1 - a_0/(2\pi^2\omega^2)}$$ Kirschner-Lipatov

with $a_0 = 4\pi\alpha_s C_F$, where $C_F = \frac{(N^2-1)}{(2N)} = 4/3$

fixed

When α_s is fixed and the signatrure is positive, DL contributions of non-ladder graphs cancel each other   Gorshkov-Lipatov-Nesterov

When it is running, non-ladder graphs become important

Besides, when α_s is running, a_0 depends on ω

$$a_0(\omega) = \frac{4\pi C_F}{b}\left[\frac{\zeta}{\zeta^2 + \pi^2} - \int_0^\infty dz \frac{e^{-z\omega}}{(z+\zeta)^2 + \pi^2}\right]$$ Ermolaev-Greco-Troyan

where $\zeta = ln\left(\mu^2/\Lambda_{QCD}{}^2\right)$, $b = (11N - 2\, n_f)/(12\, \pi)$

It is convenient to introduce the logarithmic variables

$$\rho = ln(s/\mu^2)$$

$$\xi = ln({Q_1}^2/\mu^2) + ln({Q_2}^2/\mu^2)$$

$$\eta = ln({Q_1}^2/\mu^2) - ln({Q_2}^2/\mu^2)$$

and an auxiliary amplitude $H = f_0/(2\pi^2)$

Moderately virtual photons $s\,\mu^2 \gg {Q_1}^2{Q_2}^2$

$$A^{(M)}{}_s = -e^4 \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \frac{1}{{a_0}^2}\, e^{\omega\rho}\, [W_1 + W_2]$$

where

$$W_1 = -\left(f_0 e^{|\eta|H} - a_0/\omega\right) e^{-\omega(\xi+|\eta|)/2}$$

$$W_2 = (f_0 - a_0/\omega) \frac{\omega}{\sqrt{\omega^2 - a_0/(2\pi^2)}} \left[e^{|\eta|H-\omega(\xi+|\eta|)/2} - e^{\xi(-\omega+H)}\right]$$

Deeply virtual photons $s\,\mu^2 \ll Q_1{}^2 Q_2{}^2$

$$A^{(D)}{}_s = -e^4 \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \frac{1}{a_0{}^2} e^{\omega(\xi/2 - |\eta|/2)} W$$

with

$$W = \left[\left(f_0 e^{|\eta| H} - \frac{a_0}{\omega} \right) + \left(f_0 - \frac{a_0}{\omega} \right) \frac{\omega}{\sqrt{\omega^2 - a_0/(2\pi^2)}} e^{|\eta| H} \right]$$

Particular case: $Q_1{}^2 \sim Q_2{}^2 \equiv Q^2$

$$A^{(M)}{}_s = -e^4 \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \frac{1}{a_0{}^2} x^{-\omega} \left(f_0 - \frac{a_0}{\omega} \right) \left[1 + \frac{\omega}{\sqrt{\omega^2 - a_0/(2\pi^2)}} \left(1 - e^{-y(\omega - 2H)} \right) \right]$$

$$A^{(D)}{}_s = -e^4 \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \frac{1}{a_0{}^2} x^{-\omega} \left(f_0 - \frac{a_0}{\omega} \right) \left[1 + \frac{\omega}{\sqrt{\omega^2 - a_0/(2\pi^2)}} \right]$$

$A^{(D)}{}_s$ does not depend on y i.e. on μ

where $y = ln(Q^2/\mu^2)$, $x = Q^2/s$

Forward kinematics with non-zero t: $s \gg |t| > \mu^2$

We denote a new amplitude $\bar{A}_{\gamma\gamma}(s, t, Q_1^{\,2}, Q_2^{\,2})$

It is easy to see that ***t*** acts a new IR cut-off, i.e. all previous results for Collinear kinematics can be used, with μ^2 replaced by $|t|$

There can be the following relations between $|t|$ and $Q_1^{\,2}, Q_2^{\,2}$

A. $|t| < Q_1^{\,2}, Q_2^{\,2}$
In this case amplitude $\bar{A}_{\gamma\gamma}$ does not depend on $Q_1^{\,2}, Q_2^{\,2}$

B. $|t| > Q_1^{\,2}, Q_2^{\,2}$ and $s|t| > Q_1^{\,2}, Q_2^{\,2}$ — Moderate virtual photons

$$\bar{A}_{\gamma\gamma} = A^{(M)}{}_{\gamma\gamma}(\mu^2 \to |t|)$$

C. $|t| > Q_1^{\,2}, Q_2^{\,2}$ and $s|t| < Q_1^{\,2}, Q_2^{\,2}$ — Deeply virtual photons

$$\bar{A}_{\gamma\gamma} = A^{(D)}{}_{\gamma\gamma}(\mu^2 \to |t|)$$

Small-x asymptotics

Applying the saddle-point method, we arrive at Regge asymptotics:

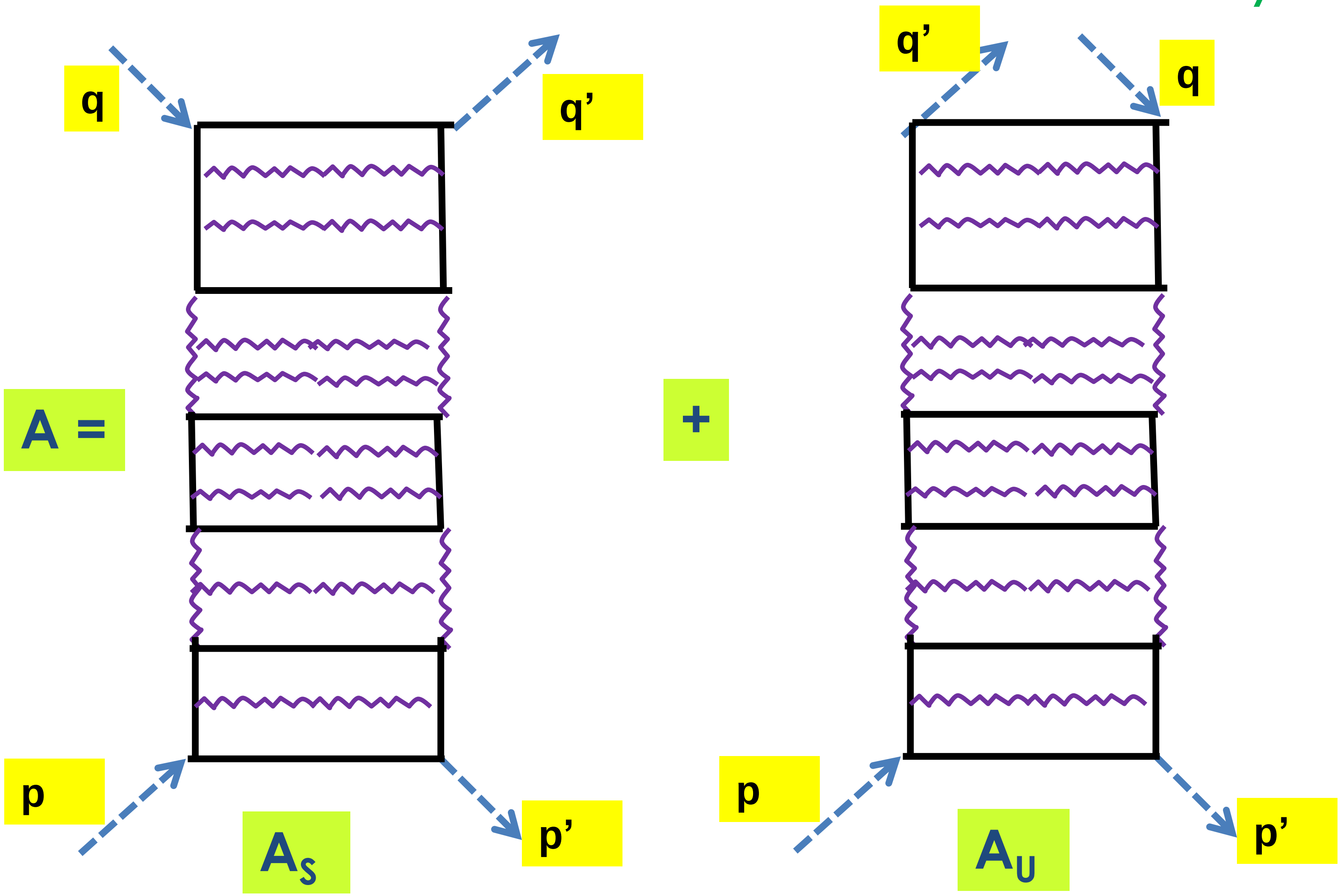
$$A^{(M,D)} \sim \Pi_{M,D} x^{-\omega_0}$$

non-vacuum Reggeon

intercept $\omega_0 \approx 0.4$

STEP 2: photons scatter via infinite succession of quark and gluon loops

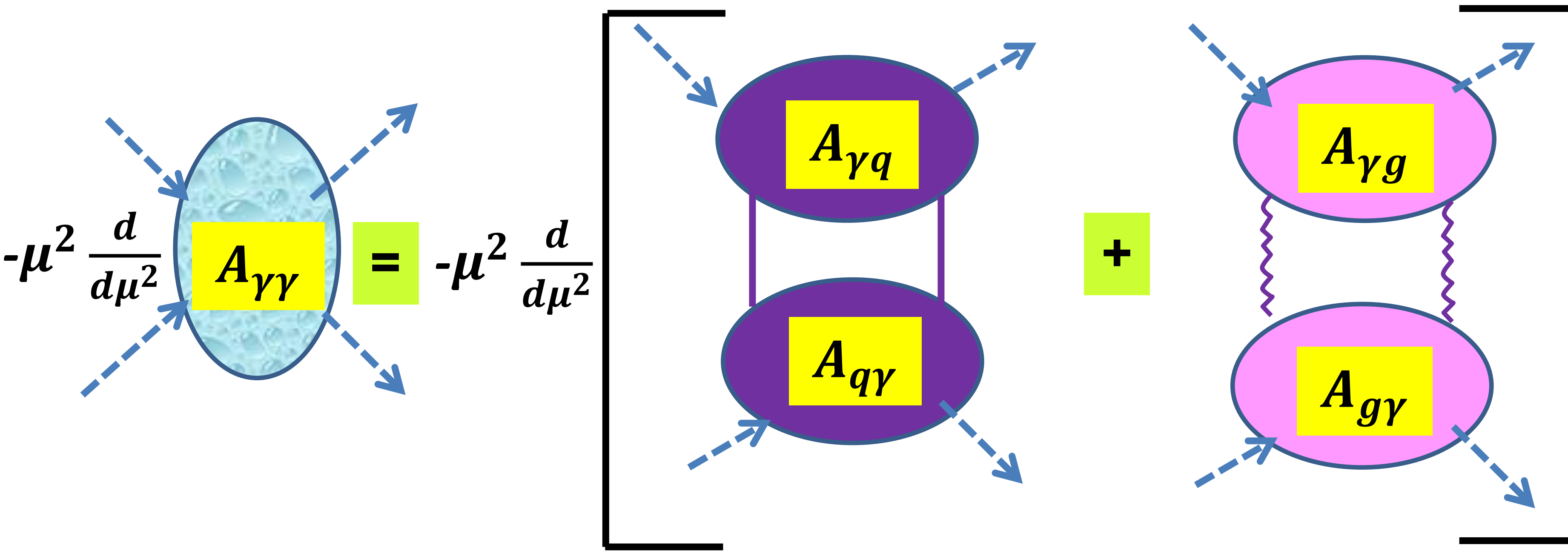
Ermolaev-Troyan



Graphs with non-ladder gluon propagators are also taken into account

InfraRed Evolution Equations

$A_{\gamma\gamma}$ is expressed through auxiliary amplitudes



IREE is a generalization of DGLAP with DL accuracy

the difference between IREE and DGLAP is that each blob now contains total resummation of DL contributions

It is convenient to use Mellin representation for writing IREE:

$$A(s, {Q_1}^2, {Q_2}^2) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} \; F(\omega, y_1, y_2)$$

$$\rho = ln(s/\mu^2)$$
$$y_1 = ln({Q_1}^2/\mu^2)$$
$$y_2 = ln({Q_2}^2/\mu^2)$$

Then

$$-\mu^2 \frac{d}{d\mu^2} A(s, {Q_1}^2, {Q_2}^2) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} \left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] F(\omega, y_1, y_2)$$

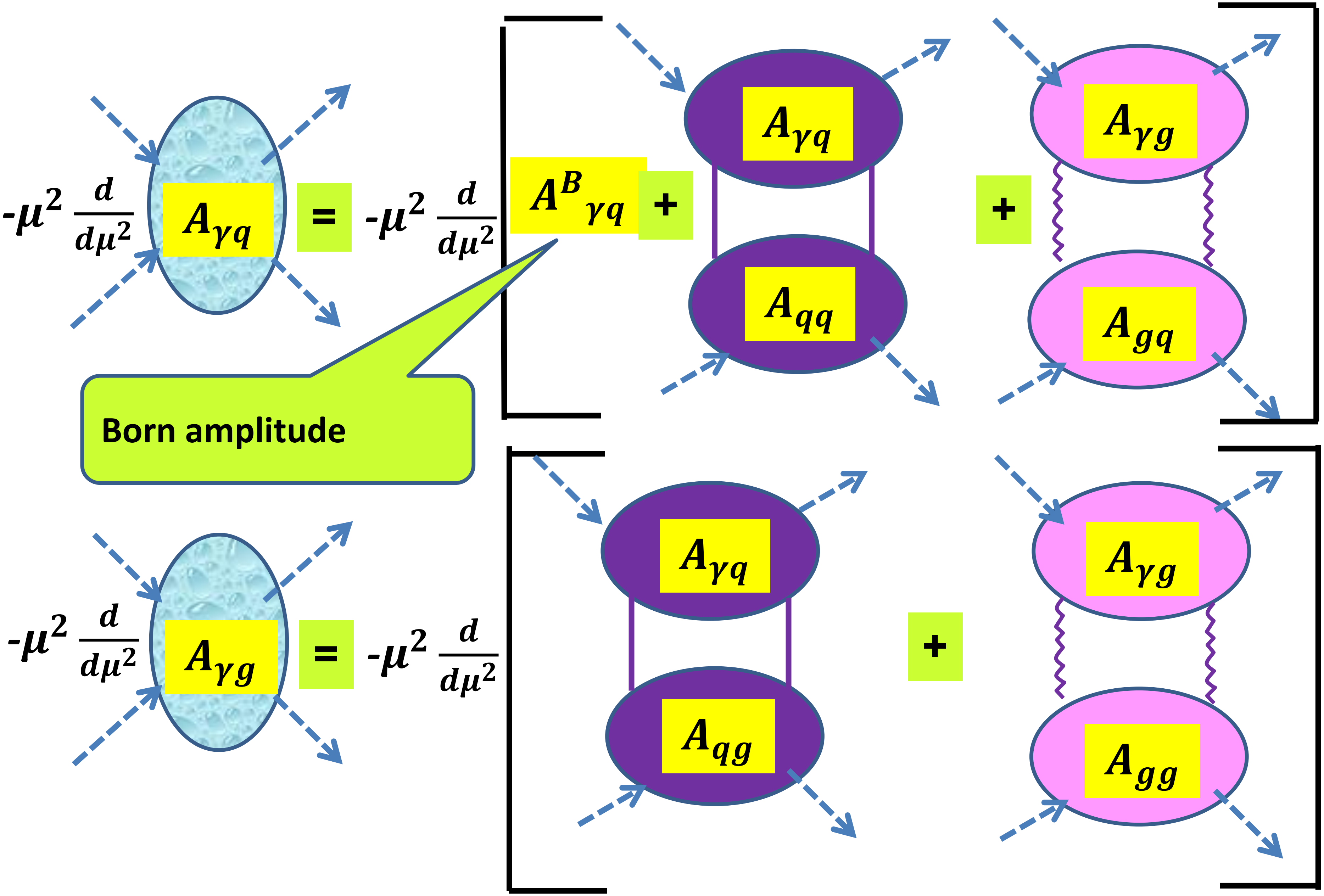
Moderately virtual photons $s\,\mu^2 \gg {Q_1}^2{Q_2}^2$

$$\left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] F^{(M)}{}_{\gamma\gamma}(\omega, y_1, y_2) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y_1) F_{q\gamma}(\omega, y_2) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y_1) F_{g\gamma}(\omega, y_2)$$

Deeply virtual photons $s\,\mu^2 \ll {Q_1}^2{Q_2}^2$

$$\left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] F^{(D)}{}_{\gamma\gamma}(\omega, y_1, y_2) = 0$$

IREE for auxiliary amplitudes



IREE for the auxiliary amplitudes are:

Born contribution

$$\left[\omega + \frac{\partial}{\partial y}\right] F_{\gamma q}(\omega, y) = e^2 + \frac{1}{8\pi^2} F_{\gamma q}(\omega, y) f_{qq}(\omega) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y) f_{gq}(\omega)$$

$$\left[\omega + \frac{\partial}{\partial y}\right] F_{\gamma g}(\omega, y) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y) f_{qg}(\omega) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y) f_{gg}(\omega)$$

parton-parton amplitudes

Solution to them allows us to express the auxiliary amplitudes in terms of the parton-parton amplitudes . They do not depend on *y* and because of that, obey algebraic albeit non-linear IREE

IREE for the parton-parton amplitudes

$$\omega f_{qq} = b_{qq} + (1/8\pi^2) \left[f_{qq} f_{qq} + f_{qg} f_{gq}\right]$$
$$\omega f_{qg} = b_{qg} + (1/8\pi^2) \left[f_{qq} f_{qg} + f_{qg} f_{gg}\right]$$
$$\omega f_{gq} = b_{gq} + (1/8\pi^2) \left[f_{gq} f_{qq} + f_{gg} f_{gq}\right]$$
$$\omega f_{gg} = b_{gg} + (1/8\pi^2) \left[f_{gq} f_{qg} + f_{gg} f_{gg}\right]$$

$$\mathrm{b}_{\mathrm{ik}}(\omega) = a_{ik}(\omega) + V_{ik}(\omega)$$

Born contributions. They depend on ω when QCD coupling is running

Contributions of the color octet (non-ladder graphs)

This system of non-linear equations can be solved exactly. Substituting the solutions in IREE for the auxiliary amplitudes and solving them, we arrive at the explicit expressions for the light-by-light amplitudes. They are quite complicated, so we do not discuss them but focus on the high-enerrgy asymptotics

Asymptotics of light-by-light amplitudes for $s \to \infty$ can be found with Saddle-Point method. The asymptotics proved to be of the Regge form.

$$A_{\gamma\gamma} \sim \lambda \rho^{-3/2} \left(s/\sqrt{Q_1{}^2 Q_1{}^2} \right)^{\omega_0}$$

Leading singularity

Impact-factors

$\rho = ln(s/\mu^2)$

Reggeon

All quantum numbers of this Reggeon are zeros, so we obtain a new Pomeron. It has nothing in common with BFKL Pomeron

ω_0 is the rightmost root of the following equation:

$$\left(\omega^2 - 2b_{qq} - 2b_{gg}\right)^2 - 4\left(b_{gg} - b_{qq}\right)^2 - 16 b_{gq} b_{qg} = 0$$

Parameters b_{ik} contain α_s so value of ω_0 depends on treatment of α_s

Estimating ω_0

The roughest estimate is to assume QCD coupling fixed,

However, assuming α_s fixed is altogether unrealistic. To account for the running coupling effects, we will use the expression where α_s is replaced by the effective coupling

$$\alpha_{eff}(\omega) = \frac{1}{b}\left[\frac{\zeta}{\zeta^2 + \pi^2} - \int_0^\infty dz \frac{e^{-z\omega}}{(z+\zeta)^2 + \pi^2}\right]$$

Ermolaev-Greco-Troyan

where $\zeta = ln\left(\mu^2 / {\Lambda_{QCD}}^2\right)$

It allows us to integrate α_s in every rung of all involved Feynman graphs
By doing so, we obtain ω_0 as a number

To compare our results with BFKL, we represent ω_0 as follows:

$$\omega_0 = 1 + \Delta$$

BFKL Pomeron is asymptotics of the SL series:

$$\left(\frac{1}{x}\right)\left[1 + c_1(\alpha_s ln(1/x)) + c_2(\alpha_s ln(1/x))^2 + c_3(\alpha_s ln(1/x))^3 + \cdots\right]$$

$$\omega_0 = 1 + \Delta$$

comes from resummation

comes from the overall factor 1/x

Our Pomeron is asymptotics of the DL series:

$$1 + c'_1\left(\alpha_s ln^2(1/x)\right) + c'_2\left(\alpha_s ln^2(1/x)\right)^2 + c'_3\left(\alpha_s ln^2(1/x)\right)^3 + \cdots$$

$$\omega_0 = 1 + \Delta$$

The both terms come from resummation

when α_s is fixed

$$\omega_{0_{fix}} = (\alpha_s/\pi)^{1/2}\left[4N + C_F + \sqrt{(4N - C_F)^2 - 8n_f C_F}\right]^{1/2}$$

α_s=0.24

Ermolaev-Greco-Troyan

A. Coupling fixed, purely gluonic Pomeron

$$\Delta_{fix} \equiv \omega_{0_{fix}} - 1 = 0.35$$

B. Coupling fixed, both gluon and quark contributions accounted for

$$\Delta_{fix} = 0.29$$

Accounting for the running α_s effects

C. Purely gluonic Pomeron $\Delta = 0.25$ ← Close to LO BFKL intercept

D. Both gluon and quark contributions are taken into account

$$\Delta = 0.066$$

← Close to NLO BFKL intercept

OBSERVATION: The higher accuracy, the less the Pomeron intercept

Δ_{fix}= 0.35 — Fixed coupling, gluons only

Δ_{fix}= 0.29 — Fixed coupling, gluons and quarks

$\Delta = 0.25$ — Coupling runs, gluons only

$\Delta = 0.07$ — Coupling runs, gluons and quarks

ASSUMPTION: this tendency suggests that eventually the intercept will go down to zero

Applicability region of the high-energy asymptotics

We introduce $R_{as} = Asympt\, A_{\gamma\gamma}/A_{\gamma\gamma\gamma}$

and plot it against *s*. The plot demonstrates that $R_{as} > 0.9$ when

$$s > s_{min} = 10^6 \sqrt{{Q_1}^2 {Q_2}^2}$$

CONCLUSIONS

We have obtained explicit expressions for light-by-light scattering amplitudes $A_{\gamma\gamma\gamma}$ in DLA, with fixed and running QCD coupling

Applying Saddle-Point method to these expressions, we arrived at the Regge asymptotics, with the Reggeon bearing the vacuum quantum numbers. It allowed us to call it a new (supercritical) Pomeron. Value of its intercept monotonically decreases with increase of accuracy of calculations

This Pomeron has nothing in common with BFKL Pomeron which is asymptotics of total resummation of single-logarithmic contributions while we deal with double logarithms

Comparing $A_{\gamma\gamma\gamma}$ to their asymptotics , we fixed the applicability region of the high-energy asymptotics