Present Status of Muonium HFS measurement in J-PARC

K. Shimomura (KEK) on behalf of the MuSEUM Collaboration
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(Muonium Spectroscopy Experiment Using Microwave)

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Muonium WS Osaka
What is Muonium?

**Muon:**
- Elementary particle (lepton)
- 200 times heavier than an electron
- Lifetime of 2.2 microseconds.

**Muonium:**
- Bound state of a positive muon and an electron.
- Hydrogen-like atom free from the finite size of the nucleon.
- Most suitable for validation of bound state quantum electrodynamics (QED).
- Theoretical and experimental precision of the hyperfine structure comparable.
Mu Energy Diagram

From K. Jungmann

$2^2S_{1/2}$

- $F=1$
- $F=0$

$2^2S_{1/2}$

$2^2P_{3/2}$

- $F=2$
- $F=1$

$2^2P_{1/2}$

- $F=1$
- $F=0$

$\lambda = 244 \text{ nm}$

$\Delta \nu (\text{Fermi}) = \frac{16}{3} \alpha^2 c R_\infty \frac{m_e}{m_\mu} \left[ 1 + \frac{m_e}{m_\mu} \right]^{-3}$.

Fine Structure explained by Dirac

Classical Lamb Shift
QED
Tomonaga, Shwinger, Feynman

Not included QED weak hadronic correction
Zeeman Splitting

\[ \mathcal{H} = h\Delta \nu I_\mu \cdot J - \mu_B^\mu g_\mu I_\mu \cdot \mathbf{H} + \mu_B^e g_J J \cdot \mathbf{H} \]

\( \Delta \nu_{\text{HFS}}: \text{Mu Hyperfine Structure} \)

\( \Delta \nu_{\text{HFS}} \approx 4463 \text{ MHz} \)

Pure lepton = point particle

Breit-Rabi diagram

\( \nu_{12} + \nu_{34} = \Delta \nu_{\text{HFS}} \)

\( \nu_{12} - \nu_{34} \propto \mu_\mu / \mu_p \propto m_\mu / m_p \)

2019/6/10

Muonium WS Osaka
Breit Rabi Diagram

\[ \nu_{12} = -\frac{\mu_B^\mu g^\mu_H}{\hbar} + \frac{\Delta \nu}{2} [(1 + x) - \sqrt{1 + x^2}], \]

\[ \nu_{34} = +\frac{\mu_B^\mu g^\mu_H}{\hbar} + \frac{\Delta \nu}{2} [(1 - x) + \sqrt{1 + x^2}], \]

\[ x = \frac{g_J \mu_B^e + g'_\mu \mu_B^\mu}{\mu_p} H / (h \Delta \nu) \]

\[ r'_e = \frac{g_J \mu_B^e}{\mu_p}, \]

\[ r'_\mu = \frac{g'_\mu \mu_B^\mu}{\mu_p}, \]

\[ x = \frac{r'_e + r'_\mu}{2} \frac{\nu_p}{\alpha \Lambda}, \]

\[ \delta \equiv \nu_{34} - \nu_{12} = \frac{\mu_B^\mu g^\mu_H \nu_p}{\mu_p} + \Delta \nu \left( \sqrt{1 + x^2} - x \right). \]

\[ \frac{\mu_\mu}{\mu_p} = \frac{r'_e g^\mu_H}{2 g'_\mu} \]

\[ = \frac{1 - (\Delta \nu)^2 + \nu_p r'_e \delta + \delta^2}{4 \nu_p (\nu_p r'_e + \delta)} \left( 1 - \frac{\alpha^2}{3} \left( 1 - \frac{3 m_e}{2 m_\mu} \right) - \frac{\alpha^2}{12 \pi} \frac{m_e}{m_\mu} + \frac{97}{108} \alpha^4 \right)^{-1} \]
# History of Mu HFS measurement

<table>
<thead>
<tr>
<th>Time</th>
<th>Group</th>
<th>$\Delta \nu$</th>
<th>ppm</th>
<th>B field (T)</th>
<th>Ref</th>
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<tbody>
<tr>
<td>1961</td>
<td>Yale-Nevis</td>
<td>$5500^{+2900}_{-1500}$ MHz</td>
<td>0.01-0.58</td>
<td>[33, 34]</td>
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<tr>
<td>1962</td>
<td>Yale-Nevis</td>
<td>4.461.3(2.0) MHz</td>
<td>450</td>
<td>1.1353</td>
<td>[35, 36]</td>
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<tr>
<td>1964</td>
<td>Yale-Nevis</td>
<td>4.463.24(12) MHz</td>
<td>27</td>
<td>0.5</td>
<td>[37, 38]</td>
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<td>1966</td>
<td>Yale-Nevis</td>
<td>4.463.18(12) MHz</td>
<td>27</td>
<td>$2.7 \times 10^{-4}$</td>
<td>[39]</td>
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<tr>
<td>1969</td>
<td>Yale-Nevis</td>
<td>4.463.26(4) MHz</td>
<td>9.0</td>
<td>$3 \times 10^{-4}$</td>
<td>[40]</td>
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<tr>
<td>1969</td>
<td>Chicago</td>
<td>4.463.317(21) MHz</td>
<td>4.7</td>
<td>1.1353</td>
<td>[36, 41]</td>
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<tr>
<td>1970</td>
<td>Chicago</td>
<td>4.463.302 2(89) MHz</td>
<td>2.0</td>
<td>1.1353</td>
<td>[42]</td>
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<tr>
<td>1971</td>
<td>Yale-Nevis</td>
<td>4.463.308(11) MHz</td>
<td>2.5</td>
<td>$3 \times 10^{-4}$ and $1 \times 10^{-6}$</td>
<td>[43]</td>
</tr>
<tr>
<td>1973</td>
<td>Chicago-SREL</td>
<td>4.463 304.4(2.3) kHz</td>
<td>0.5</td>
<td>0</td>
<td>[44]</td>
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<tr>
<td>1975</td>
<td>LAMPF</td>
<td>4.463 302.2(1.4) kHz</td>
<td>0.3</td>
<td>very weak</td>
<td>[45]</td>
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<tr>
<td>1977</td>
<td>LAMPF</td>
<td>4.463 302.35(52) kHz</td>
<td>0.12</td>
<td>1.36</td>
<td>[46, 47]</td>
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<tr>
<td>1982</td>
<td>LAMPF</td>
<td>4.463 302.88(16) kHz</td>
<td>0.036</td>
<td>1.36</td>
<td>[48]</td>
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<td>1999</td>
<td>LAMPF</td>
<td>4.463 302.765(53) kHz</td>
<td>0.012</td>
<td>1.7</td>
<td>[4]</td>
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</tbody>
</table>

Table 2.1: Comparison of Mu HFS measurements.

- **Magic Field**
- **Spin Echo**
- **Zero Field World Record**
- **Old Mu Method**
- **High Field World Record**

K.S. Tanaka thesis
Most Precise Test of Bound State QED

**Experiment:**

- $\nu_{\text{HFS}}(\text{exp}) = 4463.302765 (53) \text{ MHz}$ [12 ppb]
- $\mu_\mu/\mu_p = 3.18334524(37)$ [120 ppb]
- $m_\mu/m_e = 206.768277(24)$ [120 ppb]

**Theory:**

- $\nu_{\text{HFS}}(\text{theory}) = 4463.302868 (271) \text{ MHz}$ [61 ppb]
- $\nu_{\text{HFS}}(\text{QED}) = 4463.302720 (518) (70) (2) \text{ MHz}$
  $\left( m_\mu/m_e \right)^\text{(QED)} (\alpha)$ By Eides
- $\nu_{\text{HFS}}(\text{weak}) = -65 \text{ Hz}$
- $\nu_{\text{HFS}}(\text{had. v.p.}) = 232 (1) \text{ Hz}$
- $\nu_{\text{HFS}}(\text{had. h.o.}) = 5 (2) \text{ Hz}$

QED calculation: Effort for 10 Hz accuracy in progress (by Eides et al.)

Why Mu HFS measurement is so important?

$g$-2 E821(BNL) 0.5ppm 3.6$\sigma$ deviation

- Measurement of the deviation of muon spin direction ($\omega_s$) and muon momentum direction ($\omega_c$) $\omega_a \propto \frac{(g-2)}{2}=a_\mu$

$\Rightarrow \tilde{\omega}_a = \frac{e}{mc} \left[ a_\mu \bar{B} - \left( a_\mu - \frac{1}{\gamma^2-1} \right) \beta \times \vec{E} \right]$

$a_\mu$: an independent precise muon mass measurement is required!

- The ratio to the proton NMR frequency is important!

$\Rightarrow a_\mu = \frac{R}{\lambda - R}$

$R \equiv \frac{\omega_a}{\omega_p}$

$\lambda \equiv \frac{\mu_\mu}{\mu_p}$

$\frac{\omega_a}{\omega_L(\mu)} = \frac{a_\mu \left( \frac{eB}{mc} \right)}{g_\mu \left( \frac{eB}{2mc} \right) \left( \frac{g_\mu}{2} \right)} = \frac{a_\mu}{1 + a_\mu}$

$= \frac{\omega_a}{\omega_L(p)} \frac{\omega_L(p)}{\omega_L(\mu)} = \frac{\omega_a \mu_p}{\omega_p \mu_\mu} = \frac{R}{\lambda}$

$\frac{\mu_\mu}{\mu_p}$ accuracy from direct measurement of 120 ppb.

How to improve the accuracy of $m_\mu/m_\text{e}$?

Comparison between theoretical and experimental value of $\Delta \nu$

$$\Delta \nu (\text{Fermi}) = \frac{16}{3} \alpha^2 c R_\infty \frac{m_\text{e}}{m_\mu} \left[1 + \frac{m_\text{e}}{m_\mu}\right]^{-3}.$$  + higher order

where recoil term 800kHz (120ppm) and so on are included.

$R_\infty = 10973731.568639(91) \text{m}^{-1}(0.09\text{ppt})$

(Cs atomic beam interferomtery)

$\alpha^{-1} = 137.03599958(52) (3.8\text{ppb})$

(from electron $g$-2)

$m_\mu/m_\text{e} = 206.7682826(46) (22\text{ppb}), \mu_\mu/\mu_\text{p} = 3.183345396(94) (24\text{ppb})$

This value is used for the determination of $g$-2.
\[ a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p} \]

\[ a_\mu = \frac{\omega_a}{\omega_p} \frac{m_\mu}{m_e} \frac{\mu_p}{\mu_B} \]

\[ \mu_\mu = g_\mu \frac{e\hbar}{2m_\mu} \]

**Muonium WS Osaka**

**Muon g-2**

- hadronic contribution
- hadronic lbl contribution
- New Physics

**MUSEUM - HFS**

- \( \Delta \nu_{\text{HFS, } n=1} \)
- \( \mu_\mu \)
- \( \alpha \)
- QED corrections
- weak contribution

**Mu-MASS**

- \( \Delta \nu_{1S-2S} \)
- \( m_\mu \)
- QED corrections
- Rydberg

From K. Jungmann
Exotic particle search

a pseudo vector boson

\[ \frac{\alpha''}{r} \rightarrow \frac{\alpha''(s_1 \cdot s_2)e^{-\lambda r}}{r} \]

a massive vector boson

\[ \frac{\Delta E_{\text{hfs}}}{E_{\text{hfs}}} = \frac{8\alpha' m_e}{m_V} = \frac{8\alpha\kappa(k + g_V/e)m_e}{m_V} \]

Fig. 2 on PRL 104, 220406 (2010)


Fig. 6 on PRD90, 073004(2014).

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Test of CPT and Lorentz Invariance

CPT broken Theory $\Rightarrow$ Lorentz symmetry is broken

O.W. Greenberg, PRL 89 (2002) 231602
R. Blihm, V.A. Kosteleky, C.D. Lane, PRL 84 (2000) 1098
V.W. Hughes et al., PRL 87(2002) 111804

CPT violation search
Ex: Muon difference $g_{\mu^+}/g_{\mu^-} \approx 10^{-8}$

$g_{\mu^-}/$MuHFS precise measurement

Lorentz symmetry violating term in STE Lagrangian $b$
Corresponding MuHFS $\Delta \nu_{12/34}$
These value might change in sidereal time (23h56m)

$\sim b_{3 \mu}/\pi = -\delta \Delta \nu_{12} = \delta \Delta \nu_{34}$

LAMPF Exp. Figure of Merit

$2\sqrt{(b_{\mu^+ x})^2 + (b_{\mu^+ y})^2} / m_{\mu} < 5 \times 10^{-22}$

$m_{\mu}/M_P \sim 10^{-20}$

Plank scale sensitivity

V.A. Kostelecky, A.J. Vargas, PRD 92 (2015) 056002
J-PARC Muon Science Facility (MUSE)

H1 Area
Phase 2 (g-2/EDM, $T_{\mu S}$)

Under construction

H-Line: for particle and atomic physics large scale experiments, “precision frontier”.
Higher intensity tunable (4 – 50 MeV) $\mu^+$ & $\mu^-$ beam.
(Exp.: MuSEUM, Deeme, g-2, ...)

MLF Experimental Hall No. 1 (2018)

D-Line: Decay and Surface muon ($\mu^+$ & $\mu^-$)
Slow (50 keV) – fast (50 MeV) beam, general purpose.
MuSEUM Experimental Layout

1. Muonium formation
2. RF spin flip
3. Positron asymmetry

Upstream Counter
Experimental Procedure
Muonium
decay e+
poralized muon beam 100% ←
Online Beam Monitor
2D cross-configured fiber hodoscope
Kr Gas Chamber
1.7 T Magnet
RF Tuning Bar
RF Cavity
Positron Counter
Segmented scintillation counter

2019/6/10
Muonium WS Osaka
Zero field measurement at D Line

Important milestone for HF measurement-

Two additional components are required.
- Permalloy magnetic shield
- RF cavity for ZF
Resonance Measurement Setup

- **Fiber Beam Profile Monitor**
- **Kr Gas Chamber (RF Cavity inside)**
- **Positron Counter w/Al Absorber**
- **Muon Beam**
- **Two layers of magnetic shield**

![Image of the setup](image_url)
Magnetic Shield and Field Probe

Assembled magnetic shield without top boards

Magnetic field probe holder and its moving system

Fluxgate magnetic probe
35 mm x 35 mm x 35 mm
Expected Q-value is 10000, microwave power is up to 3 W

TM110 mode at 4.463 GHz, +/-1 MHz tuning by a piezo positioner
Gas Chamber and Gas Handling

- 425 mm length, 280 mm diameter, 100 μm Al beam window
- Gas pressure is monitored by a capacitance gauge
- Gas purity is measured by Q-Mass spectrometer
Muon Beam Profile Monitor

- Two layers of 100-μm fiber hodoscope (2x16ch).
- 3 x 3mm$^2$ active area MPPC with 15-μm pixel pitch.
- EASIROC readout

Kanda, Ueno, Toyoda
Offline 3D Beam Profile Monitor

- Scintillator
- Screw
- Kr Gas Chamber
- Actuator

Muon beam width
- ±2 mm

Muon beam center
- ±2 mm

Ueno, Kanda, Toyoda, Ito
**Positron Counter (1): Scintillation Position Detector**  
Kanda, Kojima

### Plastic scintillator + MPPC + Kaliope readout circuit

- **32ch MPPC input**
- **FPGA**
- **Ethernet**
- **Trigger input**
- **ASIC**
- **PROM**
- **+5V(A), +1.8V**

### MPPC (Multi-Pixel Photon Counter)
- 1.3 mm x 1.3 mm active area  
  (Hamamatsu)

### Segmented scintillation detector
- Scintillation counter with SiPM readout
- Unit cell: 10 mm × 10 mm × 3 mm¹
- Area: 240 mm × 240 mm
- 24x24 segments x 2 layers = 1152 ch
- High-rate capability required
- Pileup loss at 3 MHz/ch ~ 2%

---

¹ Note: The dimensions may have been approximated for clarity in the image.
First Resonance Search

2016. June. 3(24h)
Beam profile measurement

2016. June. 12-14(60h)
Muonium Resonance Search

Results

1. Good zero field condition (~100nT) was obtained.
2. Beam profile was successfully measured.
3. Enough RF power and cavity Q value were obtained.
4. All detectors were working properly.
5. After several trial, Muonium HFS resonances was obtained!
Time Dependent Spin Flip Signal

Kanda thesis

Near at Resonance
4463.1 MHz
RF frequency
1.0 Kr atm
27.4 MeV/c muon

Off-Resonance
RF frequency was far detuned
1.0 Kr atm
27.4 MeV/c muon

Signal = ON/OFF-1
Muonium HFS resonance was observed

Fitted by Lorenzian and freq. center was 4463.1±0.02 MHz

Expectation from precursor experiments is 4463.1 MHz
MuSEUM Zero Field Experiment

- Experimental Procedure
  - Muonium formation in a Kr gas chamber
  - Microwave spin flip
  - Positron asymmetry
Positron Counter (2): Silicon Strip Detector

Nishimura

<table>
<thead>
<tr>
<th>Item</th>
<th>Specification</th>
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<tbody>
<tr>
<td>Sensor type</td>
<td>single-sided, p+ on n</td>
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<tr>
<td>Size</td>
<td>98.77 mm × 98.77 mm</td>
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<tr>
<td>Active Area</td>
<td>97.28 mm × 97.28 mm</td>
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<tr>
<td>Strip pitch</td>
<td>0.19 mm</td>
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<tr>
<td>Strip length</td>
<td>48.575 mm</td>
</tr>
<tr>
<td>No. of strips</td>
<td>512 x 2 blocks</td>
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<tr>
<td>Thickness</td>
<td>0.32 mm</td>
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</table>

Silicon strip detector
- Readout chips (SliT128A, 128 ch/chip)
- Developed for J-PARC g–2/EDM experiment
- Highly-segmented
- High-rate capability
- S/N ~ 21
Zero Field Measurements at D-Line

Experimental Setup

Online Beam Profile Monitor
Positron Counters
Readout Electronics
Muon Beam

Magnetic Shield
Kr Gas Chamber

New RF Cavity for Zero Field

180 mm

Residual Magnetic Field

~ 80nT

Upstream Window
Downstream Window

RF Intensity

\[ \Delta \nu = 4.463 \text{ GHz} \]

TM220 mode
Larger cavity
More muon stop
Q-Value: 20,000 (calc.)

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Results (1): Time Integral Method

– Scintillation Position Detector Data –

Statistical uncertainty:
- 2016 Feb. ~ 20 kHz (5ppm)
- 2017 Feb. ~ 4 kHz (1ppm)
- 2017 June ~ 2 kHz (0.5ppm)
- 2018 March ~ 1kHz, measured at 0.4, 0.55, 0.7 atm.
- 2018 June ~ 1kHz, measured at 0.3 atm Kr gas pressure.

Systematic uncertainty: Estimation in progress

Previous ZF Experiment at LAMPF:
\[ \Delta v_{\text{HFS}} = 4463302.2 \pm 1.4 \text{ kHz} \ (0.3 \text{ ppm}) \]

New world record at ZF ???
Data analysis on going

2019/6/10

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Latest Experiment

- **2018 Mar 9th - 16th @ J-PARC D2-Line**
- **Purpose**: Measure the HFS value in vacuum by extrapolation
- **Measured with 0.4, 0.55 and 0.7 atm Kr gas pressure**

![Graph showing HFS frequency and Kr Gas Pressure relationship](image)

- **HFS frequency**
- **Kr Gas Pressure [atm]**
- **Pressure gauge precision**
- **Uncertainty by extrapolation**
Mu HFS Measurement in 2018

Ueno thesis

- **Kr gas pressure shift**
  - Resonance frequency is shifted due to collision of muonium & the Kr atom
  - Gas pressure in the experiment in 2018 0.3, 0.4, 1.0 atm
  - Spin flip resonance signal was obtained for each gas pressure
Mu HFS Measurement in 2018

Ueno thesis

• Kr gas pressure shift
  – Resonance frequency is shifted due to collision of muonium & the Kr atom
  – Gas pressure in the experiment in 2018 0.3, 0.4, 1.0 atm
  – Spin flip resonance signal was obtained for each gas pressure

• Recent analysis achieved 0.9kHz ! (Assume previous pressure )
Results (2): **Time Differential Method**

– Silicon Strip Detector Data –

Nishimura thesis

**Preliminary**

$\Delta \nu_{\text{HFS}} = 4\ 463\ 302.2\ \text{kHz} 
\pm 3.1 \pm 0.2\ \text{kHz}$

**Statistics:**
- less data (smaller detector area)

**Systematics (main):**
- RF power drift (200 Hz)
- gas pressure extrapolation (66 Hz) (only one pressure data !)

**Possible advantages of this method:**
- Each detuning frequency data fitted individually.
- Can determine $\Delta \nu_{\text{HFS}}$ with only one frequency data.
- Most sensitive detuning frequency is ~60 kHz.
- Can improve statistical uncertainty by 3.2 times compared to the conventional method.
- Can reduce systematics of RF power variation (free fitting parameter).
- Need high-statistics data.

**Simulation**

**Experiment (2017 June):**

$\chi^2 / \text{NDF} = 72.77 / 58$

$\Delta \nu = 23.1 \pm 28\ \text{kHz}$

$\chi^2 / \text{NDF} = 74.14 / 58$

$\Delta \nu = -223.46 \pm 13\ \text{kHz}$
Systematic Uncertainty

Nishimura

<table>
<thead>
<tr>
<th>Item</th>
<th>June 2017</th>
<th>Prospects</th>
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<td>Gas pressure fluctuation</td>
<td>7 Hz</td>
<td>7 Hz</td>
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<tr>
<td>Gas pressure extrapolation</td>
<td>66 Hz</td>
<td>7 Hz</td>
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<tr>
<td>Gas impurity</td>
<td>0 Hz</td>
<td>0 Hz</td>
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<tr>
<td>Static magnetic field</td>
<td>0 Hz</td>
<td>0 Hz</td>
</tr>
<tr>
<td>Microwave power drift (including muon beam profile)</td>
<td>200 Hz</td>
<td>1 Hz</td>
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<tr>
<td>Pileup event loss</td>
<td>1 Hz</td>
<td>1 Hz</td>
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<tr>
<td>Time Calibration</td>
<td>1 Hz</td>
<td>1 Hz</td>
</tr>
<tr>
<td>Total</td>
<td>200 Hz</td>
<td>10 Hz</td>
</tr>
</tbody>
</table>

- Systematic uncertainty was much smaller than statistical uncertainty in June 2017
- Systematic can be as small as the previous experiment
High field measurement at H Line

Two additional components are required.
- Permalloy magnetic shield
- RF cavity for ZF
Improvement of statistics

LAMPF Experiment

\[ \delta(\Delta \nu) \]

\[ \text{Statistics} \]

10.9 ppb

Kr Density/Pressure

4.4 ppb

Muon stopping

1.0 ppb

RF power

0.96 ppb

\[ \delta(\mu_\mu/\mu_p) \]

\[ \text{Statistics} \]

107 ppb

Magnetic field

56 ppb

Muon stopping

13 ppb

Kr Density/Pressure

11 ppb

RF power

9.6 ppb

MuSEUM Improvements:

Statistics:

LAMPF: DC $10^7$/s

total $10^{13}$

Pulsed $1 \times 10^8$/s

H-Line

total $2 \times 10^{15}$

Systematics:

- magnetic field accuracy & uniformity
- pressure dependence (longer cavity lower pressure)
- muon stopping distribution measurement
- RF power stability
MRI Magnet for High-Field Experiment
Sasaki, Yamaguchi, T. Tanaka

Second-hand 2.9 T MRI magnet

Long Term Stability

Field Homogeneity (after shimming)

CW-NMR Field Monitoring System
18 ppb→5.9 ppb (2017→2018)

Spheroid : r=100 mm, z=300 mm
1.4 ppm p-p→0.27 ppm p-p (2017→2018)

64 Hz / 9.7 days

0.003 ppm /h

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RF Cavity for High Field Experiment

K.S. Tanaka, Seo

MWS simulation

3D CAD

Test Cavity

Q Value

<table>
<thead>
<tr>
<th>Modes</th>
<th>Q (measured)</th>
<th>Q (simulation)</th>
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<tr>
<td>TM110</td>
<td>11,300</td>
<td>29,700</td>
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<tr>
<td>TM210</td>
<td>8,050</td>
<td>28,900</td>
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\[ \nu_{12} = 1.906 \text{ GHz} \]

\[ \nu_{34} = 2.556 \text{ GHz} \]
## Preliminary Systematic Error (HF)

<table>
<thead>
<tr>
<th>Source</th>
<th>Accuracy</th>
<th>$\nu_{12}$ and $\nu_{34}$</th>
<th>$\delta(\Delta\nu_{\text{HFS}})$</th>
<th>$\delta(\mu_{\mu}/\mu_p)$</th>
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<tbody>
<tr>
<td>Magnetic Field*</td>
<td>30 ppb</td>
<td></td>
<td>0.0 ppb</td>
<td>15 ppb</td>
</tr>
<tr>
<td>RF power*</td>
<td>0.2 %</td>
<td>4 Hz</td>
<td>0.8 ppb</td>
<td>8 ppb</td>
</tr>
<tr>
<td>Kr gas temperature</td>
<td>0.2 deg.</td>
<td>&lt; 2 Hz</td>
<td>0.4 ppb</td>
<td>4 ppb</td>
</tr>
<tr>
<td>Kr gas pressure</td>
<td>1 Pa</td>
<td>1 Hz</td>
<td>0.2 ppb</td>
<td>0 ppb</td>
</tr>
<tr>
<td>H impurity</td>
<td>&lt;50 ppm</td>
<td>1 Hz</td>
<td>0.5 ppb</td>
<td>0 ppb</td>
</tr>
<tr>
<td>Quadratic dependence</td>
<td></td>
<td>5 Hz</td>
<td>1.0 ppb</td>
<td>5 ppb</td>
</tr>
<tr>
<td>Muonium position (x,y)</td>
<td>1 mm</td>
<td>3 Hz</td>
<td>0.6 ppb</td>
<td>6 ppb</td>
</tr>
<tr>
<td>Muonium position (z)</td>
<td>1 mm</td>
<td>&lt; 1 Hz</td>
<td>0.2 ppb</td>
<td>2 ppb</td>
</tr>
<tr>
<td>Beamline</td>
<td>10(e-4)</td>
<td>&lt; 1 Hz</td>
<td>0.2 ppb</td>
<td>2 ppb</td>
</tr>
<tr>
<td>Detector pile-up</td>
<td>w/ absorber</td>
<td>0.3 Hz</td>
<td>&lt; 0.1 ppb</td>
<td>&lt; 1 ppb</td>
</tr>
</tbody>
</table>

* should be re-estimated by latest progress and further MC simulation.

* should be re-estimated by latest progress of Time differential method.

Total systematic error of $\Delta\nu_{\text{HFS}} \sim 2$ ppb, and $\mu_{\mu}/\mu_p \sim 20$ ppb
Plan for Measurement

FY2020  Zero Field Measurement H line
       50 days ？  Less than 12ppb uncertainty
FY2021  High Field Measurement at H line (1.7T)
       50 days  50 times statics
FY2022  High Field Measurement at H line (1.13T)
       50 days  50 times statics
       （magic field, rectangular cavity）
FY2023  High Field Measurement at H line (3T)
       50 days  50 times statics
       （Better condition for μμ determination, rectangular cavity）
Summary and Next Step

• Zero-field measurements at existing beamline (D-Line) in progress for engineering run of the apparatus.
  ➢ Muonium HFS resonance clearly observed !
  ➢ Soon new world record at zero field ! (data analysis in progress)
  ➢ Several Analysis Methods are in progress (Old Muonium, Time differential)
  ➢ Time-Differential Method promising to improve statistics and reduce RF power fluctuation systematics.
  ➢ Need improvement of the RF power stability (systematics) !!!
  ➢ 4 Doctor thesis  5 Master thesis

• New Precise muonium HFS measurements at high magnetic field will be carried out in a few years (H-Line).

• Present expected systematic error estimated as
  
  \[
  \begin{align*}
  \text{HFS} & \approx 2 \text{ ppb (~8Hz)} \\
  \text{Magnetic moment } (\mu_\mu/\mu_p) & \approx 20 \text{ ppb} \\
  \end{align*}
  
  \]  

preliminary
Backup Slides
**Time Integral Method**

- Signal of all positrons

\[
S_{\text{int}} = \frac{\frac{aP}{2} \cos \theta}{1 + \frac{\lambda}{\gamma} + \frac{aP}{2} \cos \theta} \frac{-2 |b|^2 \left( r^2 + 2 |b|^2 \right)}{\left( r^2 + 2 |b|^2 \right)^2 + r^2 \Delta \omega^2}
\]

Resonance spectrum | Lorentzian function

- Peak of Lorentzian is equal to Mu HFS frequency
- Mu HFS is determined by multiple frequency data
- Width and height of spectrum is changed by microwave power

\[\Delta \omega / \text{Detuning angular frequency}\]
\[|b| / \text{Microwave magnetic field intensity}\]
\[\lambda / \text{Spin relaxation rate}\]
\[\gamma / \text{Muon decay rate}\]
\[P / \text{Muon spin polarization}\]
\[\gamma' = \gamma + \lambda\]
Time Differential Method

Time dependence of signal

\[ dS_{\text{diff}} = \frac{aP}{2} \frac{(C(t) - 1) \cos \theta_s e^{-(\lambda+\gamma)t}}{\left(1 + \frac{aP}{2} e^{-\lambda t} \cos \theta_s \right) e^{-\gamma t}} \]

\[ C(t) = \frac{G_+}{\Gamma} \cos G_- t + \frac{G_-}{\Gamma} \cos G_+ t \]

Time spectrum | Summation of cos

- contains more information
- Mu HFS frequency
- Microwave power
- Spin relaxation time
- can determine Mu HFS by only one detuning frequency data

\[ G_{\pm} = \frac{\Gamma \pm \Delta \omega}{2} \quad \Gamma = \sqrt{\Delta \omega^2 + 8 |b|^2} \]

Detuning frequency dependence

Microwave power dependence

2019/6/10 Muonium WS Osaka
Time Integral Signal (Simulation)

- Simulation setup
  - Measurement point | 10 points
  - $7.8 \times 10^{10}$ muon/point

- Fitting function for Time integral method
  - Summation of Lorentzian function considering microwave power distribution felt by muonium
Time Differential Signal (Simulation)

- Time differential signal | Same statistics as time integral method
- Time spectrum are changed by the detuning frequency

- Fitting function
  - Summation of cosine considering microwave power distribution felt by muonium
Time Differential Signal (Simulation)

Mu HFS is obtained by only one detuning frequency data
Multiple Time Differential Data

- Mu HFS is determined by multiple results of time differential method

Time Integral Method  |  0.41 kHz
Time Differential Method  |  0.34 kHz

15% improvement

\[ f(x) = |x - \Delta \nu| \]

\[ \nu_0 - \Delta \nu \ (kHz) \]

\[ \chi^2/\text{ndf} = 0.14 \pm 0.034 \]
More Efficient Measurement

- Concentrating one frequency is the most efficient method

<table>
<thead>
<tr>
<th>Method</th>
<th>Error (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Integral Method</td>
<td>0.41</td>
</tr>
<tr>
<td>Time Differential Method</td>
<td>0.34</td>
</tr>
<tr>
<td>Time differential Method (Δ=60 kHz)</td>
<td>0.13</td>
</tr>
</tbody>
</table>

3.2 times improvement compared to the time integral method

\[ \chi^2 | 88.0/95 \]

\[ \Delta | 59.9 \pm 0.13 \text{ kHz} \]
Analysis by the Time Differential Method (2017)

- Time differential muon spin resonance signal was observed
  - Detuning frequency dependence is similar to the simulation results
  - Same fitting function as the simulation
  - Mu HFS was obtained from each detuning frequency data
Mu HFS Measurement (2017)

- Mu HFS was obtained from multiple data
  - Obtained $\Delta \nu_{\text{Mu}}$

4 463 302.2 ± 3.1 kHz

- It is consistent to the previous experiments

Previous experiment
ZF | 4 463 302.2(14) kHz
HF | 4 463 302.765(50)(17) kHz

Mu HFS Measurement in 2018

Nishimura

- **Kr gas pressure shift**
  - Resonance frequency is shifted due to collision of muonium & the Kr atom
  - Gas pressure in the experiment in 2018 | 0.3, 0.4, 0.7 atm
  - Spin flip resonance signal was obtained for each gas pressure
- **Analysis is ongoing**
Precision of the hyperfine structure (HFS, $\Delta \nu$):

<table>
<thead>
<tr>
<th>Hydrogen-like atom</th>
<th>Experiment</th>
<th>Theory</th>
<th>$(\Delta \nu_{\text{theo}} - \Delta \nu_{\text{exp}}) / \Delta \nu_{\text{exp}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>0.2 ppt</td>
<td>1.2 ppm</td>
<td>(-0.45 ± 1.2) ppm</td>
</tr>
<tr>
<td>Positronium</td>
<td>3.3 ppm</td>
<td>2.0 ppm</td>
<td>(15 ± 4) ppm</td>
</tr>
<tr>
<td>Muonium (Zero-Field)</td>
<td>310 ppb</td>
<td>61 ppb</td>
<td>(150 ± 320) ppb</td>
</tr>
<tr>
<td>Muonium (High-Field)</td>
<td>12 ppb</td>
<td>61 ppb</td>
<td>(23 ± 62) ppb</td>
</tr>
</tbody>
</table>
RF Cavity & Gas Chamber

- Pressure: 0.5 – 1.5 atm
- Readout precision: several ppm (crystal gauge: 0.008% of full range)
- Contamination: below 1ppm.
- gas sampling before, during and after the experiment (several weeks).
Systematic Error

Calculate transition probability.

RF field map
- fluctuation of RF power
- effect of tuning bar

B field map
- fluctuation of B homogeneity

detection efficiency map

muonium distribution map
- fluctuation of beam intensity
- statistical uncertainty

signal or no signal

Repeat calculation for every muonium.

Center of the resonance line determined by fitting.

2019/6/10
Muonium WS Osaka
Old Muonium in ZF field

![Graph showing signal height vs. detuning in kHz](image)