The muon g-2 and the MUonE project

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Balaton Limnological Research Institute of the Hungarian Academy of Sciences
status of $a_\mu = (g - 2)/2$

- E821@BNL measurement with an error of 0.54 ppm
  
  $a_\mu^{\text{exp}} = 116592089(63) \times 10^{-11}$

  G.W. Bennet et al. (Muon (g-2)), Phys. Rev. D73 (2006) 072003

- Experimental prospects $\implies$ see talk by T. Mibe

- a couple of theoretical predictions
  
  $a_\mu^{\text{SM}} = 116591783(35) \times 10^{-11}$

  F. Jegerlehner, MITP Workshop, 19-23 February 2018, Mainz

  $a_\mu^{\text{SM}} = 116591820(36) \times 10^{-11}$

  Keshavarzi, Nomura, Teubner, arXiv:1802.02995

- $\Delta(\text{Th} - \text{Exp}) = -306 \pm 72 \sim 4\sigma$ deviation

Three possible scenarios

- New Physics?
- systematics of the measurement?
- systematics of the theoretical prediction?

Let’s see what are the ingredient to make $a_\mu \neq 0 \implies$
\[ a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{HLO}} + a_{\mu}^{\text{HHO}} \]

- **QED perturbative corrections known up to 4 loops plus 5 loops partial calculation:**
  \[ a_{\mu}^{\text{QED}} = 116584718.86(30) \times 10^{-11} \sim 99.99\% \text{ of the total} \]
  
  T. Aoyama, M. Hayakawa, T. Kinoshita; S. Laporta, E. Remiddi; M. Passera

- **two loop electroweak radiative corrections:**
  \[ a_{\mu}^{\text{EW}} = 153.6(1.1) \times 10^{-11} \]
  
  Gnendiger, Stöckinger, Stöckinger-Kim

- **\( a_{\mu}^{\text{HLO}} \) = 6894.6(32.5) \times 10^{-11} \implies \text{largest source of uncertainty}**
  
  F. Jegerlehner, MITP Workshop, 19-23 February 2018, Mainz

- **Hadronic light-by-light:**
  \[ a_{\mu}^{\text{LxL}} = 103.4(28.8) \times 10^{-11} \]
  
  F. Jegerlehner, MITP Workshop, 19-23 February 2018, Mainz

- **Hadronic HO vacuum polarization:**
  \[ a_{\mu}^{\text{HHO}} = -87.0(0.6) \times 10^{-11} \]
• perturbation theory (PT) reliable for leptons and top—quark
• PT not reliable for light quark
⇒ hadronic contribution from LQCD
⇒ via optical theorem, hadronic contribution from dispersion relation involving the total hadronic cross section measured experimentally at $e^+e^-$ machines:

$$a_{\mu}^{\text{HLO}} = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\mu^2}^{\infty} ds \frac{K(s)R(s)}{s^2}$$

$$= \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \left( \int_{4m_\mu^2}^{E_{\text{cut}}} ds \frac{K(s)R^{\text{data}}(s)}{s^2} + \int_{E_{\text{cut}}^2}^{\infty} ds \frac{K(s)R^{\text{PQCD}}(s)}{s^2} \right)$$

$$R(s) = \frac{\sigma^0(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\frac{4}{3} \pi \alpha^2 s}$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m^2}} \sim \frac{1}{s}$$
• due to the $\frac{1}{s^2}$ in the dispersion relation,
Recent results:
- $G \pi^+\pi^-$ from BES-III, CMD-3 and CLEOc
- $G \pi^+\pi^-\pi^0$ from Belle
- $G K^+K^-$ from CMD-3 and SND
- $G \omega\pi^0 \rightarrow \pi^0\pi^0\gamma$ from SND
- $G K^S K^+\pi^0$, $K^S K^\pm\pi^\mp\eta$, $\pi^+\pi^-\pi^0\pi^0$, $K^S K^L\pi^0$, $K^S K^L\eta$, $K^S K^L\pi^0\pi^0$ from BaBar

Energy range:

<table>
<thead>
<tr>
<th>Energy range</th>
<th>$\rho,\omega$ (E &lt; 1 GeV)</th>
<th>$\rho,\omega$ (1 GeV &lt; E &lt; 2 GeV)</th>
<th>$\rho,\omega$ (2 GeV &lt; E &lt; $\infty$ incl pQCD)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>540.98 [78.6] (2.80)</td>
<td>0.5 % 50.7 %</td>
<td>692.5 ± 2.67 715.4 ± 18.72 711 ± 19</td>
<td>51.09 [7.4] (1.10)</td>
</tr>
</tbody>
</table>

Summary of recent LQCD results for the leading order $a_{HVP}$, in units $10^{-10}$.

Labels: $I$ marks $u$, $d$, $s$, $c$, $L$ $u$, $d$, $s$ and $S$ $u$, $d$ contributions. Individual flavor contributions from light ($u$, $d$) amount to about 90%, strange about 8% and charm about 2%.

- Integral over time-like data extremely delicate due to combination of many ($\sim 40$) exclusive channels

- several LQCD groups made recent progress, but not yet competitive in precision

see talk by S. Eidelman
4. Comparison and Discussion

This section is devoted to show aLO-HVP \( g \) − 2 contributions are included in HPQCD-17 (due to some methodological reason) while not in the results from the others.

In Fig. 7, we compare the LQCD and the dispersive method. The combined results using the LQCD and the dispersive method are also discussed. The tension comes out, we compare the experimental measurement of \( g \) − 2 and MUonE with respect to the requirements from FNAL-E989 and J-PARC-E34 experiments and consistent among all LQCD groups. The tension is on the light connected contribution \( a \( \mu \) \)LO-HVP, as shown in the upper-left panel. In turn, the LQCD published results are not fully consistent to each other. To see how the tension comes out, we compare the results, BMW-18, RBC/UKQCD-18, and HPQCD-17 with the dispersive estimates where the latter uses Eq. (19). The dispersionful theoretical treatment of the long distance behavior in \( a \( \mu \) \)LO-HVP would have to be improved more. Both statistical and IR-cuts in the correlators \( f \)C from a scale setting, lattice data cuttings, fit model dependences in the extrapolations/interpolations, are already determined with high enough precision with statistical errors.

The dispersion-full theoretical treatment of the long distance behavior in \( a \( \mu \) \)LO-HVP tends to become smaller when the higher excitation modes are taken account in the improved multi-exponential fits for the light quark connected correlator \( \omega \)C. The combined results using the LQCD and the dispersive method are also discussed. The tension comes out, we compare the experimental measurement and the dispersive SM predictions. Recently, HPQCD-17 is updated to FHM collaboration has updated their ensemble results, BMW-18, RBC/UKQCD-18 estimates. All (updated) results are consistent well to each other and no new physics (green band in the figure): the value that would have to be compared takes account of the extrapolations to the continuum limit and the physical mass point, and/or UV-cuts in the HVP \( \hat{\omega} \)C. Both statistical and IR-cuts in the correlators \( f \)C from a scale setting, lattice data cuttings, fit model dependences in the extrapolations/interpolations, are already determined with high enough precision with statistical errors.

In contrast, HPQCD-17, KNT-18, and ETM-18 estimates have observed a smaller \( a \( \mu \) \)LO-HVP, disc, disc, disc. The tension is on the light connected contribution \( a \( \mu \) \)LO-HVP, and/or UV-cuts in the HVP \( \hat{\omega} \)C. Both statistical and IR-cuts in the correlators \( f \)C from a scale setting, lattice data cuttings, fit model dependences in the extrapolations/interpolations, are already determined with high enough precision with statistical errors.

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The MUonE project


space-like evaluation of $a^{\text{HLO}}_{\mu}$

\[
a^{\text{HLO}}_{\mu} = \left( \frac{\alpha m_{\mu}}{3\pi} \right)^2 \int_{4m^2_{\pi}}^{\infty} ds \frac{K(s) R(s)}{s^2} = \frac{\alpha}{\pi} \int_0^1 dx (1 - x) \Delta \alpha_{\text{had}}(t(x))
\]

where

\[
a^{\text{HLO}}_{\mu} = -\frac{\alpha}{\pi} \int_{-\infty}^0 dt \left( \frac{1 - \beta}{1 + \beta} \right)^2 \Delta \alpha_{\text{had}}(t)
\]

\[
t(x) = \frac{x^2 m^2_{\mu}}{x - 1} \quad \beta(t) = \sqrt{1 - \frac{4m^2_{\mu}}{t}} \quad x(t) = \frac{t (1 - \beta(t))}{2m^2_{\mu}} \quad t = \begin{cases} 0^- & \text{for } x \to 0^+ \\ -\infty & \text{for } x \to 1^- \end{cases}
\]

\[\Delta \alpha_{\text{had}}(t) \text{ is the hadronic contribution to the running of } \alpha_{\text{QED}}(q^2) = \frac{\alpha}{1 - \Delta \alpha(q^2)}\]

★ $a^{\text{HLO}}_{\mu}$ can be obtained by measuring the running of $\alpha_{\text{QED}}$ in a space-like process

★ $\Delta \alpha_{\text{had}}(t)$ in the integrand is evaluated in the space-like region (negative transfer momenta) where it is a smooth function

★ Roughly, to be competitive with current time-like evaluations, $\Delta \alpha_{\text{had}}(t)$ needs to be known at some % level
General considerations

- $\Delta \alpha_{\text{had}}(t(x))$ (red) as a function of $x$
- $\Delta \alpha_{\text{lep}}(t(x))$ (blue) as a function of $x$

\[
(1 - x) \cdot \Delta \alpha_{\text{had}}(t(x)) \times 10^5
\]

\[
\Delta \alpha_{\text{had}}(t(x))
\]

• integrand function \((1 - x) \Delta \alpha_{\text{had}}(t(x))\)

$$x_{\text{peak}} \simeq 0.914$$

$$t_{\text{peak}} \simeq -0.108 \text{ GeV}^2$$
\(\mu e \rightarrow \mu e\) elastic scattering in a fixed target experiment

\[A \sim 150 \text{ GeV high-intensity} \ (\sim 1.3 \times 10^7 \mu\text{'s/s}) \mu\text{on beam available at CERN North Area}\]

Muon scattering on a low-\(Z\) target \((\mu e \rightarrow \mu e)\) looks an ideal process

\(\star\) it is a pure \(t\)-channel process \(\rightarrow\)

\[
\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt} \left| \frac{\alpha(t)}{\alpha} \right|^2
\]

\(\star\) Assuming a 150 GeV incident \(\mu\) beam we have

\[s \simeq 0.164 \text{ GeV}^2 - 0.143 \lesssim t < 0 \text{ GeV}^2 \quad 0 < x \lesssim 0.93 \quad \text{it spans the peak!}\]

\(\star\) the region \(0.9 \leq x < 1\) can be covered with LQCD + PQCD

M. Marinkovic, MITP Workshop, 19-23 February 2018, Mainz
• where is the challenge?
Our signal \( \equiv \frac{dN_{\text{data}}(O_i)}{dN_{\text{MC}}(O_i)|_{\Delta \alpha_{\text{had}}(t)=0}} \equiv \frac{dN_{\text{data}}(O_i)}{dN_{\text{MC}}^*(O_i)} = \frac{d\sigma_{\text{data}}(O_i)}{d\sigma_{\text{MC}}^*(O_i)} = \frac{N_{\text{data}}^\text{norm}}{N_{\text{data}}} \times \frac{\sigma_{\text{MC}}^\text{norm}}{d\sigma_{\text{MC}}^*(O_i)} \sim \)

\( \sim 1 + 2 [\Delta \alpha_{\text{lep}}(O_i) + \Delta \alpha_{\text{had}}(O_i)] \) (at LO)

\[ \delta(\Delta \alpha_{\text{had}}(t)) \sim 0.3\% \implies \sigma_{\text{MC}}^\text{norm} / d\sigma_{\text{MC}}^* \sim 10^{-5} \]
statistics and (main) systematic uncertainties

- **statistics**: CERN muon beam M2 \( (E = 150 \text{ GeV}) \), \( 1.3 \cdot 10^7 \mu/s \) with a target of Be layers (total thickness 60 cm) \( \Rightarrow L \sim 1.5 \cdot 10^7 \text{nb}^{-1} \Rightarrow \) statistical sensitivity \( \sim 0.3\% \) on \( a_{\mu}^{HLO} (\sim 20 \cdot 10^{-11}) \)

### Sistematics

- **theoretical**: higher order radiative corrections modify the shapes
  - order of magnitude estimate, barring infrared logs and setting \( c_{i,j} \sim 10 \)
  - \( c_{1,1} \left( \frac{\alpha}{\pi} \right) L \sim 0.2 \quad c_{1,0} \left( \frac{\alpha}{\pi} \right) \sim 2.5 \cdot 10^{-2} \)
  - \( c_{2,2} \left( \frac{\alpha}{\pi} \right)^2 L^2 \sim 5 \cdot 10^{-3} \quad c_{2,1} \left( \frac{\alpha}{\pi} \right)^2 L \sim 5 \cdot 10^{-4} \quad c_{2,0} \left( \frac{\alpha}{\pi} \right)^2 \sim 5 \cdot 10^{-5} \)
  - \( c_{3,3} \left( \frac{\alpha}{\pi} \right)^3 L^3 \sim 1.5 \cdot 10^{-4} \quad c_{3,1} \left( \frac{\alpha}{\pi} \right)^3 L^2 \sim 1.5 \cdot 10^{-5} \quad c_{3,0} \left( \frac{\alpha}{\pi} \right)^3 L \sim 1.5 \cdot 10^{-6} \)
  - the most advanced technologies for NNLO calculations and higher order resummation are needed

- **(main) experimental sources**
  - **multiple scattering**: \( E_e \) in normalization region much lower than in signal region
    Effect \( \sim 1/E \Rightarrow \) it affects signal and normalization in different way
  - absolute \( \mu \) beam energy scale, 5 MeV \( \Rightarrow 10^{-5} \) effect
  - electron pair production
  - bremsstrahlung
Theoretical status

- analytical expression for tree level

\[
\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_{\mu}^2, m_e^2)} \left[ \frac{(s - m_{\mu}^2 - m_e^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right]
\]

- VP gauge invariant subset of NLO rad. corr.
- factorized over tree-level: \( \alpha \rightarrow \alpha(t) \)

- NLO virtual diagrams


- and corresponding real emission diagrams

- NLO matrix elements calculated with finite \( m_{\mu} \) and \( m_e \) mass effects and a Monte Carlo program has been developed and taylored to the fixed target kinematics

Alacevich, Carloni Calame, Chiesa, Montagna, Nicosini, Piccinini, arXiv:1811.06743
$\theta_e - \theta_\mu$ correlation (in the lab. frame)

F. Piccinini (INFN, Pavia)
• tree-level $Z$-exchange important at the $10^{-5}$ level
• purely weak RCs (in QED NLO units) at a few $10^{-6}$ level
towards NNLO amplitudes

\begin{itemize}
  \item same diagrams needed for NNLO QCD $t\bar{t}$ production at the LHC
\end{itemize}

Mastrolia, Passera, Primo, Schubert, arXiv:1709.07435
Di Vita, Laporta, Mastrolia, Primo, Schubert, arXiv:1806.08241
towards NNLO amplitudes

- relevant on the target precision scale
- even larger contributions from lepton (mainly electron) loops
  - expected larger cancellation with real part (lepton pair emission)

Fael, Passera, arXiv:1901.03106

work in progress
On the experimental side

- a modular apparatus has been proposed

\[ \Delta \alpha_{\text{had}}(t) \] can also be measured via the elastic scattering \( \mu e \rightarrow \mu e \).

We propose to scatter a 150 GeV muon beam, available at CERN’s North Area, on a fixed electron target (Beryllium). Modular apparatus: each module has one layer of Beryllium (target) followed by several thin Silicon strip detectors.


\[ \mu \quad \text{Si} \quad \text{Si} \quad \text{Si} \quad \mu \quad e \quad \text{Be} \quad \text{Si} \quad \text{Si} \quad \text{Si} \quad \text{Be} \quad \text{ECAL} \]
First Test Beam in 2017 to study multiple scattering

- 27 September - 3 October 2017, CERN, H8 Beam Line
- adapted UA9 apparatus

<table>
<thead>
<tr>
<th>Beam</th>
<th>Target Type</th>
<th>N events×10^6</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 GeV e^-</td>
<td>8 mm C</td>
<td>15</td>
</tr>
<tr>
<td>20 GeV e^-</td>
<td>8 mm C</td>
<td>12</td>
</tr>
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G. Abbiendi et al., arXiv:1905.11677

- data well described by GEANT in the core region
- some disagreement on the tails but too low statistics
- second testbeam in 2018, analysis in progress
• International Collaboration has been setup

• Letter of Intent just submitted to CERN SPSC

• schedule
  • three week Pilot run towards the end of 2021
  • detector assembled during 2022
  • run during LHC Run 3
Summary

- $(g - 2)_\mu$ discrepancy between E821 result and SM predictions reached the 4σ level

- HLO vacuum polarization contribution is the dominant source of theoretical uncertainty

- Different methods required to allow independent cross-checks
  
  - Time-like dispersive approach: the most precise up to now
  
  - LQCD calculations: not yet competitive but improving
  
  - Space-like dispersive approach and MUonE experiment proposal: promising, provided theoretical and experimental systematics are kept under control at the level of $10^{-5}$

  - Synergic collaboration between theorists and experimentalists