The virtual-Delbruck-scattering potential for light muonic atoms

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|PCT|

Outline

1) Three types of light-by-light-scattering diagrams in muonic atoms

2) The static-muon approximation and its applicability

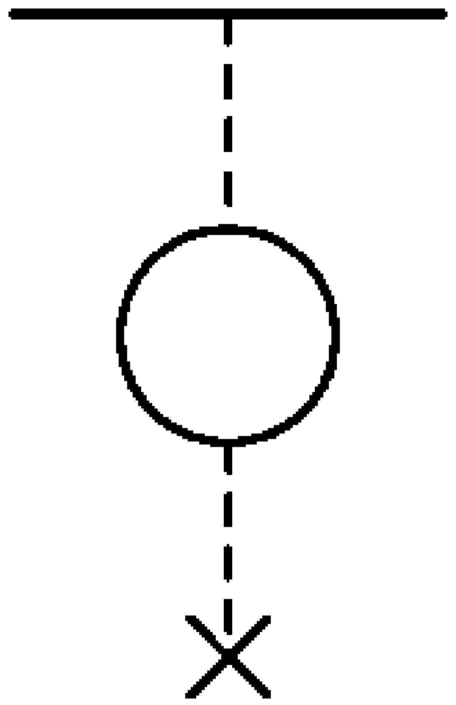
3) The effective potential for the virtual Delbrück scattering

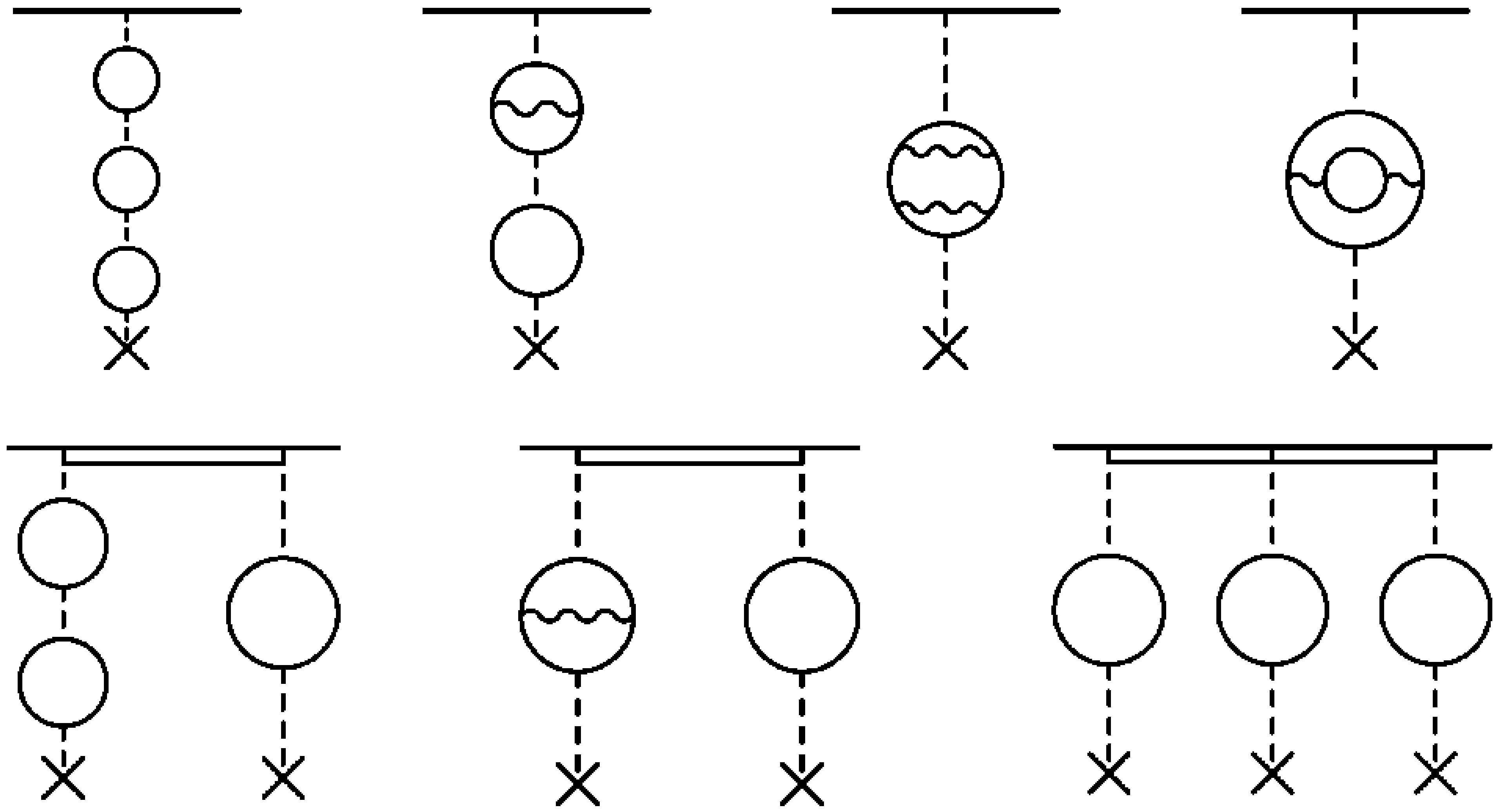
- numerical data
- asymptotics
- resulting fit

4) small summary

Light-by-light-scattering diagrams in muonic atoms

The Bohr radius in muonic atoms is comparable with the Compton wavelength of an electron. So the diagrams with the closed electron loops are enhance in QED theory of muonic atoms.





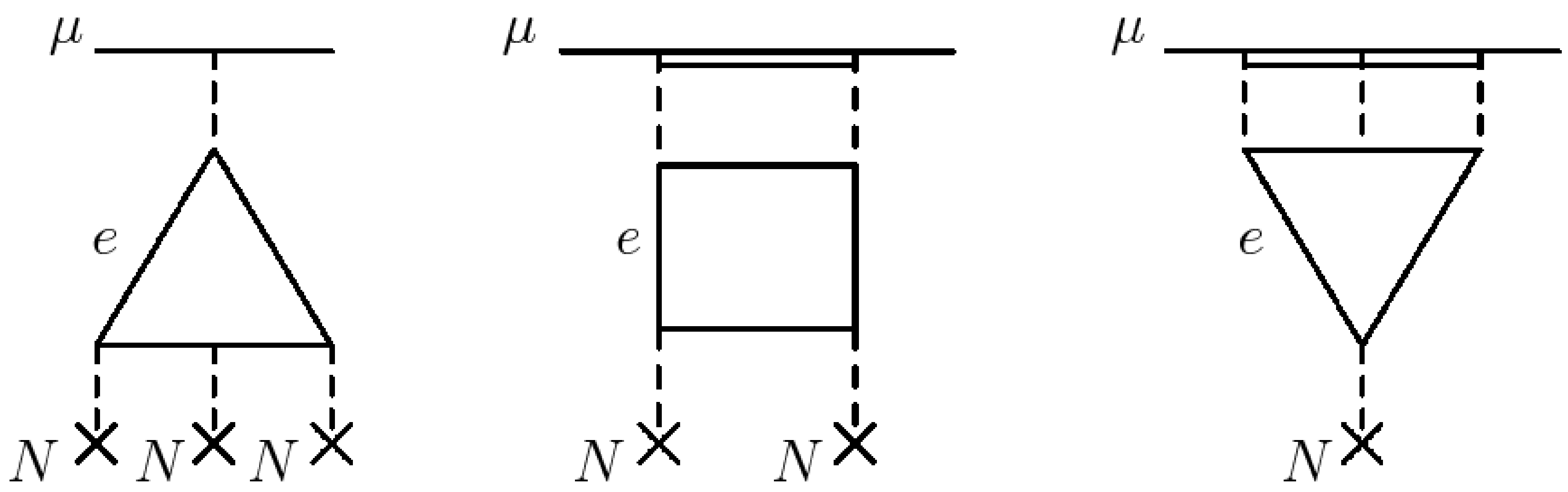
Light-by-light-scattering diagrams in muonic atoms

Specific type of the diagrams with the closed electron loops is related to Light-by-light scattering

(1:3) Wichmann-Kroll contribution

(2:2) the virtual Delbrück scattering contribution

(3:1) without special name



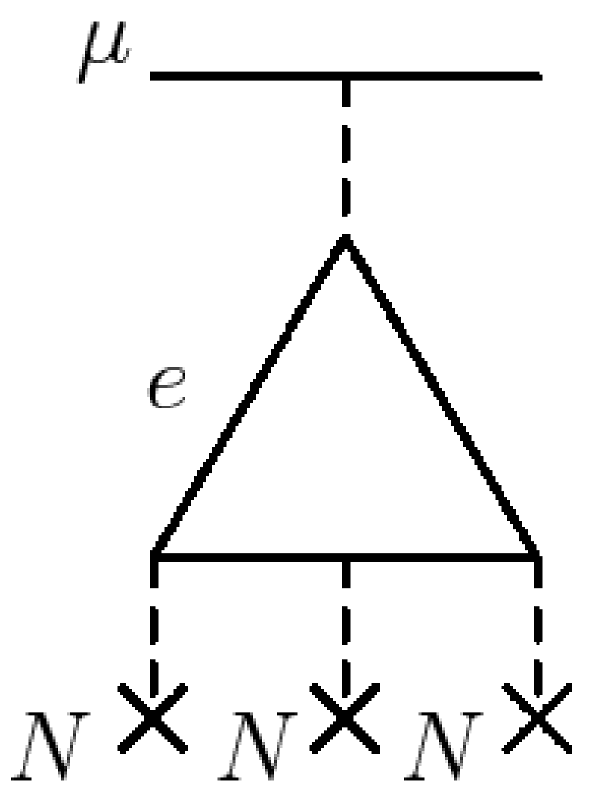
(1:3) (2:2) (3:1)

Light-by-light-scattering diagrams in muonic atoms

(1:3) Wichmann-Kroll contribution has been studied for a while.

J.Blomkwist (1972) gives analytical representation for the WK potential

K.-N. Huang (1976) and *E.Borie, G.A. Rinker (1982)* published simple approximations of WK potential

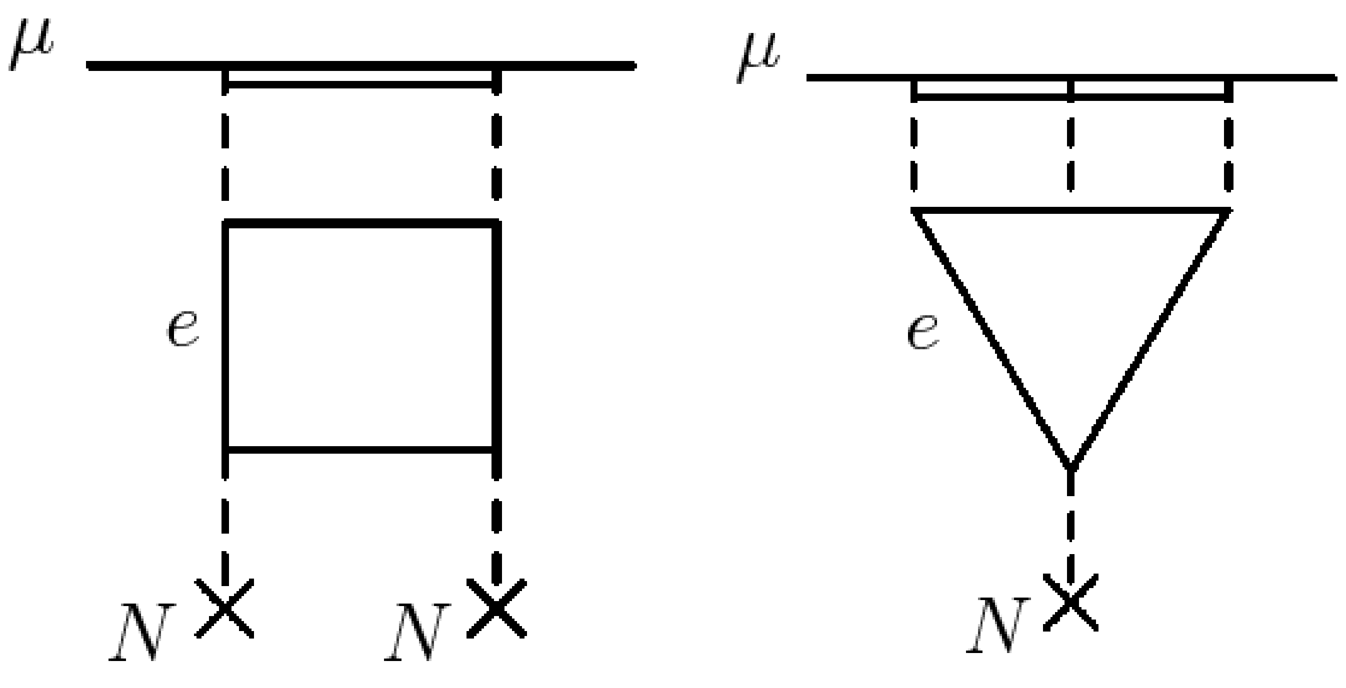


(1:3)

Light-by-light-scattering diagrams in muonic atoms

Two other diagrams contains Coulomb Green-function of the muon, that makes it difficult to calculate.

First result for (3:1) was published in 2010



(2:2) (3:1)

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Nonrelativistic contributions of order $\alpha^5 m_\mu c^2$ to the Lamb shift in muonic hydrogen and deuterium, and in the muonic helium ion

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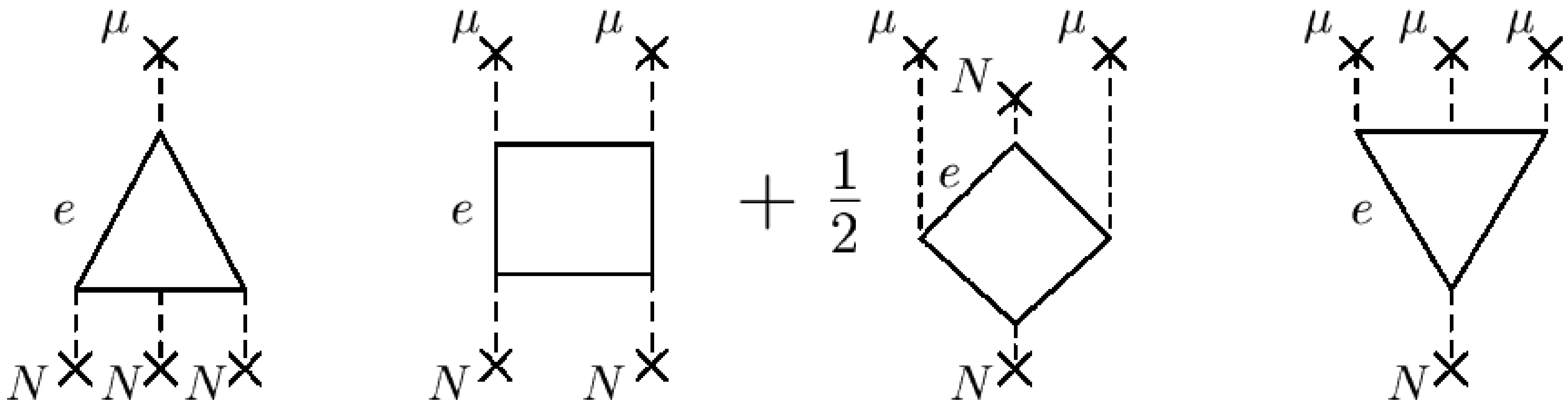
Original Russian Text © S.G. Karshenboim, E.Yu. Korzinin, V.G. Ivanov, V.A. Shelyuto, 2010, published in Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, 2010, Vol. 92, No. 1, pp. 9–15.

Contribution of Light-by-Light Scattering to Energy Levels of Light Muonic Atoms¶

S. G. Karshenboim[a, b], E. Yu. Korzinin[a], V. G. Ivanov[a, c], and V. A. Shelyuto[a]

The static-muon approximation

We reduced the contribution to the case of the static-muon approximation



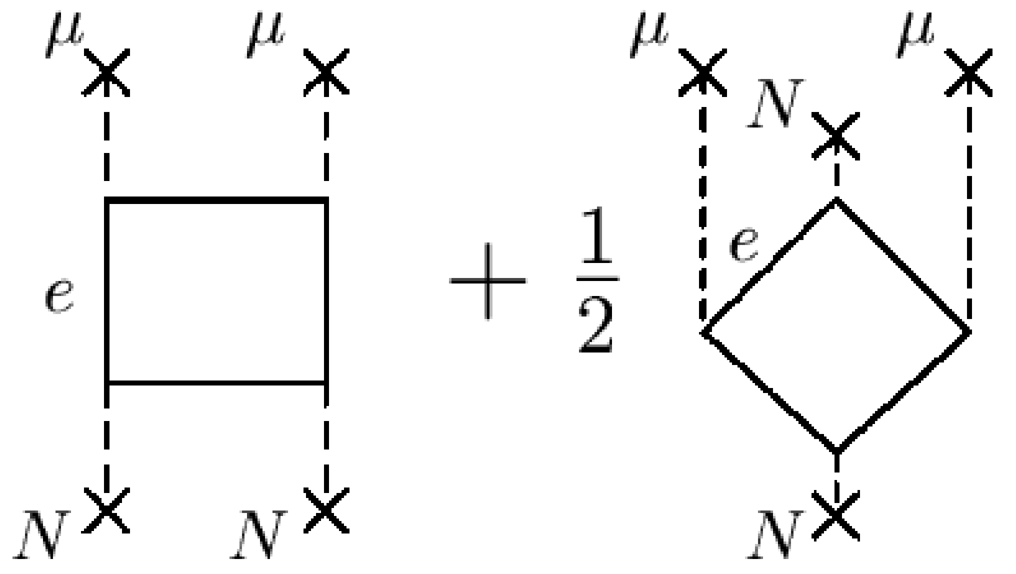
(1:3) (2:2) (3:1)

It gives simple result for the (3:1) contribution

$$\Delta E_{3:1}(ns) = \frac{1}{Z^2}\,\Delta E_{1:3}(ns)$$

The effective potential for the virtual Delbrück scattering

(2:2) was presented in the momentum space



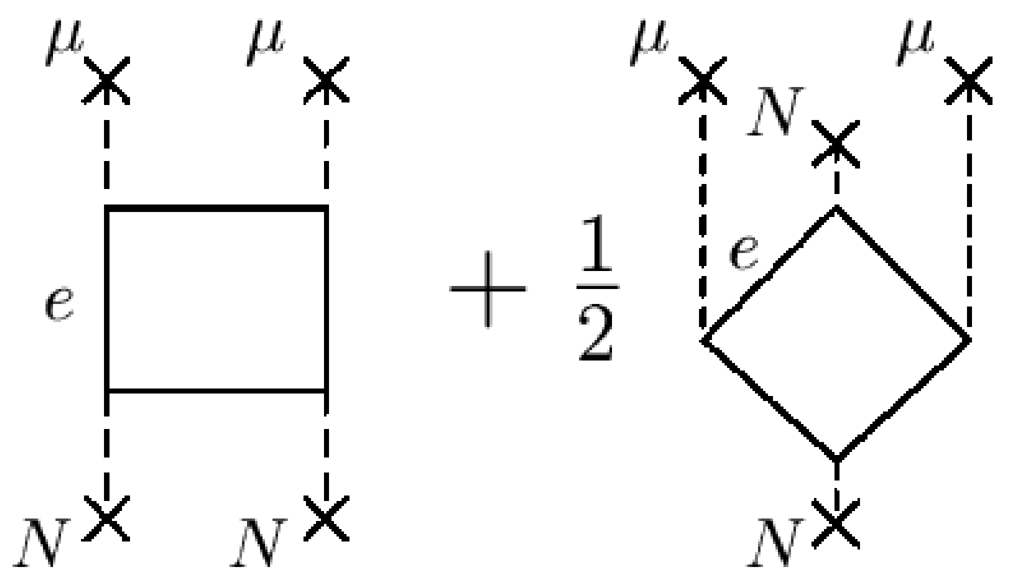
$$\Delta E_{2:2} = \int \frac{d^3\mathbf{q}}{(2\pi)^3} V_{2:2}(\mathbf{q}^2)\, F(\mathbf{q}^2),$$

$$V_{2:2}(\mathbf{q}^2) = \frac{3}{4\pi}\,\alpha^2(Z\alpha)^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 du \int_0^1 dv \int_0^1 dw \int_0^1 dt$$

$$\times \sum_{k=1,2} \left\{ \frac{\mathcal{B}_{2:2}^{(k)}}{\left(s_{2:2}^{(k)}\,\mathbf{q}^2 + m_e^2\right)} + \frac{\mathcal{C}_{2:2}^{(k)}\,\mathbf{q}^2}{\left(s_{2:2}^{(k)}\,\mathbf{q}^2 + m_e^2\right)^2} + \frac{\mathcal{D}_{2:2}^{(k)}\,\mathbf{q}^4}{\left(s_{2:2}^{(k)}\,\mathbf{q}^2 + m_e^2\right)^3} \right\},$$

The effective potential for the virtual Delbrück scattering

Fourier transformation gives result in the coordinate space

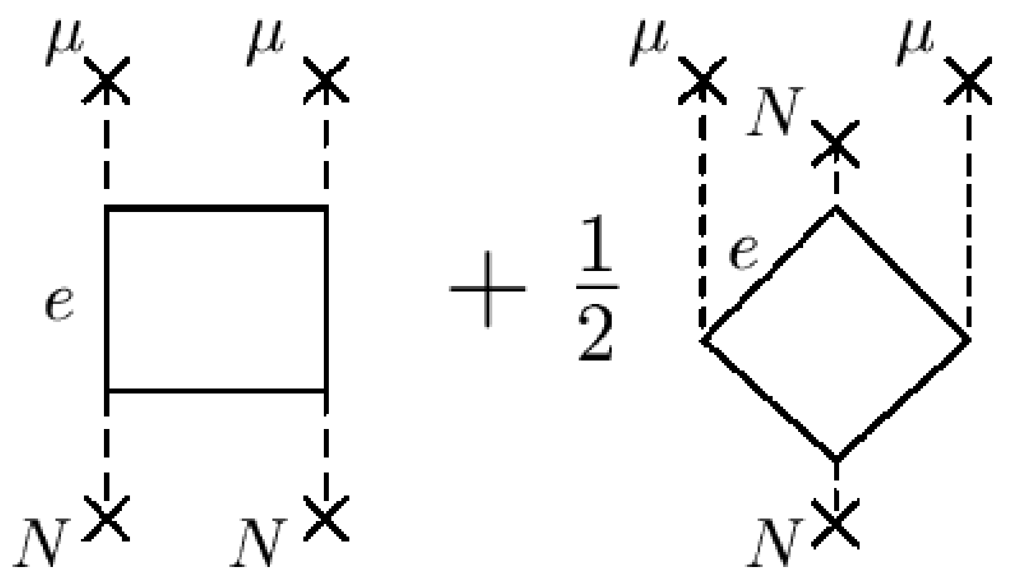


$$V_{2:2}(r) = \frac{3}{4\pi}\alpha^2(Z\alpha)^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 du \int_0^1 dv \int_0^1 dw \int_0^1 dt \sum_{k=1,2} \exp\left(-\frac{m_e r}{\sqrt{s_{2:2}^{(k)}}}\right)$$

$$\times \left\{ \frac{\mathcal{B}_{2:2}^{(k)}}{4\pi\, s_{2:2}^{(k)}\, r} + \frac{\mathcal{C}_{2:2}^{(k)}}{\left(s_{2:2}^{(k)}\right)^3} \frac{2s_{2:2}^{(k)} - m_e r\sqrt{s_{2:2}^{(k)}}}{8\pi\, r} + \frac{\mathcal{D}_{2:2}^{(k)}}{\left(s_{2:2}^{(k)}\right)^4} \frac{8s_{2:2}^{(k)} - m_e r\left(7\sqrt{s_{2:2}^{(k)}} - m_e r\right)}{32\pi\, r} \right\}.$$

Numerical problems with r→∞ and spectral parameters $m_e^2/s_{2:2}^{(k)} = 0$

The effective potential for the virtual Delbrück scattering



Asymtotics:

1) Short distances (numerical integration)

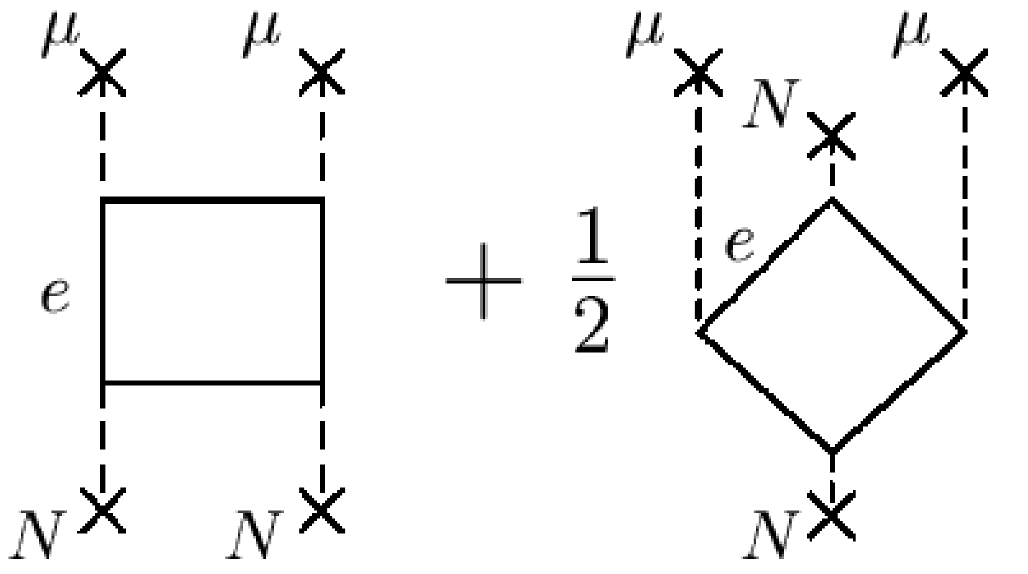
$$V_{2:2}(r \ll 1/m_e) \simeq -0.027\,565(13)\,\frac{\alpha^2(Z\alpha)^2}{r}$$

2) Large distances (soft photons limit)

$$V_{2:2}(r \gg 1/m_e) \simeq -\frac{59}{2304}\,\frac{\alpha^2(Z\alpha)^2 m_e}{(m_e r)^4}$$

$$\simeq -0.025\,61\,\frac{\alpha^2(Z\alpha)^2 m_e}{(m_e r)^4}$$

The effective potential for the virtual Delbrück scattering



Original potential

$$V_{2:2}(r) = \frac{3}{4\pi}\,\alpha^2(Z\alpha)^2 \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 du \int_0^1 dv \int_0^1 dw \int_0^1 dt \sum_{k=1,2} \exp\left(-\frac{m_e r}{\sqrt{s_{2:2}^{(k)}}}\right)$$
$$\times \left\{ \frac{\mathcal{B}_{2:2}^{(k)}}{4\pi\, s_{2:2}^{(k)}\, r} + \frac{\mathcal{C}_{2:2}^{(k)}}{\left(s_{2:2}^{(k)}\right)^3} \frac{2s_{2:2}^{(k)} - m_e r\sqrt{s_{2:2}^{(k)}}}{8\pi\, r} + \frac{\mathcal{D}_{2:2}^{(k)}}{\left(s_{2:2}^{(k)}\right)^4} \frac{8s_{2:2}^{(k)} - m_e r\left(7\sqrt{s_{2:2}^{(k)}} - m_e r\right)}{32\pi\, r} \right\}.$$

The approximation equation

$$V_{2:2}^{\mathrm{approx}}(r) = -\frac{\alpha^2(Z\alpha)^2}{r}\,\frac{7.236 + 0.3099x + 2.561x^2}{262.5 + 902.0x + 751.7x^2 + 458.6x^3 + 2.62x^4 + 100x^5}$$

$$x = m_e r.$$

Summary

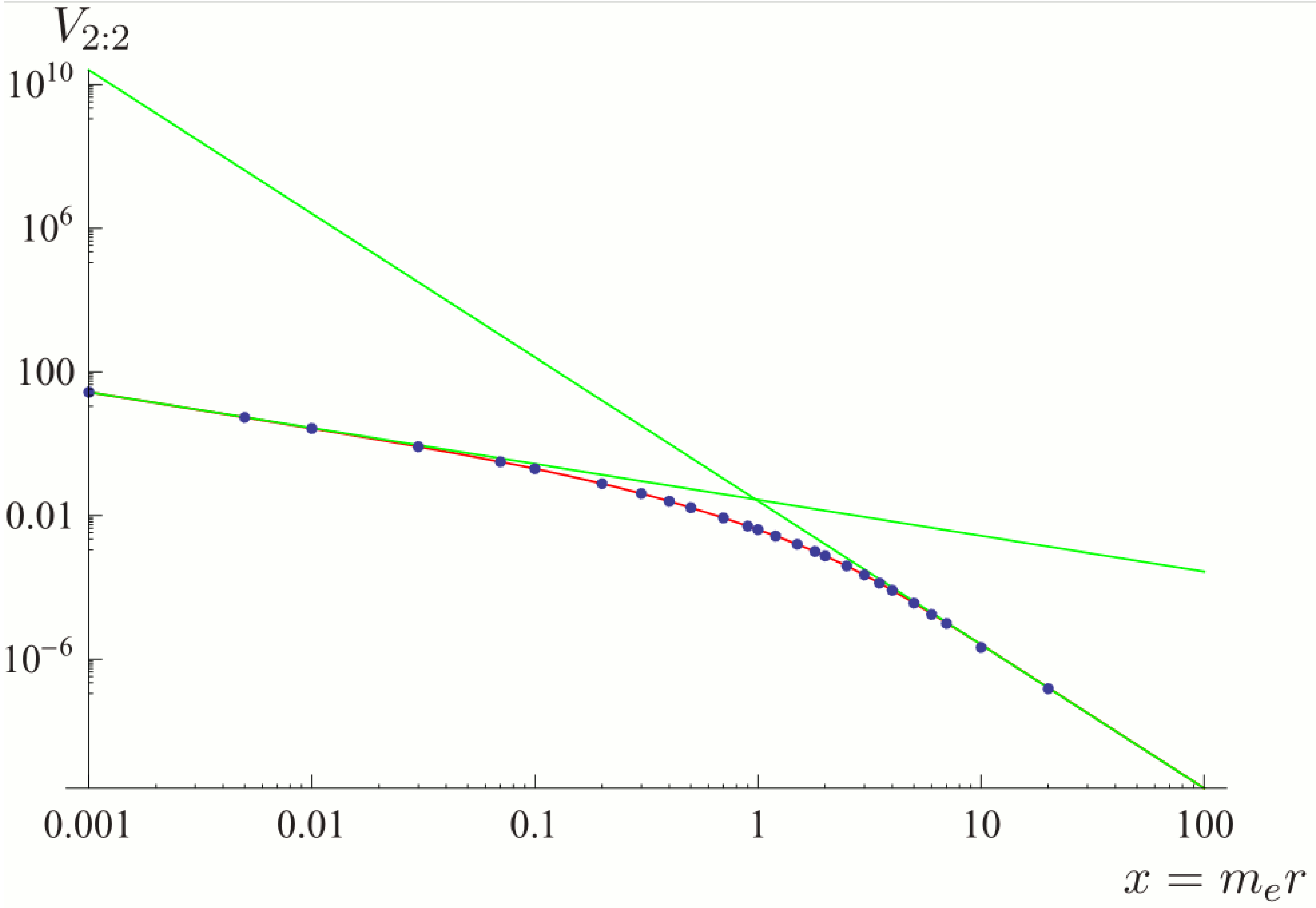
The fit for (2:2) in static-muon approximation

$$V_{2:2}^{\text{approx}}(r) = -\frac{\alpha^2 (Z\alpha)^2}{r} \frac{7.236 + 0.3099x + 2.561x^2}{262.5 + 902.0x + 751.7x^2 + 458.6x^3 + 2.62x^4 + 100x^5}$$

Accuracy of the fit:

10^{-3} for x<1

below 1-2 % for 1<x<10



Summary

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Light-by-light-scattering contributions to the Lamb shift in light muonic atoms

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