Towards testing physics beyond the Standard Model with the $g$ factor of bound electrons

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Goal and context

Search for physics beyond the Standard Model (New Physics):

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- Intensity frontier: intense beams (BaBar). Many particles. MeV-GeV
- Cosmic frontier: telescopes/detectors (ProtoDUNE). Search dark. MeV-GeV
A brief description of the project

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Search for physics beyond the Standard Model (New Physics):

- Intensity frontier: intense beams (BaBar). Many particles. MeV-GeV
- Cosmic frontier: telescopes/detectors (ProtoDUNE). Search dark. MeV-GeV
- Precision frontier: ions (small exp.). Frequency measurements. keV

Development of bound-state QED calculations and experiments

[S. Sturm, F. Köhler, J. Zatorski et al., Nature 506, 467 (2014)]
Goal and context

Search for physics beyond the Standard Model (New Physics):

- **Energy frontier**: particle colliders (LHC). High-energy collisions. TeV
- **Intensity frontier**: intense beams (BaBar). Many particles. MeV-GeV
- **Cosmic frontier**: telescopes/detectors (ProtoDUNE). Search dark. MeV-GeV
- **Precision frontier**: ions (small exp.). Frequency measurements. keV

Development of bound-state QED calculations and experiments

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Using bound-electron $g$ factor (Zeeman splitting) in search for New Physics

- ‘Direct’ method: $g$-factor measurements compared to Standard Model theory:
  → difference allowed by error bars gives upper limit on New Physics contribution
- ‘Indirect’ method: isotope shifts in the $g$ factor (data to be acquired)
  → properties of data can be used (with care) to constrain New Physics param.
  Implemented with optical transition freq. in singly-charged ions in
  [J.C. Berengut, D. Budker, C. Delaumay, V.V. Flambaum et al., Phys. Rev. Lett. 120, 091801 (2018)]
Magnetic dipole moment $\mu$ and $g$ factor of a particle:

$$\mu = g \frac{qJ}{2m}$$

$q$: charge  $m$: mass  $J$: total angular momentum
The bound-electron $g$ factor

Magnetic dipole moment $\mu$ and $g$ factor of a particle:
$$\mu = g \frac{qJ}{2m}$$
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Calculating the bound-electron $g$ factor
Relativistic quantum mechanics+QED (radiative corrections)
If several $e^-$: electron interactions
Nuclear structure corrections

Measuring the bound-electron $g$ factor
Penning trap: precision: $10^{-11}$ for medium-light H-like ions
Excellent agreement with the theory
Soon to come: same precision for medium and heavy H-like ions (e.g. Ca, Xe, Pb)
A candidate for New Physics

A proposed fifth fundamental force

- Massive spinless boson $\phi$ (mass range unknown)
- Couples electrons to neutrons according to Yukawa potential

\[ V_{\phi}(r) = -\frac{\hbar c}{\alpha_{NP}} (A - Z) e^{-m_{\phi} c \hbar |r|} \]

- $\alpha_{NP}$ coupling constant
- $m_{\phi}$ mass of the boson
A candidate for New Physics

### A proposed fifth fundamental force

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### Relevance to high-energy physics

- **Electroweak hierarchy problem**: Electroweak force $\gg$ Gravitational force  
  Linked to the mass of the Higgs boson (radiative corrections)  
  Such scalar bosons could provide a solution to this problem
- They are light (axion-like) dark matter candidates

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Yukawa potential seen by electrons

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2 Direct tests: comparing experiments and theory

3 Isotope shifts: the King representation and ‘New Physics’

4 King tests: isotope shifts in the $g$ factor

5 Outlook
Comparing experiments and theory (& testing New Physics)

Parameters of the hypothetical fifth force

- $\alpha_{NP} = y_e y_n / 4\pi$ coupling constant with $y_e$ & $y_n$ couplings of ‘new’ bosons to electron & neutron
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Regions above the curves are excluded by corresponding measurements

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Consider experimental and theoretical values for the $g$ factor of a given ion
Find the largest discrepancy allowed by the error bars
Set that discrepancy as largest value possible for New Physics contribution to $g$ factor

Implementation with H-like $^{28}$Si$^{13+}$

Experiment: $g = 1.995 \ 348 \ 959 \ 10(7)(7)(6)$

Theory: $g = 1.995 \ 348 \ 958 \ 109(584)$
[A. Czarnecki et al., Phys. Rev. Lett. 120, 043203 (2018)]

Contrib. from New Physics is bounded by $1.7 \times 10^{-9}$
Comparing experiments and theory (& testing New Physics)

Parameters of the hypothetical fifth force

- \( \alpha_{NP} = \frac{y_ey_n}{4\pi} \) coupling constant
  - with \( y_e \) and \( y_n \) couplings of 'new' bosons to electron and neutron

![Graph showing the coupling constant \( \alpha \) as a function of mediator mass \( m_\phi \) in units of \( m_e \).]

- Regions above the curves are excluded by corresponding measurements.
- Consider experimental and theoretical values for the \( g \) factor of a given ion.
- Find the largest discrepancy allowed by the error bars.
- Set that discrepancy as the largest value possible for the New Physics contribution to the \( g \) factor.

Experiment:
- \( g = 1.99534895910(7)(7)(6) \)

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Isotope shifts and the King representation

Isotope shift

Isotope shift measures difference in a given quantity between two isotopes of given ion
Isotope shifts and the King representation

Isotope shifts and the King representation

Bounds from the isotope shift in the Li-like Ca\textsuperscript{17+} $g$ factor

Experiment: [F. Köhler et al., Nat. Commun. 7, 10246 (2016)]

Idea to measure isotope shift in $g$ factor of highly charged ions with super-high precision:

Isotope shifts and the King representation

**Isotope shift**

Isotope shift measures difference in a given quantity between two isotopes of given ion.

**King representation**

Take a certain number (≥ 3) of pairs of isotopes.

King plot: Isotope Shift in 2 quantities (x/y axis) 1 point for each pair of isotopes.

E.g. 2 g factors in an ion \( \left( g_A^1 - g_A'^1 \right) / \mu_{AA'} \) and \( \left( g_A^2 - g_A'^2 \right) / \mu_{AA'} \)

Where \( \mu_{AA'} = 1/M_A - 1/M_A' \) inverse reduced mass of nuclei.

(from M. Avgoulea et al., Hyperfine Interact. 171, 217 (2006))

Experimental graph gives a straight line: why?
Isotope shifts at the (Standard Model) leading order

Shift in the $g$ factor of a level $i$ between isotopes $A$ and $A'$

$$g_i^{AA'} = g_i^A - g_i^{A'}$$

Two largest Standard Model contributions to the isotope shift

- **Leading order** contribution to the mass shift:
  $$K_i \mu_{AA'}$$ where $\mu_{AA'} = 1/M_A - 1/M_{A'}$ inverse reduced mass of nuclei

- **Leading order** contribution to the field shift:
  $$F_i \delta \langle r^2 \rangle_{AA'}$$ where $\delta \langle r^2 \rangle_{AA'}$ difference in nuclear charge radii

$F_i$ and $K_i$ are purely electronic coefficients
Isotope shifts at the (Standard Model) leading order

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King plot at the leading order

Take four different isotopes $A, A'_1, A'_2, A'_3$ & two different $e^-$ states 1 and 2

At the SM leading order

$$\frac{g_{2}^{AA'}}{\mu_{AA'}} = \frac{F_2}{F_1} \frac{g_{1}^{AA'}}{\mu_{AA'}} + \left( K_2 - \frac{F_2}{F_1} K_1 \right)$$

which explains why the Isotope Shift data is linear (previous slide)

If isotope shift $g_{i}^{AA'}$ calculated at leading order → linear graph
The hypothetical fifth force which we consider acts between neutrons and electrons!

Introduce New Physics contribution to $g$ factor

$$g_{i}^{AA'} = K_{i} \mu_{AA'} + F_{i} \delta \langle r^2 \rangle_{AA'} + \alpha_{NP} X_{i} (A - A')$$

where $X_{i}$ is a purely electronic factor (computed on earlier slide).
The hypothetical fifth force which we consider acts between *neutrons* and electrons!

**Introduce New Physics contribution to $g$ factor**

$$g_i^{AA'} = K_i \mu_{AA'} + F_i \delta \langle r^2 \rangle_{AA'} + \alpha_{NP} X_i (A - A')$$

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**King plot in the presence of New Physics at the Standard Model leading order**

$$\frac{g_2^{AA'}}{\mu_{AA'}} = \frac{F_2}{F_1} \frac{g_1^{AA'}}{\mu_{AA'}} + \left( K_2 - \frac{F_2}{F_1} K_1 \right) + \alpha_{NP} \left( \frac{X_2}{X_1} - \frac{F_2}{F_1} \right) \frac{A - A'}{\mu_{AA'}}$$

At SM leading order: King nonlinearity is a signature of New Physics

→ New Physics can be constrained from

- Experiment: Isotope Shift data
- Theory: New Physics contrib. to $g$ factor

At SM leading order: better exp. precision always

→ better bounds on New Physics
The hypothetical fifth force which we consider acts between neutrons and electrons!

Introduce New Physics contribution to $g$ factor

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For extremely high experimental accuracy: King nonlinearities can be expected to be caused by subleading Standard Model nuclear corrections to the $g$ factor

Higher-order finite nuclear size correction

Nuclear polarisation

Nuclear shape deformation

Higher-order nuclear mass correction

→ Should not be interpreted as New Physics!
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Tests with the isotope shift:

![Graph showing coupling constant vs mediator mass](image)

- **Si$^{11+/13+}$ WD [Wagner+Yerokhin]**
- **Ca$^{17+/19+}$ IS–NL [@10$^{-11}$]**
- **Ni$^{25+/27+}$ IS–NL [@10$^{-11}$]**
- **Ca$^{17+}$ IS [Köhler]**
- **Si$^{13+}$ [Sturm+Theory]**
- **Ca$^{17+/19+}$ IS–NL [@10$^{-13}$]**
- **Ni$^{23+/27+}$ IS–NL [@10$^{-11}$]**
- **Ca$^{+}$ Freq. IS–NL [Berengut]**
- **Proj. Si$^{13+}$[ALPHATRAP+Theory]**
- **Ca$^{15+/19+}$ IS–NL [@10$^{-11}$]**
- **Ca$^{17+/19+}$ IS–NL [@10$^{-15}$]**
- **Proj. Si$^{11+/13+}$ WD**

**Mediator mass $m_\phi$ [units of $m_e$]**

**Coupling constant $\alpha$**
The (specific) weighted difference

\[
\delta \xi_s g = g_{2s_{1/2}} - \xi_s g_{1s_{1/2}} \\
\delta \xi_p g = g_{2p_{1/2}} - \xi_p g_{1s_{1/2}}
\]

\(\xi_s\) and \(\xi_p\) coefficients optimised to cancel the finite-nuclear-size contributions in \(\delta \xi g\)


Goal: more stringent tests of QED

\[\xi_{s_{1/2}} \sim \frac{1}{8}\]  \[\xi_{s_{1/2}} \sim \frac{3}{128} (Z\alpha)^2\]
Tests with the isotope shift and the weighted difference
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### Summary

Two main methods to obtain bounds on New Physics from $g$-factor spectroscopy

- **‘Direct’ method**: $g$-factor measurements compared to Standard Model theory:
  - some existing bounds but less stringent than other atomic results
  - Improvements envisioned but demand strong progress from theory

- **‘Indirect’ method**: isotope shifts in the $g$ factor
  - data is to be acquired and requires several energy levels and isotopes
  - Competitive bounds possible with realistic exp. precision
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**Perspectives**

Other proposed types of new particles and interactions

\( \rightarrow \) (they need to affect the bound-electron \( g \) factor)

\( e.g. \): \( B - L \) gauged symmetry, chameleon models
Thank You