# *QED with heavy ions: on the way from strong to supercritical fields*

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# *Outline of the talk*

- Introduction
- QED at strong electromagnetic fields
  - Lamb shift in heavy ions
  - Hyperfine splitting in heavy ions
  - g factor of highly charged ions
- QED at supercritical fields
  - Schwinger mechanism
  - Pair creation in the $U^{92+}-U^{92+}$ collision
  - How to observe the vacuum decay
- Conclusion

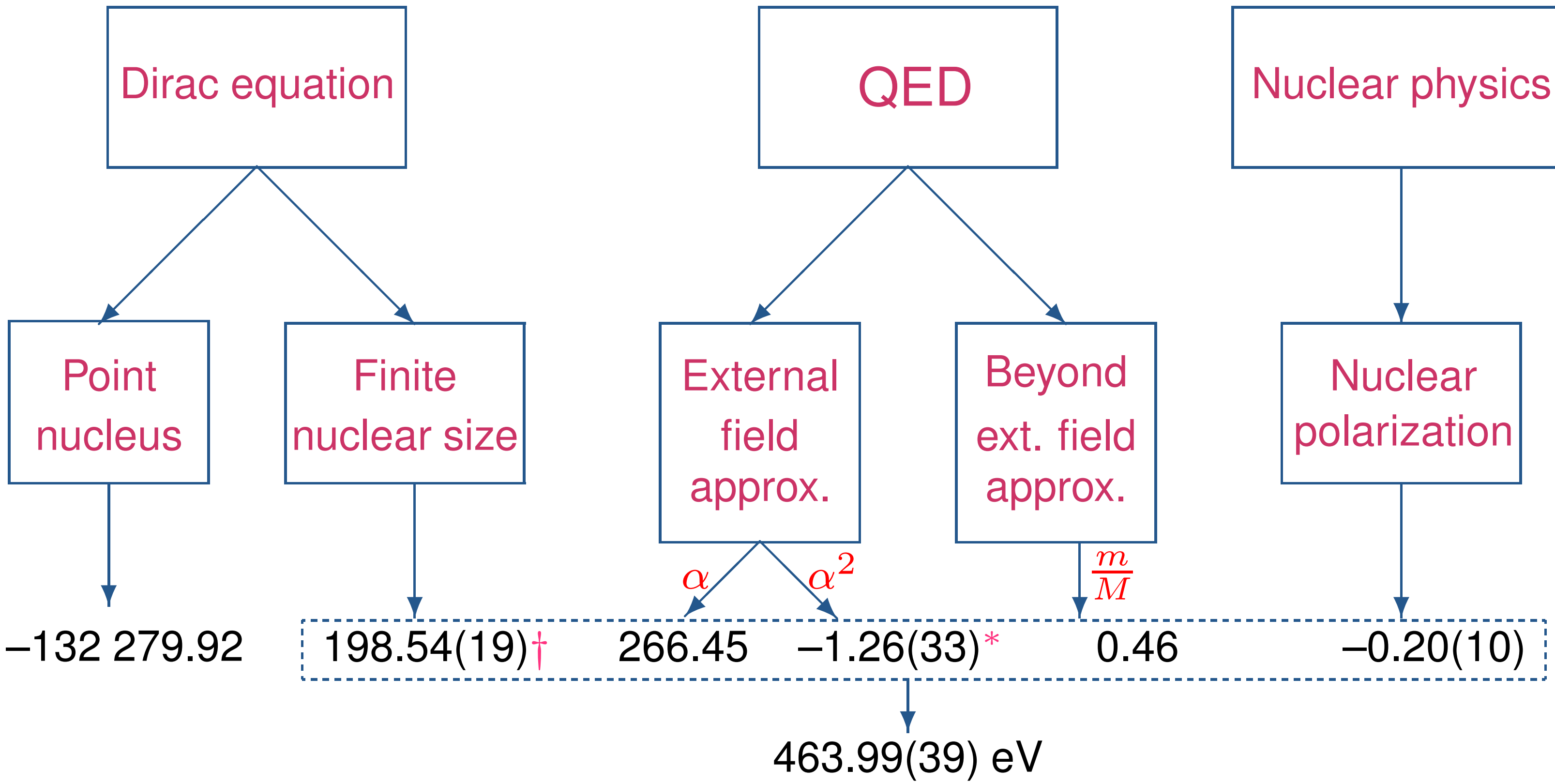
# *Introduction: tests of QED with atomic systems*

Light atoms ($\alpha Z \ll 1$, weak fields):
Tests of QED to lowest orders in $\alpha Z$.

Heavy few-electron ions ($\alpha Z \sim 1$, strong fields):
Tests of QED in nonperturbative in $\alpha Z$ regime.

Low-energy heavy-ion collisions at $Z_1 + Z_2 > 173$ (supercritical fields):
Tests of QED in supercritical regime.

# *1s Lamb shift in H-like uranium, in eV*




Experiment: 460.2(4.6) eV

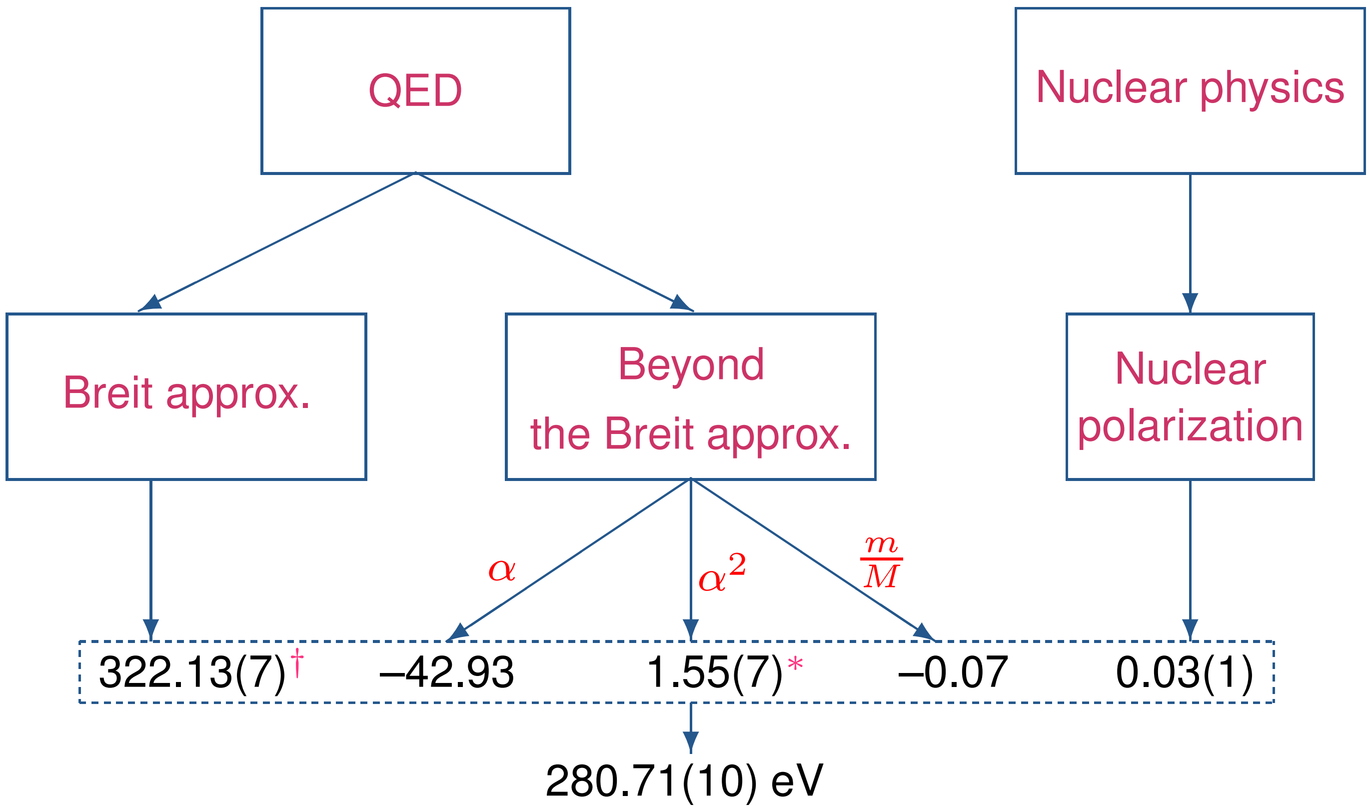
(A. Gumberidze, T. Stöhlker, D. Banas et al., PRL, 2005)

Test of QED: $\sim$ 2%

* V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006

† Y.S. Kozhedub, O.V. Andreev, V.M. Shabaev et al., PRA, 2008

# $2p_{1/2}$-$2s$ *transition energy in Li-like uranium, in eV*




Experiment: 280.59(10) eV (J. Schweppe et al., PRL, 1991)
280.52(10) eV (C. Brandau et al., PRL, 2003)
280.645(15) eV (P. Beiersdorfer et al., PRL, 2005)

Test of QED: $\sim$ 0.2%

* V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006

† Y.S. Kozhedub, O.V. Andreev, V.M. Shabaev et al., PRA, 2008

# *Current value for the specific HFS difference in Bi*

Theoretical contributions to $\Delta' E = \Delta E^{(2s)} - \xi \Delta E^{(1s)}$ (in meV) for $\mu/\mu_N = 4.1106(2)$ *(A.V. Volotka et al., PRL, 2012; O.V. Andreev et al., PRA, 2012)*

| | |
|---|---|
| Dirac value | -31.809 |
| Interel. inter., $\sim 1/Z$ | -29.995 |
| Interel. inter., $\sim 1/Z^2$ and h.o. | 0.255(3) |
| One-electron QED | 0.036 |
| Screened QED | 0.193(2) |
| Total | -61.320(6) |
| Experiment *[1]* | -61.012 (5)(21) |

*[1] J. Ullmann et al., Nature Communications, 2017.*

New calculations of the shielding constant and new NMR measurements in $Bi(NO_3)_3$ and $BiF_6^-$ yielded $\mu/\mu_N = 4.092(2)$ *(L. Skripnikov et al., PRL, 2018)*, which gave $\Delta' E = -61.043(5)(30)$ meV.

# *Future prospects for the g-factor investigations*

1) Tests of bound-state QED at strong fields

For stringent tests of QED in the g-factor experiments, one should study specific differences of the g factors of H-, Li- and B-like ions.

2) Tests of QED beyond the Furry picture *(A.V. Malyshev, V.M. Shabaev, D.A. Glazov, and I.I. Tupitsyn, JETP Letters, 2017).*

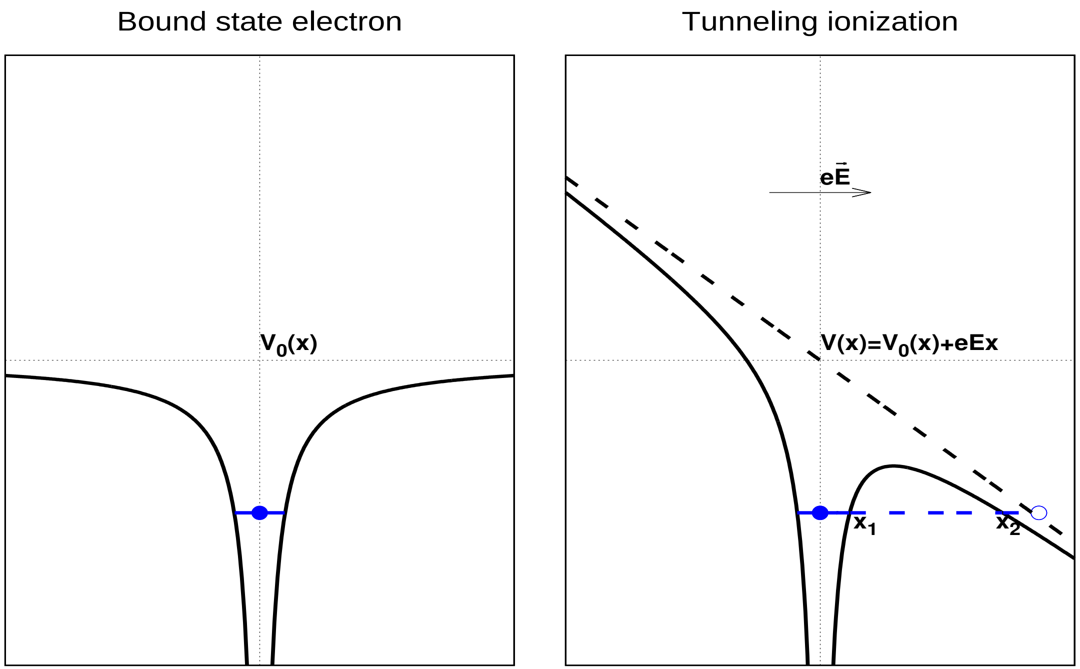
3) Determination of the nuclear magnetic moments

$$
\begin{aligned}
g_{\mathrm{atom}} \quad &= \quad g^{(e)}\frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)} \\
&\quad -\frac{m_e}{m_p}g^{(N)}\frac{F(F+1)+I(I+1)-J(J+1)}{2F(F+1)}\,.
\end{aligned}
$$

4) Determination of the fine structure constant by studying the g factors of H-, Li-, and B-like ions *(V.M. Shabaev et al., PRL, 2006; V.A. Yerokhin et al., PRL, 2016).*

# *QED at supercritical fields*

## Tunneling ionization in quantum mechanics



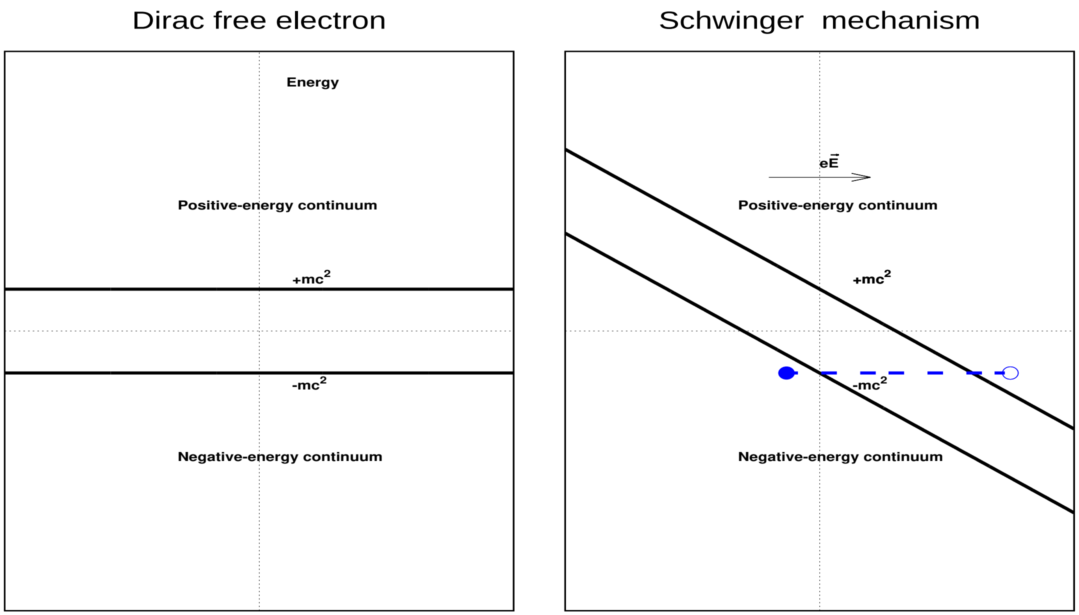

The tunneling probability for a static uniform electric field $E$:

$$W \sim \exp\Big\{-\frac{4\pi}{\hbar}\int_{x_1}^{x_2} dx \sqrt{2m(V(x)-\mathcal{E})}\Big\}$$

where $V(x) = V_0(x) + eEx$ and $\mathcal{E}$ is the electron energy.

# *QED at supercritical fields*

## Electron-positron pair creation by a static uniform electric field




The rate of pair production for a static uniform electric field $E$:

$$\frac{d^4 n_{e^+e^-}}{d^3x\,dt} \sim \frac{c}{4\pi^3 {\lambda_{\rm C}}^4} \exp\left(-\pi \frac{E_c}{E}\right)$$

where $\lambda_{\rm C} = \hbar/(mc)$ and $E_c = m^2c^3/(e\hbar) \approx 1.3 \times 10^{16} {\rm V/cm}$.

## *QED at supercritical fields*

Electron-positron pair creation by a static electric field

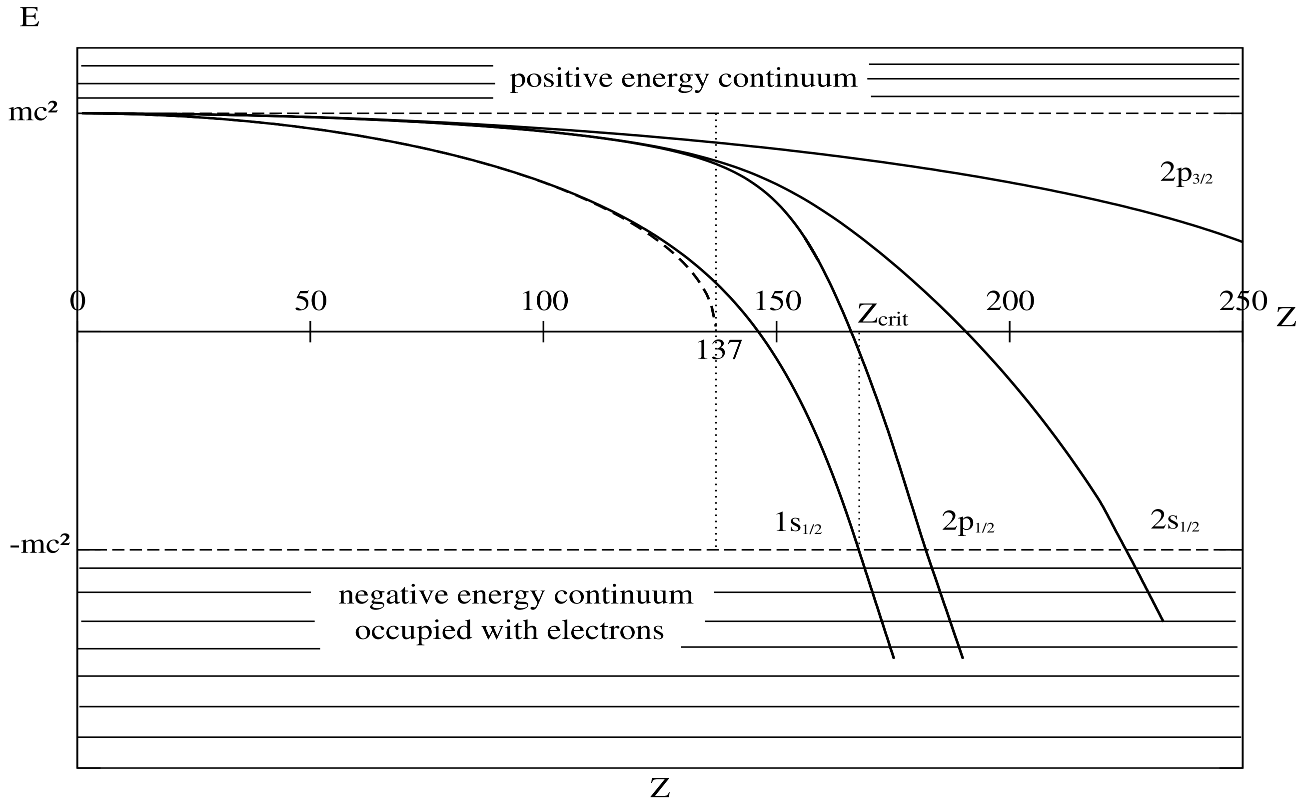
The Schwinger effect has never been observed experimentally as the required field strength, $E_c \approx 1.3 \times 10^{16} \mathrm{V/cm}$, is extremely large. The recent developments of the laser technologies have triggered a great interest to theoretical calculations of this effect for various scenarios. The scenario emplyong two counter-propagating laser pulses is considered as most favorable. For the recent progress on these calculations we refer to *[I.A. Aleksandrov, G. Plunien, and V.M. Shabaev, PRD, 2017; PRD, 2018].*

However, even in the most promising scenarios the electric field strength reached with new laser technologies in the not too distant future is expected to be two orders of magnitude smaller than the critical value. So, this does not seem very encouraging.

# *Low-energy heavy-ion collisions*

Access to supercritical fields

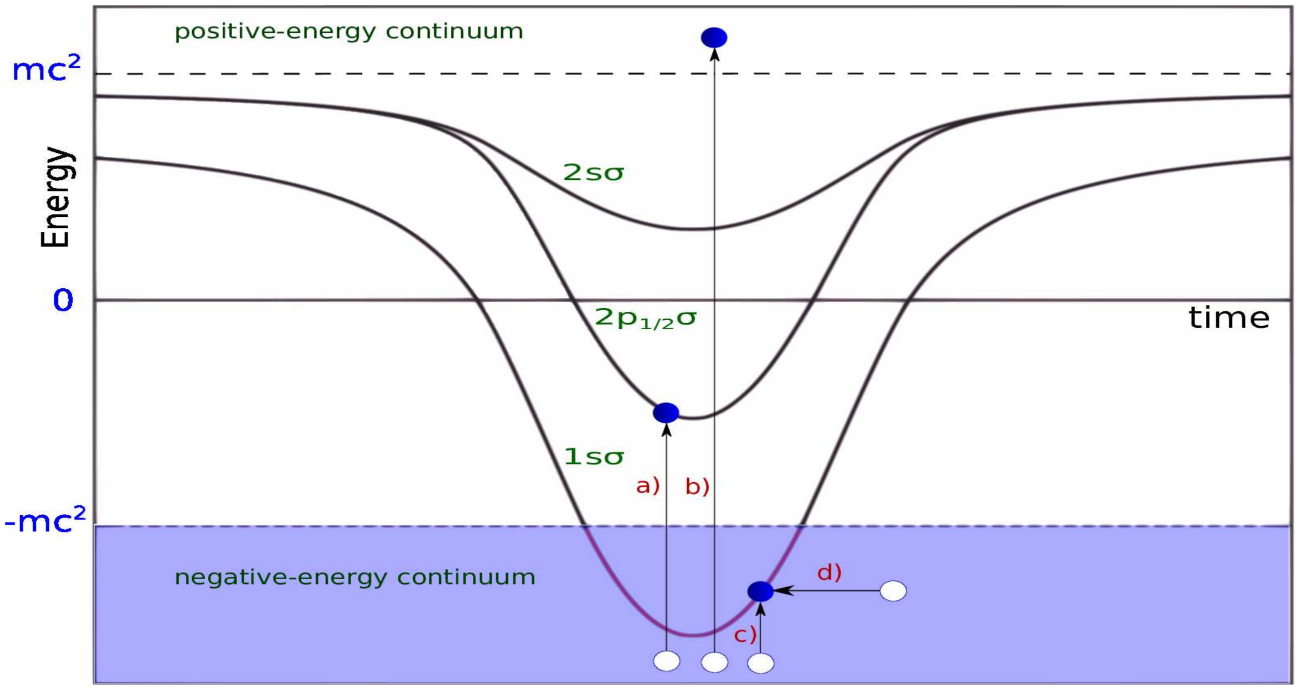
S.S. Gershtein, Ya.B. Zel'dovich, 1969; W. Pieper, W. Greiner, 1969




The $1s$ level dives into the negative-energy continuum at $Z_{\rm crit} \approx 173$.

# *Low-energy heavy-ion collisions*

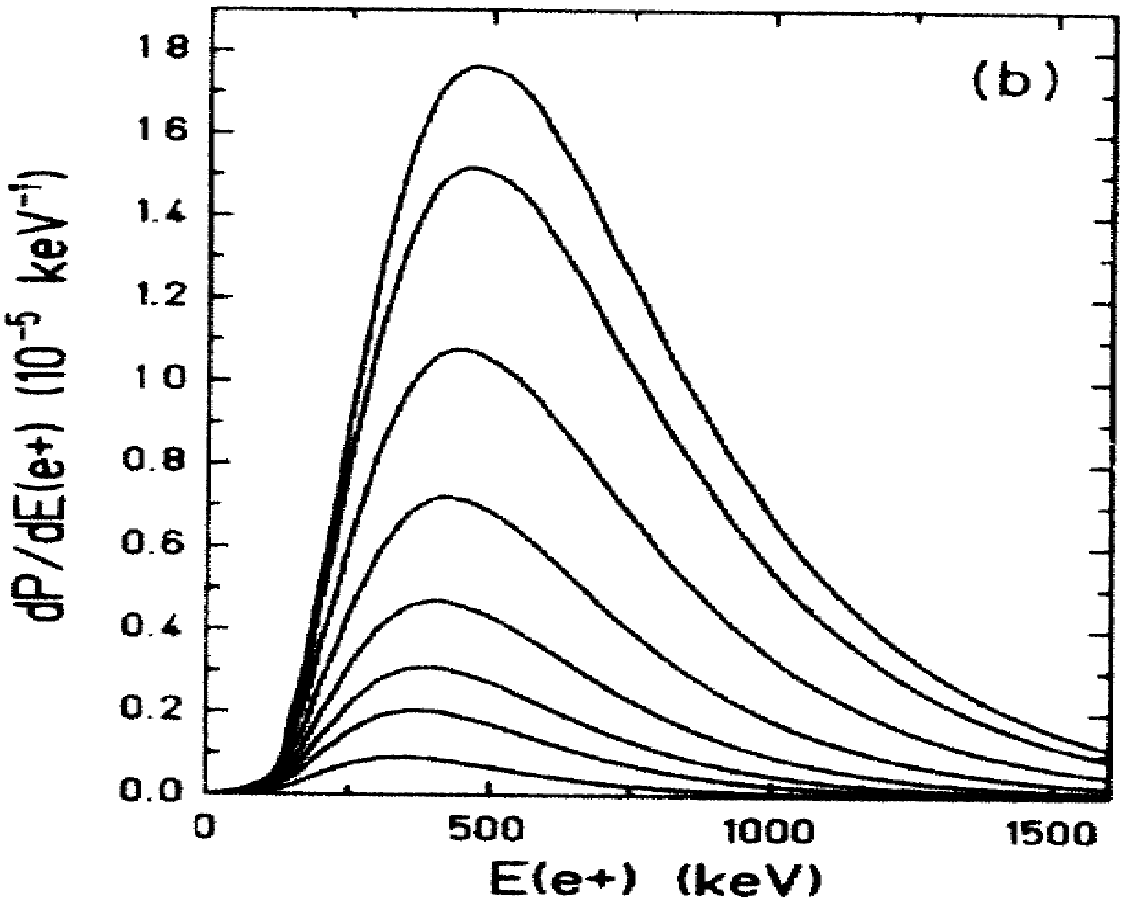
Creation of electron-positron pairs in low-energy heavy-ion collisions, with $Z_1 + Z_2 > 173$




Dynamical mechanism: a),b),c). Spontaneous mechanism (vacuum decay): d). The ground state dives into the negative-energy continuum for about $10^{-21}$ sec.

# *Low-energy heavy-ion collisions*

Electron-positron pair production in low-energy U-U collisions




Energy distribution of positrons emitted in U-U collisions at energy $E$=6.2 MeV/u for the impact parameter in the range: $b = 0 - 40$ fm *(U. Müller, T. de Reus, J. Reinhardt et al., Phys. Rev. A, 1988)*.

# *Low-energy heavy-ion collisions*

New method for solving the time-dependent two-center Dirac equation (*I.I. Tupitsyn, Y.S. Kozhedub, V.M. Shabaev et al., PRA, 2010*):

$$i\frac{\partial\Psi(\vec{r},t)}{\partial t} = c(\vec{\alpha}\cdot\vec{p}) + \beta\, mc^2 + V_{\rm nucl}^{(A)}(\vec{r}_A) + V_{\rm nucl}^{(B)}(\vec{r}_B)\,,$$

where $\vec{r}_A = \vec{r} - \vec{R}_A$, $\vec{r}_B = \vec{r} - \vec{R}_B$.

The time-dependent Dirac wave function is presented as a sum of atomic-like Dirac-Sturm orbitals localized at the ions.

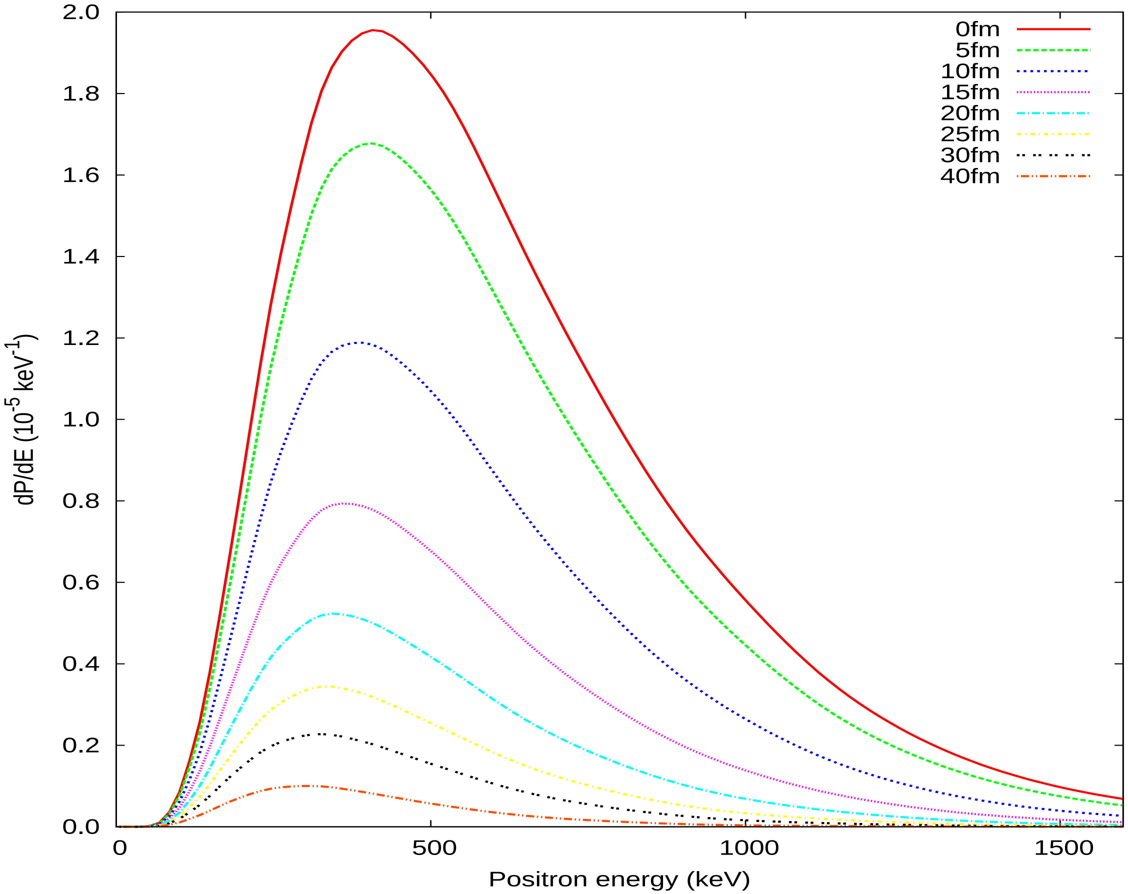
The method has been tested by calculation of the charge-transfer and ionization probabilities for low-Z systems and comparison with the related nonrelativistic results.

Extention of the method to collisions of neutral atoms with H-like ions and comparison with related experiments: *I.I. Tupitsyn et al., PRA, 2012*.

An independent method based on solving the time-dependent two-center Dirac equation in a B-spline basis: *I.A. Maltsev et al., Phys. Scr. 2013; PRA, 2015; PRA 2018*.

# *Low-energy heavy-ion collisions*

Electron-positron pair production in low-energy U-U collisions




Energy distribution of positrons emitted in U-U collisions at energy $E$=6.2 MeV/u for the impact parameter in the range: $b = 0 - 40$ fm *(I.A. Maltsev et al., PRA, 2015)*.

# *Low-energy heavy-ion collisions*

Pair creation beyond the monopole approximation

U-U, $E_{\rm cm}$ = 740 MeV

Expected number of created pairs as a function of the impact parameter b

*(I.A. Maltsev et al., PRA, 2018)* .

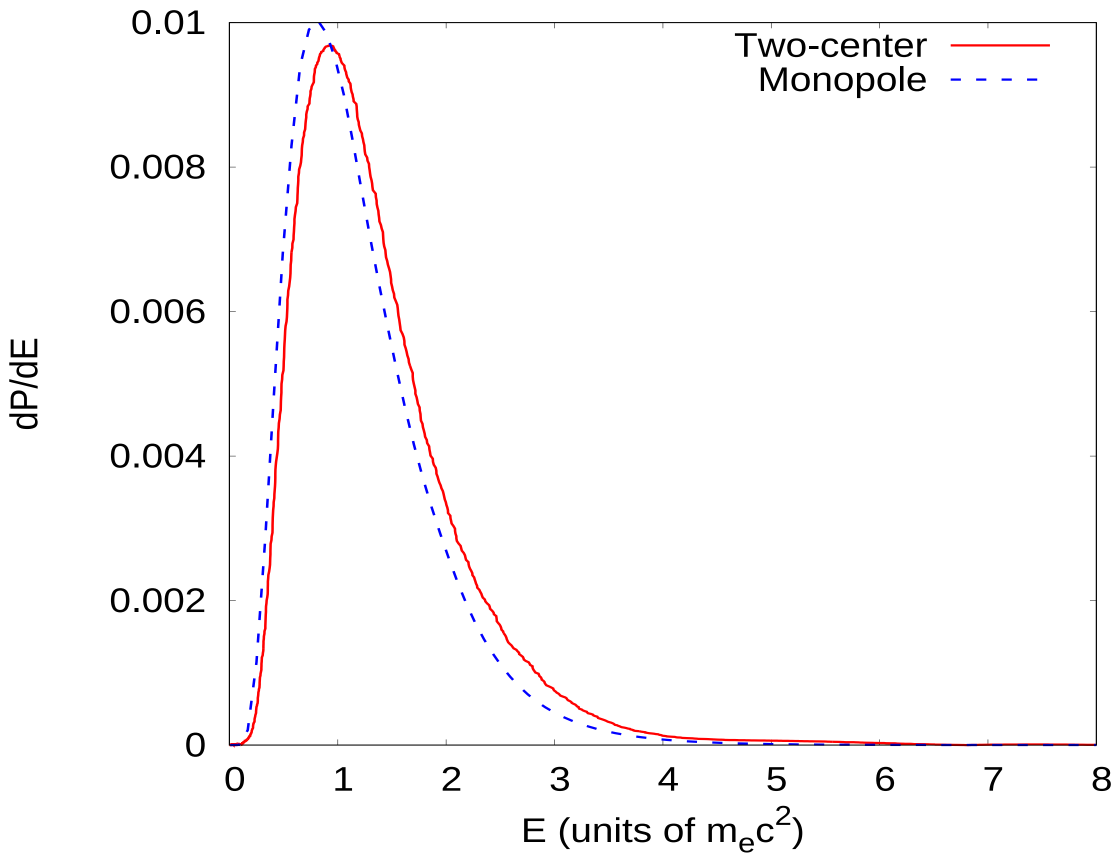
| $b$ (fm) | Monopole approximation | Two-center approach |
|---|---|---|
| 0 | 1.29 $\times 10^{-2}$ | 1.38 $\times 10^{-2}$ |
| 10 | 7.26 $\times 10^{-3}$ | 8.01 $\times 10^{-3}$ |
| 20 | 2.75 $\times 10^{-3}$ | 3.46 $\times 10^{-3}$ |
| 30 | 1.04 $\times 10^{-3}$ | 1.42 $\times 10^{-3}$ |
| 40 | 4.12 $\times 10^{-4}$ | 7.04 $\times 10^{-4}$ |

The two-center result for $b = 0$ has been confirmed by a different method *(R.V. Popov et al., EPJD, 2018)* .

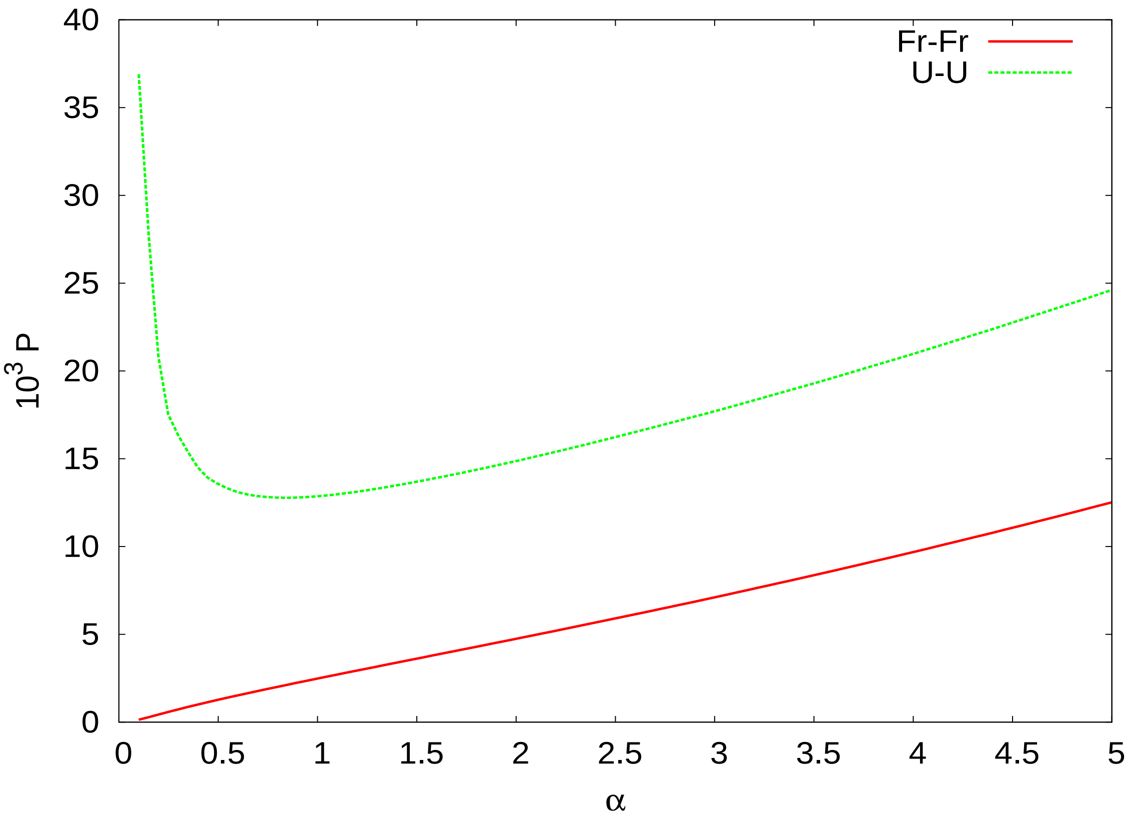
# *Low-energy heavy-ion collisions*

Pair creation beyond the monopole approximation

Positron energy spectrum for the U−U head-on collision at energy $E_{\rm cm}$ = 740 MeV *(I.A. Maltsev et al., PRA, 2018)*.

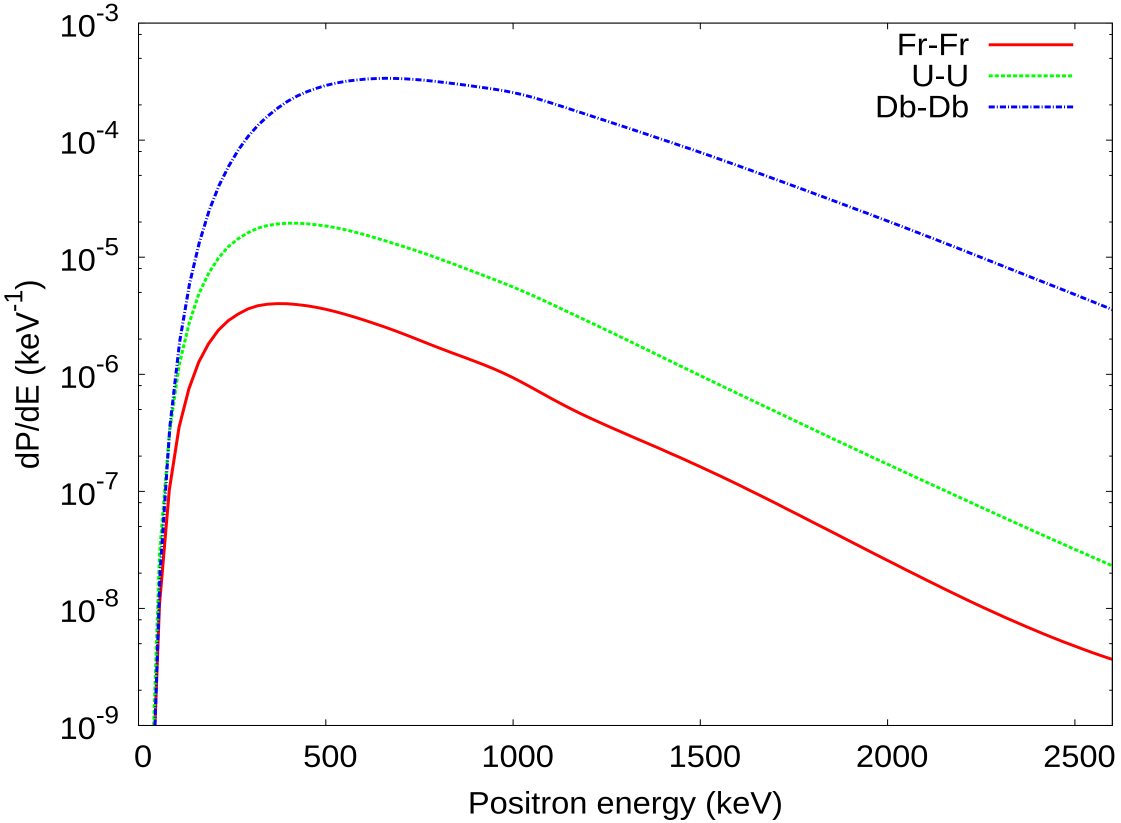


# *Low-energy heavy-ion collisions*



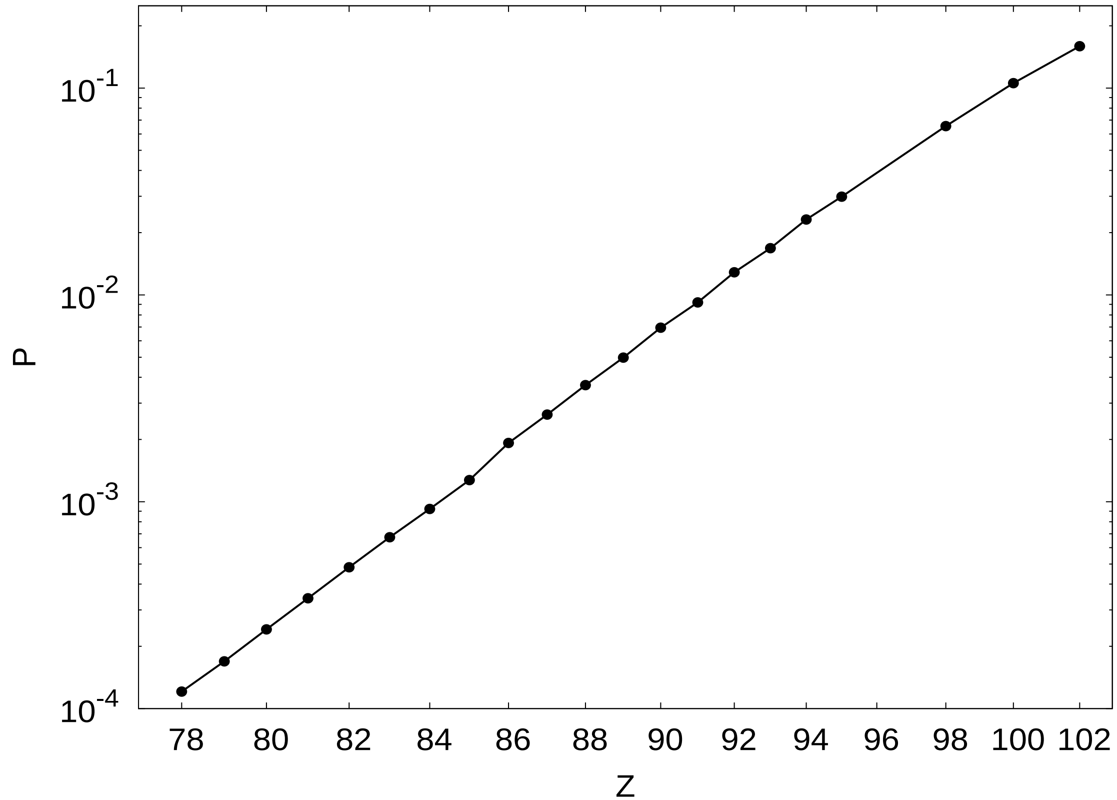

Pair creation with artificial trajectories for the supercritical U−U and subcritical Fr−Fr head-on collisions at $E_{\rm cm} = 674.5$ and $E_{\rm cm} = 740$ MeV, respectively. The trajectory $R_\alpha(t)$ is defined by $\dot{R}_\alpha(t) = \alpha \dot{R}(t)$, where $R(t)$ is the classical Rutherford trajectory *(I.A. Maltsev et al., PRA, 2015)*.

# *Low-energy heavy-ion collisions*




Positron energy spectrum for the Fr−Fr, U−U, and Db−Db head-on collisions at energies 674.5, 740, and 928.4 MeV, respectively *(I.A. Maltsev et al., 2015)*.

# *Low-energy heavy-ion collisions*




Number of created pairs P in the head-on collision of identical nuclei as a function of the nuclear charge $Z_A = Z_B = Z$ for the projectile energy $E_0 = 6.2$ MeV/u in the nuclear rest frame *(I.A. Maltsev et al., PRA, 2015)*.

# *Low-energy heavy-ion collisions*

## How to observe the vacuum decay

*(I.A. Maltsev, V.M. Shabaev, R.V. Popov, Yu.S. Kozhedub, G. Plunien, X. Ma, and Th. Stöhlker, arXiv: 1903.08546.)*

Let us choose the impact parameter $b(E)$ in such a way, that the minimal internuclear distance $r_{\min}$ remains the same (in a.u.):

$$b^2 = r_{\min}^2 - r_{\min}\frac{Z_1 Z_2}{E},$$

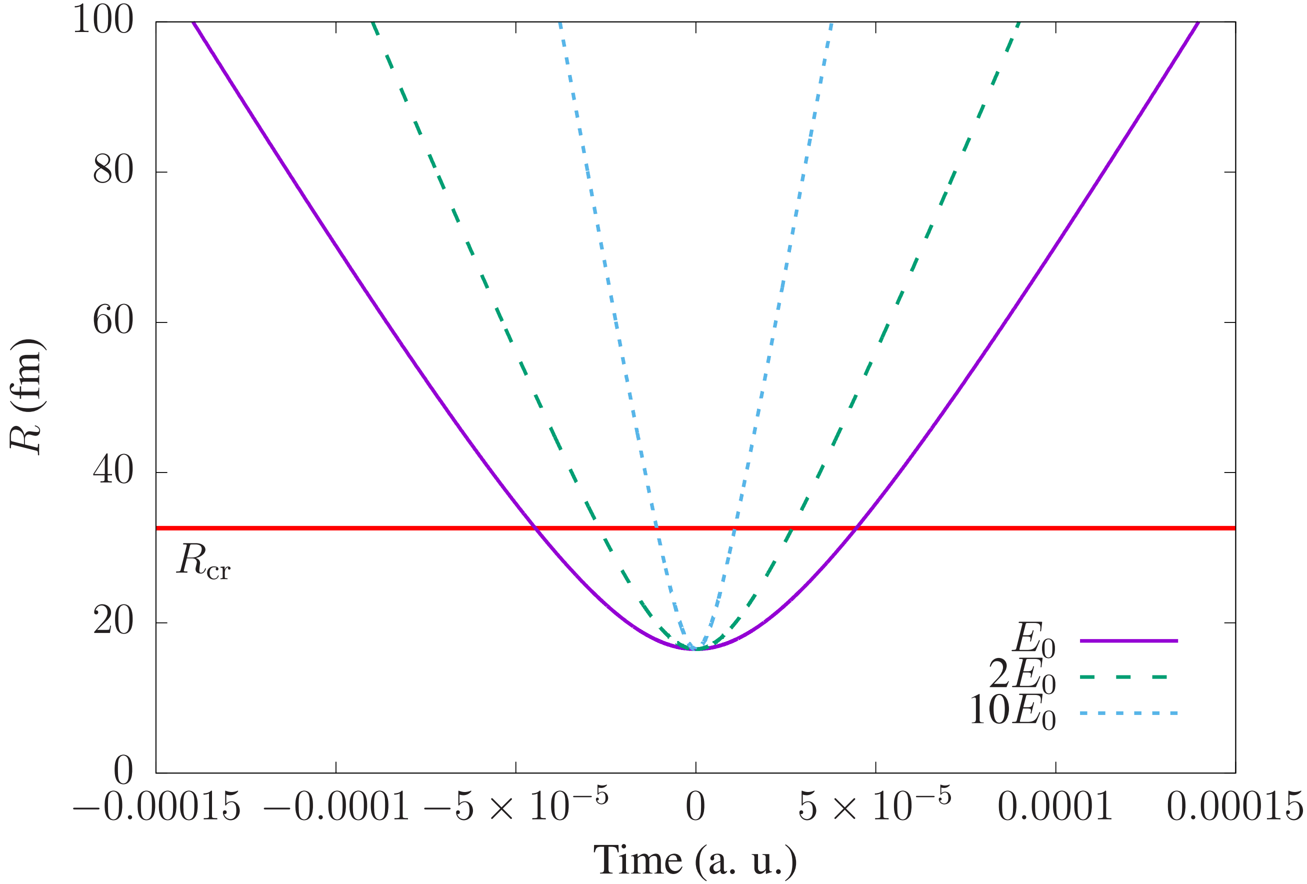
where

$$E \geq \frac{Z_1 Z_2}{r_{\min}}.$$

For head-on collision ($b$ = 0):

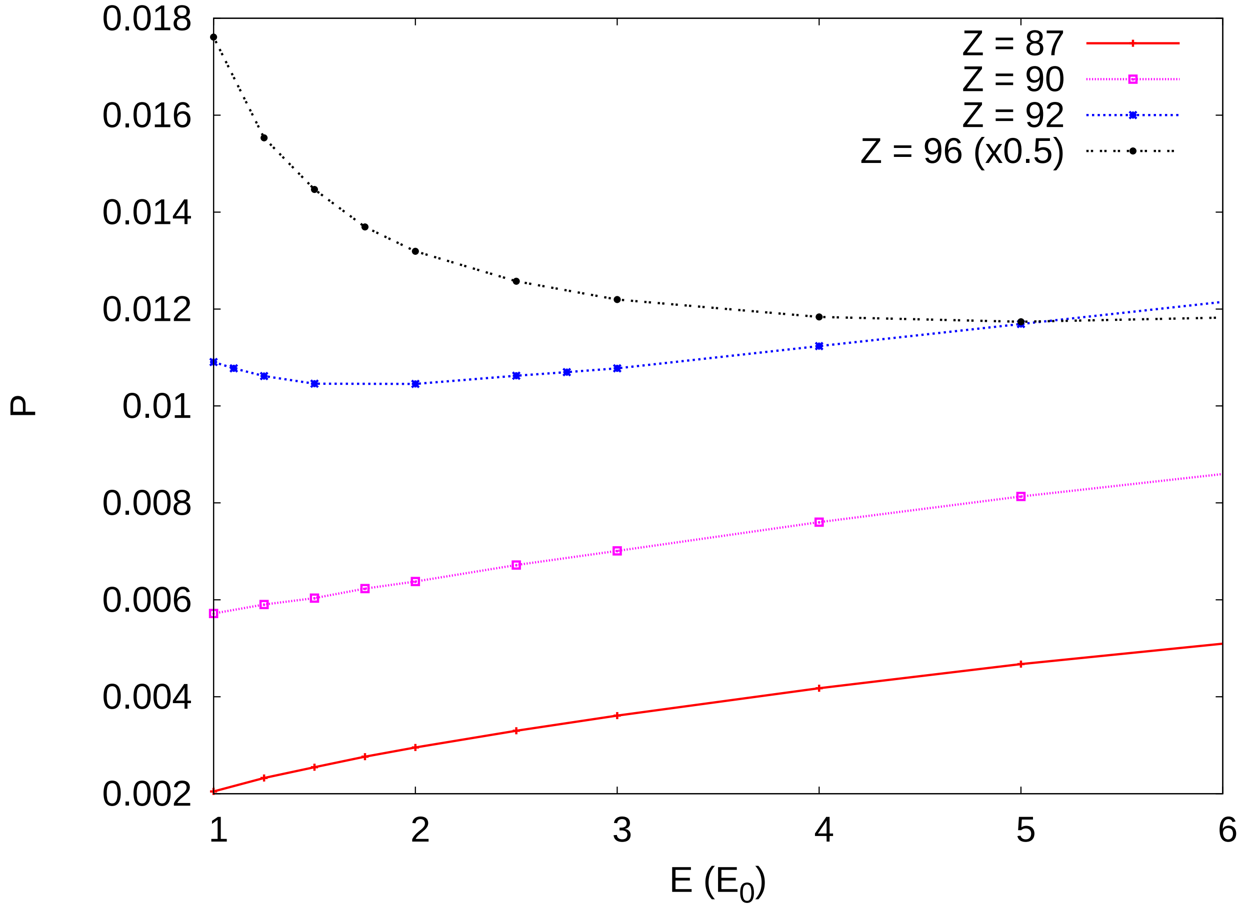
$$E_0 = \frac{Z_1 Z_2}{r_{\min}}.$$

# *Low-energy heavy-ion collisions*



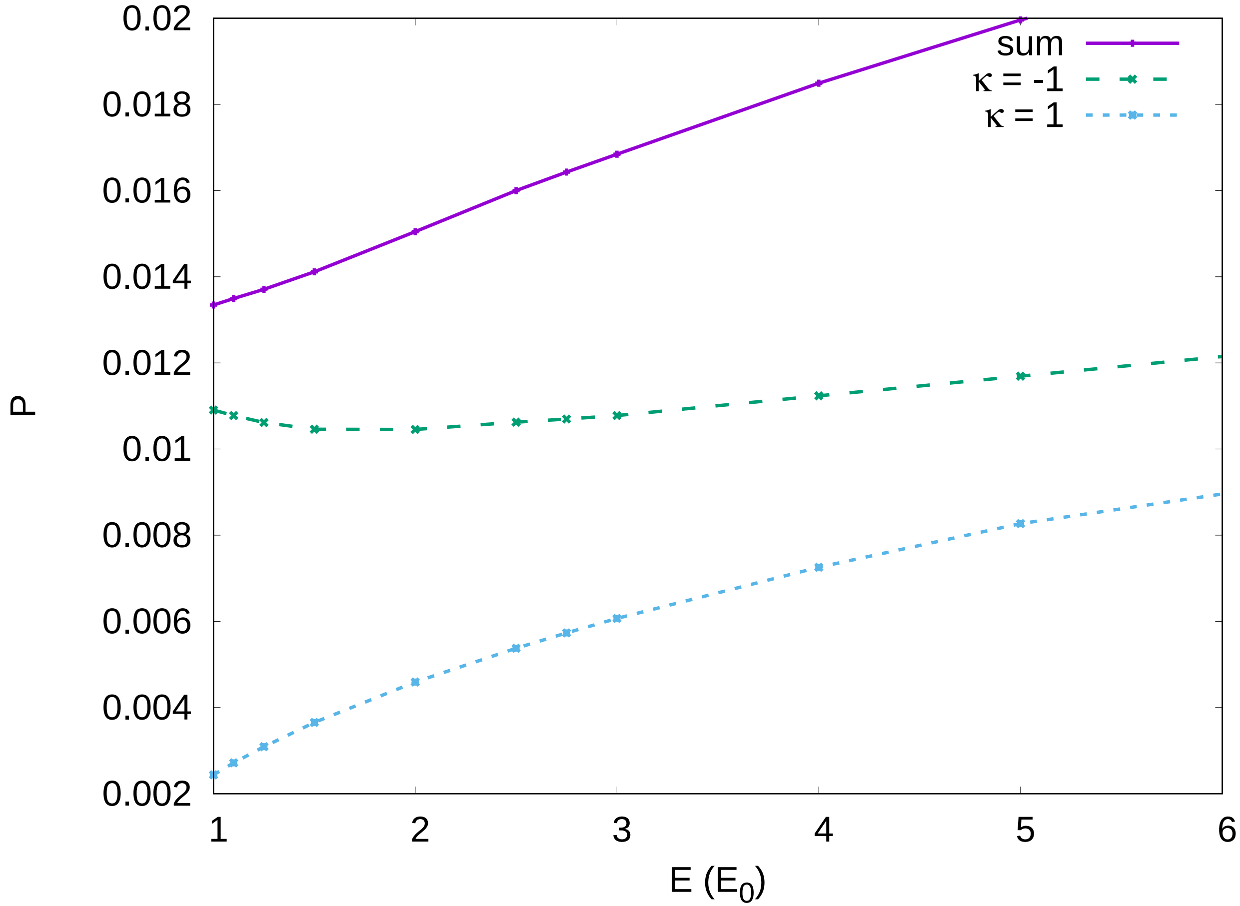

The internuclear distance $R$ for U–U collision as a function of time for different values of the collision energy with the fixed distance of the closest approach $R_{\min} = 16.5$ fm, $E_0$ is the energy of the head-on collision. The red horizontal line corresponds to the critical distance $R_{cr} = 32.6$ fm.

# *Low-energy heavy-ion collisions*



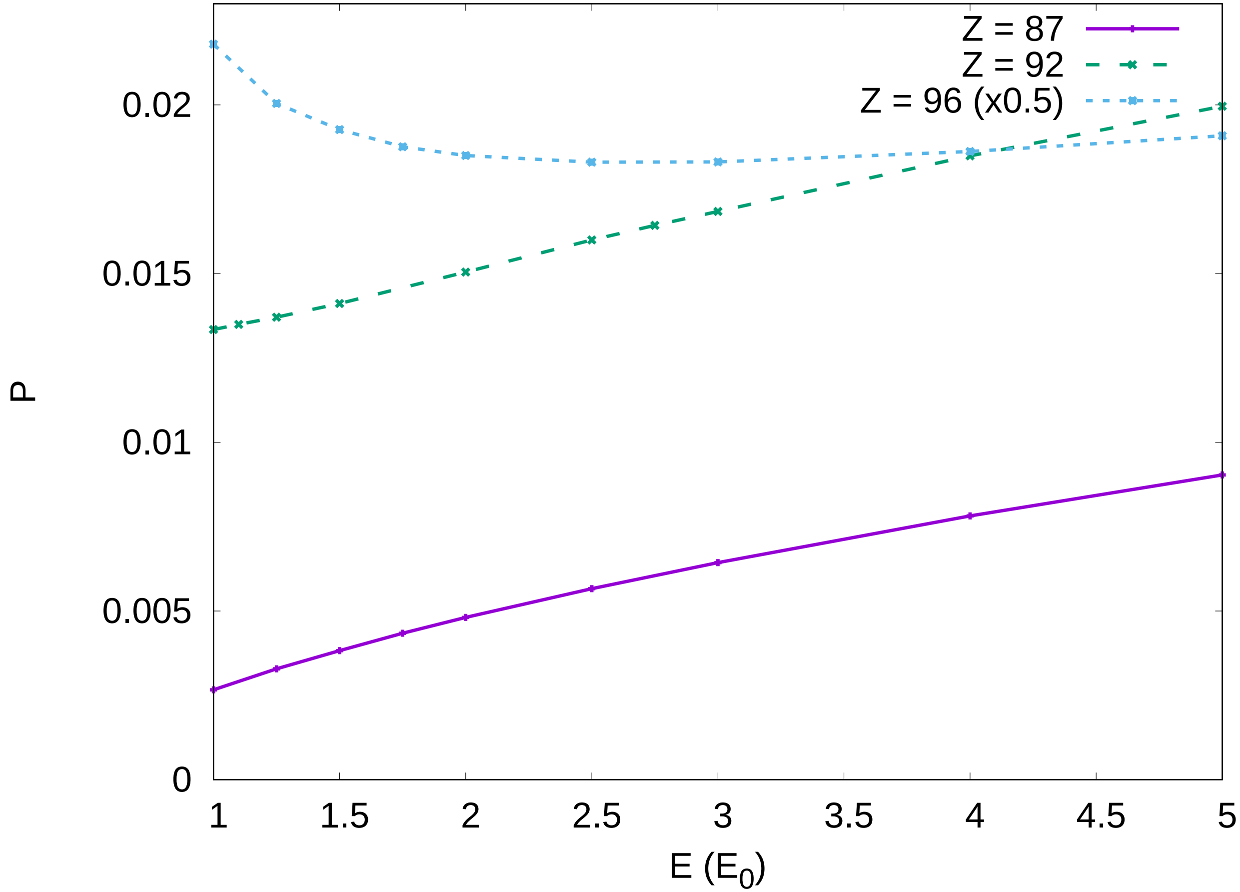

The $\kappa = -1$ contribution to pair-production probability as a function of the collision energy.

## *Low-energy heavy-ion collisions*



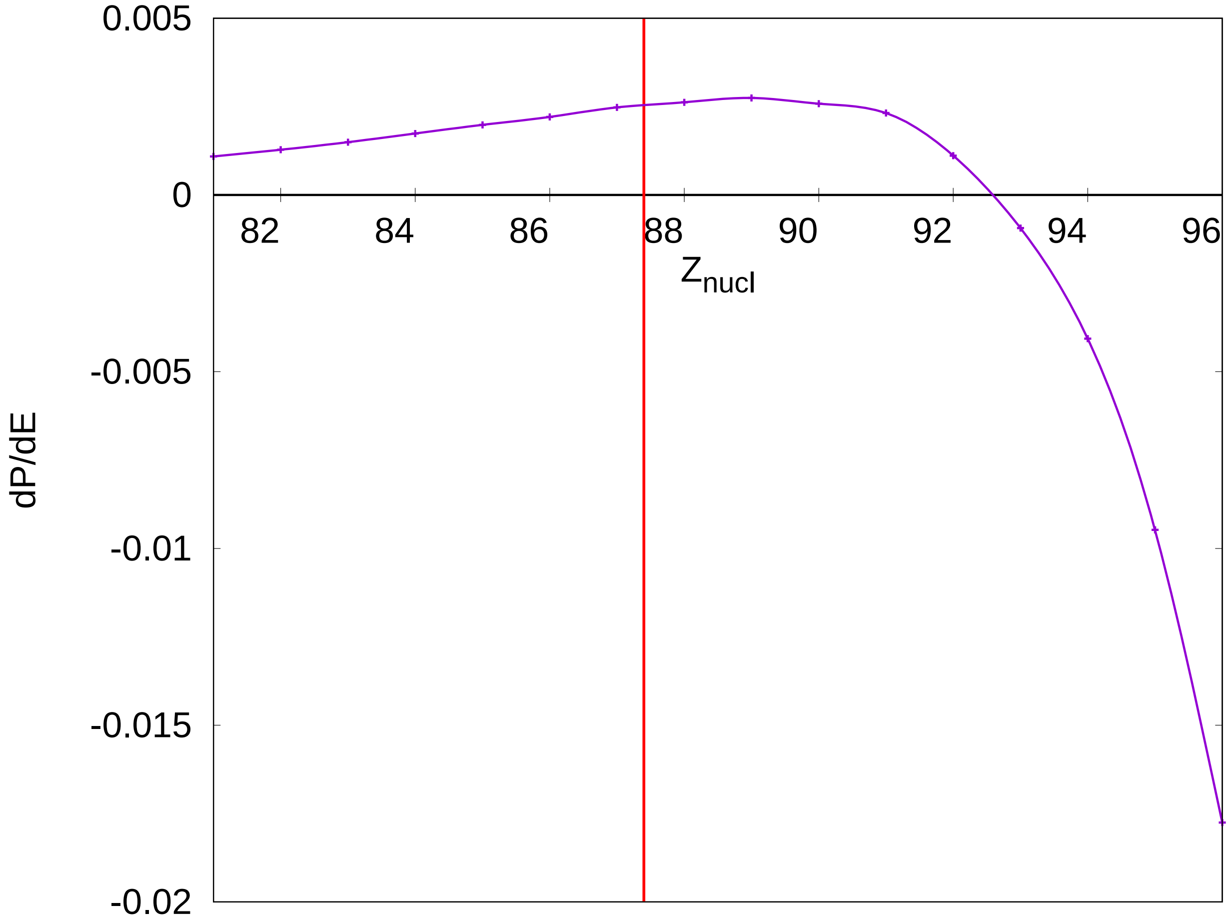

The $\kappa = -1$ and $\kappa = 1$ contributions to the pair-production probability for the U$-$U collision as a function of the collision energy.

# *Low-energy heavy-ion collisions*




Total pair-production probability as a function of the collision energy.

# *Low-energy heavy-ion collisions*




The derivative of the pair-production probability with respect to the energy $dP/dE$ at the point $E = E_0$ as a function of the nuclear charge number $Z_{\mathrm{nucl}}$.

## *Conclusion*

Investigations of heavy ions at low-energy regime can provide:

- Tests of QED at strong coupling regime within and beyond the Furry picture
- Determination of the fundamental constants
- Observing the vacuum decay in supercritical fields