Precise determinations of the strong coupling in lepton collisions

Zoltán Trócsányi

Eötvös University and MTA-DE Particle Physics Research Group

based on
arXiv:1603.08927, 1606.03453, 1708.04093, 1804.09146, 1807.11472, 1902.08158
and unpublished results of ongoing work

FFK-2019, Tihany
12 June 2019
Outline

- Status of the strong coupling
- New measurements of $\alpha_s$
- Conclusions and Outlook
Status of the strong coupling
PDG 2016 on $\alpha_s$

Dominated by lattice

$\Delta \alpha_S(M_Z) = 0.9\%$

PDG 1992: 2.4%
Lattice unbeatable?

recent prevailing view: lattice is unbeatable
Lattice unbeatable?

- recent prevailing view: lattice is unbeatable
- yet determination of $\alpha_s$ from experiments remains desirable
  (or at least a fancy)

$\alpha_s = 0.1181 \pm 0.0011$

D. d’Enterria, arXiv: 1806.06156
Lattice unbeatable?

- recent prevailing view: lattice is unbeatable
- yet determination of $\alpha_s$ from experiments remains desirable (or at least a fancy)
- $e^+e^-$ event shapes, jets
  - ✓ are sensitive to $\alpha_s$
  - ✓ are measured extensively
  - ✓ can almost be computed from first principles (assuming local parton-hadron duality)

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**Table 1** summarizes all high-precision $\alpha_s$ values extracted so far. The $c^2$-averaging of the six subgroups of observables currently in the PDG-2017 yields $\alpha_s(m_Z^2) = 0.1181 \pm 0.0011$. Inclusion of the newly derived (red-italics) values has almost no impact in four subclasses (lattice QCD, PDF, $e^+e^-$, $Z$ decays) but would change by $0.4\%$ ($\pm 2\%$) the $t$-based pre-averages (Fig. 1). The updated world-average, combining all results, would thereby be $\alpha_s(m_Z^2) = 0.1183 \pm 0.0008$ with slightly increased central value and decreased uncertainty ($\sim 0.7\%$).

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D. d’Enterria, arXiv: 1806.06156
Our picture of a high-energy particle collision
Our picture of a high-energy particle collision

1. High-$Q^2$ Scattering

2. Parton Shower

- where new physics lies
- process dependent
- first principles description
- it can be systematically improved

3. Hadronization

4. Underlying Event
Our picture of a high-energy particle collision

1. High-$Q^2$ Scattering

- QCD - "known physics"
- universal/ process independent
- first principles description

2. Parton Shower

3. Hadronization

4. Underlying Event
Our picture of a high-energy particle collision
Our picture of a high-energy particle collision

1. High-$Q^2$ Scattering

   - low $Q^2$ physics
   - energy and process dependent
   - model dependent

   absent in lepton collisions

2. Parton Shower

3. Hadronization

4. Underlying Event
Shapes at NLO+NLL+power corr.+had. mass at LEP

Figure 9: 1-σ confidence-level contours from fits to event-shape variable $a$.

(a) Fits in the default schemes (normal hadron level).
(b) Fits in the $E$-scheme (normal hadron level), with arrows indicating the motion of the contour in going from the default to the $E$-scheme.
(c) Fits in the $E$-scheme at resonance level, with arrows indicating the motion of the contour from the decay-scheme, to the hadron-level $E$-scheme, to the resonance $E$-scheme — here the correction to resonance level has carried out using only events with light primary quarks.
(d) Fits in the $E$-scheme at resonance level where the correction to resonance level now includes events with heavy primary quarks as well — the arrows indicate the motion from the 'uds' resonance level.
$T = \text{thrust: how pencil-like is the event}$
Three-jet event shapes at LEP

- LO vs. NLO vs. data:
  - suffer large perturbative & hadronization corrections

- new since LEP:
  - ✓ NNLO corrections
  - ✓ $N^2$LL or $N^3$LL resummation
New measurements of $a_s$
NNLO is not enough

\[
\frac{\tau \, d\sigma}{\sigma \, d\tau} = \left( \frac{\alpha_s}{2\pi} \right) A(\tau) + \left( \frac{\alpha_s}{2\pi} \right)^2 B(\tau) + \left( \frac{\alpha_s}{2\pi} \right)^3 C(\tau)
\]

A, B and C computed with MCCSM (=Monte Carlo for CoLoRFuNNLO Subtraction Method)

\[ T = \max_{\vec{n}} \left( \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \right) \]
The analytic structure of perturbative expansion is given by:

\[
\frac{\tau}{\sigma} \frac{d\sigma}{d\tau} = \left( \frac{\alpha_s}{2\pi} \right) A(\tau) + \left( \frac{\alpha_s}{2\pi} \right)^2 B(\tau) + \left( \frac{\alpha_s}{2\pi} \right)^3 C(\tau)
\]

where

\[
A(\tau) = A_1 L - A_0,
\]
\[
B(\tau) = B_3 L^3 + B_2 L^2 + B_1 L + B_0,
\]
\[
C(\tau) = C_5 L^5 + C_4 L^4 + C_3 L^3 + C_2 L^2 + C_1 L + C_0
\]

and \( L = -\ln \tau \).

The expansion includes terms up to \( N^3_{\text{LL}} \), with \( N_{\text{LL}} \) indicating leading logarithmic order, \( N_{\text{NLL}} \) next-to-leading logarithmic order, etc. The \( L = -\ln \tau \) terms need resummation for all orders.
✓ Match to approximate predictions that resum large logarithms of the event shapes

precise predictions are available, e.g.:

- $N^3LL$ for thrust ($\tau$), C-parameter and heavy jet mass ($\rho$)
- $N^2LL$ for broadenings and EEC
Matching NNLO with $N^3$LL

Works for $\tau > 0.1$, fails in peak regions
✓ Match to approximate predictions that resum large logarithms of the event shapes

precise predictions are available, e.g.:

- $N^3LL$ for thrust ($\tau$), C-parameter and heavy jet mass ($\rho$)
- $N^2LL$ for broadenings and EEC

✓ Correct for hadronisation

two options:

- estimate of hadronisation using modern MC tools
- use analytic model for power corrections, e.g.:

$$\frac{\tau}{\sigma} \frac{d\sigma}{d\tau} (\tau) \rightarrow \frac{\tau}{\sigma} \frac{d\sigma}{d\tau} (\tau - 2a_0)$$
Fit to data with NNLO+N^3LL+PC

Works down to the peak, but
Fit data on heavy jet mass with NNLO+N^3LL+PC

not as expected
Fit to data with PC

... and look marginally universal
Fit to data with PC

...and look marginally universal

but $a_0$ and $\alpha_s$ are strongly anticorrelated
EEC @ fixed orders

\[ EEC(\chi) = \frac{1}{\sigma_{\text{had}}} \sum_{i,j} \int \frac{E_i E_j}{Q^2} \]
\[ \times d\sigma_{e^+e^- \rightarrow i + j + X} \delta(\cos \chi + \cos \theta_{ij}) \]

\[ \sin^2 \Delta \text{EEC} \]
\[ \sqrt{Q^2} = 91.2 \text{ GeV} \]
\[ \alpha_s(Q^2) = 0.118 \]

only MCCSM can compute NNLO

large corrections
\[ \alpha_S(M_Z) = 0.121^{+0.001}_{-0.003} \] \[ a_1 = 2.47^{+0.48}_{-2.38} \text{ GeV}^2 \] \[ a_2 = 0.31^{+0.27}_{-0.05} \text{ GeV} \]

\[
\text{corr}(\alpha_S, a_1, a_2) = \begin{pmatrix}
1 & 0.05 & -0.97 \\
0.05 & 1 & -0.07 \\
-0.97 & -0.07 & 1 \\
\end{pmatrix}
\]
\( \alpha_S(M_Z) = 0.121^{+0.001}_{-0.003} \)  
\( a_1 = 2.47^{+0.48}_{-2.38} \text{ GeV}^2 \)  
\( a_2 = 0.31^{+0.27}_{-0.05} \text{ GeV} \)

\[ \text{corr}(\alpha_S, a_1, a_2) = \begin{pmatrix} 1 & 0.05 & -0.97 \\ 0.05 & 1 & -0.07 \\ -0.97 & -0.07 & 1 \end{pmatrix} \]

**parameters are strongly anticorrelated**
How to improve?

✓ Correct for hadronisation, 2nd option:
  - estimate of hadronisation using modern MC tools

\[
\frac{1}{\sigma_t} \frac{d\Sigma}{d \cos \chi} = \frac{1}{\sigma_t} \int \sum_{i,j} \frac{E_i E_j}{Q^2} d\sigma_{e^+e^- \rightarrow ij} + \chi \delta(\cos \chi + \cos \theta_{ij})
\]

Hadronization corrections are parametrized using smooth functions to tame statistical fluctuations

Parametrization is valid only in the fit range

Z. Tulipánt et al, arXiv: 1804.09146
Global fit at NNLL+NLO:

$$\alpha_S(M_Z) = 0.12200 \pm 0.00023(exp.) \pm 0.00113(hadr.) \pm 0.00433(ren.) \pm 0.00293(res.)$$

with combined uncertainty: $$\alpha_S(M_Z) = 0.12200 \pm 0.00535$$

Global fit at NNLL+NNLO:

$$\alpha_S(M_Z) = 0.11750 \pm 0.00018(exp.) \pm 0.00102(hadr.) \pm 0.00257(ren.) \pm 0.00078(res.)$$

with combined uncertainty: $$\alpha_S(M_Z) = 0.11750 \pm 0.00287$$

The effect of NNLO on central value is moderate but not negligible, ren. uncertainty down by a factor of 2, res. uncertainty down by a factor of 3

The overall uncertainty is dominated by theoretical uncertainty (ren. and res.)
Global fit at NNLL+ NNLO:

\[ \alpha_s(M_Z) = 0.11881 \pm 0.00063(\text{exp.}) \pm 0.00101(\text{hadr.}) \pm 0.00045(\text{ren.}) \pm 0.00034(\text{res.}) \]

with combined uncertainty: \[ \alpha_s(M_Z) = 0.11881 \pm 0.00131(\text{comb.}) \]

compared to result using energy-energy correlation:

Global fit at NNLL+NNLO:

\[ \alpha_S(M_Z) = 0.11750 \pm 0.00018(\text{exp.}) \pm 0.00102(\text{hadr.}) \pm 0.00257(\text{ren.}) \pm 0.00078(\text{res.}) \]

with combined uncertainty: \[ \alpha_S(M_Z) = 0.11750 \pm 0.00287 \]
Conclusions
Precise determination of the strong coupling using hadronic final states in electron-positron annihilation requires

- careful selection of observables (and data — not discussed here)
- methods to reduce hadronisation corrections
- estimation of the hadronisation corrections with modern MCs

New determination based on NNLO+NNLL+non-perturbative corrections from MC simulation gives

\[ \alpha_s(M_Z) = 0.11881 \pm 0.00131 \text{(comb.)} \]

with uncertainty competitive with other determinations.
Outlook
## Outlook: prospects for $\alpha_s$

<table>
<thead>
<tr>
<th>Method</th>
<th>Current relative precision</th>
<th>Future relative precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^-$ evt shapes</td>
<td>expt $\sim 1%$ (LEP)</td>
<td>$&lt; 1%$ possible (ILC/TLEP)</td>
</tr>
<tr>
<td></td>
<td>thry $\sim 1$–$3%$ (NNLO+up to $N^3$LL, n.p. signif.) [27]</td>
<td>$\sim 1%$ (control n.p. via $Q^2$-dep.)</td>
</tr>
<tr>
<td>$e^+e^-$ jet rates</td>
<td>expt $\sim 2%$ (LEP)</td>
<td>$&lt; 1%$ possible (ILC/TLEP)</td>
</tr>
<tr>
<td></td>
<td>thry $\sim 1%$ (NNLO, n.p. moderate) [28]</td>
<td>$\sim 0.5%$ (NLL missing)</td>
</tr>
<tr>
<td>precision EW</td>
<td>expt $\sim 3%$ ($R_Z$, LEP)</td>
<td>0.1% (TLEP [10]), 0.5% (ILC [11])</td>
</tr>
<tr>
<td></td>
<td>thry $\sim 0.5%$ ($N^3$LO, n.p. small) [9,29]</td>
<td>$\sim 0.3%$ (N$^4$LO feasible, $\sim 10$ yrs)</td>
</tr>
<tr>
<td>$\tau$ decays</td>
<td>expt $\sim 0.5%$ (LEP, B-factories)</td>
<td>$&lt; 0.2%$ possible (ILC/TLEP)</td>
</tr>
<tr>
<td></td>
<td>thry $\sim 2%$ ($N^3$LO, n.p. small) [8]</td>
<td>$\sim 1%$ (N$^4$LO feasible, $\sim 10$ yrs)</td>
</tr>
<tr>
<td>$ep$ colliders</td>
<td>$\sim 1$–$2%$ (pdf fit dependent) [30,31], (mostly theory, NNLO) [32,33]</td>
<td>0.1% (LHeC + HERA [23])</td>
</tr>
<tr>
<td></td>
<td>$\sim 10$ yrs)</td>
<td>$\sim 0.5%$ (at least N$^3$LO required)</td>
</tr>
<tr>
<td>hadron colliders</td>
<td>$\sim 4%$ (Tev. jets), $\sim 3%$ (LHC $tt$)</td>
<td>$&lt; 1%$ challenging</td>
</tr>
<tr>
<td></td>
<td>(NLO jets, NNLO $tt$, gluon uncert.) [17,21,34]</td>
<td>(NNLO jets imminent [22])</td>
</tr>
<tr>
<td>lattice</td>
<td>$\sim 0.5%$ (Wilson loops, correlators, ...)</td>
<td>$\sim 0.3%$</td>
</tr>
<tr>
<td></td>
<td>(limited by accuracy of pert. th.) [35–37]</td>
<td>($\sim 5$ yrs [38])</td>
</tr>
</tbody>
</table>

### Determination of strong coupling from $e^+e^-$ data with decreased theoretical uncertainty might be possible
How to improve?

✓ Correct for hadronisation, 2nd option:
   – estimate of hadronisation using modern MC tools

✓ Find observable quantities with small perturbative and hadronisation corrections:
   motto: “large uncertainty in small quantity is small uncertainty”

V. Del Duca et al, arXiv:1606.03453
✓ Correct for hadronisation, 2nd option:
  – estimate of hadronisation using modern MC tools
✓ Find observable quantities with small perturbative and hadronisation corrections:
  motto: “large uncertainty in small quantity is small uncertainty”

jet cone energy fraction:

\[
\frac{d\Sigma_{JCEF}}{d\cos\chi} = \sum_i \int \frac{E_i}{Q} d\sigma_{e^+e^\to i+X} \delta \left( \cos\chi - \frac{\vec{p}_i \cdot \vec{n}_T}{|\vec{p}_i|} \right)
\]

V. Del Duca et al, arXiv:1606.03453
How to improve?

✓ Correct for hadronisation, 2nd option:
   – estimate of hadronisation using modern MC tools

✓ Find observable quantities with small perturbative and hadronisation corrections:
   motto: “large uncertainty in small quantity is small uncertainty”
   – precluster hadrons and compute shapes from jets
Preclustering reduces hadronization corrections

Old: without, New: with plecustering (requiring 5 jets)

A. Verbytskyi, private communication
✓ Correct for hadronisation, 2nd option:
  – estimate of hadronisation using modern MC tools
✓ Find observable quantities with small perturbative and hadronisation corrections:

motto: “large uncertainty in small quantity is small uncertainty”
  – precluster hadrons and compute shapes from jets
  – groomed (soft drop) event shapes, designed to reduce contamination from non-perturbative effects


Work in progress...