Isotope shift and atomic parity violation in the search for new physics

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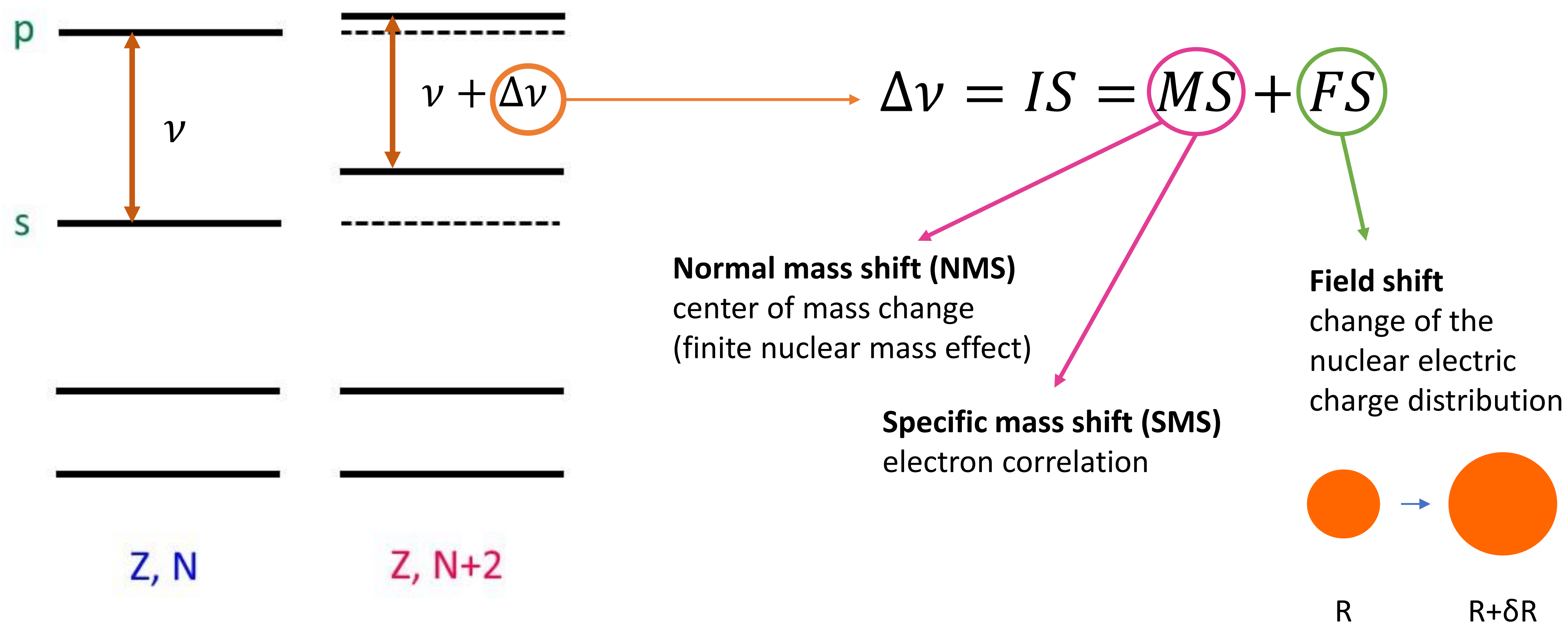
Outline

- Isotope shifts & King plots
- Atomic parity violation & Yb experiment in Mainz

Isotope shift & King plot

Isotope shift



$$\Delta\nu = IS = MS + FS$$

Normal mass shift (NMS)
center of mass change
(finite nuclear mass effect)

Specific mass shift (SMS)
electron correlation

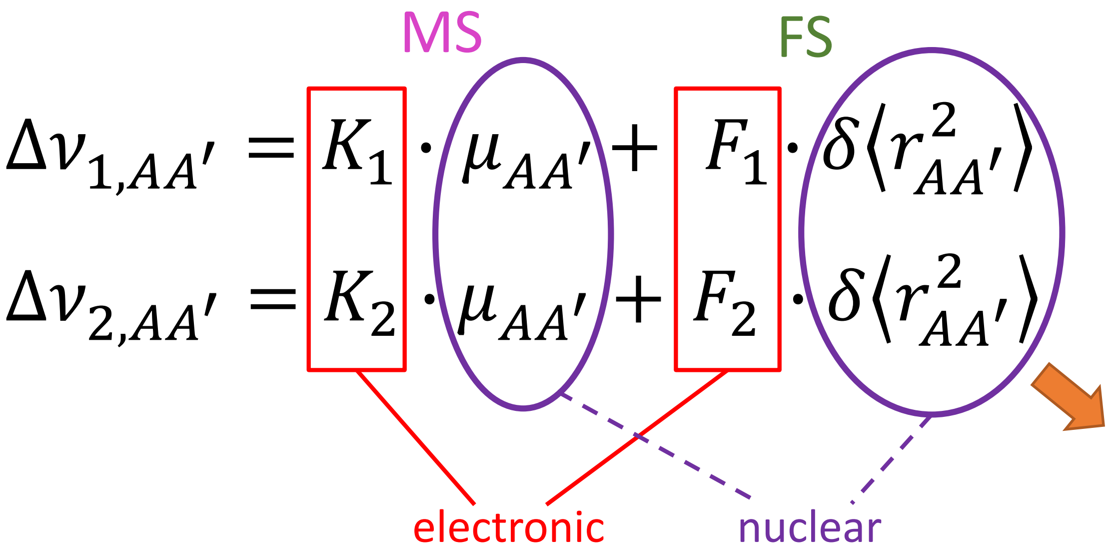
Field shift
change of the
nuclear electric
charge distribution

King plot

$$IS = MS + FS$$

Let's look at a pair of transitions in two isotopes A and A'.

First-order non-relativistic case gives:



$$\Delta\nu_{1,AA'} = K_1 \cdot \mu_{AA'} + F_1 \cdot \delta\langle r^2_{AA'}\rangle$$

$$\Delta\nu_{2,AA'} = K_2 \cdot \mu_{AA'} + F_2 \cdot \delta\langle r^2_{AA'}\rangle$$

, here $\mu_{AA'} = \frac{1}{M_A} - \frac{1}{M_{A'}}$

$$n_{1,AA'} = {}^{\Delta\nu_{1,AA'}}/_{\mu_{AA'}} = K_1 + F_1 x_{AA'}$$

$$n_{2,AA'} = {}^{\Delta\nu_{2,AA'}}/_{\mu_{AA'}} = K_2 + F_2 x_{AA'}$$

$, x_{AA'} = \frac{\delta\langle r^2_{AA'}\rangle}{\mu_{AA'}}$

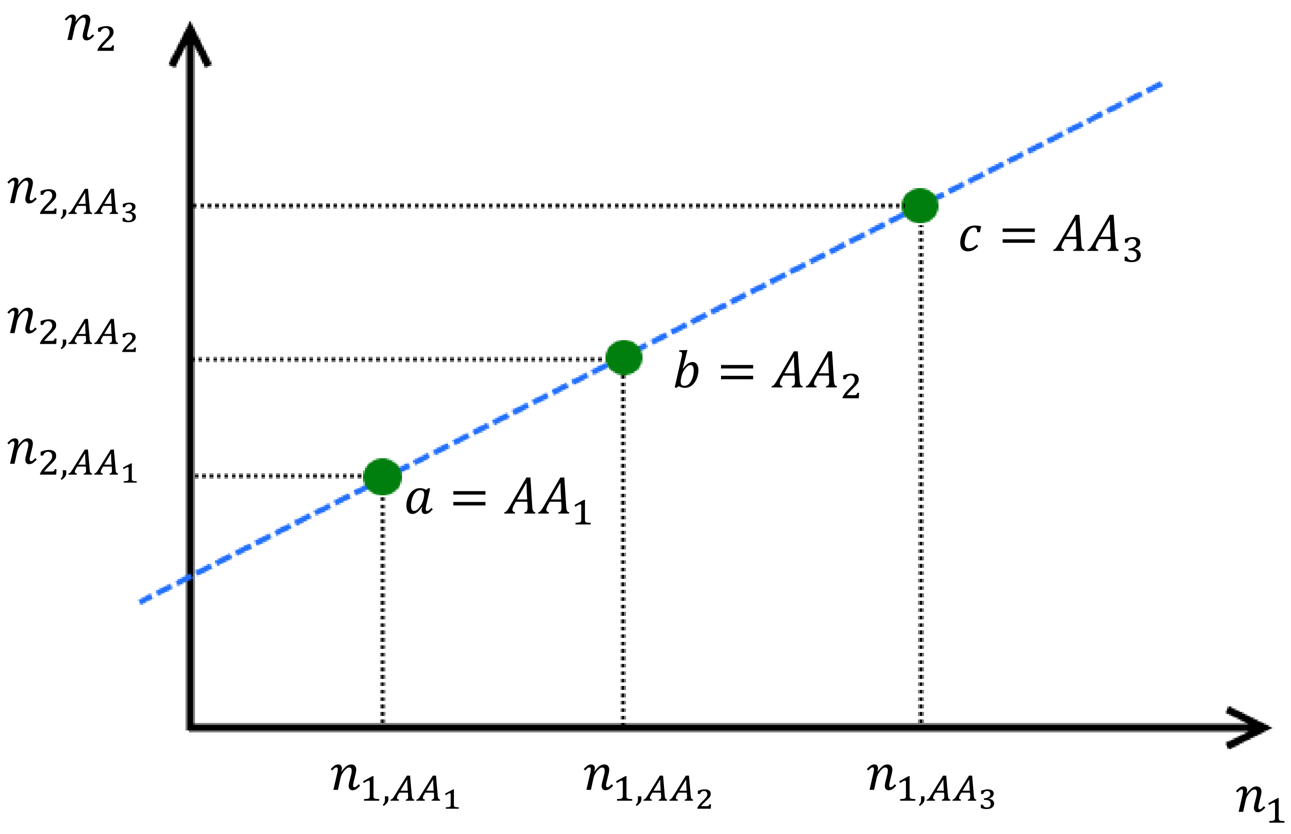
$$n_{2,AA'} = K_2 + \frac{F_2}{F_1}(n_{1,AA'} - K_1)$$

[1] King, W. H. (1963)

King plot

$$n_{2,AA'} = K_2 + \frac{F_2}{F_1}(n_{1,AA'} - K_1)$$

A, A_1, A_2, A_3 - isotopes of an element Z



King plot non-linearity

$IS = MS + FS$

$$\Delta\nu_{1,AA'} = \overbrace{K_1\mu_{AA'}}^{\text{MS}} + \overbrace{F_1\delta\langle r_{AA'}^{2\gamma_1}\rangle \boxed{+ G_1\delta\langle r_{AA'}^{2\gamma_2}\rangle + \cdots}}^{\text{FS}}$$

$$\Delta\nu_{2,AA'} = K_2\mu_{AA'} + F_2\delta\langle r_{AA'}^{2\gamma_1}\rangle \boxed{+ G_2\delta\langle r_{AA'}^{2\gamma_2}\rangle + \cdots}$$

$$\mu_{AA'} = \frac{1}{M_A} - \frac{1}{M_{A'}}$$

$$\gamma_1 = \sqrt{{\kappa_1}^2 - (\alpha Z)^2}$$

$$\kappa_1 = -1, 1$$

$$\gamma_2 = \sqrt{{\kappa_2}^2 - (\alpha Z)^2}$$

$$\kappa_2 = -2, 2, -3, 3, \ldots$$

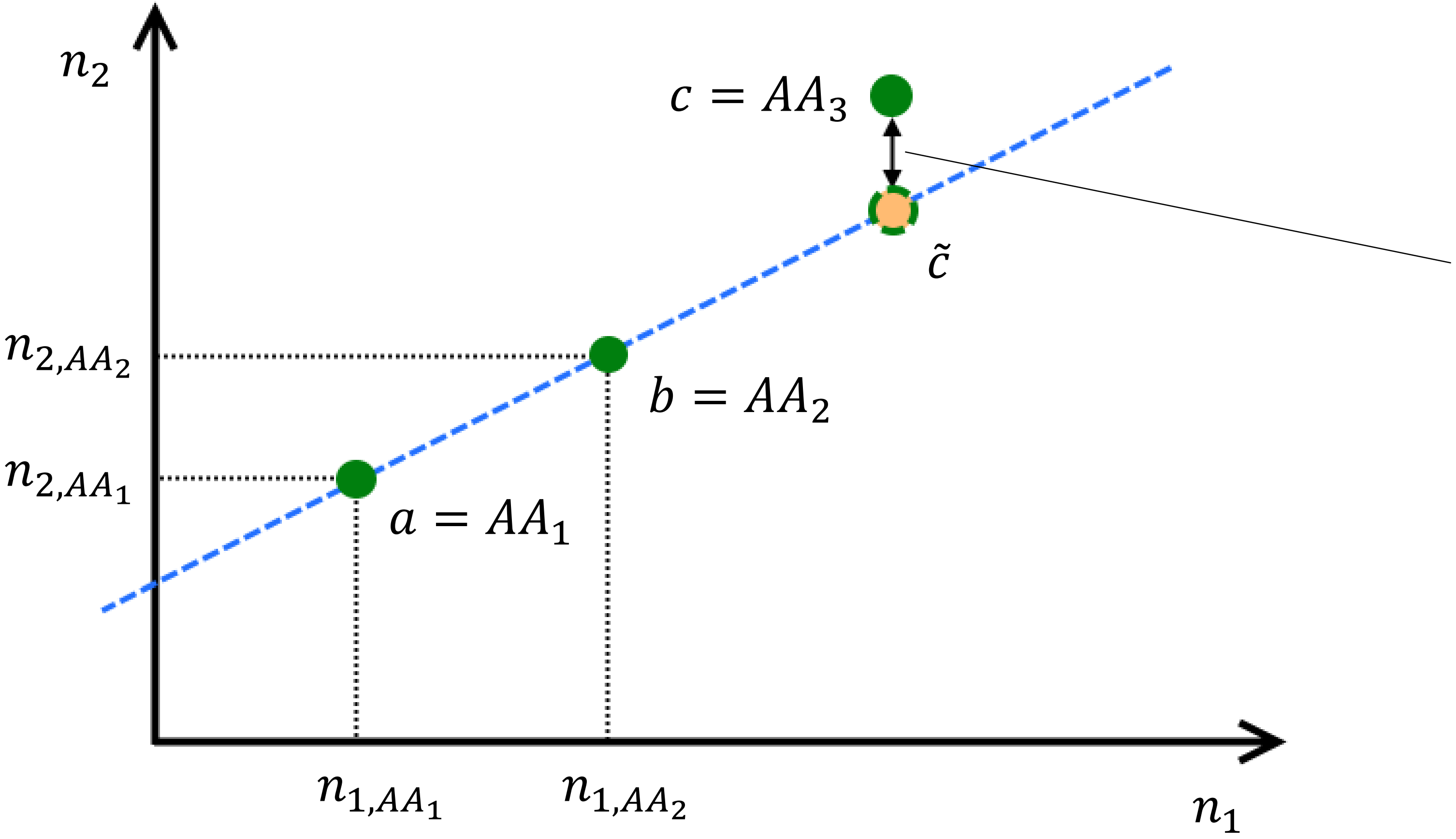
↘ divided both by $\mu_{AA'}$

$$n_{1,AA'} = K_1 + F_1 x_{AA'} \boxed{+ G_1 y_{AA'} + \cdots}$$

$$n_{2,AA'} = K_2 + F_2 x_{AA'} \boxed{+ G_2 y_{AA'} + \cdots}$$

???

King plot non-linearity



- **New Physics ?!?**

or

- **Nonlinearities from higher-order SM contributions ?**

New Physics in King plot?

Effective potential:

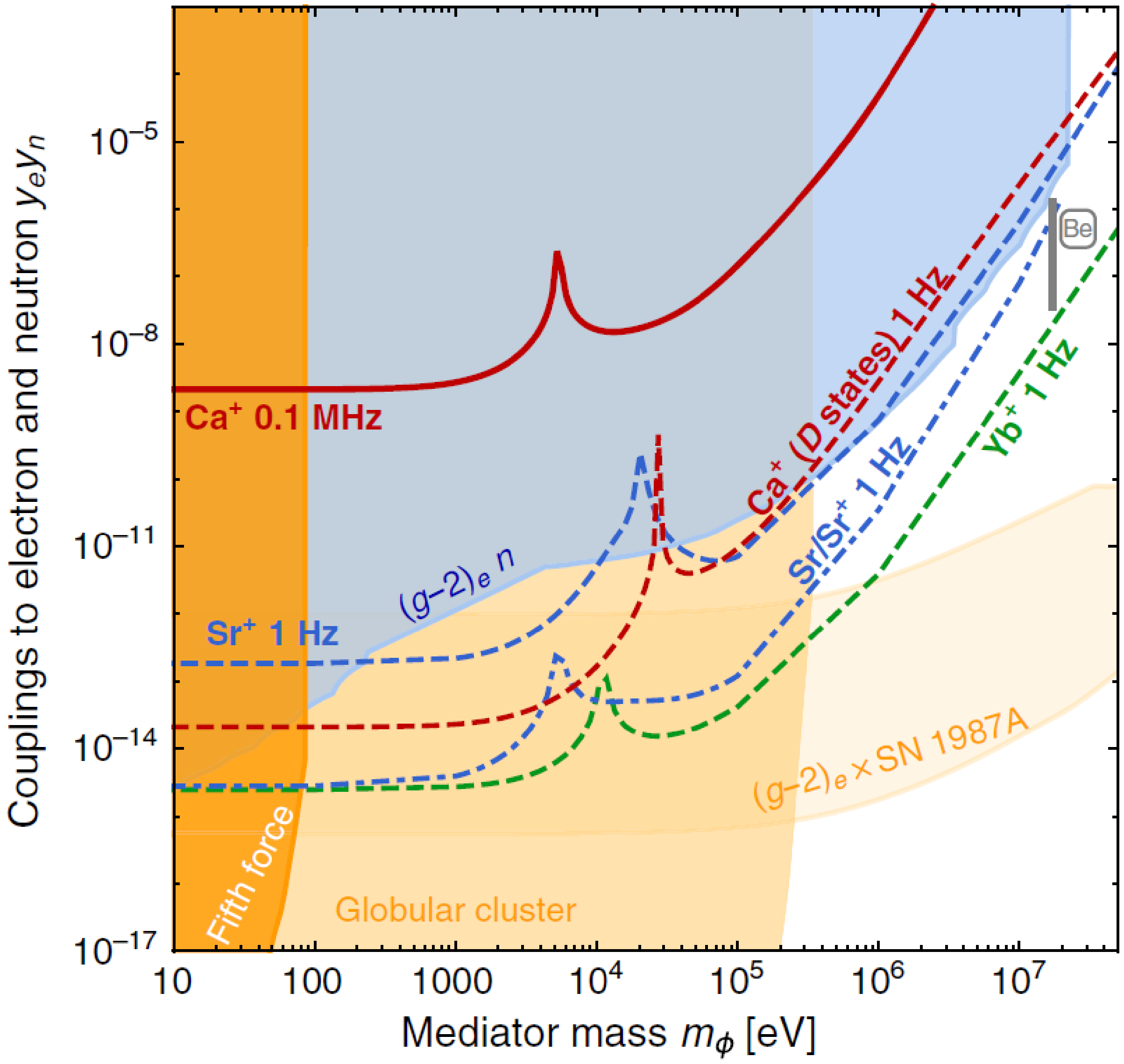
$$V_\varphi = -q_e q_n N \frac{e^{-kr}}{r} \qquad k = \frac{m_\varphi c}{\hbar}$$

Particle's coupling **to electron** **to neutron**

Energy shift:

$$\delta E_{NP} \approx -q_e q_n \Delta N \int_0^\infty \rho_\kappa\,(r)\,\frac{e^{-kr}}{r}\,r^2 dr$$

Electron wave function density



Berengut et al. (2018) *PRL*, 120(9), 091801.

Higher-order SM contributions

- FS affected by higher waves in the transition ($p_{3/2}$, $d_{3/2}$, $d_{5/2}$ …)
- Nuclear polarizability
- Many-body effects
- …

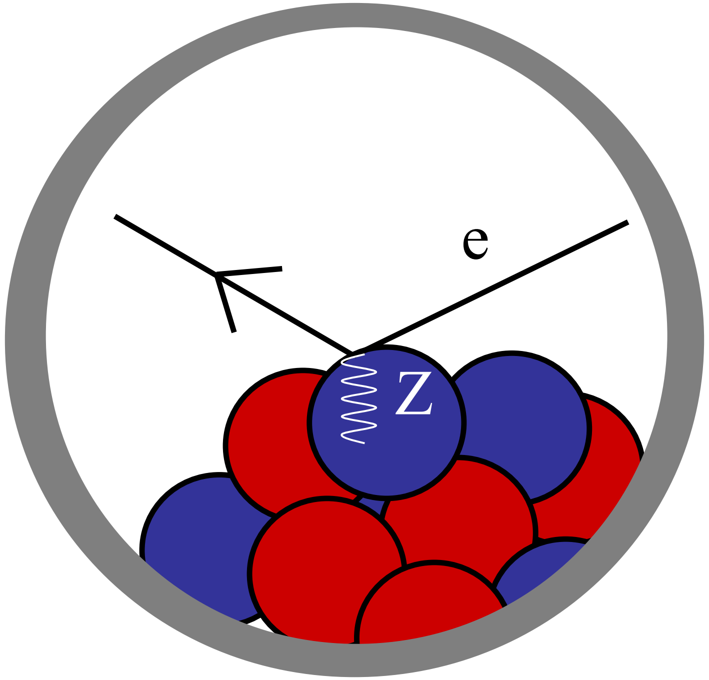
V. V. Flambaum, A. Geddes, A. V. Viatkina *Phys. Rev. A 97* 032510 (2018).

Estimates for the non-linearities

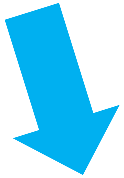
							Nonlinearity (Hz)		
Ion	Z	A	A_1	A_2	A_3	Pair of transitions	Higher waves	Polarizability	Many-body
Ca^+	20	40	42	44	48	$3p^64s\ ^2S_{1/2} \rightarrow 3p^63d\ ^2D_{3/2}$ $3p^64s\ ^2S_{1/2} \rightarrow 3p^63d\ ^2D_{5/2}$	3.0×10^{-4}	-6.6×10^{-2}	$\pm 2.7 \times 10^{-3}$
Sr^+	38	84	86	88	90	$4p^65s\ ^2S_{1/2} \rightarrow 4p^64d\ ^2D_{3/2}$ $4p^65s\ ^2S_{1/2} \rightarrow 4p^64d\ ^2D_{5/2}$	1.1×10^{-2}	-2.6	± 0.25
Ba^+	56	132	134	136	138	$5p^66s^1\ ^2S_{1/2} \rightarrow 5p^65d\ ^2D_{3/2}$ $5p^66s^1\ ^2S_{1/2} \rightarrow 5p^65d\ ^2D_{5/2}$	-3.9×10^{-2}	7.4	∓ 1.9
Yb^+	70	168	170	172	176	$4f^{14}6s\ ^2S_{1/2} \rightarrow 4f^{13}6s^2\ ^2F^o_{7/2}$ $4f^{14}6s\ ^2S_{1/2} \rightarrow 4f^{14}5d\ ^2D_{3/2}$	-3.1	39	± 12130
						$4f^{14}6s\ ^2S_{1/2} \rightarrow 4f^{14}5d\ ^2D_{3/2}$ $4f^{14}6s\ ^2S_{1/2} \rightarrow 4f^{14}5d\ ^2D_{5/2}$	3.1	-18	± 386
Hg^+	80	196	198	200	204	$5d^{10}6s\ ^2S_{1/2} \rightarrow 5d^96s^2\ ^2D_{3/2}$ $5d^{10}6s\ ^2S_{1/2} \rightarrow 5d^96s^2\ ^2D_{5/2}$	3.0	-13	± 2382

Atomic Parity Violation

Weak interaction between electrons and nucleus via Z boson



Z boson is very heavy ($M_Z = 91.2$ GeV)



Point-like interaction between electrons and nucleus

Violates parity!

$$\hat{h}_{\mathrm{PNC}} = \frac{G_f}{\sqrt{2}} \sum_B [C_{1n}\, \overline{e}\gamma_\mu\gamma_5 e\, \overline{n}\gamma_\mu n + C_{2n}\, \overline{e}\gamma_\mu e\, \overline{n}\gamma_\mu\gamma_5 n]$$

$\mathrm{e} =$ electrons

$n =$ nucleons: protons, neutrons

$$G_f \approx 1.17 \times 10^{-5}\ (\hbar c)^3 \mathrm{GeV}^{-2}$$

Weak interaction between electrons and nucleus via Z boson

$$\hat{h}_{\text{PNC}} = \frac{G_f}{\sqrt{2}} \sum_B [\underbrace{C_{1n}\, \overline{e}\gamma_\mu\gamma_5 e\, \overline{n}\gamma_\mu n}_{\substack{\text{spin-independent}\\ \text{larger}}} + \underbrace{C_{2n}\, \overline{e}\gamma_\mu e\, \overline{n}\gamma_\mu\gamma_5 n}_{\substack{\text{spin-dependent}\\ \text{smaller}}}]$$

Also Spin-Dependent & Parity Violating: anapole moment

At tree-level (Standard Model):

$$C_{1p} = \frac{1}{2}(1 - 4\sin^2\theta_W) \approx 0.04,$$

$$C_{1n} = -\frac{1}{2},$$

$$C_{2p} = -C_{2n} = \frac{1}{2}(1 - 4\sin^2\theta_W) g_A \approx 0.05\ .$$

$$\sin^2\theta_W \approx 0.23$$

$$g_A \approx 1.26$$

Spin-independent PNC interaction

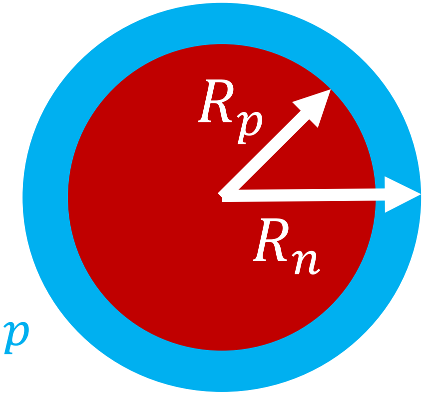
spin-independent part, if nucleons are considered non-relativistic :

$$\hat{h}_{SI} = \frac{G_f}{\sqrt{2}} \left[C_{1p} Z \underline{\rho_p(r)} + C_{1n} N \underline{\rho_n(r)} \right] \gamma_5$$

proton density

neutron density

Generally $\rho_p(r) \neq \rho_n(r)$



neutron skin

$R_n - R_p$

Matrix element with atomic wave functions:

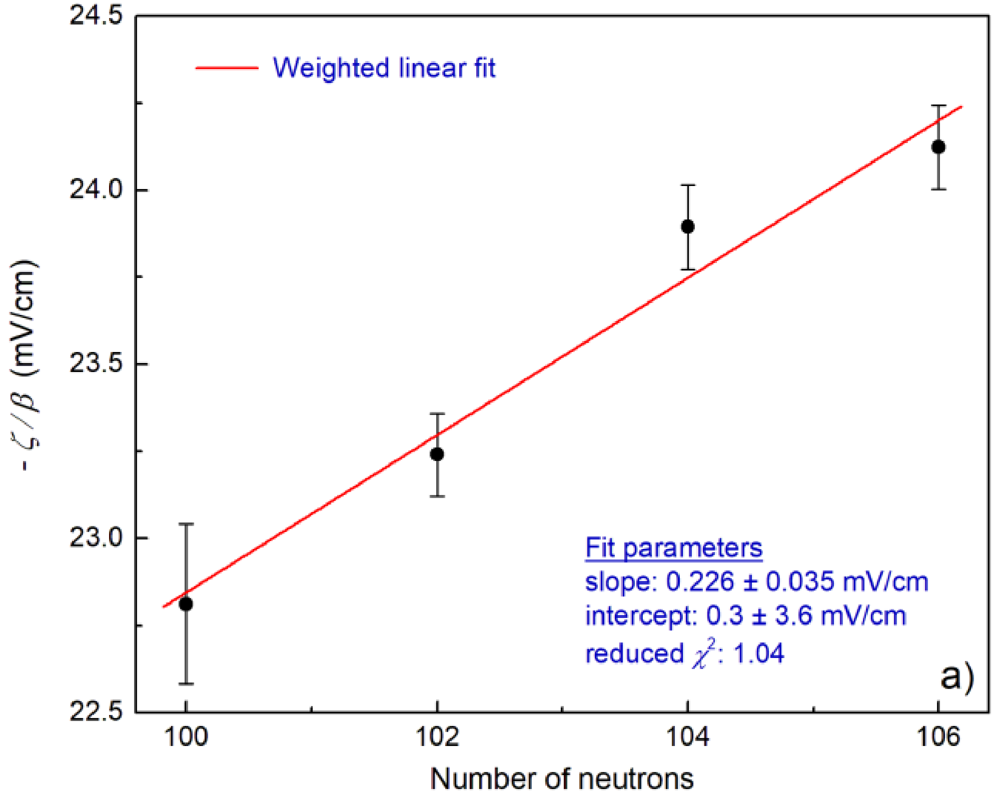
$$\mathcal{M} = \langle j | \hat{h}_{SI} | i \rangle$$

$s, p_{1/2}$ waves

$$= \frac{G_f}{2\sqrt{2}} \left[2C_{1p} Z \int \underline{\rho_p(r) \psi_j^\dagger \gamma_5 \psi_i d^3 r} + 2C_{1n} N \int \underline{\rho_n(r) \psi_j^\dagger \gamma_5 \psi_i d^3 r} \right]$$

Experiments

- [Wood, C. S. et al. *Science* 275, 1759 (1997)] in ^{133}Cs with 0.35% accuracy
- [Antypas, D. et al. *Nature Physics*, 15(2), 120 (2018)] in Yb isotopes with 0.5% accuracy



Atomic calculations

$$\mathcal{M} = \frac{G_f}{2\sqrt{2}}\mathcal{A}_{ps}R_p^{2\gamma-2}\left[Q_W q_p + \underline{N\eta(Z\alpha)^2 y} + \underline{Z\Delta Q_P + N\Delta Q_N}\right]$$

extra Z’ bosons, dark forces, …

Nuclear charge radius

neutron skin contrib.

$$y \approx 2\frac{\Delta R_{np}}{R_p}$$

Neutron skin parameter

$$q_p = \int \rho_p(r) f(r) d^3 r,$$

Q_w - nuclear weak charge (standard model).

$f(r)$ - spatial variation of electron WFs over nucleus

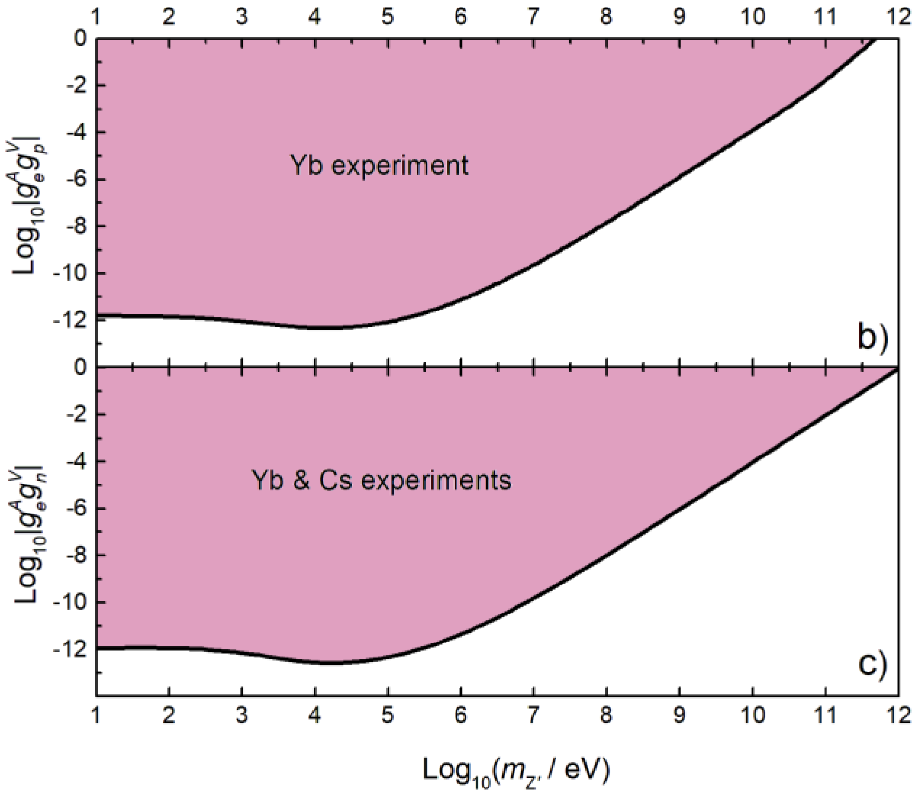
New physics in PNC interaction?

$$\mathcal{M} = \frac{G_f}{2\sqrt{2}} \mathcal{A}_{ps} R_p^{2\gamma-2} \left[Q_W q_p + N\eta(Z\alpha)^2 y + \underline{Z\Delta Q_P + N\Delta Q_N} \right]$$

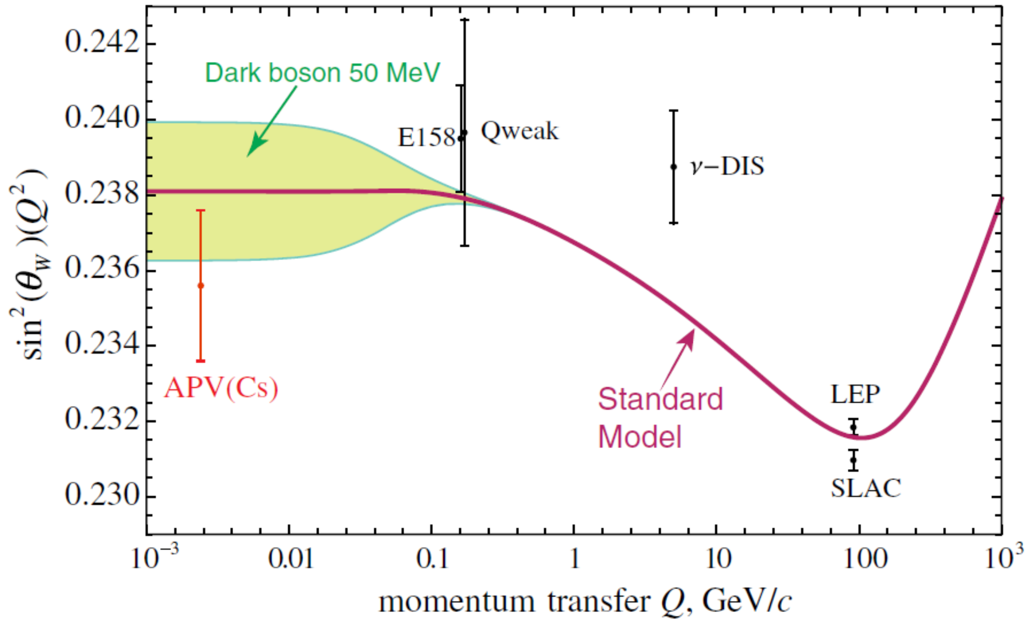
extra Z’ bosons, dark forces, …

Z’ between
e and proton

Z’ between
e and neutron



[D. Antypas et al. *Nature Physics*, 15(2), 120 (2018)]



[M. Safronova, D. Budker, D. DeMille, D. F. J. Kimball, A. Derevianko, and C. W. Clark,
Reviews of Modern Physics 90, 025008 (2018)]

Detect neutron skin?

$$y \approx 2\frac{\Delta R_{np}}{R_p}$$

$$\mathcal{M} = \frac{G_f}{2\sqrt{2}} \mathcal{A}_{ps} R_p^{2\gamma-2} \left[Q_W q_p + N\eta (Z\alpha)^2 y \right]$$

below 1%

Obstacles:

- Nuclear model uncertainty
- Nuclear charge radius uncertainty

Typically $y = 0.04\ - 0.09$

Larger with higher Z

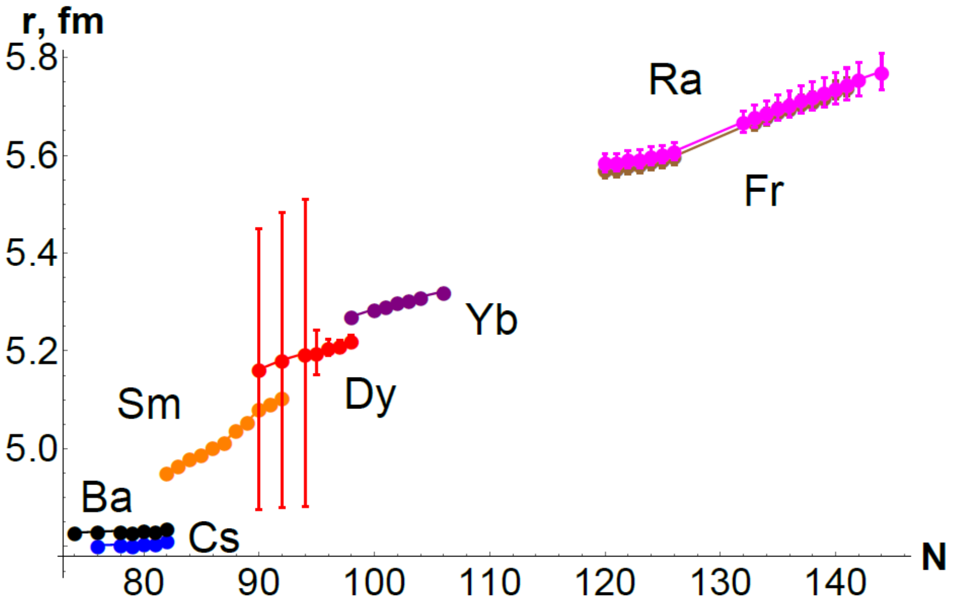
[B. A. Brown et al. Phys. Rev. C 79, 035501 (2009)]

If we knew nuclear part $R^{2\gamma-2}[\dots]\sim 0.1\%$ and $Q_W \sim 0.1\%$, then

$$\Delta y = 0.03\ - 0.05$$

Less with higher Z

[A. V. Viatkina, D. Antypas et al. Arxiv 1903.00123]



[I. Angeli and K. P. Marinova, Atomic Data and Nuclear Data Tables 99, 69 (2013)]

Isotopic ratio

$$\mathcal{M} = \frac{G_f}{2\sqrt{2}} \mathcal{A}_{ps} R_p^{2\gamma-2} \left[Q_W q_p + N\eta(Z\alpha)^2 y + Z\Delta Q_P + N\Delta Q_N \right]$$

Let us take a ratio with 2 isotopes.

In sharp-edge sphere approximation, with tree-level Q_W:

$$\frac{\mathcal{M}'}{\mathcal{M}} = \left(\frac{R_p'}{R_p}\right)^{2\gamma-2} \frac{N'}{N}\left[1+\frac{Z\Delta N}{NN'}\left(\xi+\frac{\Delta Q_P}{q_{p,0}}\right) - \frac{\eta_0}{q_{p,0}}(Z\alpha)^2(y'-y)\right]$$

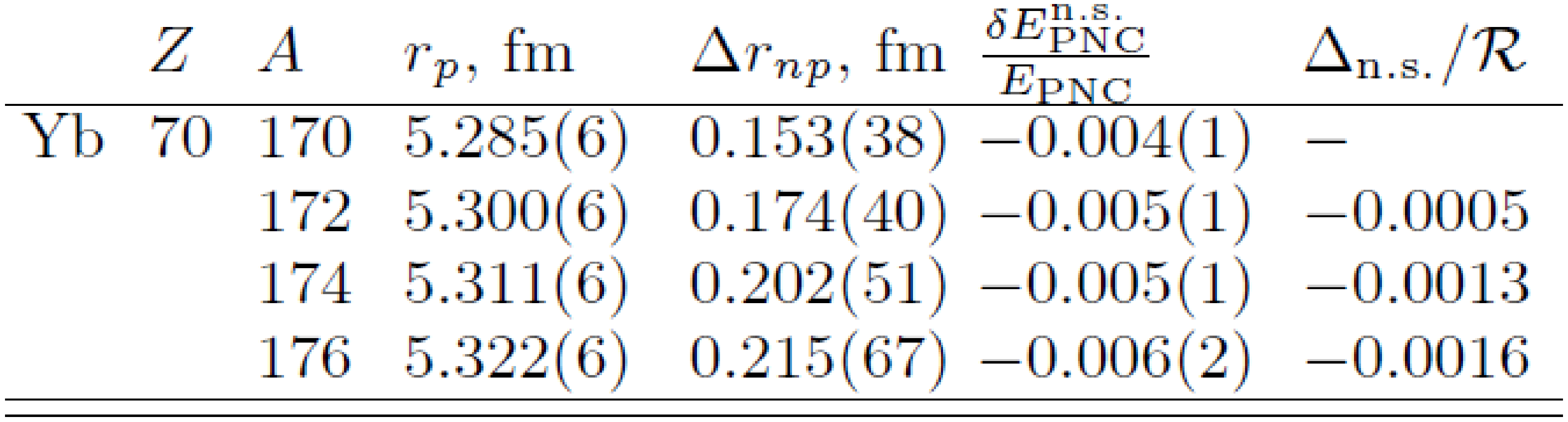
No atomic part!

Only new couplings of electrons and protons (=corrections to Weinberg angle).

$$\xi \equiv 1 - 4\sin^2\theta$$

Difference of neutron skins

Ytterbium



			Charge radius	Neutron Skin (n.s.)	n.s. effect in single isotope	n.s. effect in isotopic ratio
	Z	A	r_p, fm	Δr_{np}, fm	$\frac{\delta E^{\rm n.s.}_{\rm PNC}}{E_{\rm PNC}}$	$\Delta_{\rm n.s.}/\mathcal{R}$
Yb	70	170	5.285(6)	0.153(38)	−0.004(1)	–
		172	5.300(6)	0.174(40)	−0.005(1)	−0.0005
		174	5.311(6)	0.202(51)	−0.005(1)	−0.0013
		176	5.322(6)	0.215(67)	−0.006(2)	−0.0016

[A. V. Viatkina, D. Antypas et al. Arxiv 1903.00123]

In a nutshell…

- Neutron skin can be seen at 0.1% accuracy for isotopic ratios (in Yb and elsewhere).
- For constraining new physics: isotopic ratios in lighter atoms; proton new couplings are isolated in ratios and errors of y may cancel.

Thank you for attention!

Backup Slides

Spin-independent PNC interaction

spin-independent part, if nucleons are considered non-relativistic :

$$\hat{h}_{SI} = \frac{G_f}{\sqrt{2}} \left[C_{1p} Z \rho_p(r) + C_{1n} N \rho_n(r) \right] \gamma_5$$

proton density

neutron density

Usually, one takes $\rho_p(r) = \rho_n(r) = \rho(r)$, then:

$$\hat{h}_{SI} = \frac{G_f}{2\sqrt{2}} Q_W \rho(r) \gamma_5$$

$$\boxed{Q_W \equiv 2C_{1p} Z + 2C_{1n} N}$$

NUCLEAR WEAK CHARGE

$$\boxed{Q_W \equiv 2C_{1p}Z + 2C_{1n}N}$$

NUCLEAR WEAK CHARGE

Tree level :

$$Q_W = -N + Z(1 - 4\sin^2\theta_W) \approx -N + 0.08Z$$

With radiative corrections (accurate at the 0.1%):

$$Q_W = -0.9889N + 0.071Z$$

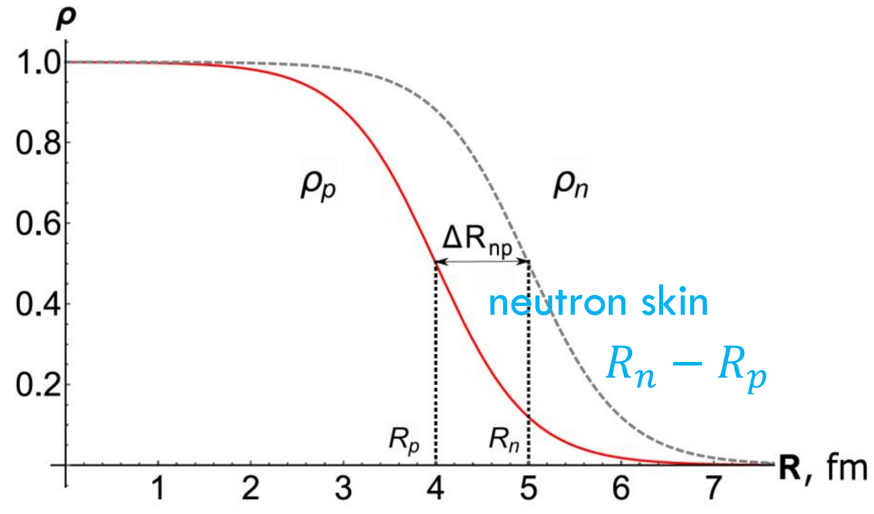
[Tanabashi, M. et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)]

Evaluation of integrals

Model:

- Nucleus has Fermi-like density

$$\rho_F(R, R_0) = \mathcal{B}\frac{1}{1+\exp\left(\frac{R-R_0}{z}\right)}$$



neutron skin

$R_n - R_p$

diffuseness $z \approx 0.5$ fm

$$\mathcal{M} = \frac{G_f}{2\sqrt{2}}\mathcal{A}_{ps}R_p^{2\gamma-2}\left[Zq_p(1-4\sin^2\theta_W) - Nq_n\right]$$

$$\gamma = \sqrt{1-(Z\alpha)^2}$$

$$q_p = \int \rho_p(r)f(r)d^3r,$$

$$q_n = \int \rho_n(r)f(r)d^3r.$$

$\psi_j^\dagger\gamma_5\psi_i$ → $f(r)$ - spatial variation of electron WFs over nucleus