Two-loop QED corrections to the bound-electron g factor involving the magnetic loop

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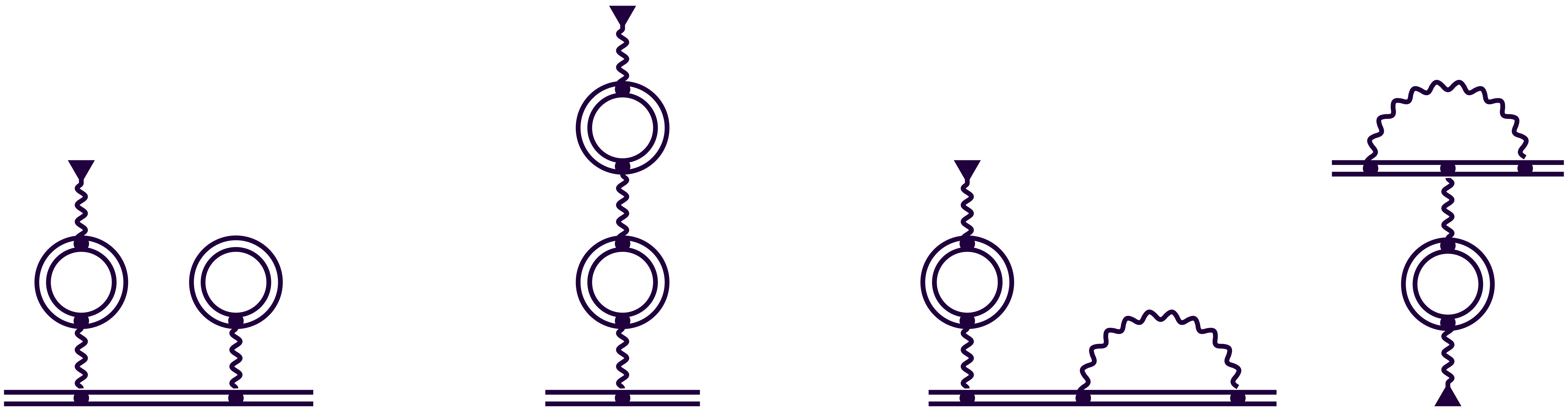
Diagrams to be computed

Electric loop + magnetic loop	Magnetic loop-after-loop	(Non-vertex) self-energy + magnetic loop	(Vertex) self-energy + magnetic loop



Interest for the g factor

Provides high-precision tests of quantum electrodynamics and the Standard Model.
Can help improve determination of fundamental constants.

Basics

There are twenty-nine (29) different diagrams [1, 2, 3] that contribute to the bound-electron g factor at the two-loop level. Four diagrams feature the so-called magnetic loop, which vanishes at the free-loop level.
→ Lowest nonvanishing order: the external magnetic potential is scattered in the Coulomb field of the nucleus before interacting with the bound electron.

The one-loop magnetic loop correction has been calculated [4, 5]. In the process, Lee *et al.* derived an analytical expression for the light-by-light scattering amplitude of a low-energy photon in the Coulomb field, which we use here.

We focus on the simplest case of the $1s$ state.

Electric loop+magnetic loop correction

The treatment is based on the simpler case of the one-loop magnetic loop correction [4, 5], with a modified electronic current. The electric loop-corrected wave function is computed numerically at the Uehling approximation (*i.e.* lowest-order within the vacuum polarisation loop). The Uehling potential is well known and the correction to the wave functions can be computed with relative ease.

Magnetic loop-after-loop correction

We also start from the one-loop magnetic loop correction [4, 5], but with an extra vacuum polarisation loop computed at the free-loop (Uehling) approximation. The vacuum polarisation-dressed photon propagator is well known [2].

Non-vertex self-energy+magnetic loop correction

This diagram is split [6] between a reducible contribution (the intermediate state of the bound electron between the self-energy loop and the magnetic loop is the reference state $1s$) and an irreducible contribution (all other allowed intermediate states).

The treatment of the reducible correction is based on previously computed diagrams:

$$\Delta g^{\mathrm{SE-ML}}_{1s(\mathrm{red})} = \frac{\Delta g^{\mathrm{ML}}_{1s}}{g_{1s}} \Delta g^{\mathrm{SE}}_{1s(\mathrm{red})},$$

where $\Delta g^{\mathrm{ML}}_{1s}$ is the one-loop magnetic loop correction [4, 5], $\Delta g^{\mathrm{SE}}_{1s(\mathrm{red})}$ is the reducible one-loop self-energy correction [6], and g_{1s} is the Dirac value of the bound g factor.

The treatment of the irreducible correction is similar to that of the electric loop+magnetic loop correction. The electric loop-corrected wave function is replaced by the self-energy-corrected wave function, the computation of which is a major numerical challenge [7].

Vertex self-energy+magnetic loop correction

The vertex correction is split [6] between a zero-potential term $\Delta g^{(\mathrm{SE-ML})(0)}_{1s(\mathrm{ver})}$ (wherein the electron does not interact with the Coulomb field of the nucleus under the self-energy loop) and a many-potential term $\Delta g^{(\mathrm{SE-ML})(1+)}_{1s(\mathrm{ver})}$ (which is a sum over all strictly positive numbers of interactions with the Coulomb field under the loop).

The zero-potential term is ultraviolet-divergent, before renormalisation, which is carried out by using the renormalised vertex function $\mathbf{\Gamma}_R$ [6]:

$$\Delta g^{(\mathrm{SE-ML})(0)}_{1s(\mathrm{ver})} = \frac{2m_e}{m\,|\mathbf{B}|} \int \frac{\mathrm{d}\mathbf{p}}{(2\pi)^3} \int \frac{\mathrm{d}\mathbf{p}'}{(2\pi)^3} \bar{\psi}(\mathbf{p})\, \mathbf{\Gamma}_R(p,p') \cdot \mathbf{A}_{\mathrm{ML}}(\mathbf{p}-\mathbf{p}')\, \psi(\mathbf{p}'),$$

where $m = \pm 1/2$ is the projection of the total (= spin) electron angular momentum, $\mathbf{B}$ is the (homogeneous, constant) external magnetic field, ψ is the bound wave function of the $1sm$ state and $\mathbf{A}_{\mathrm{ML}}$ is the light-by-light-scattered vector potential, expressed in terms of the light-by-light scattering amplitude [5]. We compute integrals of products of up to five spherical harmonics over two different solid angles. We are left with a triple spatial integral (2 radial, 1 angular) to be performed numerically, on top of a numerical 1-Feynman parameter integral.

The many-potential term is computed numerically similarly to the corresponding term in the one-loop self-energy correction [6]. We exploit the fact that the magnetic loop preserves the angular structure of the external vector potential.

Some preliminary results

Upcoming experiments have been announced on heavy hydrogenlike ions at HITRAP [8] and ALPHATRAP [9]. The corrections computed here increase strongly with increasing Z. For experimental relevance we present results for two high-Z hydrogenlike ions (Xe^{53+} and Pb^{81+}).

Z	$\Delta g^{\mathrm{EL-ML}}_{1s}$	$\Delta g^{\mathrm{MLAL}}_{1s}$	$\Delta g^{\mathrm{SE-ML}}_{1s(\mathrm{irr})}$	$\Delta g^{\mathrm{SE-ML}}_{1s(\mathrm{ver+red})}$
54	$1.4344(26)\times10^{-9}$	$6.019(1)\times10^{-10}$	$-5.3135(20)\times10^{-9}$	$1.7572(25)\times10^{-9}$
82	$2.0982(8)\times10^{-8}$	$6.845(3)\times10^{-9}$	$-5.5379(20)\times10^{-8}$	$-9.435(23)\times10^{-9}$

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