Two-loop self-energy corrections to the $g$-factor of bound electrons

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g-factor — definition

$g$-factor of bound electron in ground state $|1s\rangle$ of hydrogenlike ion:

$$\mu_e = g \frac{eJ}{2m_e} ; \quad \delta E = -\langle 1s | \mu_e B | 1s \rangle = -g \frac{eB}{4m_e}$$

$\delta E$: energy splitting due to external magnetic field $B$ (Zeeman effect)
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**Contributions to the bound-electron \( g \)-factor**

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g_D = \frac{2}{3} + \frac{4}{3} \sqrt{1 - (Z\alpha)^2}
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Largest uncertainty in theoretical $g$-factor predictions due to uncalculated SESE diagrams

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Done  Ongoing