

Determination of the dissociation energy of para-H₂

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S. Hu^c, Ch. Jungen^d

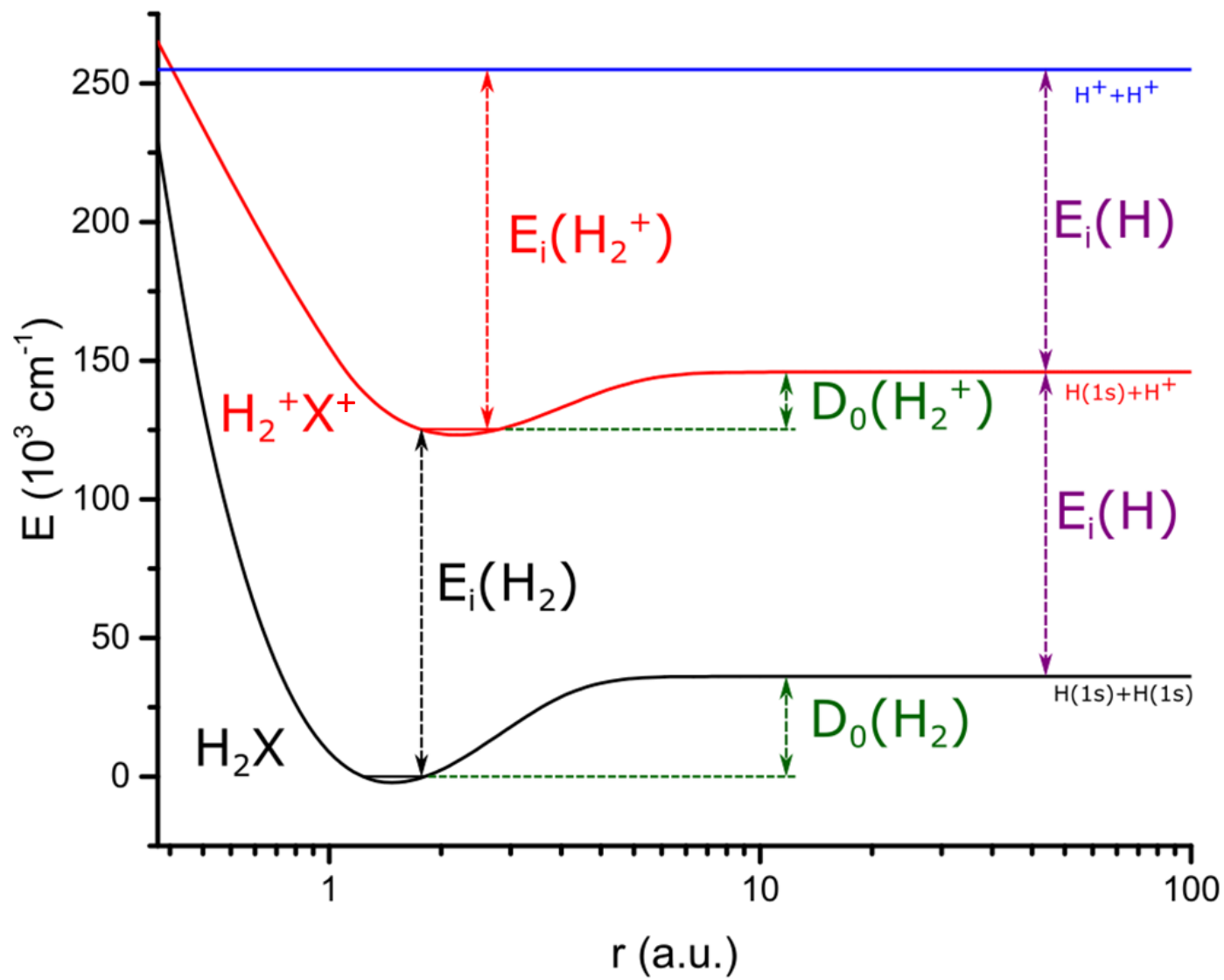
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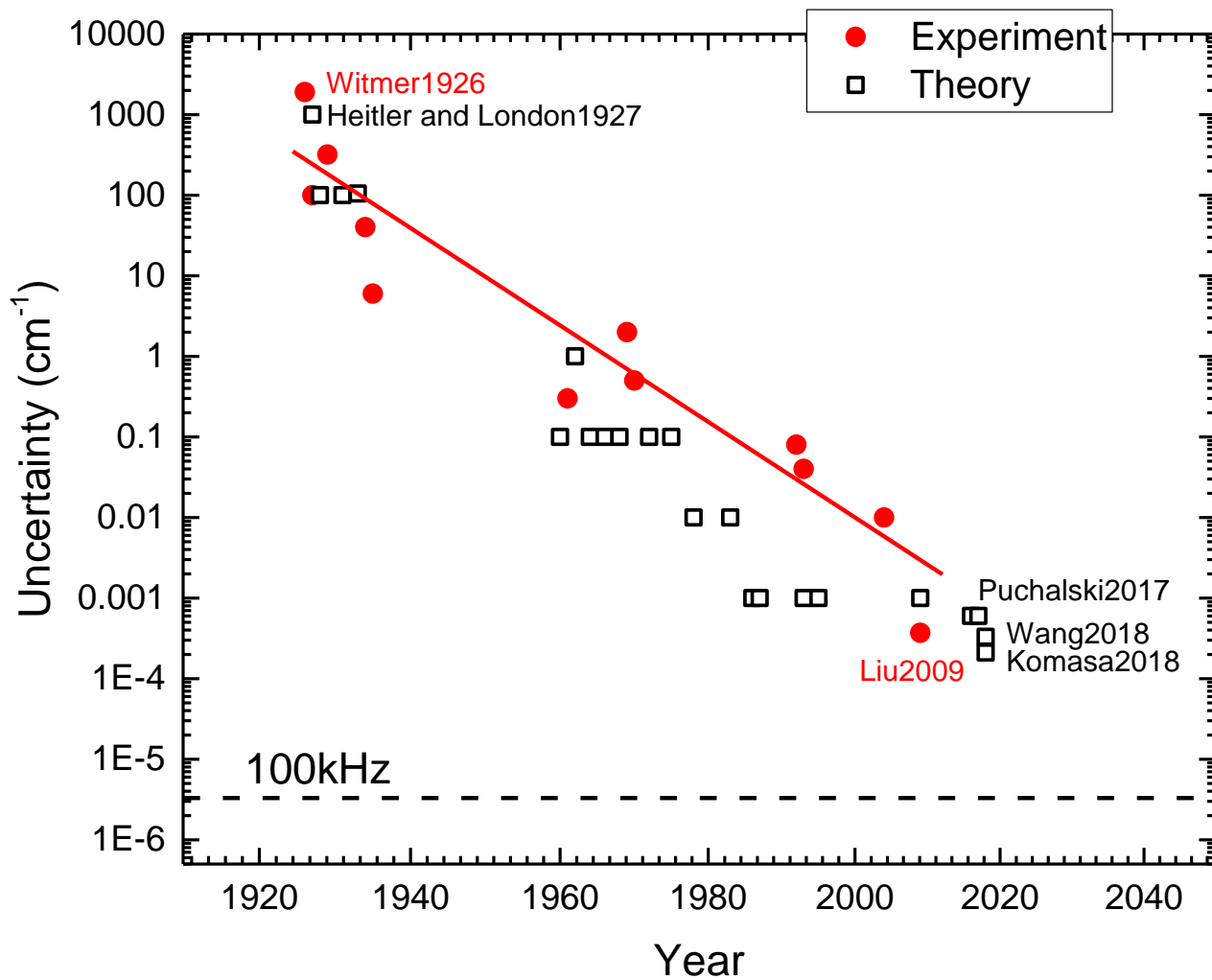
^c Department of Chemical Physics, University of Science and Technology of China

^d Laboratoire Aimé Cotton, CNRS, Orsay

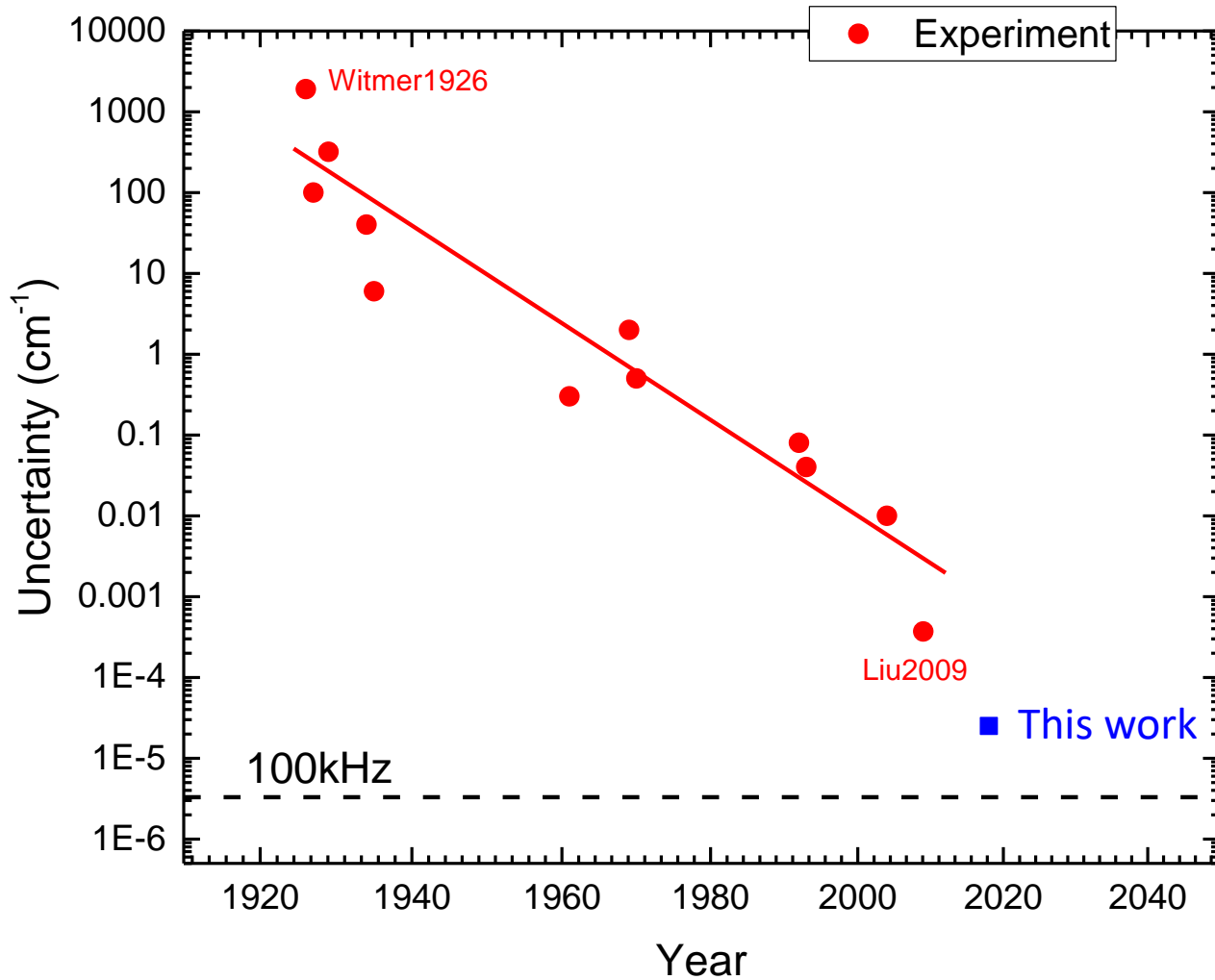
Determining D_0



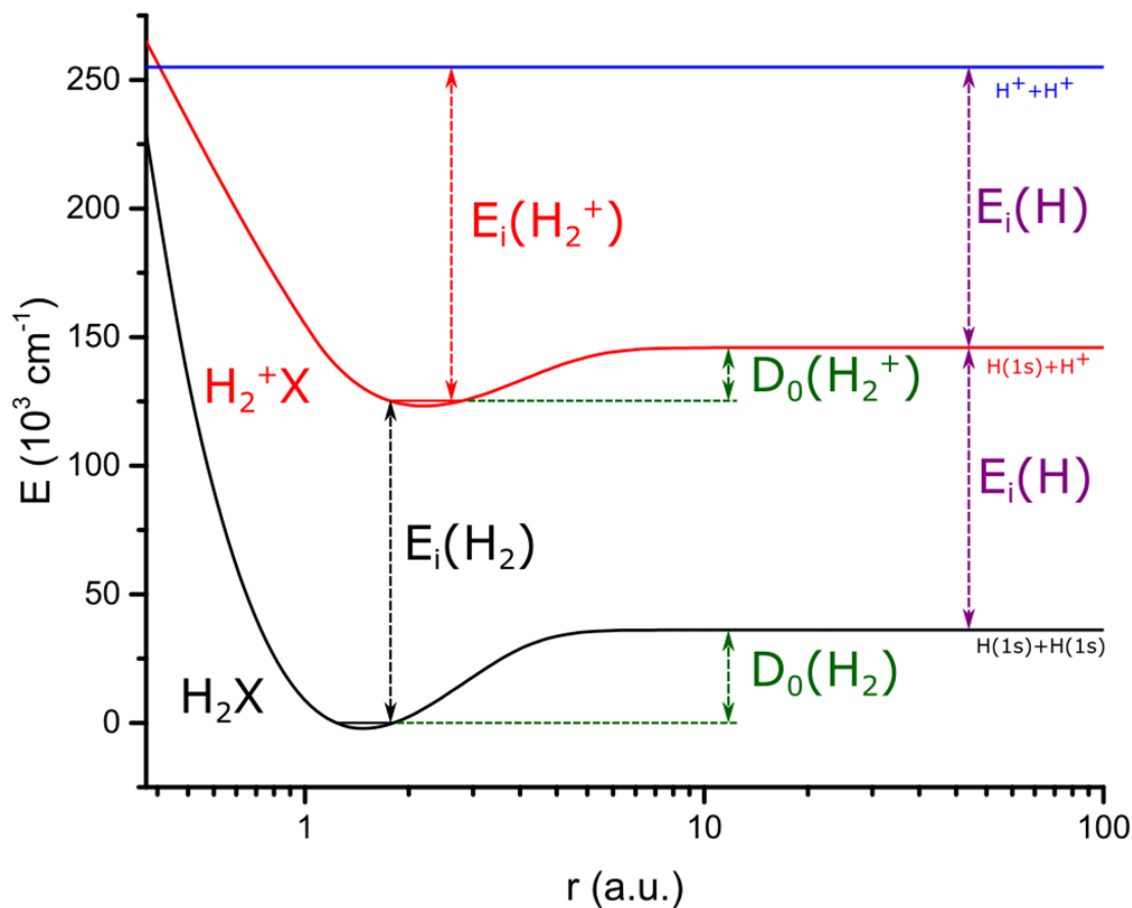
Test of molecular theory with D_0



Test of molecular theory with D_0



Determining $D_0(\text{Ortho-H}_2)$



$$D_0(\text{H}_2) = E_i(\text{H}_2) + E_i(\text{H}_2^+) - 2E_i(\text{H})$$

$$E_i(\text{H}) = 109678.77174307(10) \text{ cm}^{-1} \\ \sim 3 \text{ kHz}$$

$$E_i(\text{H}_2^+) = 131058.1219937(6) \text{ cm}^{-1} \\ \sim 18 \text{ kHz}$$

$$E_i(\text{H}_2) = 124417.49113(37) \text{ cm}^{-1} \\ \sim 11 \text{ MHz}$$

$$D_0(\text{H}_2) = 36118.0696(4) \text{ cm}^{-1} \\ \sim 11 \text{ MHz}$$

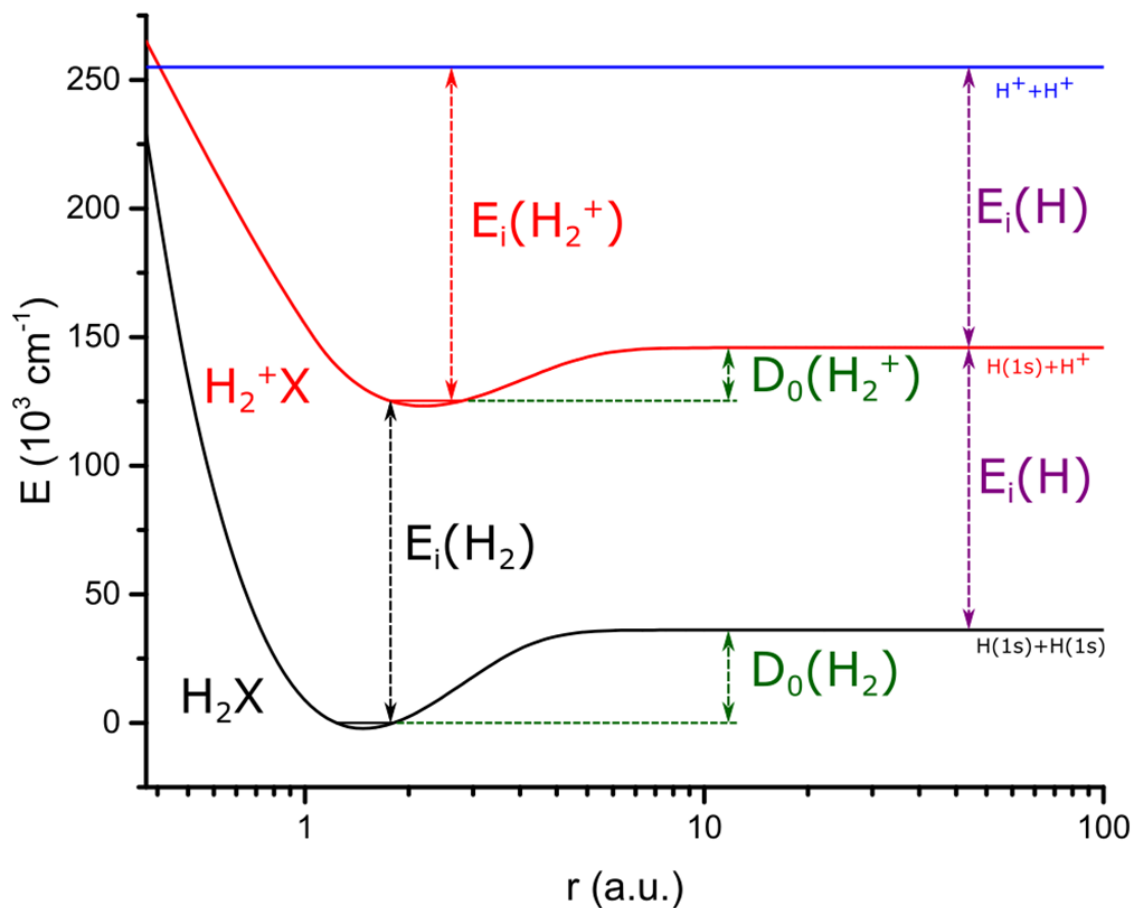
Korobov, PRL 2017

Mohr, RMP 2016

Liu, JCP 2009

Determining $D_0(\text{Ortho-H}_2)$

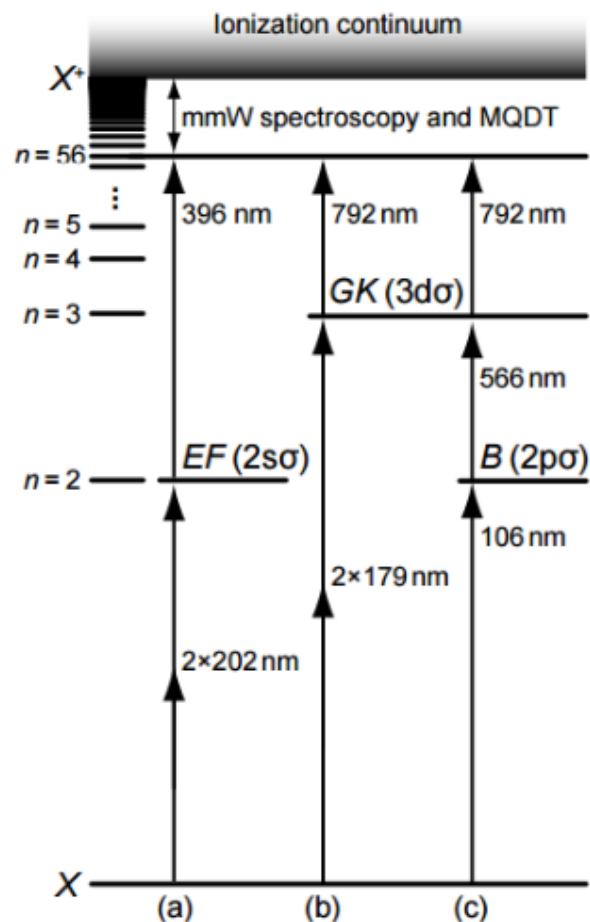
$$D_0(\text{H}_2) = E_i(\text{H}_2) + E_i(\text{H}_2^+) - 2E_i(\text{H})$$



Value	Uncertainty
$E_i(\text{H})$	3 kHz
$E_i(\text{H}_2^+)$	18 kHz
$E_i(\text{H}_2)$	11 MHz
$D_0(\text{H}_2)$	11 MHz

Korobov, PRL 2017
 Mohr, RMP 2016
 Liu, JCP 2009

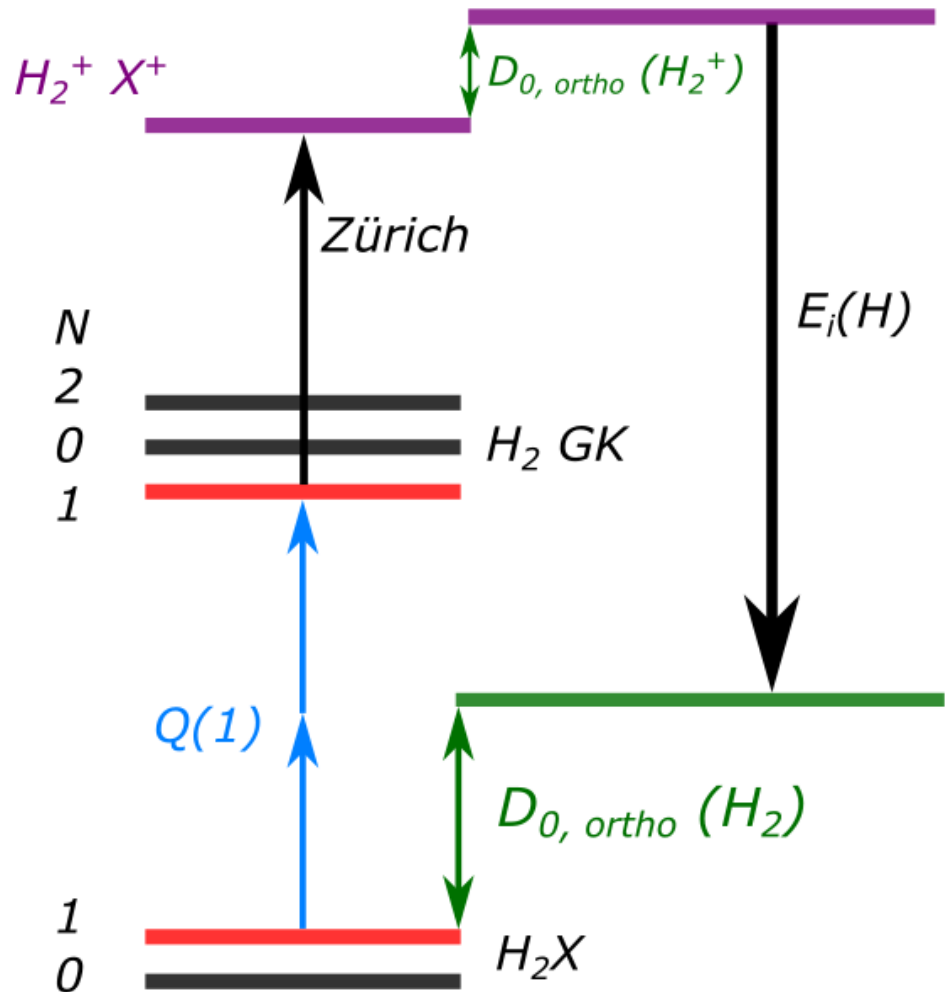
The ionization energy



Ortho- and Para-Hydrogen

Ortho:

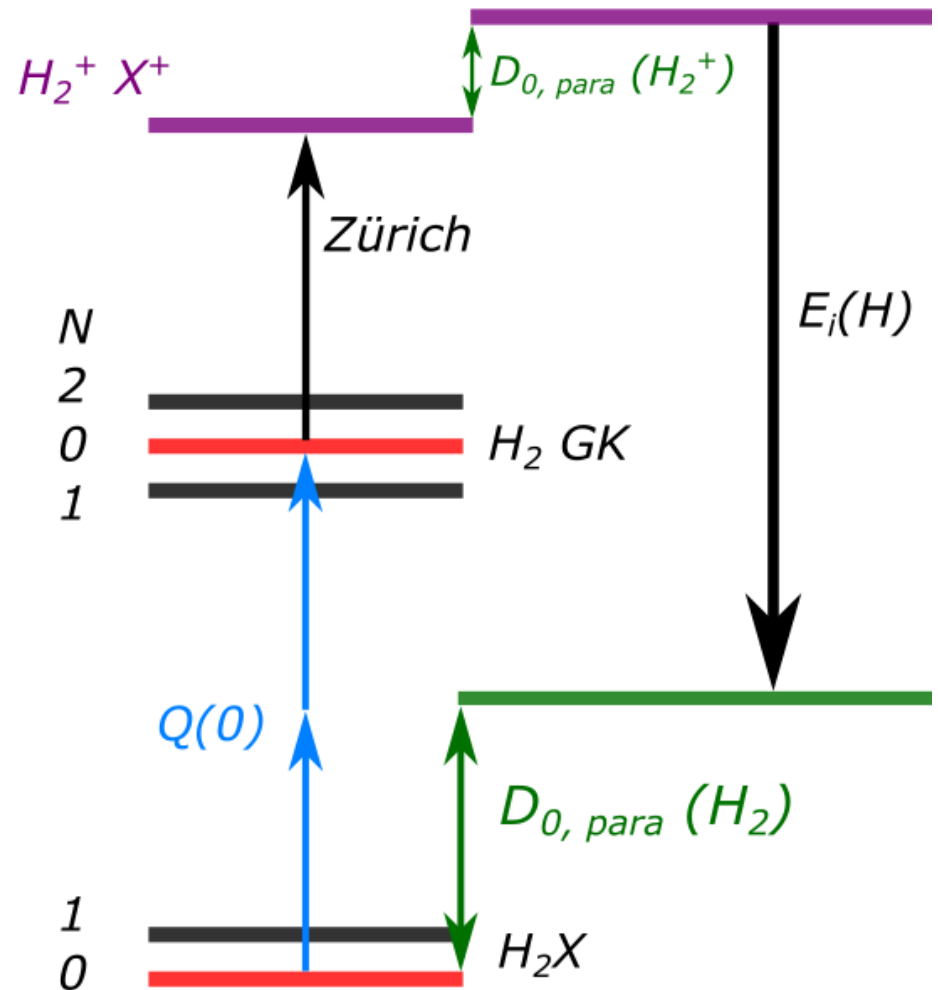
- Nuclear spins parallel
- More signal
- Previous measurements



Ortho- and Para-Hydrogen

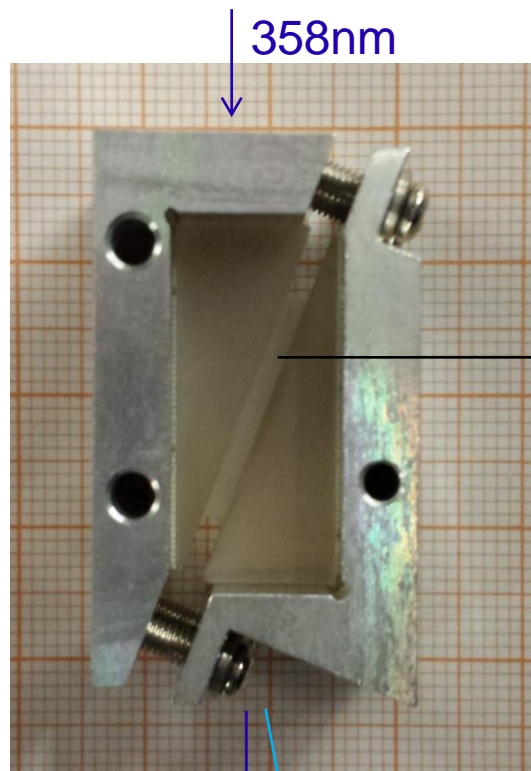
Para:

- Nuclear spins antiparallel
- No hyperfine structure
- "Real" Dissociation Energy



VUV generation: KBBF

2nd harmonic from 716 nm laser

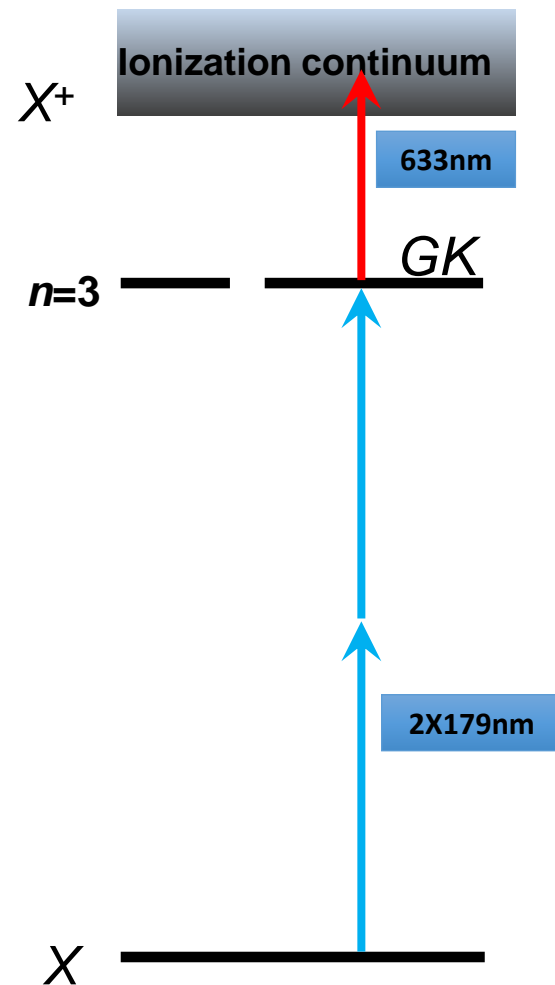
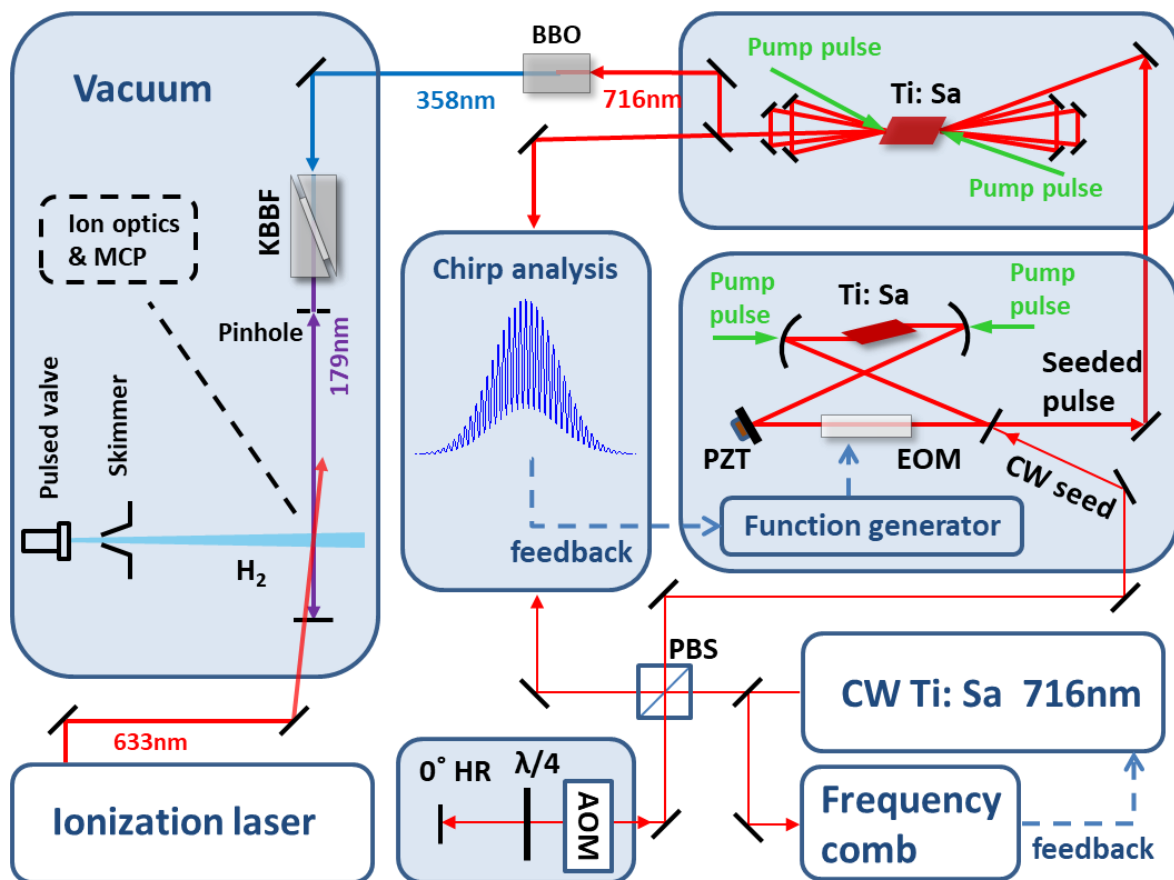


- $\text{KBe}_2\text{BO}_3\text{F}_2$
- Used in Vacuum
- Low efficiency (0.01% to 0.1%)
- Low damage threshold

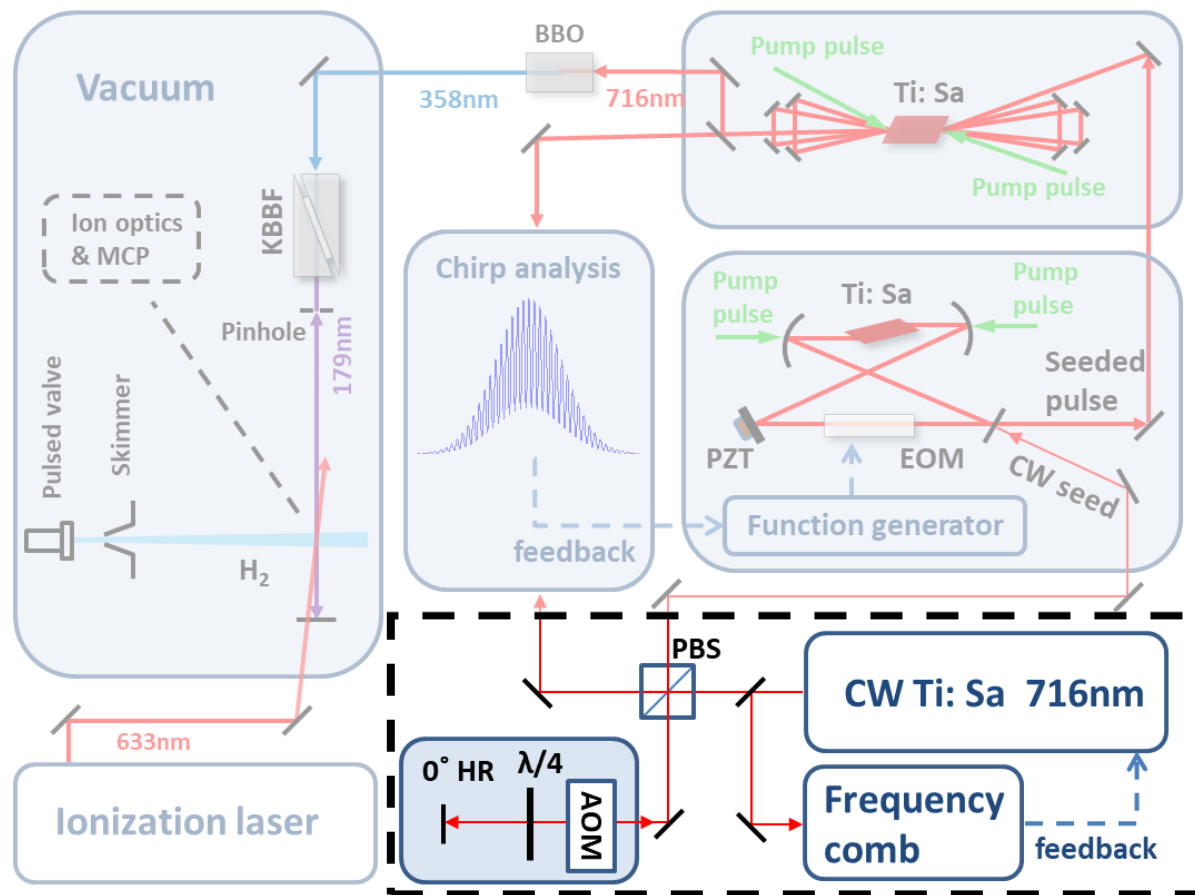
358nm
179nm

Provided by prof. dr. S. Hu,
Department of Chemical Physics,
University of Science and Technology of China

Experimental setup



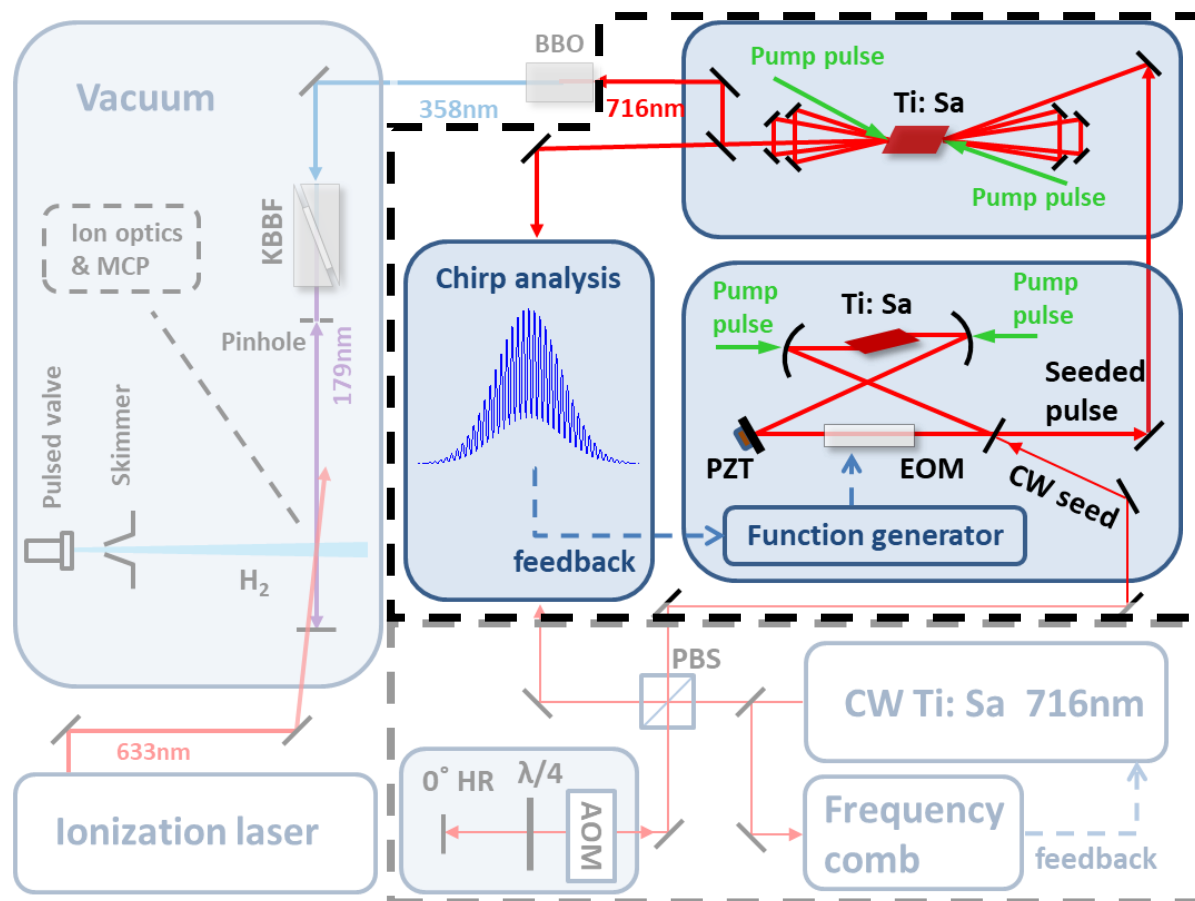
Experimental setup



Frequency accuracy $< 10^{-12}$

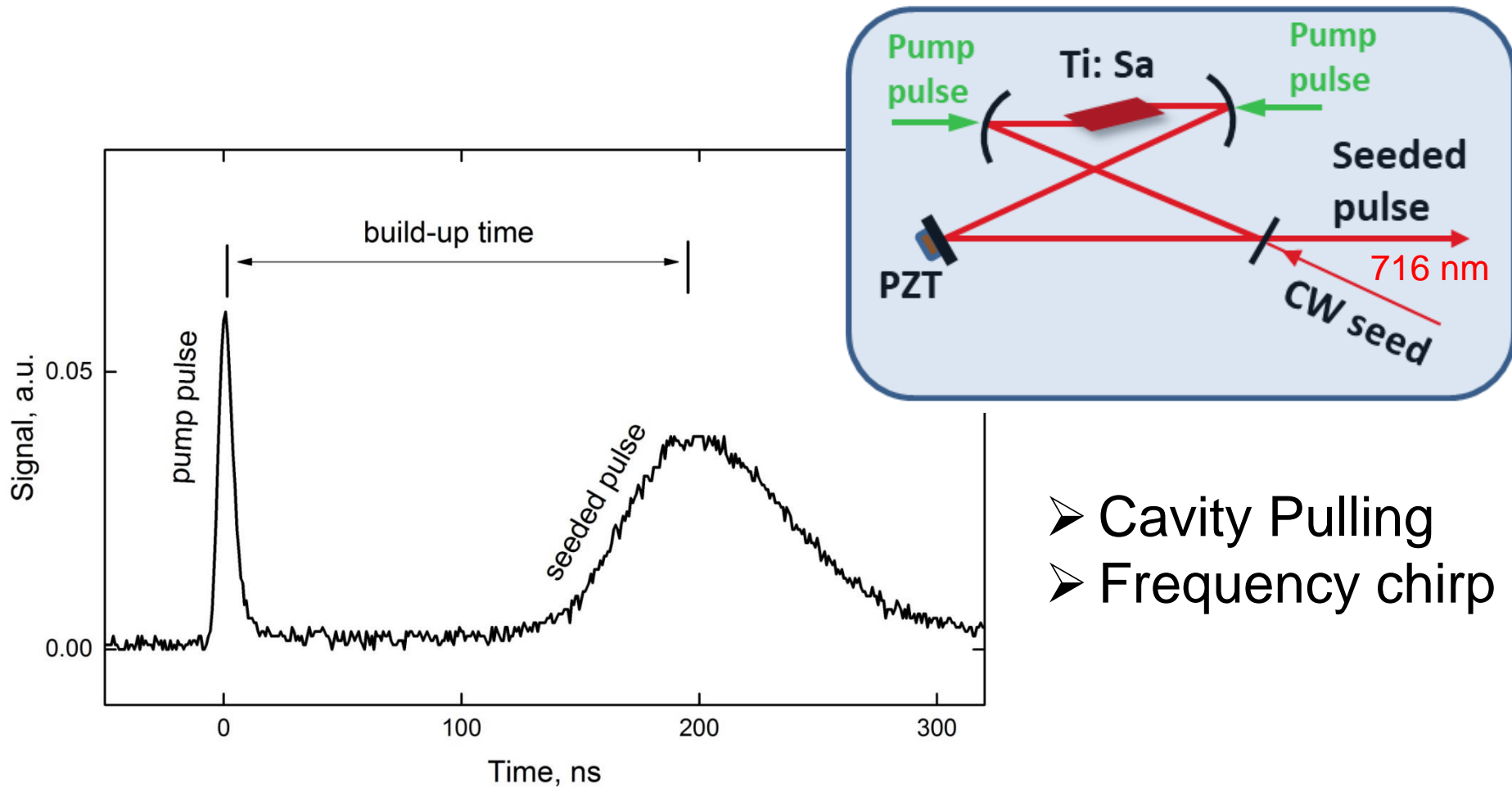
Experimental setup

High power, high accuracy



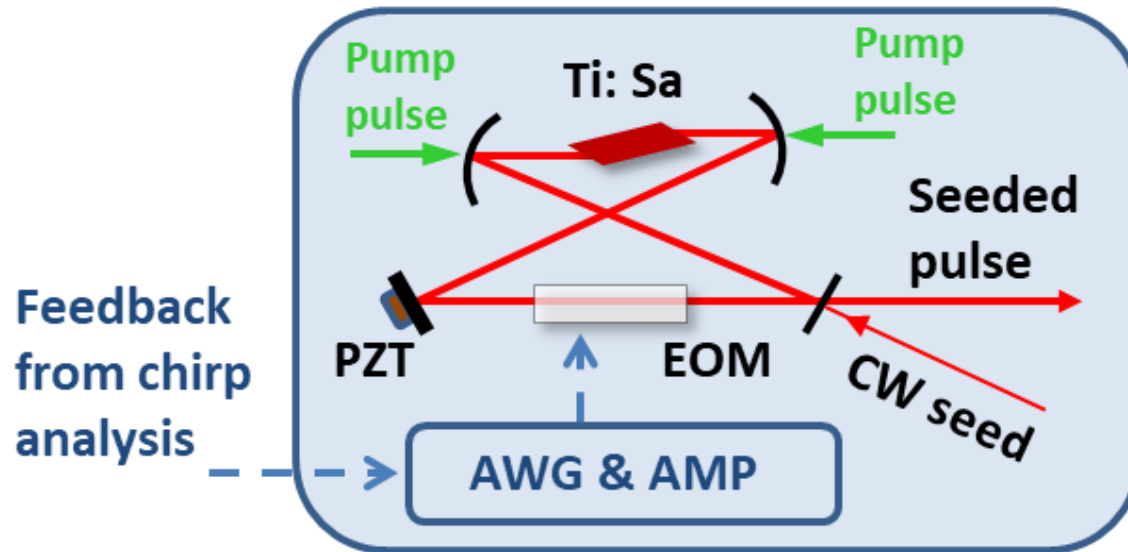
Frequency accuracy $< 10^{-12}$

Chirp



- Cavity Pulling
- Frequency chirp

Anti-chirp

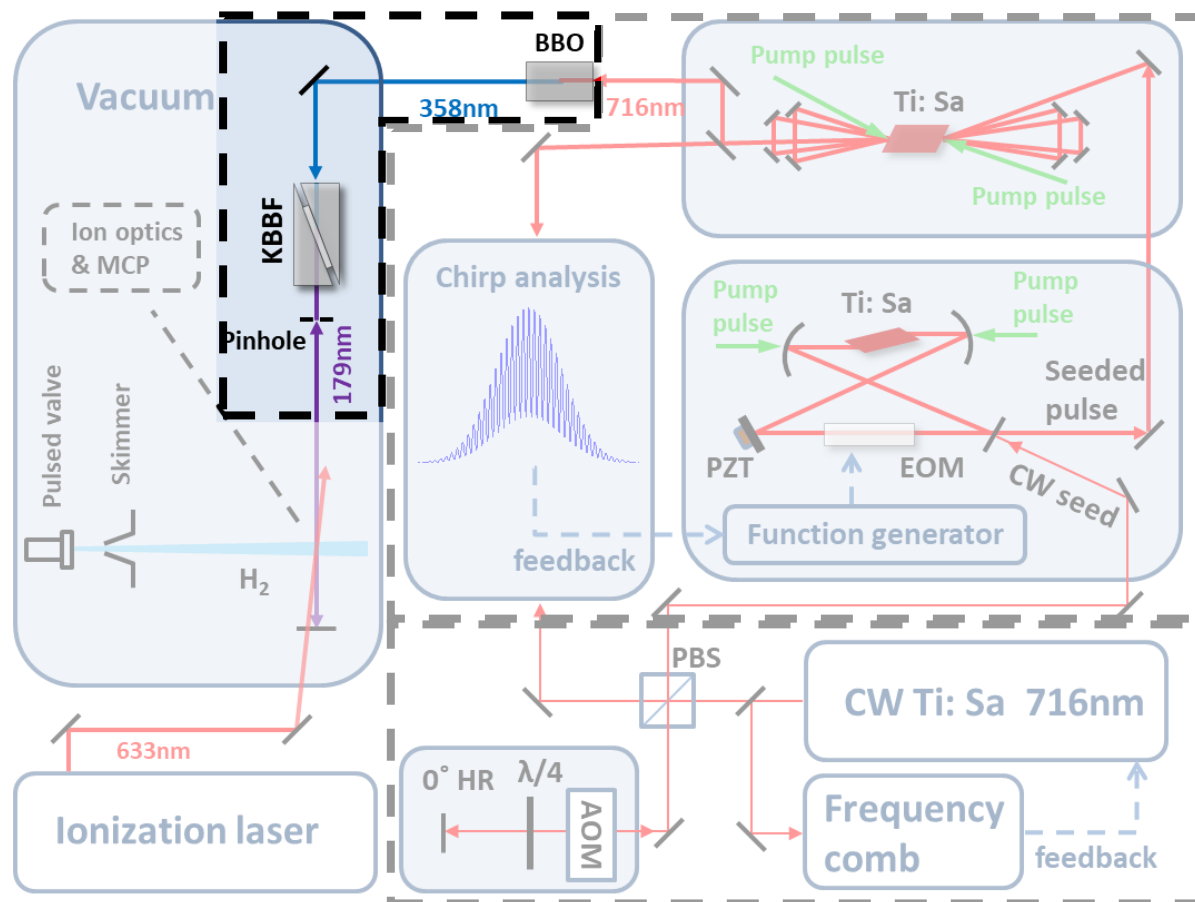


- Sensitive to temperature and stress
- Arbitrary function generator to drive
- Amplified to max 150V

Experimental setup

VUV generation

High power, high accuracy

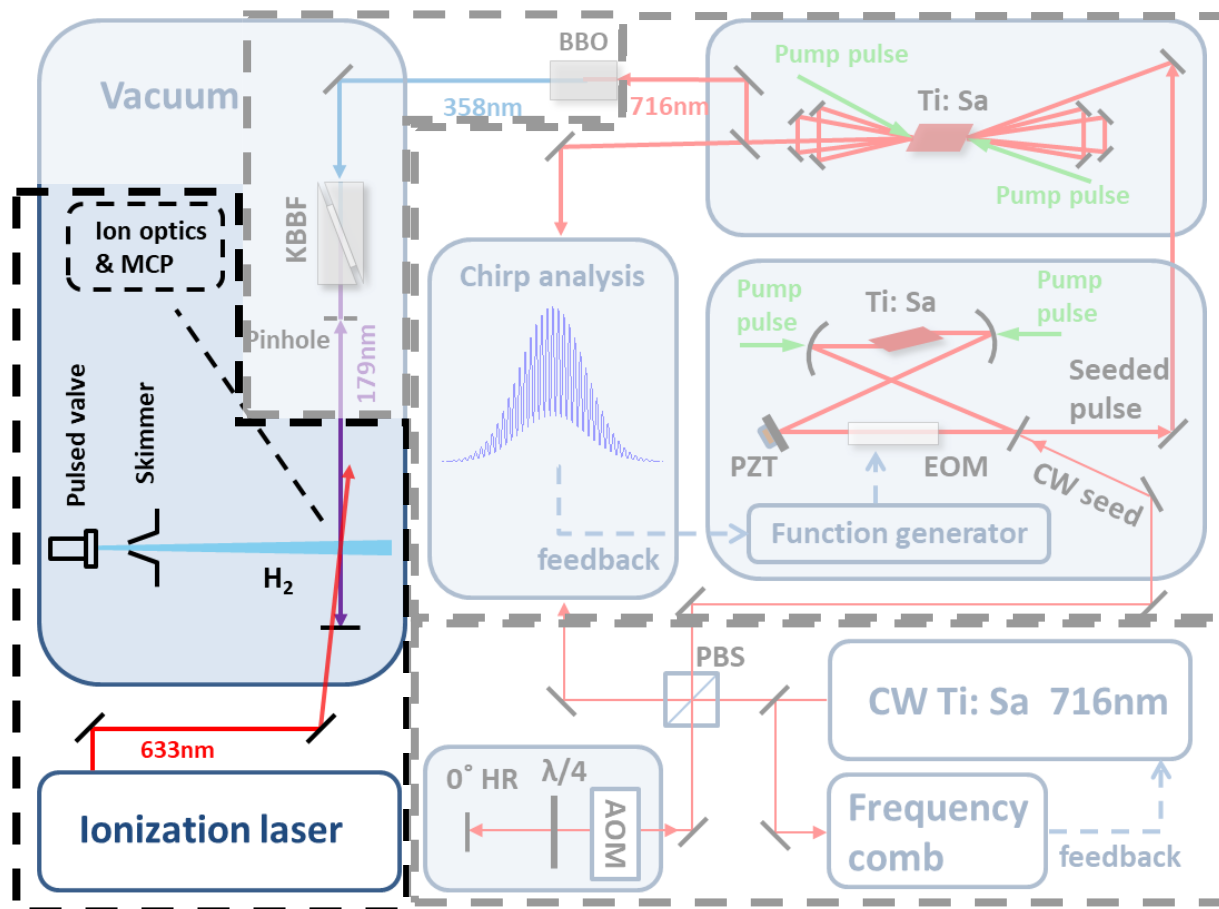


Frequency accuracy $< 10^{-12}$

Experimental setup

VUV generation

High power, high accuracy

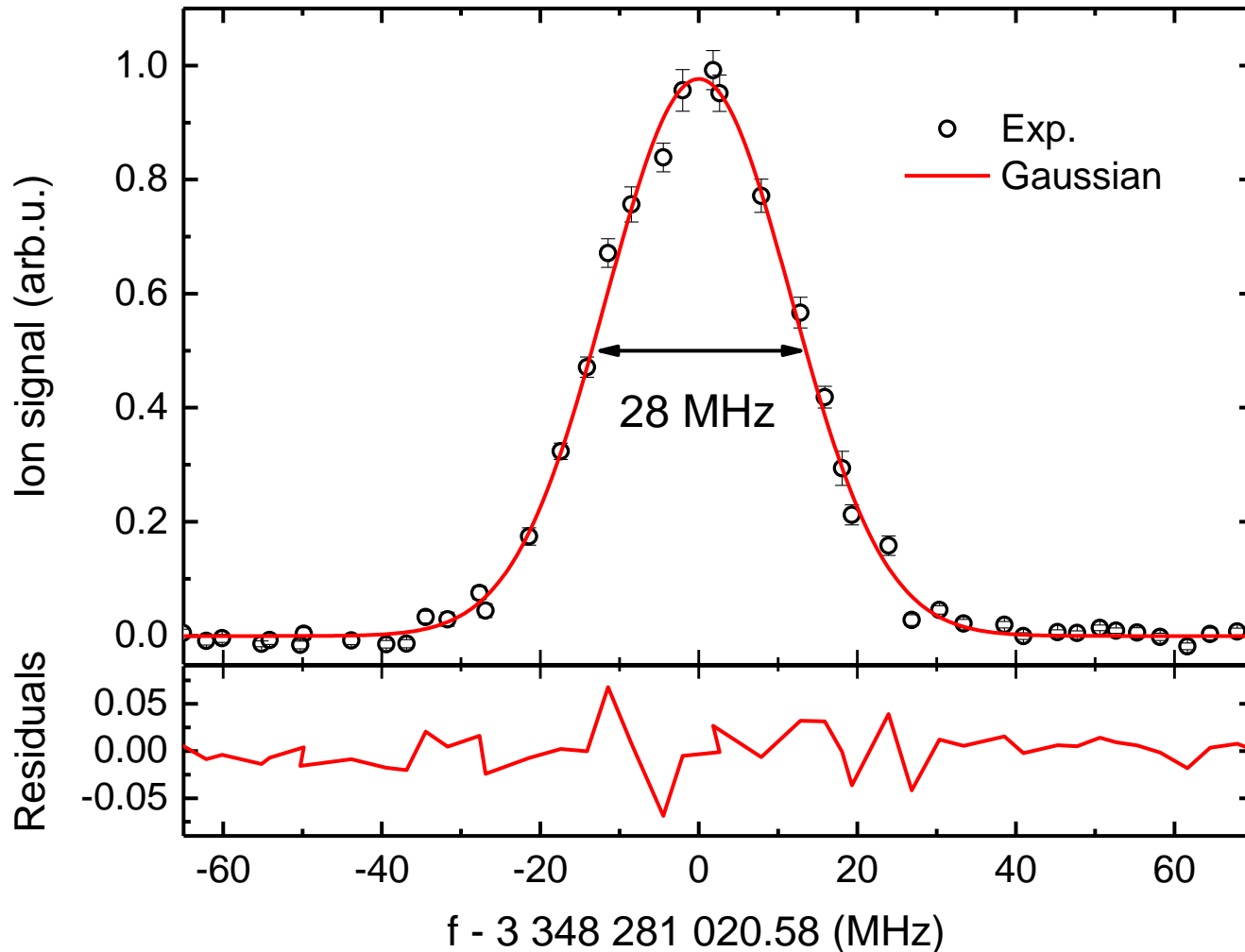


Ionize and detect

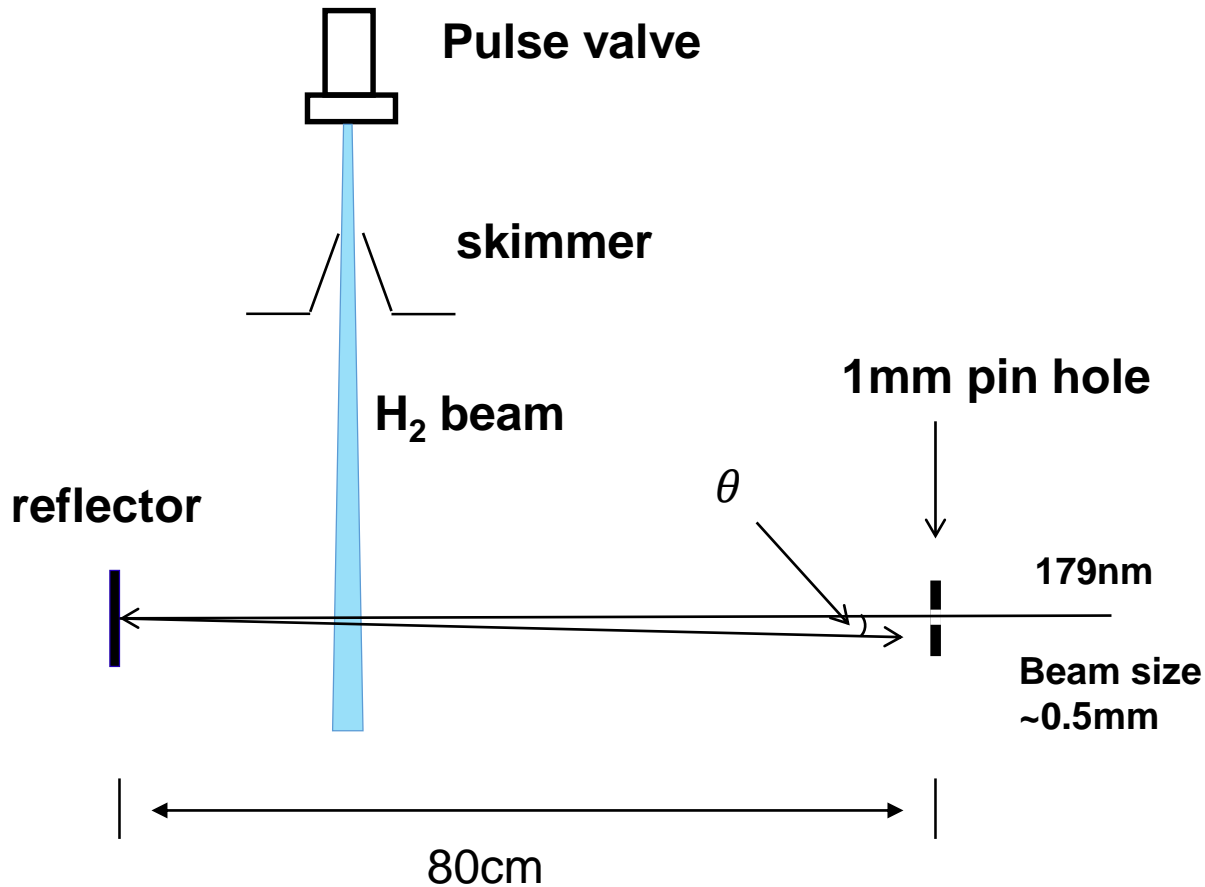
Frequency accuracy < 10⁻¹²

Scan of GK-X Q(1) in H₂

GK($v=1, N=1$) - X($v=0, N=1$), chirp compensated shot-by-shot.



Systematic check: Residual Doppler



$$\frac{\delta f}{f} = \frac{v}{c} \sin \frac{\theta}{2}$$

$$v = 2900(300) \text{ m/s}$$

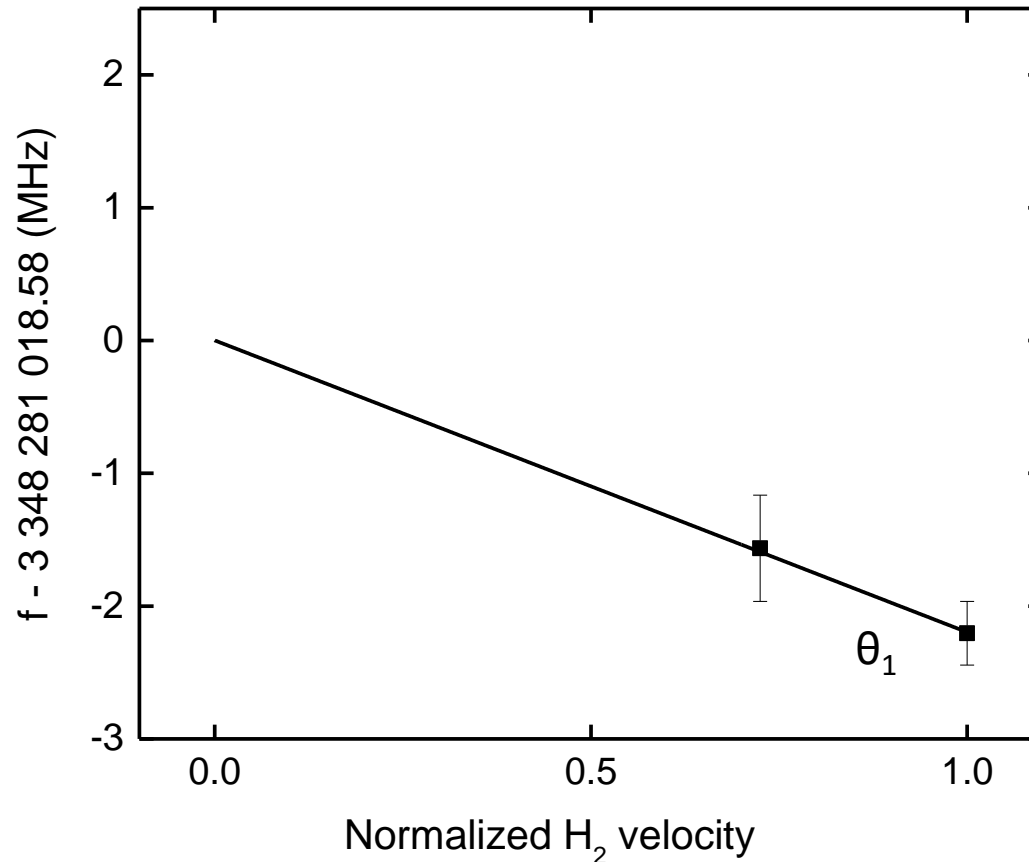
$$\theta < 0.3 \text{ mrad}$$

$$\rightarrow \delta f < 3 \text{ MHz}$$

Systematic check: Residual Doppler



$$v_{norm} = \frac{v_{mix}}{v_{pure}} = \sqrt{\frac{m_{H_2}}{n_{H_2} \cdot m_{H_2} + n_{Ne} \cdot m_{Ne}}}$$

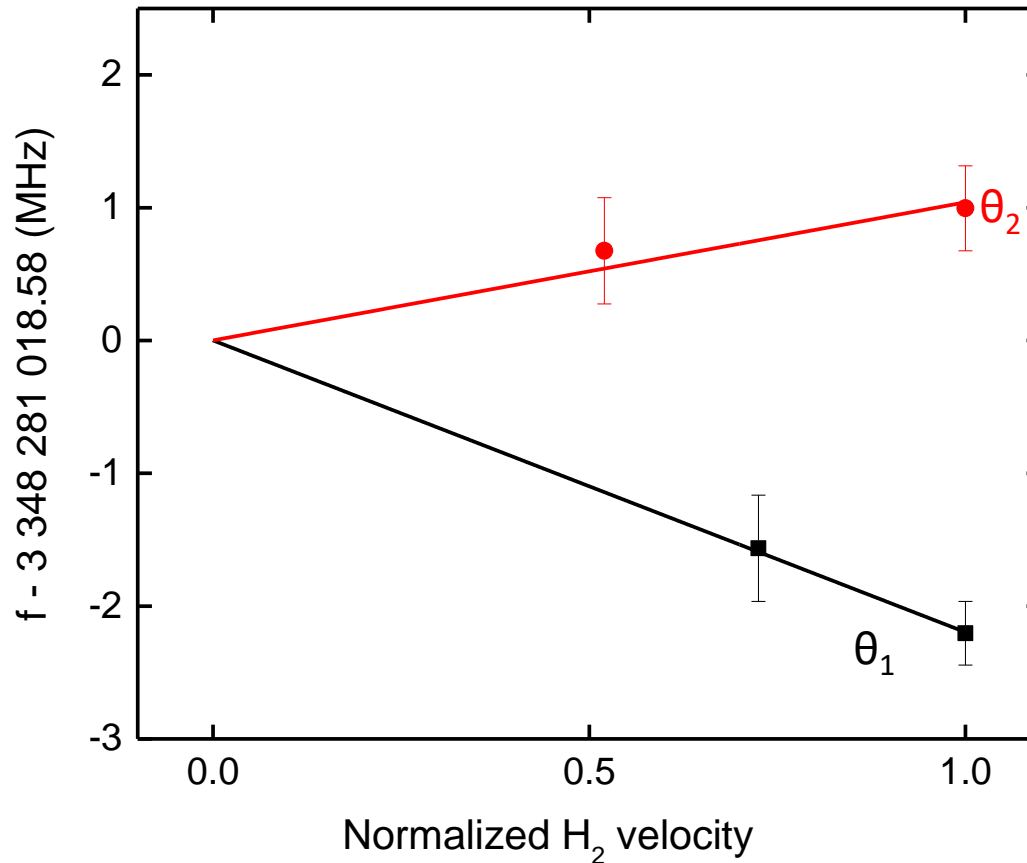


Systematic check: Residual Doppler



$$v_{norm} = \frac{v_{mix}}{v_{pure}} = \sqrt{\frac{m_{H_2}}{n_{H_2} \cdot m_{H_2} + n_{Ne} \cdot m_{Ne}}}$$

Changing θ

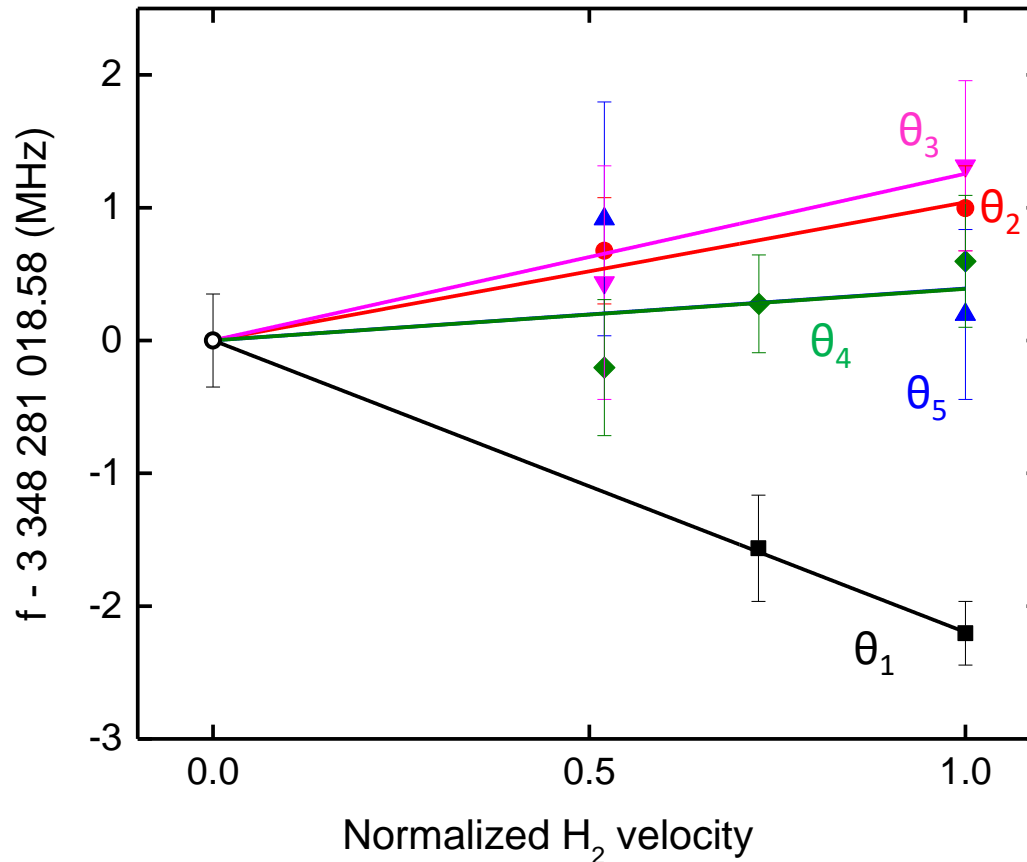


Systematic check: Residual Doppler

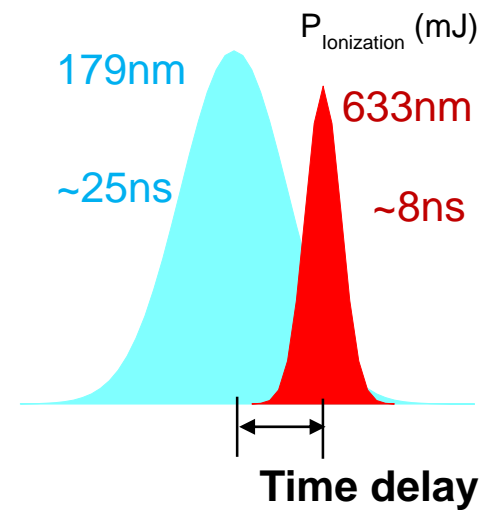
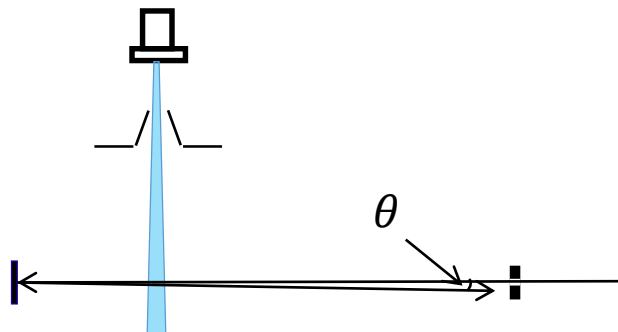
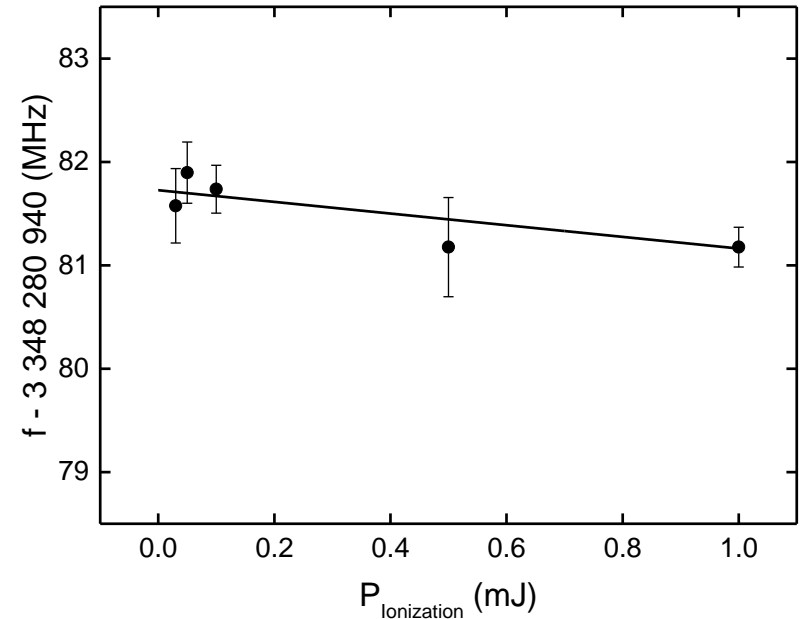
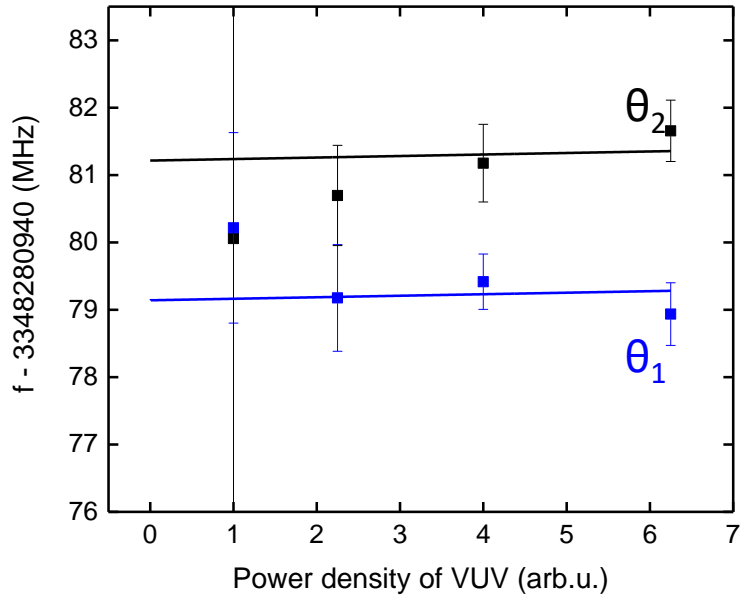


$$v_{norm} = \frac{v_{mix}}{v_{pure}} = \sqrt{\frac{m_{H_2}}{n_{H_2} \cdot m_{H_2} + n_{Ne} \cdot m_{Ne}}}$$

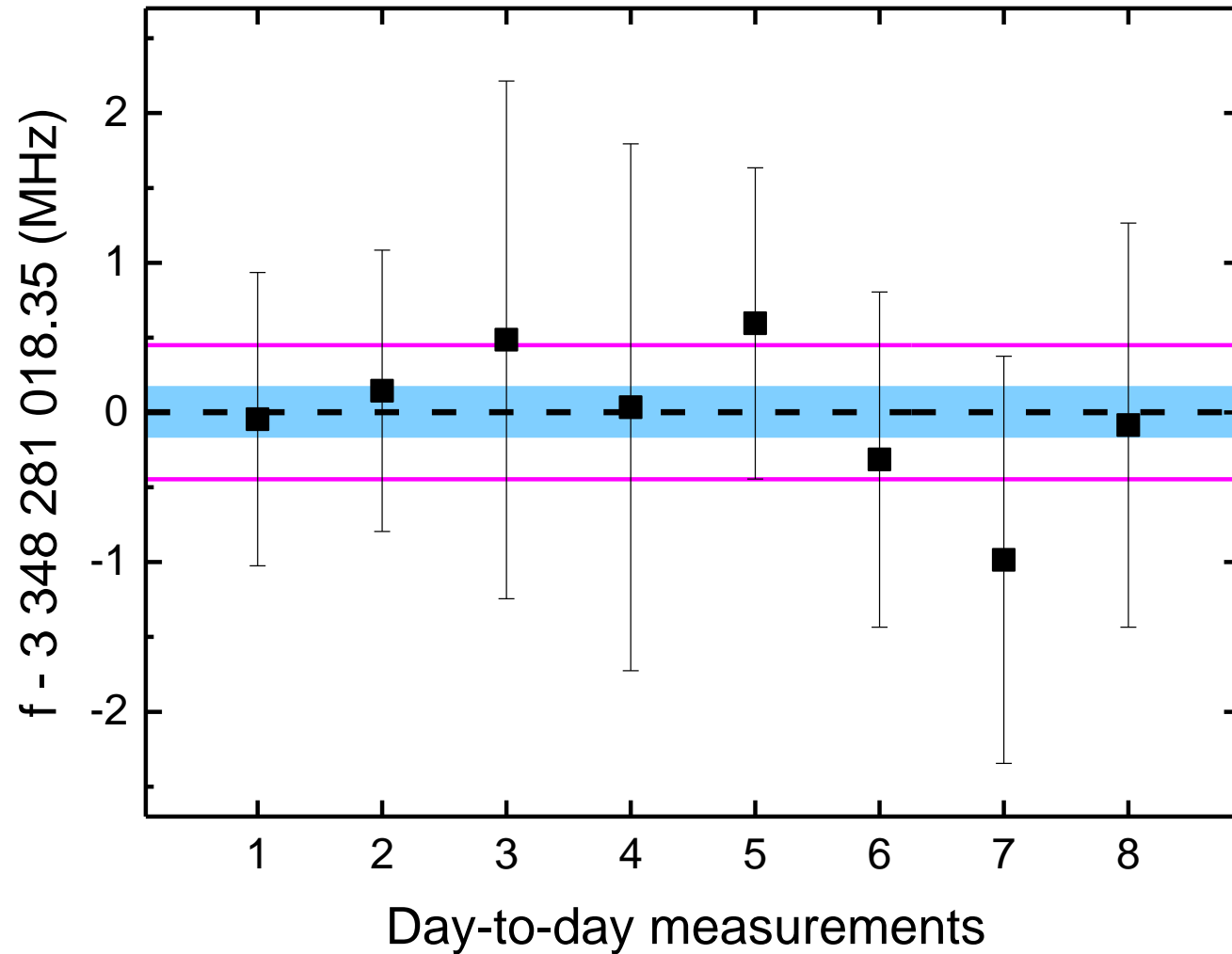
Changing θ



Systematic check: AC Stark



Statistical uncertainty



Error budget ortho-hydrogen

For GK($v=1, N=1$) - X($v=0, N=1$) (ortho- H_2)

Measured frequency	3348281018.58(49) MHz	
Effect	Correction	Uncertainty
Chirp		(<490kHz) _{stat}
Residual 1 st order Doppler		350kHz
2 nd order Doppler		<30kHz
AC Stark red	-40kHz	90kHz
AC Stark VUV	-190kHz	200kHz
Hyperfine structure		<100kHz
Total systematic uncertainty		426kHz
Final frequency	3348281018.35(49) _{stat} (43) _{sys} MHz	

Error budget ortho-hydrogen

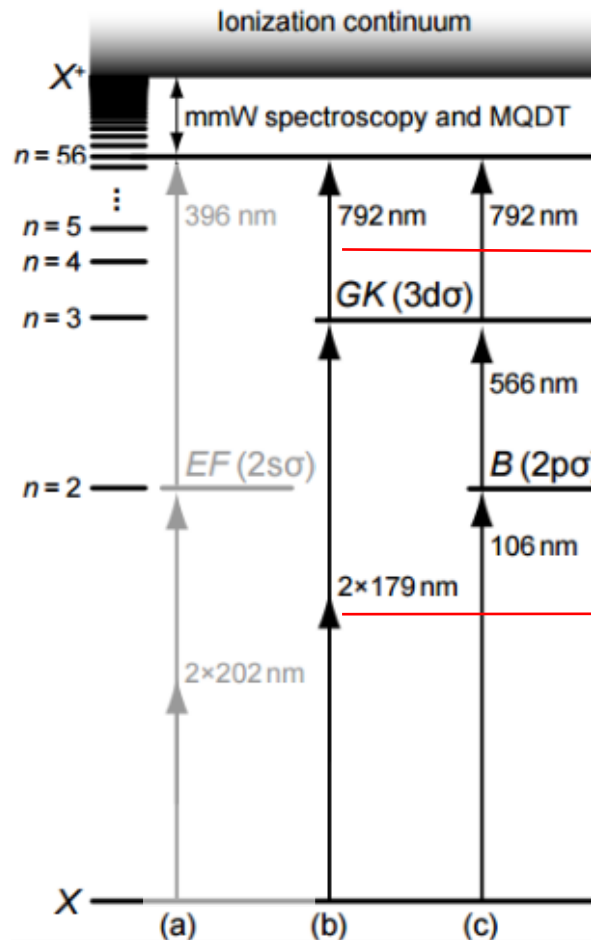
For GK($v=1, N=1$) - X($v=0, N=1$) (ortho-H₂)

Measured frequency	3348281018.58(49) MHz	
Effect	Correction	Uncertainty
Chirp		(≤ 400 kHz)

Total accuracy is 650 kHz:
 2×10^{-10} for GK-X

Total systematic uncertainty	426 kHz	
Final frequency	3348281018.35(49) _{stat} (43) _{sys} MHz	

Ionization energy of H₂ at 750 kHz



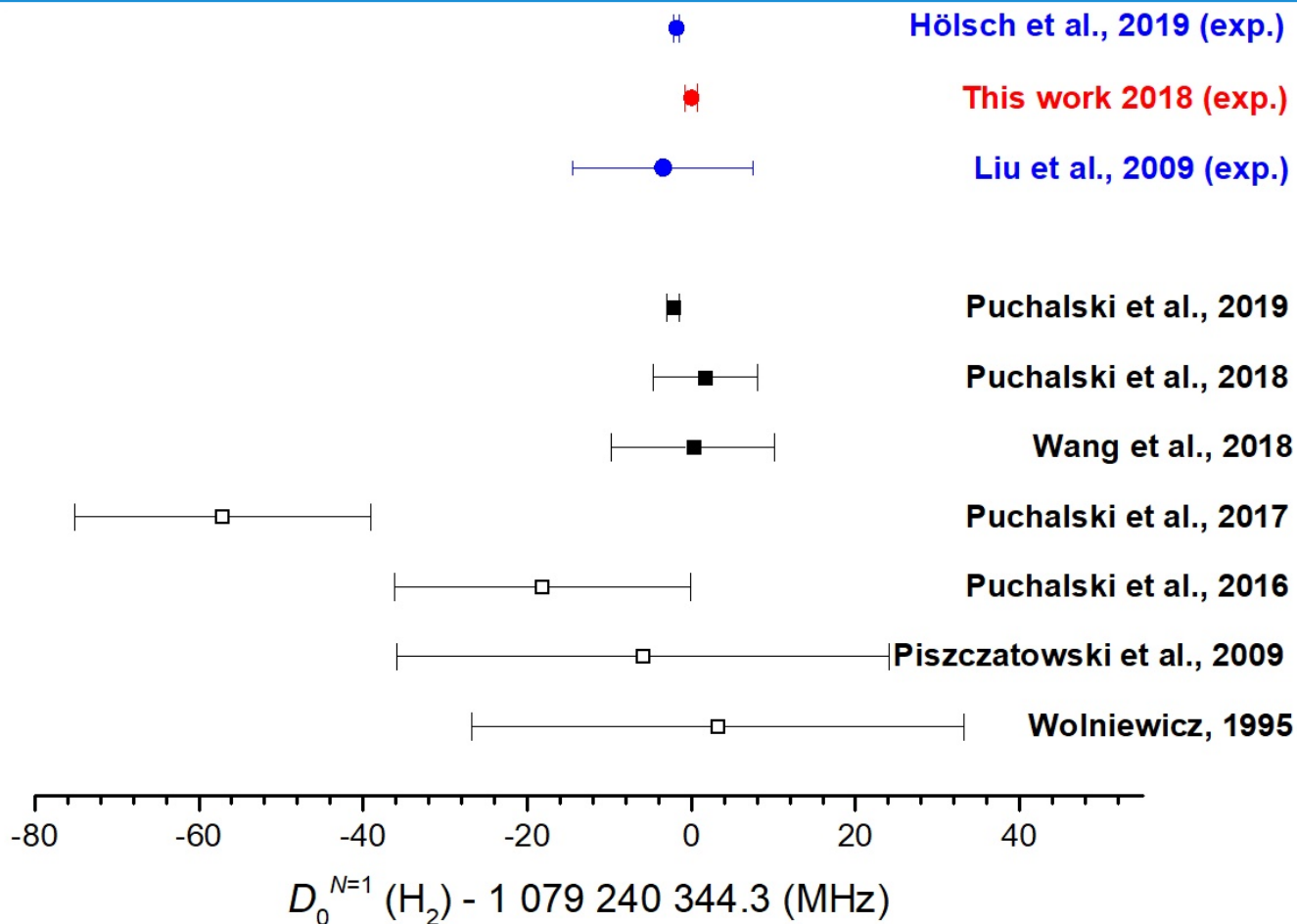
$\Delta f = 100 \text{ kHz}$

$\Delta f = 360 \text{ kHz}$

M. Beyer, N. Höltsch, J. A. Agner,
F. Merkt, ETH Zürich.

$\Delta f = 650 \text{ kHz}$

Dissociation Energy of ortho-H₂

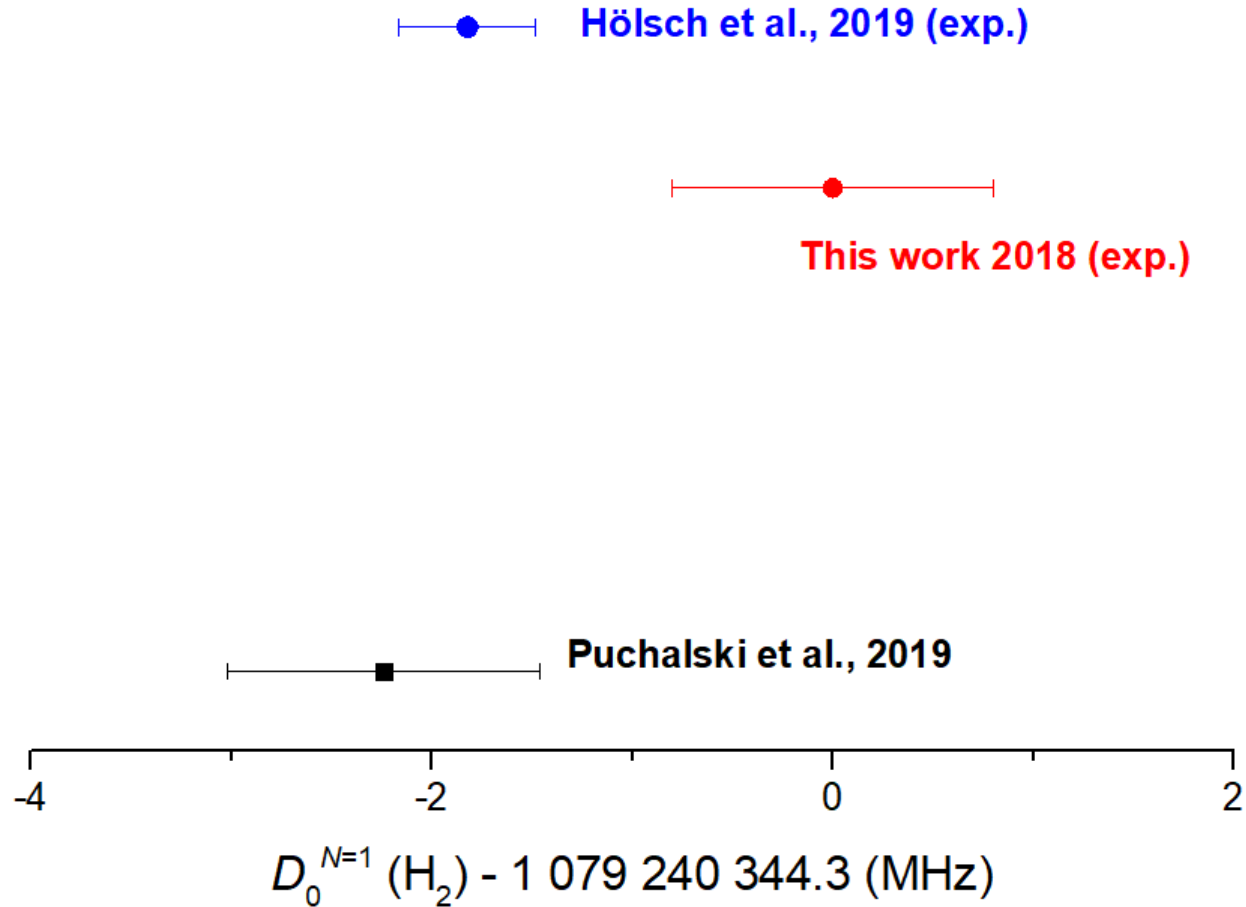


This work:

“Dissociation energy of the hydrogen molecule at 10^{-9} accuracy”

C. Cheng, J. Hussels, et al. *Phys. Rev. Lett.* 121.013001 (2018)

Dissociation Energy of ortho-H₂

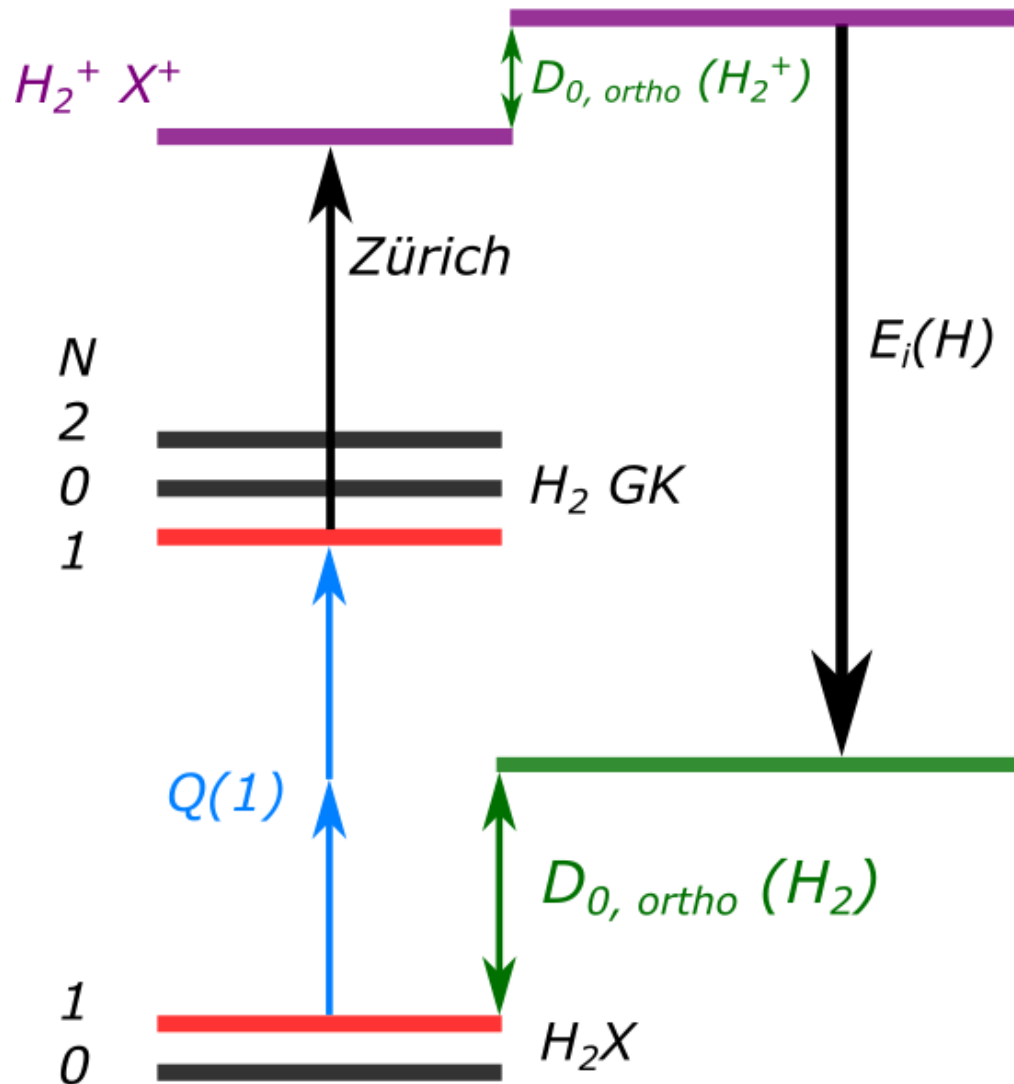


This work:

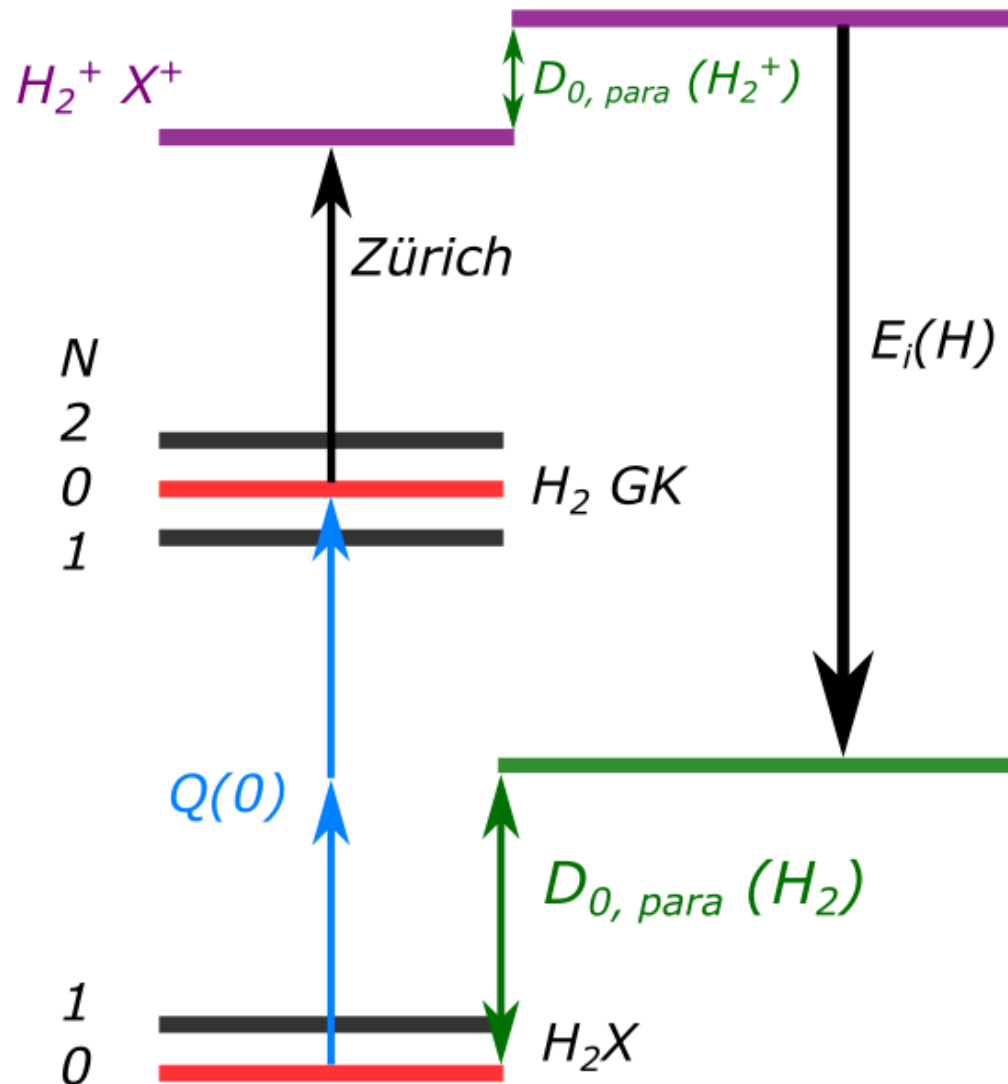
“Dissociation energy of the hydrogen molecule at 10^{-9} accuracy”

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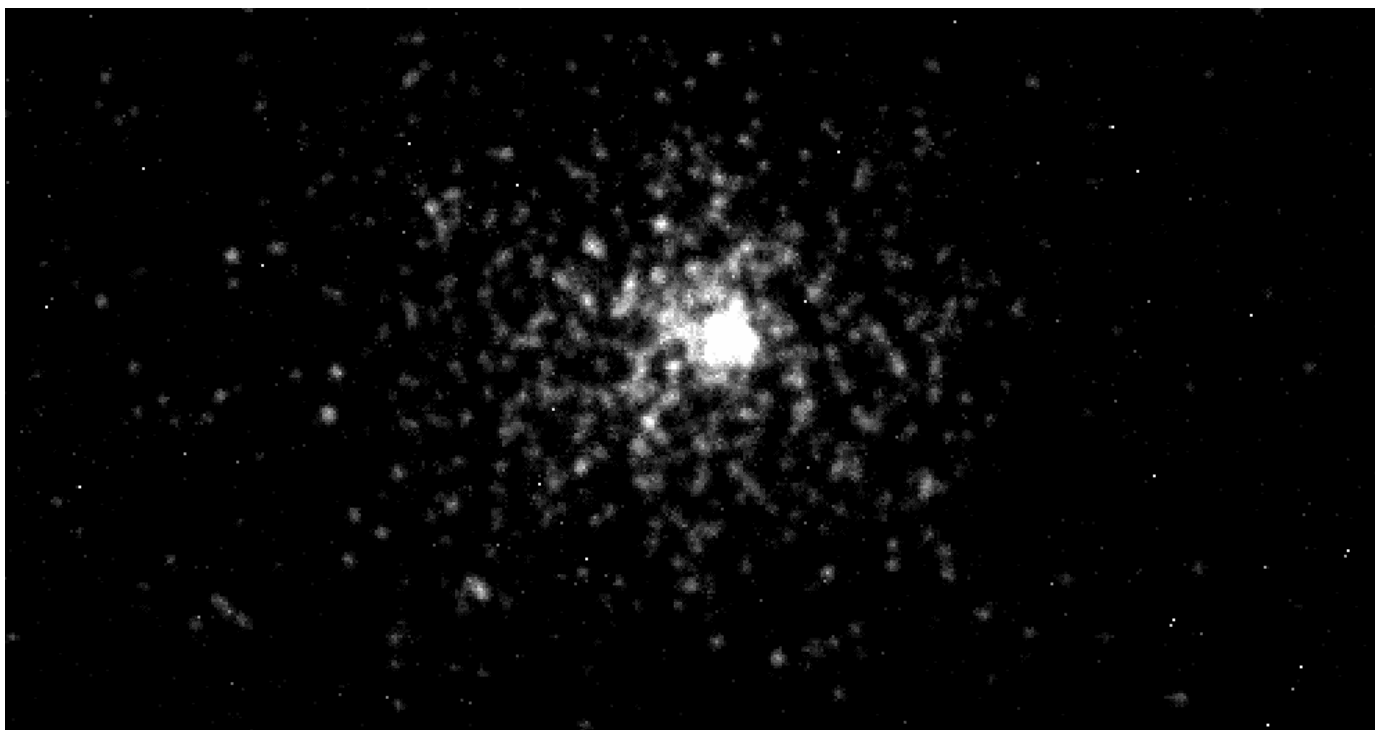
Ortho Hydrogen



Para Hydrogen

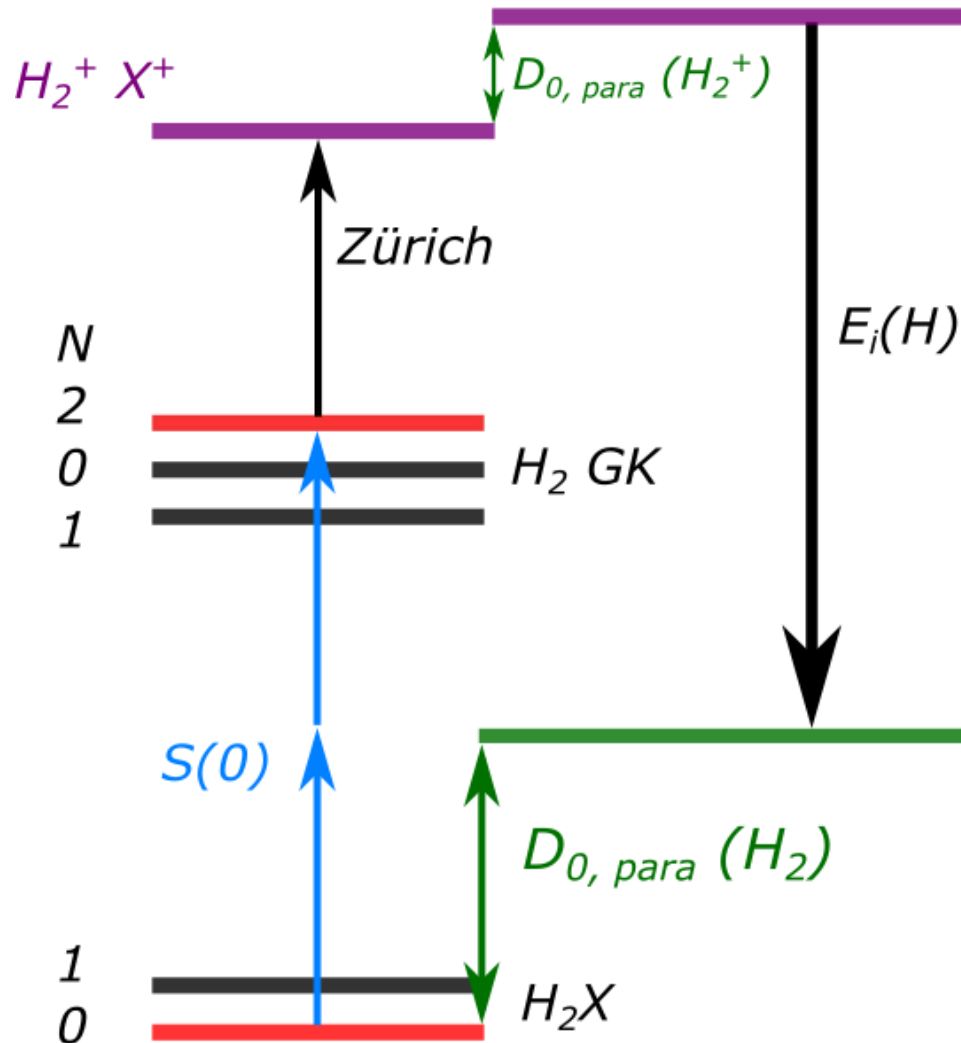


The search for $Q(0)$

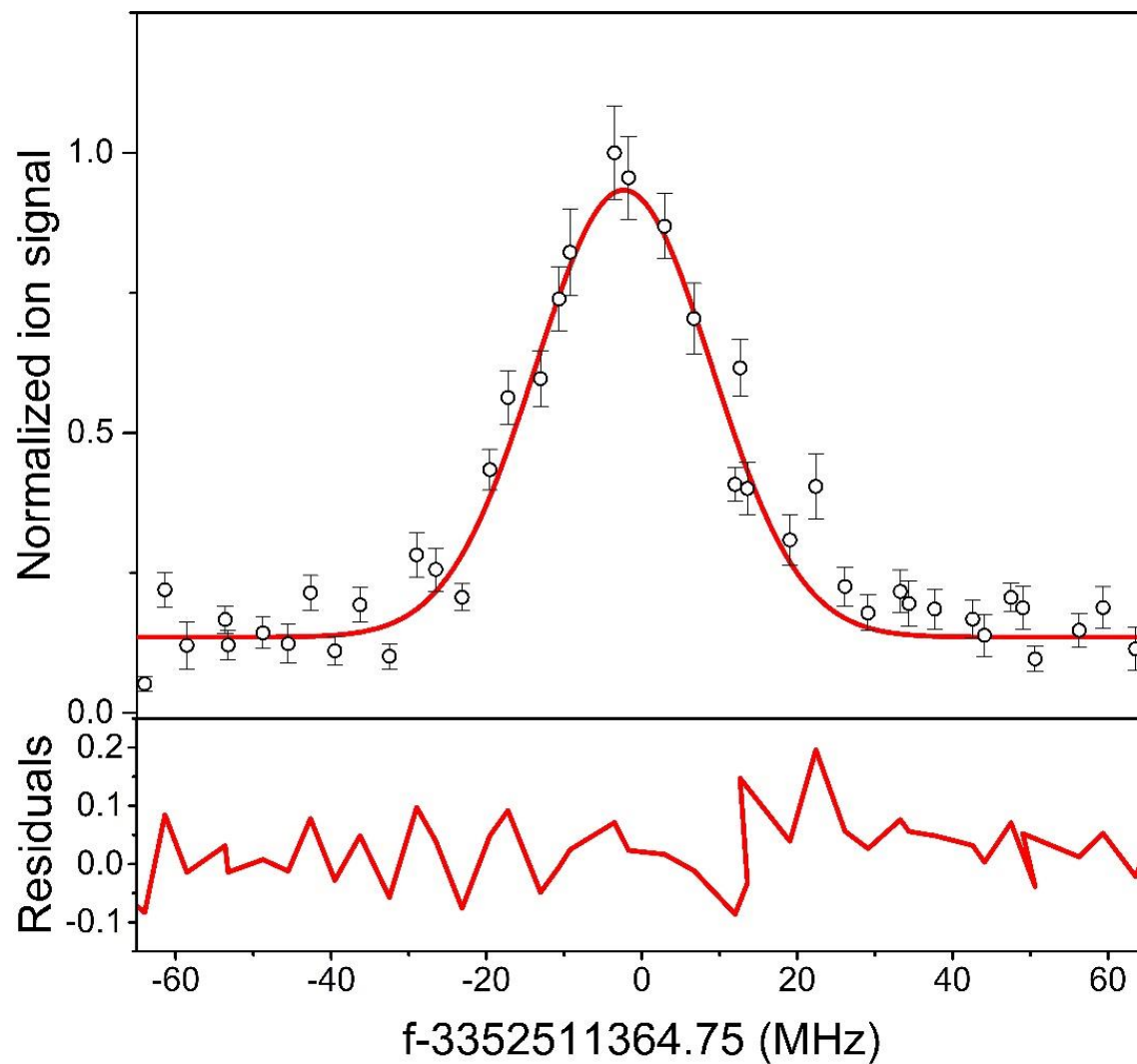


Signal after 60 seconds averaging with VMI
More than 100 times smaller compared to $Q(1)$?!?

S(0)



$S(0)$



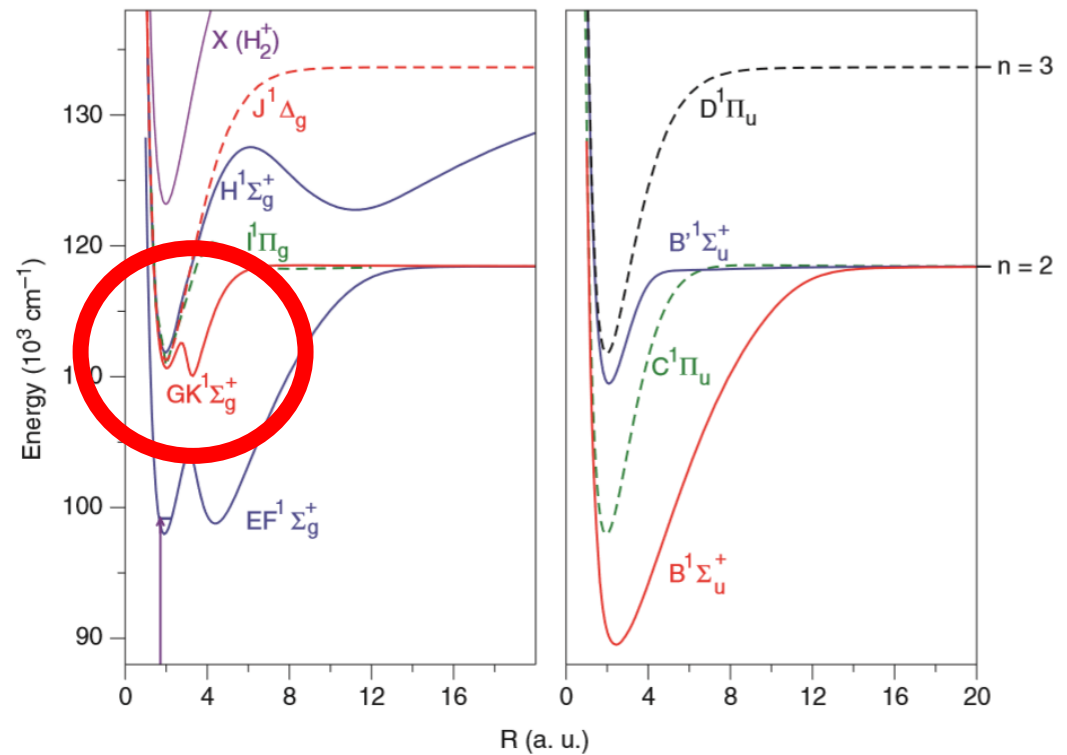
S(0)

Π -state mixing?

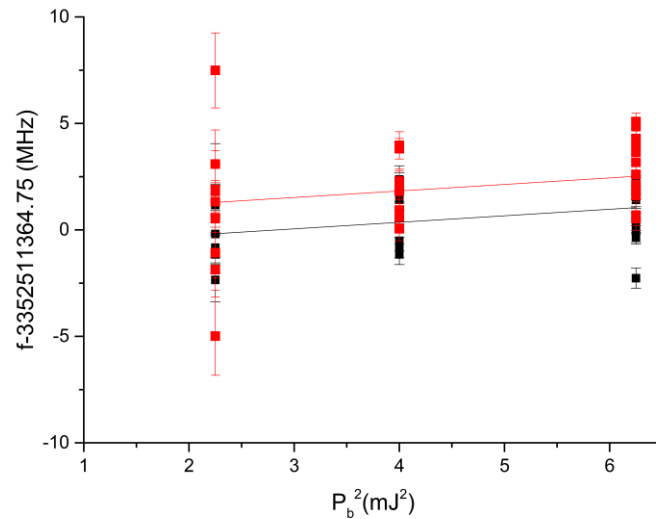
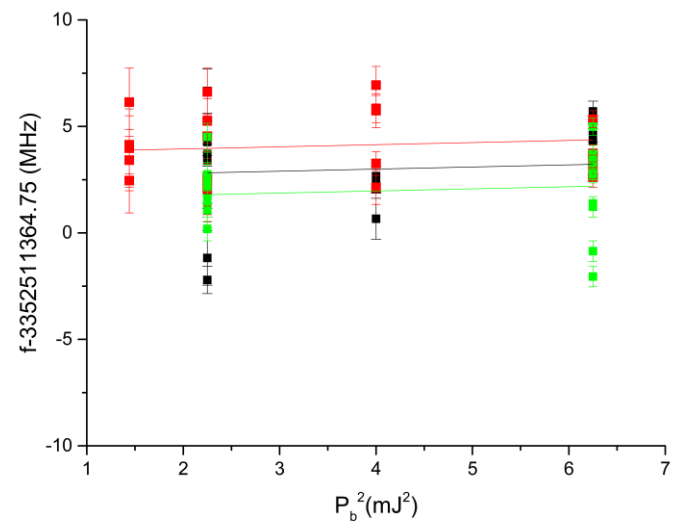
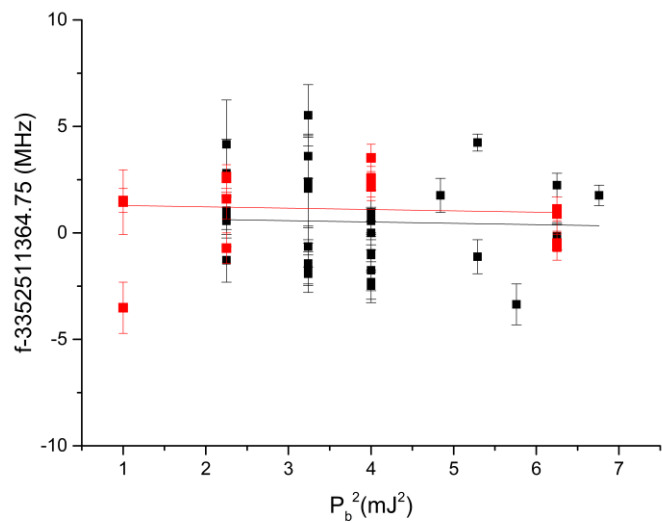
GK(v=1, N)	P(GK)	P(I)
N=0	0.96291	0
N=1	0.78205	0.16964
N=2	0.67133	0.25516

S. Yu and K. Dressler (1994)

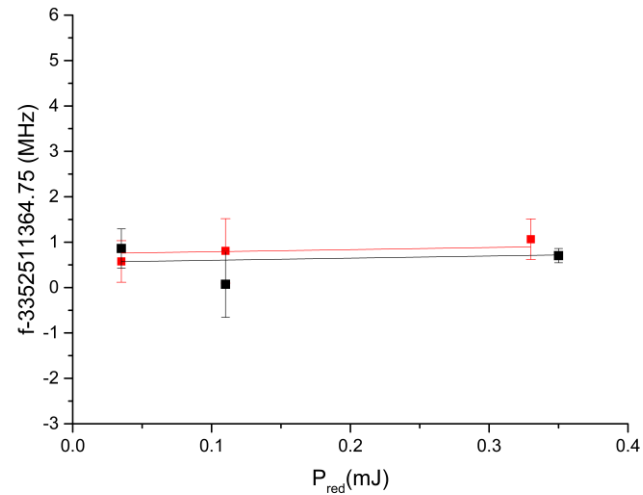
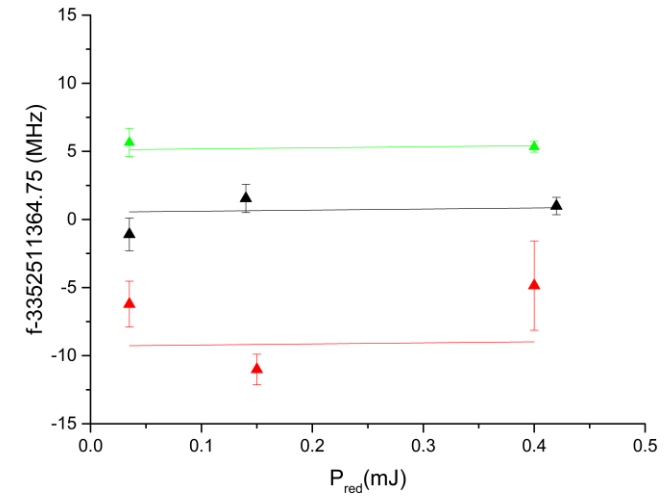
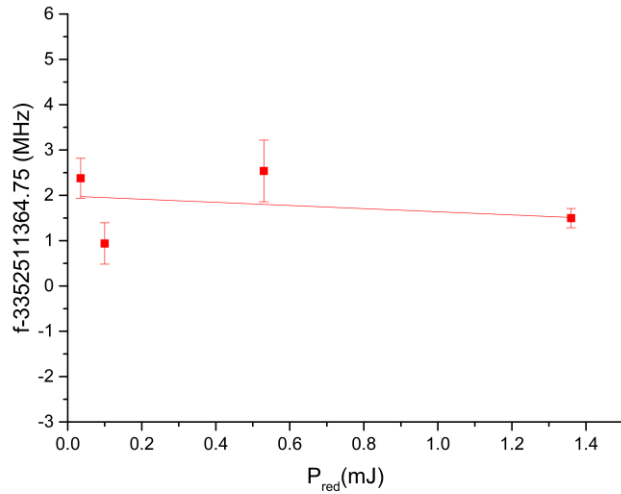
D. Bailly et al.



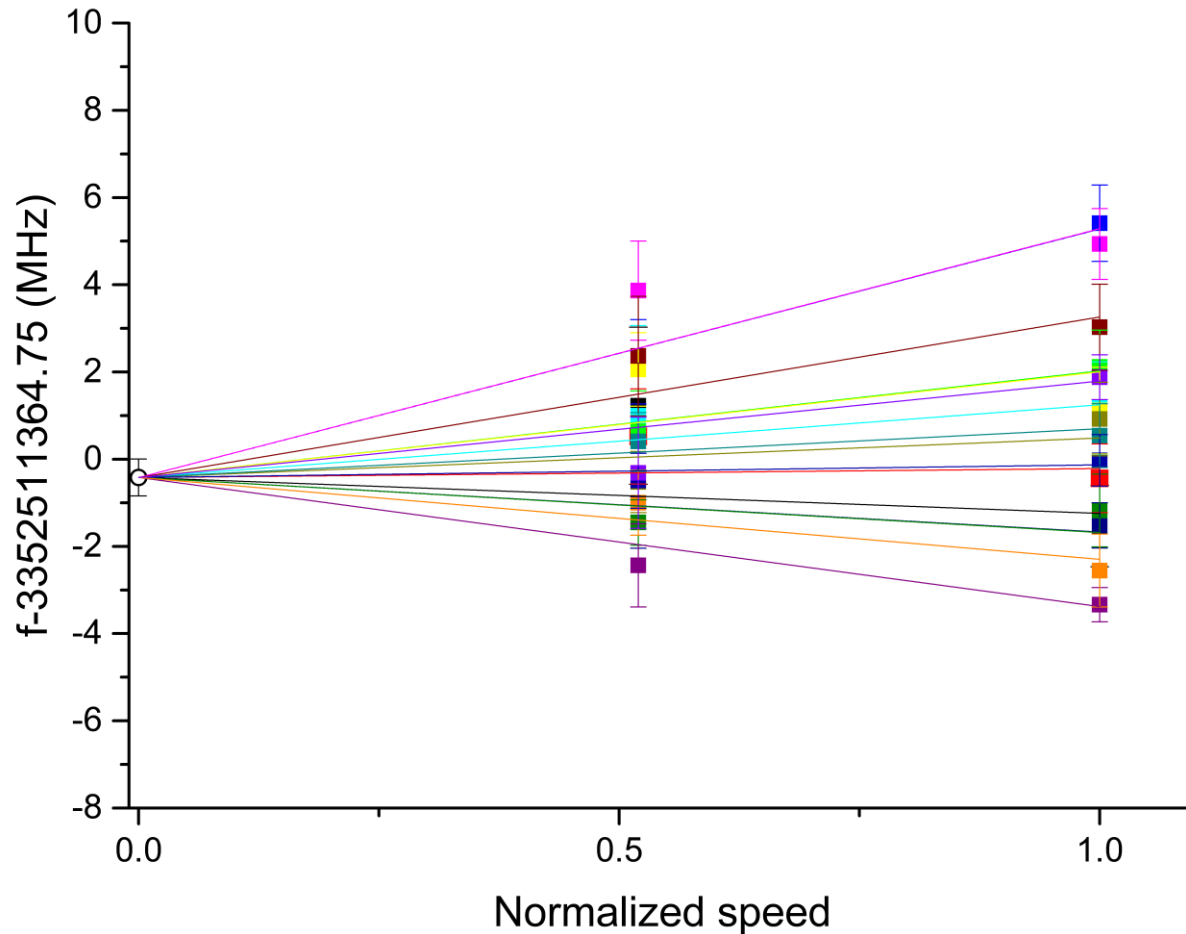
S(0): 3 rounds of Stark VUV



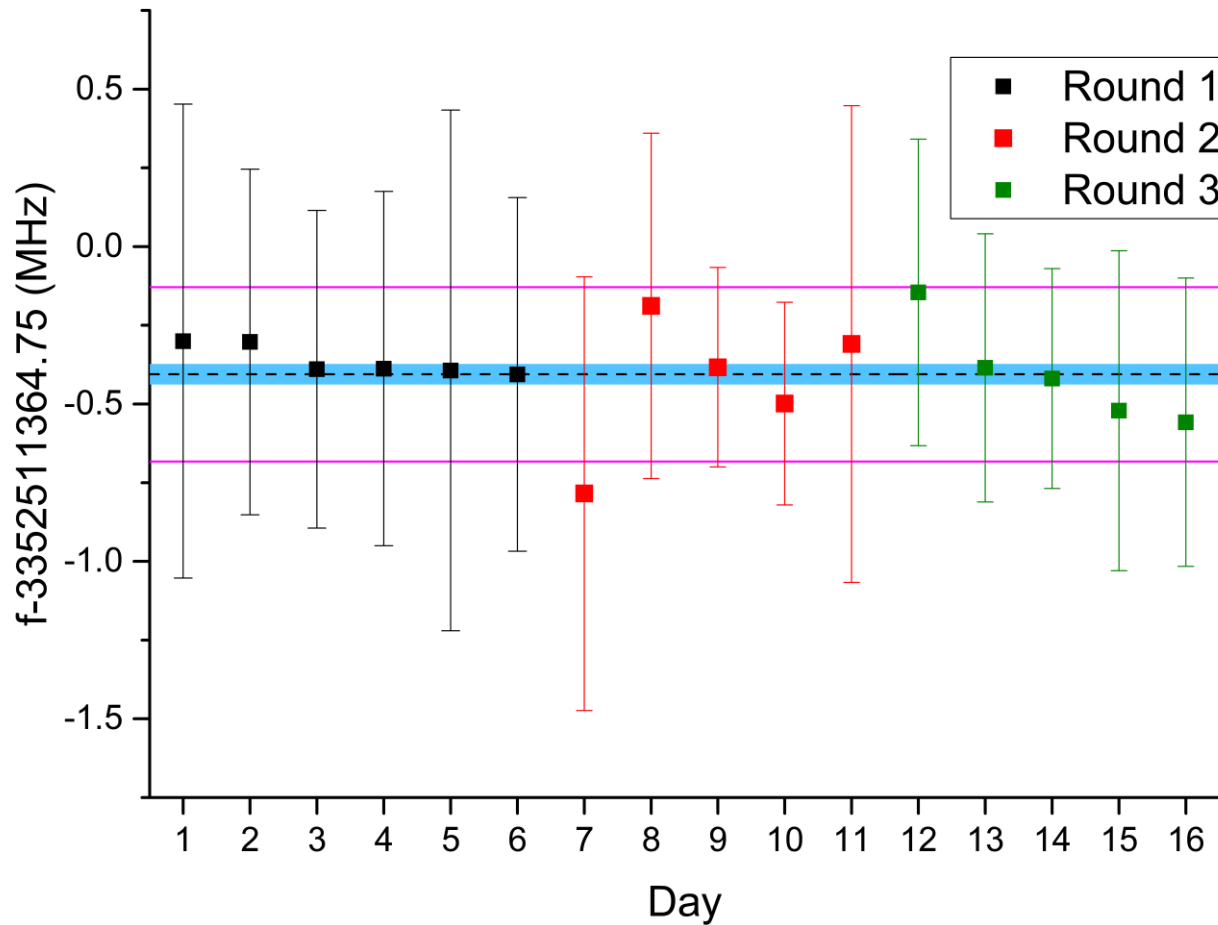
S(0): 3 rounds of Stark ionization



S(0): 3 rounds Doppler analysis



S(0): 3 rounds statistical analysis



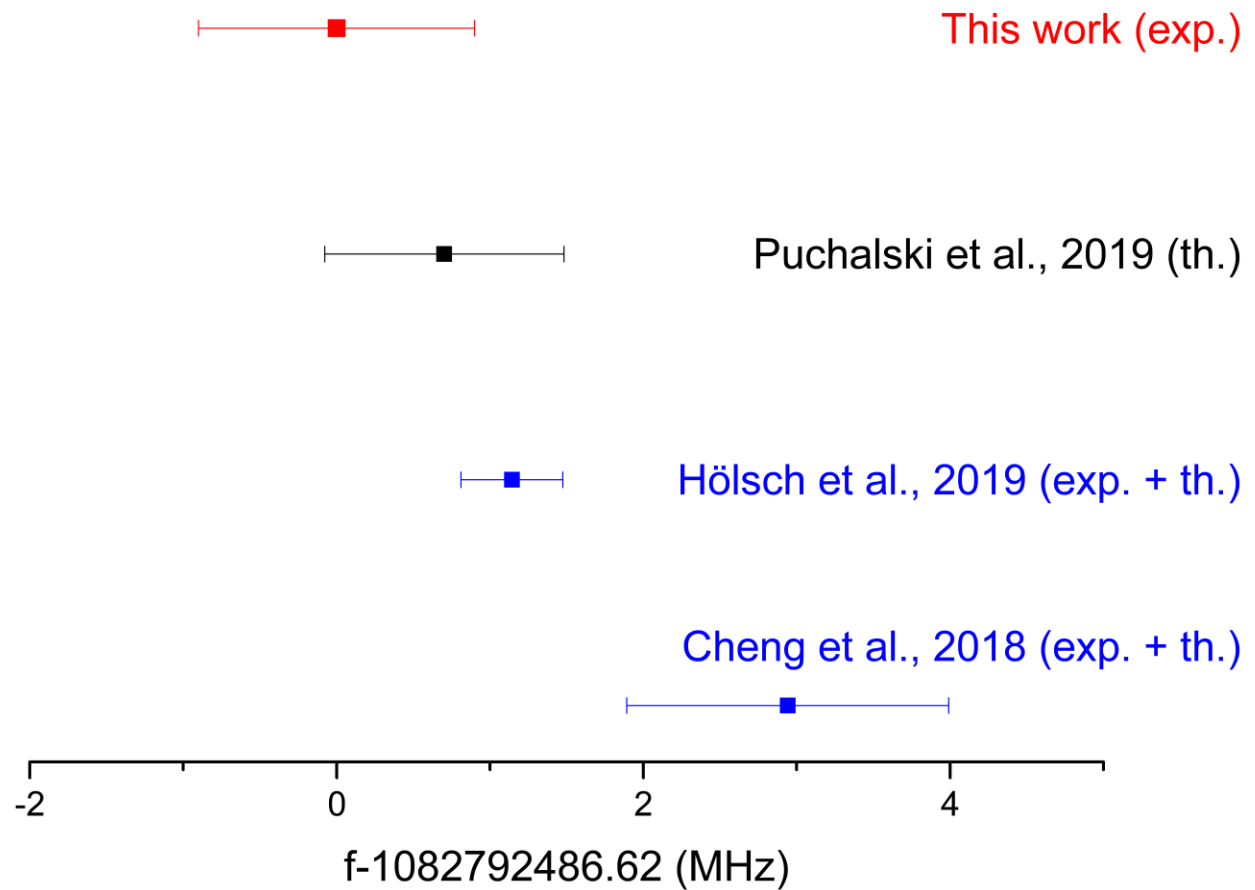
Error budget

For GK($v=1, N=2$) - X($v=0, N=0$) (para- H_2)

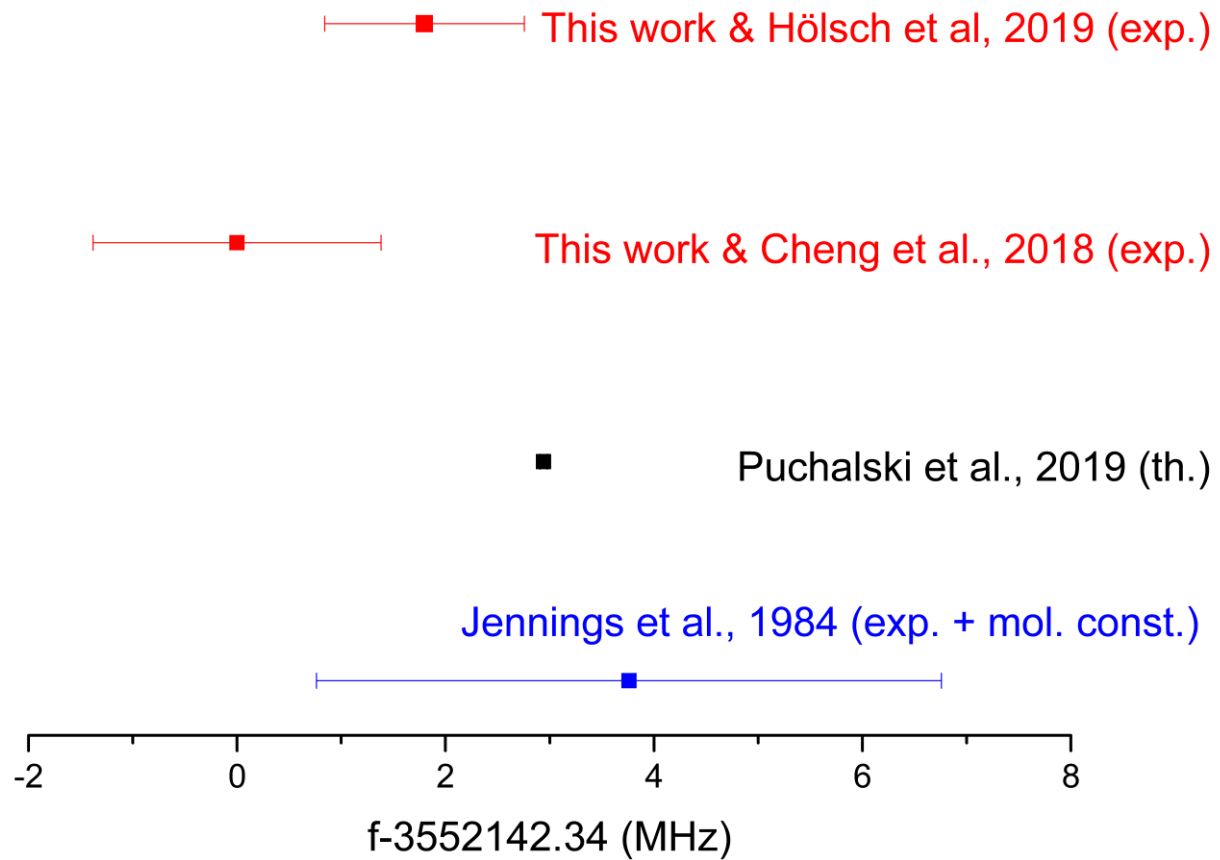
Measured frequency	3 352 511 364.34(3) MHz
Effect	Uncertainty
Chirp	(<32kHz) _{stat}
Residual 1 st order Doppler	425kHz
2 nd order Doppler	<16kHz
AC Stark red	120kHz
AC Stark VUV	452kHz
Hyperfine structure	-
Total systematic uncertainty	632kHz
Final frequency	3 352 511 364.34(3) _{stat} (63) _{sys} MHz

Total accuracy is 640kHz

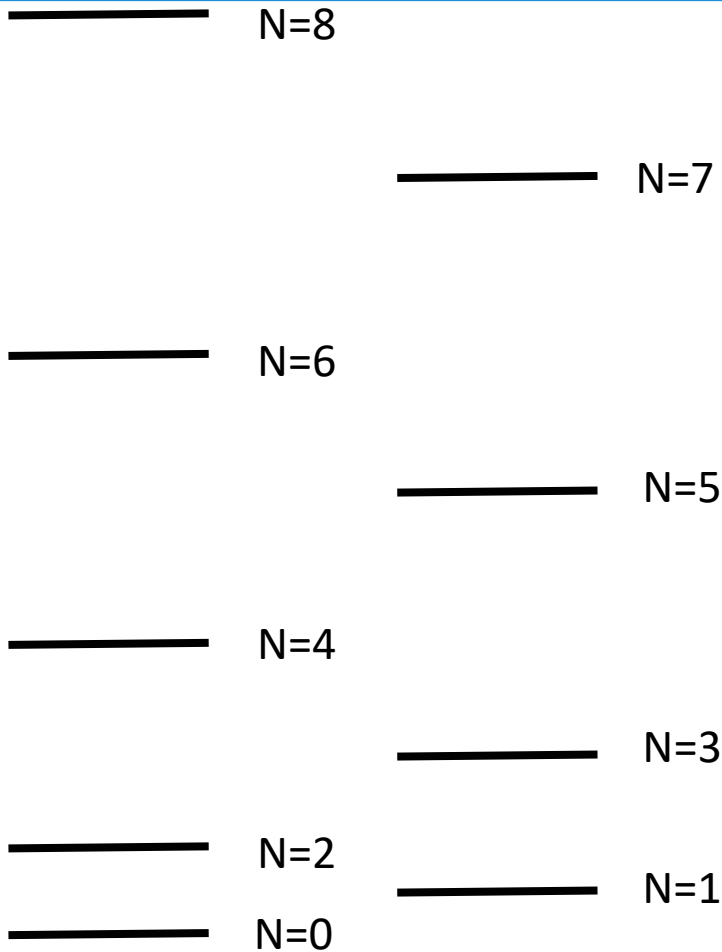
Dissociation Energy of para-H₂



Ortho-para splitting



Ortho-para splitting

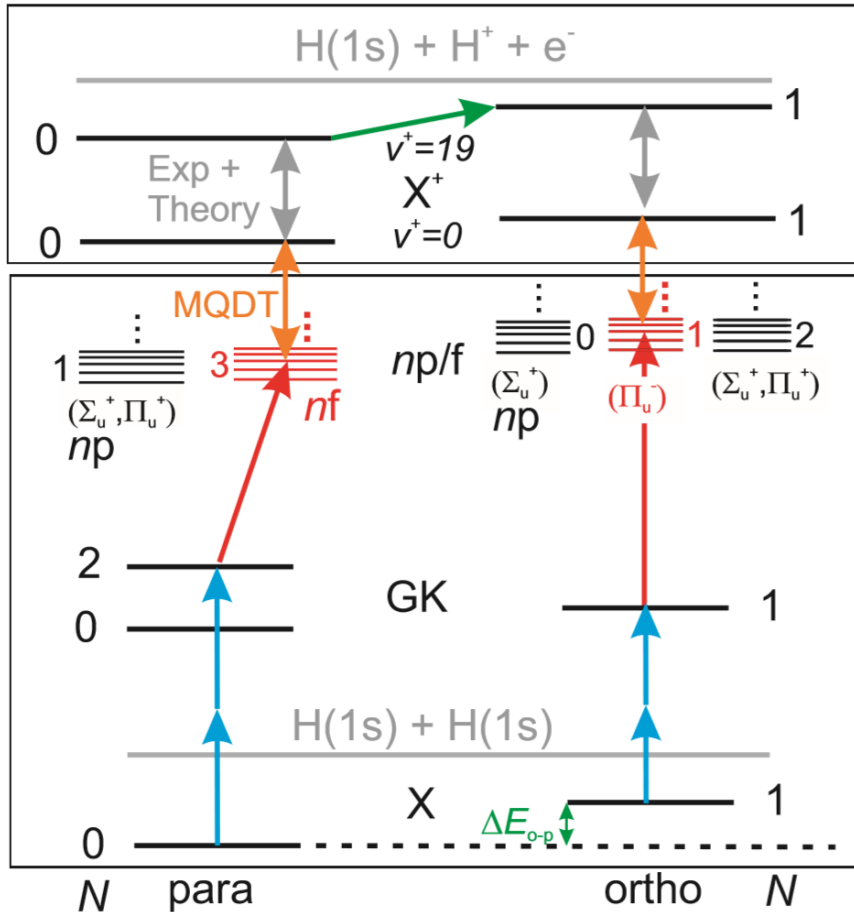


Assuming

$$E(N) = B * N(N+1) - D * N^2(N+1)^2 + \dots$$

(Jennings et al., 1984)

Ortho-para splitting



H_2^+

First ortho-para interaction in H_2^+ :
 Critchley et al., PRL 86, 1725 (2001)

Now possible to connect to H_2

H_2

Conclusion & Outlook

- GK ($v=1, N=1$) – X($v=0, N=1$) and GK ($v=1, N=2$) – X($v=0, N=2$) have been measured directly for the first time. An accuracy of 0.65 MHz and 0.64 MHz was achieved.
- $D_0(\text{ortho-H}_2)$ determination has been improved by an order of magnitude to 0.75 MHz.
- $D_0(\text{para-H}_2)$ determination has been improved to 0.78 MHz.

Conclusion & Outlook

- GK ($v=1, N=1$) – X($v=0, N=1$) and GK ($v=1, N=2$) – X($v=0, N=2$) have been measured directly for the first time. An accuracy of 0.65 MHz and 0.64 MHz was achieved.
- $D_0(\text{ortho-H}_2)$ determination has been improved by an order of magnitude to 0.75 MHz.
- $D_0(\text{para-H}_2)$ determination has been improved to 0.78 MHz.
- The splitting between ortho- and para- H_2 could be derived with an accuracy of 0.99 MHz.
- Ortho- and para levels in H_2 can be linked for the first time
- Future: Similar measurements for HD and D_2

Thanks to:

LaserLaB, VU Amsterdam:

Dr. Cunfeng Cheng

Dr. Ming Li Niu

Dr. Edcel Salumbides

Dr. Hendrick Bethlem

Prof. dr. Kjeld Eikema

Prof. dr. Wim Ubachs

ETH, Zürich:

Maximilian Beyer

Nicolas Hölsch

Josef Agner

Prof. dr. Frédéric Merkt

CNRS, Orsay:

Prof. dr. Christian Jungen

USTC, Hefei:

Lei-Gang Tao

Prof. dr. Shui-Ming Hu

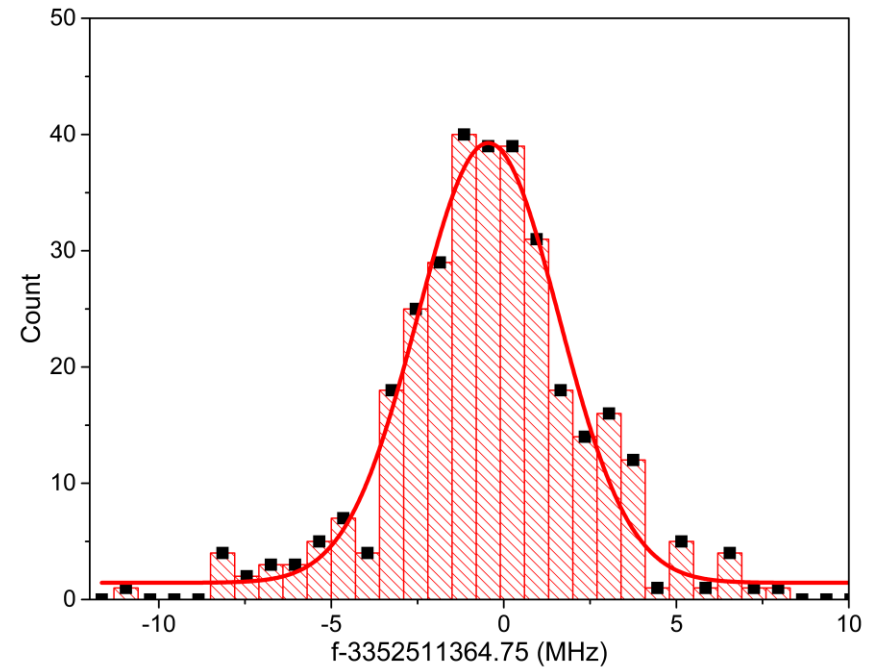
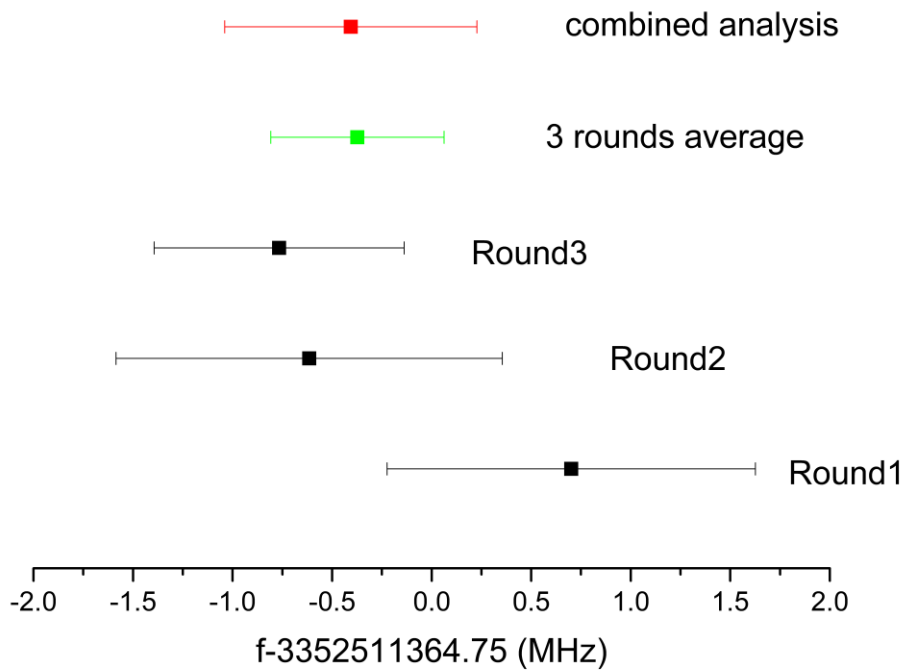
Rob Kortekaas (Technician)



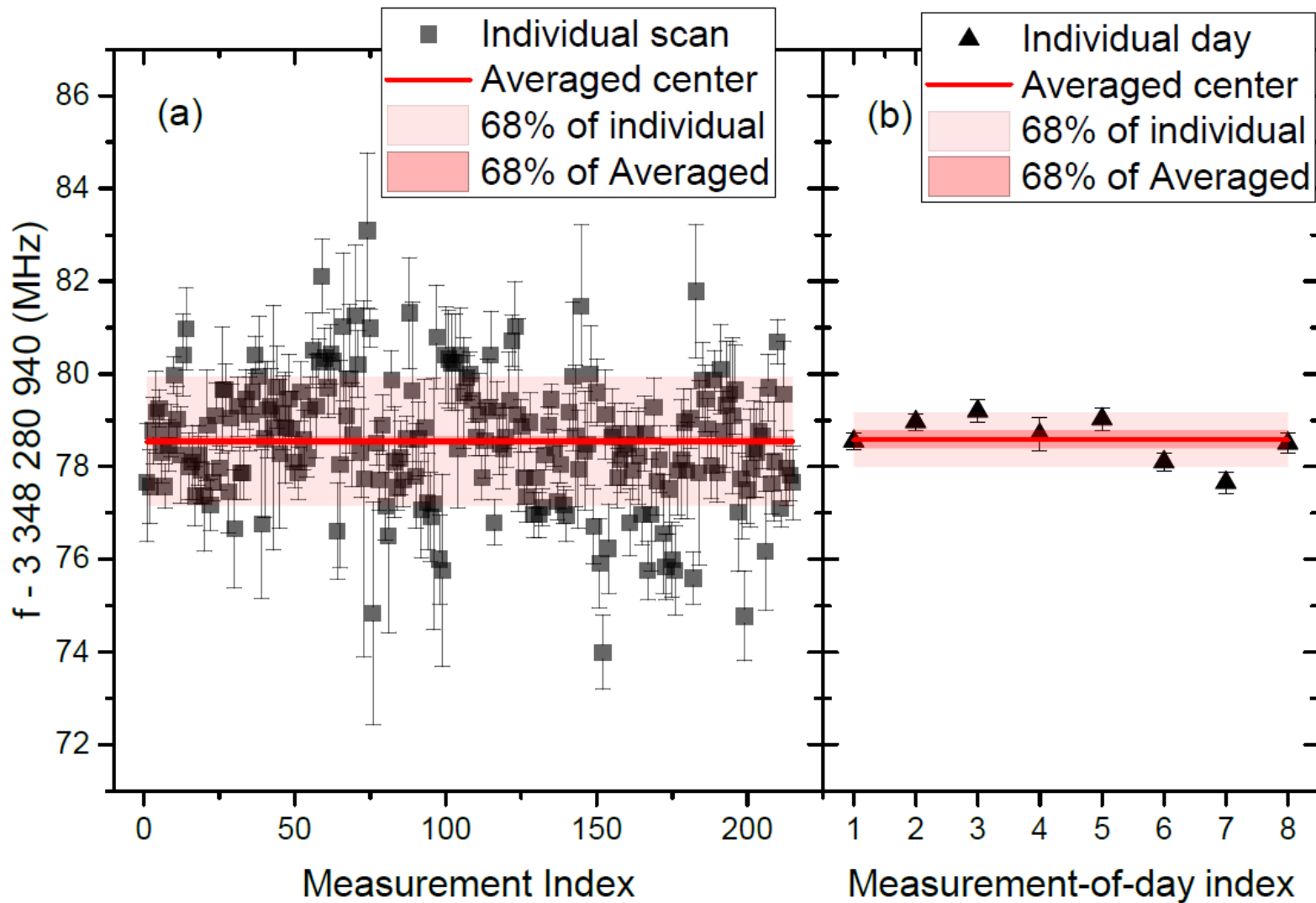
European Research Council



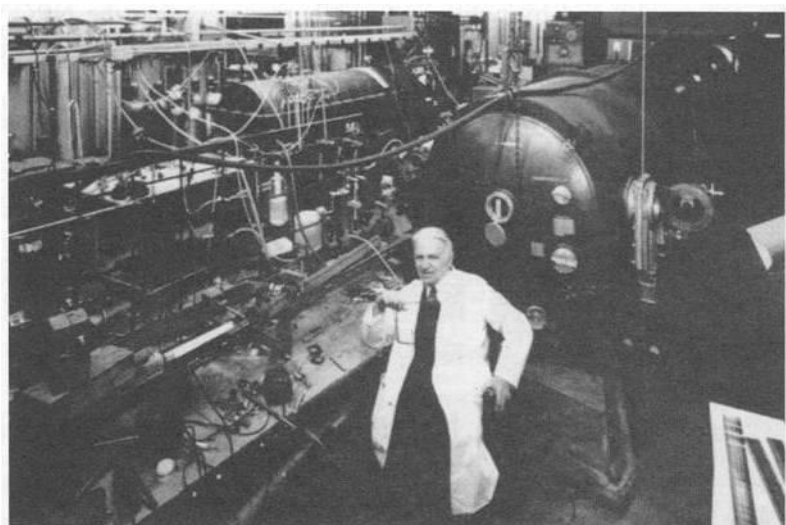
S(0): 3 rounds statistical analysis



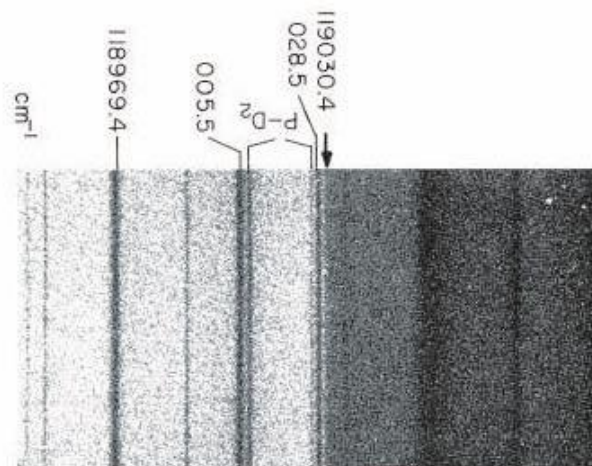
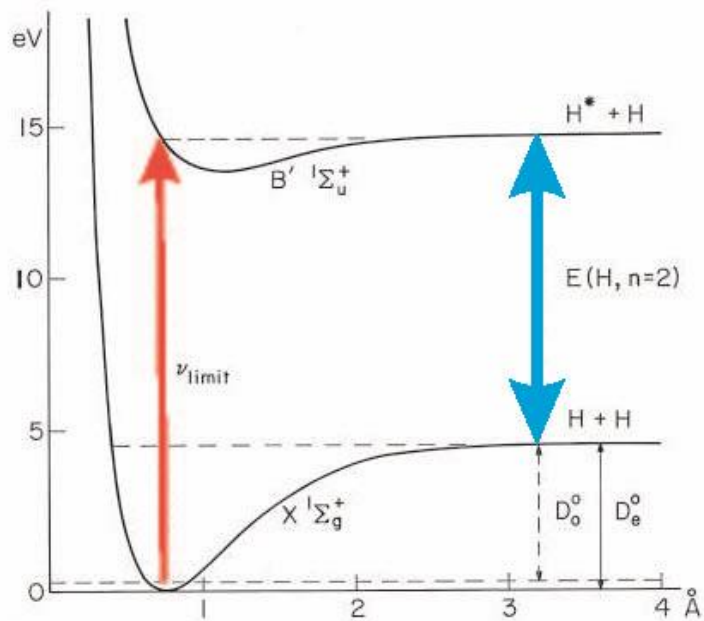
Residual systematic and statistical Q(1)



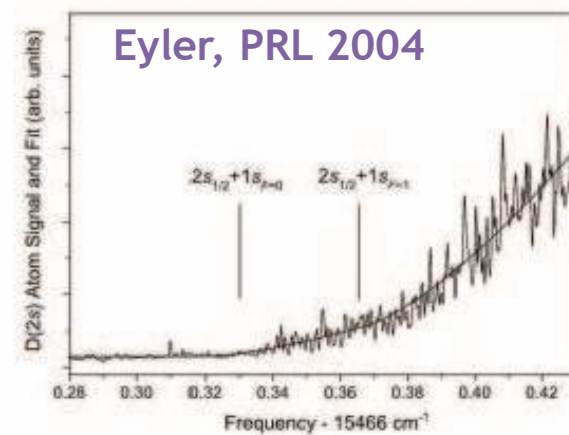
H₂ Dissociation energy; benchmark



Herzberg in his laboratory.

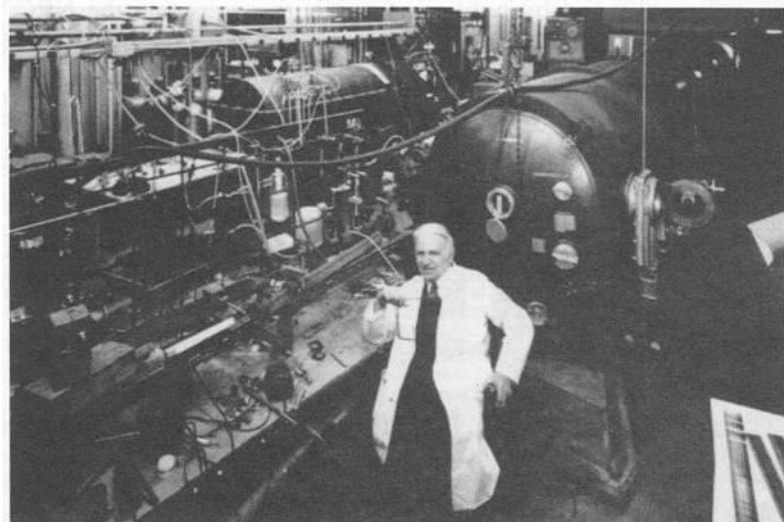


Herzberg
Nobel
Lecture



Eyler, PRL 2004

H₂ Dissociation energy; measurement of IP



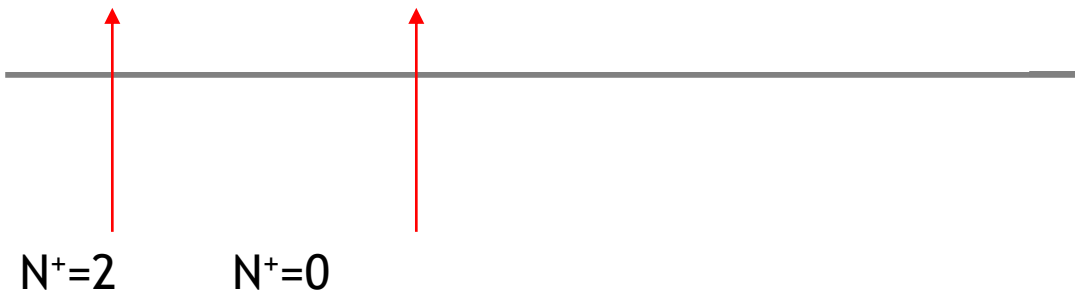
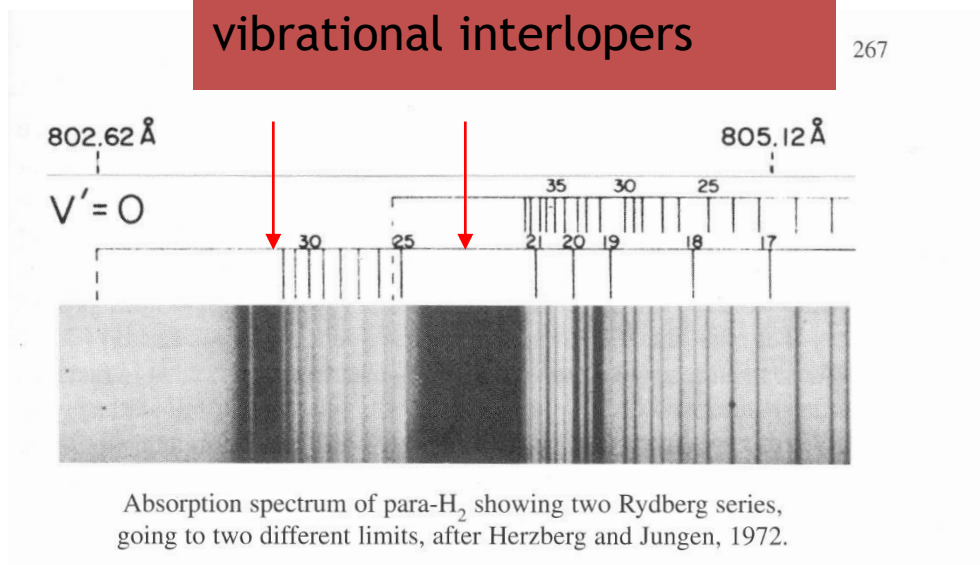
Herzberg in his laboratory.



Christian Jungen

vibrational interlopers

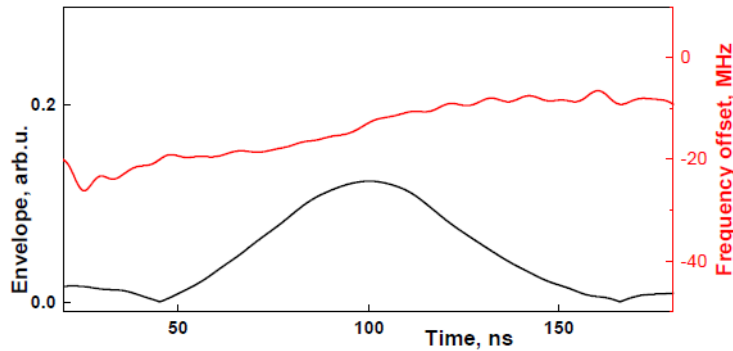
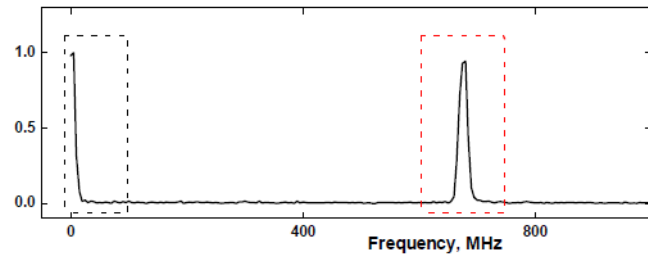
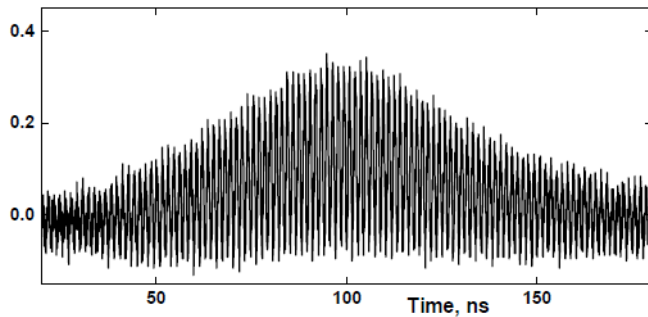
267



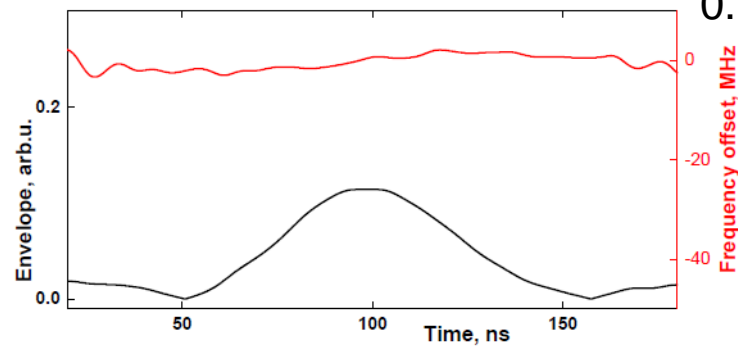
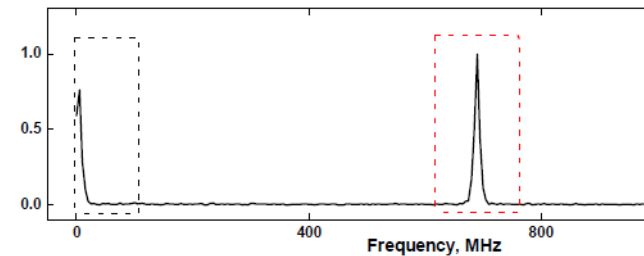
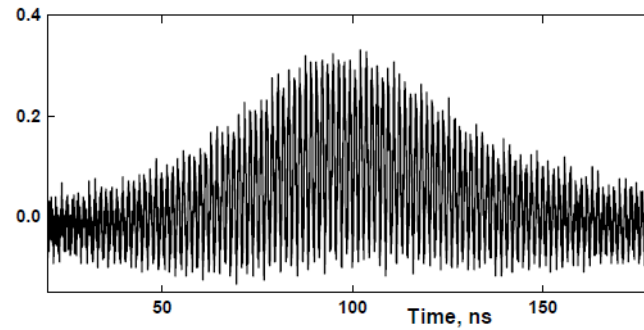
Herzberg and Jungen 1972
MQDT of molecules

Chirp measurement

w/o anti-chirp



with anti-chirp



Cavity pulling:

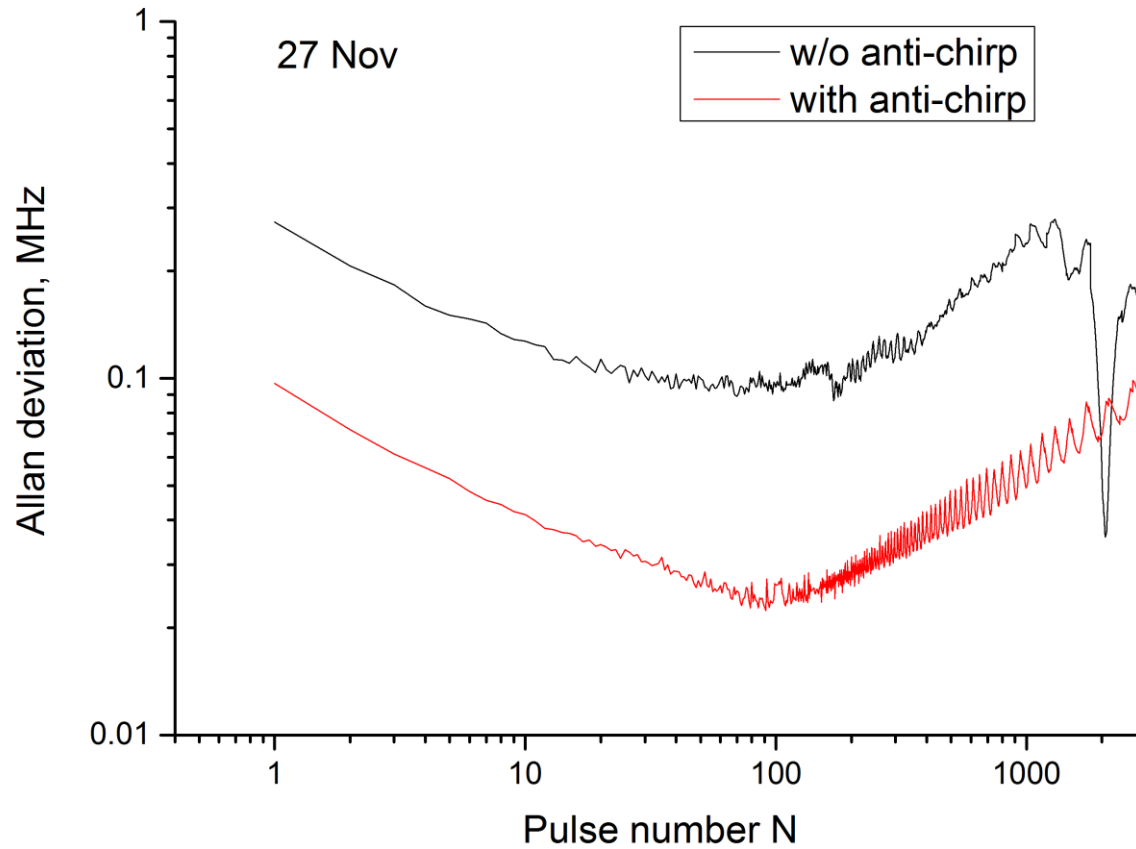
~ 0 MHz

Freq chirp:

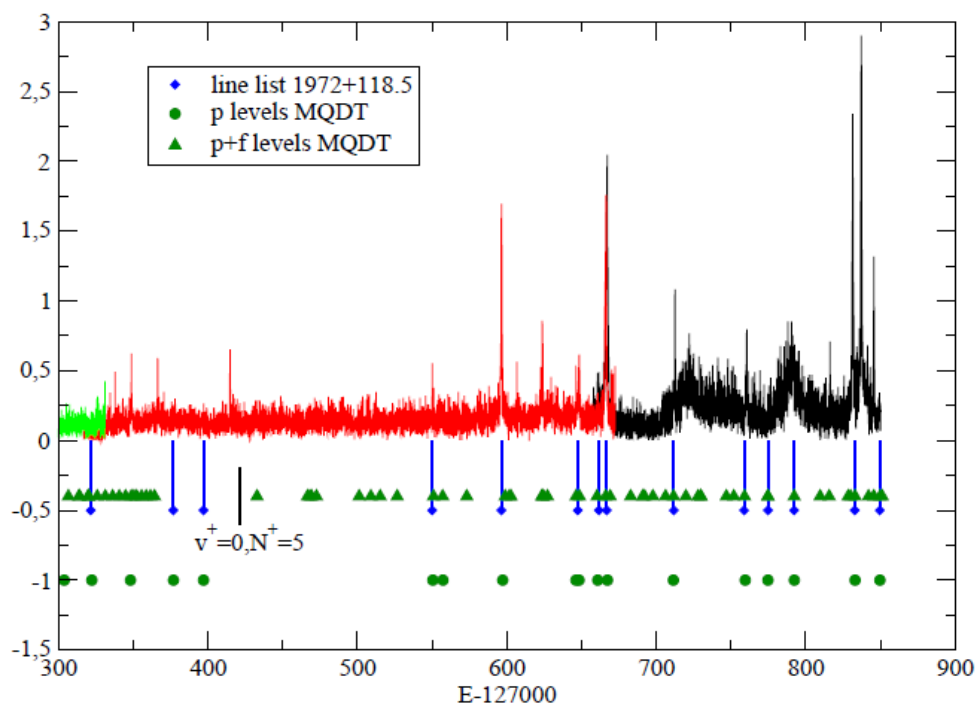
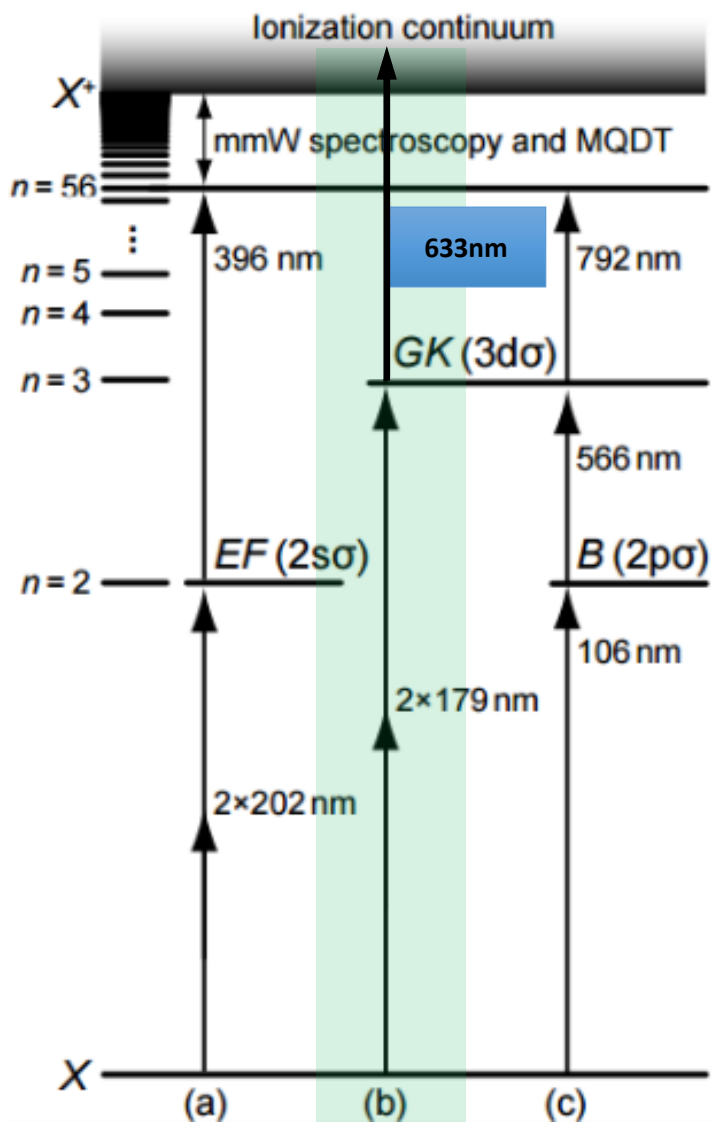
< 1.5 MHz/50 ns

0.1 MHz uncertainty

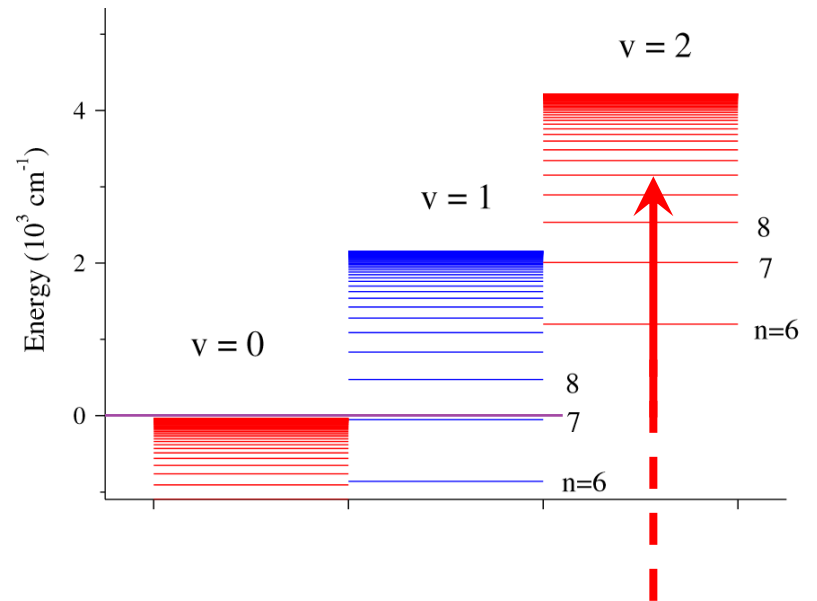
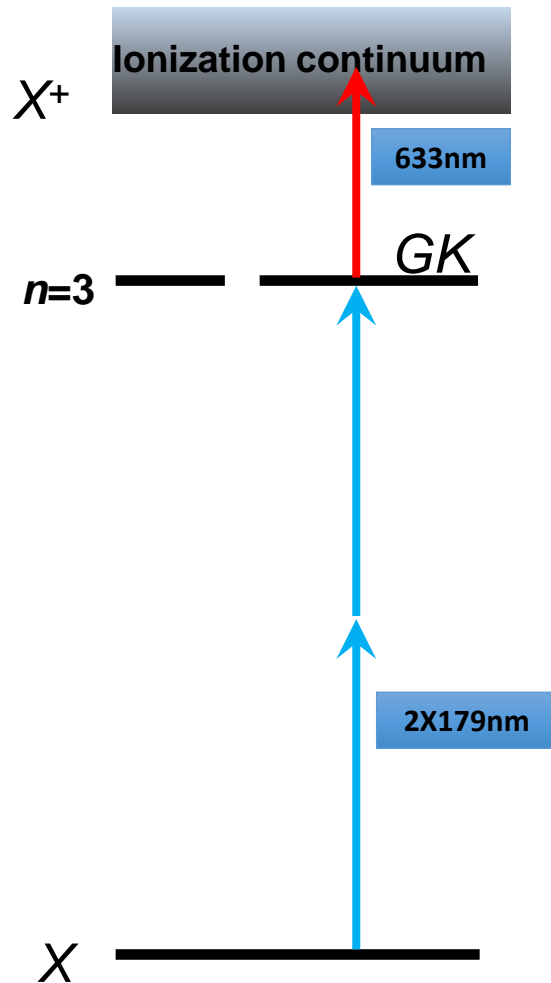
Development of anti-chirp



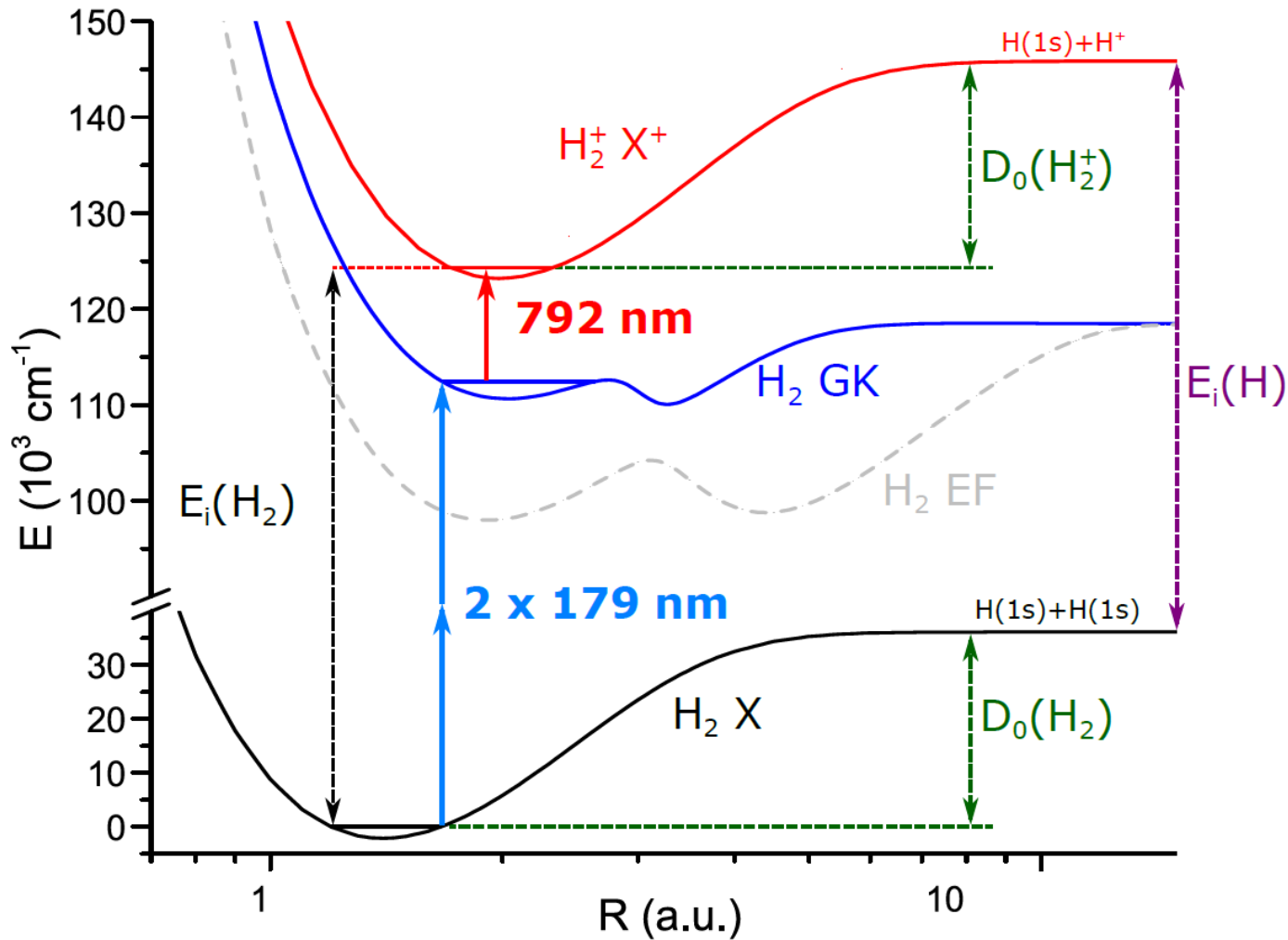
Auto-ionization



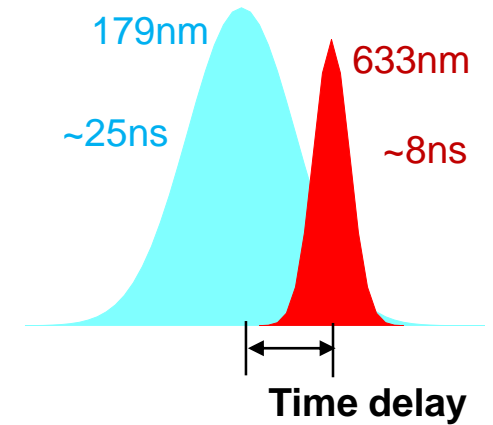
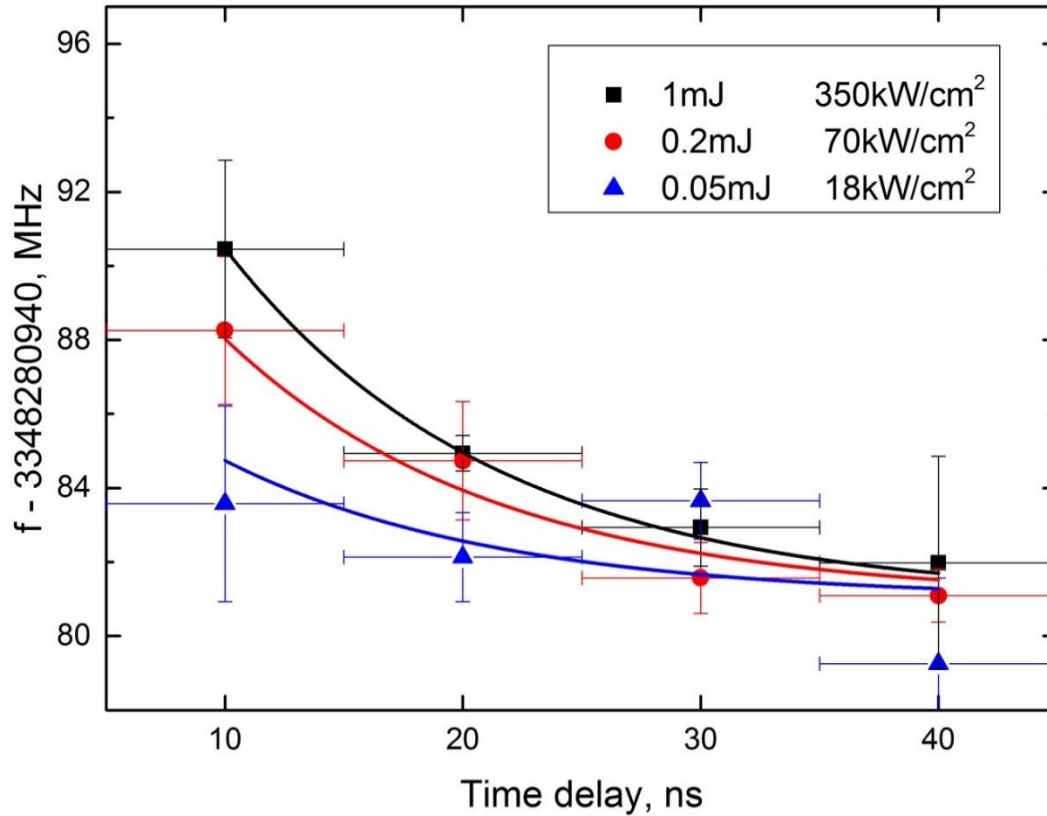
Measurement of GK-X



H₂ Potentials

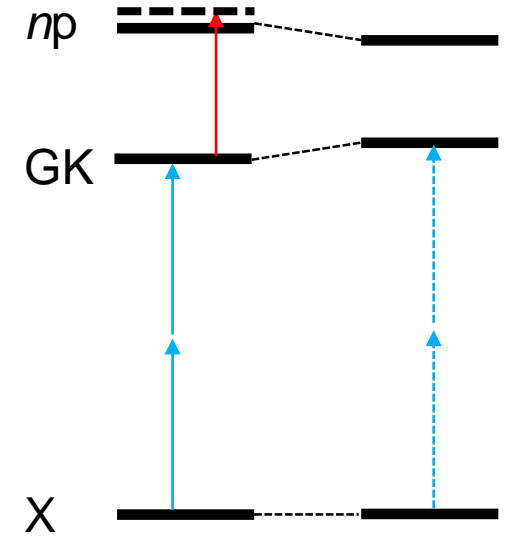
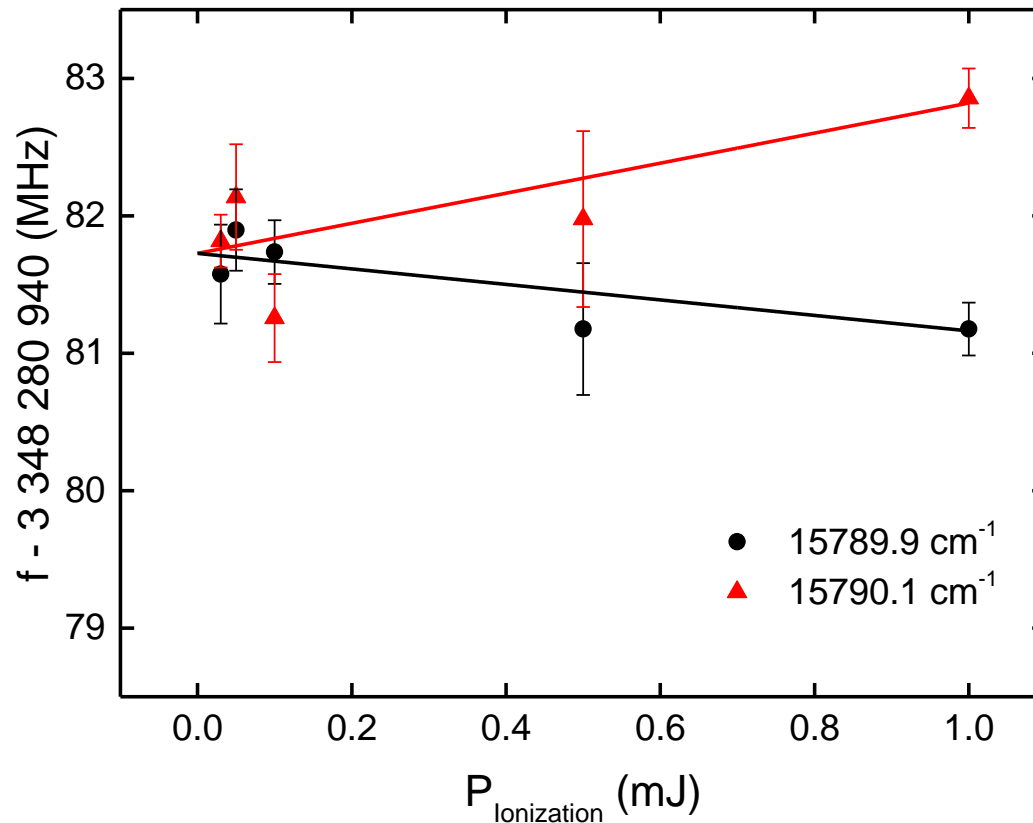


Systematic check: AC-stark of ionization light



Systematic check: AC-stark of ionization light

The time delay: 30ns



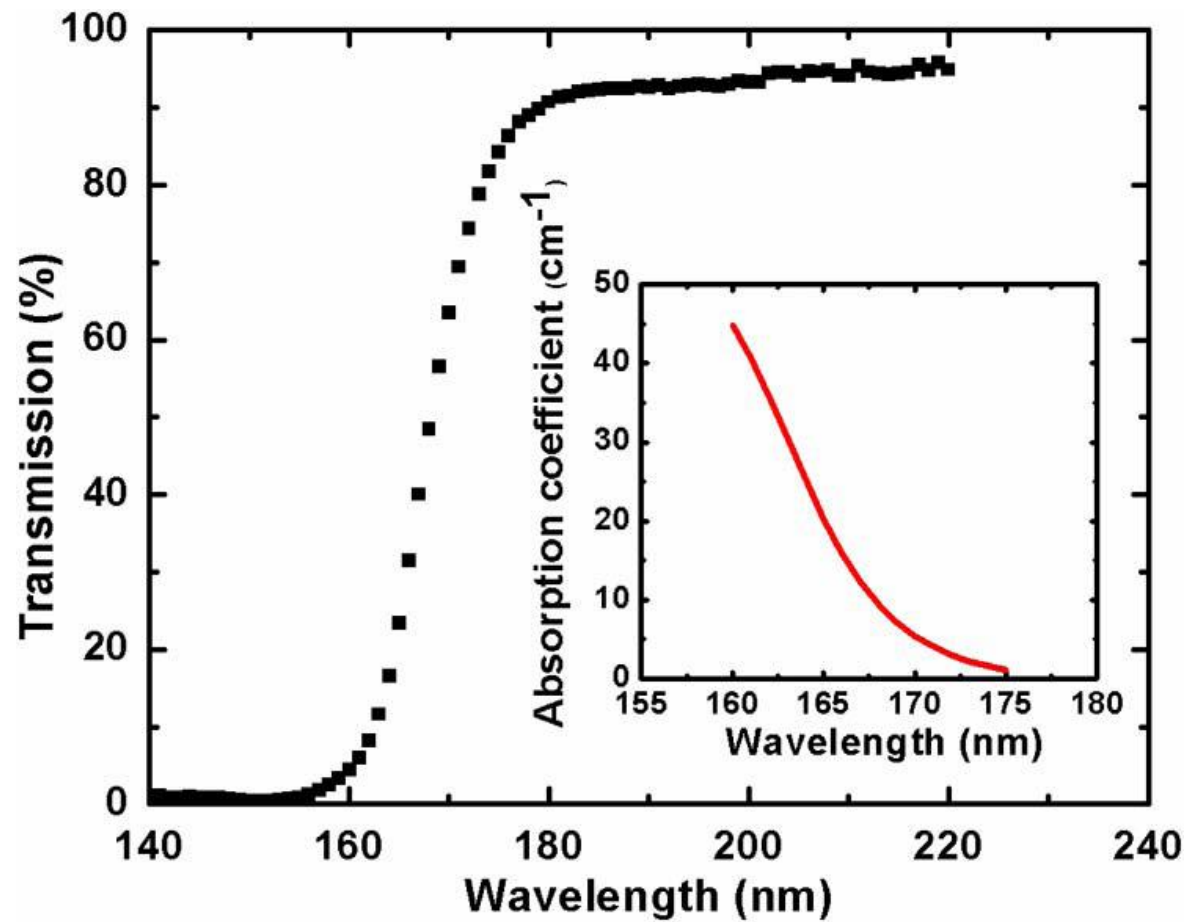
Power range:

0.05 – 1.0mJ (633nm)

About

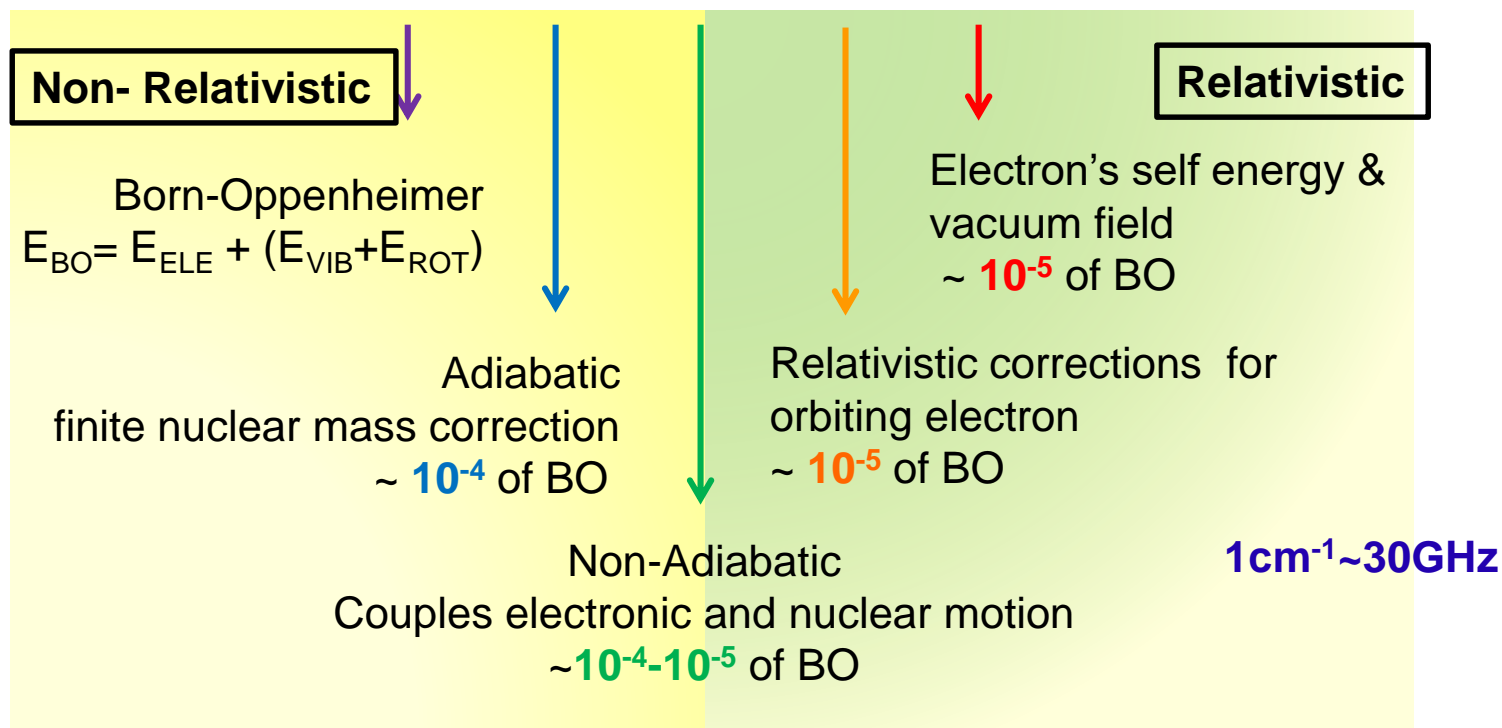
18 – 350 kW/cm²

KBBF



Test QED in H₂ system

Ground state $E_{\text{level}} = E_{\text{BO}} + E_{\text{AD}} + E_{\text{NAD}} + E_{\text{REL}} + E_{\text{QED}}$



- ◆ Current calculation accuracy 3~30MHz, including QED
- ◆ Proton size kicks in when accuracy reaches 0.01~0.1MHz

Systematic check: AC Stark VUV

Power range:

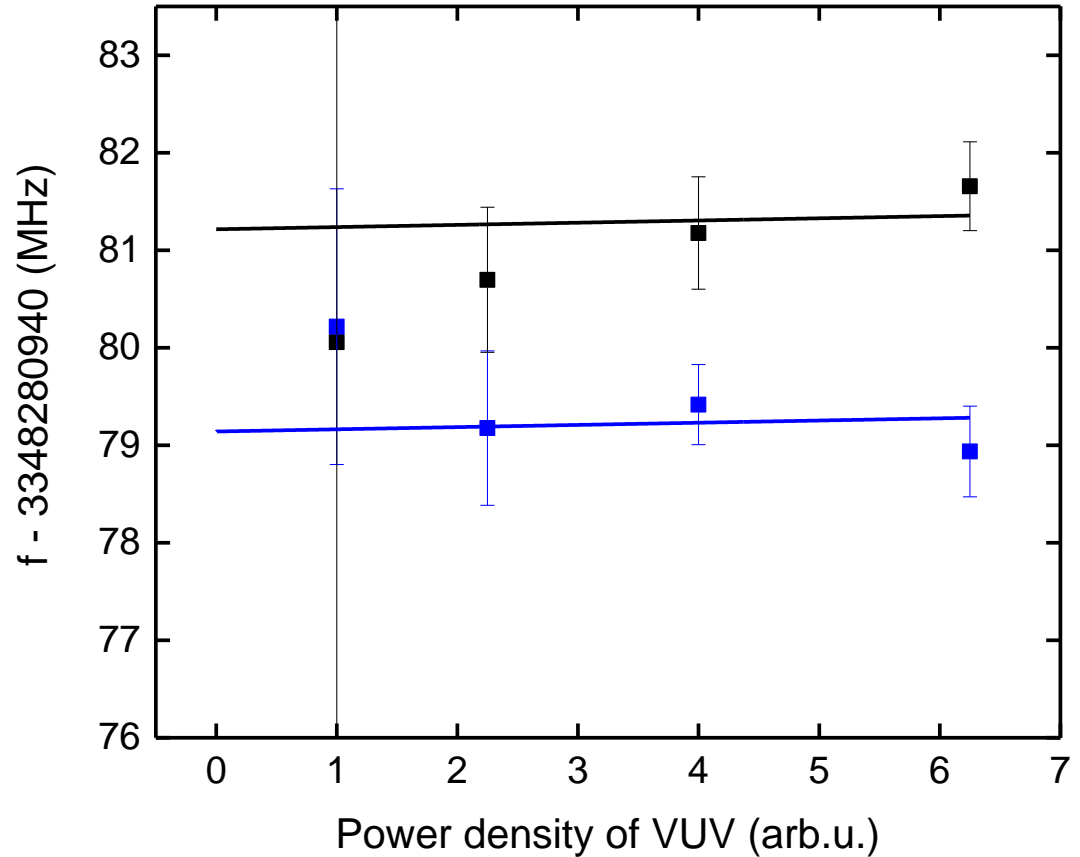
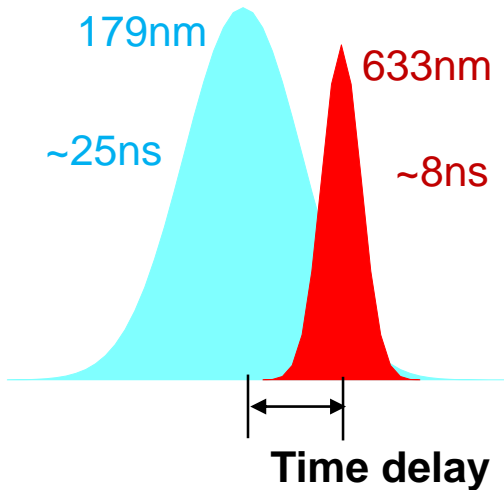
1 – 2.5mJ (358nm)

About

1 – 10uJ (179nm)

4 – 40 kW/cm²

Two different “Doppler” alignments



Systematic check: AC Stark ionization

Power range:

0.05 – 1.0 mJ (633 nm)

About

18 – 350 kW/cm²

