Hyperfine Splitting in Muonium: Theory Status Report

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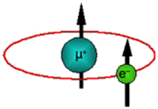
Outline

Energy Levels



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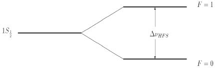

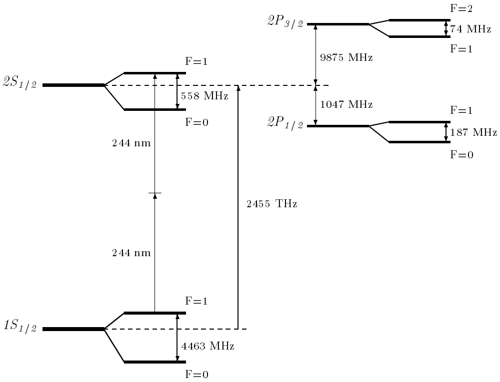

Experimental Results

- Mariam et al, 1982:
  - $\Delta\nu_{HFS}(Mu) = 4\ 463\ 302\ 88\ (16)$ Hz, $\quad \delta = 3.6 \cdot 10^{-8}$
  - $\frac{\mu_\mu}{\mu_p} = 3.183\ 346\ 1(11), \quad \delta = 3.6 \cdot 10^{-7}$
- Liu et al, 1999:
  - $\Delta\nu_{HFS}(Mu) = 4\ 463\ 302\ 765\ (53)$ Hz, $\quad \delta = 1.2 \times 10^{-8}$
  - $\frac{\mu_\mu}{\mu_p} = 3.183\ 345\ 24\ (37), \quad \delta = 1.2 \times 10^{-7}$
- $\frac{m_\mu}{m_e} = \left(\frac{\mu_e}{\mu_p}\right)\left(\frac{\mu_\mu}{\mu_p}\right)^{-1}\left(\frac{g_\mu}{g_e}\right)$
- **Combined result**
  - $\Delta\nu_{HFS}(Mu) = 4\ 463\ 302\ 776\ (51)$ Hz, $\quad \delta = 1.1 \times 10^{-8}$
  - $\frac{m_\mu}{m_e} = 206.768\ 277\ (24), \quad \delta = 1.2 \times 10^{-7}$
- **MuSEUM experiment at J-PARC. Goal: reduce the experimental uncertainties of $\Delta\nu_{HFS}$ and m_e/m_μ by about an order of magnitude (*Shimomura, 2018*)**

Can theory match the present and future experimental accuracy?

Intro to Theory

- Hyperfine interval: $\Delta\nu_{HFS} = \nu_F \left[1 + F\left(\alpha, Z\alpha, \frac{m_e}{m_\mu}\right)\right] + \Delta\nu_{Weak}$
- Fermi energy:

  $\nu_F = \frac{8}{3}(Z\alpha)^4 \frac{m_e}{m_\mu}\left(\frac{m_r}{m_e}\right)^3 \frac{m_e c^2}{h} = \frac{16}{3} Z^4 \alpha^2 \frac{m_e}{m_\mu}\left(\frac{m_r}{m_e}\right)^3 c\, R_\infty$
- HFS interval is linear in m_e/m_μ and R_∞, and quadratic in α
- Theoretical accuracy is determined by the intrinsic accuracy of the theoretical formula and accuracy of m_e/m_μ, R_∞ and α

Small parameters: $\alpha \sim 1/137$, $m_e/m_\mu \sim 1/207$, $Z\alpha$ $(Z = 1)$

- Nonrecoil Corrections
  1. **Binding (relativistic) corrections: expansion in $Z\alpha$**
  2. **Radiative (quantum electrodynamic) corrections: combined expansion in α/π and $Z\alpha$**
     The same physics corresponds to the corrections of different order in α/π at fixed order of $Z\alpha$
- Recoil Corrections
  1. **Recoil corrections: expansion in m_e/m_μ and $Z\alpha$**
  2. **Radiative-recoil corrections: expansion in m_e/m_μ, α, and $Z\alpha$**
- **Heavy particles loops (τ-lepton, strongly interacting particle closed loops)**
- **Weak interaction contributions (Z-boson exchange, radiative corrections)**

- *Nonrecoil corrections arise in external field approximation*
- *Recoil corrections are due to truly two-body effects*

What remains to be calculated for muonium HFS?

Largest Unknown Contributions

- **Nonlogarithmic recoil corrections of order** $(Z\alpha)^3(m/M)E_F$ **(uncertainty 27 Hz)**
- **Nonlogarithmic radiative-recoil corrections of order** $\alpha(Z\alpha)^2(m/M)E_F$ **(uncertainty 27 Hz)**
- **Radiative-recoil corrections of order** $\alpha^2(Z\alpha)(m_e/m_\mu)E_f$ **(estimate 10-15 Hz)** (*M.E., Shelyuto, work in progress*)
- Radiative corrections of order $\alpha^3(Z\alpha)E_F$ (estimate 3-5 Hz) (*M.E., Shelyuto, work in progress*)
- Nonlogarithmic radiative corrections of order $\alpha^2(Z\alpha)^2E_F$ (uncertainty 3 Hz)

Estimate of yet uncalculated terms: 70 Hz

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Estimate of yet uncalculated terms: 70 Hz

Theoretical prediction and its uncertainty

Theoretical formula

- $\Delta\nu_{HFS} = \nu_F \left[1 + F\left(\alpha, Z\alpha, \frac{m_e}{m_\mu}\right)\right] + \Delta\nu_{Weak} + \Delta\nu_{th}$
- $\nu_F = \frac{16}{3} Z^4 \alpha^2 \frac{m_e}{m_\mu} \left(\frac{m_r}{m_e}\right)^3 c\, R_\infty$

How well do we know relevant constants? Relative uncertainties

- Theoretical error $\Delta\nu_{th} \sim 70$ Hz, $\Delta\nu_{th}/\Delta\nu_{HFS} \sim 1.6 \times 10^{-8}$
- $\delta_\alpha = \Delta\alpha/\alpha = 2.4 \times 10^{-10}$
- $\delta_R = \Delta R_\infty / R_\infty = 5.9 \times 10^{-12}$
- **Experimental mass ratio supplies by far the largest contribution to the uncertainty:** $\frac{m_\mu}{m_e}_{ex} = 206.768\,277\,(24), \quad \delta = 1.2 \times 10^{-7}$

Theoretical prediction and its uncertainty

Theoretical prediction

$$\Delta\nu_{HFS}^{th}(Mu) = 4\ 463\ 302\ 872\ (511)\ (70)\ (2)\ \text{Hz}$$

- First uncertainty is due to the uncertainty of $(m_\mu/m_e)_{ex}$
- Second uncertainty is due to the uncalculated theoretical terms
- Third uncertainty is due to the uncertainty of α
- **Combine uncertainties:**
  $\boldsymbol{\Delta\nu_{HFS}^{th}(Mu) = 4\ 463\ 302\ 872\ (515)\ \text{Hz}, \delta = 1.2 \times 10^{-7}}$

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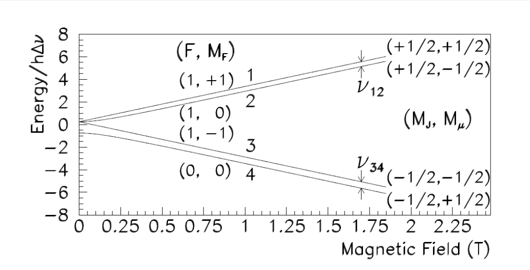
Surprise

- Theoretical number for HFS in CODATA 2014 adjustment eq.(216):
  $\boldsymbol{\Delta\nu^{th}_{HFS}(Mu) = 4\ 463\ 302\ 868\ (271)\ \text{Hz}, \delta = 6.1 \times 10^{-8}}$
- **The uncertainty due to $(m_\mu/m_e)_{ex}$ is 511 Hz, roughly two times larger than the total CODATA error!**
- **What happened?**

LAMPF experiments



- LAMPF (1999) measured two Zeeman frequencies ν_{12} and ν_{34}
- **Zeeman effect theory (Breit-Rabi formula):**

$$\nu_{12} = -\frac{\mu_\mu B}{h} + \frac{\Delta\nu}{2}\left[(1+x) - \sqrt{1+x^2}\right]$$

$$\nu_{34} = \frac{\mu_\mu B}{h} + \frac{\Delta\nu}{2}\left[(1-x) + \sqrt{1+x^2}\right]$$

  - $x = (\mu_\mu - \mu_e)B/(h\Delta\nu)$, magnetic field B from $h\nu_p = 2\mu_p B$
  - $\Delta\nu$ – HFS at zero field and μ_μ/μ_p - **unknown parameters in the Breit-Rabi formula**

LAMPF experiments

Experimental HFS and m_e/m_μ

- Find unknown parameters in Breit-Rabi formula
  $$\Delta\nu = \nu_{12} + \nu_{34}$$
  $$\frac{\mu_\mu}{\mu_p} = \frac{4\nu_{12}\nu_{34} + \nu_p \frac{\mu_e}{\mu_p}(\nu_{34} - \nu_{12})}{\nu_p \left[\nu_p \frac{\mu_e}{\mu_p} - (\nu_{34} - \nu_{12})\right]}$$
- $\Delta\nu$ is the experimental HFS
  $$\Delta\nu^{ex}_{HFS}(Mu) = 4\ 463\ 302\ 776\ (51)\ \mathrm{Hz}, \quad \delta = 1.1 \times 10^{-8}$$
- μ_μ/μ_p together with high accuracy experimental μ_e/μ_p determines experimental m_e/m_μ
  $$\left(\frac{m_\mu}{m_e}\right)_{ex} = 206.768\ 277\ (24), \quad \delta = 1.2 \times 10^{-7}$$
- Theory plus $m_\mu/m_{e\,ex}$ leads to
  $$\Delta\nu^{th}_{HFS}(Mu) = 4\ 463\ 302\ 868\ (271)\ \mathrm{Hz}, \delta = 6.1 \times 10^{-8}$$
- **What about CODATA value with two times lower error bars?**

CODATA theoretical error bars

- Rewrite solution of Breit-Rabi formula as
  $\boldsymbol{\Delta\nu_{HFS}(Mu) = \nu_{12} + \nu_{34}}$, $\boldsymbol{\frac{\mu_\mu}{\mu_p} = \frac{\Delta\nu^2 - \nu^2(f_p) + 2s_e f_p \nu(f_p)}{4s_e f_p^2 - 2f_p\nu(f_p)} \frac{g_\mu}{g_\mu(Mu)}}$
- $\nu = \nu_{34} - \nu_{12}$, $\boldsymbol{\Delta\nu = \nu_{34} + \nu_{12}}$, f_p - proton NMR frequency, $s_e = \frac{\mu_e}{\mu_p}\frac{g_e(Mu)}{g_e}$
- **CODATA: one cannot use μ_μ/μ_p (and respective m_e/m_μ) to calculate theoretical HFS because then $\Delta\nu^{th}$ is calculated through $\Delta\nu^{ex}$!**
- **Wrong! Only experimental frequencies ν_{12} and ν_{34} are used!**

How two times lower error bars are obtained by CODATA

- **CODATA 1st step: plug theoretical QED formula for HFS in Breit-Rabi solution above instead of $\Delta\nu$**
- It turns into equation for m_e/m_μ
- **Non CODATA approach: solve equation, calculate respective m_e/m_μ and compare with experiment**
- **This is a test of QED HFS theory**

CODATA theoretical error bars

- After substitution of the QED theoretical formula equation for ratio of magnetic moments has the form
  $\frac{m_e}{m_\mu} = f\left(\frac{m_e}{m_\mu}\right)$
- **Function $f(m_e/m_\mu)$ is quadratic in m_e/m_μ**
- **CODATA 2nd step: obtain value of m_e/m_μ with two times lower uncertainty and plug into theoretical QED HFS formula**
- **Two many reasons why this wrong: QED theoretical formula was considered to be exact, one obtains an entry in QED formula using this formula and then plugs this parameter in this very formula**
- **This is a closed circle. One cannot use QED formula twice: to obtain a value of a parameter and then plug this parameter in the formula to check it!**
- **"Theoretical prediction" obtained in this was not only does not have two times lower uncertainty, but has an uncontrollable uncertainty!**

What can we learn from experiment and theory?

- Experimental value of m_μ/m_e is the worst known constant in the theoretical formula for $\Delta\nu_{HFS}^{th}$
- Experimental value of $\Delta\nu_{HFS}^{exp}$ is an order of magnitude more accurate than $(m_\mu/m_e)_{ex}$
- Use experiment and theory to obtain a more accurate value of the mass ratio
  $\frac{m_\mu}{m_e} = \mathbf{206.768\ 281\ (2)(3)}$
- First uncertainty from uncertainty of $\Delta\nu_{HFS}^{ex}$
- Second uncertainty from uncalculated terms in $\Delta\nu_{HFS}^{th}$
- We combine uncertainties
  $\frac{m_\mu}{m_e} = \mathbf{206.768\ 281\ (4)}, \qquad \delta = 2\times 10^{-8}$
- **An order of magnitude more accurate than current $(m_\mu/m_e)_{ex}$**
- **Hyperfine splitting in muonium is the best source for a precise value of the electron-muon mass ratio**
- **Improving accuracy of $\Delta\nu_{HFS}^{ex}$ and theory by an order of magnitude reduces uncertainty of m_μ/m_e by an order of magnitude to 2×10^{-9}!**

Theory versus experiment for HFS

Can we use comparison between theory and experiment for $\Delta\nu_{HFS}(Mu)$ to discover weak interactions contribution to atomic level shift, look for new physics, etc.?

- Theoretical prediction for HFS
  $\boldsymbol{\Delta\nu^{th}_{HFS}(Mu) = 4\ 463\ 302\ 872\ (511)\ (70)\ (2)\ \text{Hz}}$
- First uncertainty from uncertainty of $(m_\mu/m_e)_{ex}$, second from uncalculated theoretical terms, third from uncertainty of α
- **The dominant contribution is due to experimental accuracy of $(m_\mu/m_e)_{ex}$!**
- The second largest uncertainty is due to the theory of HFS splitting
- Combine uncertainties
  $\boldsymbol{\Delta\nu^{th}_{HFS}(Mu) = 4\ 463\ 302\ 872\ (515)\ \text{Hz}, \quad \delta = 1.2 \times 10^{-7}}$
- Compare
  $\boldsymbol{\Delta\nu^{ex}_{HFS}(Mu) = 4\ 463\ 302\ 776\ (51)\ \text{Hz}, \quad \delta = 1.1 \times 10^{-8}}$
- **Theory and experiment are compatible but theoretical error bars are too large due to $(m_\mu/m_e)_{ex}$**

The Road Ahead

- Improve accuracy of $(m_\mu/m_e)_{ex}$ by an order of magnitude $\delta = 1.16 \times 10^{-7} \to 1.16 \times 10^{-8}$
- Then $(m_\mu/m_e)_{ex}$ contribution to the uncertainty of $\Delta\nu^{th}_{HFS}(Mu)$ is 52 Hz, total uncertainty of $\Delta\nu^{th}_{HFS}(Mu)$ reduces to 87 Hz, $\delta = 2 \times 10^{-8}$
- Recall $\Delta\nu^{ex}_{HFS}(Mu) = 4\ 463\ 302\ 776\ (51)$ **Hz**, $\delta = 1.1 \times 10^{-8}$
- Comparison between theory and experiment becomes much more critical!
- **Weak interaction level shift (-65 Hz) is comparable to the new experimental accuracy**
- **An unexpected contribution to HFS larger than 100 Hz can be detected**
- Improve theoretical accuracy of HFS interval by an order of magnitude, uncertainty 70 Hz $\Longrightarrow$ 0.7 Hz and total uncertainty of $\Delta\nu^{th}_{HFS}(Mu)$ will be completely determined by experimental accuracy of m_μ/m_e

The Road Ahead

Theoretical dreams

- Improve accuracy of $(m_\mu/m_e)_{ex}$ by two orders of magnitude $\delta = 1.16 \times 10^{-7} \to 1.16 \times 10^{-9}$ or $\Delta(m_\mu/m_e) = 2.4 \times 10^{-5} \to 2.4 \times 10^{-7}$
- Then $(m_\mu/m_e)_{ex}$ contribution to the uncertainty of $\boldsymbol{\Delta\nu^{th}_{HFS}(Mu)}$ is $5.2Hz$
- **Improve theoretical accuracy of HFS interval by an order of magnitude, uncertainty** 70 **Hz** $\Longrightarrow 0.7$ **Hz**
- **Then total uncertainty of $\boldsymbol{\Delta\nu^{th}_{HFS}(Mu)}$ reduces to 6 Hz, or** $\delta = 1.3 \times 10^{-9}$
- **Discovery of weak interaction contribution to HFS splitting is guaranteed**
- **New physics (if it exists) can be discovered**

Practical goals

- **Experiment: improvement of experimental accuracy of $\Delta\nu_{HFS}(Mu)$ and m_μ/m_e by an order of magnitude**
- **Theory: calculation of all corrections of order 1-10 Hz**

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Thank You!