New quantum SI units
(active starting right now)

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Ludwig-Maximilians-Universität München
Pulkovo Observatory (ГАО РАН) (St. Petersburg)
Max-Planck-Institut für Quantenoptik (Garching)
Outline

- Proton vs. kilogram (as yesterday)
- Macroscopic quantum phenomena and electrical units
- $m/h$
- 4 units to change
- CODATA specials: the last determination of $h$, $e$, $k$, and $N_A$
- Silicon unit
- Kibble’s watt balance
- New and old constants
- Does it really matter?
- What is next?
- Are atomic physics and QED involved?
Who deal with the units? And why?

Who?
- Non-physicists.
- Many have physical education, but different priorities and responsibilities.
- Not for physicists.

Why?
- For better standards to facilitate better measurements of practical importance.
- Nothing to do with fundamental physics.
When did all that [re]start?

Redefinition of the kilogram: a decision whose time has come

Ian M Mills¹, Peter J Mohr², Terry J Quinn³, Barry N Taylor² and Edwin R Williams²

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² National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, MD 20899, USA
³ Emeritus Director, Bureau International des Poids et Mesures, Pavillon de Breteuil, F-92312 Sèvres, Cedex, France

Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to implementing CIPM recommendation 1 (CI-2005)

Ian M Mills¹, Peter J Mohr², Terry J Quinn³, Barry N Taylor² and Edwin R Williams²

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A proton vs. the kilogram
(a yesterday’s slide)
Proton mass and the kilogram: microscopic, macroscopic and electrical

- Proton mass $m_p$
- Reduced Planck constant $\hbar/m_p$
- Mass of carbon $m(C)$
- Mass of silicon $m(Si)$
- Avogadro constant $N_A$
- Watt balance

$10^{-10}$, $10^{-8}$, $10^{-9}$
Macroscopic quantum phenomena and quantum electrical standards

**ac Josephson effect**
- volt standard in the terms of the Josephson constant
  
  \[ K_J = \frac{2e}{h} \]

**quantum Hall effect**
- ohm standard in the terms of the von Klitzing constant
  
  \[ R_K = \frac{h}{e^2} \]

- The measures of those kinds are sufficient for the maintainance.
- As for the reproduction, we had to determine the values of two fundamental constants: \( K_J \) and \( R_K \).
- These constants are fundamental and might be determined through various phenomena.
ac Josephson effect and quantum volt standard

Irradiation with microwave:
- Cooper pairs synchronize with radiation
- Voltage steps appear

\[ V_n = \frac{n}{2e} f \]

Shapiro step, 1963

\[ V_1 \sim 145 \mu V @ 70 \text{ GHz} \]

\[ K_J = \frac{2e}{h} \]
Quantum Hall Effect and quantum ohm standard

Steps:

\[ R_n = \frac{R_K}{n} \]
Atomic masses in frequency units: measuring m/h

- A back door for h
- Recoil spectroscopy
  - caesium (Berkley)
  - rubidium (LKB)
- Needs now lengthy theoretical expressions
- Energy conservation
- Recoil
  \[ \sim f^2/(M c^2/h) \]

- Rydberg constant and \( \alpha \)

\[
R_\infty = \frac{e^4 m_e}{8 \epsilon_0^2 h^3 c} = \frac{c}{2} \alpha^2 \left( \frac{m_e}{h} \right)
\]
International system of units, SI

- 6 base physical units
  - second
  - metre

- 4 base physical units to change
  - kilogram
  - ampere
  - kelvin
  - mole
4 base physical units to change: yesterday’s definition

- **kilogram** (mass of IPK)
  - The physical mass of IPK may change, but not its value of 1 kg (exactly).

- **ampere**
  - $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$ (exactly)

- **kelvin**
  - $T_{\text{triple}} = 273.16 \text{ K}$ (exactly)

- **mole**
  - molar mass of carbon $M(^{12}\text{C}) = 12 \text{ g/mol}$ (exactly)
4 base physical units to change: what was wrong with them

- **kilogram**
- **kelvin**
  - isotopic composition of water
  - used seldom (ITS-90 instead)
- **mole**
  - molar mass of a pure carbon is a fiction,
  - but that is not a problem
- **ampere**
  - hard to access ohm and volt
4 base physical units to change: today’s definition

- kilogram
  - value of the Planck constant \( h \)

- ampere
  - value of the elementary charge \( e \)

- kelvin
  - value of the Boltzmann constant \( k \)

- mole
  - value of the Avogadro constant \( N_A \)
4 base physical units to change: today’s definition

- Kilogram
  - Value of the Planck constant $\hbar$

- Ampere
  - Value of the elementary charge $e$

- Kelvin
  - Value of the Boltzmann constant $k$

- Mole
  - Value of the Avogadro constant $N_A$

Continuity: the new values of the constants and the old ones should be consistent within their uncertainty. That would mean that the realized units, old and new, are consistent as well.
Data and analysis for the CODATA 2017 special fundamental constants adjustment*

Peter J Mohr, David B Newell, Barry N Taylor and Eite Tiesinga
National Institute of Standards and Technology, Gaithersburg, MD 20899-8420, United States of America

Short Communication

The CODATA 2017 values of $h$, $e$, $k$, and $N_A$ for the revision of the SI


Committee on Data for Science and Technology (CODATA) Task Group on Fundamental Constants
Special CODATA adjustment of fundamental constants 2017

Short Communication

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Committee on Data for Science and Technology (CODATA) Task Group on Fundamental Constants

Table 3. The CODATA 2017 values of $h$, $e$, $k$, and $N_A$ for the revision of the SI.

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The [really] last determination of $h$, $e$, $k$, and $N_A$ (in yesterday's SI)

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4 base physical units to change: determination of $h$ (in yesterday’s kg)

Methods:
- Kibble watt balance
- enriched perfect silicon crystal

Results

Figure 1. Values of the Planck constant $h$ inferred from the input data in table 1 and the CODATA 2017 value in chronological order from top to bottom. The inner green band is $\pm 20$ parts in $10^9$ and the outer grey band is $\pm 50$ parts in $10^9$. KB: Kibble balance; XRCD: x-ray-crystal-density.
4 base physical units to change: determination of $e$ (in yesterday’s A)

- Relation between $e$ and $h$
- Relation between $\alpha$, $\varepsilon_0$, $e$, and $h$

Figure 1. Comparison of input data B21 (HarvU-08) and B39 (LKB-11) through their inferred values of $\alpha$ (see table 9). Both B21 and B39 have the same value as in the 2010 and 2014 adjustments and are essentially the sole determinants of the recommended value of $\alpha$ in each. The grey band is ±5 parts in 10$^{10}$. 

\[ \alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{2\varepsilon_0 c} \cdot \frac{e^2}{\hbar} \]
4 base physical units to change: determination of \( k \) (in yesterday’s \( K \))

Methods

- Acoustic Gas Thermometry
- Dielectric Constant Gas Thermometry
- Johnson Noise Thermometry

Results

![Graph showing the determination of \( k \) with various acronyms for different methods and data sets.](image)

**Figure 2.** Values of the Boltzmann constant \( k \) inferred from the key input data in table 1 and the CODATA 2017 value in chronological order from top to bottom. The inner green band is \( \pm 5 \) parts in \( 10^7 \) and the outer grey band is \( \pm 15 \) parts in \( 10^7 \). AGT: acoustic gas thermometry; DCGT: dielectric constant gas thermometry; JNT: Johnson noise thermometry.
The kilogram and ampere: today’s definitions & reproduction

- The kilogram and ampere will be defined with adopted values of \( h \) and \( e \).
- The realization of the kilogram will be based on Kibble watt balance or silicon crystal.

- The ohm is on the base of QHE and \( h/e^2 \).
- The volt is on base of ac JF and \( 2e/h \).
- The ampere is from the Ohm law.
Silicon crystal, atomic masses, and determination of \( h \) (in yesterday’s kg)

- Relation between \( h, \alpha, R_\infty \), and the atomic masses [in atomic units and kilograms]

\[
m(Si) = \frac{A_r(Si)}{A_r(e)} \times m_e/h \times h
\]

\[
R_\infty = \frac{e^4 m_e}{8 \varepsilon_0^2 h^3 c} = \frac{c}{2} \alpha^2 \left( \frac{m_e}{h} \right)
\]
Kibble’s watt balance and determination of $h$ (in yesterday’s kg)

**WB Principle (1): static phase / weighing mode**

\[ I \cdot \int d\ell \times B = mg \]

\[ I = \frac{U_i}{R} \quad \text{(QHE & JVS)} \]

**WB Principle (2): dynamic phase / velocity mode**

\[ U_{ind} = v \cdot \int d\ell \times B \]

**WB Principle (3): combination of modes**

Only if $G_m = G_e$

\[ G(B, \ell) = \frac{mg}{I} = \frac{U}{v} \quad \Rightarrow \quad UI = mgv \]

B. Jeanneret, Les Houches, 2007
Kibble’s watt balance and determination of $\hbar$ (in yesterday’s kg)

The watt balance is a comparator for the force, energy or power. It compares the mechanical ones, linked to the kilogram, to the electric ones, linked to the maintained electrical units, whatever they are.

**WB Principle (1): static phase**

**WB Principle (3): combination of modes**

$$G(B, \ell) = \frac{mg}{I} = \frac{U}{v} \Rightarrow UI = mgv$$

Only if $G_m = G_e$

- Electrical power
- Mechanical power

B. Jeanneret, Les Houches, 2007
Kibble’s watt balance and determination of $\hbar$ (in yesterday’s kg)

The watt balance is a comparator for the force, energy or power. It compares the mechanical ones, linked to the kilogram, to the electric ones, linked to the maintained electrical units, whatever they are.

If one uses ac JE and QHE, then ...

$$G(B, \ell) = \frac{mg}{I} = \frac{U}{v} \Rightarrow UI = mgv$$
Kibble's watt balance and determination of $h$ (in yesterday's kg)

WB Principle (1): static phase / weighing mode

\[ K_J = \frac{2e}{h} \]

WB Principle (2): determination of $R_K$

\[ R_K = \frac{h}{e^2} \]

Only if $G_m = G_e$

\[
\frac{1}{R_K K_J^2} = \frac{h}{4}
\]

\[ U = mgv \implies UI = mgv \]

electrical power

mechanical power

B. Jeanneret, Les Houches, 2007
The kilogram and ampere: today’s definitions & reproduction

- The kilogram and ampere will be defined with adopted values of $h$ and $e$.

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- The ohm is on the base of QHE and $\frac{h}{e^2}$.

- The volt is on base of ac JE and $\frac{2e}{h}$.

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## Determination of fundamental constants (in past & present)

<table>
<thead>
<tr>
<th>Yesterday:</th>
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<td>• measurable</td>
<td></td>
</tr>
<tr>
<td>- e</td>
<td></td>
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<tr>
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<tr>
<td>- $N_A$</td>
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Proton mass and the kilogram: microscopic, macroscopic and electrical

- Micro: $m_p$
- Macro: $h/m_p$
- Electrical: $R_K$, $K_j$

- $10^{-10}$ for $m(Si)$
- $10^{-8}$ for $N_A$
- $10^{-9}$ for $m(C)$

Balance: $10^{-8}$
Does it matter: electron-Volt?

Photons

- energy expressed in eV
- while measured through frequency and wave length

The today’s conversion factor from eV to Hz and m⁻¹ is known exactly (cf. $5 \times 10^9$).

Note:

- the former conversion factor was inaccurate and ‘time-dependent’ because of progress in the field.
- ‘Old’ results need corrections but for the last time.
Does it matter: electron-Volt?

Photons

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The today’s conversion factor from ev to Hz and m⁻¹ is known exactly (cf. \(5 \times 10^{-9}\)).

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\[
\hbar c R_\infty = 13.605693122994(26) \text{ ev } [1.9 \times 10^{-12}] 
\]
Does it matter: electron-Volt?

Atomic and nuclear masses;
mass excesses
(nuclear binding energy)
- The today’s conversion factor from eV/c² to u is known with essentially higher accuracy than ever.

Note:
- the former conversion factor was inaccurate and ‘time-dependent’.
- ‘Old’ results need corrections but afterwards the conversion will have accuracy \( \sim 5 \times 10^{-10} \) (cf. \( 5 \times 10^{-9} \)).
Does it matter: electron-Volt?

Atomic and nuclear masses;
mass excesses
(nuclear binding energy)

- The today’s conversion factor from eV/c² to u is known with essentially higher accuracy than ever.

Note:
- the former conversion factor was inaccurate and ‘time-dependent’.
- ‘Old’ results need corrections but afterwards the conversion will have accuracy ~ $5 \times 10^{-10}$ (cf. $6 \times 10^{-9}$).

$m_e = 0.51099895000(15) \text{ MeV/c}^2$ [3.0 $\times 10^{-10}$]
Do we need quantum electrodynamics for new SI?

Table 3. The CODATA 2017 values of \(h\), \(e\), \(k\), and \(N_A\) for the revision of the SI.

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Table 9. Inferred values of the fine-structure constant $\alpha$ in order of increasing standard uncertainty obtained from the indicated experimental data in table 4.

<table>
<thead>
<tr>
<th>Primary source</th>
<th>Item number</th>
<th>Identification</th>
<th>$\alpha^{-1}$</th>
<th>Relative standard uncertainty $u_r$</th>
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<tbody>
<tr>
<td>$a_e$</td>
<td>$B21$</td>
<td>HarvU-08</td>
<td>$137.035 999 150(33)$</td>
<td>$2.4 \times 10^{-10}$</td>
</tr>
<tr>
<td>$h/m(^{87}\text{Rb})$</td>
<td>$B39$</td>
<td>LKB-11</td>
<td>$137.035 998 995(85)$</td>
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<tr>
<td>$\gamma$</td>
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Do we need quantum electrodynamics for new SI?

α obtained from QED is required for
- continuity of ohm, farad and ε₀;
- continuity of kg/A² and \( h/e² \);
- continuity of kg/mol and \( hN_A \);
- continuity of kg/mol and molar masses;
- continuity of A²/mol and \( e²N_A \)

Derived units are sometimes dealt with a higher fractional accuracy than the base units.

System of the standards does not follow system of the units.
CHANGE AHEAD

EVERYTHING is going to be OK
What’s new?

The brand new CODATA’s values in terms of brand new SI units have been available from the NIST web site.

[Table of fundamental constants]

*Defining constants of the International System of Units (SI).

[Link to NIST website]