



FFK-2019
Tihany, Hungary
13 June 2019



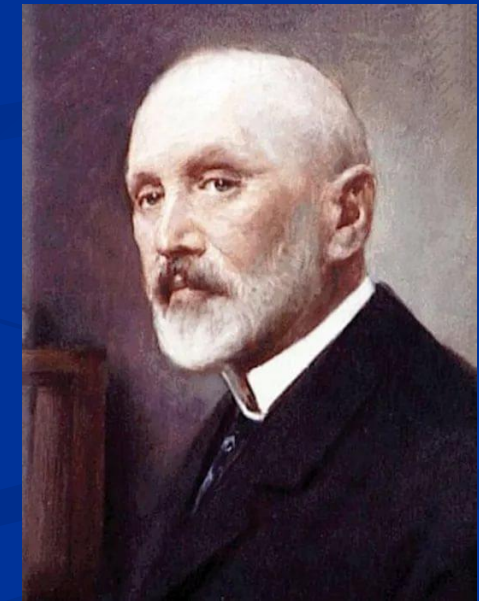
Newtonian Gravitation Constant: History of Measurement and New Results



Henry Cavendish (1731–1810),

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Lomonosov MSU
Sun Yat sen University



Loránd Eötvös (1848-1919)



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United Nations
Educational, Scientific and
Cultural Organization

**100th anniversary of Roland Eötvös
(1848-1919), physicist, geophysicist,
and innovator of higher education**

Commemorated in association with UNESCO

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Subject: Invitation to Eötvös 100
Honorary Board
Reference: 626/2019/NKF

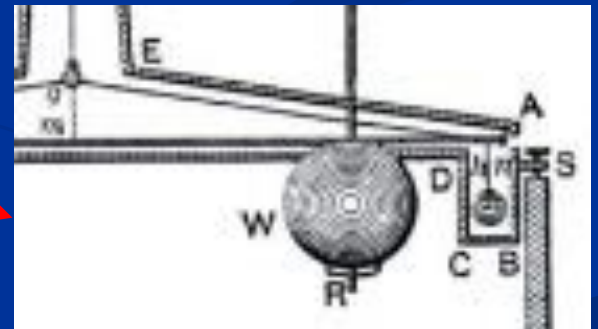
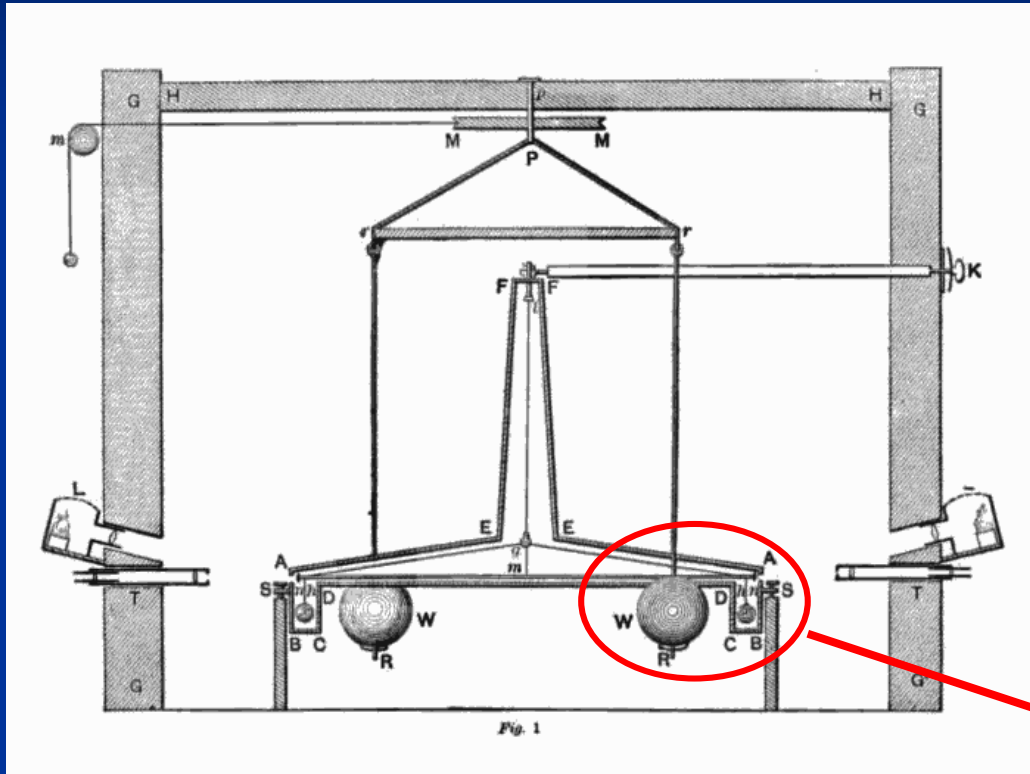
Budapest, 23 January, 2019

In association with UNESCO, the world's scientific community commemorates this year the 100th anniversary of the death of Roland Eötvös (1848-1919), a pioneer of high precision gravitational physics, founding father of geophysics and innovator of higher education.

Throughout the centenary year a series of scientific events and exhibitions will be organized, and special publications will be presented in Hungary and worldwide, all in remembrance of the genius of Baron Eötvös. A central event will take place in Budapest on the day of his passing away (8 April), and at the same time an "Eötvös 100" exhibition will open at the 2019 Annual Meeting of the European Geosciences Union, to be held in Vienna. Roland Eötvös will also be remembered at the General Assembly of the International Union of Geodesy and Geophysics (IUGG), an organization which also celebrates its 100th anniversary in 2019.



Cavendish experiment 1797-1798



- Wooden rod 1,8 m
- Spherical lead test masses 0.73 kg, $D=51$ mm
- Spherical lead attracting masses 158 kg, $D=300$ mm



Cavendish experiment



Earth's density:

$$\sigma = (5.448 \pm 0.033) \text{ gcm}^{-3}$$

Cavendish, H. *Phil. Trans. R.Soc. B* **88**, 469–526 (1798).

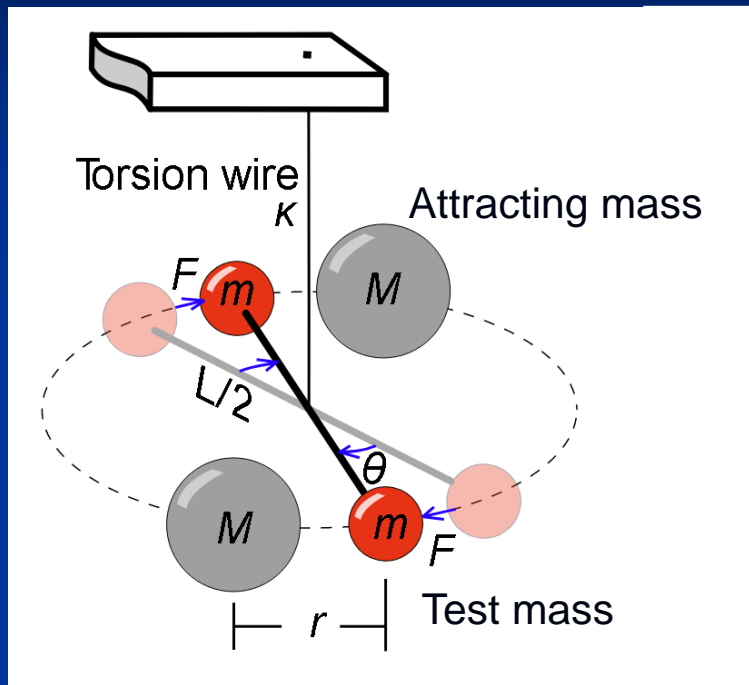
First G value:

$$G = (6.67 \pm 0.07) \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$$

$$\Delta G / G \approx 10^4 \text{ ppm}$$



Time – of – swing method



$$J\ddot{\varphi} + [k + G(\partial\Gamma / \partial\varphi)]\varphi = 0$$
$$\omega_1^2 = \frac{k + G(\partial\Gamma / \partial\varphi)}{J}$$
$$G = \frac{J[(\omega_1^2)_1 - (\omega_1^2)_2]}{(\partial\Gamma / \partial\varphi)_1 - (\partial\Gamma / \partial\varphi)_2}$$

To determine G by time-of swing method it is necessary to perform the following measurements:

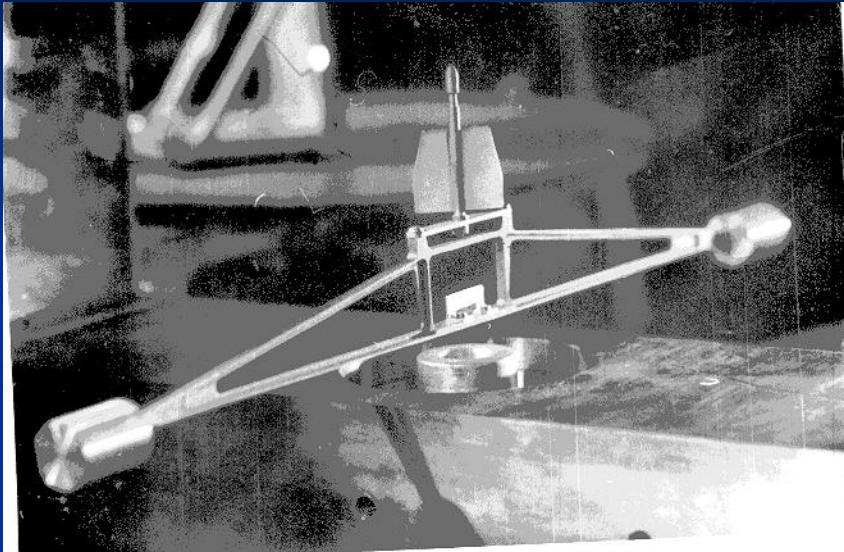
- frequencies of torsional oscillations at two different positions of the attracting masses M;
- geometric dimensions and densities of the torsion balance;
- geometric dimensions and density of the attracting mass M;
- the mutual location of the torsion balance and test masses M.



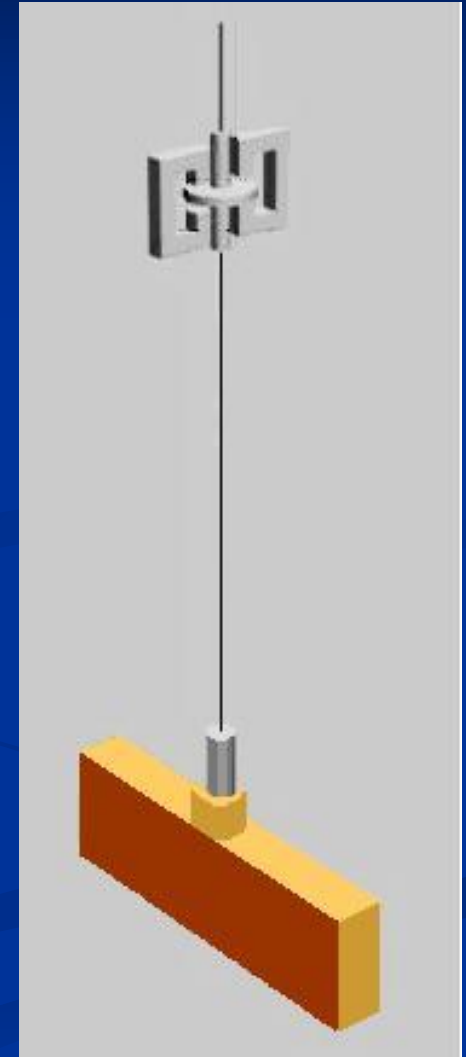
The torsion balances and time of swing method



No 1

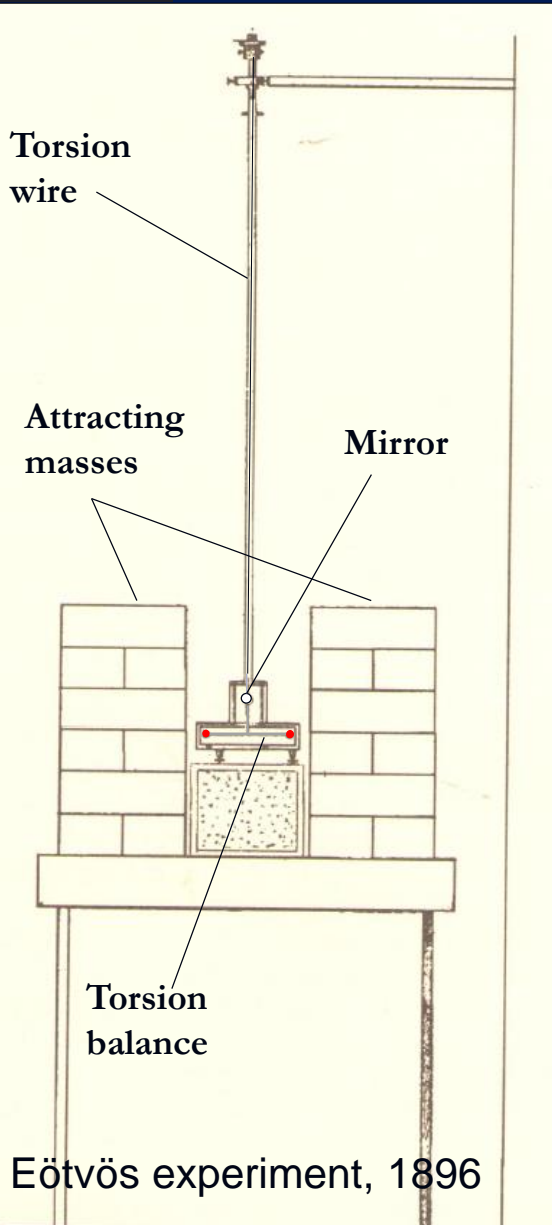


No 2





Eötvös experiment



Eötvös experiment, 1896

The attracting masses: two lead parallelepipeds with a base of **30x30 cm** and a height of **60 cm**

Period of oscillation in the longitudinal position:

$$T_1 = 641 \text{ s};$$

Period of oscillation in the transversal position:

$$T_2 = 860 \text{ s}.$$

$$\left(\frac{1}{T_1^2} - \frac{1}{T_2^2} \right) = G \frac{8\rho}{\pi} (1 - \epsilon)$$

Eötvös G value:

$$G = (6.657 \pm 0.013) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$\Delta G / G = 2 \times 10^3 \text{ ppm}$$

Lorand Eötvös. *Annalen der Phys. und Chem.*, N.F., **59**, 354-400 (1896)



The most reliable measurements of the Newtonian gravitation constant (1 -2 stage)

Experiment: Author, year of publication, Place of experiment	Technique	$G \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
H. Cavendish, 1798 Clapham, England	Torsion balance, static	$6.67 \div 0.07 \quad \sim 10^4 \text{ ppm}$
F. Reich, 1838, 1852 Freiburg, Germany	Torsion balance, dynamic	$6.64 \div 0.06$
R. Eötvös, 1896 Budapest, Hungary	Torsion balance, dynamic	$6.657 \div 0.013$
C. Braun, 1897 Marienstein, Austria	Torsion balance, static and dynamic	$6.649 \div 0.02$
P.Heil, 1930 Washington, USA	Torsion balance, dynamic	$6.670 \div 0.005 \quad \sim 10^3 \text{ ppm}$
P.Heil, Chrzanowski, 1942 Washington, USA	Torsion balance, dynamic	$6.673 \div 0.005$
R. Rose, H.Parker, K. Lowry, A. Kuhlhan, I. Beams, 1969 Charlotenscille, USA	Torsion balance, compensative	$6.674 \div 0.004$
J. Renner, 1973 Budapest, Hungary	Torsion balance, dynamic	$6.670 \div 0.008$



CODATA values of the gravitational constant



The Task Group on Fundamental Constants of the Committee on Data for Science and Technology (CODATA) was established in 1969 to periodically provide the scientific and technological communities with a self-consistent set of internationally recommended values of the basic constants and conversion factors of physics and chemistry based on all relevant data available at a given point in time.

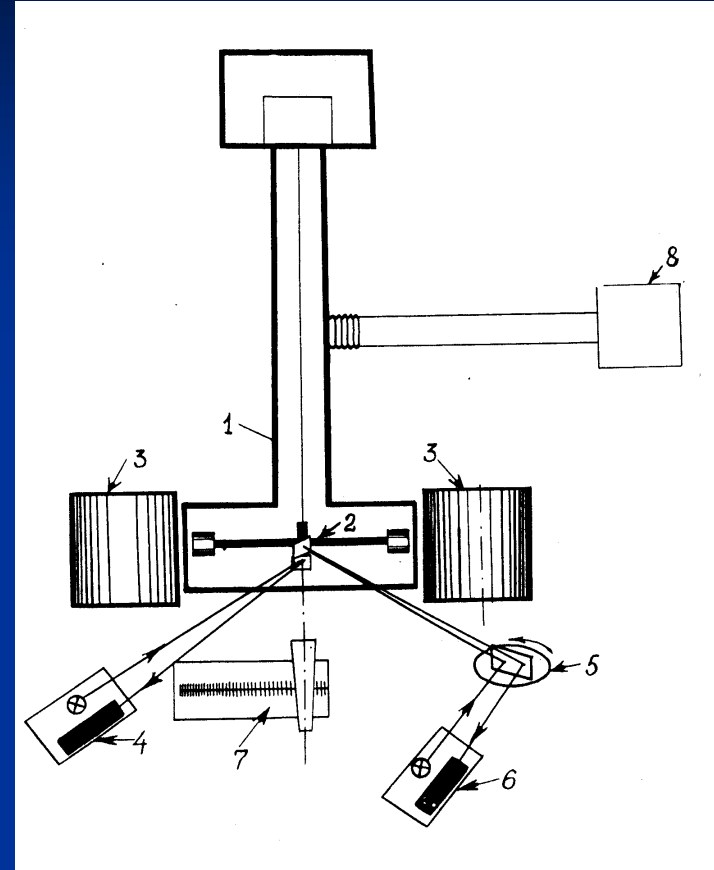
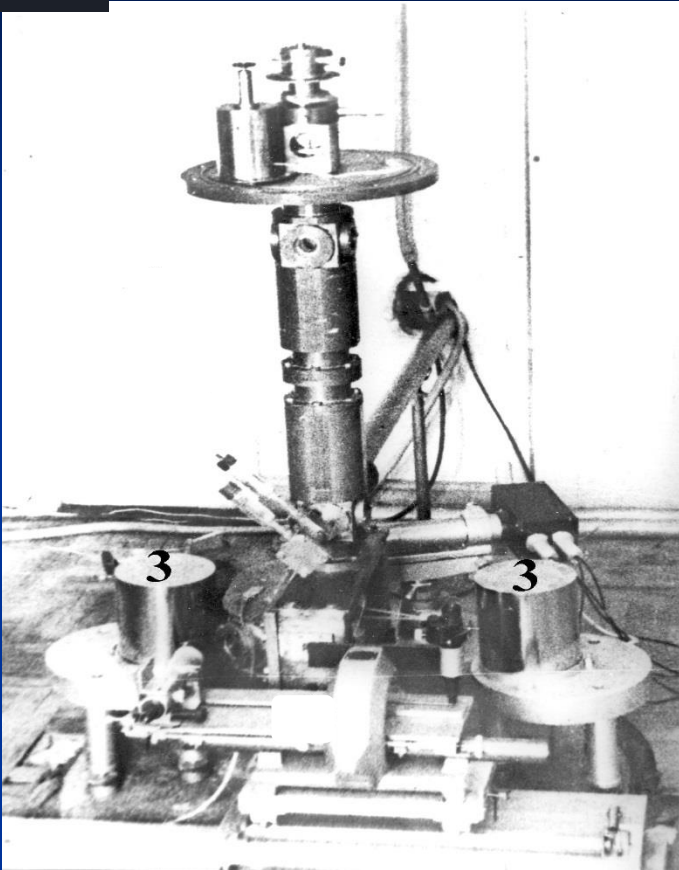
CODATA - 1973 :

The recommended value of the Newtonian gravitation constant

$$G = (6.6720 \pm 0.0041) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \text{ (615 ppm)}$$

was mainly based on the Heil and Chrzanowski results, obtained in 1942

SAI MSU experiment (1975-1978)



General view and sketch of the SAI MSU experimental setup:

1 – housing of device; 2 – torsion balance; 3 – attracting masses; 4, 6 – optical readout systems; 5 – rotating mirror; 7 – device for linear measurements; 8 – vacuum pump.



The value of the Newtonian gravitational constant, obtained in the MSU experiment

Four series of the experiments:

1. March - May 1975,
2. March - June 1976,
3. January - March 1977
4. October 1977 – January 1978

$$G = (6.6745 \pm 0.0008) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

with relative standard uncertainties of 120 ppm

Сагитов М.У, Миллюков В.К. и др. Новое определение кавендишевой гравитационной постоянной // Доклады АН СССР, 1979, т.245, № 3, с.567 -569 (Sov. Phys. Dokl. 245(1-6), 20-22 (1981))



THE CONSTANT OF GRAVITATION (Result of the MSU experiment)



APPENDIX

DETERMINATION OF THE GRAVITY CONSTANT

LORÁND EÖTVÖS

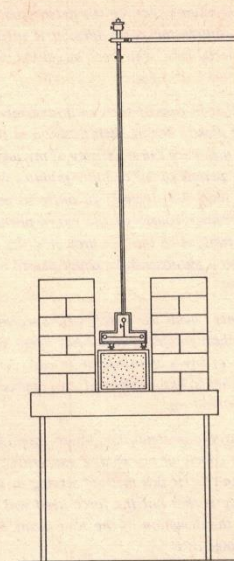
An English translation of the classical paper of Eötvös, presented to the Hungarian Academy of Sciences, on the 20th April 1896. The paper was published as Chapter III of the study "Studies concerning gravity and terrestrial magnetism".

Bibliography:

Math. és Term.-tud. Ért., 14 (1896), 221-266 (Hung.)

Math. u. Naturwiss. Ber. aus Ungarn, 13 (1896), 193-243 (Germ.)

Ann. d. Phys. u. Chem., Neue Folge, 59 (1896), 354-400 (Germ.)



The oscillations of the beam were observed about two equilibrium positions perpendicular to each other: one of them was a longitudinal one falling into the longitudinal direction of the wall, while the other – perpendicular to it – had a transversal orientation. In the longitudinal position the oscillation time was 641 sec, in the transversal one 860 sec.

The choice of this particular arrangement of the attracting masses was made owing to the following reasoning taking into account an ideal case. Let us consider a homogeneous, vertical wall of parallelepipedic form being

First HUST experiment (1998)



$$G=(6.6699 \pm 0.0007) \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \quad \text{with rel. unc.: 105 ppm}$$

Luo, J. et al. *Phys. Rev. D* **59**, 042001 (1998).



Second HUST experiment (2009)



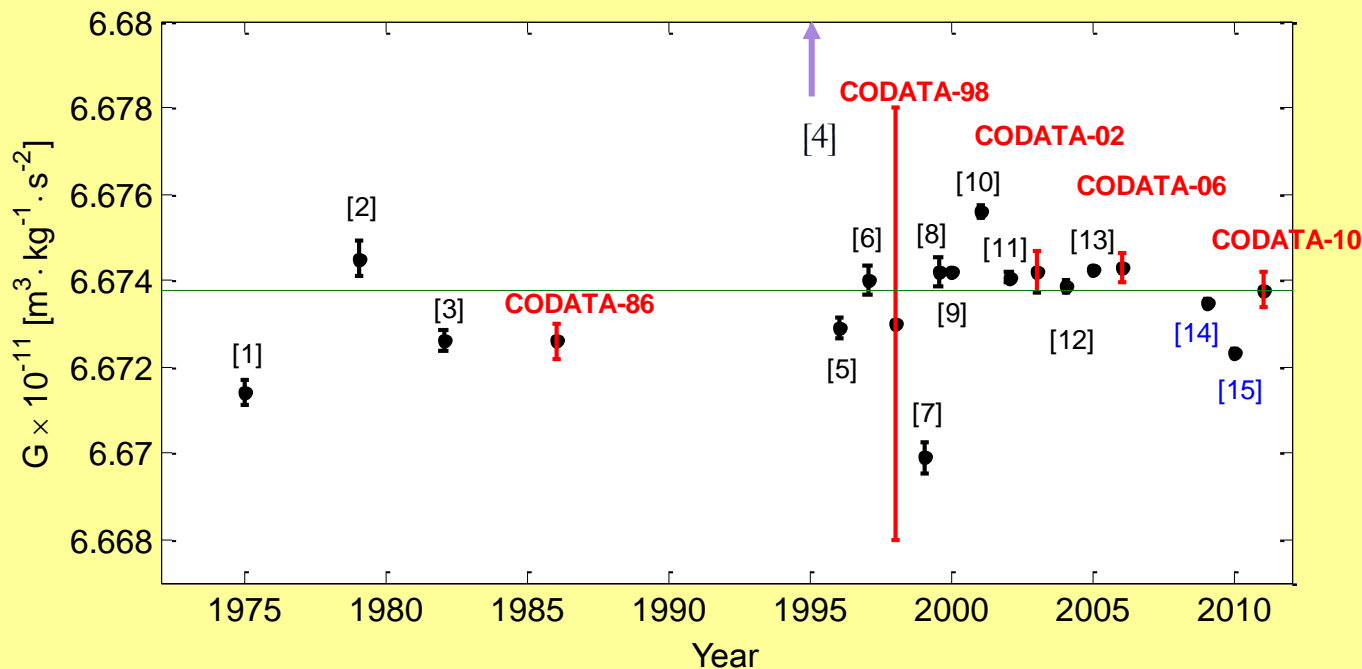
$$G=(6.67349 \pm 0.00018)\times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$$

with rel. unc.: 26 ppm



The most reliable measurements of the Newtonian gravitation constant (3 -4 stage)

Authors, year of publication	Value of $G \times 10^{-11}$ $m^3 kg^{-1} s^{-2}$	STD $\times 10^{-11}$ $m^3 kg^{-1} s^{-2}$	ppm	
Stage 3: ~100 ppm				
[1] <i>Facy and Ponticis, France</i>	1972	6.6714	0.0006	90
[2] <i>Sagitov, Milyukov, et al. Moscow University</i>	1979	6.6745	0.0008	120
[3] <i>Luther and Towler, Nat. Bur. of Stand., Washington</i>	1982	6.6726	0.0005	75
[4] <i>Michaelis, et al., Physik Technische Bundesanstalt</i>	1995	6.7154	0.0006	90
[5] <i>Karagioz, Izmailov, Committee of Standards, Moscow</i>	1996	6.6729	0.0005	75
[6] <i>Bagley and Luther, Los Alamos National Lab</i>	1997	6.6740	0.0007	105
[7] <i>Jun Luo, et al., HUST, China</i>	1999	6.6699	0.0007	105
[8] <i>Fitzgerald and Armstrong Meas.St.Lab, New Zealand</i>	1999	6.6742	0.0007	105
Stage 4: ~15÷40 ppm				
[9] <i>Gundlach and Merkowich, University of Washington,</i>	2000	6.674215	0.000092	14
[10] <i>Quinn, Speake et al. University of Birmingham</i>	2001	6.67559	0.00027	41
[11] <i>Schlamminger et al. University of Zurich</i>	2002	6.67407	0.00022	33
[12] <i>Armstrong and Fitzgerald, Meas.St.Lab, New Zealand</i>	2003	6.67387	0.00027	40
[13] <i>Schlamminger et al. University of Zurich</i>	2006	6.674252	0.000109	16
[14] <i>Jun Luo, et al. HUST, China</i>	2009	6.67349	0.00018	26
[15] <i>Parks and Faller, University of Colorado</i>	2010	6.67234	0.00014	21



Authors, year of publication	Value of $G \times 10^{-11}$ $m^3 kg^{-1} s^{-2}$	STD $\times 10^{-11}$ $m^3 kg^{-1} s^{-2}$	ppm
[1] Facy and Ponticis, France 1972	6.6714	0.0006	90
[2] Sagitov, Milyukov, et al. Moscow University 1979	6.6745	0.0008	120
[3] Luther and Towler, Nat. Bur. of Stand., Washington 1982	6.6726	0.0005	75
CODATA 1986	6.67259	0.00085	128
[4] Michaelis, et al., Physik Technische Bundesanstalt 1995	6.7154	0.0006	90
[5] Karagioz, Izmailov, Committee of Standards, Moscow 1996	6.6729	0.0005	75
[6] Bagley and Luther, Los Alamos National Lab 1997	6.6740	0.0007	105
CODATA 1998	6.673	0.010	1500
[7] Jun Luo, et al., HUST, China 1999	6.6699	0.0007	105
[8] Fitzgerald and Armstrong Meas.St.Lab, New Zealand 1999	6.6742	0.0007	105
[9] Gundlach and Merkowich, University of Washington, 2000	6.674215	0.000092	14
[10] Quinn, Speake et al. University of Birmingham 2001	6.67559	0.00027	41
[11] Schlamminger et al. University of Zurich 2002	6.67407	0.00022	33
CODATA 2002	6.6742	0.0010	150
[12] Armstrong and Fitzgerald, Meas.St.Lab, New Zealand 2003	6.67387	0.00027	40
[13] Schlamminger et al. University of Zurich 2006	6.674252	0.000109	16
CODATA 2006	6.67428	0.00067	100
[14] Jun Luo, et al. HUST, China 2009	6.67349	0.00018	26
[15] Parks and Fallor, University of Colorado 2010	6.67234	0.00014	21
CODATA 2010	6.67384	0.00080	120



Effect of unelasticity of torsion fiber (Kuroda effect)

Time-of-swing method

$$G = \frac{I(\omega_n^2 - \omega_f^2) - (k_n - k_f)}{C_{gn} - C_{gf}} = \frac{I\Delta(\omega^2)}{\Delta C_g}$$

In ideal case $k_n = k_f = \text{const}$

$$(k_n - k_f) = 0$$

In general case, spring constant k is a complex value, and not only dissipation, but also k depends on the frequency

$$G = \frac{I(\omega_n^2 - \omega_f^2) - (k_n - k_f)}{C_{gn} - C_{gf}} = \frac{I\Delta(\omega^2)}{\Delta C_g} \left[1 - \frac{\Delta k}{I\Delta(\omega^2)} \right]$$



$$\Delta G/G \approx 1/\pi Q$$

For $Q \approx 1700$, $\Rightarrow \Delta G/G \approx 187 \text{ ppm}$;
For $Q \approx 10^5$, $\Rightarrow \Delta G/G \approx 3 \text{ ppm}$



Ways to overcome Kuroda effect :

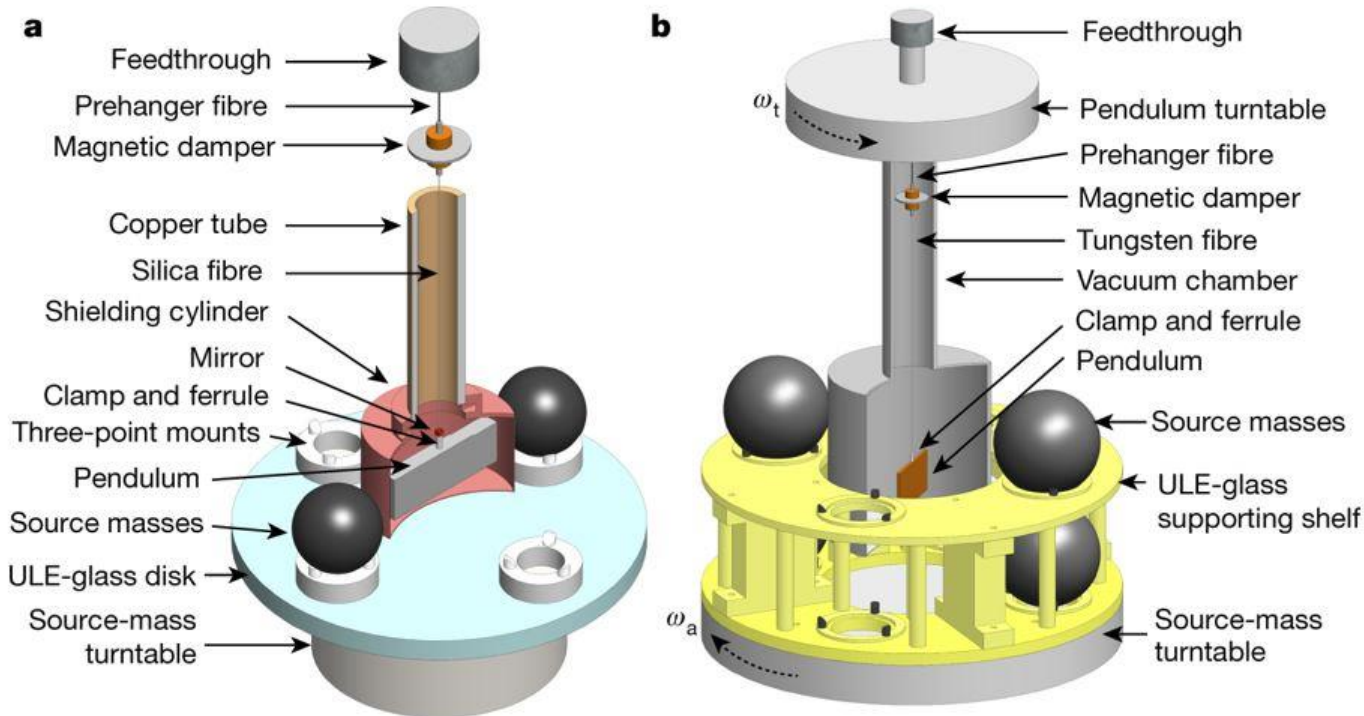
- Torsional balance with high Q
- The method of the experiment, in which the torsion wire is not twisted



- Time-of-swing (TOS) method;
- Angular-acceleration-feedback method (AAF)



Ideology of TOS and AAF methods



See next presentations:

- Qing Li, et al. Determination of G with time-of-swing method by using high Q silica fiber.
- Chao Xue, et al. Determination of G with Angular-Acceleration-Feedback Methods



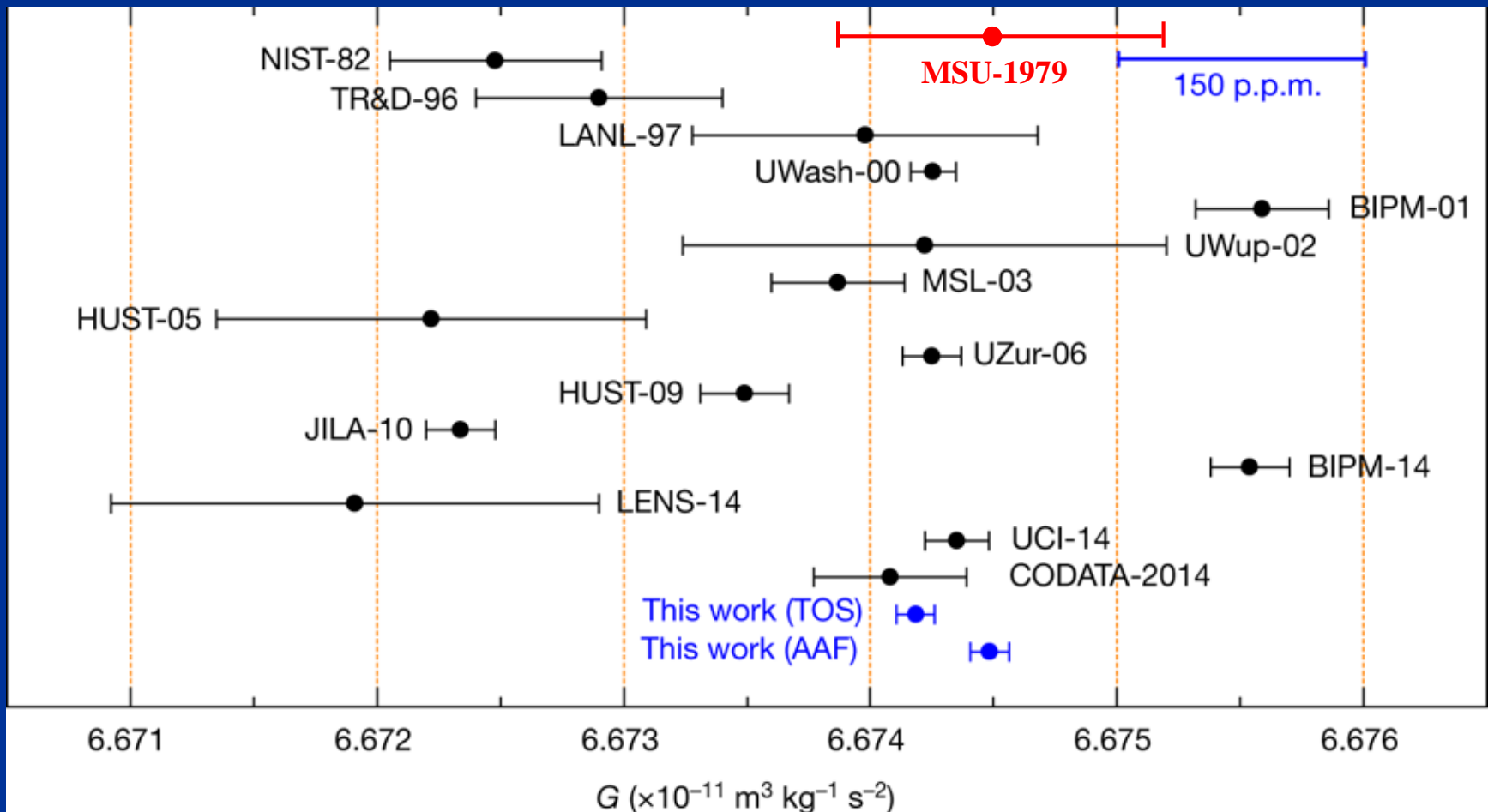
The most reliable measurements of the Newtonian gravitation constant

Luo Jun, et al (TOS) $G=(6.674184(78)\times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2})$

with rel. unc.: 11.64 ppm

Luo Jun, et al (AAF) $G=(6.674484(78)\times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2})$

with rel. unc.: 11.61 ppm





Nature volume 560, pages 582–588 (2018)

doi.org/10.1038/s41586-018-0431-5

ARTICLE

<https://doi.org/10.1038/s41586-018-0431-5>

Measurements of the gravitational constant using two independent methods

Qing Li^{1,8}, Chao Xue^{2,3,8}, Jian-Ping Liu^{1,8}, Jun-Fei Wu^{1,8}, Shan-Qing Yang^{1*}, Cheng-Gang Shao^{1*}, Li-Di Quan⁴, Wen-Hai Tan¹, Liang-Cheng Tu^{1,2}, Qi Liu^{2,3}, Hao Xu¹, Lin-Xia Liu⁵, Qing-Lan Wang⁶, Zhong-Kun Hu¹, Ze-Bing Zhou¹, Peng-Shun Luo¹, Shu-Chao Wu¹, Vadim Milyukov⁷ & Jun Luo^{1,2,3*}

The Newtonian gravitational constant, G , is one of the most fundamental constants of nature, but we still do not have an accurate value for it. Despite two centuries of experimental effort, the value of G remains the least precisely known of the fundamental constants. A discrepancy of up to 0.05 per cent in recent determinations of G suggests that there may be undiscovered systematic errors in the various existing methods. One way to resolve this issue is to measure G using a number of methods that are unlikely to involve the same systematic effects. Here we report two independent determinations of G using torsion pendulum experiments with the time-of-swing method and the angular-acceleration-feedback method. We obtain G values of 6.674184×10^{-11} and 6.674484×10^{-11} cubic metres per kilogram per second squared, with relative standard uncertainties of 11.64 and 11.61 parts per million, respectively. These values have the smallest uncertainties reported until now, and both agree with the latest recommended value within two standard deviations.



Conclusion

1798



Henry Cavendish : “The apparatus is very simple”
(*Philos. Trans. R. Soc. London*, **88**, 469, 1798)

2010



James Faller:

“Big G is the Mt. Everest of precision
measurement science, and it should be climbed.”
(*Phys. Rev. Lett.*, **105**, 2010)

2018



Stefan Schlamminger:

“Gravity measured with record precision”
562 (Nature, 560, 2018)

