High-precision tests of the Standard Model at future e^+e^- colliders

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Outline

Motivation

Motivation:

- Development of the physical program for future high-energy e^+e^- colliders
- Having high-precision theoretical description of Bhabha scattering and e^+e^- annihilation processes is of crucial importance
- Many of the future e^+e^- colliders foresee running with polarized beam(s)

QUESTIONS:

- What we have?
- What we need?
- What to do?
- How to do?

FUTURE e^+e^- COLLIDER PROJECTS

Linear Colliders

- ILC, CLIC
- ILC: technology is ready, to be built in Japan (?)

E_{tot}

- ILC: 91; 250 GeV — 1 TeV
- CLIC: 500 GeV — 3 TeV

$\mathcal{L} \approx 2 \cdot 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

Stat. uncertainty $\sim 10^{-3}$

Beam polarization:
e^-beam: $P = 80 - 90\%$
e^+beam: $P = 30 - 60\%$

Circular Colliders

- FCC-ee, TLEP
- CEPC
- muon collider (?)

E_{tot}

- 91; 160; 240; 350 GeV

$\mathcal{L} \approx 2 \cdot 10^{36}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$ (4 exp.)

Stat. uncertainty $< 10^{-3}$

Beam polarization: desirable

SUPER CHARM-TAU FACTORY PROJECTS

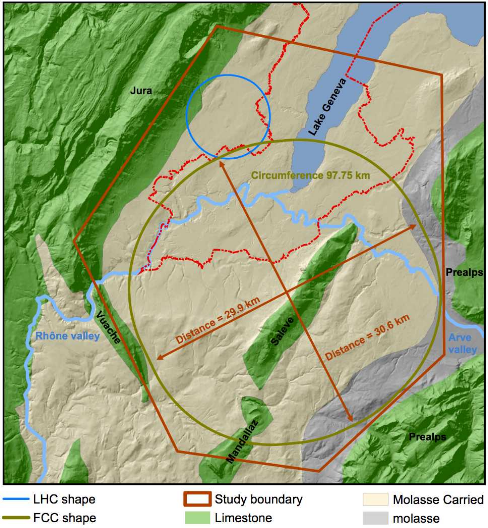
Budker Institute of Nuclear Physics in Novosibirsk and/or China

Colliding electron-positron beams with c.m.s. energies from 2 to 5 GeV with unprecedented high luminosity $10^{35} cm^{-2} c^{-1}$

The electron beam will be longitudinally polarized

The main goal of experiments at the Super Charm-Tau factory is to study the processes charmed mesons and tau leptons, using a data set that is 2 orders of magnitude more than the one collected by BESIII

FCC-EE PROJECT AT CERN (2038 ?)



PHYSICS MILESTONES AT HIGH ENERGIES

Exploration of the SM and looking for its applicability limits

Measurements at Z boson peak: improve the (LEP) precision in studies of EW physics by one order of magnitude

$E \sim 240$ GeV: Higgsstrahlung $e^+e^- \to ZH$

$E \sim 350$ GeV: $e^+e^- \to t\bar{t}$

$E > 500$ GeV: high-precision EW measurements and searches for new physics

Post-LHC HEP Physics Goals

• Properties of the Higgs and EW gauge bosons, pinning down their interactions with an accuracy order(s) of magnitude better than today, and acquiring sensitivity to the processes that at $\sim 10^{12}$ s after the Big Bang led to the formation of todays Higgs vacuum field.

• Improve by at least an order of magnitude the discovery reach for new particles at the highest masses.

• Improve by orders of magnitude the sensitivity to rare and elusive phenomena at low energies, including the possible discovery of (**dark matter**) particles with very small couplings.

• Probe energy scales beyond the direct kinematic reach, via an extensive campaign of precision measurements, sensitive to tiny deviations from the Standard Model behaviour.

[A. Blondel et al. arXiv:1906.02693]

HIGGS COUPLING CONSTANTS

Collider	HL-LHC	ILC_{250}	$CLIC_{380}$	$CEPC_{240}$	FCC-$ee_{240\to365}$
$\mathcal{L}$ (ab^{-1})	3	2	1	5.6	5 + 0.2 + 1.5
Years		11.5 [5]	8	7	3 + 1 + 4
g_{HZZ} (%)	1.5 / 3.6	0.29 / 0.47	0.44 / 0.66	0.18 / 0.52	0.17 / 0.26
g_{HWW} (%)	1.7 / 3.2	1.1 / 0.48	0.75 / 0.65	0.95 / 0.51	0.41 / 0.27
g_{Hbb} (%)	3.7 / 5.1	1.2 / 0.83	1.2 / 1.0	0.92 / 0.67	0.64 / 0.56
g_{Hcc} (%)	SM / SM	2.0 / 1.8	4.1 / 4.0	2.0 / 1.9	1.3 / 1.3
g_{Hgg} (%)	2.5 / 2.2	1.4 / 1.1	1.5 / 1.3	1.1 / 0.79	0.89 / 0.82
$g_{H\tau\tau}$ (%)	1.9 / 3.5	1.1 / 0.85	1.4 / 1.3	1.0 / 0.70	0.66 / 0.57
$g_{H\mu\mu}$ (%)	4.3 / 5.5	4.2 / 4.1	4.4 / 4.3	3.9 / 3.8	3.9 / 3.8
$g_{H\gamma\gamma}$ (%)	1.8 / 3.7	1.3 / 1.3	1.5 / 1.4	1.2 / 1.2	1.2 / 1.2
$g_{HZ\gamma}$ (%)	11. / 11.	11. / 10.	11. / 9.8	6.3 / 6.3	10. / 9.4
g_{Htt} (%)	3.4 / 2.9	2.7 / 2.6	2.7 / 2.7	2.6 / 2.6	2.6 / 2.6
g_{HHH} (%)	50. / 52.	28. / 49.	45. / 50.	17. / 49.	19. / 34.
Γ_H (%)	SM	2.4	2.6	1.9	1.2

[J. de Blas et al. arXiv:1905.03764; A. Blondel et al. arXiv:1906.02693]

FCC-ee Tera-Z

The huge improvements will allow the FCC-ee Tera-Z stage to test the Standard Model at an unprecedented precision level. The step in precision corresponds to the step which was represented by LEP/SLC at their time; they tested the SM at a precision which needed complete one-loop corrections, plus leading higher order terms. The FCC-ee at the Z peak needs complete two-loop corrections, plus leading higher order terms. Even without a reference to New Physics, the FCC-ee Tera-Z stage lets us expect exciting, qualitiatively new results.

A. Blondel et al., *Standard Model Theory for the FCC-ee: The Tera-Z*, arXiv:1809.01830 [hep-ph] (251 pages).

A. Blondel, J. Gluza, S. Jadach, P. Janot and T. Riemann (conv.) et al., *Theory report on the 11th FCC-ee workshop*, arXiv:1905.05078 [hep-ph] (290 pages).

Estimated experimental precision

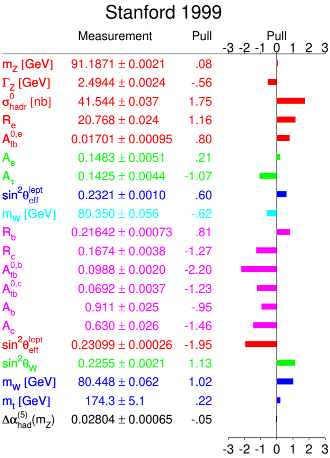
Now:

Quantity	Theory error	Exp. error
M_W [MeV]	4	15
$\sin^2\theta^l_{eff}[10^{-5}]$	4.5	16
Γ_Z [MeV]	0.5	2.3
$R_b[10^{-5}]$	15	66

Quantity	ILC	FCC-ee	CEPC	Projected theory error
M_W [MeV]	3–4	1	3	1
$\sin^2\theta^l_{eff}[10^{-5}]$	1	0.6	2.3	1.5
Γ_Z [MeV]	0.8	0.1	0.5	0.2
$R_b[10^{-5}]$	14	6	17	5–10

The estimated error for the theoretical predictions of these quantities is given, under the assumption that $O(\alpha\alpha_s^2)$, fermionic $O(\alpha^2\alpha_s)$, fermionic $O(\alpha^3)$, and leading four-loop corrections entering through the ρ-parameter will become available.

Where we are: LEP results



LEPEWWG with the help of ZFITTER

N.B. LHC is approaching LEP precision for some SM parameters

INTRODUCTION TO SANC

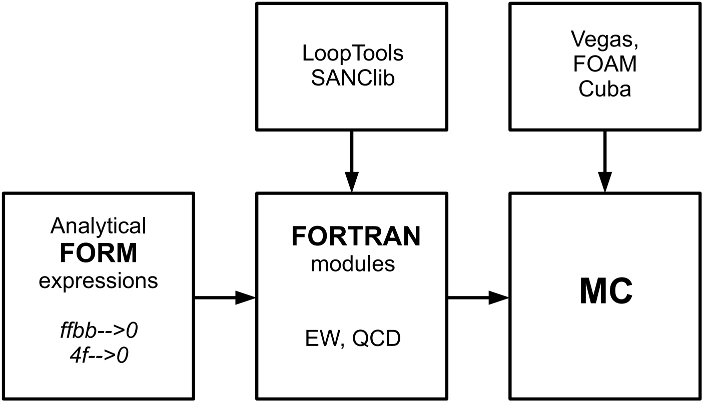
- The SANC system implements calculations of complete (real and virtual) NLO QCD and EW corrections for various processes at the partonic level

- All calculations are performed within the OMS (on-mass-shell) renormalization scheme in the R_ξ gauge which allows an explicit control of the gauge invariance by examining cancellation of the gauge parameters in the analytic expression of the matrix element

- Cross-sections of the processes at hadron level obtained by convolution the partonic level cross-sections with PDFs

- The list of processes implemented in the MCSANC integrator includes Drell-Yan processes (inclusive), associated Higgs and gauge boson production and single-top quark production in s- and t-channel (v1.01 – CPC 184 (2013) 2343), photon-induced contribution, EW corrections beyond NLO approximation to DY (v1.20 – JETP Lett. 103 (2016) 131)

SANC FRAMEWORK SCHEME



SANC for processes with polarized beams

- NLO EW corrections for polarized e^+e^- scattering:
  - Bhabha scattering (PRD 2018)
  - $e^+e^- \to ZH$ (arXiv:1812.10965)
  - $e^+e^- \to \mu^+\mu^-$(or $\tau^+\tau^-$) (preliminary)
  - $e^+e^- \to Z\gamma$ (preliminary)
  - $e^+e^- \to \gamma\gamma$ (preliminary)
  - $e^+e^- \to t\bar{t}$ (in progress)
  - $e^+e^- \to ZZ$ (in progress)
  - $e^+e^- \to f\bar{f}\gamma$ (future plans)
  - $e^+e^- \to f\bar{f}H$ (future plans)
- NLO EW corrections for polarized $\gamma\gamma$ scattering:
  - $\gamma\gamma \to \gamma\gamma$ (future plans)
  - $\gamma\gamma \to Z\gamma$ (future plans)
  - $\gamma\gamma \to ZZ$ (future plans)

BHABHA SCATTERING: GENERAL REMARKS

- Bhabha scattering is the basic QED process which is widely used for luminosity measurements at e^+e^- colliders (but no only)
- We got a record accuracy in small-angle Bhabha measurement and description at LEP. That was one of keystones for the high-precision verification of the Standard Model at LEP
- Since that time a considerable progress has been achieved:
  1) calculations of higher order radiative corrections
  2) computer tools, including adaptive Monte Carlo
  3) detector techniques, data analysis, etc.
- Still a lot has to be done to meet requirements of experiments at future e^+e^- colliders
- The aim of the talk is to overview the past achievements and to indicate and discuss the present problems (=tasks)

BHABHA IN SANC: HA FOR BORN AND VIRTUAL CONTRIBUTIONS

At one-loop level we have 6 non-zero HAs (4 independent):

$$\mathcal{H}_{++++} = \mathcal{H}_{----} = -2e^2\,\frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) - \chi^t_Z\delta_e\mathcal{F}^Z_{QL}(t,s,u)\right],$$

$$\mathcal{H}_{+-+-} = \mathcal{H}_{-+-+} = -e^2\,c_-\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) - \chi^s_Z\delta_e\mathcal{F}^Z_{QL}(s,t,u)\right],$$

$$\mathcal{H}_{+--+} = -e^2\,c_+\Bigg(\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) + \chi^s_Z\left(\mathcal{F}^Z_{LL}(s,t,u) - 2\delta_e\mathcal{F}^Z_{QL}(s,t,u)\right)\right] + \frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) + \chi^t_Z\left(\mathcal{F}^Z_{LL}(t,s,u) - 2\delta_e\mathcal{F}^Z_{QL}(t,s,u)\right)\right]\Bigg),$$

$$\mathcal{H}_{-++-} = -e^2\,c_+\Bigg(\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u)\right] + \frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u)\right]\Bigg),$$

where $c_+ = 1 + \cos\theta, \quad c_- = 1 - \cos\theta$

$$\chi^s_Z = \frac{1}{4s^2_W c^2_W}\,\frac{s}{s - M^2_Z + iM_Z\Gamma_Z}, \quad \chi^t_Z = \frac{1}{4s^2_W c^2_W}\,\frac{t}{t - M^2_Z}, \quad \delta_e = v_e - a_e = 2s^2_W,$$

$$\mathcal{F}^{(\gamma,Z)}_{QQ}(a,b,c) = \mathcal{F}^{\gamma}_{QQ}(a,b,c) + \chi^a_Z\delta^2_e\mathcal{F}^Z_{QQ}(a,b,c)$$

We get the Born level HAs by replacing $\mathcal{F}^Z_{LL} \to 1$, $\mathcal{F}^Z_{QL} \to 1$, $\mathcal{F}^Z_{QQ} \to 1$ and $\mathcal{F}^\gamma_{QQ} \to 1$

SANC Monte Carlo generator for Bhabha

We created a Monte Carlo generator of unweighted events for polarized Bhabha scattering $e^+e^- \to e^+e^-$ with complete one-loop EW corrections and with possibility to produce events in the standard Les Houches format.

This generator uses adaptive algorithm `mFOAM` (CPC 177:441-458,2007) which is part of the `ROOT` program

SETUP FOR TUNED COMPARISON

We performed a tuned comparison of polarized Born and hard Bremsstrahlung by WHIZARD. The unpolarized soft and virtual parts were compared with the results of AÏtalc.

Initial parameters

$$\alpha^{-1}(0) = 137.03599976,$$

$$M_W = 80.451495 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV}, \quad \Gamma_Z = 2.49977 \text{ GeV},$$
$$m_e = 0.5109990 \text{ MeV}, \quad m_\mu = 0.105658 \text{ GeV}, \quad m_\tau = 1.77705 \text{ GeV},$$
$$m_d = 0.083 \text{ GeV}, \quad m_s = 0.215 \text{ GeV}, \quad m_b = 4.7 \text{ GeV},$$
$$m_u = 0.062 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_t = 173.8 \text{ GeV}.$$

Cuts

$$|\cos\theta| < 0.9,$$
$$E_\gamma > 1 \text{ GeV} \quad \text{(for comparison of hard Bremsstrahlung)}.$$

$e^+e^- \to e^+e^-$: WHIZARD VS SANC (HARD)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	48.62(1)	49.58(1)	48.74(1)	50.40(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	48.65(1)	49.56(1)	48.78(1)	50.44(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	15.14(1)	15.81(1)	13.54(1)	18.07(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	15.12(1)	15.79(1)	13.55(1)	18.11(2)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{hard}}_{e^+e^-}$, pb	4.693(1)	4.976(1)	3.912(1)	6.041(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb	4.694(1)	4.975(1)	3.913(1)	6.043(1)

$e^+e^- \to e^+e^-$: AI̊TALC VS SANC $\sqrt{s} = 500 GeV$

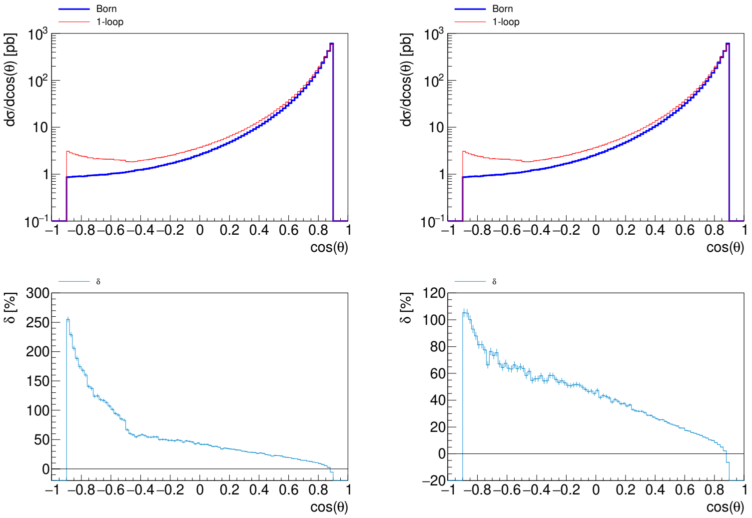
$\cos\theta$	$\sigma^{\text{Born}}_{e^+e^-}$, pb	$\sigma^{\text{Born+virt+soft}}_{e^+e^-}$, pb
-0.9	$2.16999 \cdot 10^{-1}$ $2.16999 \cdot 10^{-1}$	$1.93445 \cdot 10^{-1}$ $1.93445 \cdot 10^{-1}$
-0.5	$2.61360 \cdot 10^{-1}$ $2.61360 \cdot 10^{-1}$	$2.38707 \cdot 10^{-1}$ $2.38707 \cdot 10^{-1}$
0	$5.98142 \cdot 10^{-1}$ $5.98142 \cdot 10^{-1}$	$5.46677 \cdot 10^{-1}$ $5.46677 \cdot 10^{-1}$
$+0.5$	$4.21273 \cdot 10^{0}$ $4.21273 \cdot 10^{0}$	$3.81301 \cdot 10^{0}$ $3.81301 \cdot 10^{0}$
$+0.9$	$1.89160 \cdot 10^{2}$ $1.89160 \cdot 10^{2}$	$1.72928 \cdot 10^{2}$ $1.72928 \cdot 10^{2}$
$+0.99$	$2.06556 \cdot 10^{4}$ $2.06555 \cdot 10^{4}$	$1.90607 \cdot 10^{4}$ $1.90607 \cdot 10^{4}$
$+0.999$	$2.08236 \cdot 10^{6}$ $2.08236 \cdot 10^{6}$	$1.91624 \cdot 10^{6}$ $1.91624 \cdot 10^{6}$
$+0.9999$	$2.08429 \cdot 10^{8}$ $2.08429 \cdot 10^{8}$	$1.91402 \cdot 10^{8}$ $1.91402 \cdot 10^{8}$

BORN VS 1-LOOP (SANC)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.6763(1)	57.7738(1)	56.2725(4)	59.2753(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	61.731(6)	62.587(6)	61.878(6)	63.287(7)
δ, %	8.92(1)	8.33(1)	9.96(1)	6.77(1)
$\sqrt{s} = 500$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	14.3789(1)	15.0305(1)	12.7061(1)	17.3550(2)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	15.465(2)	15.870(2)	13.861(1)	17.884(2)
δ, %	7.56(1)	5.59(1)	9.09(1)	3.05(1)
$\sqrt{s} = 1000$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb	3.67921(1)	3.90568(1)	3.03577(3)	4.77562(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	3.8637(4)	3.9445(4)	3.2332(3)	4.6542(7)
δ, %	5.02(1)	0.99(1)	6.50(1)	-2.54(1)

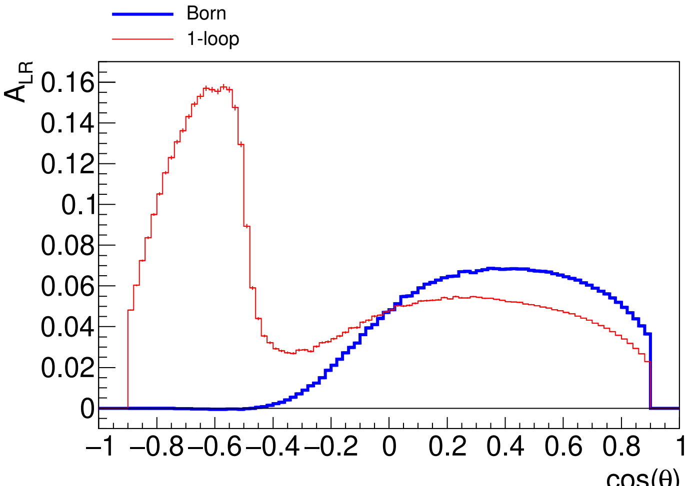
Distributions in $\cos\theta$

$\sqrt{s} = 250$ GeV $\sqrt{s} = 500$ GeV



Left-Right Asymmetry ($E_{\rm CM} = 250$ GeV)

$$A_{LR} \equiv \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$$



PROCESS $e^+e^- \to ZH$: BORN VS 1-LOOP

P_{e-}, P_{e+}	$\sigma_{ZH}^{\text{Born}}$, pb	$\sigma_{ZH}^{1-\text{loop}}$, pb	δ,%
	$\sqrt{s} = 250$ GeV		
0, 0	205.64(1)	186.6(1)	-9.24(1)
-0.8, 0	242.55(1)	201.5(1)	-16.94(1)
-0.8, -0.6	116.16(1)	100.8(1)	-13.25(1)
-0.8, $+0.6$	368.93(1)	302.2(1)	-18.10(1)
	$\sqrt{s} = 500$ GeV		
0, 0	51.447(1)	57.44(1)	11.65(1)
-0.8, 0	60.680(1)	62.71(1)	3.35(2)
-0.8, -0.6	29.061(1)	31.25(1)	7.54(1)
-0.8, $+0.6$	92.299(1)	94.17(2)	2.03(2)
	$\sqrt{s} = 1000$ GeV		
0, 0	11.783(1)	12.92(1)	9.68(1)
-0.8, 0	13.898(1)	13.91(1)	0.10(2)
-0.8, -0.6	6.6559(1)	6.995(1)	5.09(2)
-0.8, $+0.6$	21.140(1)	20.83(1)	-1.47(2)

A.A. et al. in "Theory report on the 11th FCC-ee workshop" arXiv:1905.05078

PRELIMINARY: $e^+e^- \to \mu^+\mu^-;\ \ \tau^+\tau^-$

$P_{e^-},\ P_{e^+}$	$\sigma^{\text{Born}}_{\mu^+\mu^-}$, pb	$\sigma^{1-\text{loop}}_{\mu^+\mu^-}$, pb	δ,%	$\sigma^{\text{Born}}_{\tau^+\tau^-}$, pb	$\sigma^{1-\text{loop}}_{\tau^+\tau^-}$, pb	δ,%
			$\sqrt{s} = 250$ GeV			
0, 0	1.417(1)	2.397(1)	69.1(1)	1.417(1)	2.360(1)	66.5(1)
−0.8, 0	1.546(1)	2.614(1)	69.1(1)	1.546(1)	2.575(1)	66.5(1)
−0.8, −0.6	0.7690(2)	1.301(1)	69.2(1)	0.7692(1)	1.298(1)	68.8(1)
−0.8, +0.6	2.323(1)	3.927(1)	69.1(1)	2.324(1)	3.850(1)	65.7(1)
			$\sqrt{s} = 500$ GeV			
0, 0	0.3436(1)	0.4696(1)	36.7(1)	0.3436(1)	0.4606(1)	34.0(3)
−0.8, 0	0.3716(1)	0.4953(1)	33.3(1)	0.3715(1)	0.4861(1)	30.8(1)
−0.8, −0.6	0.1857(1)	0.2506(1)	35.0(1)	0.1857(1)	0.2466(1)	32.8(1)
−0.8, +0.6	0.5575(1)	0.7399(1)	32.7(1)	0.5575(1)	0.7257(1)	30.1(1)
			$\sqrt{s} = 1000$ GeV			
0, 0	0.08535(1)	0.1163(1)	36.2(1)	0.08534(2)	0.1134(1)	33.6(1)
−0.8, 0	0.09213(1)	0.1212(1)	31.6(1)	0.09213(1)	0.11885(2)	29.0(1)
−0.8, −0.6	0.04608(1)	0.06169(1)	33.9(1)	0.04608(1)	0.06067(1)	31.7(1)
−0.8, +0.6	0.1382(1)	0.1807(1)	30.8(1)	0.1382(1)	0.1770(1)	28.1(1)

A.A. et al. in "Theory report on the 11th FCC-ee workshop" arXiv:1905.05078

Leading and next-to-leading logs in QED

The QED leading (LO) logarithmic corrections

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^n \frac{s}{m_e^2}$$

were relevant for LEP measurements of Bhabha, $e^+e^- \to \mu^+\mu^-$ etc. for $n \leq 3$ since $\ln(M_Z^2/m_e^2) \approx 24$

NLO contributions

$$\sim \left(\frac{\alpha}{2\pi}\right)^n \ln^{n-1} \frac{s}{m_e^2}$$

with $n = 3$ are required for future e^+e^- colliders

In the collinear approximation we can get them within the NLO QED structure function formalism

- F.A.Berends, W.L. van Neerven, G.J.Burgers, NPB'1988
- A.A., K.Melnikov, PRD'2002; A.A. JHEP'2003

QED NLO master formula

The NLO Bhabha cross section reads

$$
\begin{aligned}
d\sigma &= \sum_{a,b,c,d=e,\bar{e},\gamma} \int_{\bar{z}_1}^{1} dz_1 \int_{\bar{z}_2}^{1} dz_2 \mathcal{D}^{\rm str}_{ae}(z_1)\mathcal{D}^{\rm str}_{b\bar{e}}(z_2) \\
&\times \left[d\sigma^{(0)}_{ab\to cd}(z_1,z_2) + d\bar{\sigma}^{(1)}_{ab\to cd}(z_1,z_2) \right] \\
&\times \int_{\bar{y}_1}^{1} \frac{dy_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{dy_2}{Y_2} \mathcal{D}^{\rm frg}_{ec}\left(\frac{y_1}{Y_1}\right) \mathcal{D}^{\rm frg}_{\bar{e}d}\left(\frac{y_2}{Y_2}\right) + \mathcal{O}(\alpha^n L^{n-2})
\end{aligned}
$$

$\alpha^2 L^1$ terms are completely reproduced [A.A., E.Scherbakova, JETP Lett. 2006; PLB 2008]

$\alpha^3 L^2$ terms are required at the Z peak and for e^+e^- annihilation channels at higher energies

QED NLO EVOLUTION

$$\frac{d\mathcal{D}_{ee}(x,\mu_f,m_e)}{d\ln\mu_f^2} = \sum_{a=e,\gamma,\bar{e}} \int\limits_z^1 \frac{dz}{z} P_{ea}(z,\bar{\alpha}(\mu_f))\mathcal{D}_{ae}\left(\frac{x}{z},\mu_f,m_e\right)$$

with perturbative splitting functions

$$P_{ea}(z,\bar{\alpha}(\mu_f)) = \frac{\bar{\alpha}(\mu_f)}{2\pi}P^{(0)}_{ea}(z) + \left(\frac{\bar{\alpha}(\mu_f)}{2\pi}\right)^2 P^{(1)}_{ea}(z) + \mathcal{O}(\alpha^3)$$

and initial conditions

$$\begin{aligned}
\mathcal{D}^{\rm ini}_{ee}(x,\mu_0,m_e) &= \delta(1-x) + \frac{\bar{\alpha}(\mu_0)}{2\pi} d_1(x,\mu_0,m_e) + \mathrm{O}(\alpha^2) \\
d_1(x,\mu_0,m_e) &= \left[\frac{1+x^2}{1-x}\left(\ln\frac{\mu_0^2}{m_e^2} - 2\ln(1-x) - 1\right)\right]_+ \\
\mathcal{D}^{\rm ini}_{\gamma e}(x,\mu_0,m_e) &= \frac{\bar{\alpha}(\mu_0)}{2\pi} P^{(0)}_{\gamma e}(x) + \mathrm{O}(\alpha^2)
\end{aligned}$$

Iterative solution

The pure photonic non-singlet part of the electron structure function

$$\mathcal{D}_{ee}^{(\gamma)}(x,\mu_f,m_e) = \delta(1-x) + \frac{\alpha}{2\pi} d_1(x,\mu_0,m_e) + \frac{\alpha}{2\pi} L_f P_{ee}^{(0)}(x)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^2 \left(\frac{1}{2} L_f^2 P_{ee}^{(0)} \otimes P_{ee}^{(0)}(x) + L_f P_{ee}^{(0)} \otimes d_1(x,\mu_0,m_e) + L_f P_{ee}^{(1,\gamma)}(x)\right)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^3 \left(\frac{1}{6} L_f^3 P_{ee}^{(0)} \otimes P_{ee}^{(0)} \otimes P_{ee}^{(0)}(x) + L_f^2 P_{ee}^{(0)} \otimes P_{ee}^{(1,\gamma)}(x)\right.$$
$$\left. + L_f^2 P_{ee}^{(0)} \otimes P_{ee}^{(0)} \otimes d_1(x,\mu_0,m_e)\right) + \mathcal{O}(\alpha^3 L^1)$$

The large logarithm $L_f \equiv \ln \frac{\mu_f^2}{m_e^2}$ with factorization scale $\mu_f^2 \sim s$ or $\sim -t$

NLO contributions of e^+e^- pairs are recovered in the same way

Required convolution integrals are listed in [A.A. hep-ph/0304063]

Example of convolution (I)

$$P_{ee}^{(0)}(x) = \left[\frac{1+x^2}{1-x}\right]_+$$

$$P_{ee}^{(1,\gamma)\mathrm{str}}(x) = \delta(1-x)\left(\frac{3}{8} - 3\zeta(2) + 6\zeta(3)\right)$$
$$+\frac{1+x^2}{1-x}\left(-2\ln x\ln(1-x) + \ln^2 x + 2\mathrm{Li}_2\,(1-x)\right)$$
$$-\frac{1}{2}(1+x)\ln^2 x + 2\ln x + 3 - 2x$$

Plus prescription

$$\int_{x_{\min}}^{1} dx\,[V(x)]_+\,W(x) = \int_0^1 dx V(x)\Big[W(x)\Theta(x-x_{\min}) - W(1)\Big]$$

Convolution

$$A\otimes B(x) = \int_0^1 dz\int_0^1 dz'\delta(x-zz')A(z)B(z')$$

EXAMPLE OF CONVOLUTION (II) PRELIMINARY

$$
\begin{aligned}
P_{ee}^{(0)} \otimes P_{ee}^{(1,\gamma)\mathrm{str}}(x) = \Bigg[& \frac{1+x^2}{1-x} \Bigg(-4\mathrm{S}_{12}(1-x) + 4\mathrm{Li}_2(1-x)(\ln(1-x) - \ln(x)) \\
& -4\ln(x)\ln^2(1-x) + 4\ln^2(x)\ln(1-x) - \frac{2}{3}\ln^3(x) + 3\mathrm{Li}_2(1-x) \\
& -3\ln(x)\ln(1-x) + \frac{3}{2}\ln^2(x) + 4\zeta(2)\ln(x) + 6\zeta(3) - 3\zeta(2) + \frac{3}{8} \Bigg) \\
& +4(1+x)\mathrm{S}_{12}(x) + \frac{1+x}{2}\ln^3(x) - 2(1+x)\ln^2(x)\ln(1-x) \\
& +(6x-2)\mathrm{Li}_2(1-x) + (6-2x)\ln(x)\ln(1-x) + \left(\frac{11}{4}x - \frac{9}{4}\right)\ln^2(x) \\
& +(6-4x)\ln(1-x) + (5x-3)\ln(x) + 2x - \frac{1}{2} \Bigg]_+
\end{aligned}
$$

This is a part of universal collinear radiation factors

Outlook

- Having high theoretical precision for the normalization processes $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$, and $e^+e^- \to 2\gamma$ with polarized beams is crucial for future e^+e^- colliders
- There are several two-loop QED and EW results, but most of them are w/o polarization yet
- Numerical calculations of three-loop EW RC are in progress
- The SANC computer system is being upgraded
- New Monte Carlo codes are required
- $O(\alpha^3 L^2)$ collinear radiator factors are derived. First, they will be implemented into the ZFITTER code

STATEMENT OF THE PROBLEM

General task: we need a high-precision description of (polarized) e^+e^- interactions at high energies

General results: Monte Carlo generator(s), Monte Carlo integrator(s), and semi-analytic codes for cross-checks

Input: the beam energy distribution (for linear colliders), hadronic vacuum polarization, analytic results from the literature, and new calculations

Output: various distributions and inclusive observables with estimates of uncertainties

Set-up: collaboration of several groups is definitely required to solve the problems