Muonic atoms and radii of the lightest nuclei

Randolf Pohl

Uni Mainz
MPQ Garching

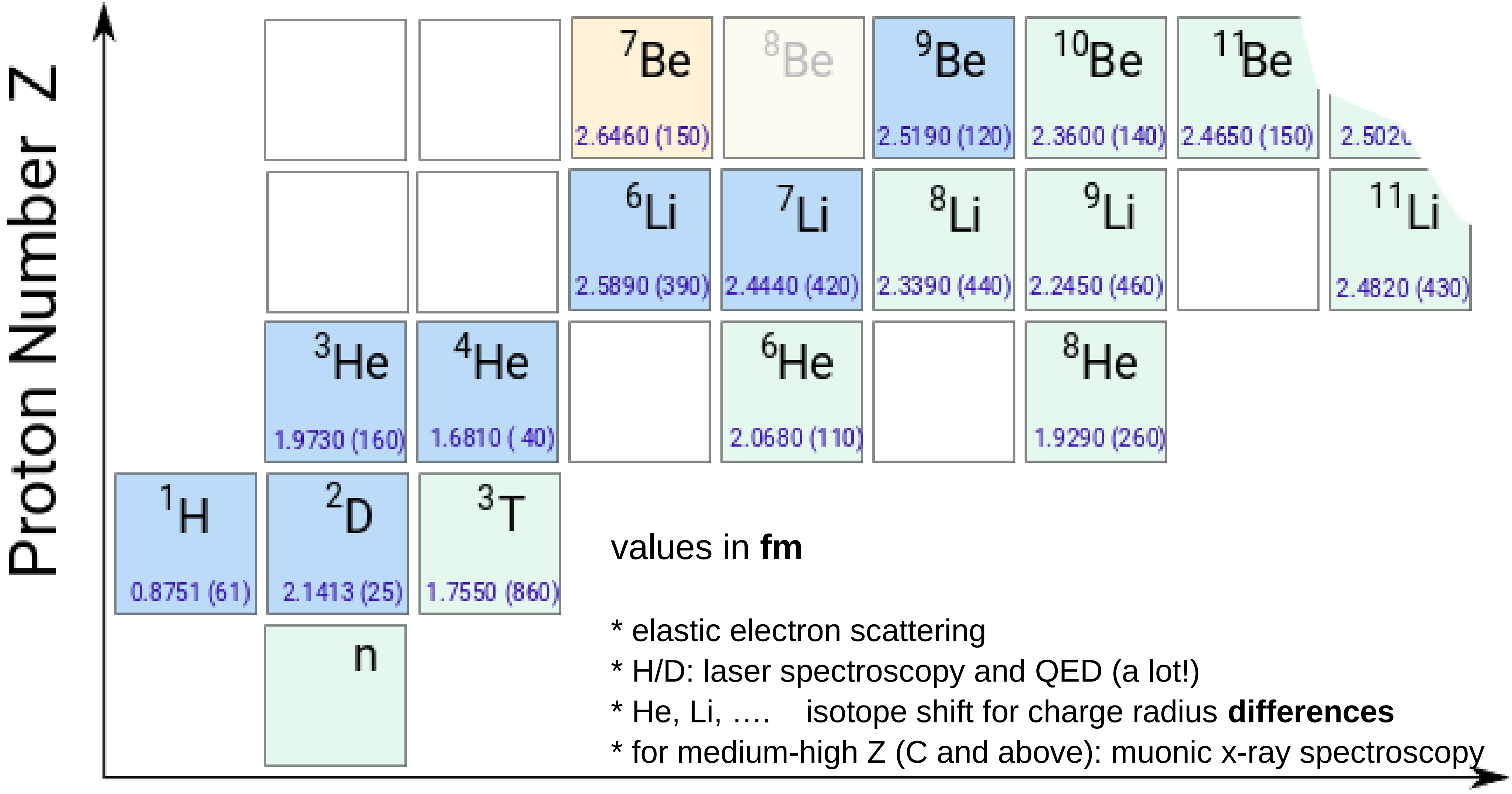
Tihany
10 June 2019

Outline

- Muonic hydrogen, deuterium and the Proton Radius Puzzle
  New results from H spectroscopy, e-p scattering
- Muonic helium-3 and -4
  Charge radii and the isotope shift
- Muonic **present**: HFS in μH, μ^3He
  10x better (magnetic) Zemach radii
- Muonic **future**: muonic Li, Be
  10-100x better charge radii
- **Ongoing**: Triton charge radius from atomic T(1S-2S)
  400fold improved triton charge radius

Nuclear rms charge radii

from measurements with **electrons**



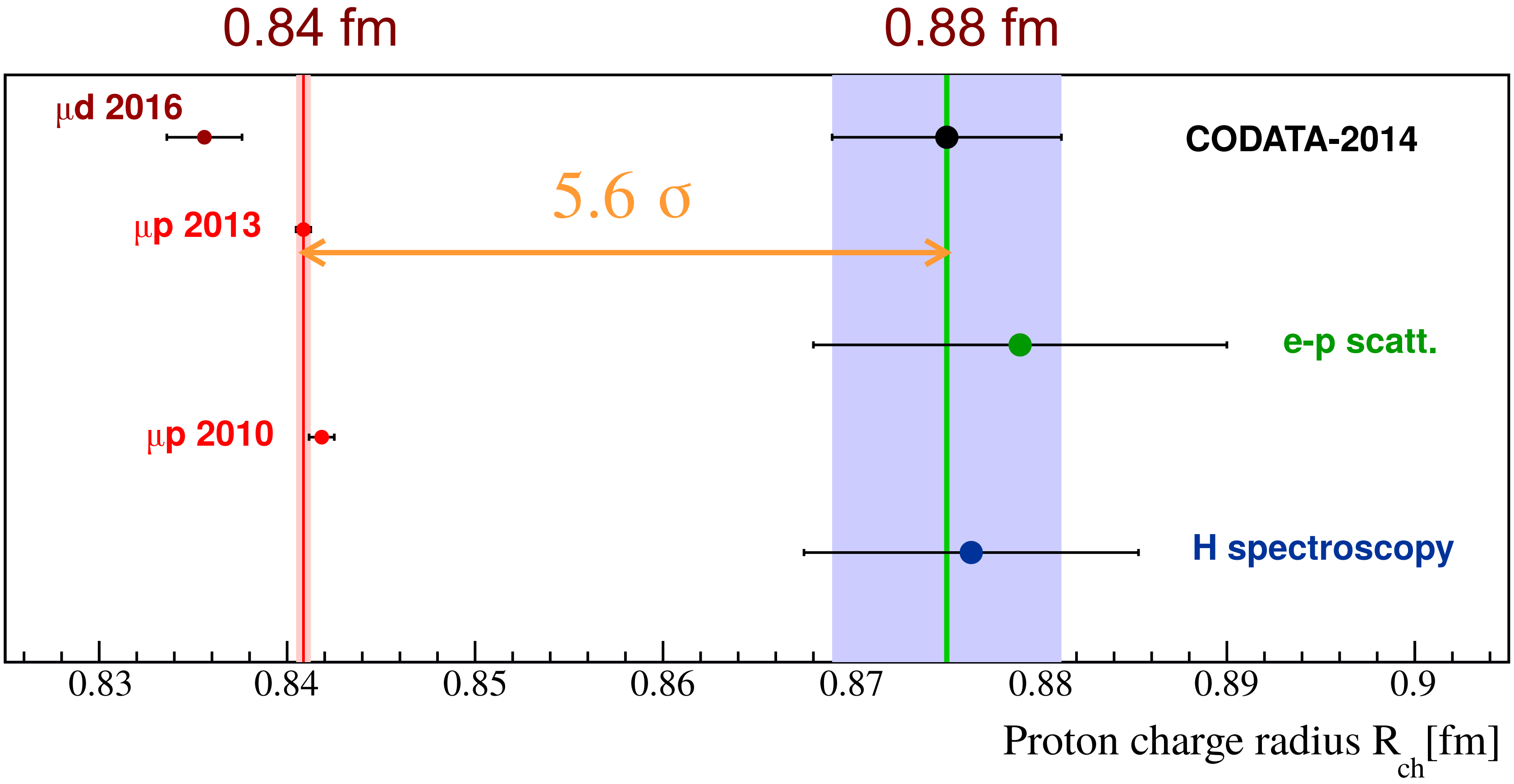
values in **fm**

- * elastic electron scattering
- * H/D: laser spectroscopy and QED (a lot!)
- * He, Li, …. isotope shift for charge radius **differences**
- * for medium-high Z (C and above): muonic x-ray spectroscopy

sources: * p,d: CODATA-2014
* t: Amroun et al. (Saclay) , NPA 579, 596 (1994)
* ^{3,4}He: Sick, J.Phys.Chem.Ref Data 44, 031213 (2015)
* Angeli, At. Data Nucl. Data Tab. 99, 69 (2013)

The “Proton Radius Puzzle”

Measuring R_p using electrons: 0.88 fm (+- 0.7%)
using muons: 0.84 fm (+- 0.05%)



μd 2016: RP et al (CREMA Coll.) Science 353, 669 (2016)
μp 2013: A. Antognini, RP et al (CREMA Coll.) Science 339, 417 (2013)

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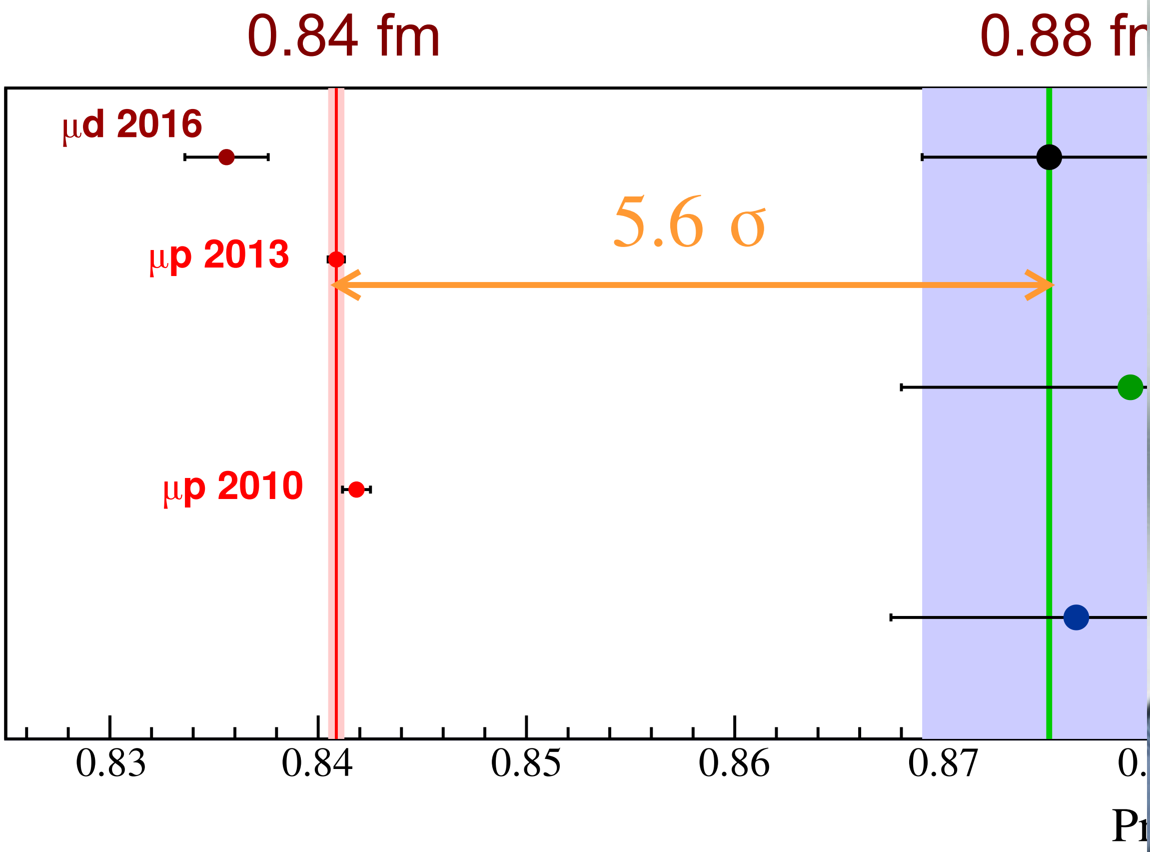

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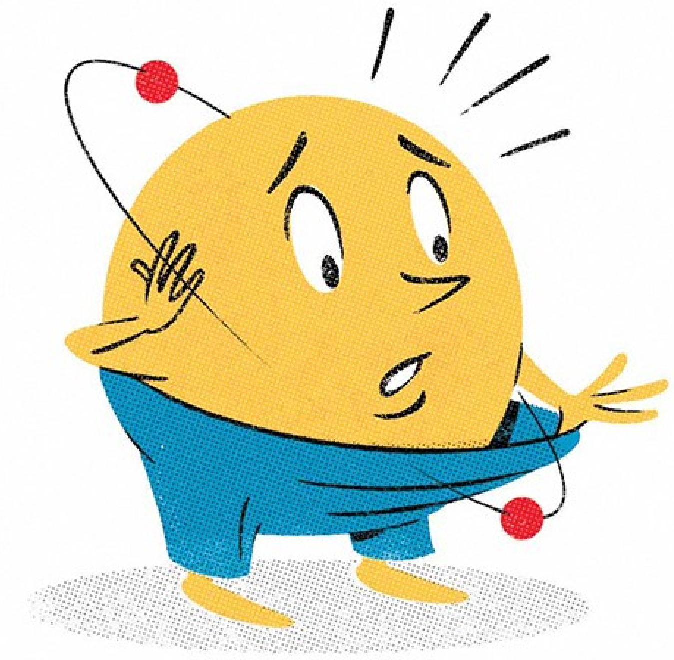
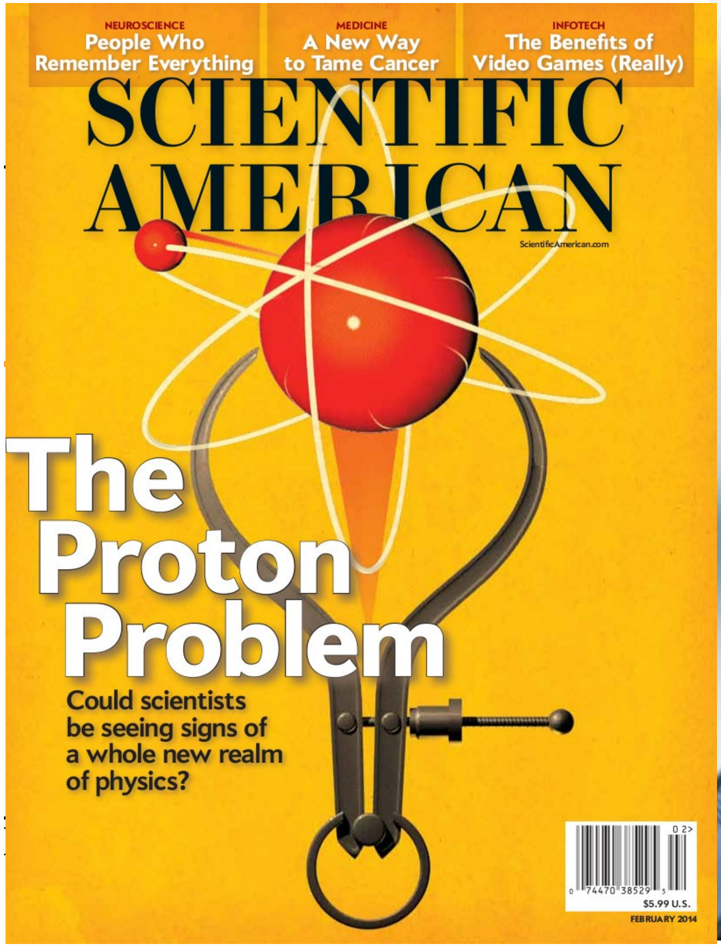
0.84 fm



The New York Times



NEUROSCIENCE
People Who Remember Everything

MEDICINE
A New Way to Tame Cancer

INFOTECH
The Benefits of Video Games (Really)

SCIENTIFIC AMERICAN

ScientificAmerican.com

The Proton Problem

Could scientists be seeing signs of a whole new realm of physics?

$5.99 U.S.
FEBRUARY 2014

8 July 2010 | www.nature.com/nature | $10

THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE

nature

OIL SPILLS
There's more to come

PLAGIARISM
It's worse than you think

CHIMPANZEES
The battle for survival

SHRINKING THE PROTON
New value from exotic atom trims radius by four per cent

NATUREJOBS
Researchers for hire

μd 2016: RP et al (CREMA Coll.) Science 353, 669 (2016)
μp 2013: A. Antognini, RP et al (CREMA Coll.) Science 339, 417 (2013)

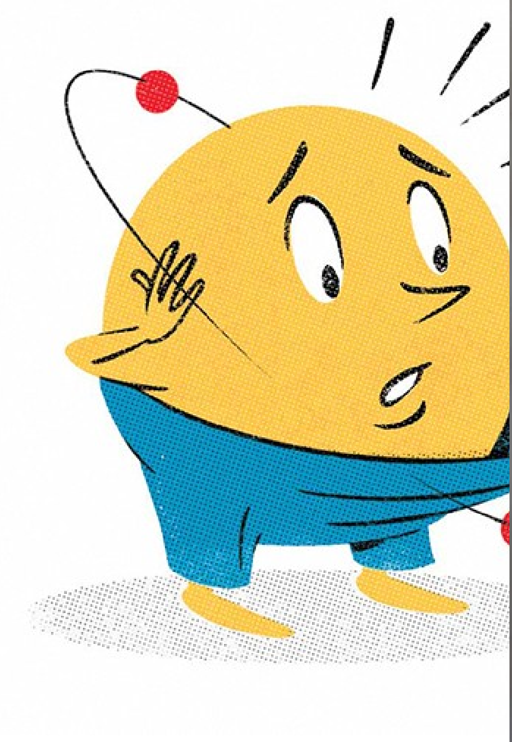
The “Proton Radius Puzzle”

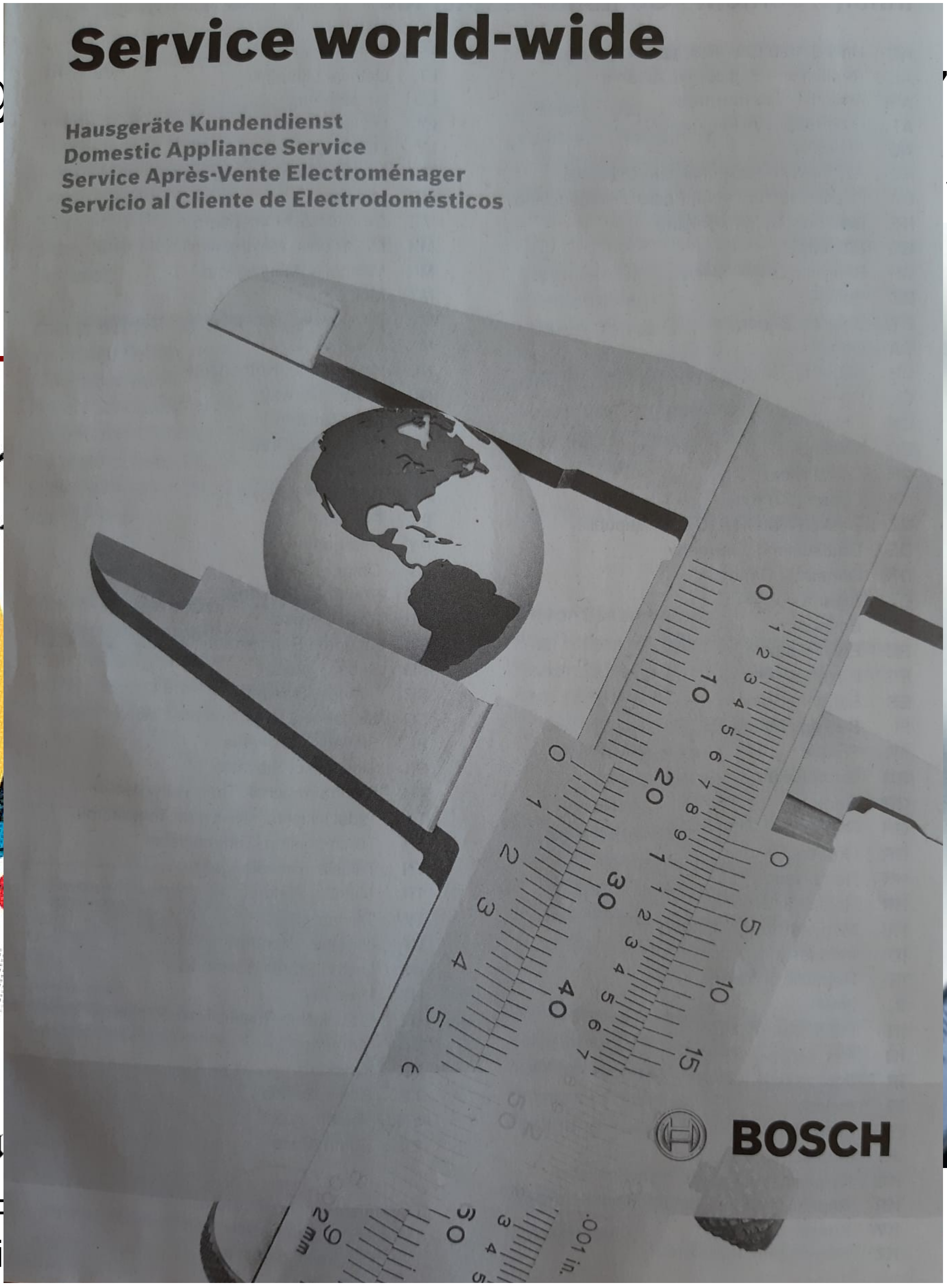
Measuring

7%)

5%)

0.





Service world-wide

Hausgeräte Kundendienst
Domestic Appliance Service
Service Après-Vente Electroménager
Servicio al Cliente de Electrodomésticos

BOSCH

8 July 2010 | www.nature.com/nature | $10

THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE

nature

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The New York Tim

µd 2016: RP et al (CR
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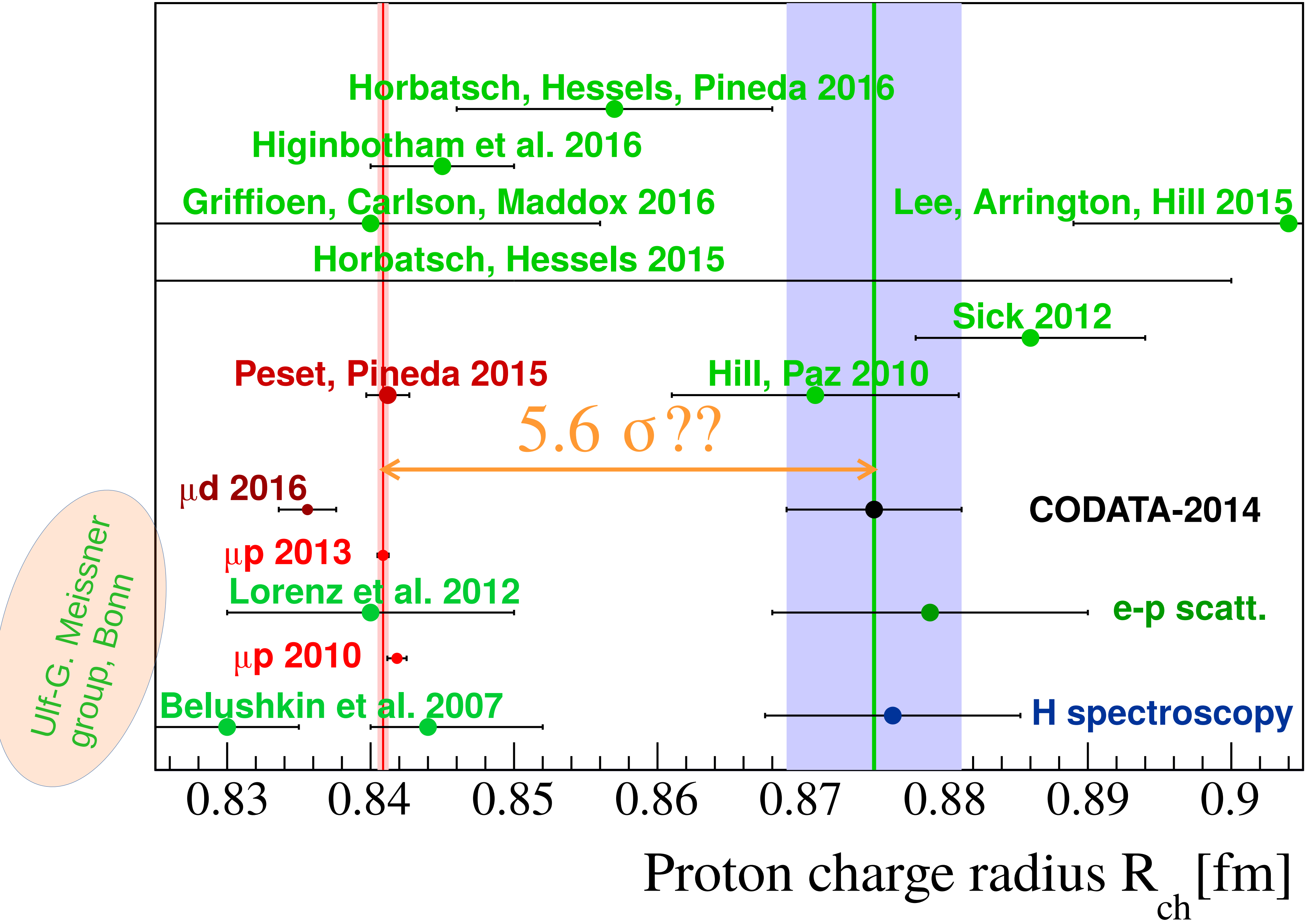
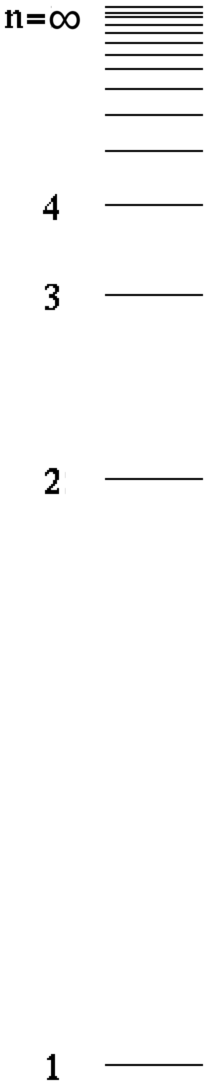
en

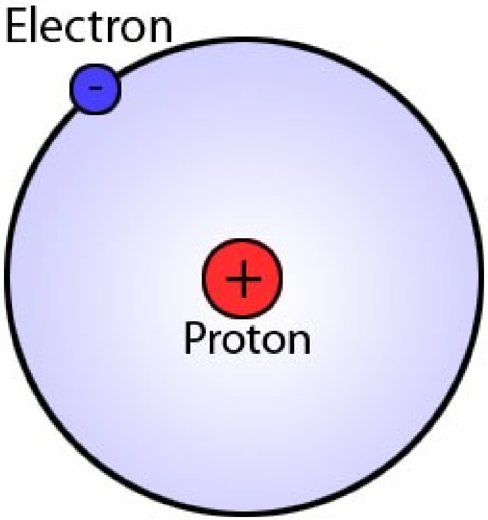
A “Proton Radius **Puzzle**” ??



Hydrogen

Energy levels of hydrogen

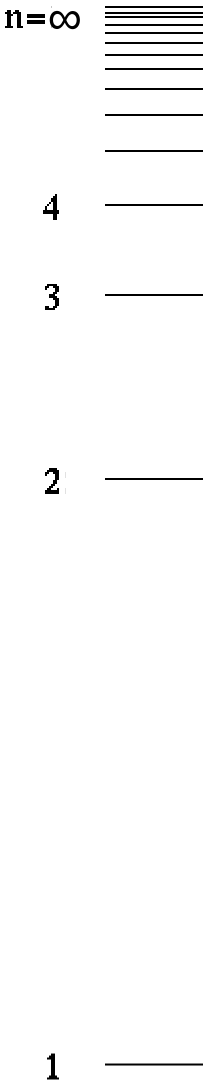




$$E_n \approx -\frac{R_\infty}{n^2}$$

Bohr formula

Energy levels of hydrogen

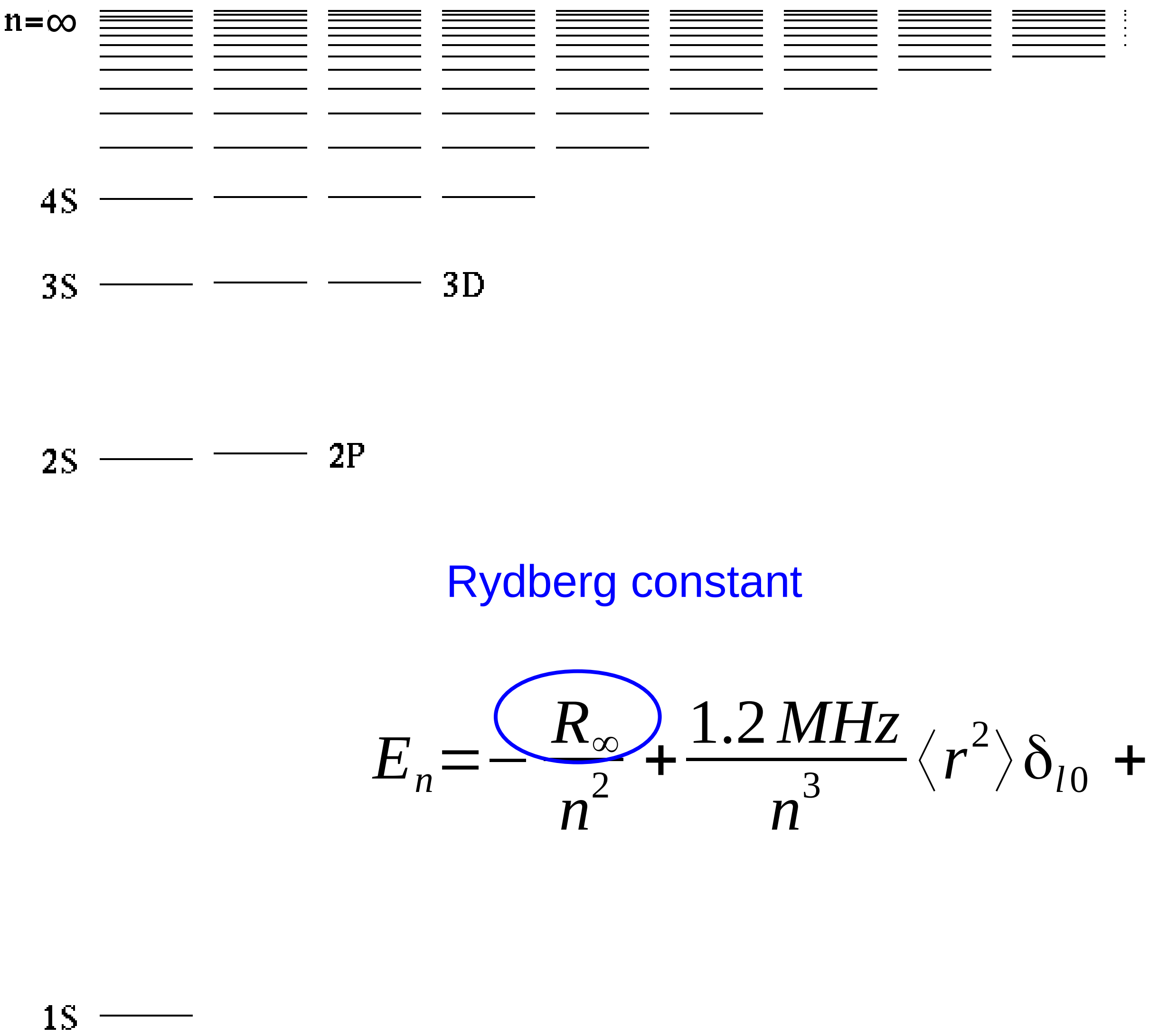


Rydberg constant

$$E_n \approx -\frac{R_\infty}{n^2}$$

Bohr formula

Energy levels of hydrogen

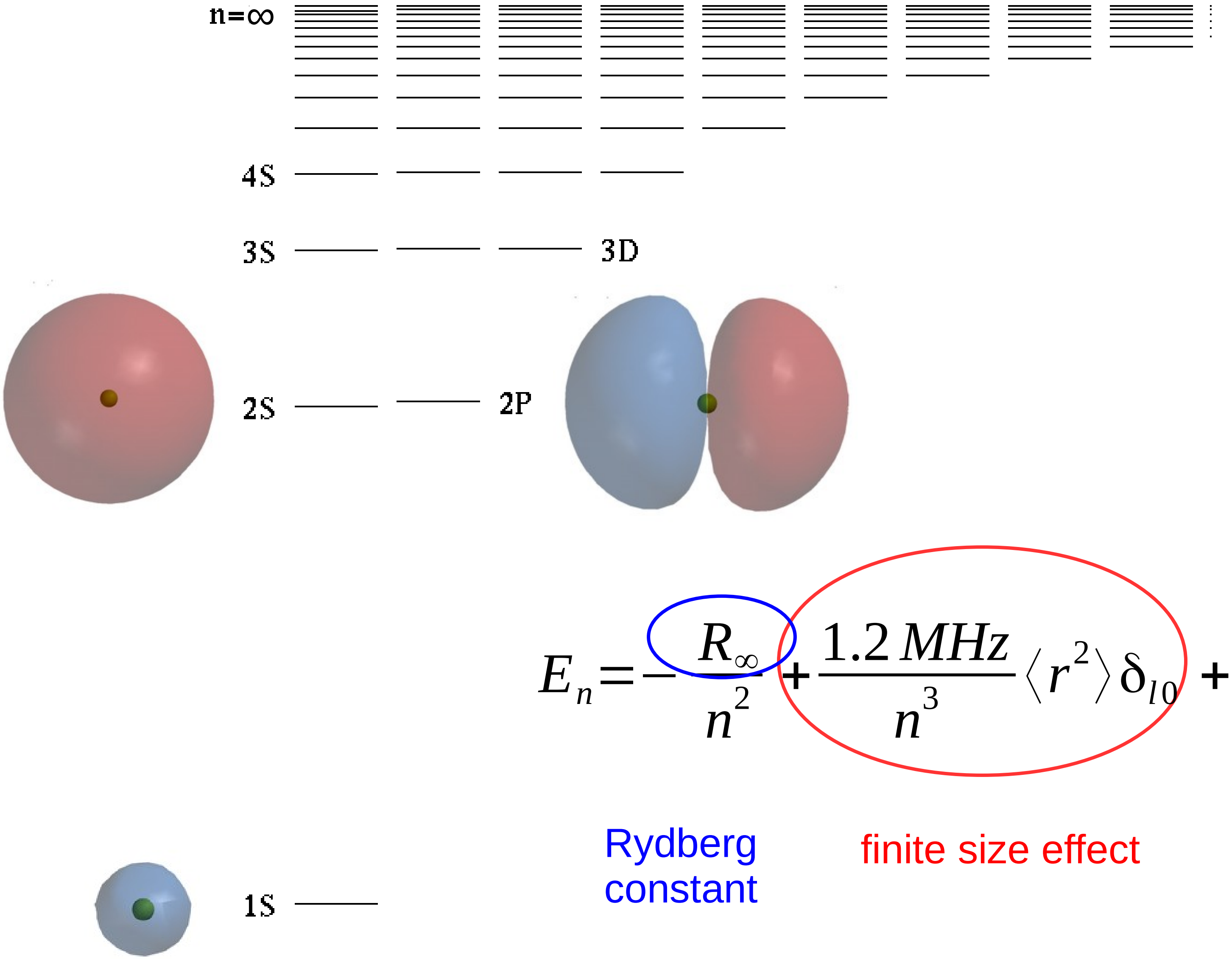


Rydberg constant

$$E_n = -\frac{\boxed{R_\infty}}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n, l, j)$$

RP et al., Metrologia 54, L1 (2017)

Energy levels of hydrogen



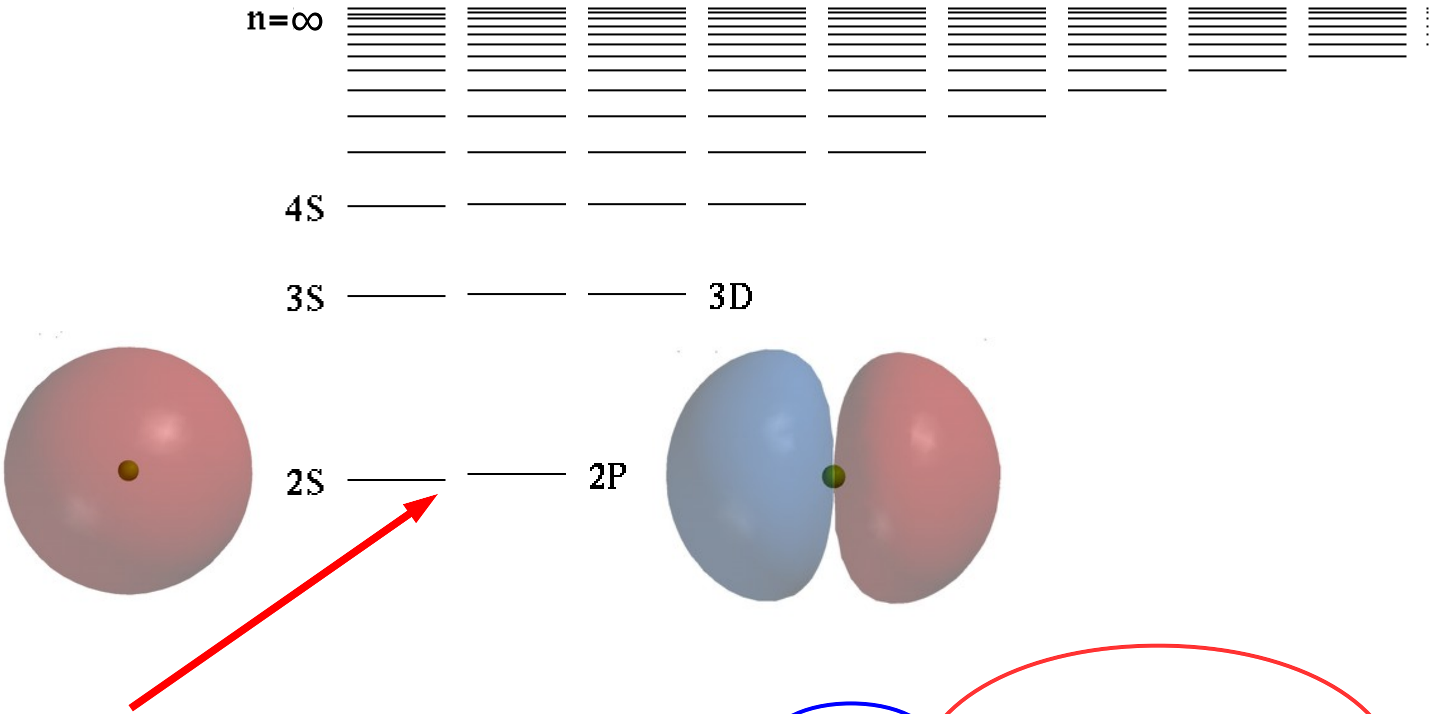
$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle \delta_{l0} + \Delta(n,l,j)$$

Rydberg constant

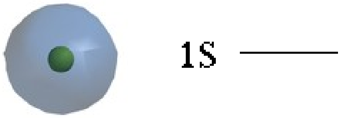
finite size effect

RP et al., Metrologia 54, L1 (2017)

Energy levels of hydrogen



2S-2P Lamb shift



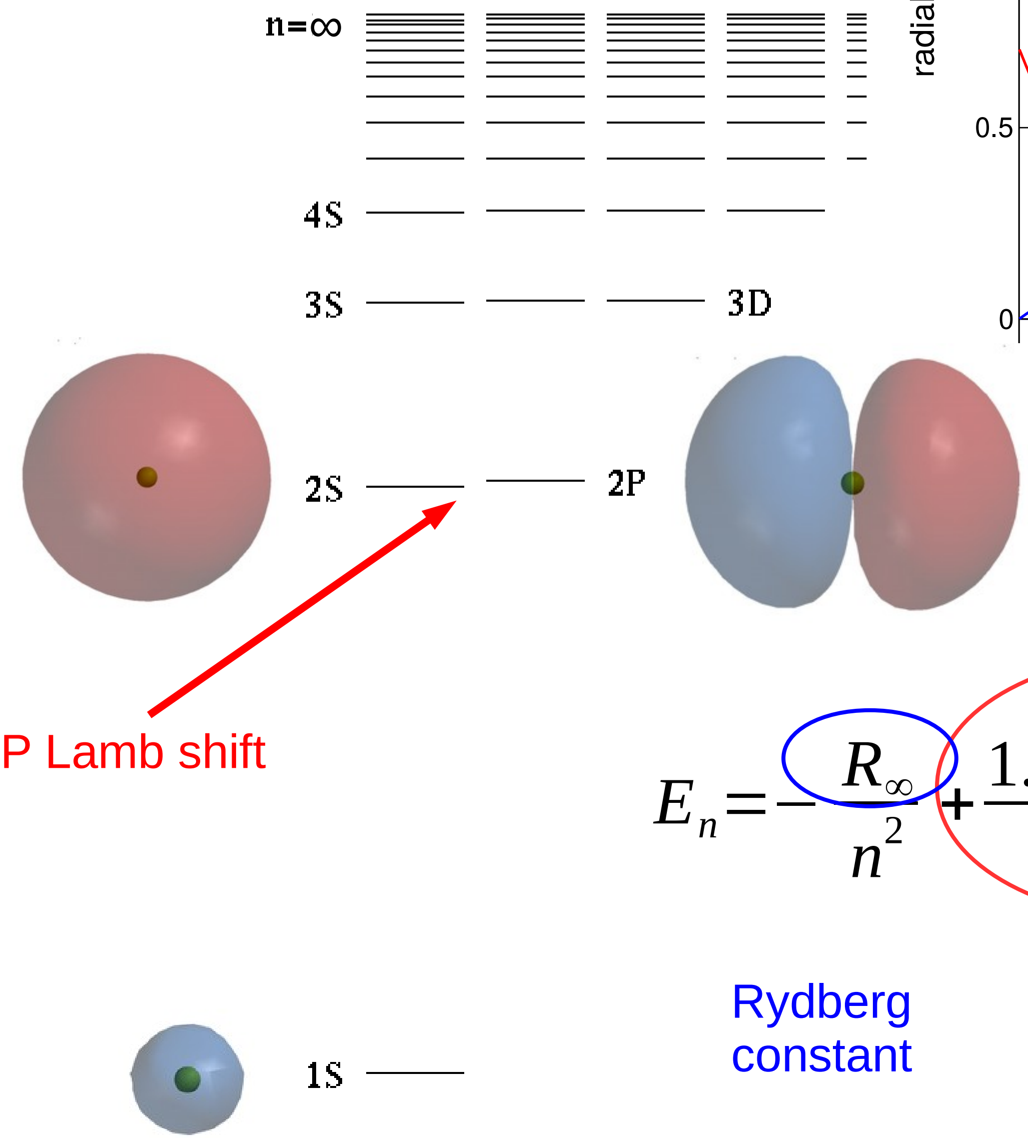
$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle\delta_{l0} + \Delta(n,l,j)$$

Rydberg
constant

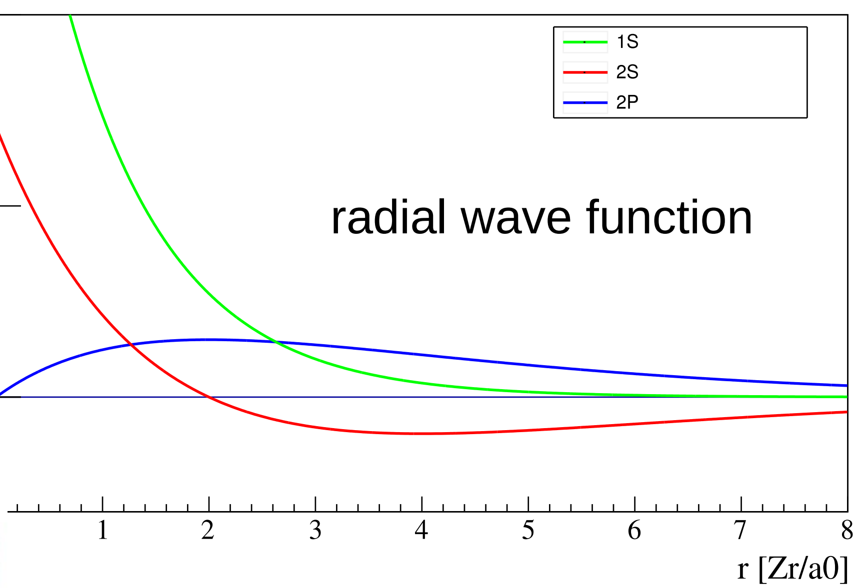
finite size effect

RP et al., Metrologia 54, L1 (2017)

Energy levels of hydrogen



2S-2P Lamb shift



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Rydberg constant

finite size effect

RP et al., Metrologia 54, L1 (2017)

Muonic Hydrogen

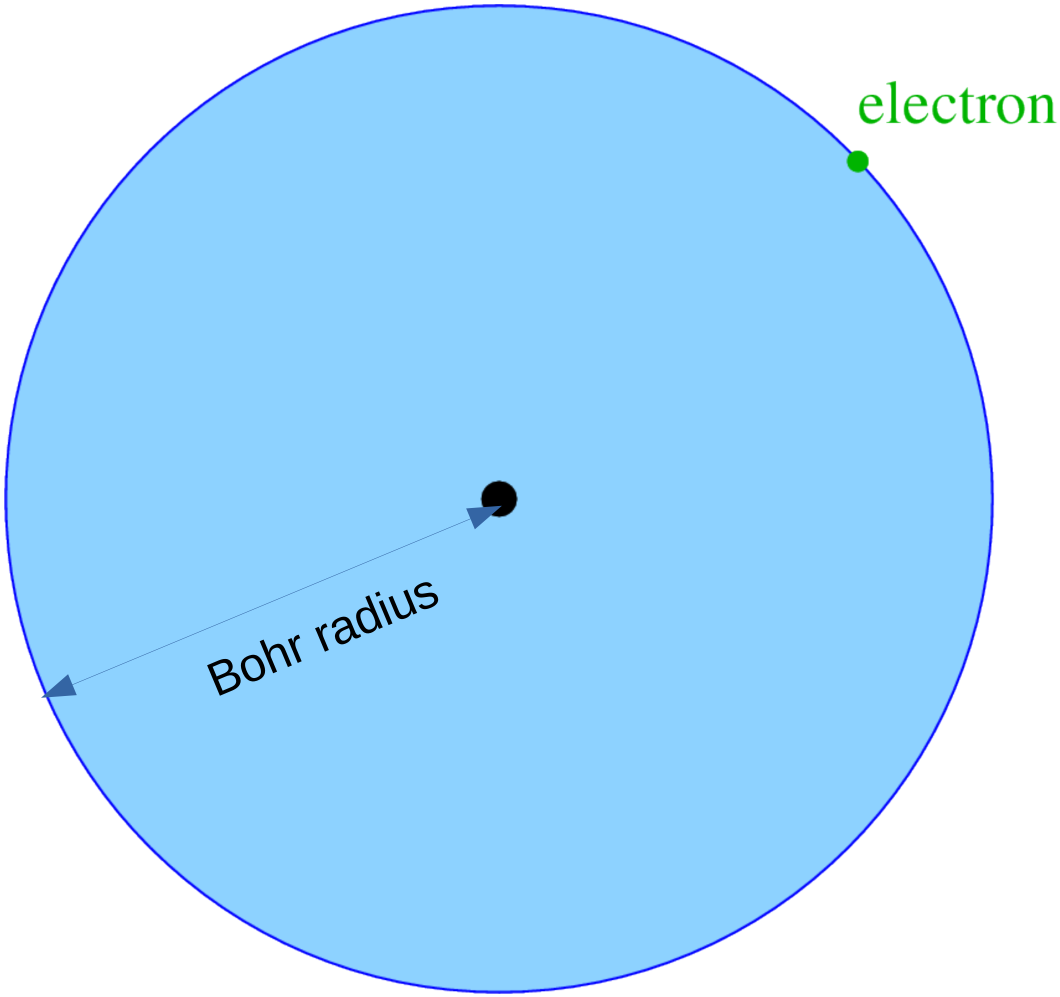
Muonic Hydrogen

A proton, orbited by a **negative muon.**

Electronic and muonic atoms

Regular hydrogen:

Proton + Electron



Muonic hydrogen:

Proton + Muon

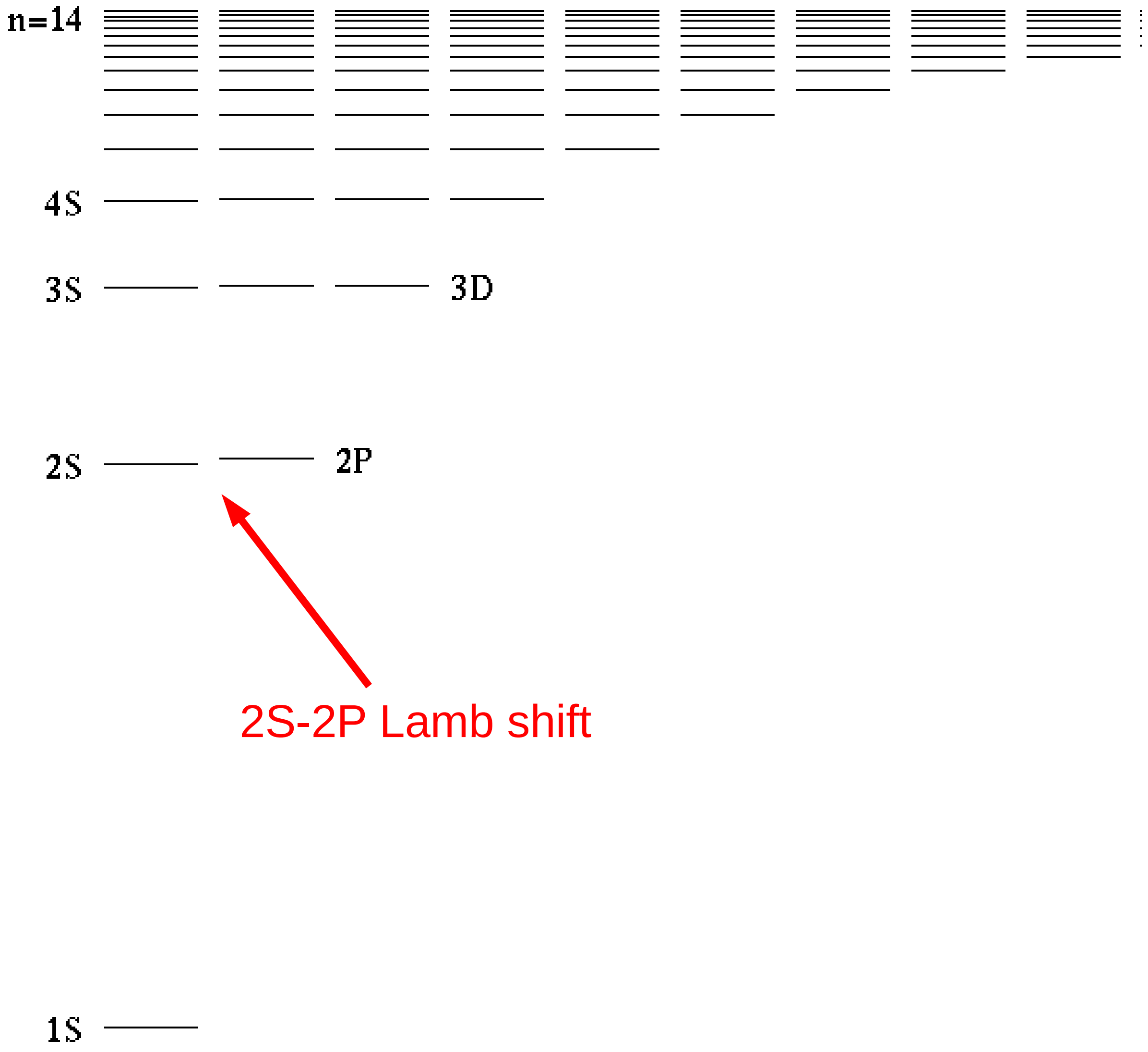
Muon **mass** = **200** * electron mass

Bohr **radius** = **1/200** of H

200^3 = a **few million times** more sensitive to proton size

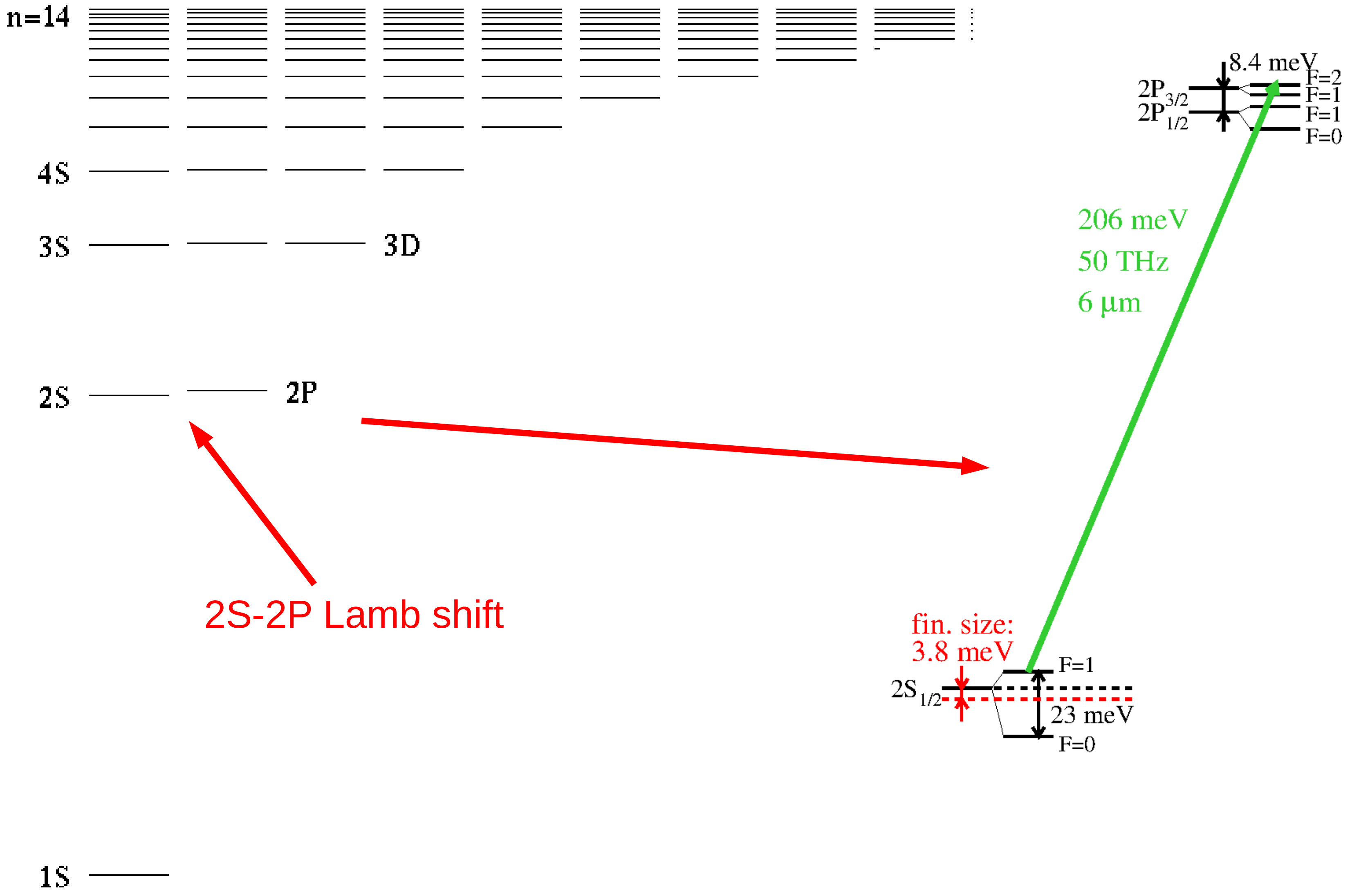
Vastly not to scale!!

Muonic Hydrogen



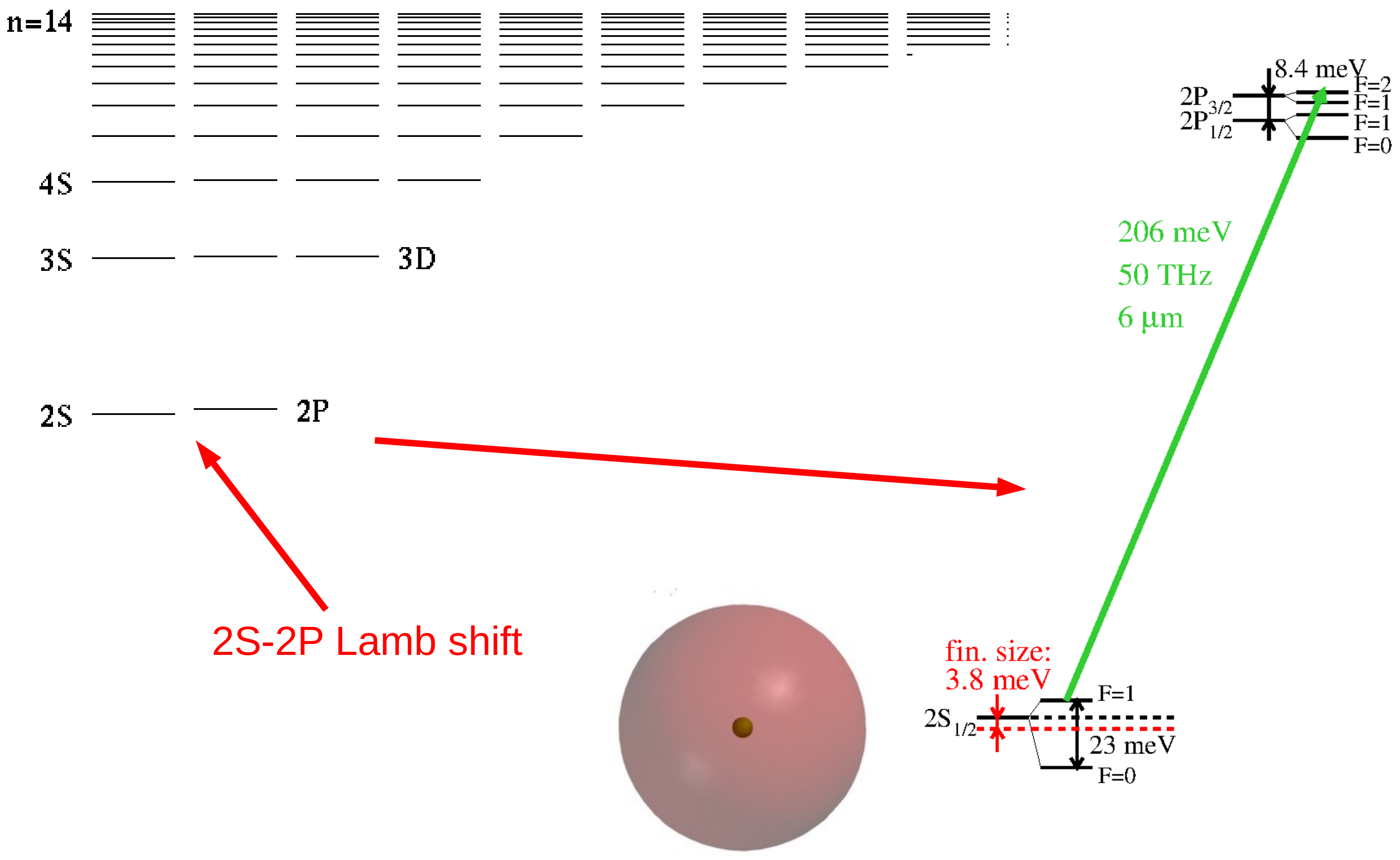
2S-2P Lamb shift

Muonic Hydrogen



2S-2P Lamb shift

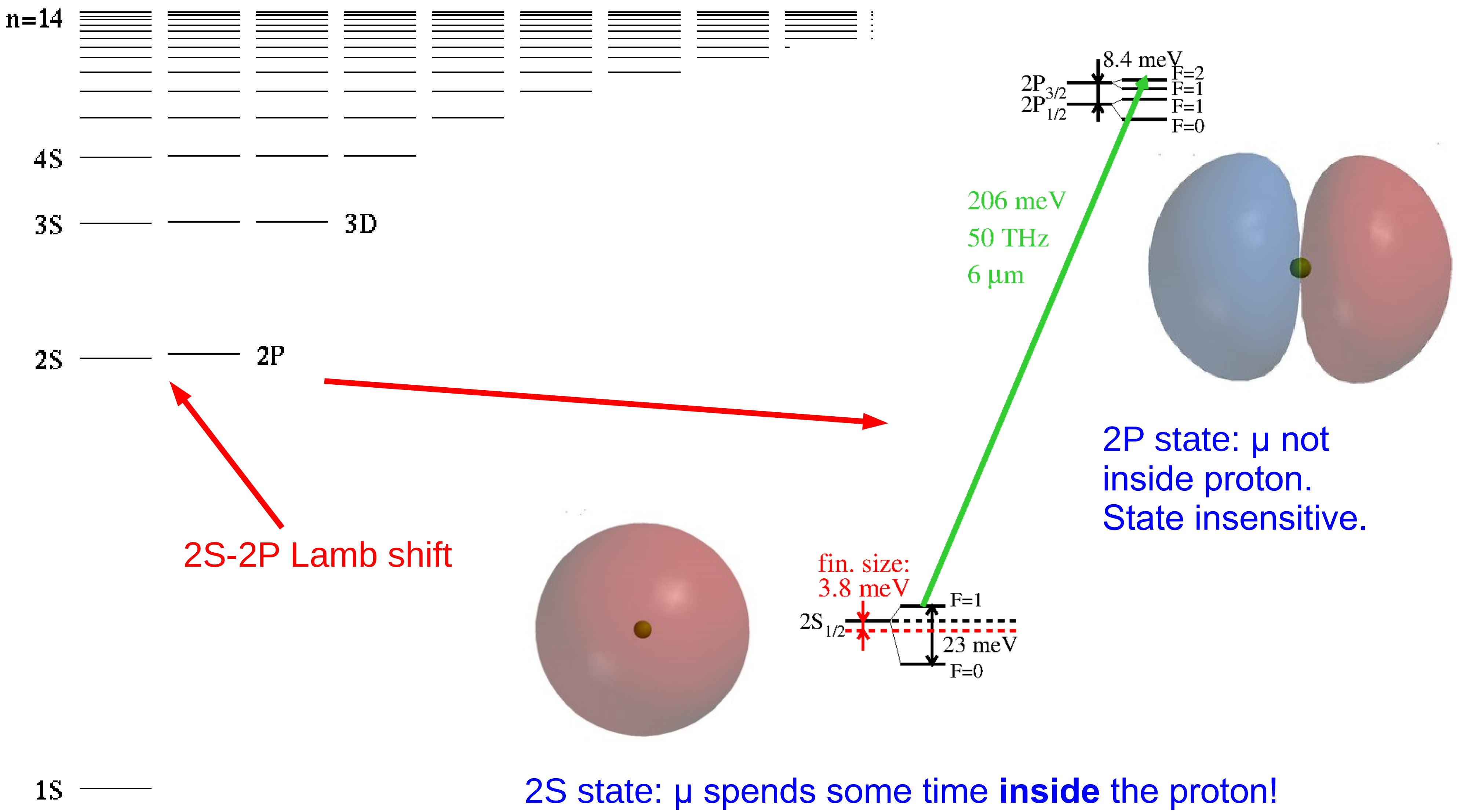
Muonic Hydrogen



2S-2P Lamb shift

2S state: μ spends some time **inside** the proton!
State is sensitive to the proton size.

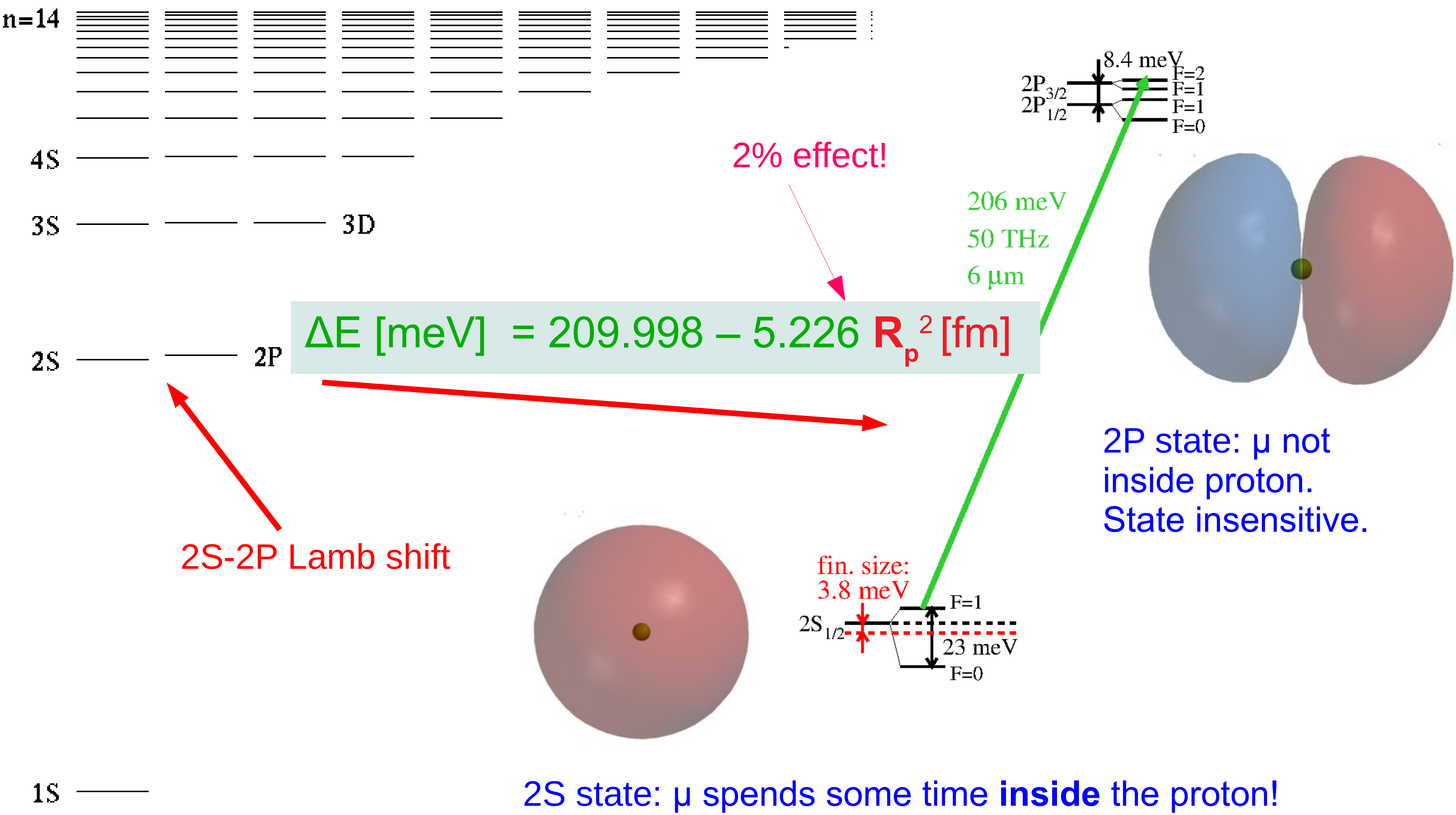
Muonic Hydrogen



2P state: µ not inside proton. State insensitive.

2S state: µ spends some time **inside** the proton! State is sensitive to the proton size.

Muonic Hydrogen



2% effect!

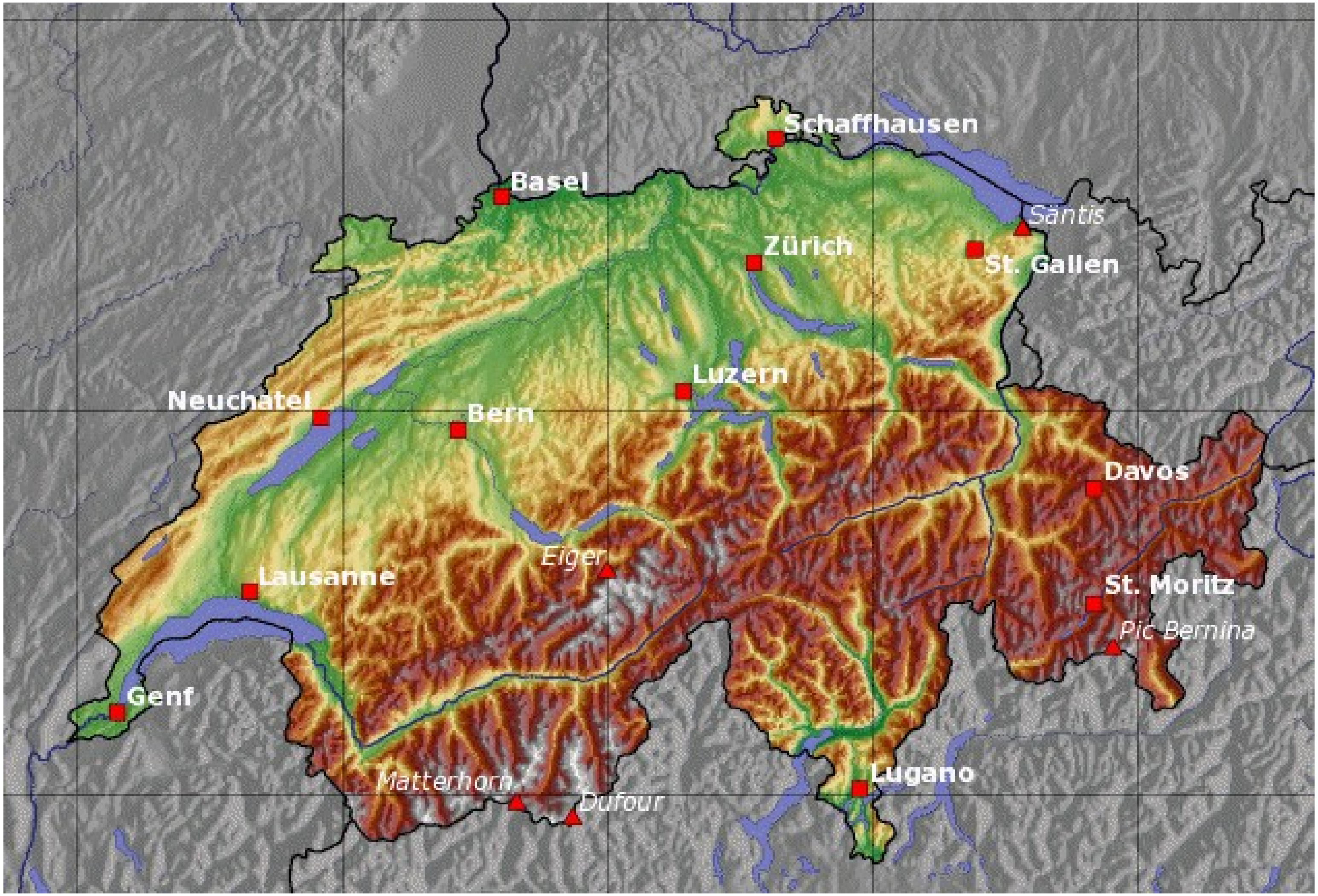
$$\Delta E\ [\text{meV}] = 209.998 - 5.226\ \mathbf{R_p}^2\ [\text{fm}]$$

2S-2P Lamb shift

2P state: µ not inside proton. State insensitive.

2S state: µ spends some time **inside** the proton!
State is sensitive to the proton size.

The accelerator at PSI



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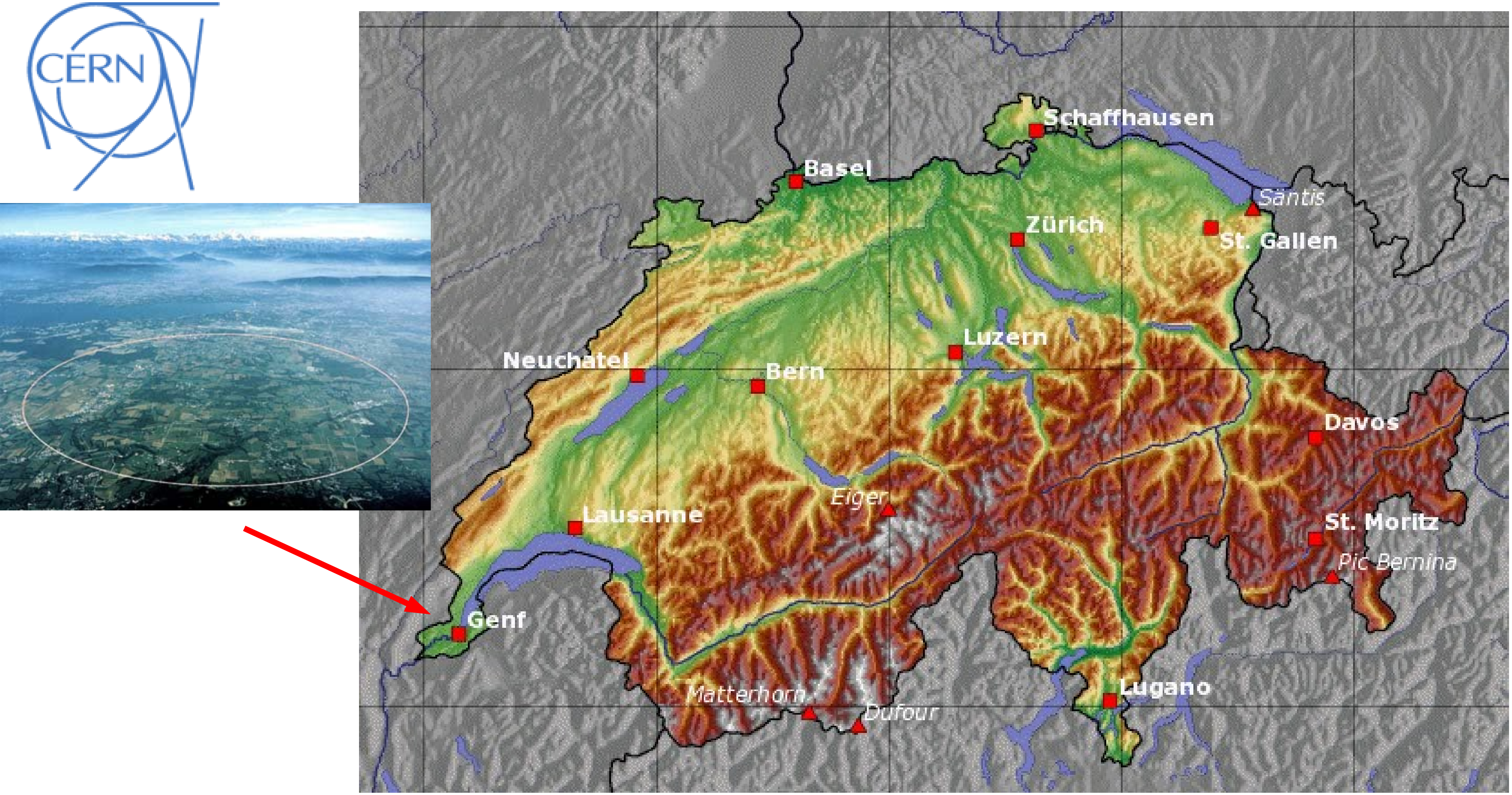

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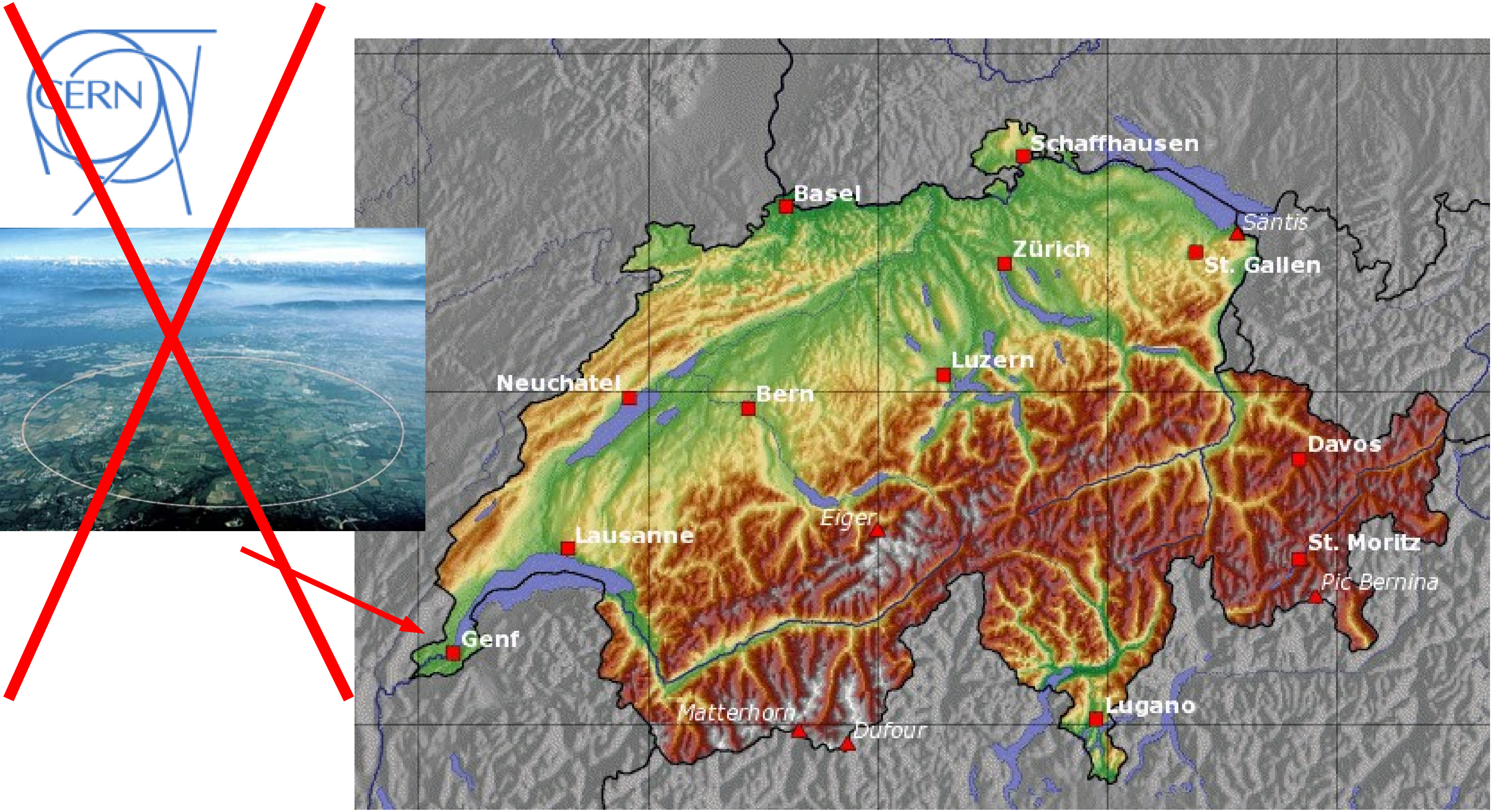
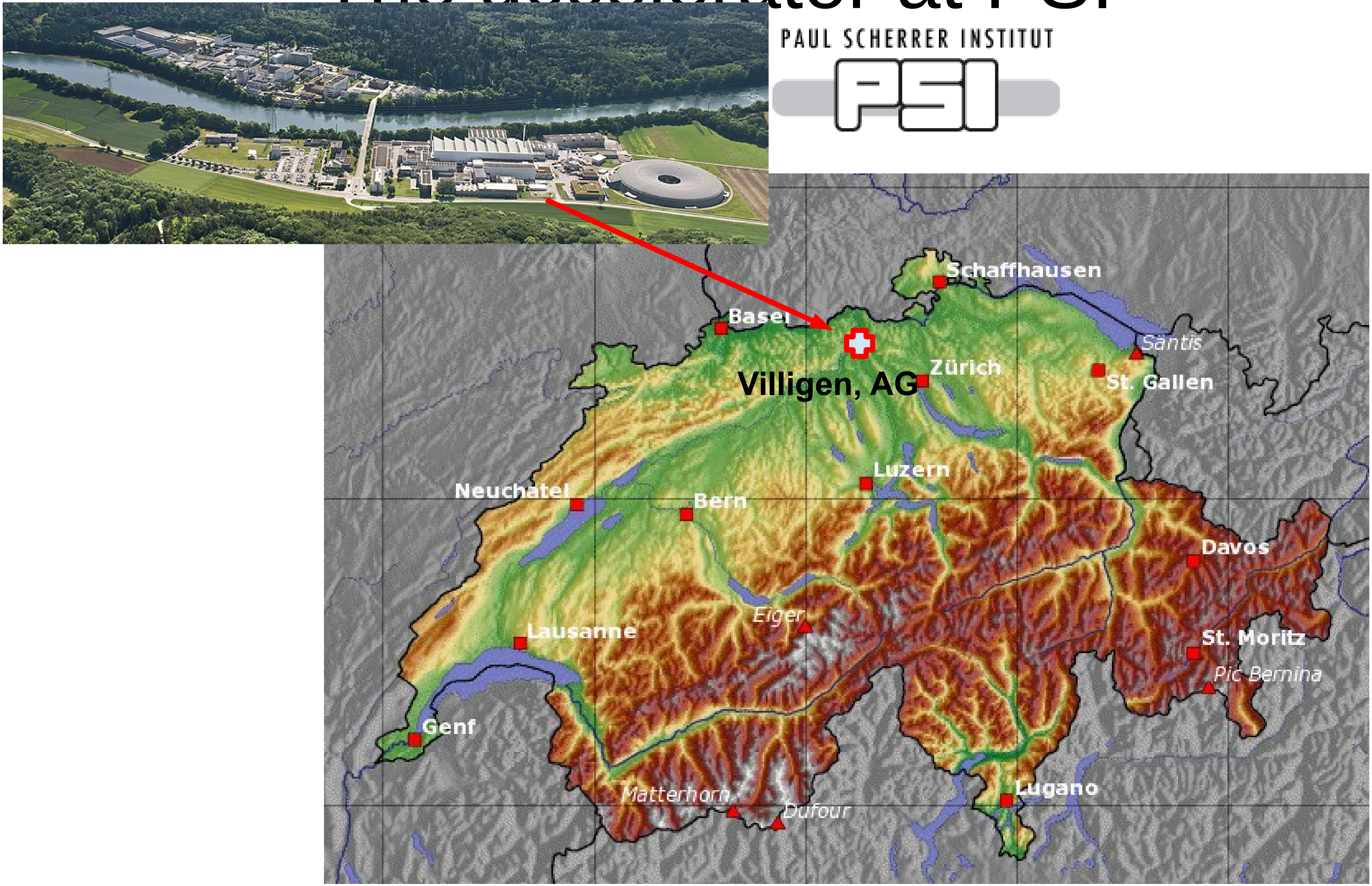


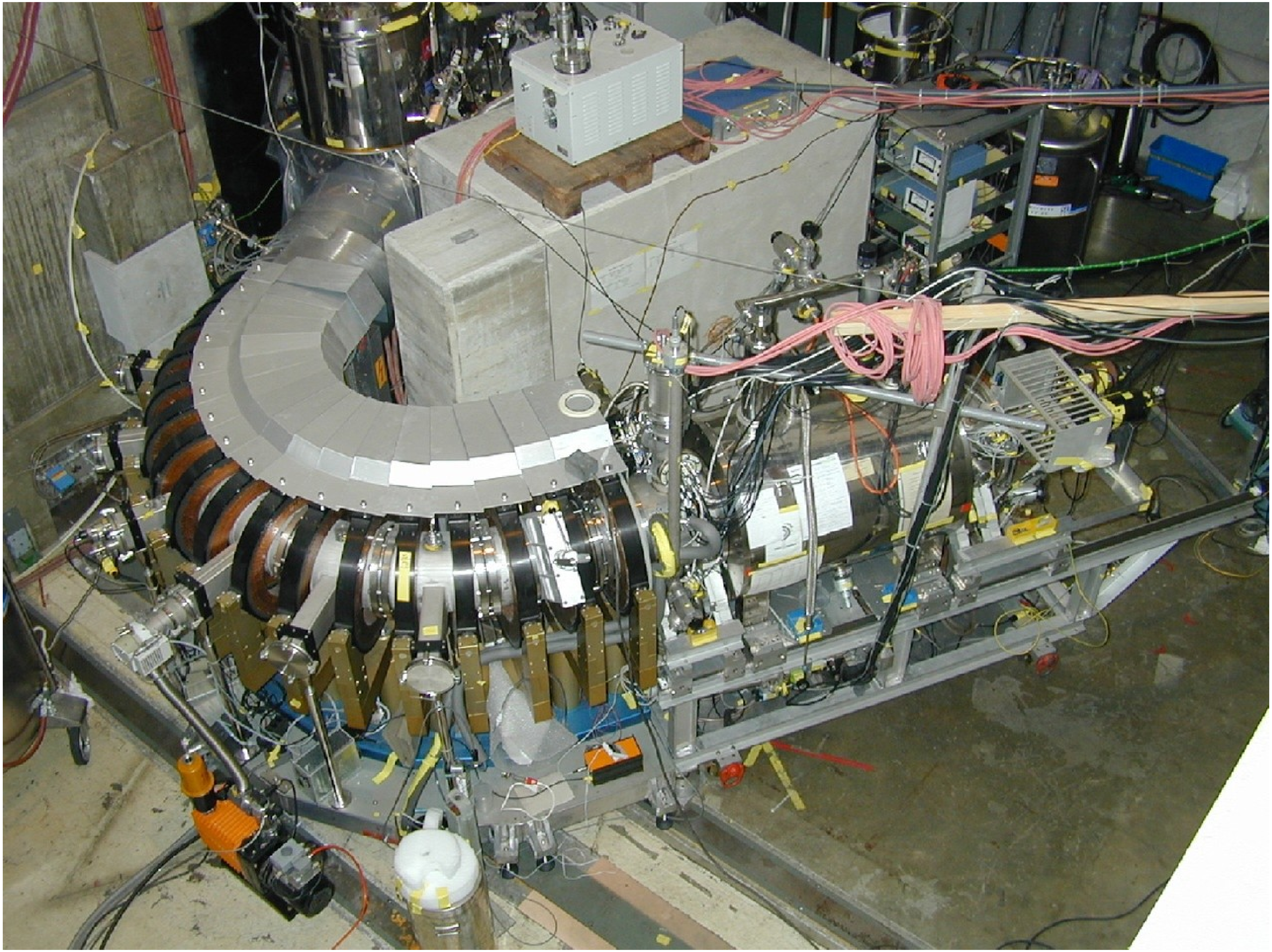
The accelerator at PSI

PAUL SCHERRER INSTITUT

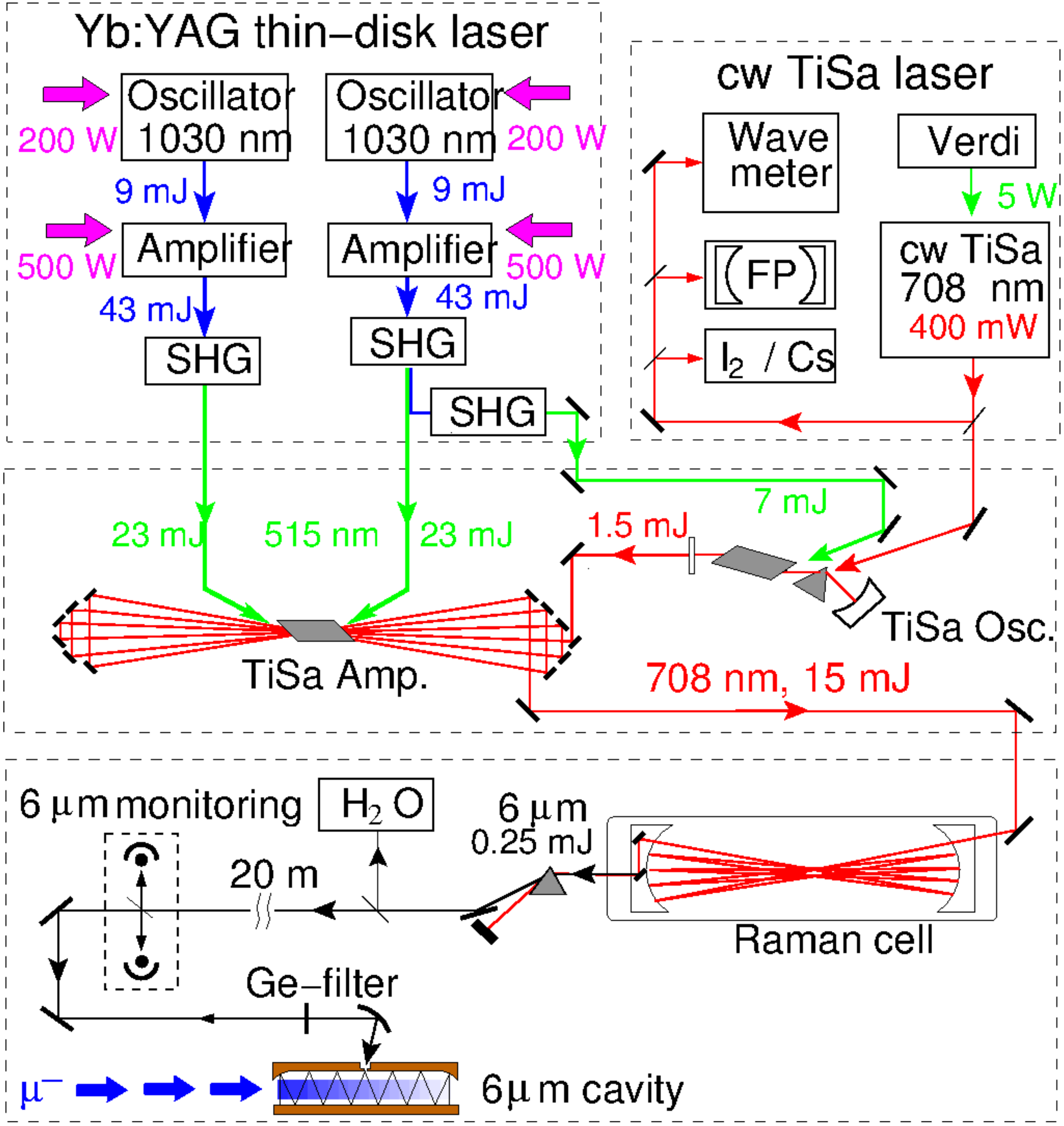
PSI



The muon beam line in πE5



The laser system



Yb:YAG Disk laser
→ fast response on μ

Frequency doubling (SHG)
→ green light to pump Ti:sapphire laser

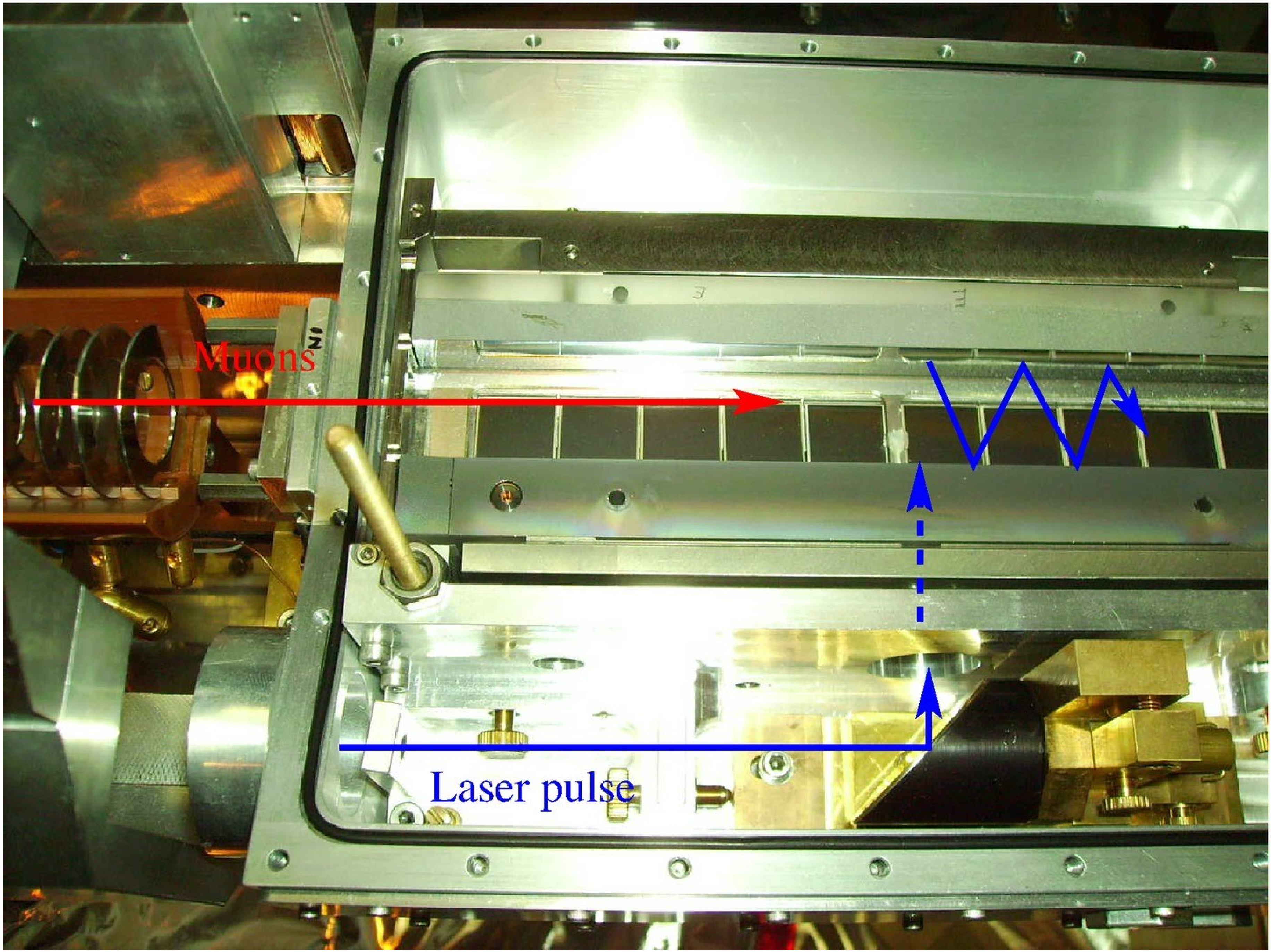
Ti:sapphire cw laser
→ determines laser frequency

Ti:sapphire MOPA
→ high pulse energy (15 mJ)

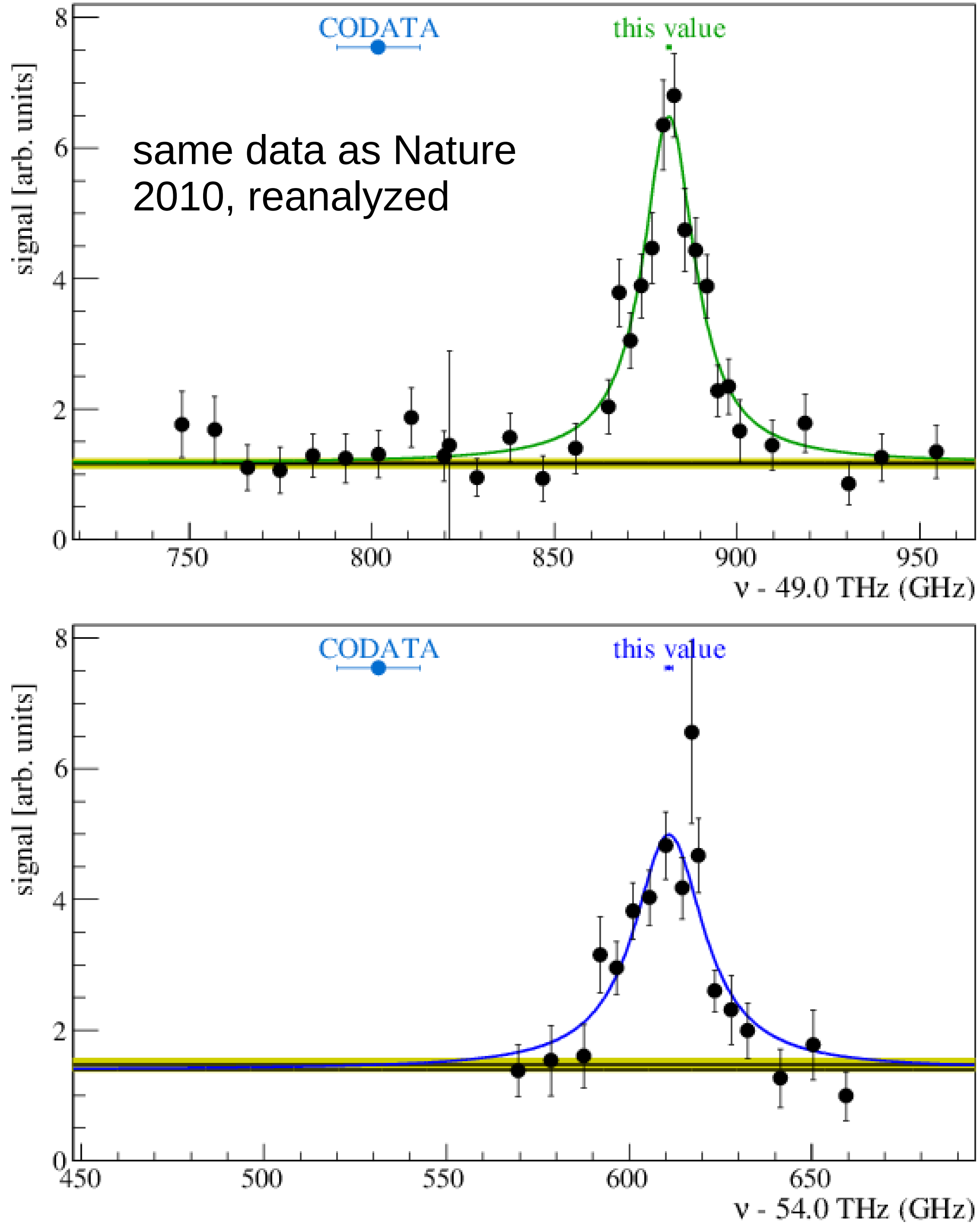
Raman cell
→ 3 sequential stimulated Raman Stokes shifts Laser wave length → 6 μm

Target Cavity
→ Mirror system to fill the muon stop volume (H_2)

The hydrogen target

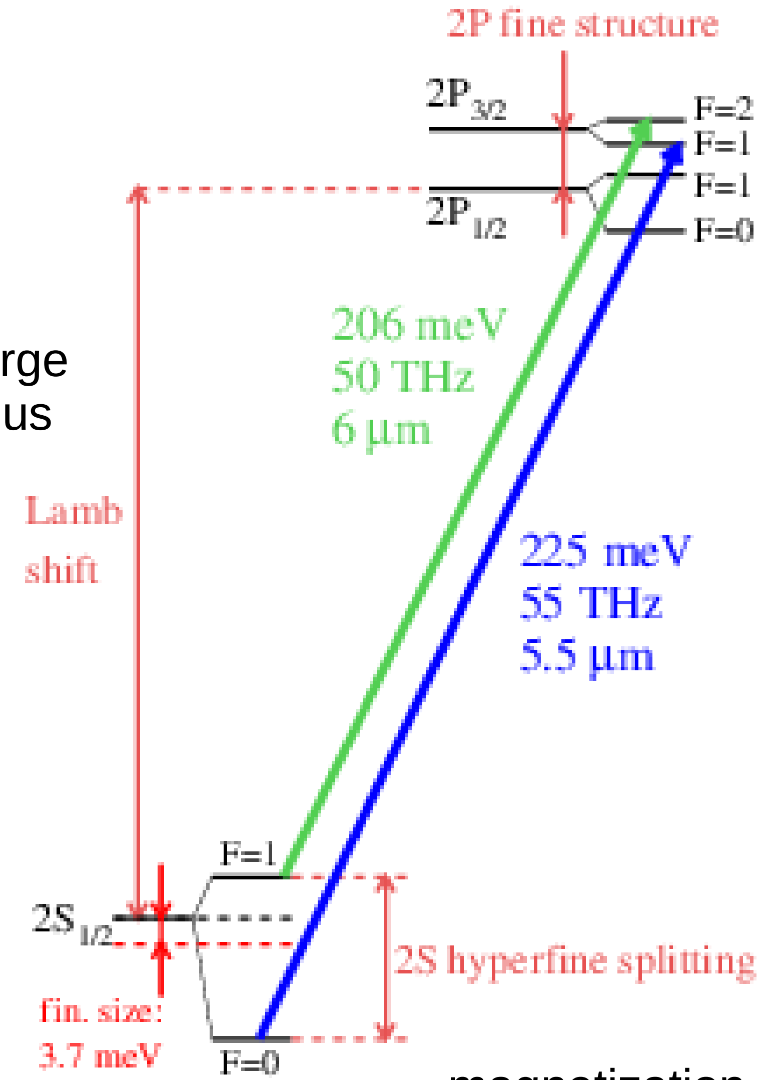


2 transitions in muonic H



same data as Nature 2010, reanalyzed

charge radius



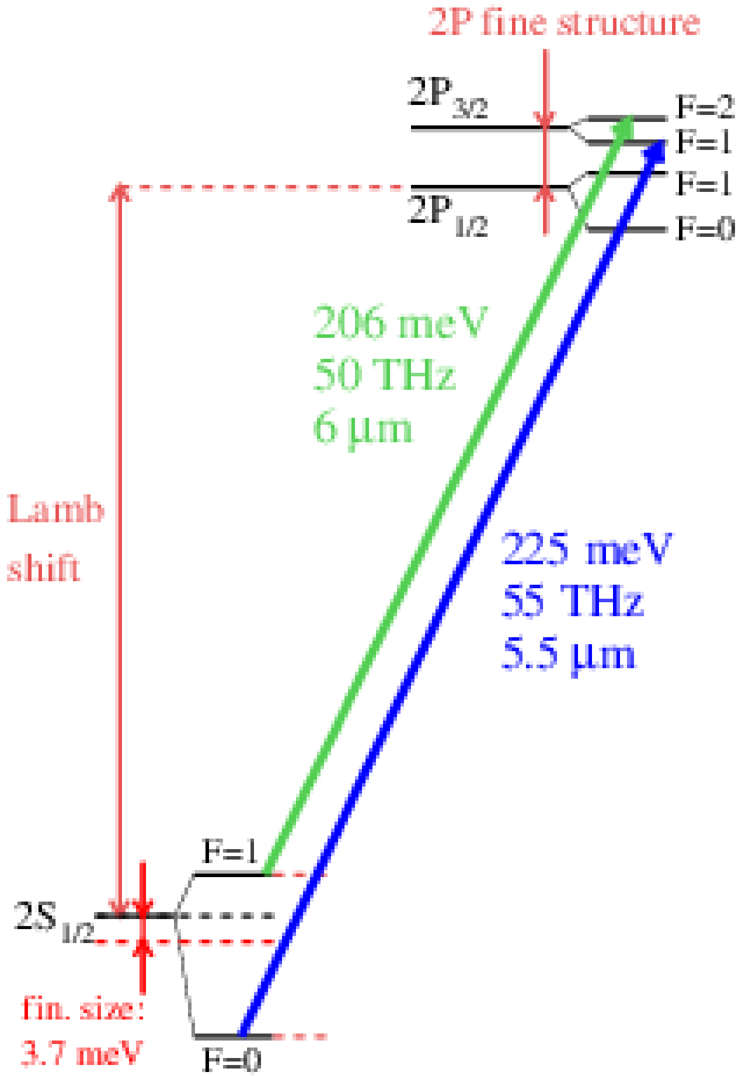
magnetization distribution (Zemach radius)

Antognini et al. (CREMA Coll.), Science 339, 417 (2013)

Theory in muonic H

$$\Delta E_{\text{Lamb}} = 206.0336\,(15)\ \text{meV}_{\text{QED}} + 0.0332\,(20)\ \text{meV}_{\text{TPE}} - 5.2275\,(10)\ \text{meV/fm}^2 * \mathbf{R_p}^{\mathbf{2}}$$

Simple-looking formula
based on decades of work by
E. Borie, M.C. Birse, P. Blunden, C.E. Carlson,
M.I. Eides, R. Faustov, J.L. Friar, G. Paz,
A. Pineda, J. McGovern, K. Griffioen, H. Grotch,
F. Hagelstein, H.-W. Hammer, R.J Hill, P.Indelicato,
U.D. Jentschura, S.G. Karshenboim, E.Y. Korzinin,
V.G. Ivanov, I.T. Lorenz, A.P. Martynenko,
G.A. Miller, U.-G. Meissner, P.J. Mohr,
K. Pachucki, V. Pascalutsa, J. Rafelski,
V.A. Shelyuto, I. Sick, A.W. Thomas,
M. Vanderhaeghen, V. Yerokhin,
…..
(shout if I missed your name!)

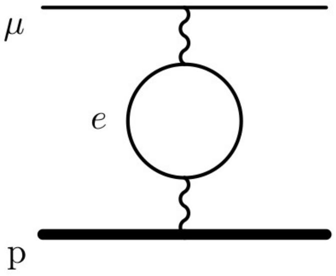


Antognini, RP at al., Ann. Phys. (N.Y.) 331, 127 (2013)

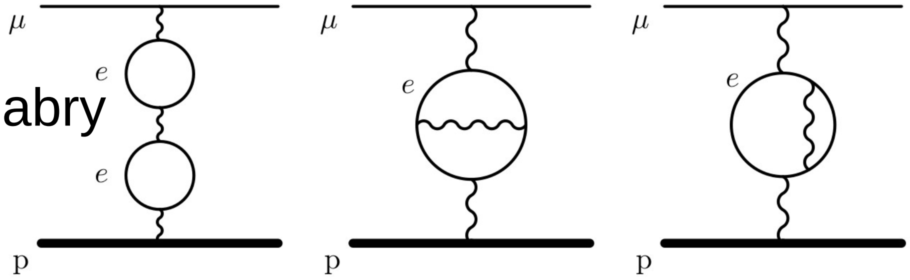
Theory in muonic H

$$\Delta E_{Lamb} = 206.0336\ (15)\ \mathrm{meV}_{QED} + 0.0332\ (20)\ \mathrm{meV}_{TPE} - 5.2275\ (10)\ \mathrm{meV/fm}^2 * \mathbf{R_p}^{\mathbf{2}}$$

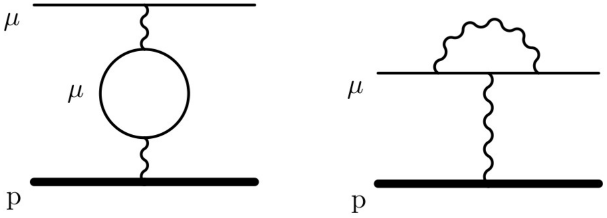
Uehling



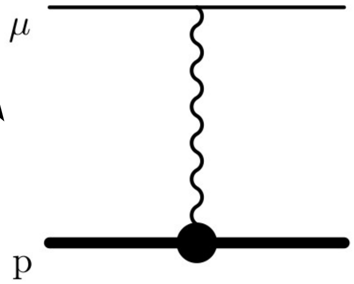
Källen-Sabry



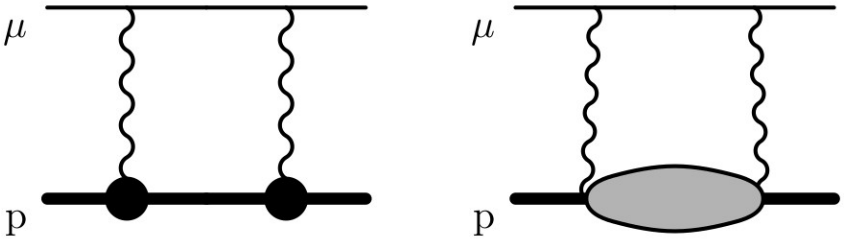
Muon SE+VP



and 20+ more….



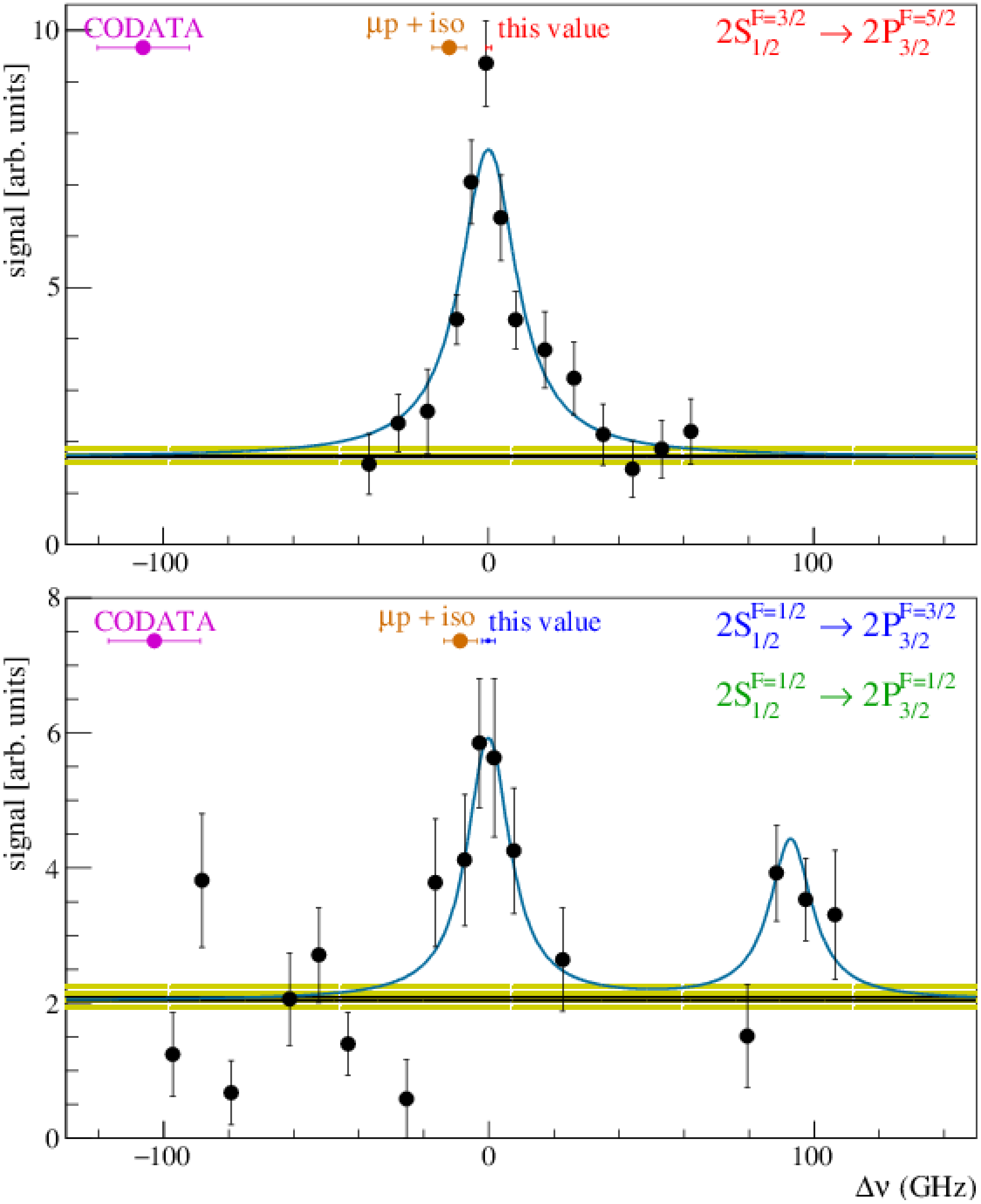
Proton form factor

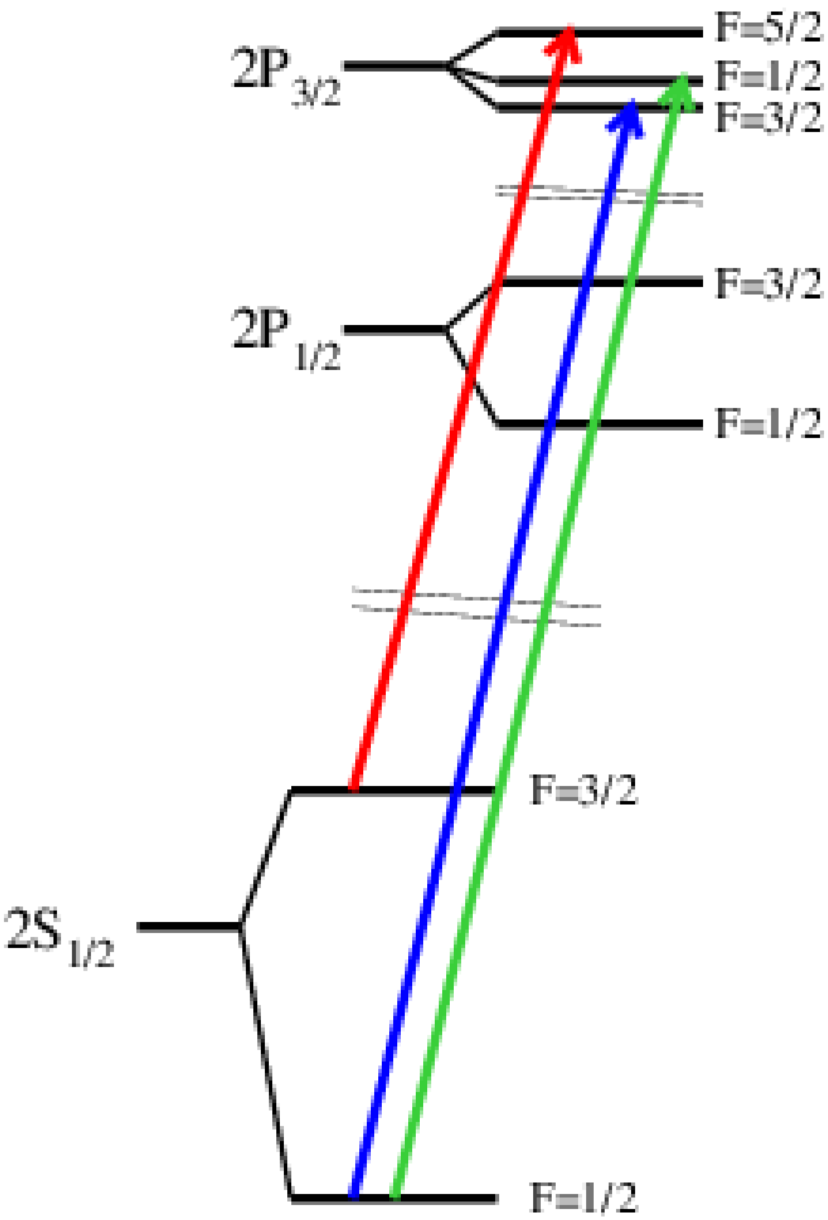


elastic and inelastic two-photon exchange
(Friar moment and polarizability)

Muonic Deuterium

2.5 transitions in muonic D



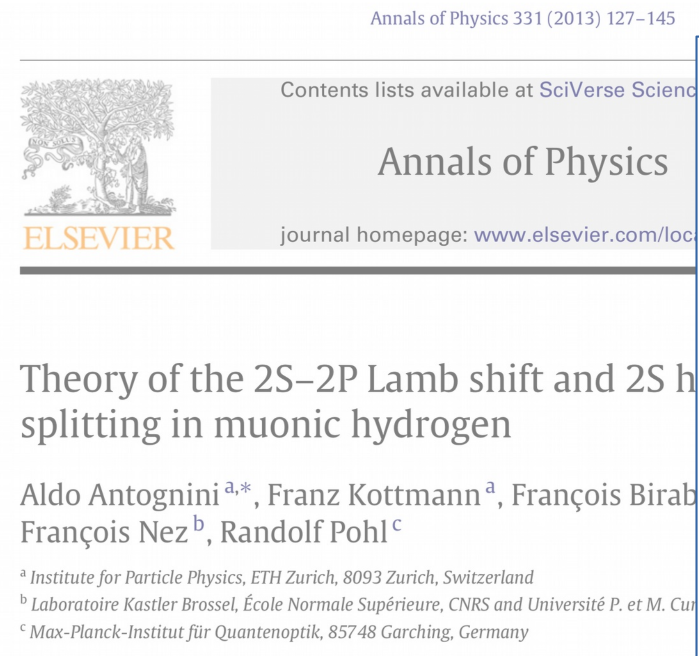


Pohl et al. (CREMA Coll.), Science 353, 669 (2016)

Theory in muonic D

$$\Delta E^{\mu D}_{\text{Lamb}} = 228.7854\ (13)\ \text{meV}_{\text{QED}} + 1.7500\ (\mathbf{210})\ \text{meV}_{\text{TPE}} - 6.1103\ (3)\ \text{meV/fm}^2 * \mathbf{R_d}^{\mathbf{2}}$$

$$\Delta E^{\mu H}_{\text{Lamb}} = 206.0336\ (15)\ \text{meV}_{\text{QED}} + 0.0332\ (\ \ 20)\ \text{meV}_{\text{TPE}} - 5.2275\ (10)\ \text{meV/fm}^2 * \mathbf{R_p}^{\mathbf{2}}$$



Annals of Physics 331 (2013) 127–145

Contents lists available at SciVerse Scienc

Annals of Physics

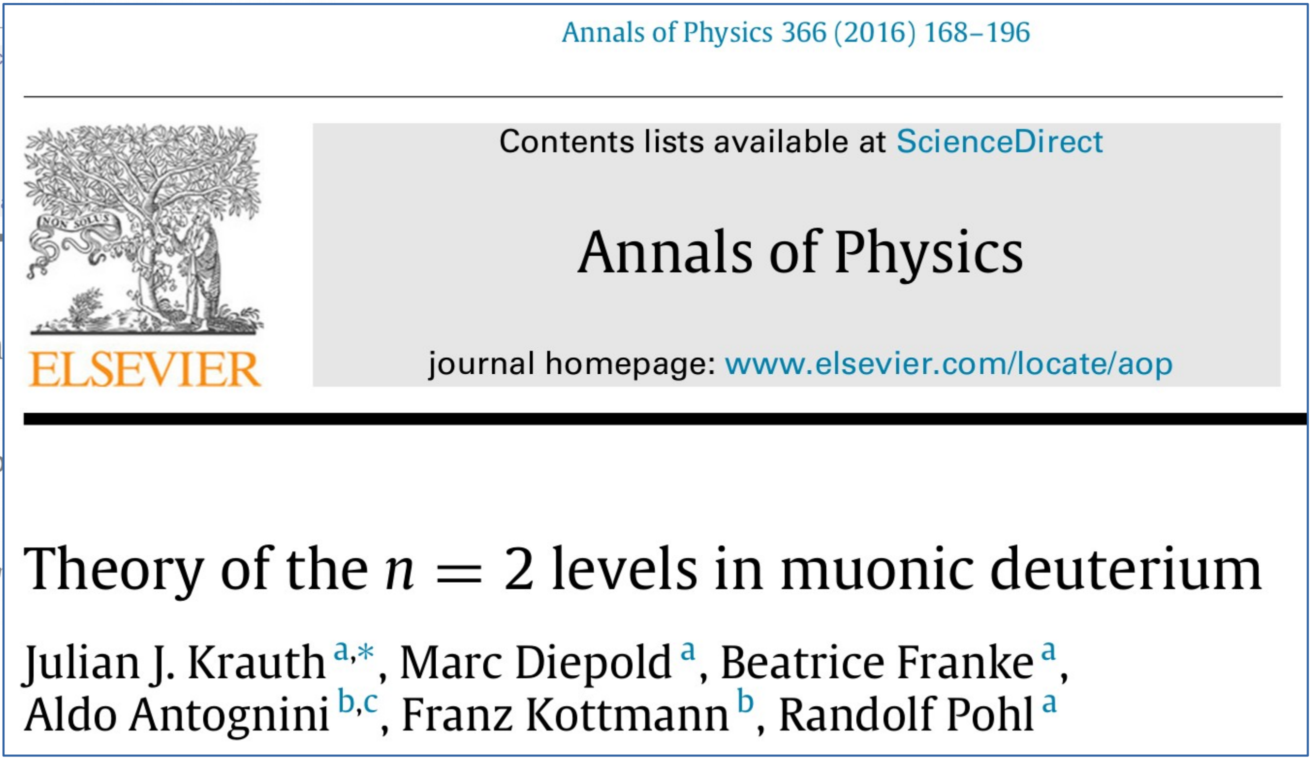
journal homepage: www.elsevier.com/loc

ELSEVIER

Theory of the 2S–2P Lamb shift and 2S h
splitting in muonic hydrogen

Aldo Antognini [a,*], Franz Kottmann [a], François Birab
François Nez [b], Randolf Pohl [c]

[a] Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland
[b] Laboratoire Kastler Brossel, École Normale Supérieure, CNRS and Université P. et M. Cu
[c] Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany



Annals of Physics 366 (2016) 168–196

Contents lists available at ScienceDirect

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ELSEVIER

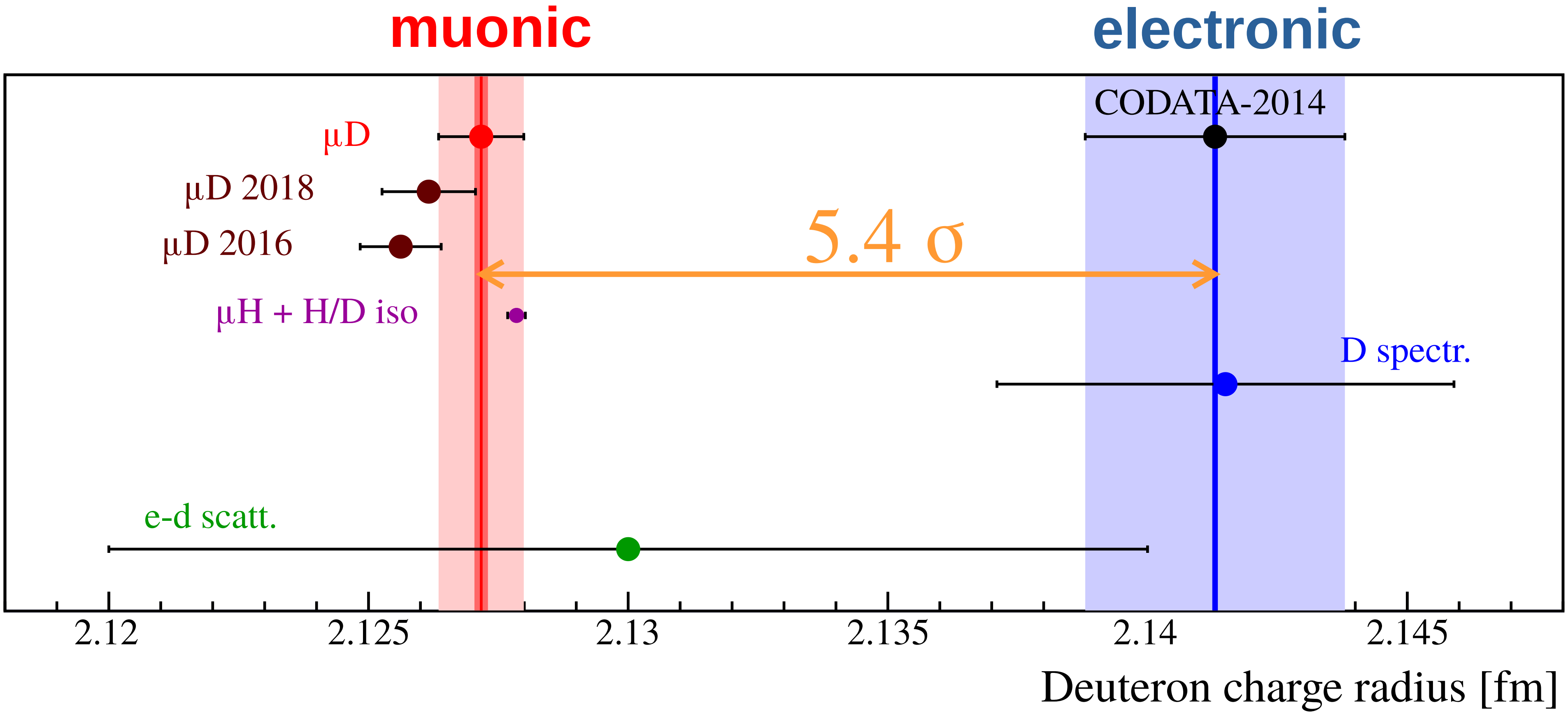
Theory of the $n = 2$ levels in muonic deuterium

Julian J. Krauth [a,*], Marc Diepold [a], Beatrice Franke [a],
Aldo Antognini [b,c], Franz Kottmann [b], Randolf Pohl [a]

Summarizes original work by: Bacca, Barnea, Birse, Borie, Carlson, Eides, Faustov, Friar, Gorchtein, Hernandez, Ivanov, Jentschura, Ji, Karshenboim, Korzinin, Krutov, Martynenko, McGovern, Nevo~Dinur, Pachucki, Shelyuto, Sick, Vanderhaeghen, et al.

Newer work: Pachucki et al., PRA 97, 062511 (2018), Hernandez et al., PLB 778, 377 (2018)

Muonic Deuterium



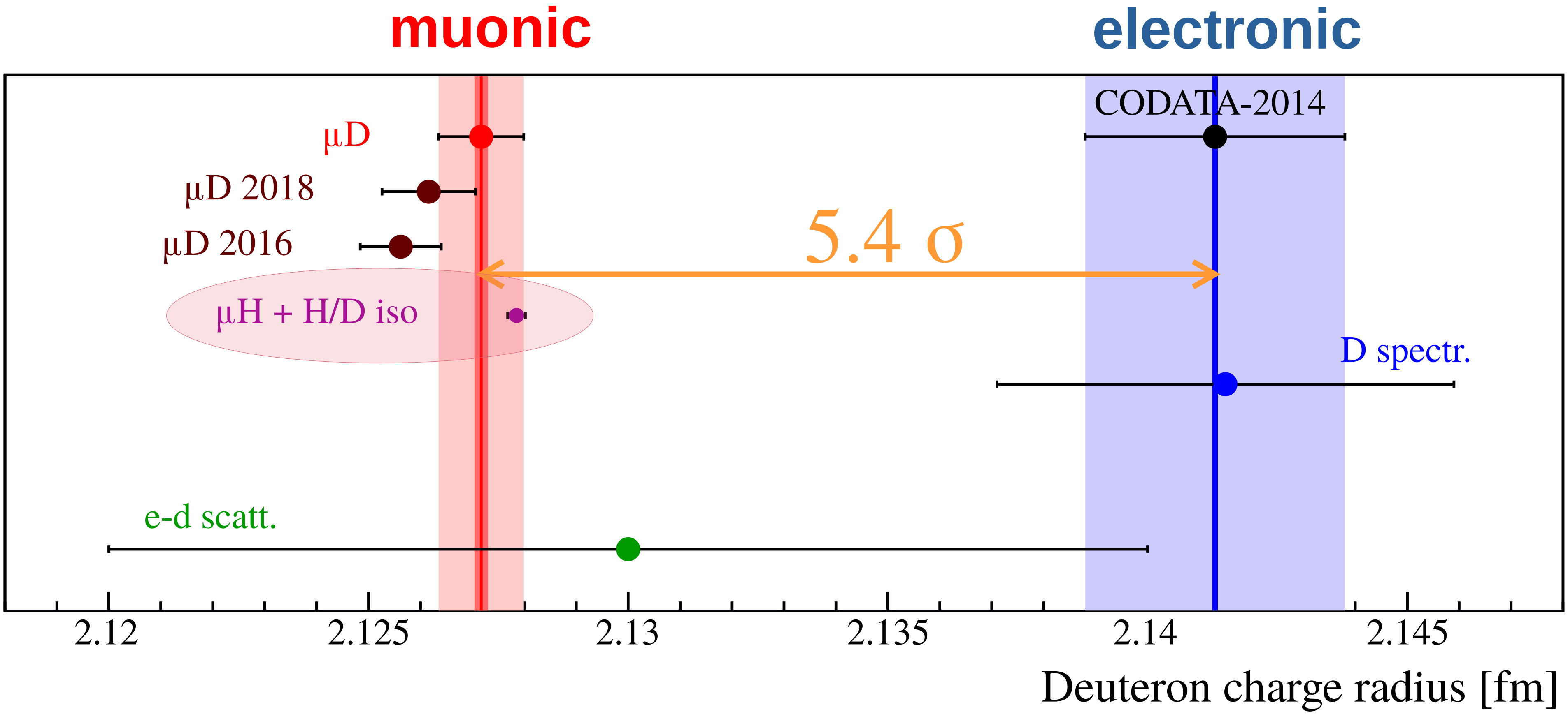
μD: 2.12717 $(13)_{exp}$ $(81)_{theo}$ fm (theo = nucl. polarizability)

CODATA-2014: 2.1**4**130 (250) fm

RP et al. (CREMA Coll.), Science 353, 559 (2016)

Krauth, RP at al., Ann. Phys. (N.Y.) 366, 168 (2016)
+ Pachucki et al., PRA 97, 062511 (2018)
+ Hernandez et al., PLB 778, 377 (2018)
+ Kalinowski, arXiv 1812.10993

Muonic Deuterium



μD: 2.12717 $(13)_{exp}$ $(82)_{theo}$ fm (theo = nucl. polarizability)

μH + H/D(1S-2S): 2.12785 (17) fm

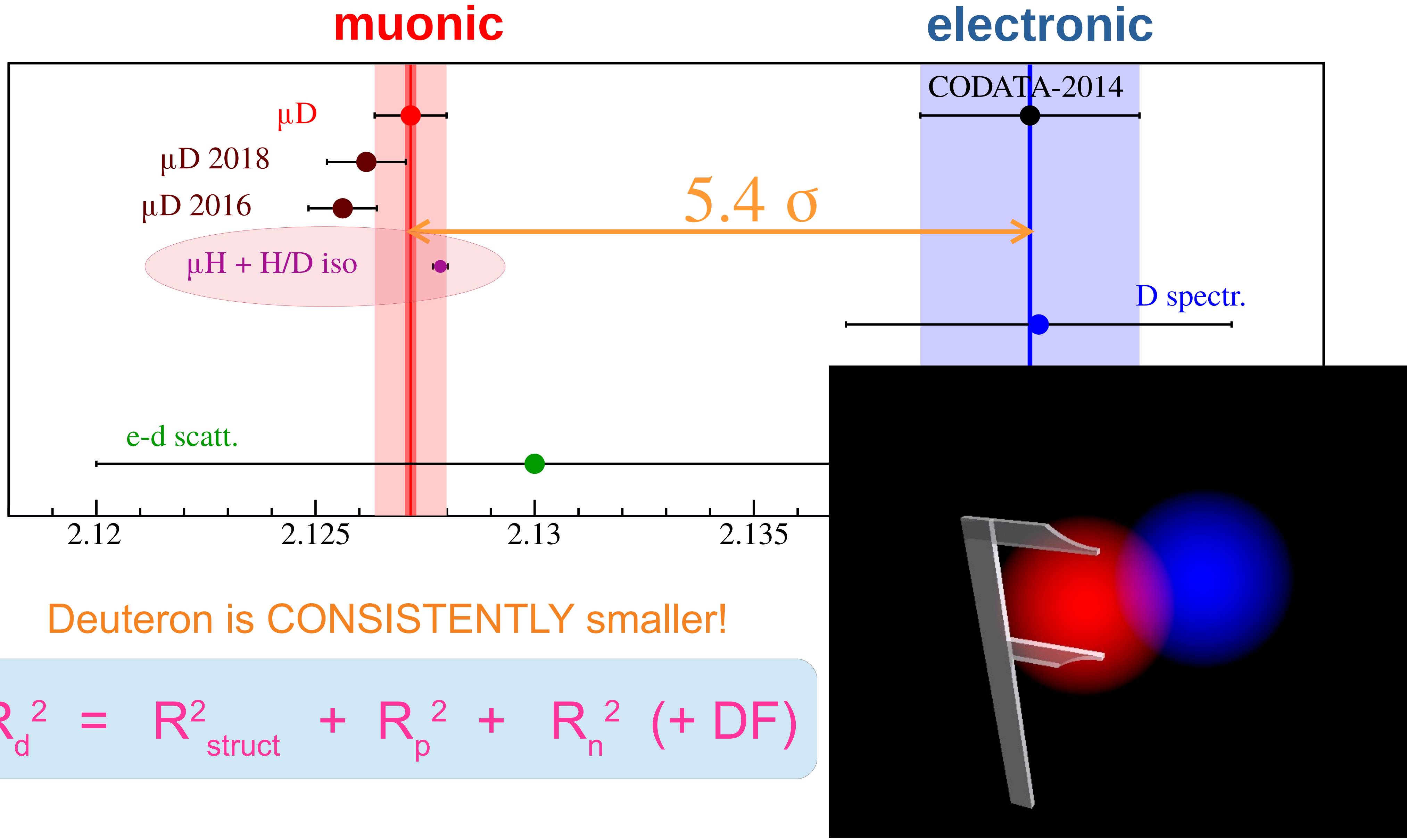
CODATA-2014: 2.1**4**130 (250) fm

H/D 1S-2S isotope shift:
$r_d^2 - r_p^2 = 3.82070(31)\ \mathrm{fm}^2$

Pachucki et al., PRA 97, 062511 (2018)

H/D 1S-2S. Parthey, RP et al. (MPQ Garching), PRL 104, 233001 (2010)
PRL 107, 203001 (2011)

Muonic Deuterium



Deuteron is CONSISTENTLY smaller!

$$R_d^2 = R^2_{struct} + R_p^2 + R_n^2 \quad (+ \mathrm{DF})$$

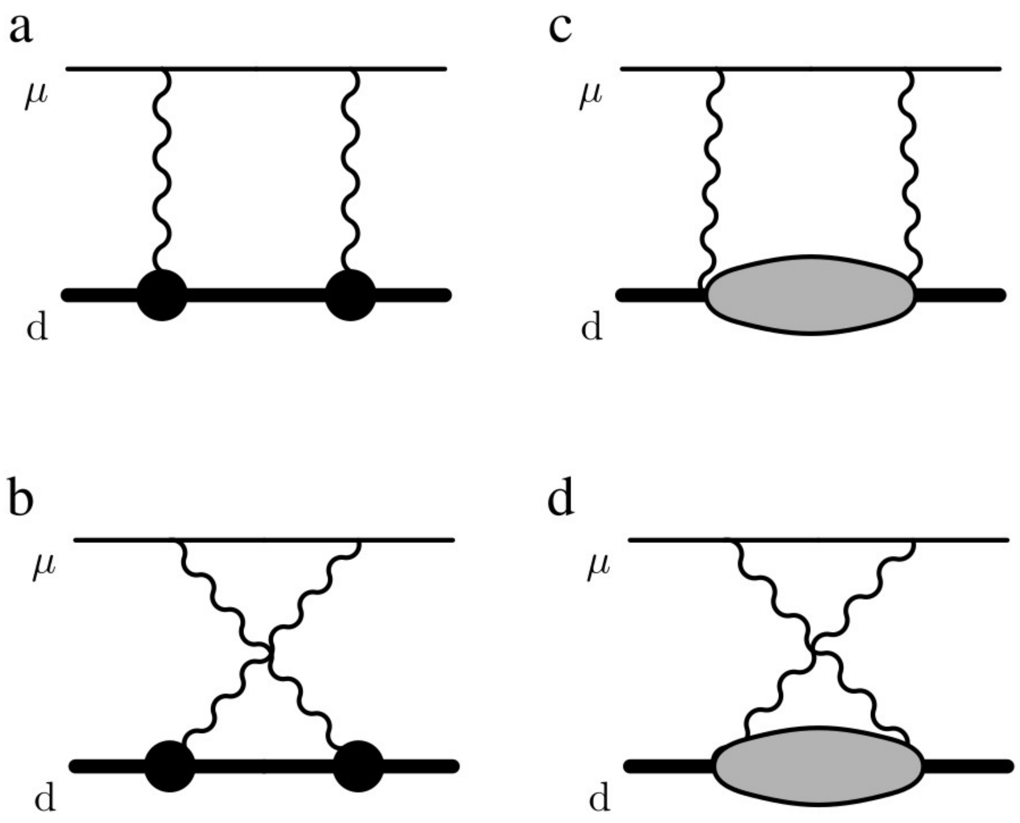
Pohl et al. (CREMA), Science 353, 669 (2016)

Theory in muonic D

$$\Delta E^{\mu D}_{\mathrm{Lamb}} = 228.7854\ (13)\ \mathrm{meV}_{\mathrm{QED}} + 1.7500\ (\mathbf{210})\ \mathrm{meV}_{\mathrm{TPE}} - 6.1103\ (3)\ \mathrm{meV/fm}^2 * \mathbf{R_d}^{\mathbf{2}}$$

Krauth, RP at al., Ann. Phys. (N.Y.) 366, 168 (2016)
+ Pachucki et al., PRA 97, 062511 (2018)
+ Hernandez et al., PLB 778, 377 (2018)
+ Kalinowski, arXiv 1812.10993

Two-photon nuclear structure contributions to the Lamb shift in muonic deuterium.



Theory in muonic D

$$\Delta E^{\mathbf{\mu D}}_{\mathrm{Lamb}} = 228.7854\ (13)\ \mathrm{meV}_{\mathrm{QED}} + 1.7500\ (\mathbf{210})\ \mathrm{meV}_{\mathrm{TPE}} - 6.1103\ (3)\ \mathrm{meV/fm}^2 * \mathbf{R_d^2}$$

ΔE_{TPE} (theo) = 1.7500 +- 0.0210 meV (Kalinowski, 2018)

vs. +- 0.0034 meV experimental uncertainty

(1) **charge radius**, using **calculated TPE**

r_d (μD) = 2.12717 (13) $_{\mathrm{exp}}$ (82) $_{\mathrm{theo}}$ fm vs.

r_d (CODATA-14) = 2.1**4**130 (250) fm

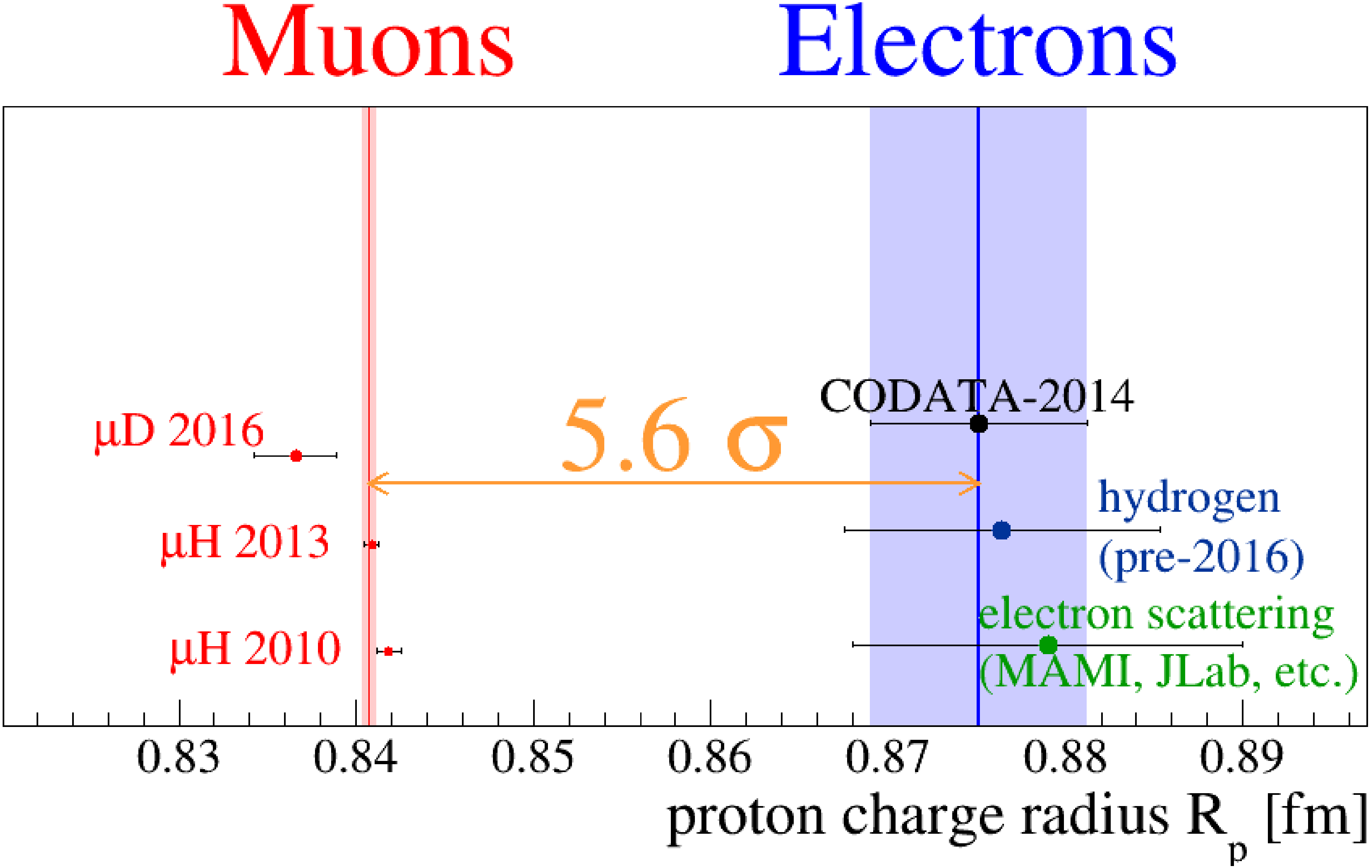
(2) **polarizability**, using **charge radius from isotope shift**

ΔE_{TPE} (theo) = 1.7500 (210) meV vs.

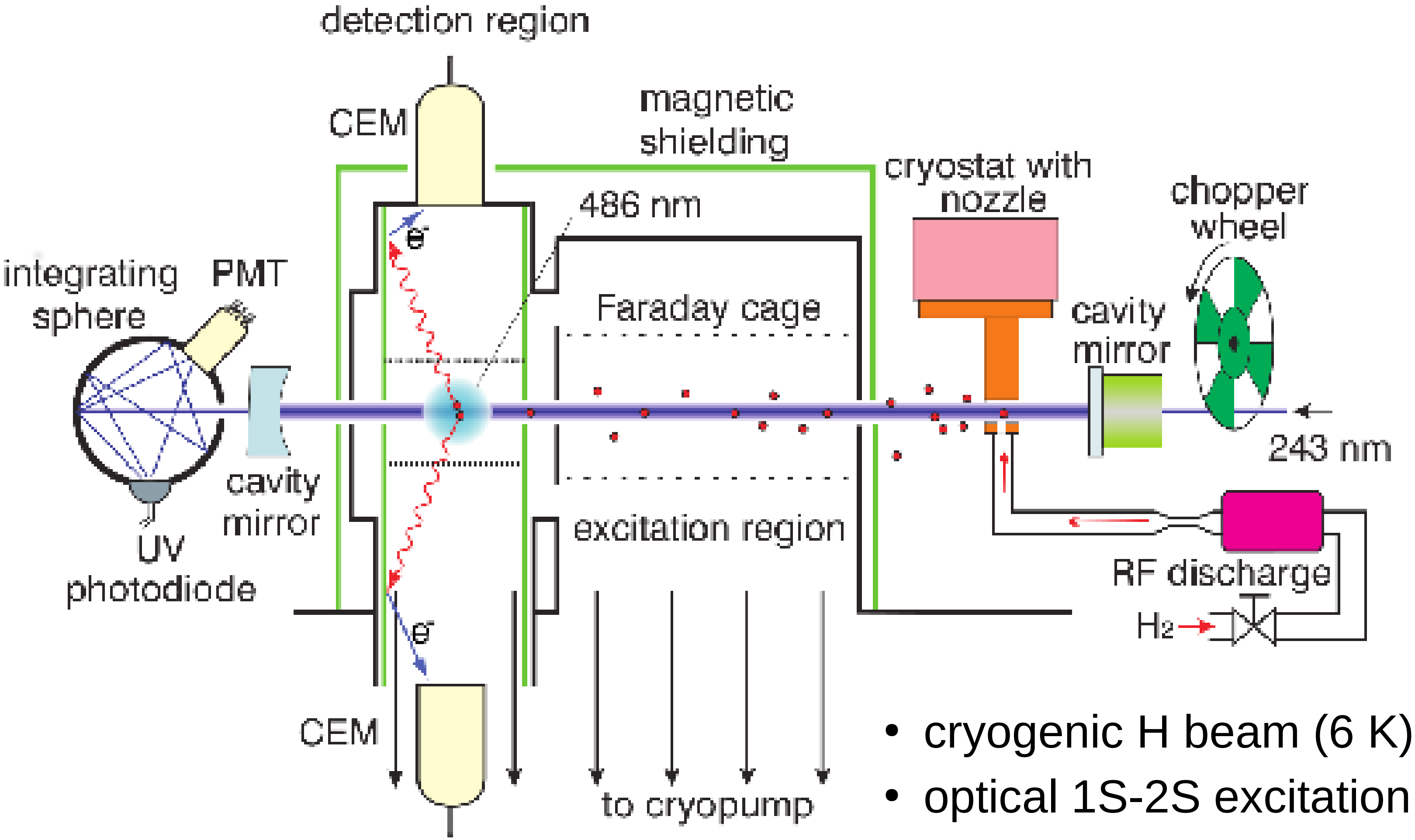
ΔE_{TPE} (exp) = 1.7591 (59) meV 3.5x more accurate

Krauth et al. (2016) + Pachucki et al. (2018) + Hernandez et al. (2018) + Kalinowski (2018)

The “Proton Radius Puzzle”



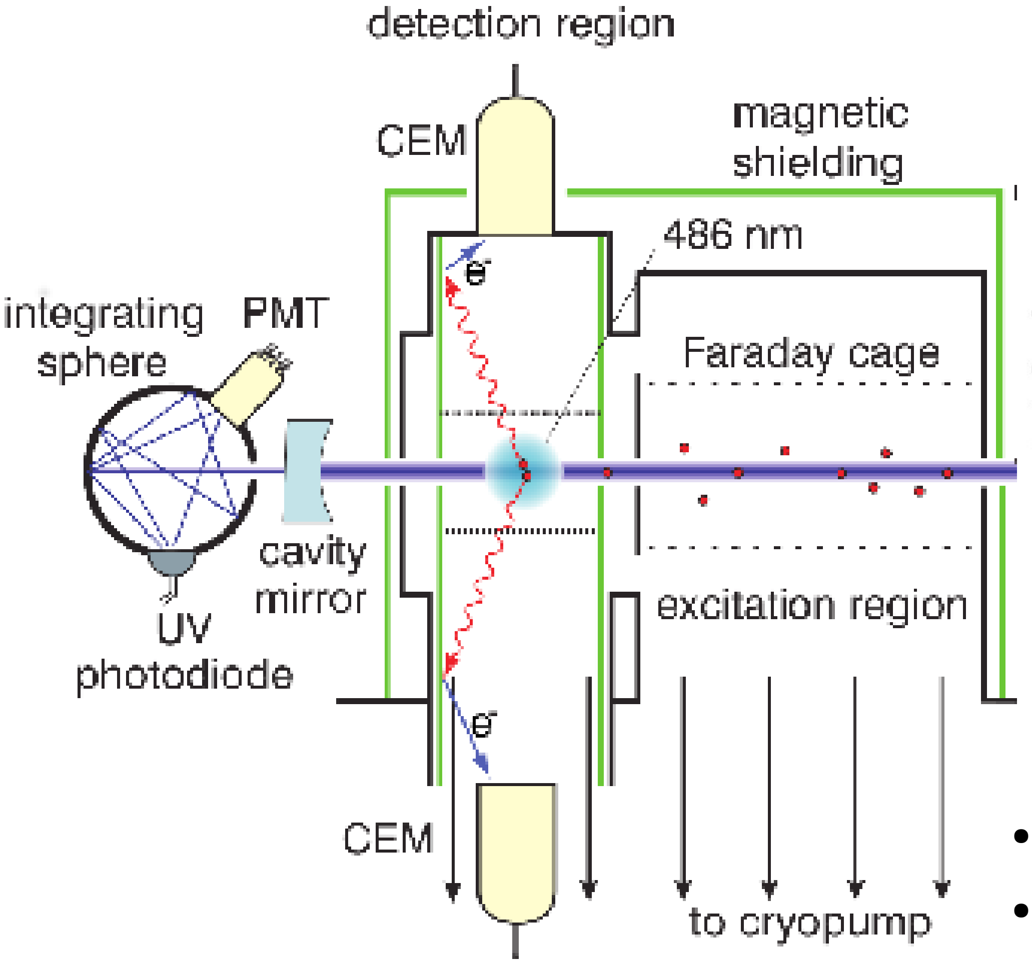
Garching H(2S-4P)

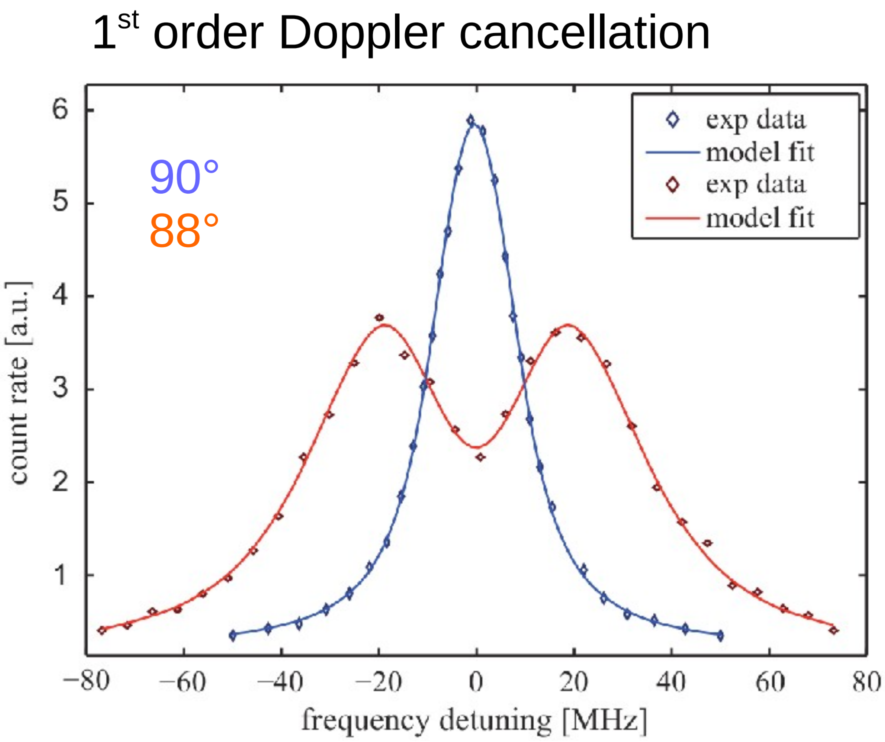


- cryogenic H beam (6 K)
- optical 1S-2S excitation (2S, F=0)
- 2S-4P transition is 1-photon: retroreflector
- split line to 10^{-4} !!!
- 2.3 kHz vs. 9 kHz PRP
- large systematics

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Garching H(2S-4P)





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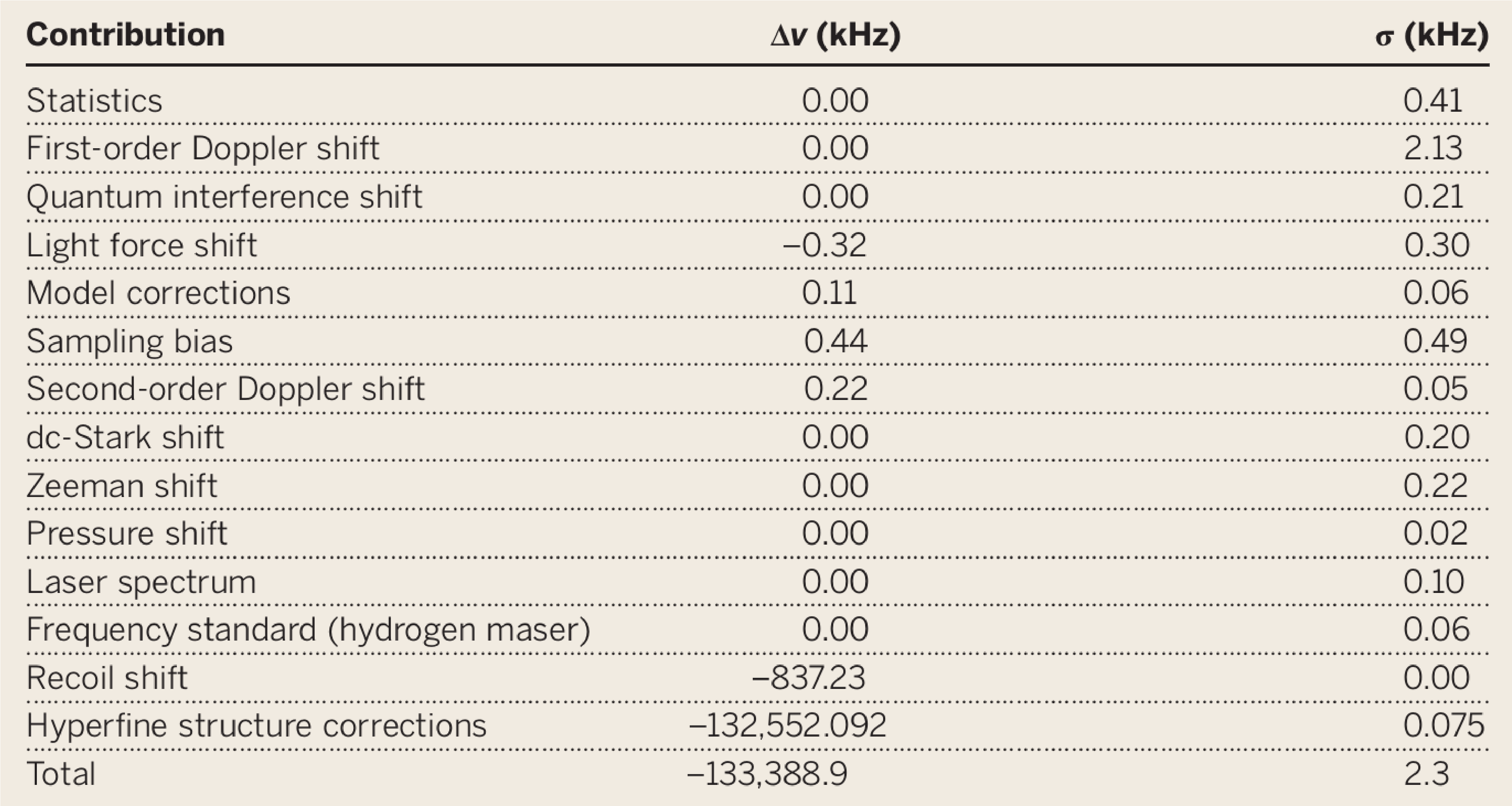
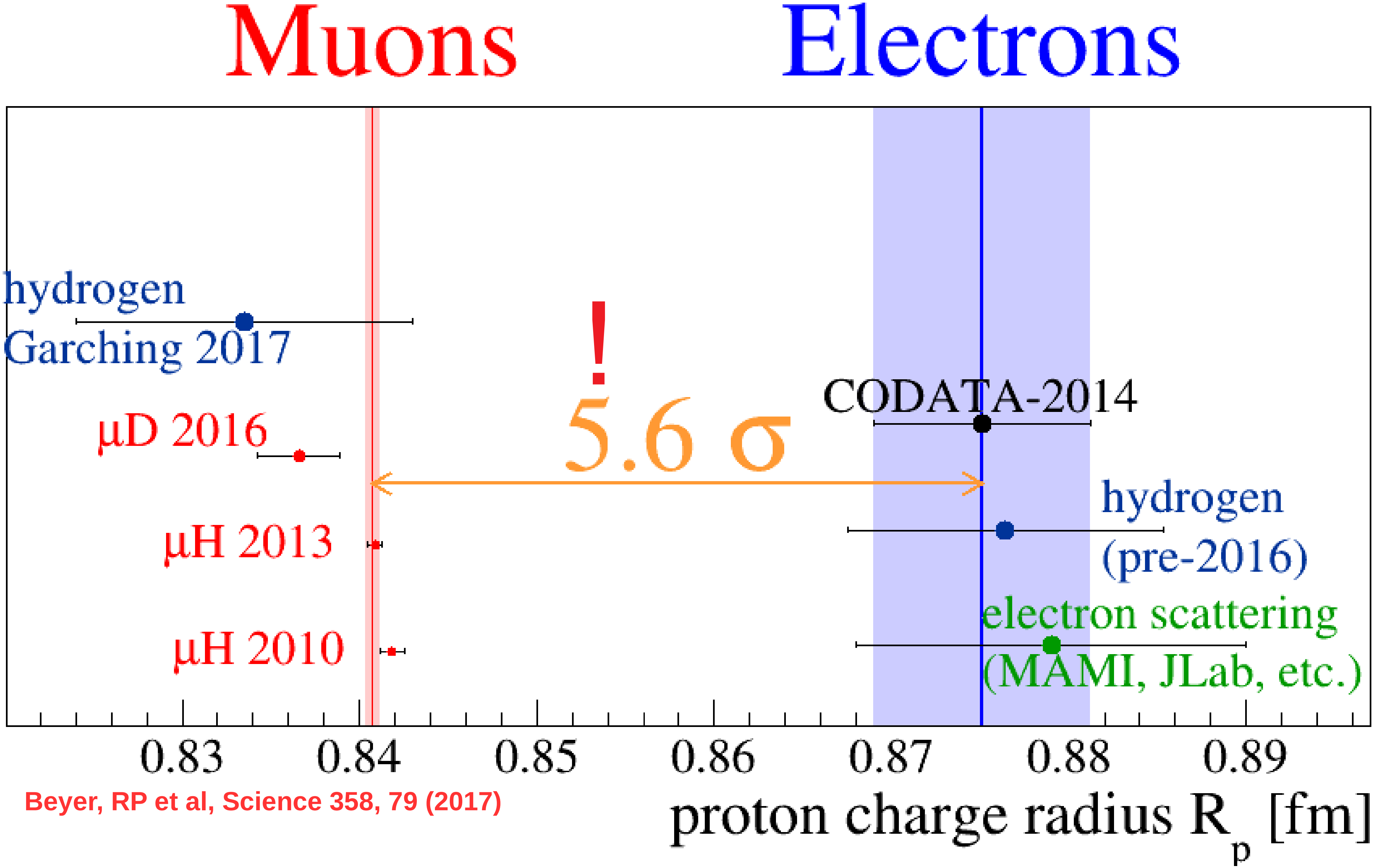
Systematics



Contribution	**Δν (kHz)**	**σ (kHz)**
Statistics	0.00	0.41
First-order Doppler shift	0.00	2.13
Quantum interference shift	0.00	0.21
Light force shift	–0.32	0.30
Model corrections	0.11	0.06
Sampling bias	0.44	0.49
Second-order Doppler shift	0.22	0.05
dc-Stark shift	0.00	0.20
Zeeman shift	0.00	0.22
Pressure shift	0.00	0.02
Laser spectrum	0.00	0.10
Frequency standard (hydrogen maser)	0.00	0.06
Recoil shift	–837.23	0.00
Hyperfine structure corrections	–132,552.092	0.075
Total	–133,388.9	2.3

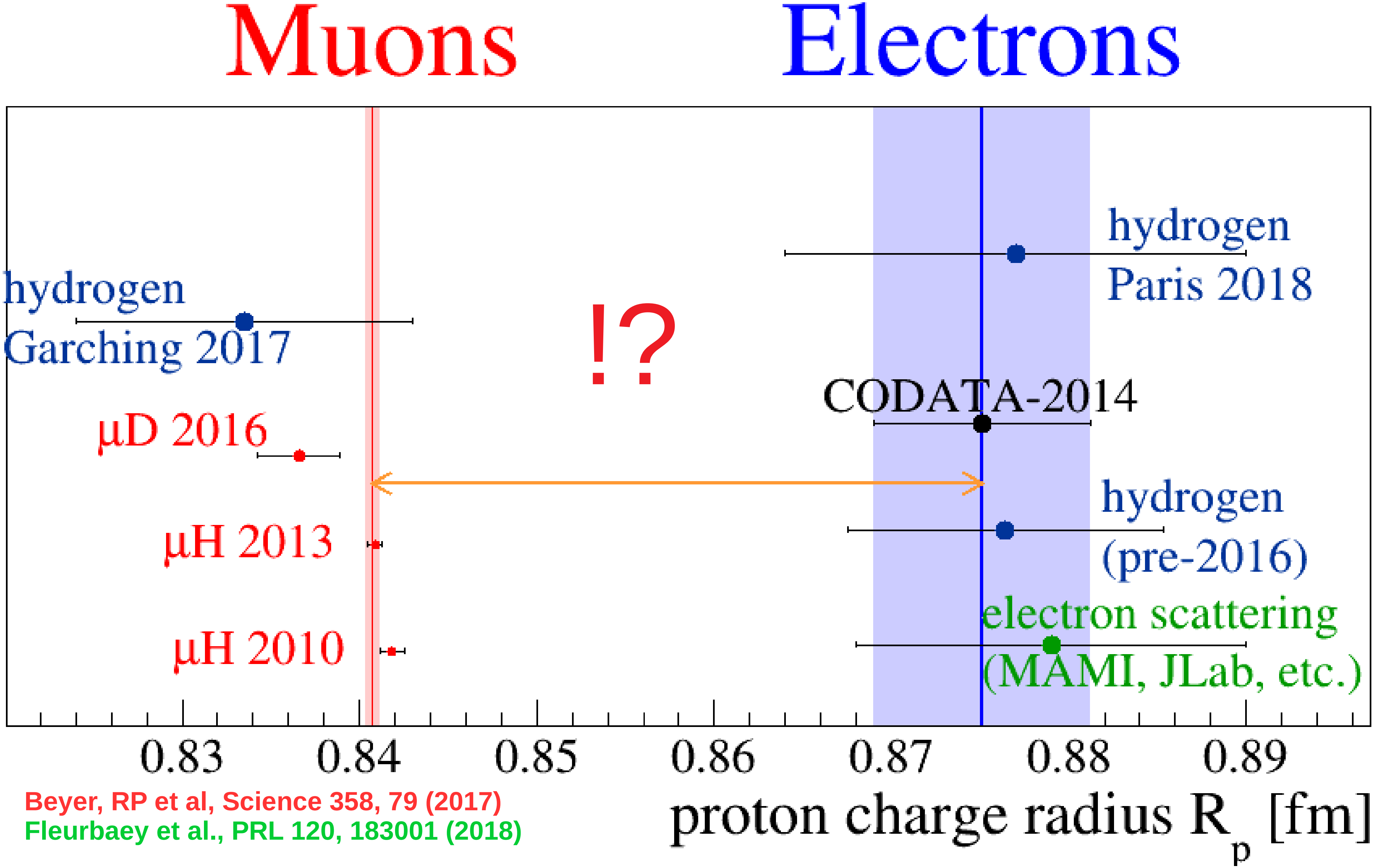
Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

New Measurements: Garching 2S-4P



Beyer, RP et al, Science 358, 79 (2017)

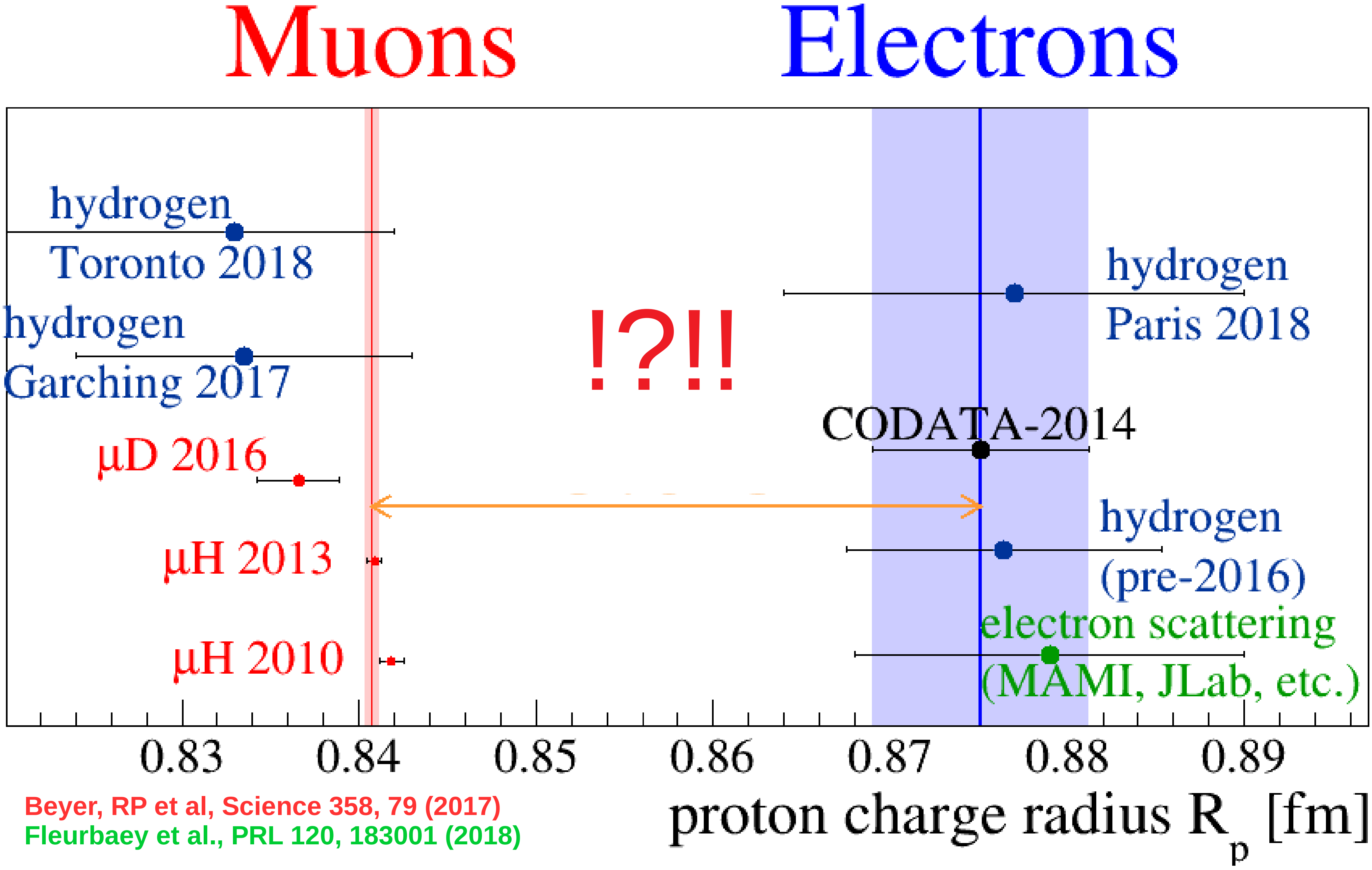
New Measurements: Paris 1S-3S



Beyer, RP et al, Science 358, 79 (2017)

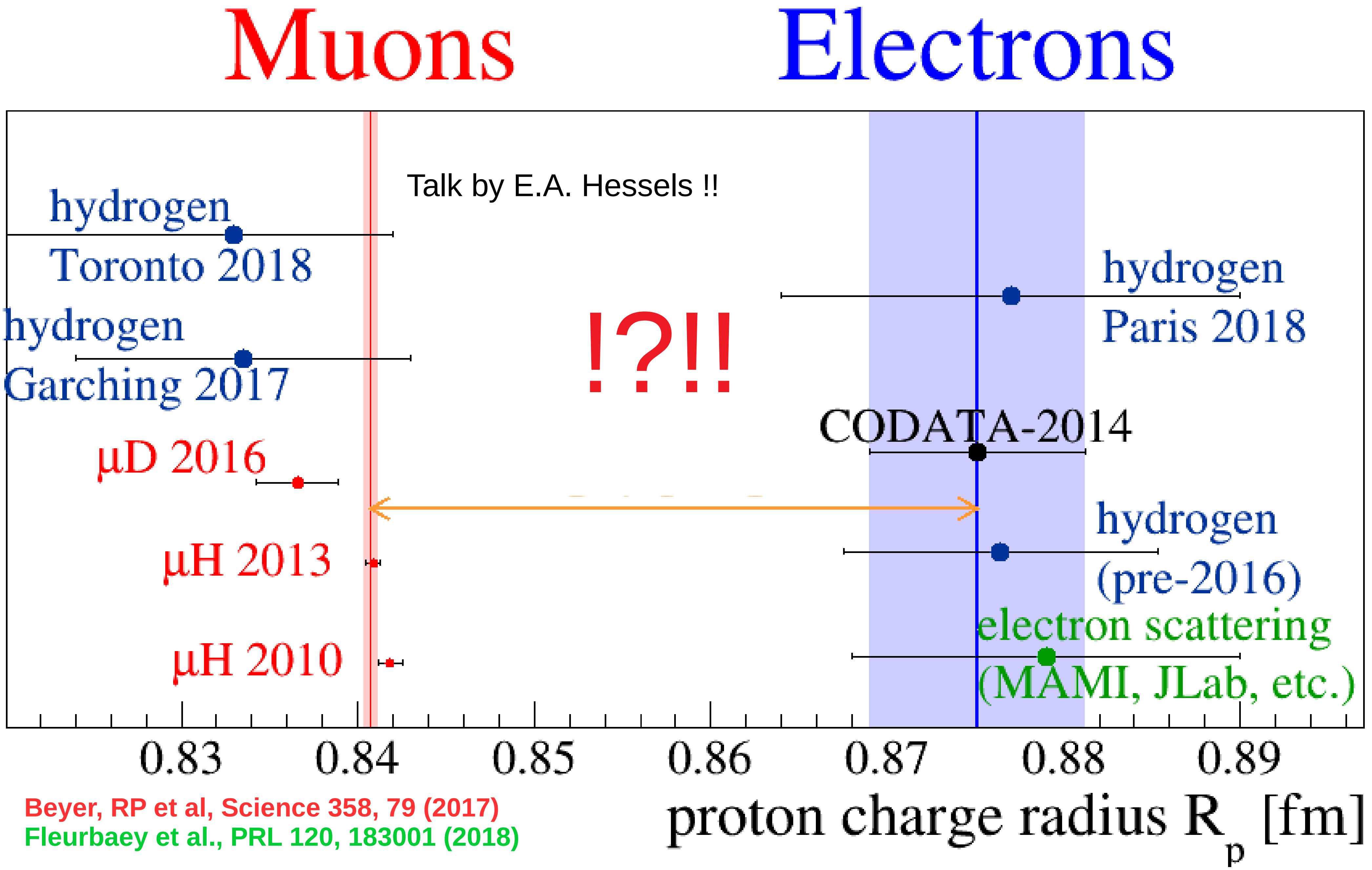
Fleurbaey et al., PRL 120, 183001 (2018)

New Measurements: Toronto 2S-2P



Beyer, RP et al, Science 358, 79 (2017)
Fleurbaey et al., PRL 120, 183001 (2018)

New Measurements: Toronto 2S-2P



Talk by E.A. Hessels !!

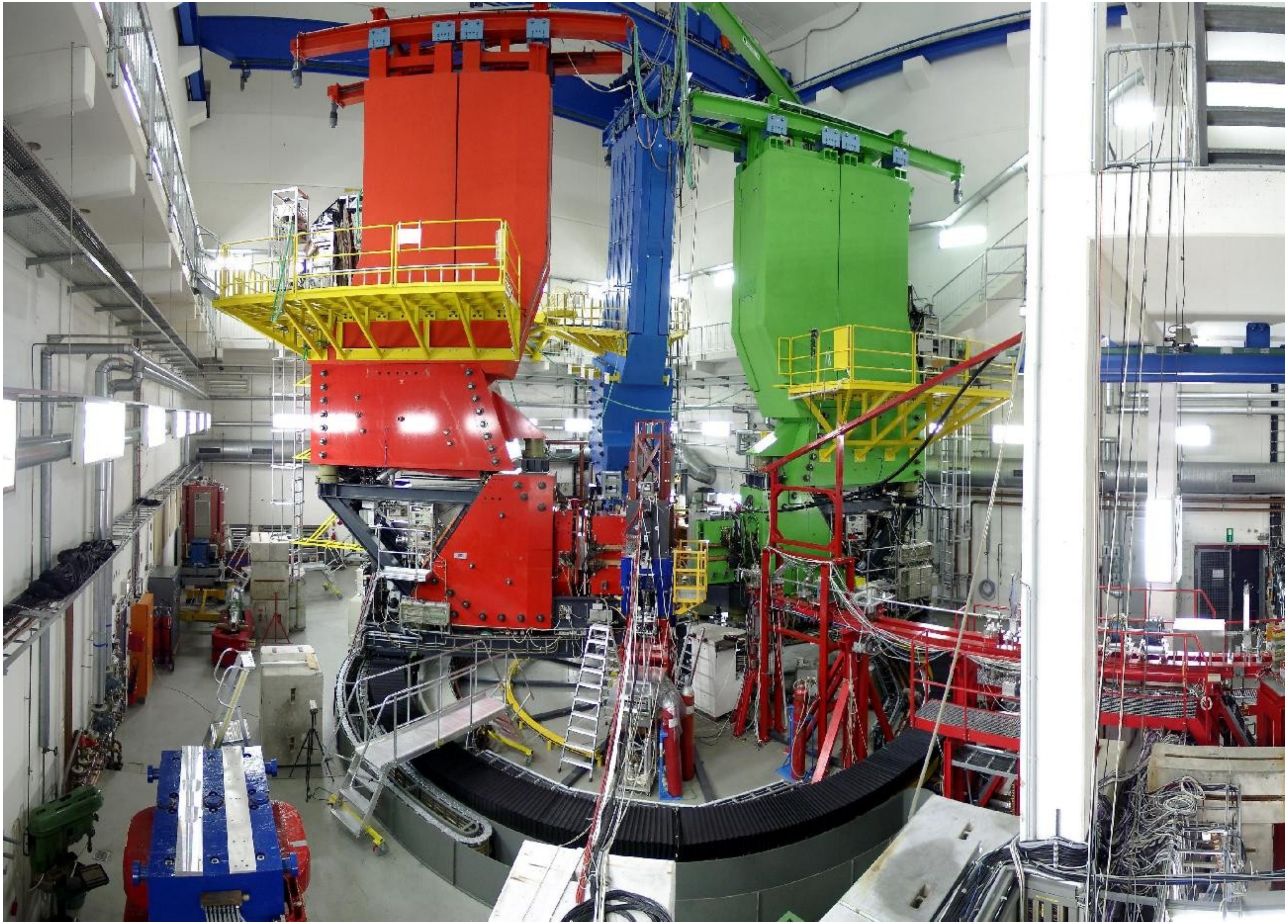
!?!!

Beyer, RP et al, Science 358, 79 (2017)

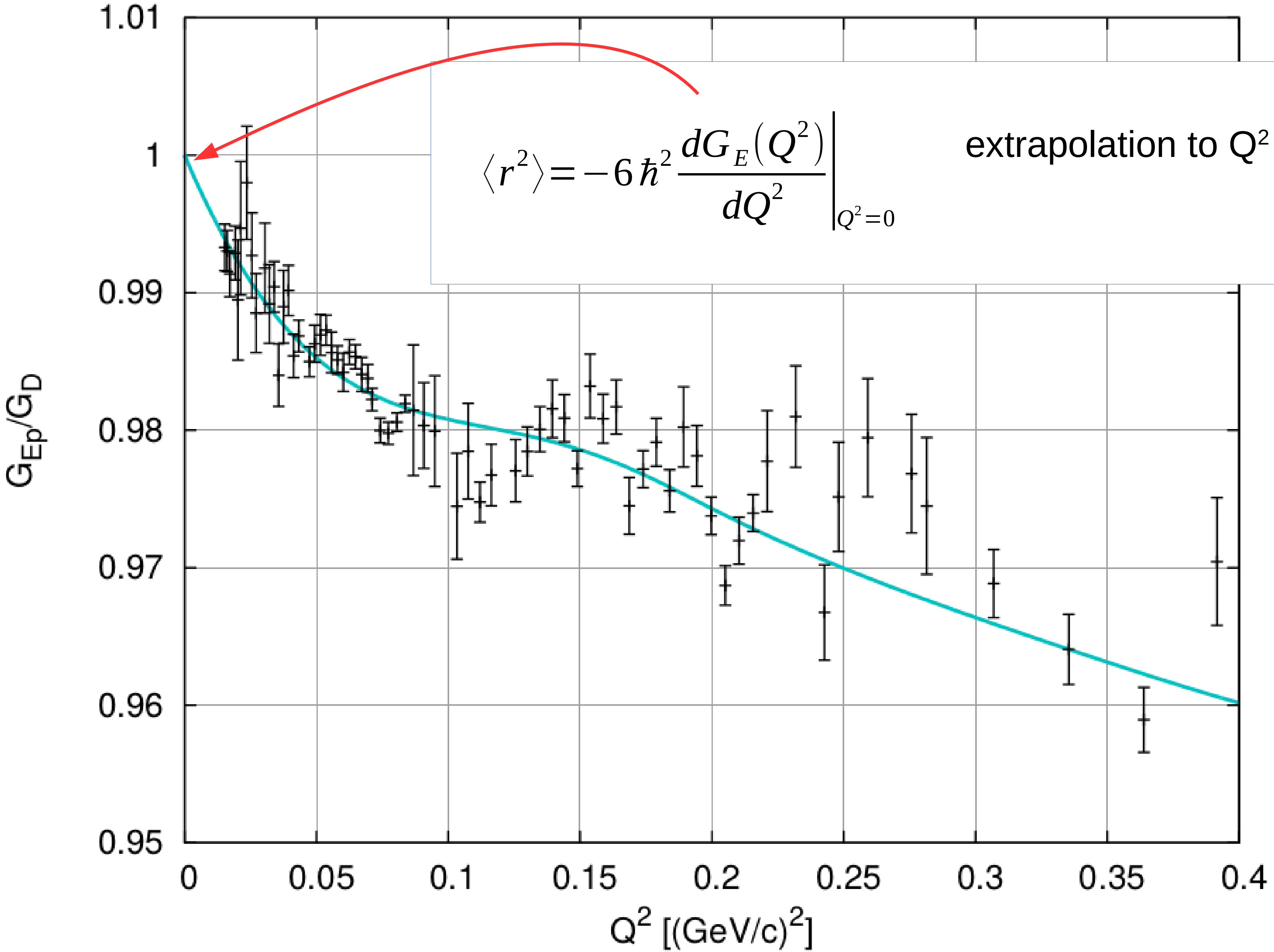
Fleurbaey et al., PRL 120, 183001 (2018)

Electron scattering

Mainzer Microtron MAMI



Electron scattering



$$\langle r^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

extrapolation to $Q^2 = 0$ required

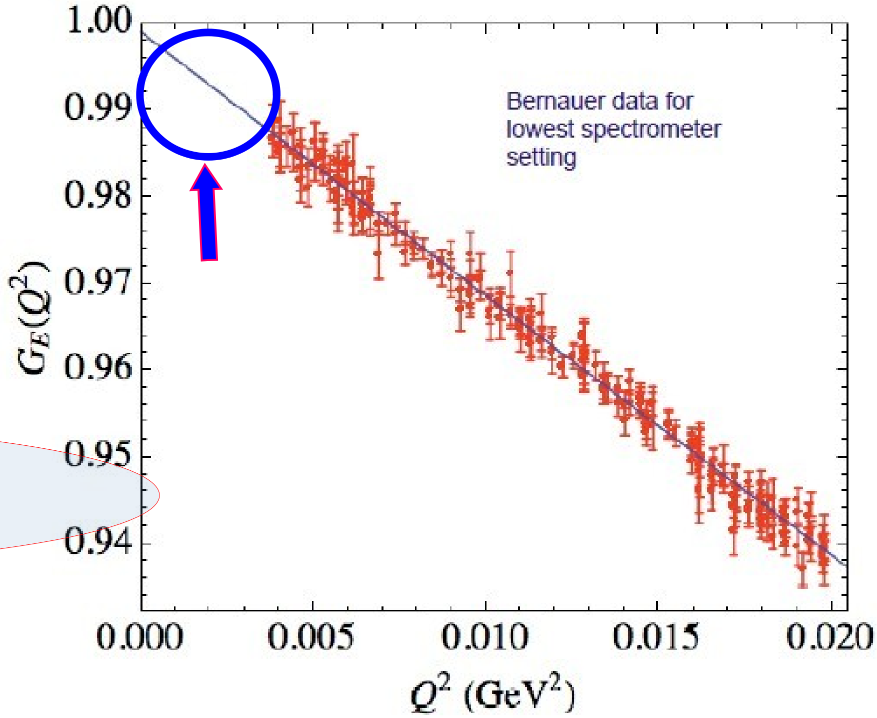
Mainz MAMI data 2010

Vanderhaeghen, Walcher: 1008.4225

The PRad Experimental Approach

- PRad initial goals:
  - large Q^2 range in one experimental setting
  - reach to very low Q^2 range (~ 10^{-4} GeV/C^2)
  - reach to sub-percent precision in cross section

- PRad suggested solutions:
  - use high resolution high acceptance calorimeter:
    - ✓ reach smaller scattering angles: ($\theta_e = 0.7^0 – 7.0^0$)
      ($Q^2 = 2x10^{-4} \div 6x10^{-2}$) GeV/c^2 ;
    - ✓ large Q^2 range in one experimental setting!;
    - ✓ simultaneous detection of *ee* → *ee* Moller scattering (best known control of systematics).
  - use high density windowless H_2 gas flow target:
    - ✓ beam background under control;
    - ✓ minimize experimental background.



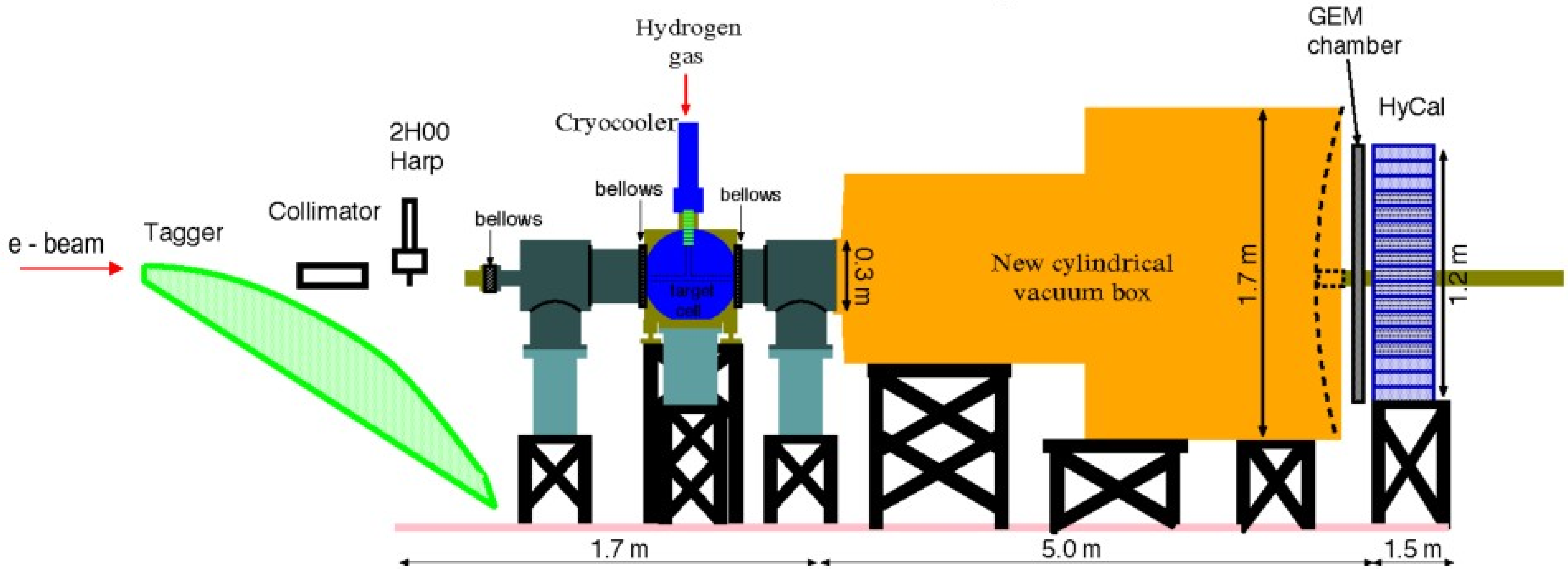
Mainz low Q^2 data set
Phys. Rev. C 93, 065207, 2016

- Two beam energies: E_0 = 1.1 GeV and 2.2 GeV to increase Q^2 range.
- Approved by JLab PAC39 (June, 2012) with high “A” scientific rating.

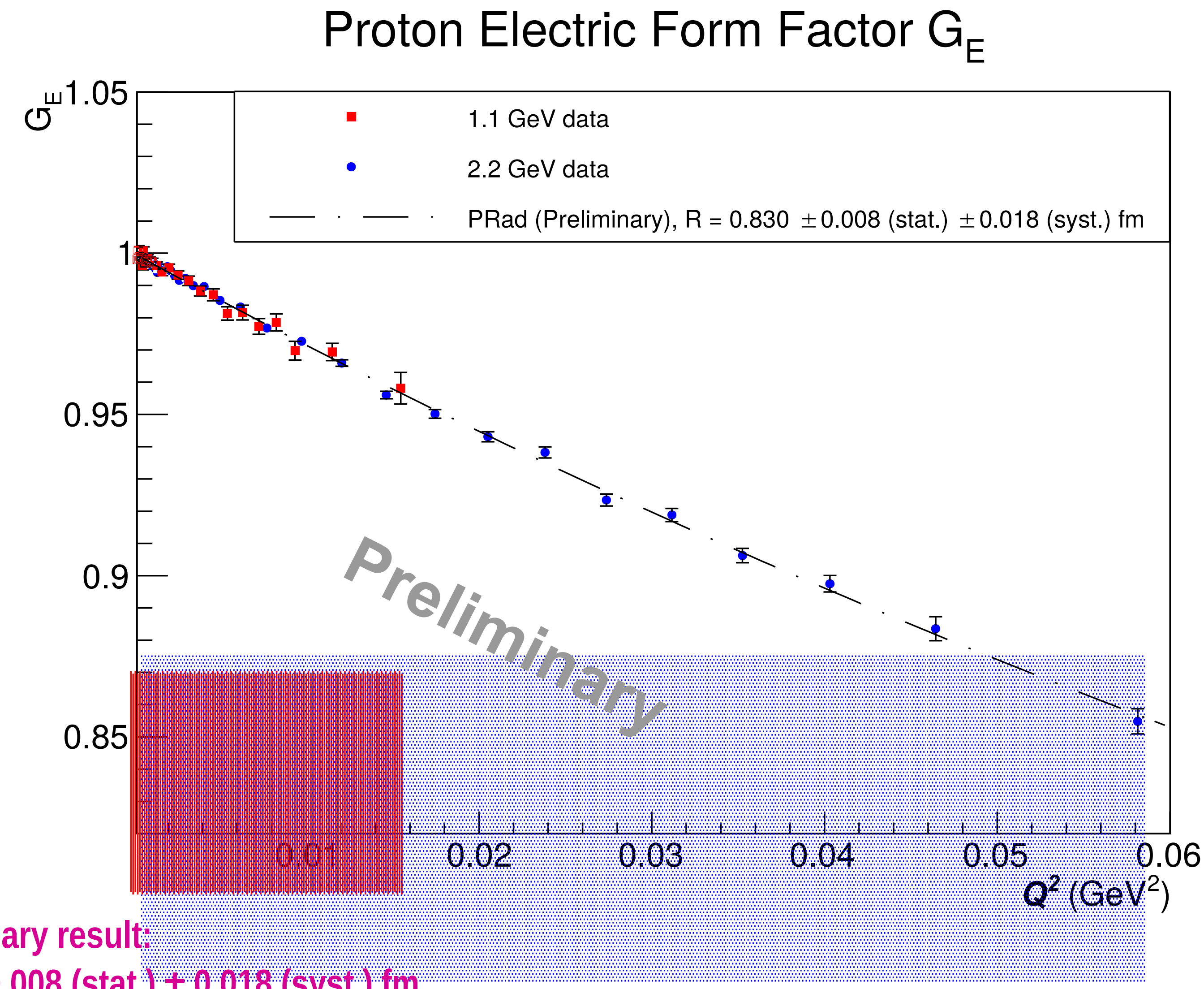
PRad Experimental Setup in Hall B at JLab (schematics)

- Main detector elements:
  - windowless H_2 gas flow target
  - PrimEx HyCal calorimeter
  - vacuum box with one thin window at HyCal end
  - X,Y – GEM detectors on front of HyCal
- Beam line equipment:
  - standard beam line elements (0.1 – 50 nA)
  - photon tagger for HyCal calibration
  - collimator box (6.4 mm collimator for photon beam, 12.7 mm for e^- beam halo “cleanup”)
  - Harp 2H00
  - pipe connecting Vacuum Window through HyCal

PRad Setup (Side View)

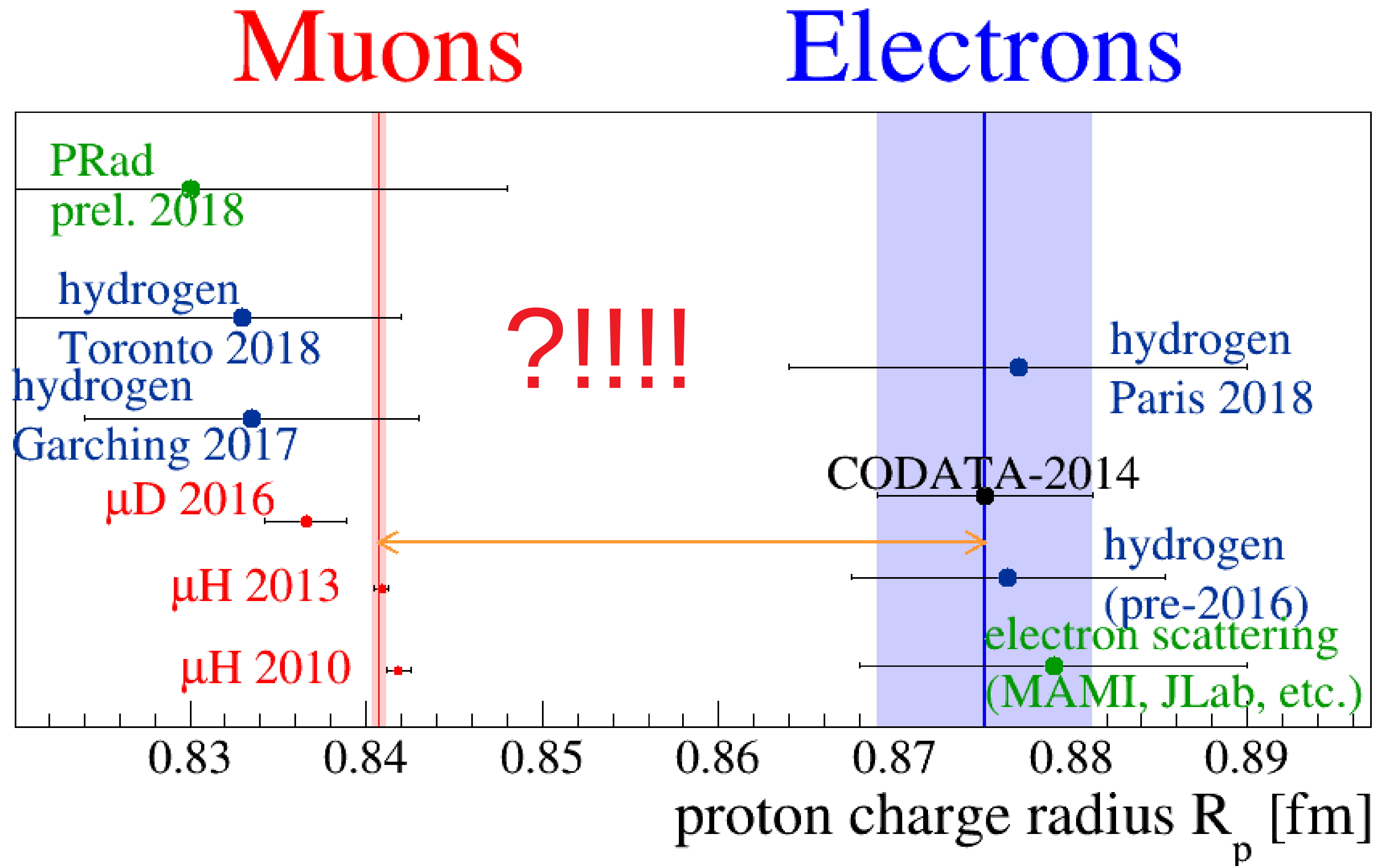


Our Fit of the Extracted G_E (Preliminary)



PRad Preliminary result:
R_p = 0.830 ± 0.008 (stat.) ± 0.018 (syst.) fm

New Measurements: PRad

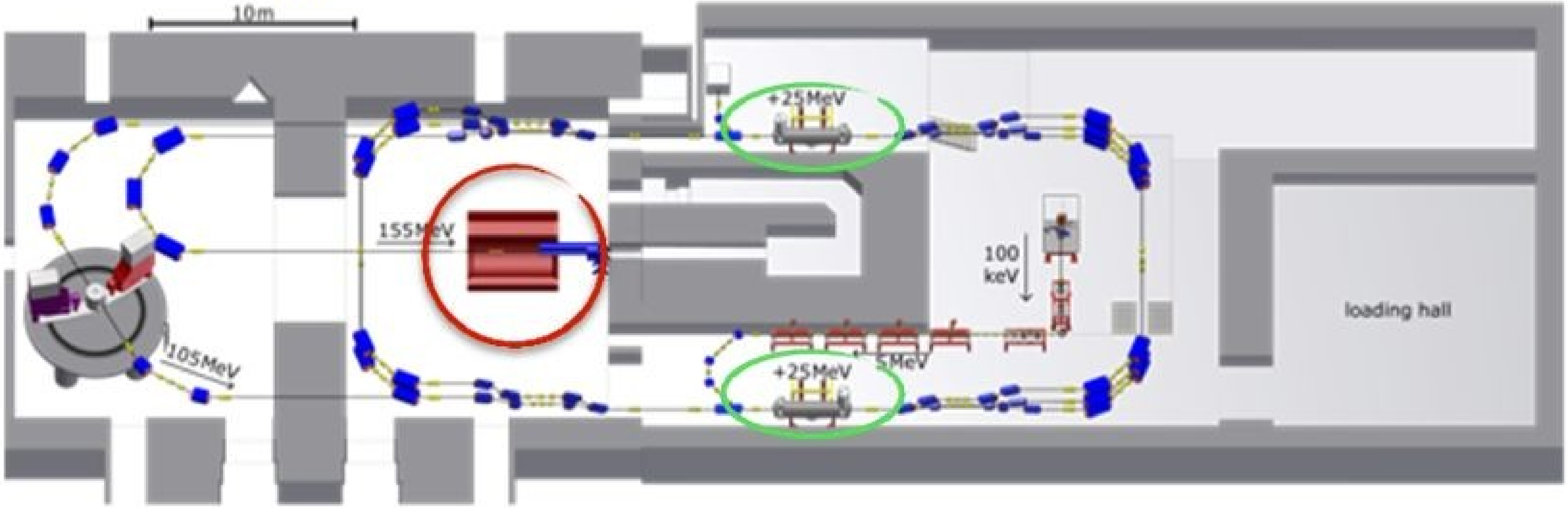


New Mainz electron accelerator **MESA**

Kurt Aulenbacher

MESA — “**M**ainz **E**nergy-Recovering **S**uperconducting **A**ccelerator

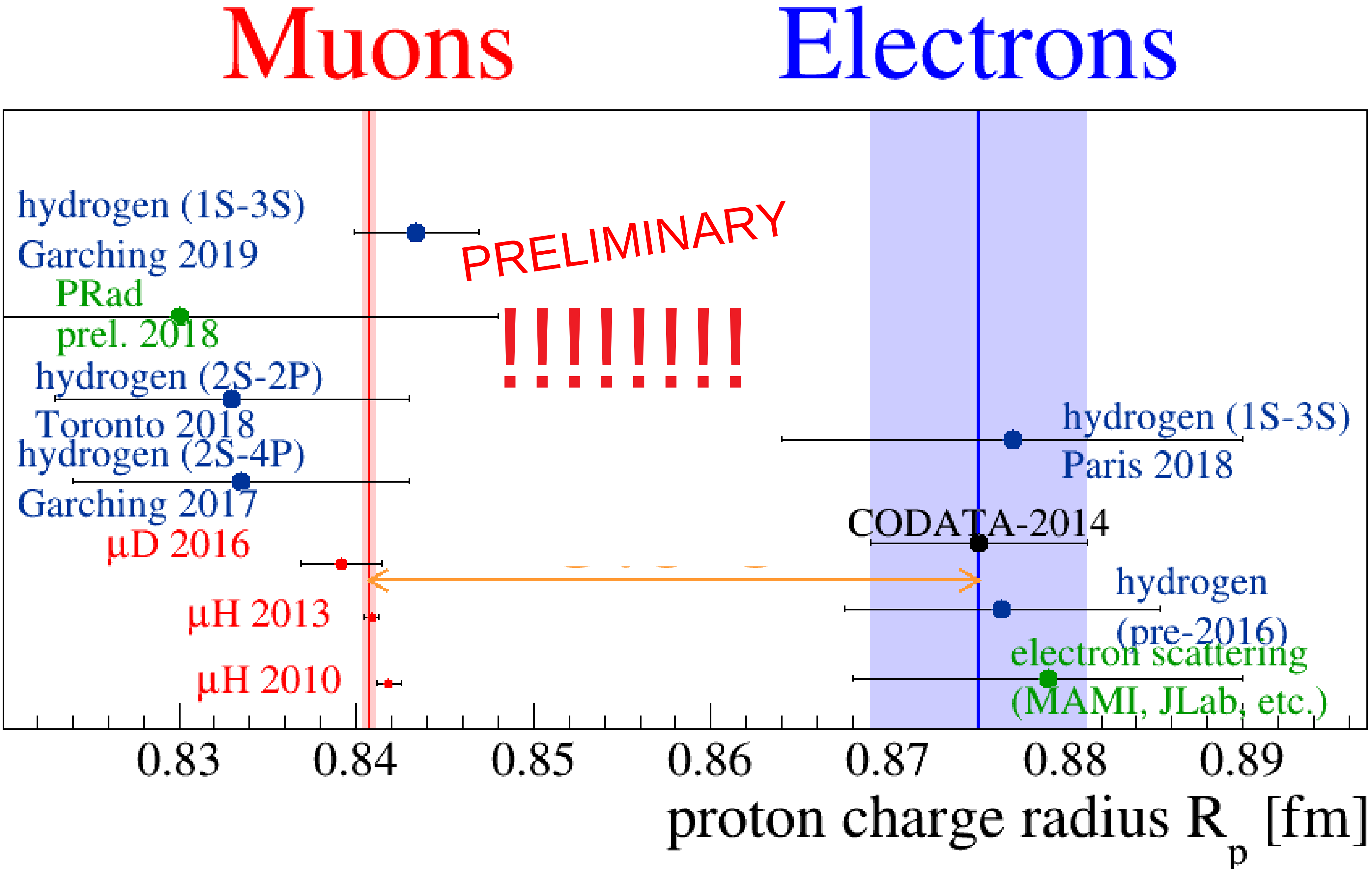
Beam energy: 105 MeV / 155 MeV Current: 1–2 mA



Being built on **Campus of JGU Mainz**

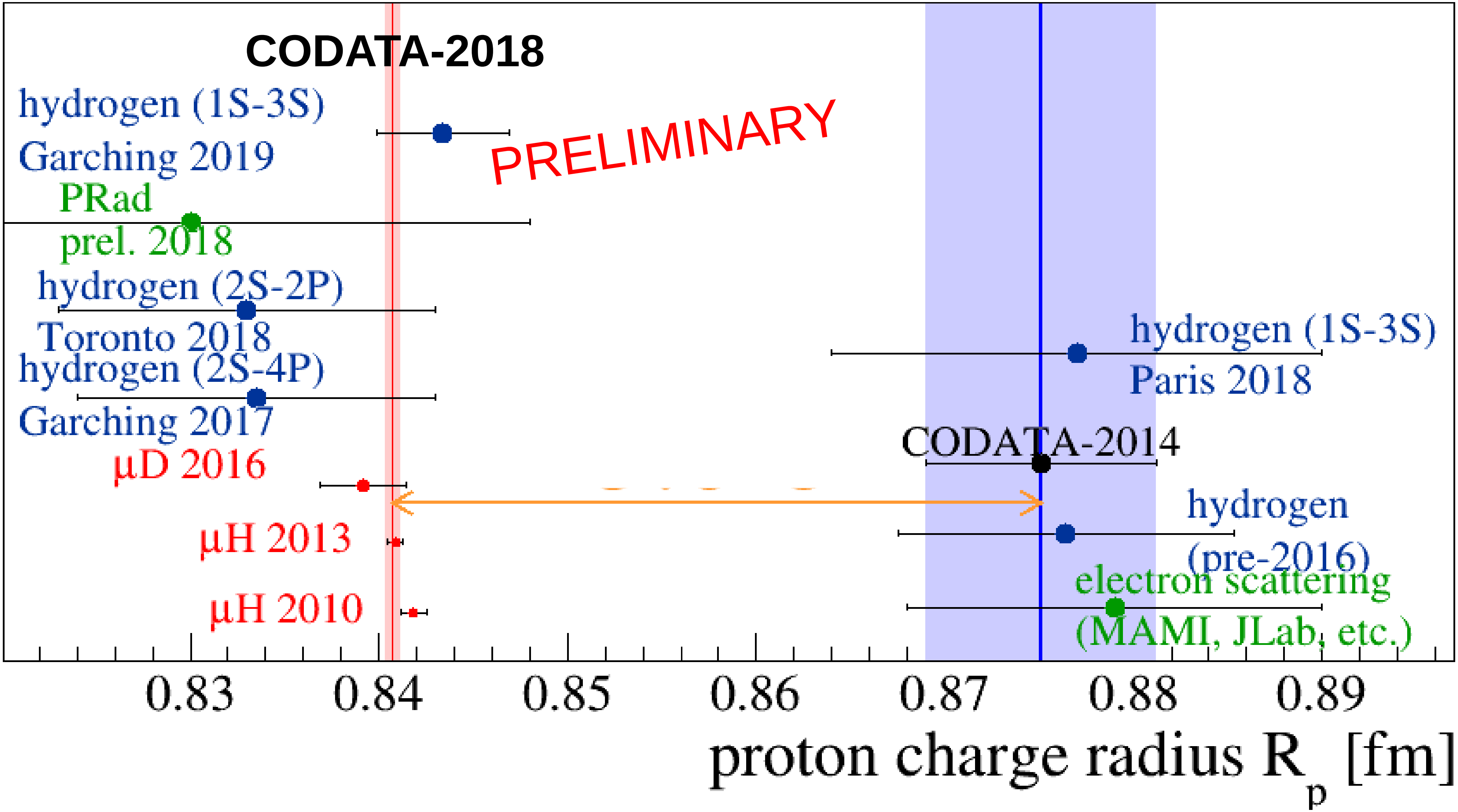
Cluster of Excellence **PRISMA**, since 27.9. also **PRISMA+ !!!**

New Measurements: Garching 1S-3S

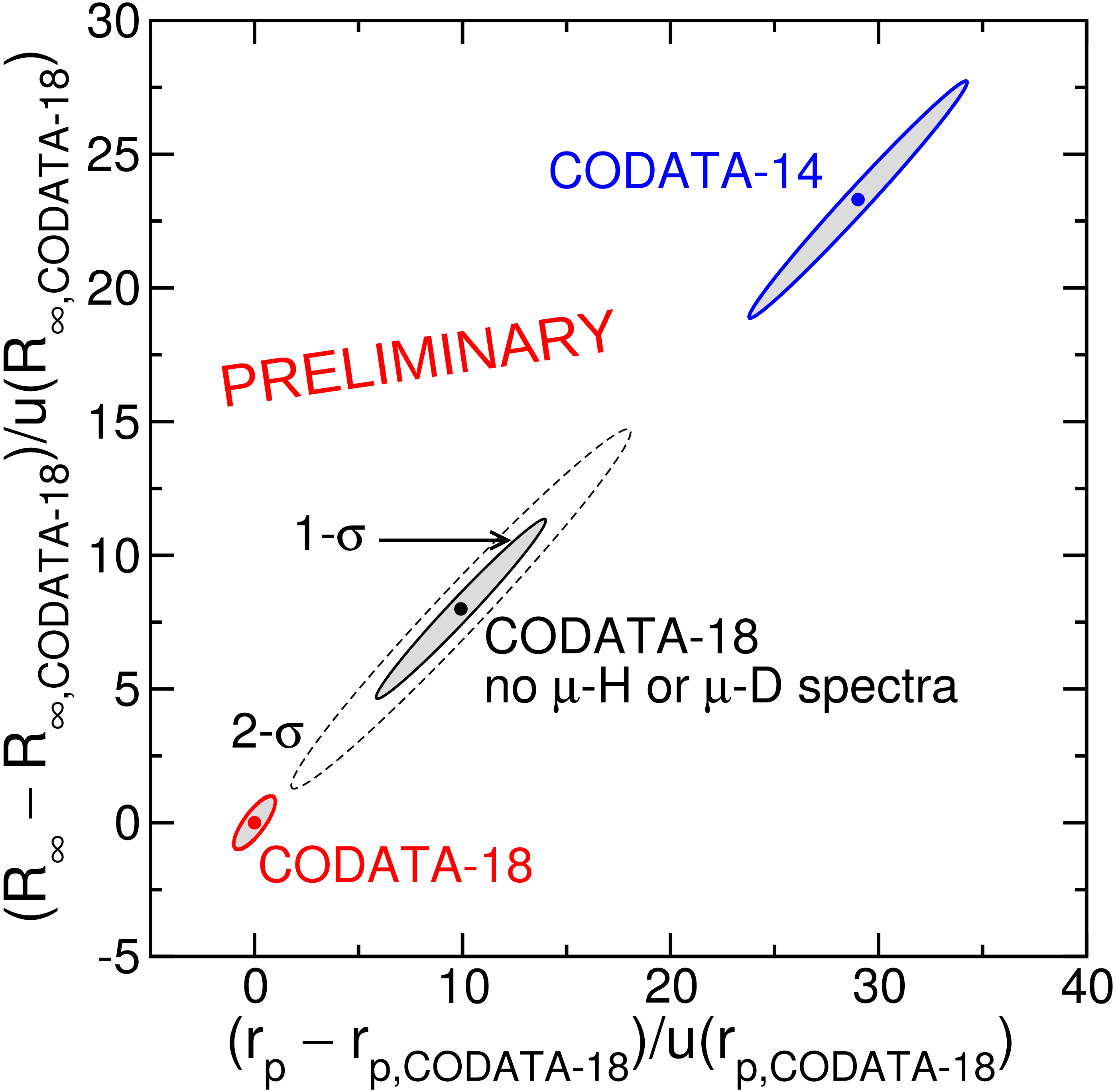


New: CODATA-2018

Muons **Electrons**



New: CODATA-2018



Proton radius,

Deuteron radius

and

Rydberg constant

smaller by >5σ

☺

courtesy K. Pachucki

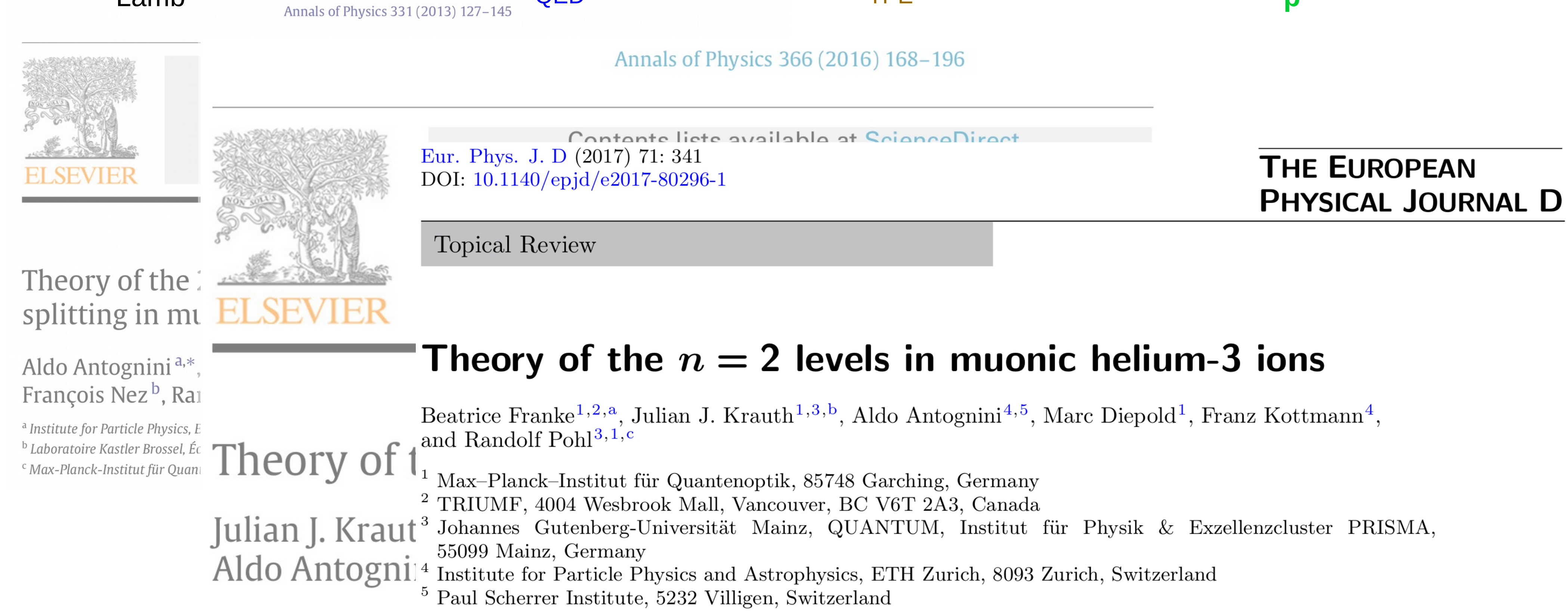
Muonic Helium-3 and -4

Theory in muonic He-3

$$\Delta E^{\mu^3\mathrm{He}}_{\mathrm{Lamb}} = 1644.4820(149)_{\mathrm{QED}} + 15.3000(\mathbf{5200})_{\mathrm{TPE}} - 103.5184(10) * \mathbf{R_h}^{2} /\mathrm{fm}^2 \quad [\mathrm{meV}]$$

$$\Delta E^{\mu\mathrm{D}}_{\mathrm{Lamb}} = 228.7854\ (13)_{\mathrm{QED}} + 1.7500(\ 210)_{\mathrm{TPE}} - 6.1103\ (\ 3) * \mathbf{R_d}^{2} /\mathrm{fm}^2 \quad [\mathrm{meV}]$$

$$\Delta E^{\mu\mathrm{H}}_{\mathrm{Lamb}} = 206.0336\ (15)_{\mathrm{QED}} + 0.0332(\ 20)_{\mathrm{TPE}} - 5.2275(10) * \mathbf{R_p}^{2} /\mathrm{fm}^2 \quad [\mathrm{meV}]$$



Annals of Physics 331 (2013) 127–145

ELSEVIER

Theory of the 2

splitting in mu

Aldo Antognini[a,*],

François Nez[b], Ra

[a] Institute for Particle Physics, E

[b] Laboratoire Kastler Brossel, Éc

[c] Max-Planck-Institut für Quant

Annals of Physics 366 (2016) 168–196

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Theory of t

Julian J. Kraut

Aldo Antogni

Eur. Phys. J. D (2017) 71: 341
DOI: 10.1140/epjd/e2017-80296-1

THE EUROPEAN PHYSICAL JOURNAL D

Topical Review

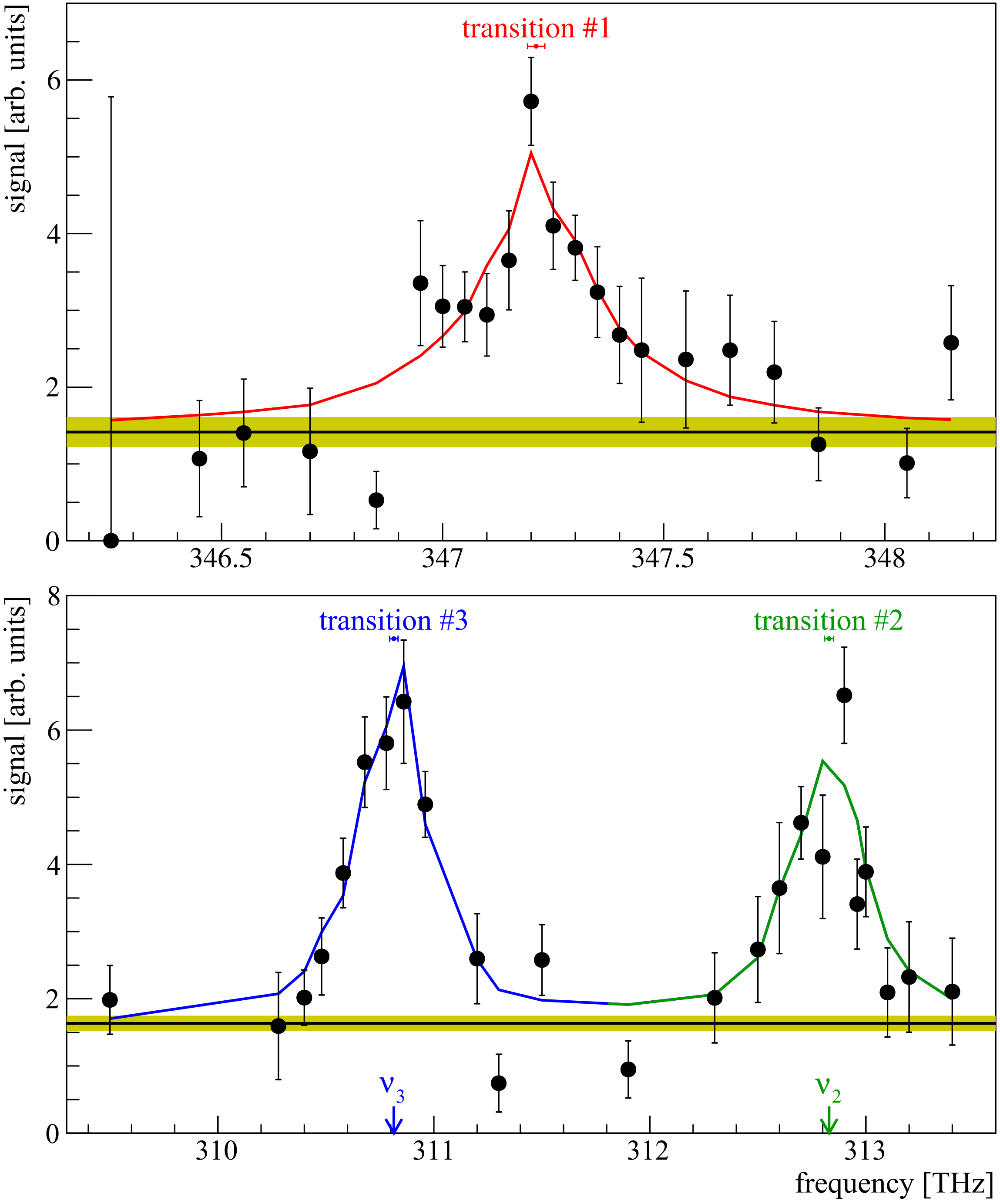
Theory of the $n = 2$ levels in muonic helium-3 ions

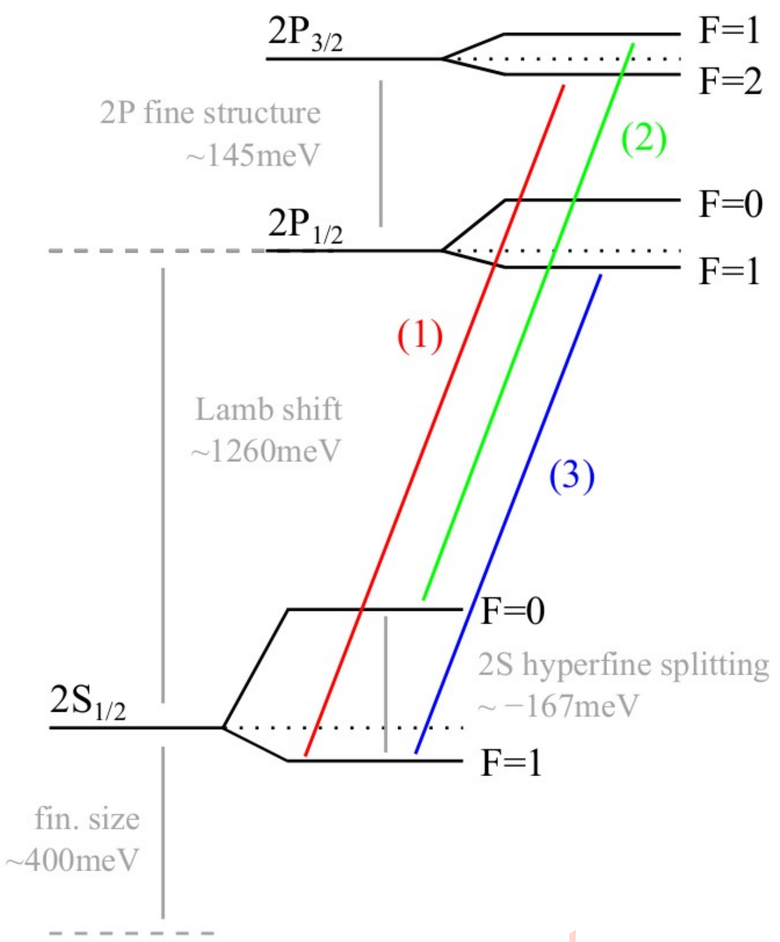
Beatrice Franke[1,2,a], Julian J. Krauth[1,3,b], Aldo Antognini[4,5], Marc Diepold[1], Franz Kottmann[4], and Randolf Pohl[3,1,c]

[1] Max–Planck–Institut für Quantenoptik, 85748 Garching, Germany
[2] TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada
[3] Johannes Gutenberg-Universität Mainz, QUANTUM, Institut für Physik & Exzellenzcluster PRISMA, 55099 Mainz, Germany
[4] Institute for Particle Physics and Astrophysics, ETH Zurich, 8093 Zurich, Switzerland
[5] Paul Scherrer Institute, 5232 Villigen, Switzerland

Three-photon contribution still missing (Pachucki et al., PRA 97, 052511 (2018))

muonic ^{3}He ions

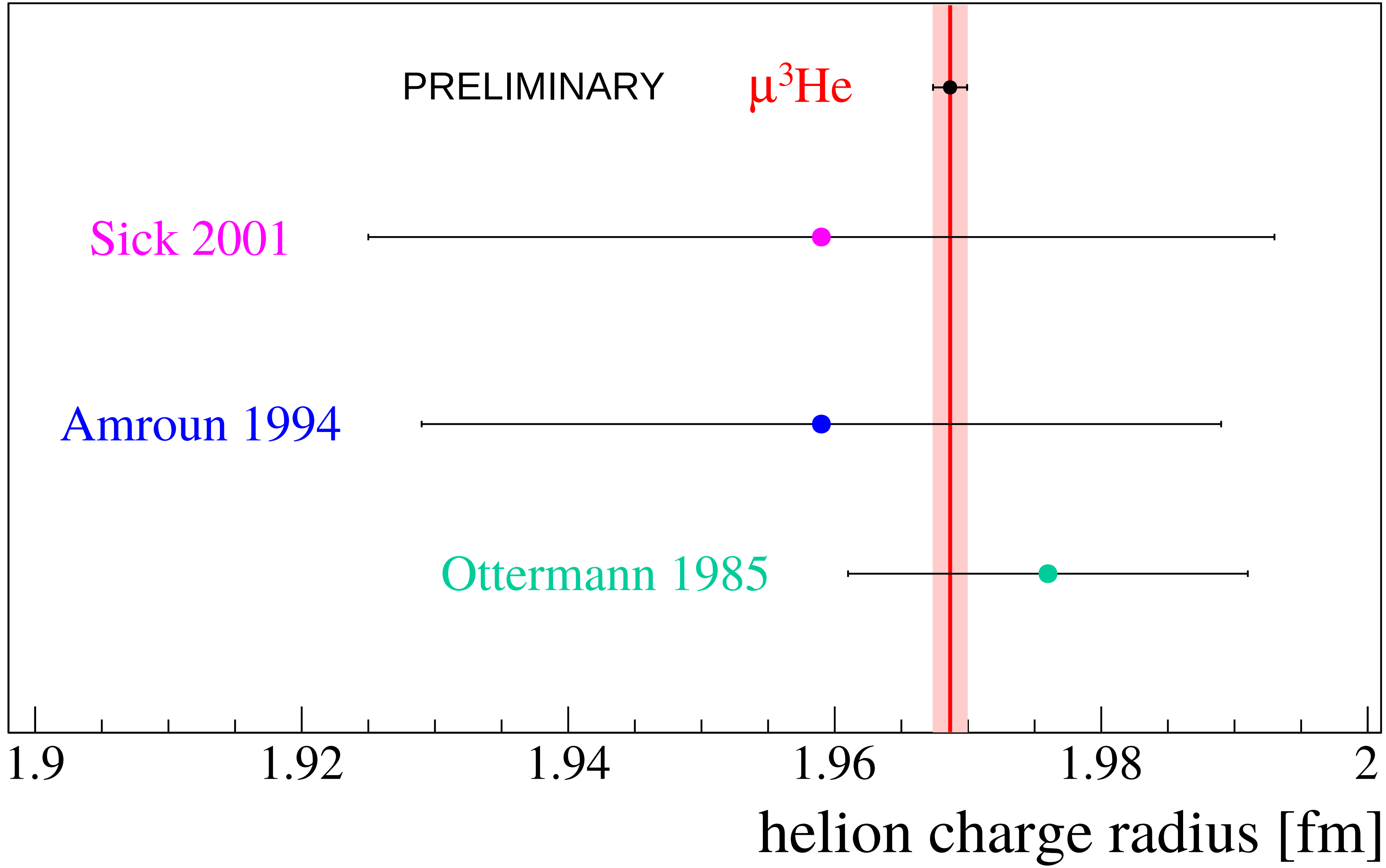




$$R(^3\text{He}) = 1.96866\ (12)_{\text{exp}}\ (128)_{\text{theo}}\ \text{fm}$$

Theory: Franke et al., EPJD (2017),
but 3-photon (Pachucki et al.) ?!?!

Muonic Helium-3



prel. accuracy: exp **+- 0.00012** fm, theo **+- 0.00128** fm (nucl. polarizability)

Theory: see Franke et al. EPJ D 71, 341 (2017) [1705.00352]

Theory in muonic He-4

$$\Delta E^{\mu^4\mathrm{He}}_{\mathrm{Lamb}} = 1668.5670(178)_{\mathrm{QED}} + 9.9000(\mathbf{2800})_{\mathrm{TPE}} - 106.3540(80) * \mathbf{R_{\alpha}}^{\mathbf{2}} /\mathrm{fm}^2 \quad [\mathrm{meV}]$$

$$\Delta E^{\mu^3\mathrm{He}}_{\mathrm{Lamb}} = 1644.4820(149)_{\mathrm{QED}} + 15.3000(5200)_{\mathrm{TPE}} - 103.5184(10) * \mathbf{R_{h}}^{\mathbf{2}} /\mathrm{fm}^2 \quad [\mathrm{meV}]$$

$$\Delta E^{\mu\mathrm{D}}_{\mathrm{Lamb}} = 228.7854\,(13)_{\mathrm{QED}} + 1.7500(\,210)_{\mathrm{TPE}} - 6.1103\,(\;3) * \mathbf{R_{d}}^{\mathbf{2}} /\mathrm{fm}^2 \quad [\mathrm{meV}]$$

$$\Delta E^{\mu\mathrm{H}}_{\mathrm{Lamb}} = 206.0336\,(15)\;_{\mathrm{QED}} + 0.0332(\;\;20)_{\mathrm{TPE}} - 5.2275(10) * \mathbf{R_{p}}^{\mathbf{2}} /\mathrm{fm}^2 \quad [\mathrm{meV}]$$

Annals of Physics 331 (2013) 127–145

ELSEVIER

Theory of the
splitting in mu

Aldo Antognini[a,*],
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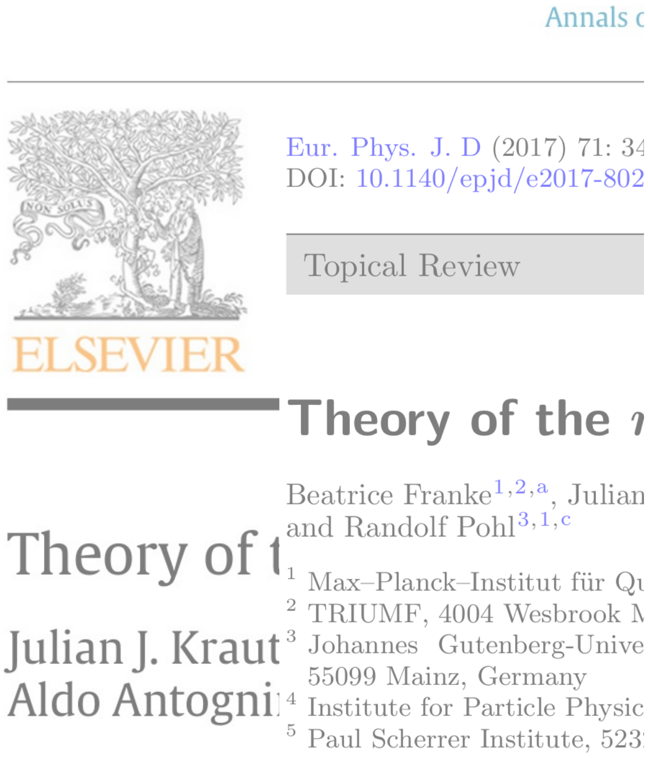
[c] Max-Planck-Institut für Quan



Annals c

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Eur. Phys. J. D (2017) 71: 34
DOI: 10.1140/epjd/e2017-802

Topical Review

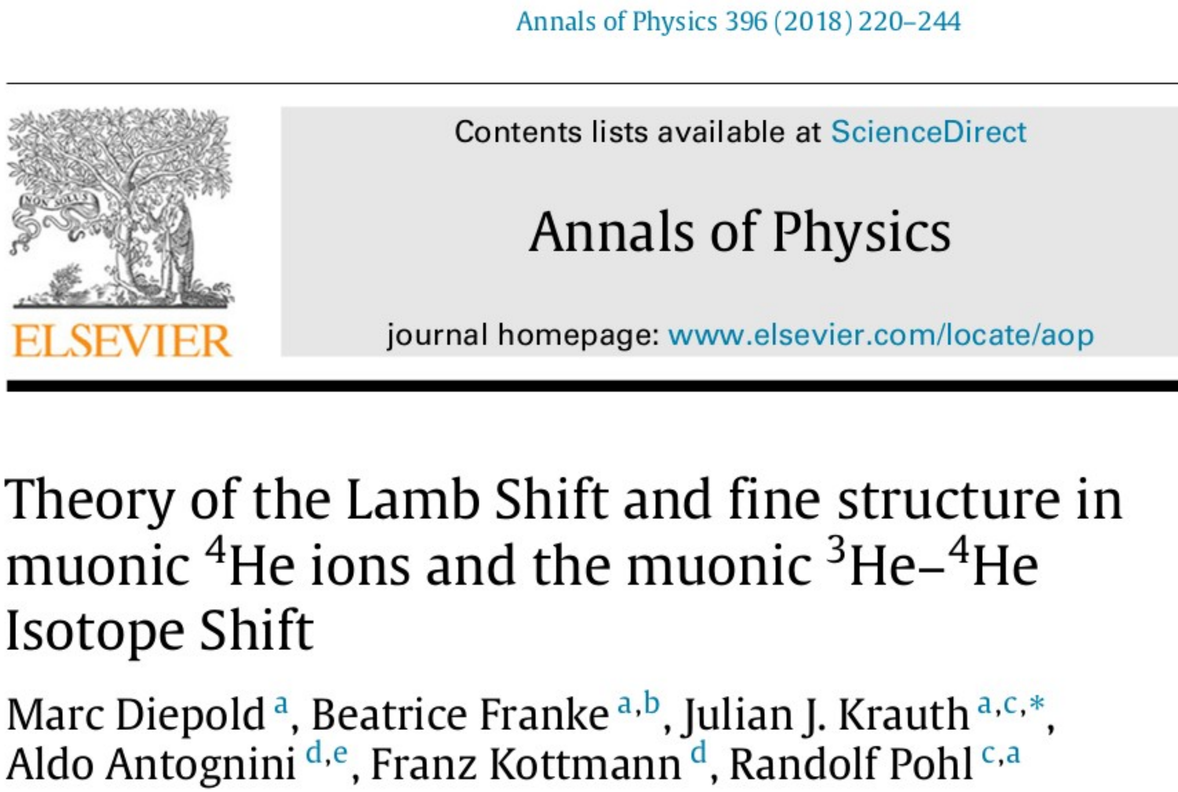
Theory of the

Beatrice Franke[1,2,a], Julian
and Randolf Pohl[3,1,c]

[1] Max–Planck–Institut für Qu
[2] TRIUMF, 4004 Wesbrook M
[3] Johannes Gutenberg-Unive
55099 Mainz, Germany
[4] Institute for Particle Physic
[5] Paul Scherrer Institute, 523

Theory of t

Julian J. Kraut
Aldo Antogni



Annals of Physics 396 (2018) 220–244

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journal homepage: www.elsevier.com/locate/aop

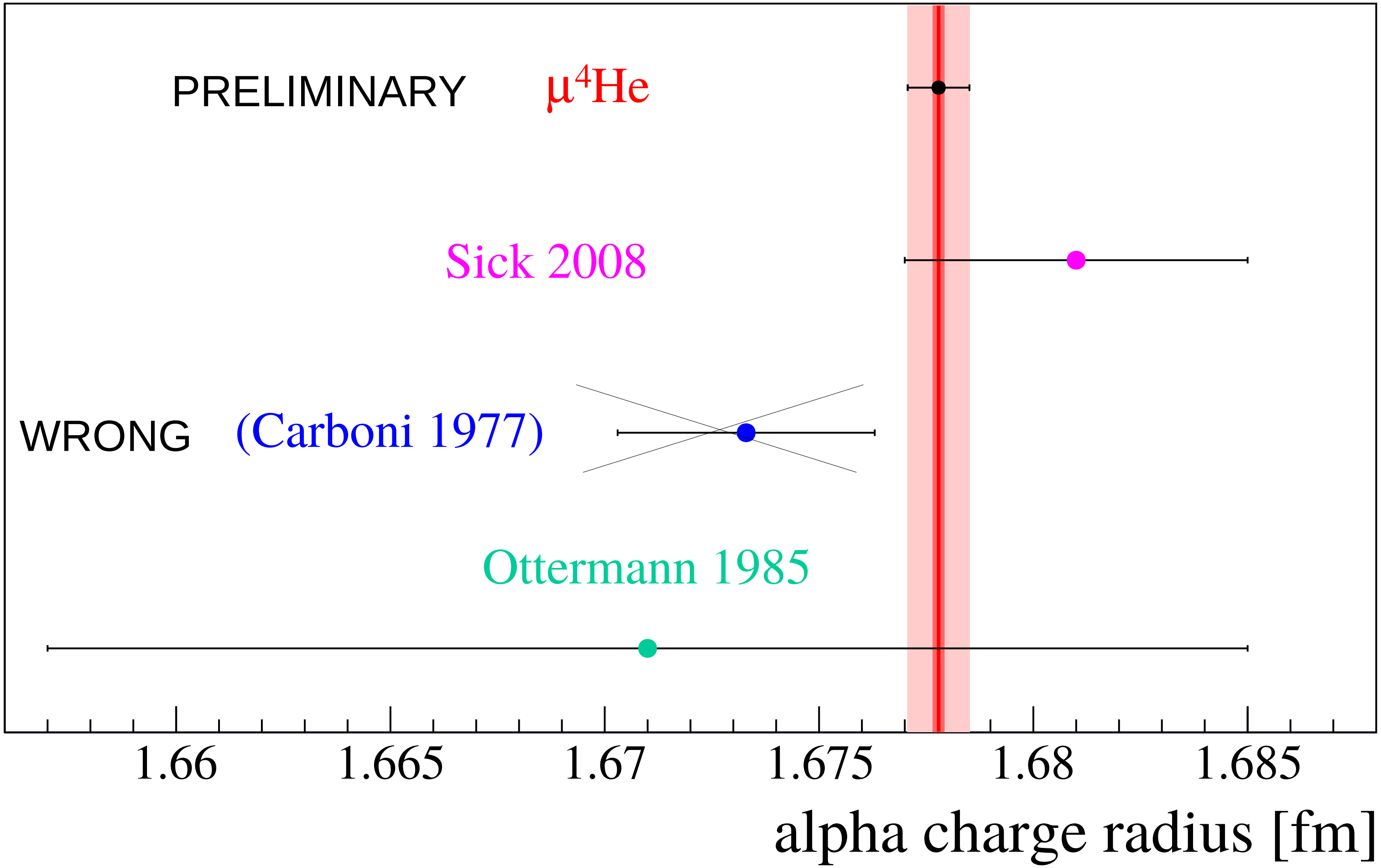
ELSEVIER

Theory of the Lamb Shift and fine structure in muonic ^{4}He ions and the muonic ^{3}He–^{4}He Isotope Shift

Marc Diepold[a], Beatrice Franke[a,b], Julian J. Krauth[a,c,*],
Aldo Antognini[d,e], Franz Kottmann[d], Randolf Pohl[c,a]

Three-photon contribution still missing (Pachucki et al., PRA 97, 052511 (2018))

Muonic Helium-4



prel. accuracy: exp **+- 0.00019** fm, theo **+- 0.00058** fm (nucl. polarizability)

Theory: M. Diepold, RP et al. Ann. Phys. (N.Y.) 396, 220 (2018)
(arxiv 1606.05231 (sic!))

Conclusions

Muonic atoms / ions provide:

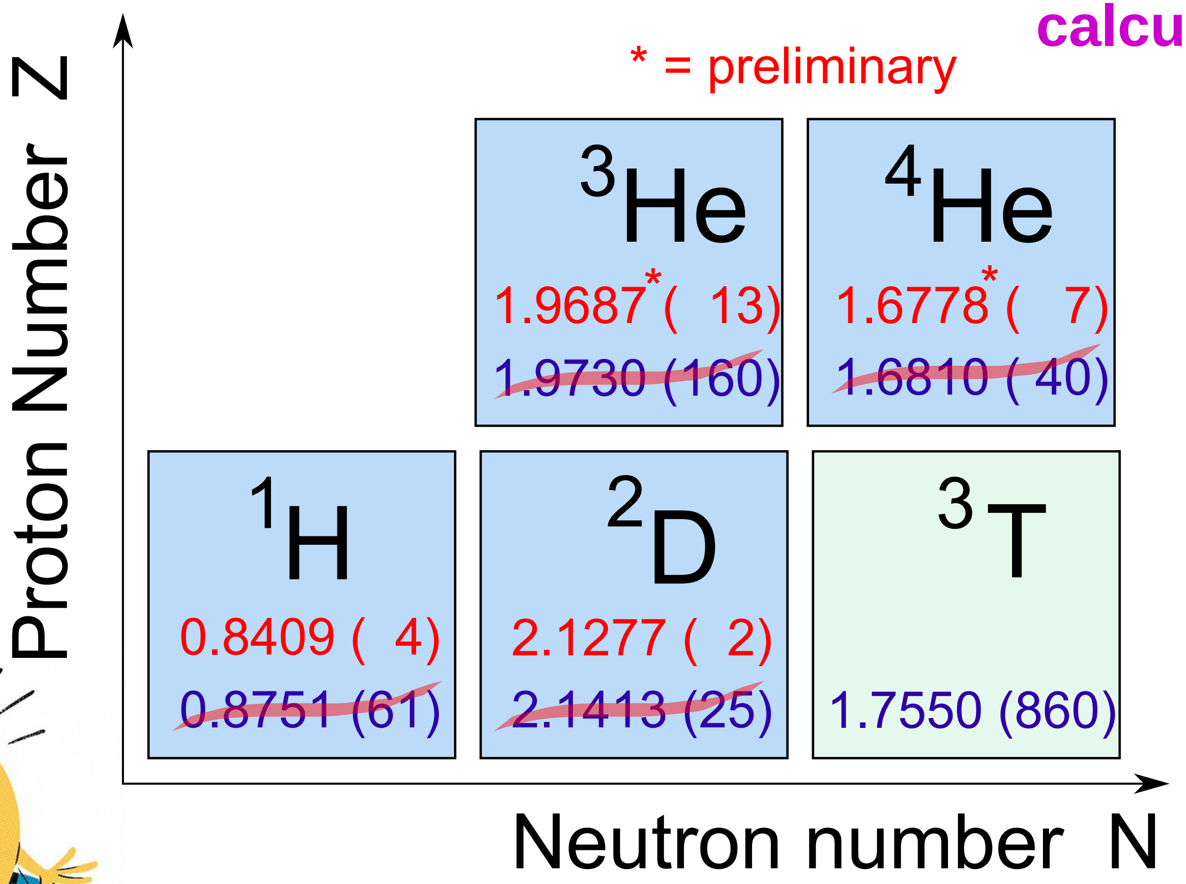
- **~10x more accurate charge radii**, when combined with **calculated polarizability**

Conclusions

Muonic atoms / ions provide:

- **~10x more accurate charge radii**, when combined with

calculated polarizability



^{3}He
1.9687*(13)
~~1.9730 (160)~~

^{4}He
1.6778*(7)
~~1.6810 (40)~~

^{1}H
0.8409 (4)
~~0.8751 (61)~~

^{2}D
2.1277 (2)
~~2.1413 (25)~~

^{3}T
1.7550 (860)

Intermediate conclusions

Muonic atoms / ions provide:

- **~10x more accurate charge radii**, when combined with **calculated polarizability**

- few times more accurate **nuclear polarizability**, when combined with **charge radius from regular atoms**

Muonic atoms are a novel tool for proton and new-nucleon properties!

Intermediate conclusions

Proton radius situation:

- smaller radii from muonic **hydrogen** and **deuterium** imply a smaller Rydberg constant
- new H(2S-4P) gives a smaller proton radius
- new H(1S-3S) however confirms large proton radius

More data coming in!

- H(2S – 6P, 8P, 9P, …) and D(2S-nl) underway in Garching and Colorado
- H(1S – 3S, 4S, ..) underway in Paris and Garching
- H(2S-2P) (Hessels @ Toronto)
- Muonium at PSI, J-PARC
- Positronium (Cassidy @ UCL, Crivelli @ ETH)
- He^+(1S-2S) underway in Garching (Udem) and Amsterdam (Eikema)
- HD^+, H_2, etc. in Amsterdam (Ubachs) and Paris (Hilico, Karr)
- He (Vassen @ Amsterdam), Li^+ (Udem @ Garching)
- HCI, e.g. H-like Ne (Tan @ NIST)
- Rydberg-atoms, e.g. Rb (Raithel @ Ann Arbor)
- new low-Q^2 electron scattering at MAMI, JLab, MESA
- muon scattering: MUSE @ PSI, COMPASS @ CERN

Compare Rydberg values to test QED and SM

The Future

Up next: Hyperfine structure in μp

The 21 cm line in hydrogen (1S hyperfine splitting) has been **measured** to 12 digits (0.001 Hz) in 1971:

$$\boldsymbol{\nu_{\text{exp}} = 1\ 420\ 405.\ 751\ 766\ 7\ \pm 0.000\ 001\ \text{kHz}}$$

Essen et al., Nature 229, 110 (1971)

Up next: Hyperfine structure in μp

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$$\nu_{\mathrm{exp}} = 1\ 420\ 405.\ 751\ 766\ 7 \ \pm 0.000\ 001\ \mathrm{kHz}$$

Essen et al., Nature 229, 110 (1971)

QED test is limited to 6 digits (800 Hz) because of proton structure effects:

$$\nu_{\mathrm{theo}} = 1\ 420\ 403.\ 1 \ \pm 0.6_{\mathrm{proton\ size}} \ \pm 0.4_{\mathrm{polarizability}}\ \mathrm{kHz}$$

Eides et al., Springer Tracts 222, 217 (2007)

Proton Zemach radius

HFS depends on “Zemach” radius:

$$\Delta E = -2(Z\alpha) m \langle r \rangle_{(2)} E_F$$

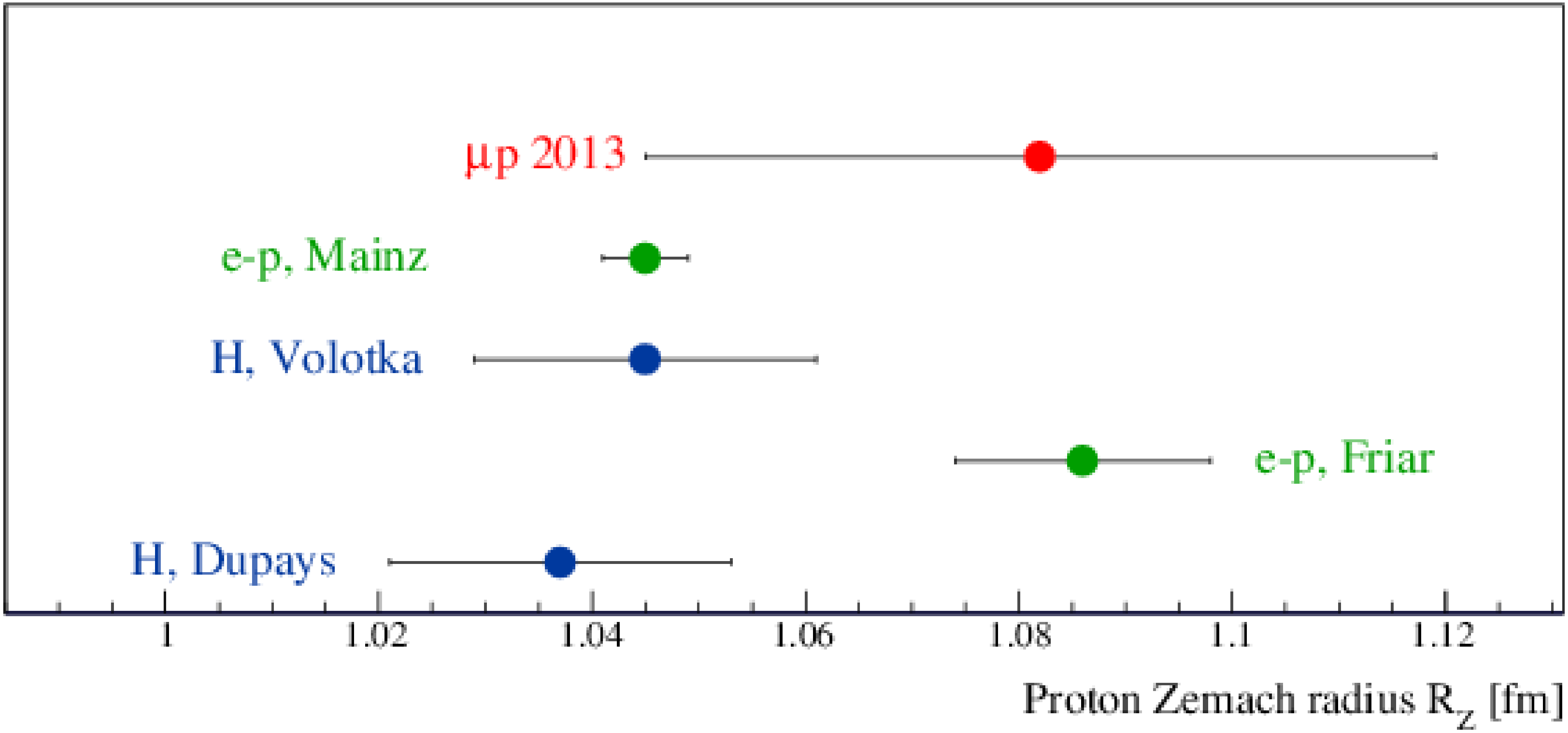
$$\langle r \rangle_{(2)} = \int d^3 r\, d^3 r' \rho_E(r) \rho_M(r') |r - r'|$$

Zemach, Phys. Rev. 104, 1771 (1956)

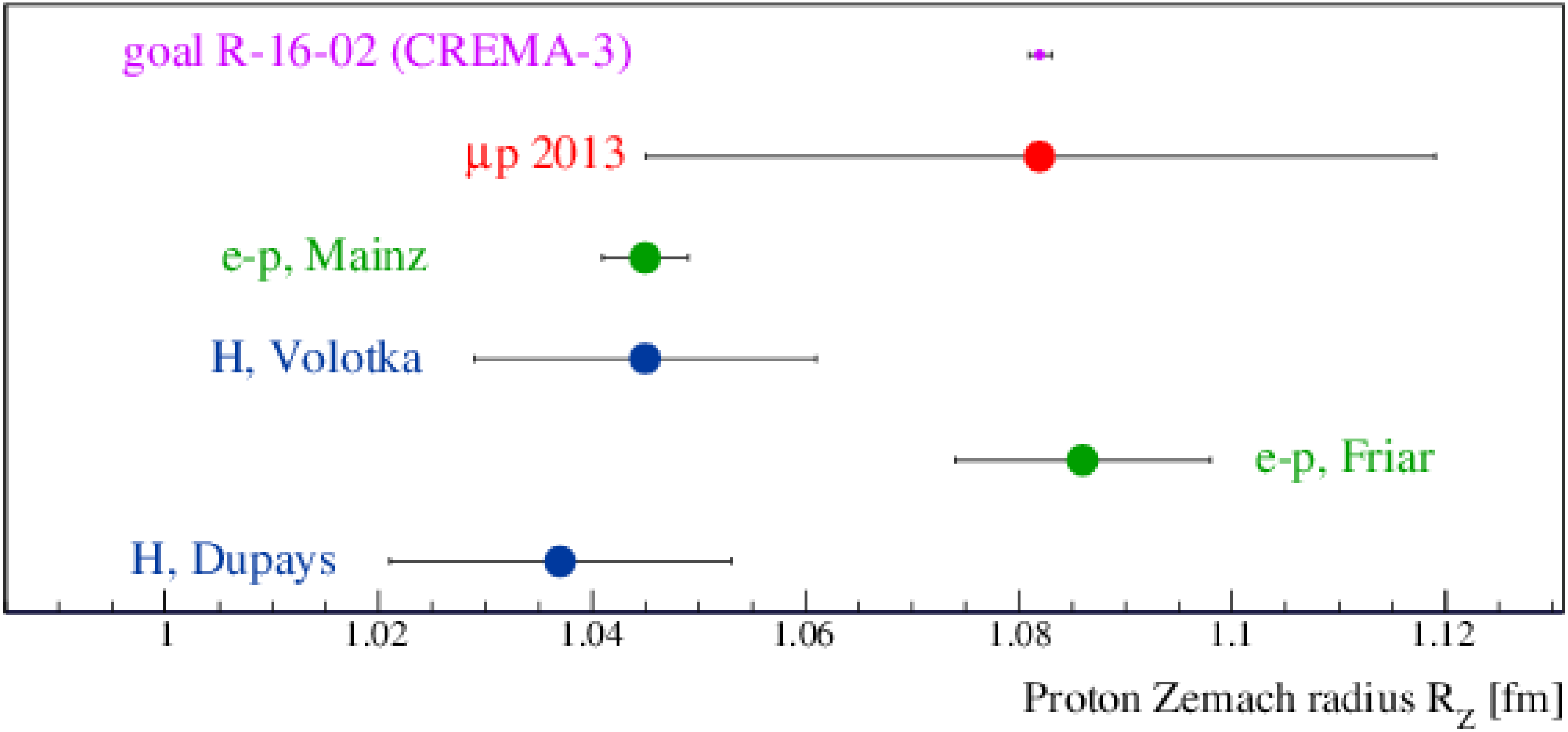
Form factors and momentum space

$$\Delta E = \frac{8(Z\alpha)m}{\pi n^3} E_F \int_0^\infty \frac{dk}{k^2} \left[\frac{G_E(-k^2) G_M(-k^2)}{1+\kappa} \right]$$

Proton Zemach radius from μp



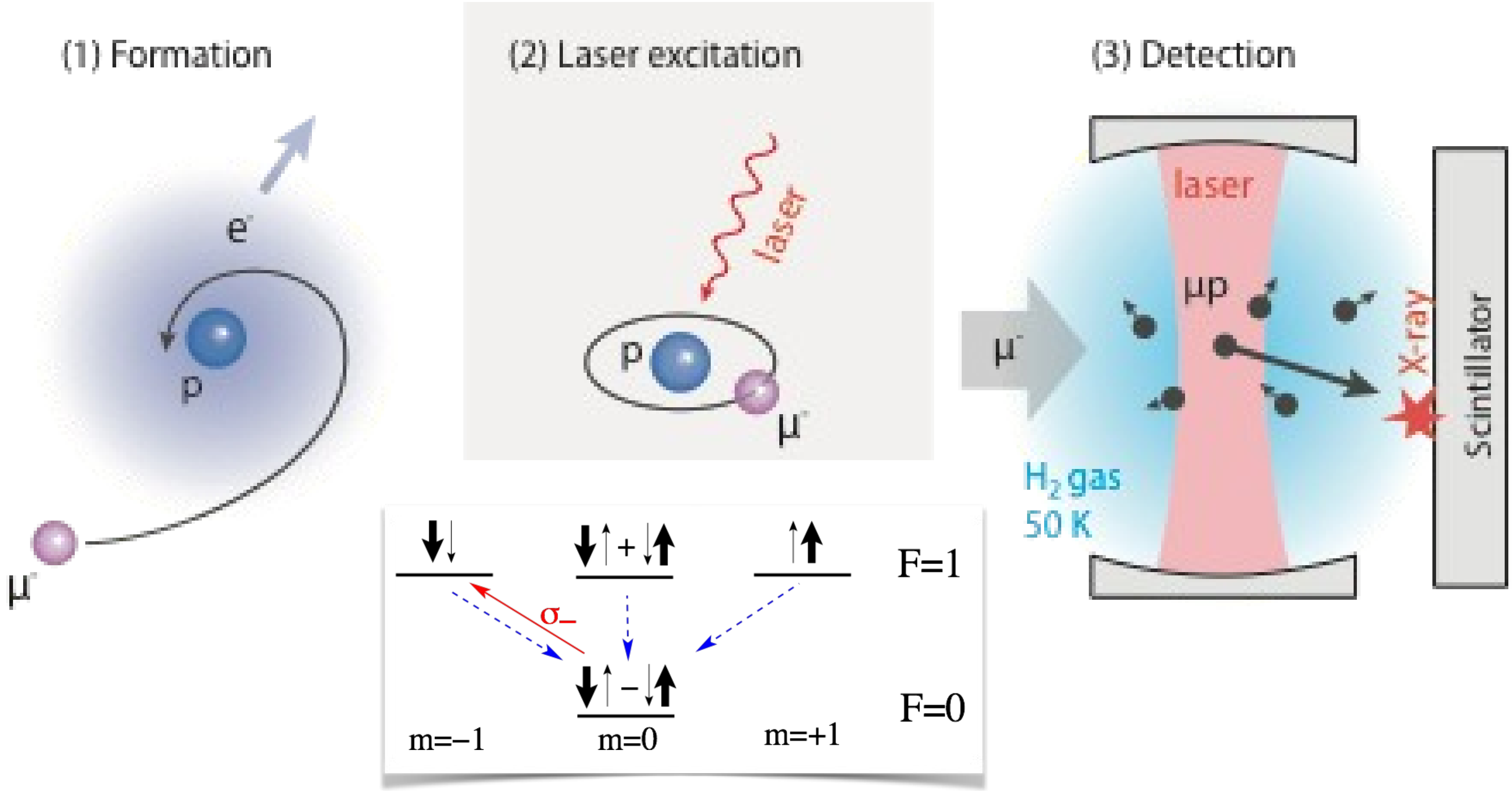
Proton Zemach radius from μp



PSI Exp. R-16-02: Antognini, RP et al. (CREMA-3 / HyperMu)

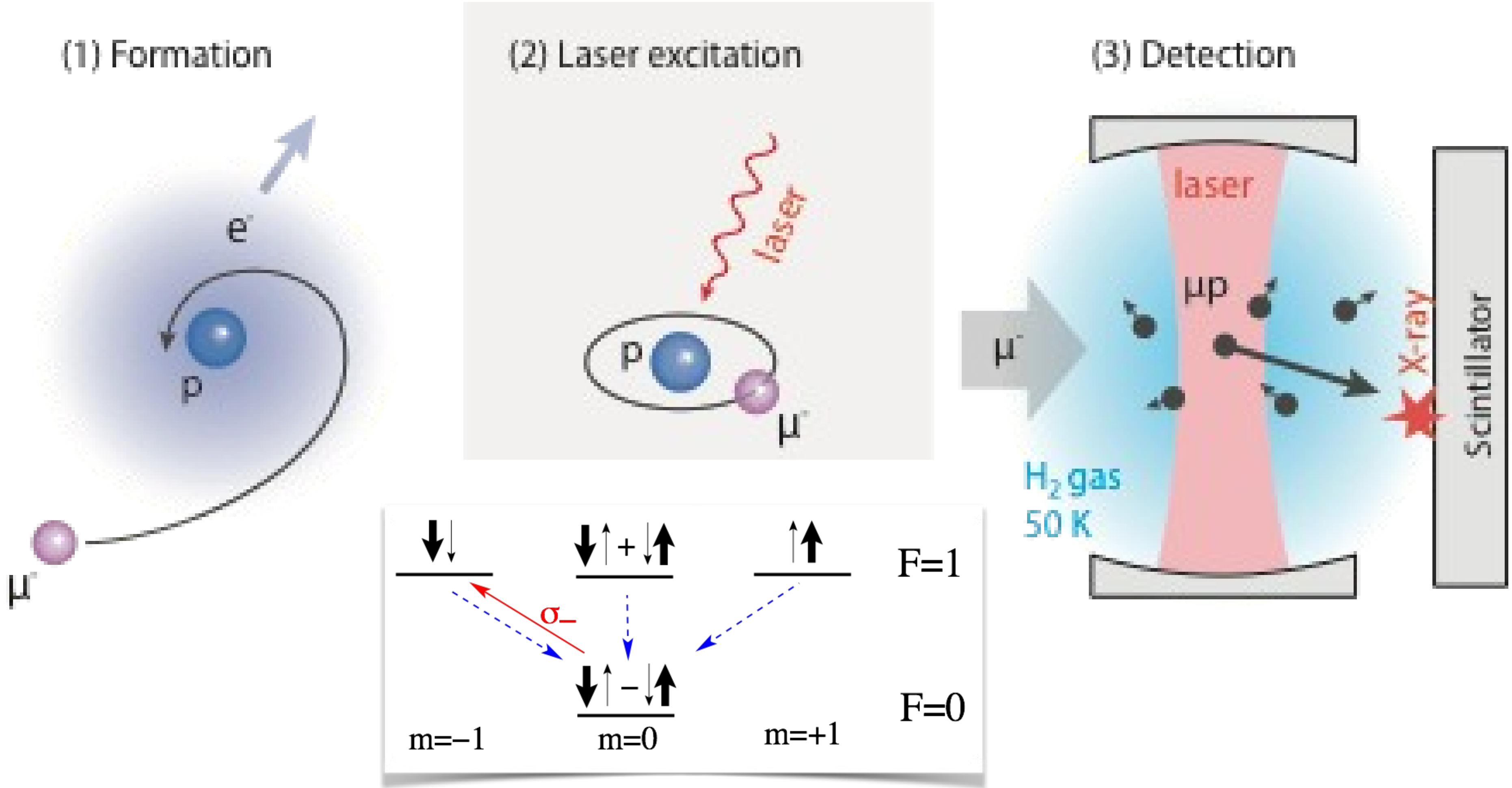
see e.g. Schmidt, RP et al., J. Phys. Conf. Ser 1138, 012010 (2018); arXiv 1808.07240

CREMA-3/HyperMu @ PSI



- Laser pulse: $\mu p(F=0) + \gamma \rightarrow \mu p(F=1)$
- De-excitation: $\mu p(F=1) + H_2 \rightarrow \mu p(F=0) + H_2 + E_{kin}$
- Diffusion: X-rays produced at target walls
- Resonance: Number of X-rays vs laser freq.

CREMA-3/HyperMu @ PSI



- Laser pulse: μp(F=0) + γ → μp(F=1)
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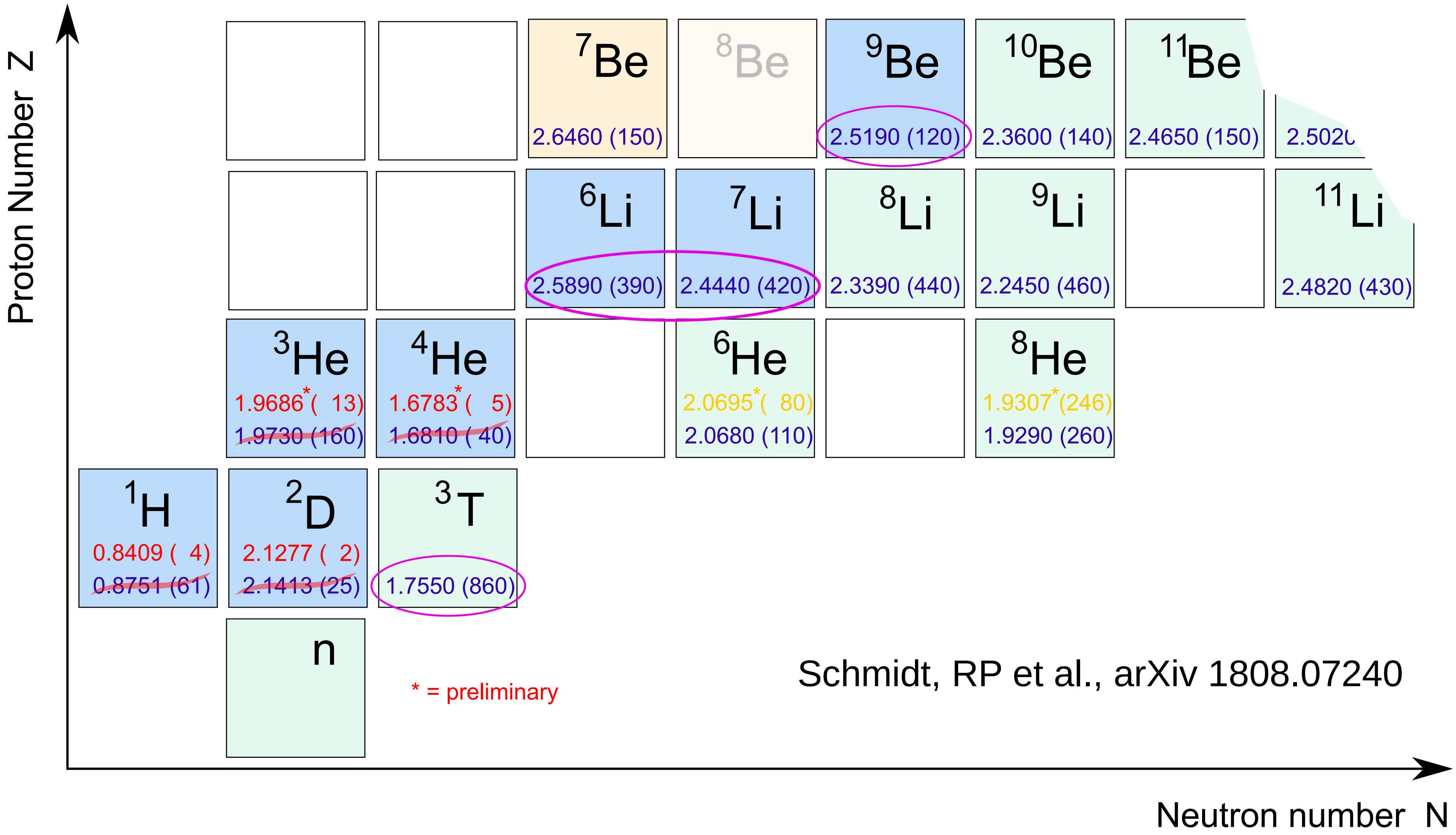
- Diffusion: X-rays produced at target walls
- Resonance: Number of X-rays vs laser freq.

also:
FAMU@RIKEN-RAL
J-PARC Collab.

Charge radii: The future



			^{7}Be 2.6460 (150)	^{8}Be	^{9}Be 2.5190 (120)	^{10}Be 2.3600 (140)	^{11}Be 2.4650 (150)	2.502
			^{6}Li 2.5890 (390)	^{7}Li 2.4440 (420)	^{8}Li 2.3390 (440)	^{9}Li 2.2450 (460)		^{11}Li 2.4820 (430)
	^{3}He 1.9686*(13) ~~1.9730 (160)~~	^{4}He 1.6783*(5) ~~1.6810 (40)~~		^{6}He 2.0695*(80) 2.0680 (110)		^{8}He 1.9307*(246) 1.9290 (260)		
^{1}H 0.8409 (4) ~~0.8751 (61)~~	^{2}D 2.1277 (2) ~~2.1413 (25)~~	^{3}T 1.7550 (860)						
	n							

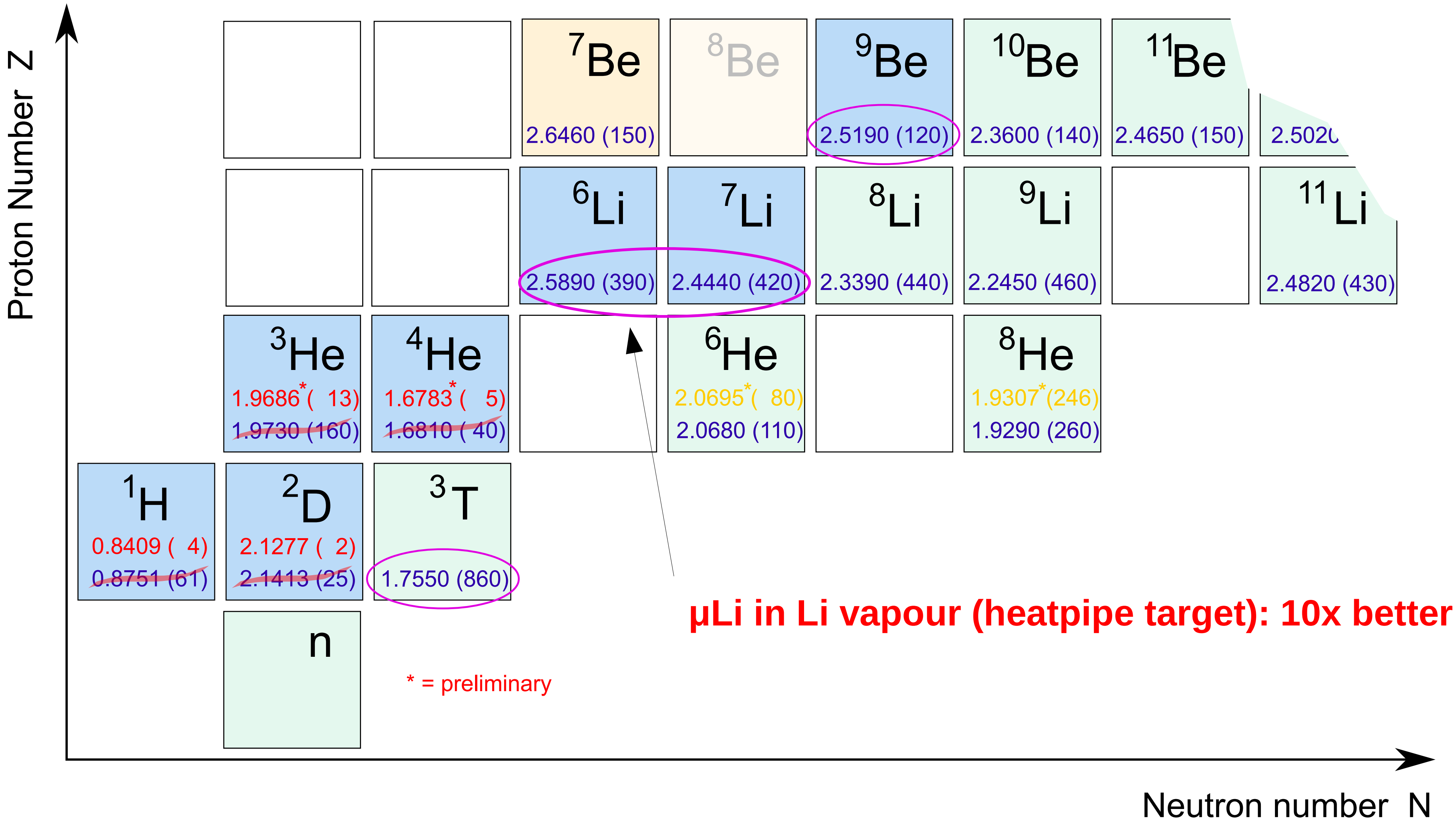
Proton Number Z

Neutron number N

* = preliminary

Schmidt, RP et al., arXiv 1808.07240

Charge radii: The future



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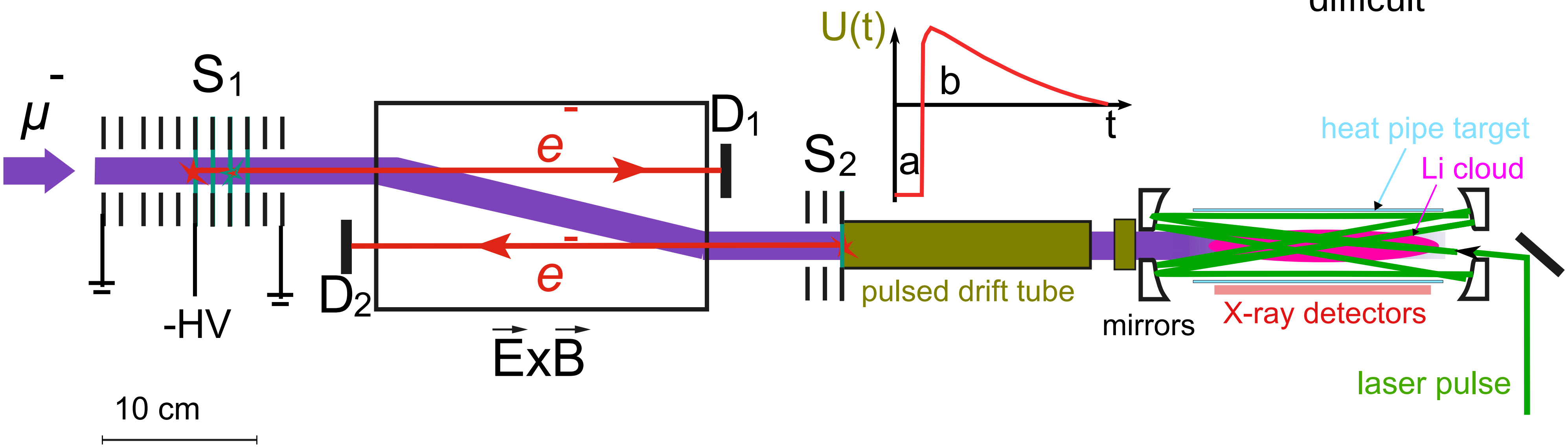
^{1}H 0.8409 (4) ~~0.8751 (61)~~	^{2}D 2.1277 (2) ~~2.1413 (25)~~	^{3}T 1.7550 (860)						
	n							

μLi in Li vapour (heatpipe target): 10x better

* = preliminary

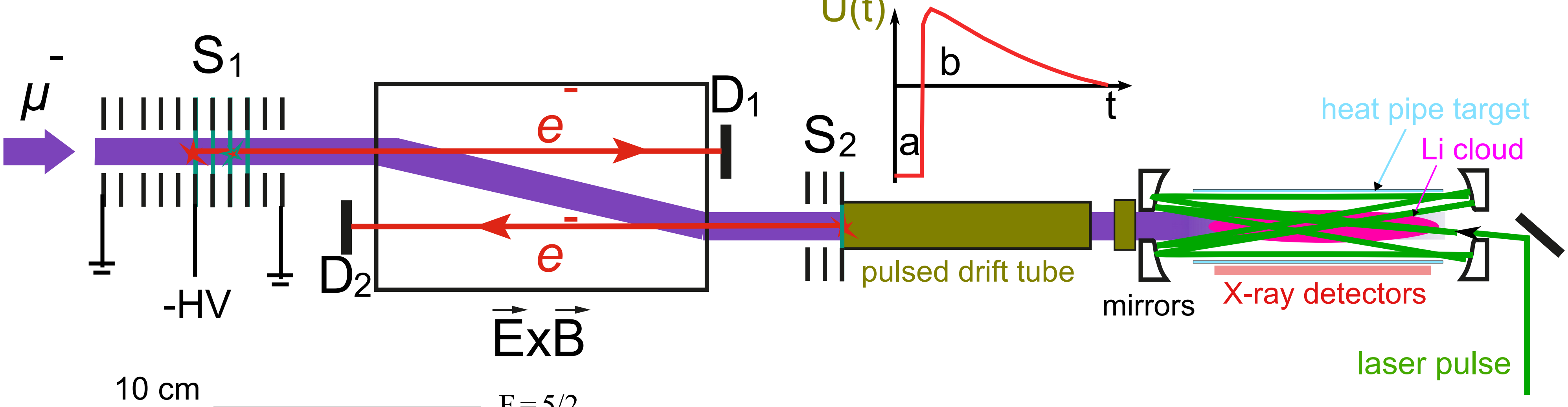
muonic Li with heat pipe target

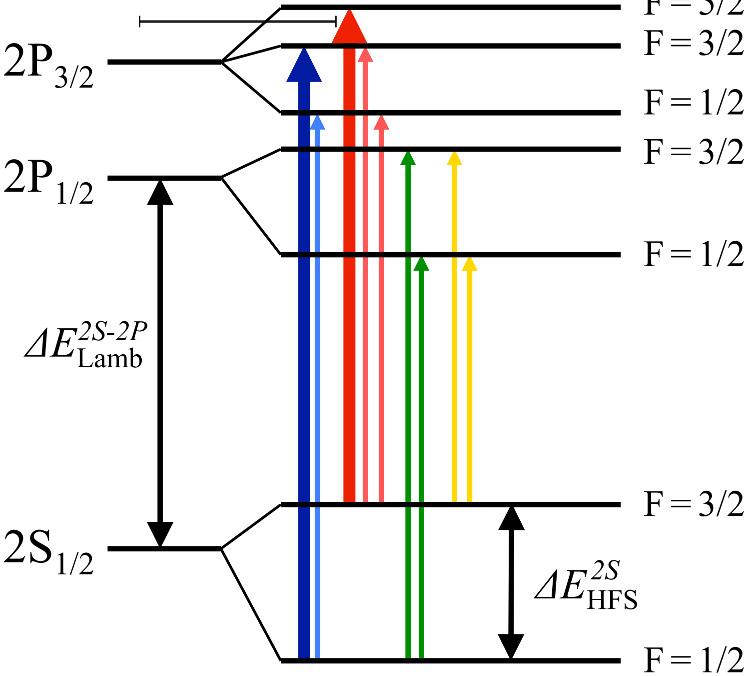
difficult



muonic Li with heat pipe target

difficult





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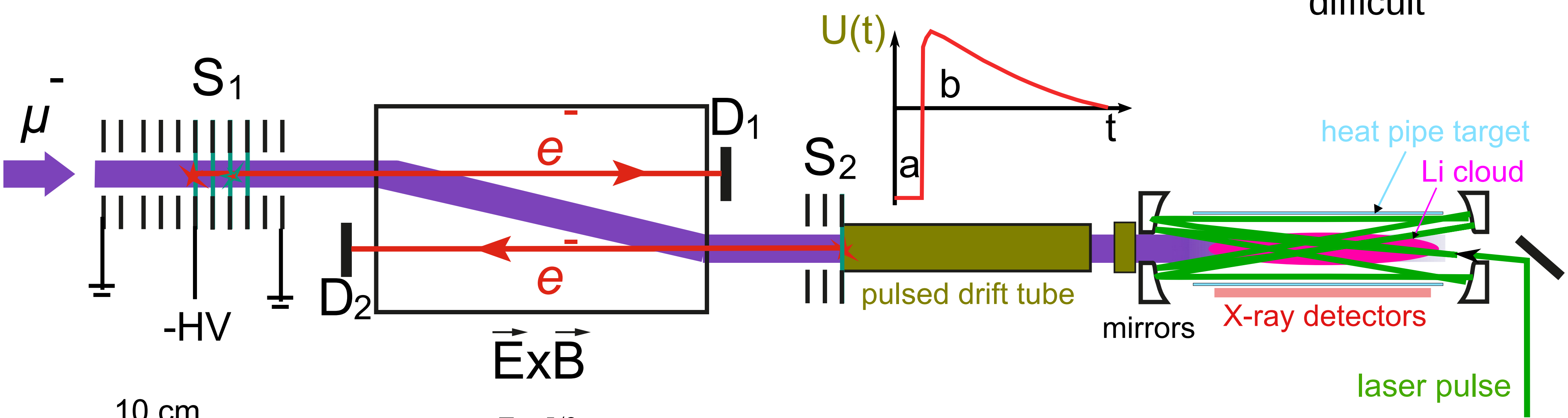
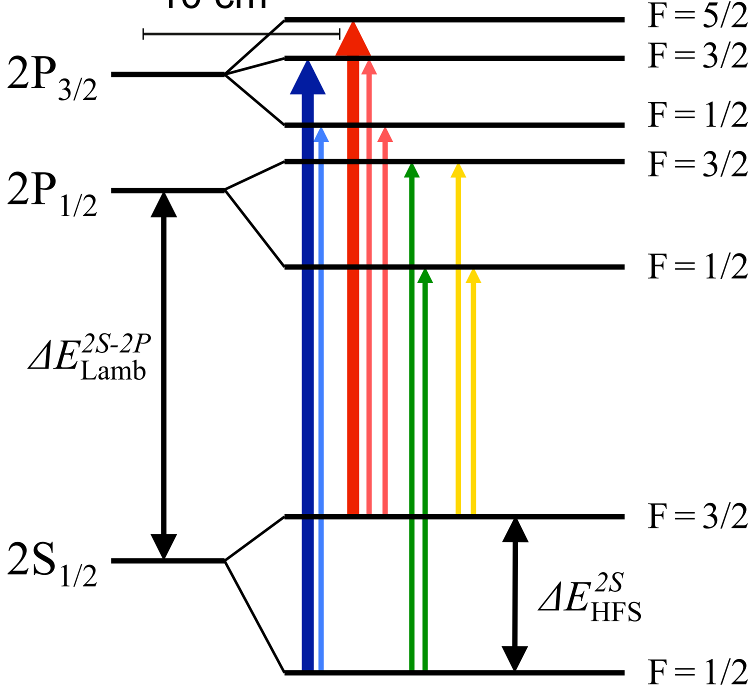
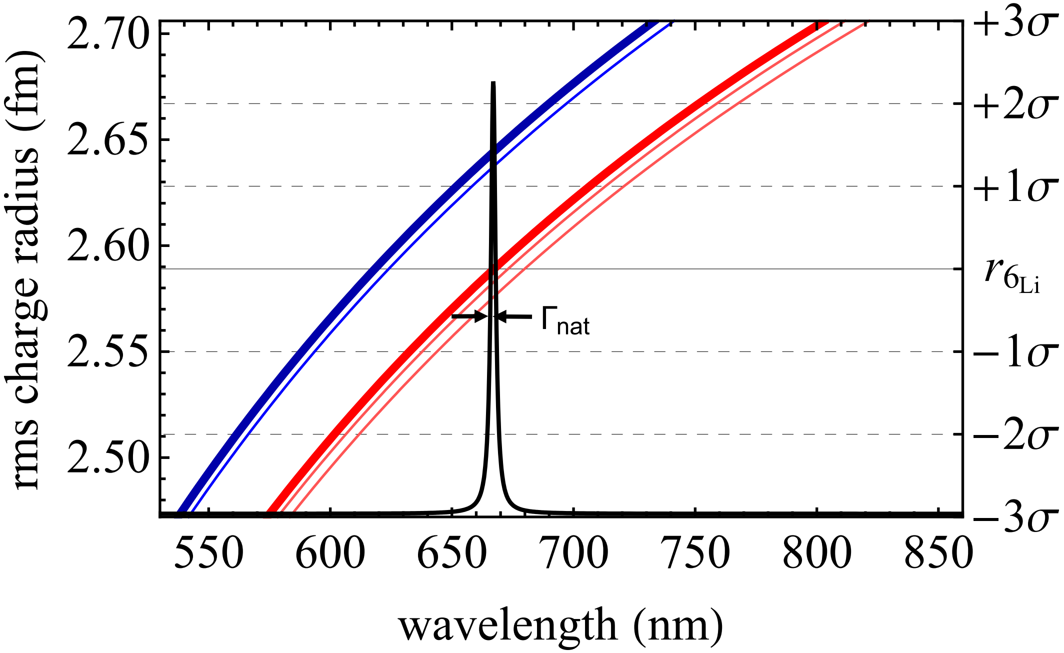
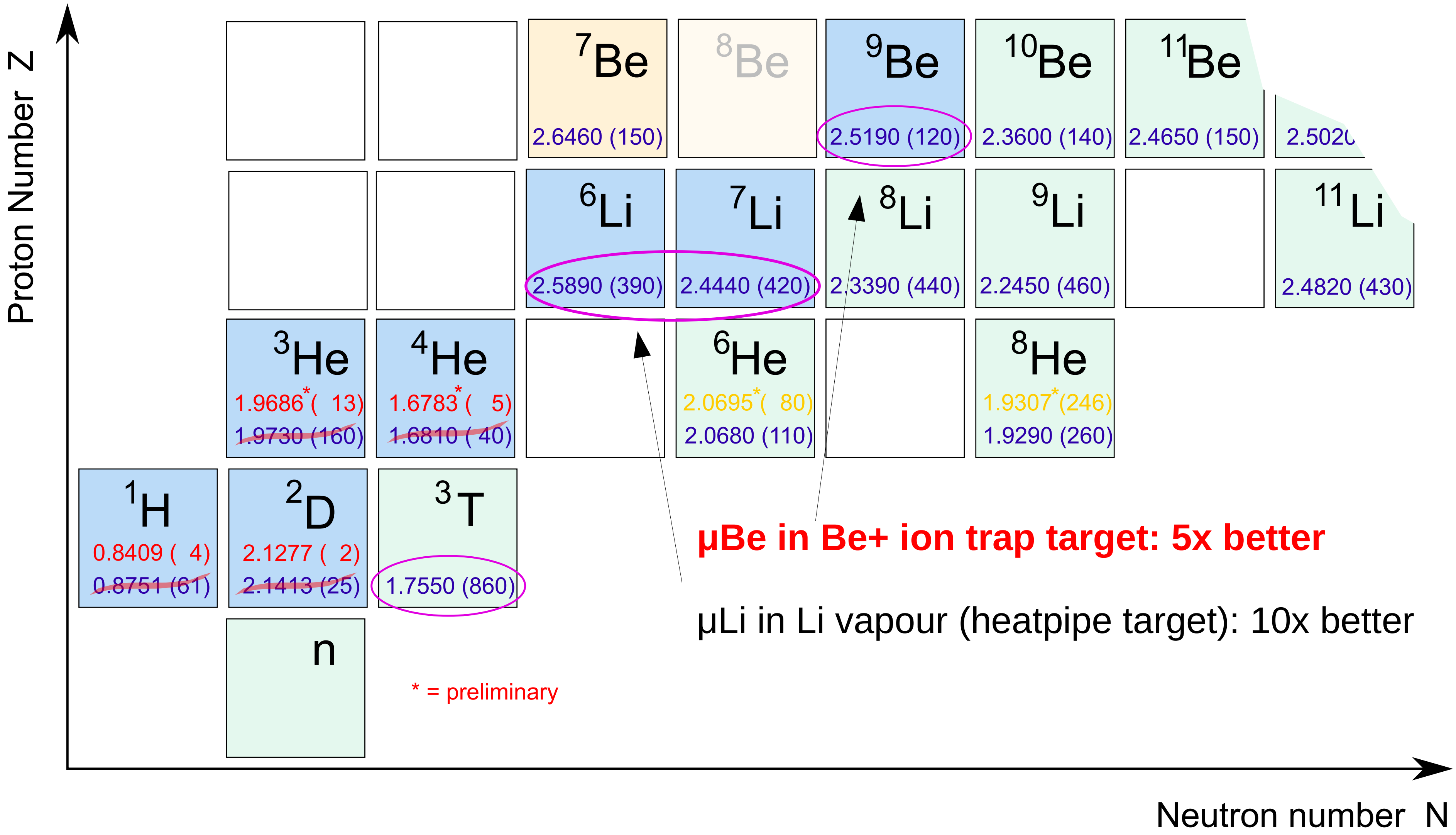






Charge radii: The future



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	n							

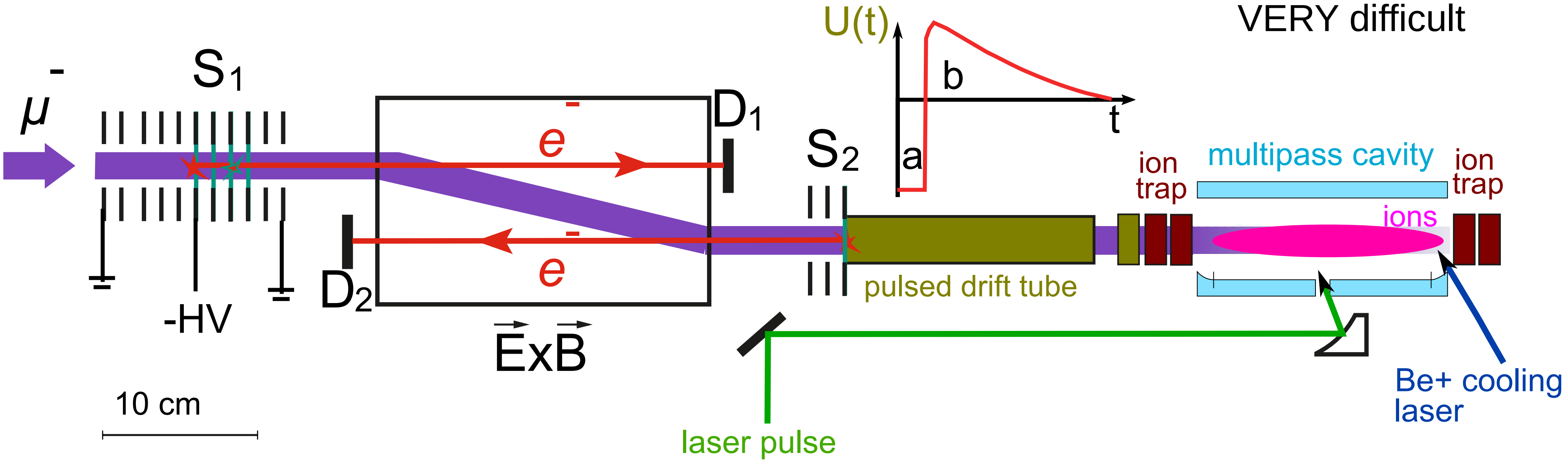
* = preliminary

μBe in Be+ ion trap target: 5x better

μLi in Li vapour (heatpipe target): 10x better

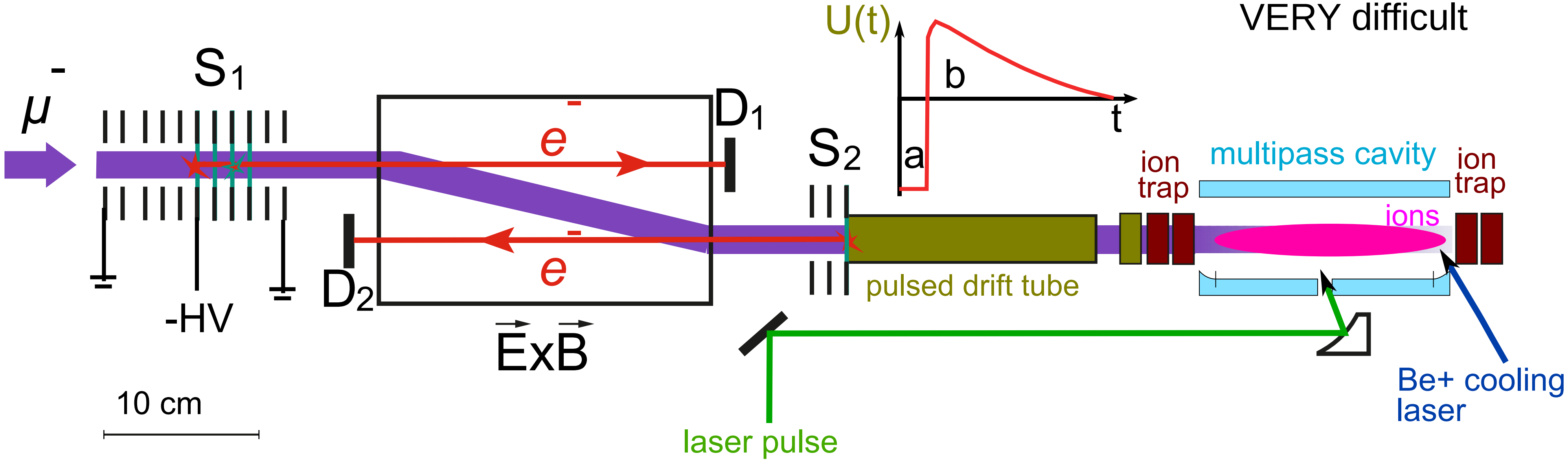
muonic Be with Penning trap target

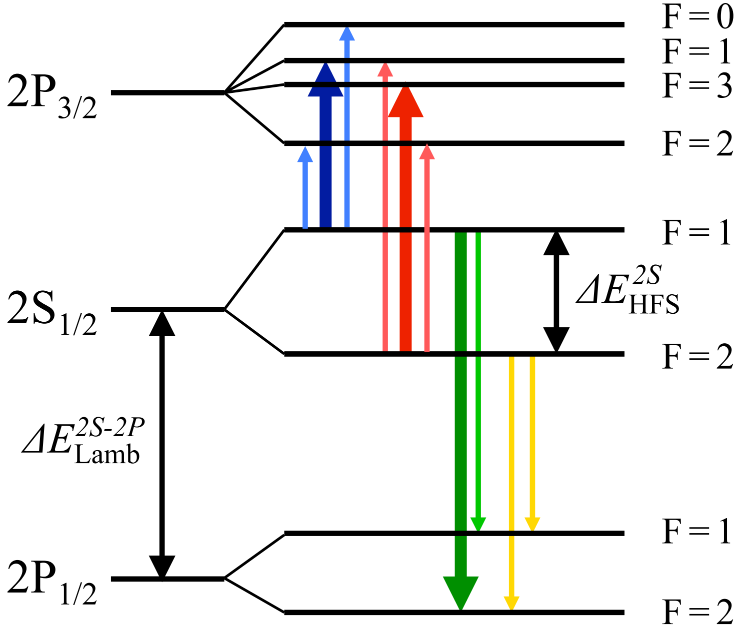
VERY difficult

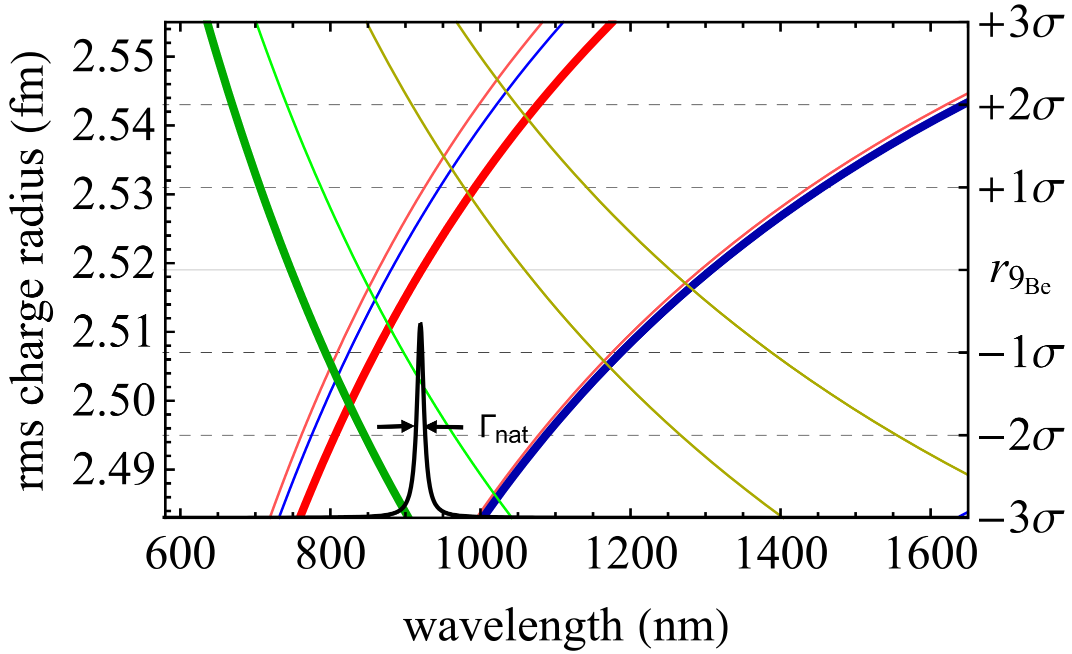


muonic Be with Penning trap target

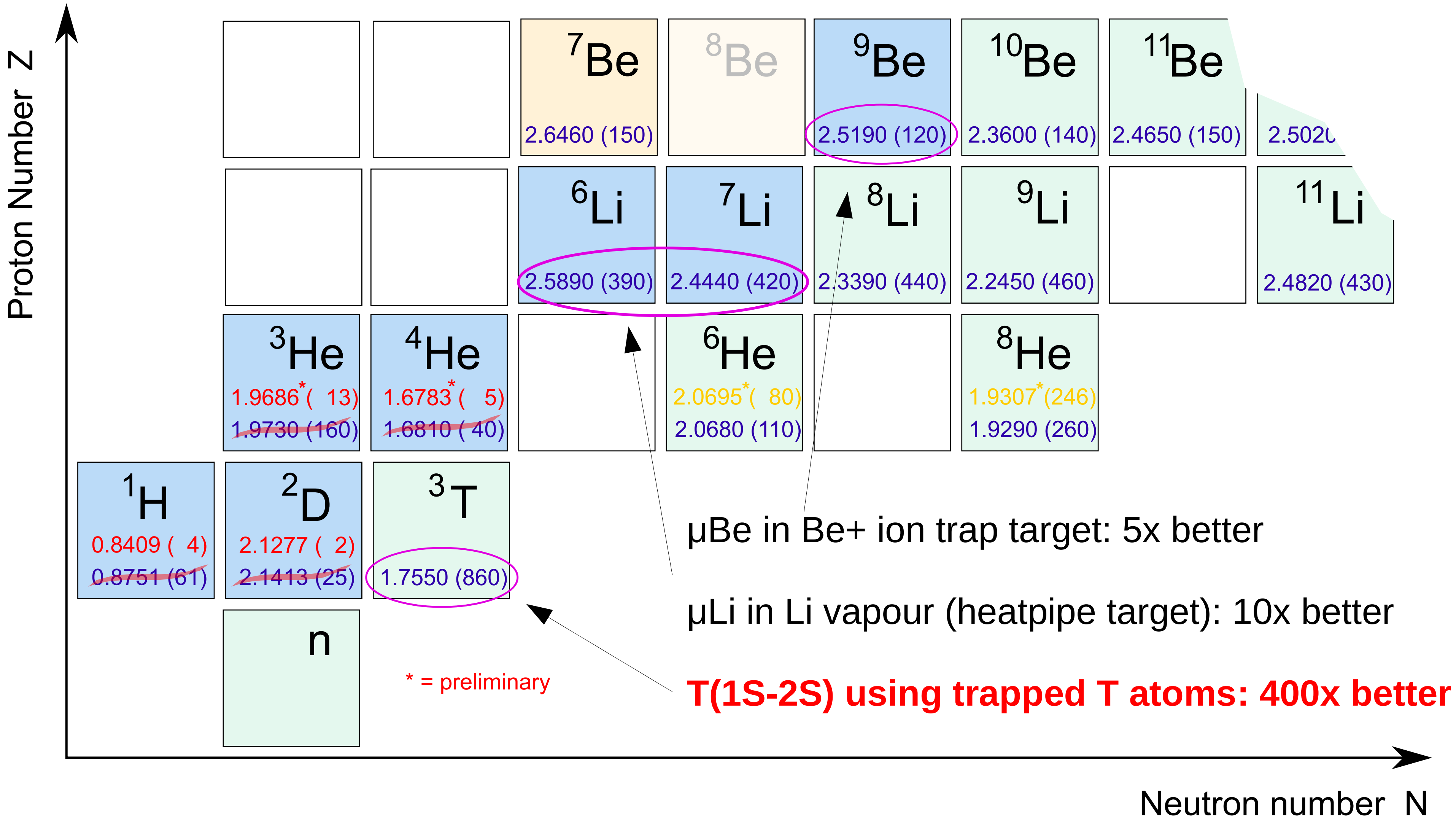
VERY difficult







Charge radii: The future



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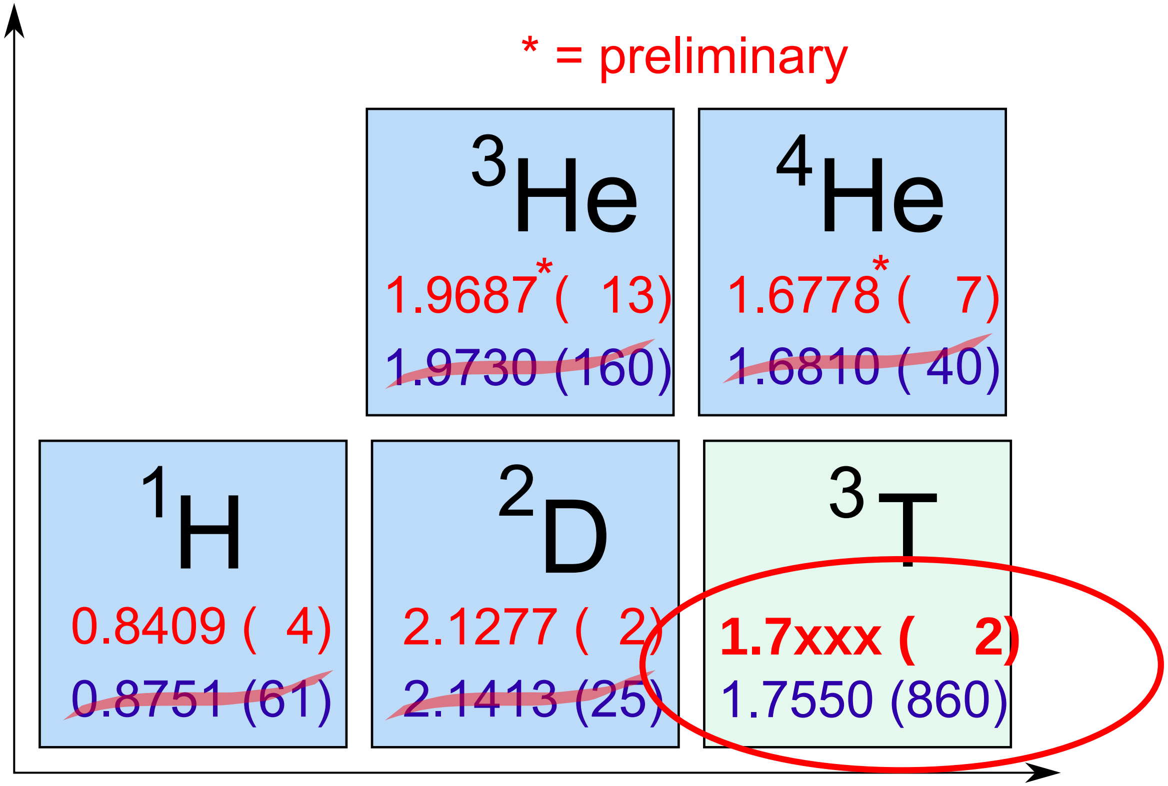
μBe in Be+ ion trap target: 5x better

μLi in Li vapour (heatpipe target): 10x better

T(1S-2S) using trapped T atoms: 400x better

Tritium 1S-2S in a trap



* = preliminary

^{3}He
1.9687* (13)
~~1.9730 (160)~~

^{4}He
1.6778* (7)
~~1.6810 (40)~~

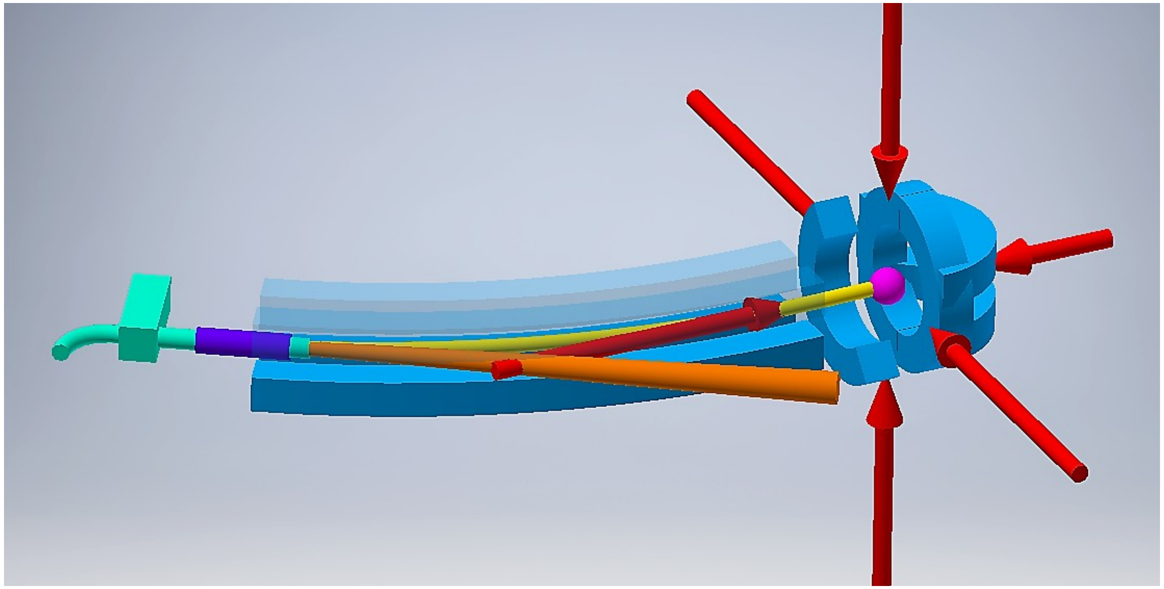
^{1}H
0.8409 (4)
~~0.8751 (61)~~

^{2}D
2.1277 (2)
~~2.1413 (25)~~

^{3}T
1.7xxx (2)
1.7550 (860)

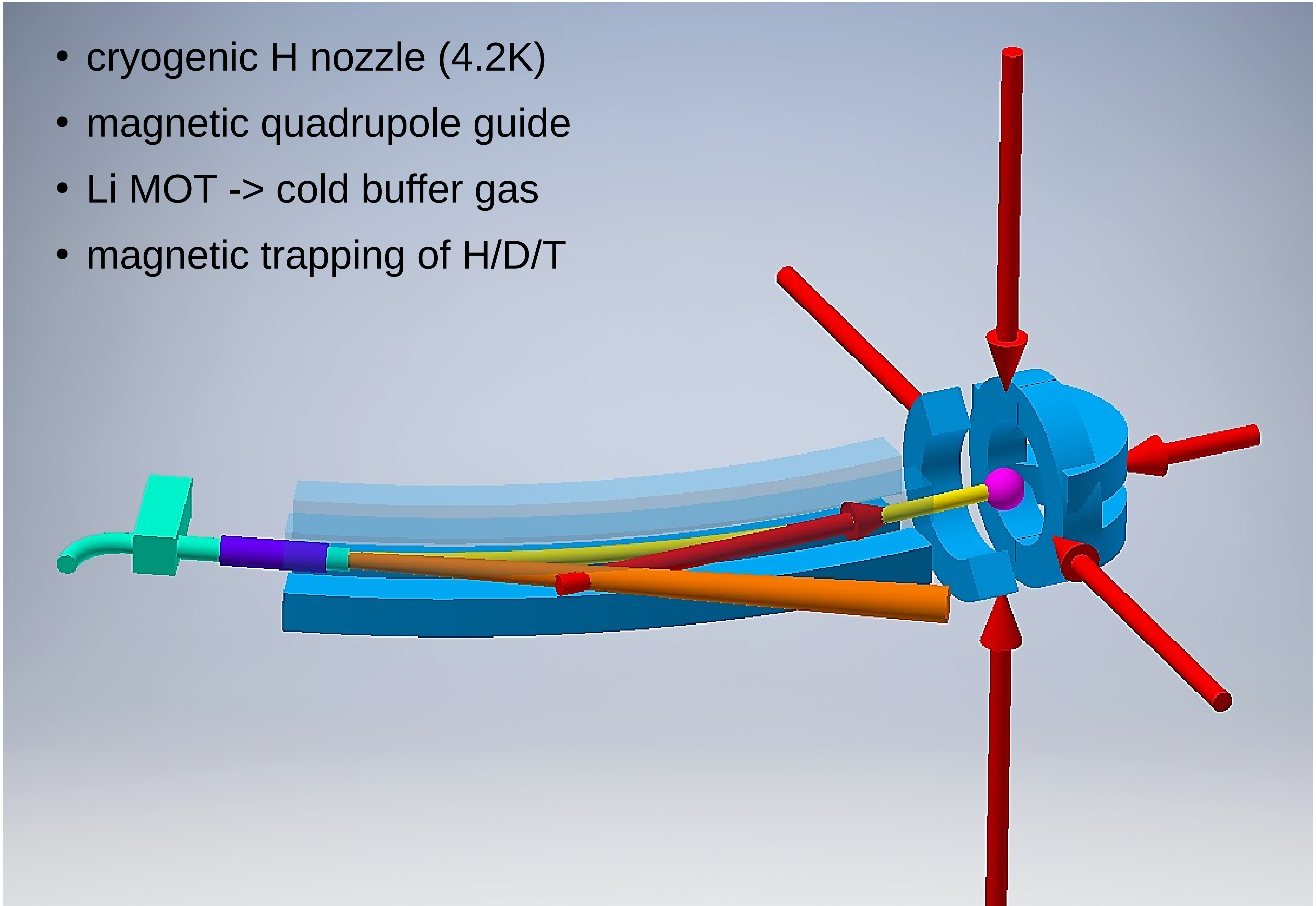
Triton-Radius EXperiment
Mainz

400x better radius
with 1 kHz measurement
(vs. 0.01 kHz for H, D)



- cryogenic H nozzle (4.2K)
- magnetic quadrupole guide
- Li MOT -> cold buffer gas
- magnetic trapping of H/D/T

Tritium 1S-2S in a trap



- cryogenic H nozzle (4.2K)
- magnetic quadrupole guide
- Li MOT -> cold buffer gas
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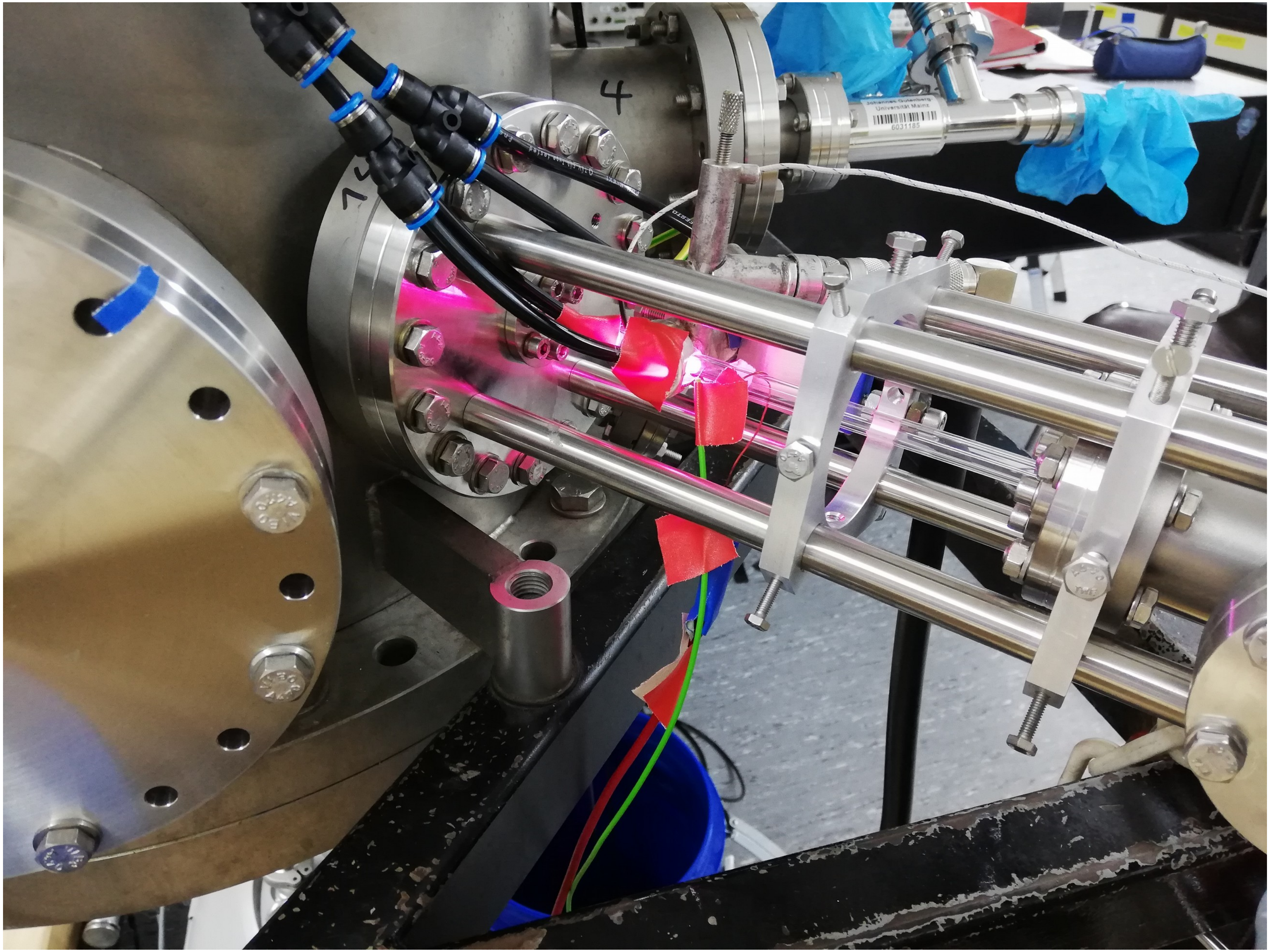
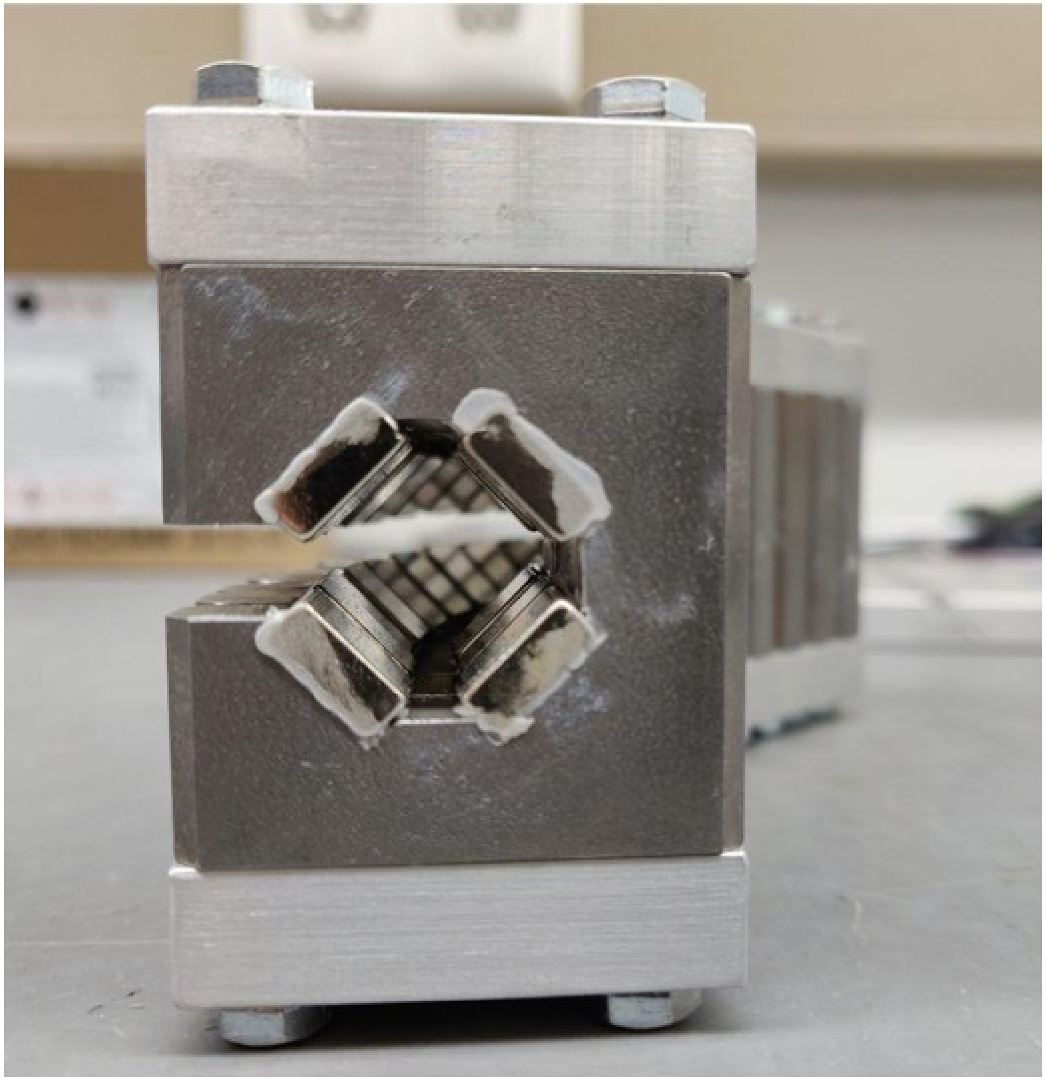
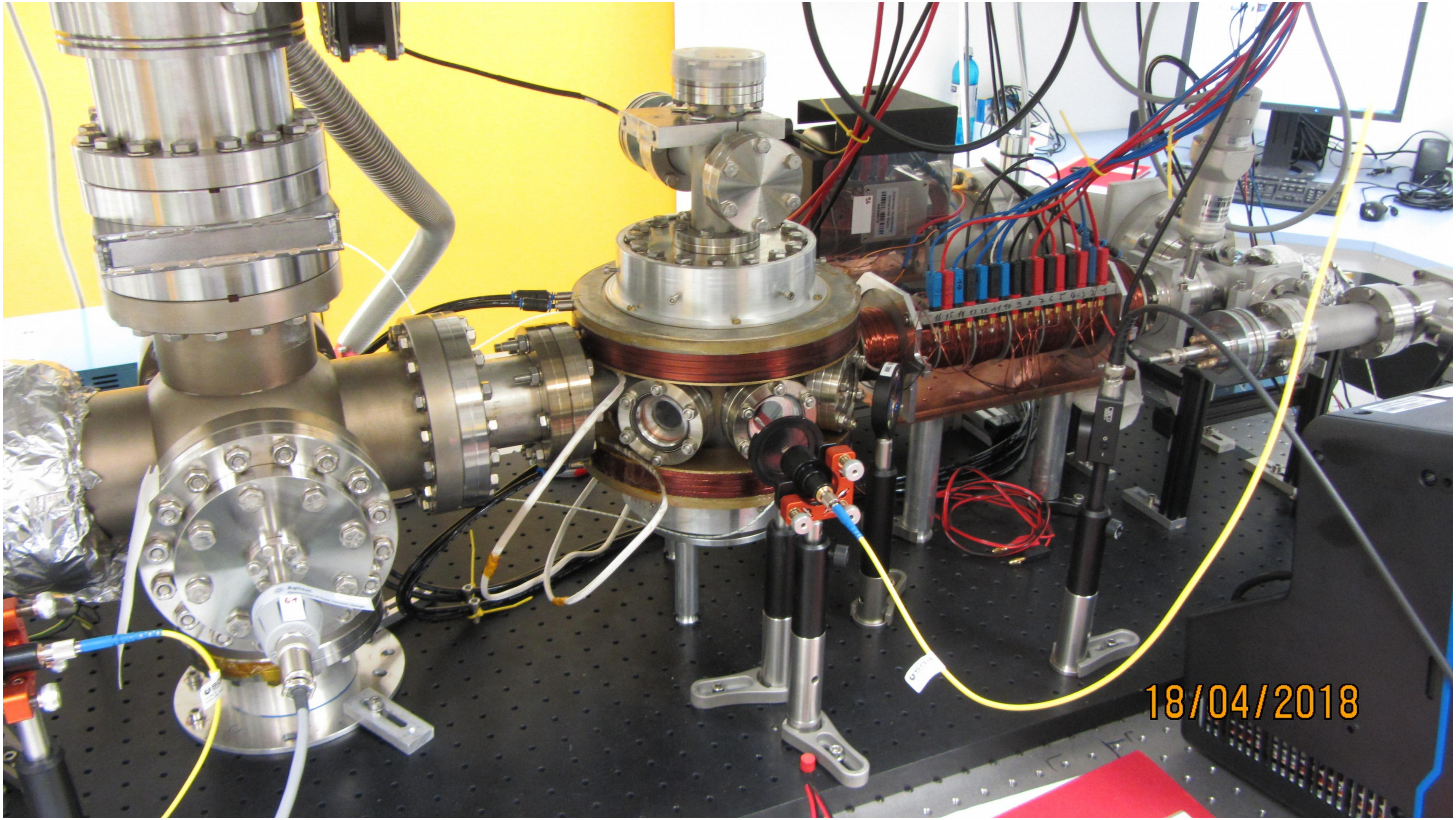
Atomic H source



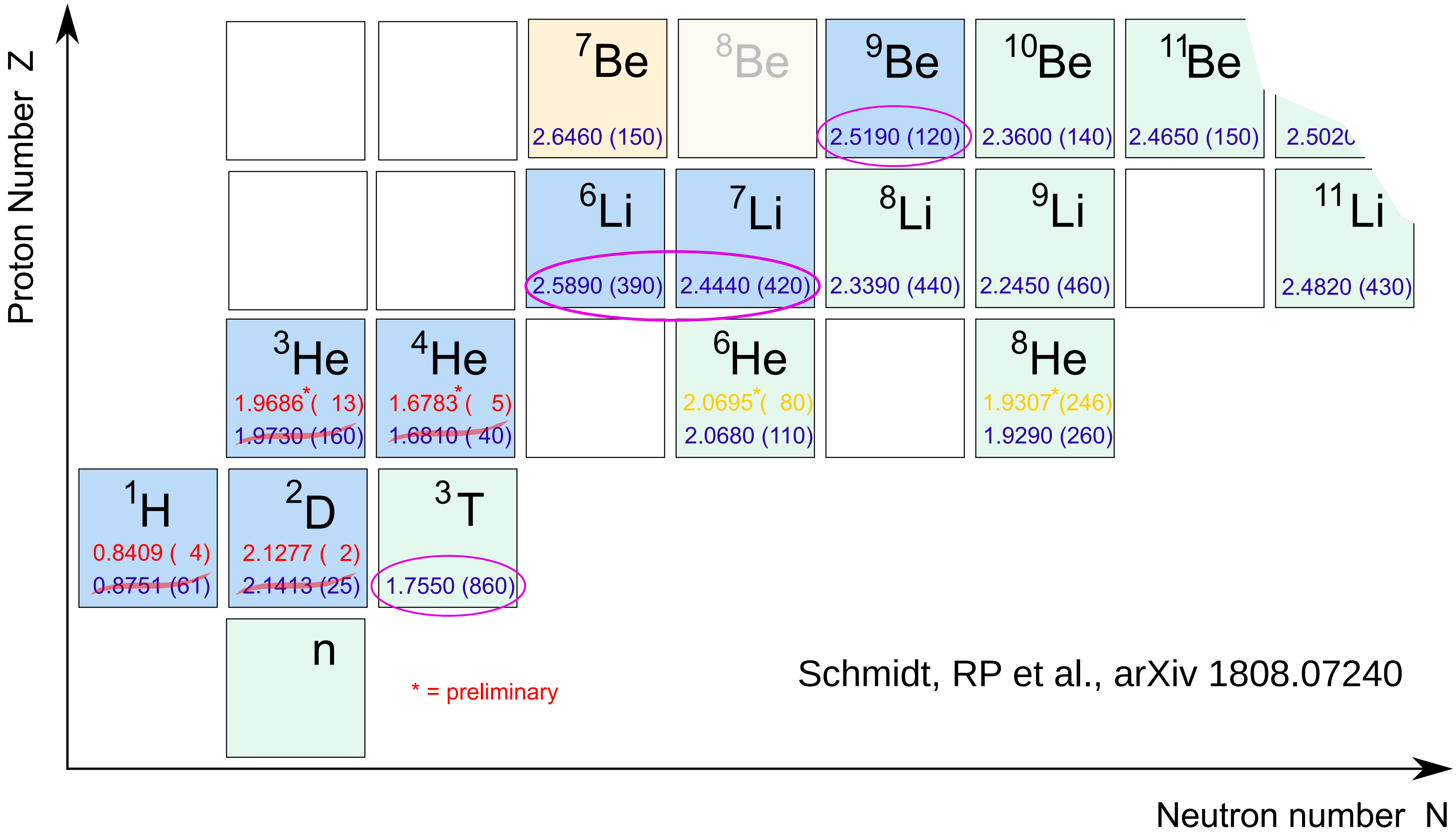
Magnetic quadrupole guide



Li MOT apparatus



Charge radii: The future



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	n							

* = preliminary

Schmidt, RP et al., arXiv 1808.07240

Thanks a lot for your attention

The Garching Hydrogen Team:

Axel Beyer, Lothar Maisenbacher, Arthur Matveev, RP,
Ksenia Khabarova, Alexey Grinin, Tobias Lamour, Dylan C. Yost,
Theodor W. Hänsch, Nikolai Kolachevsky, Thomas Udem

The CREMA Collaboration:

Aldo Antognini, Fernando D. Amaro, François Biraben, João M. R. Cardoso, Daniel S. Covita, Andreas Dax, Satish Dhawan, Marc Diepold, Luis M. P. Fernandes, Adolf Giesen, Andrea L. Gouvea, Thomas Graf, Theodor W. Hänsch, Paul Indelicato, Lucile Julien, Paul Knowles,Franz Kottmann, Juilian J. Krauth, Eric-Olivier Le Bigot, Yi-Wei Liu, José A. M. Lopes, Livia Ludhova, Cristina M. B. Monteiro, Françoise Mulhauser, Tobias Nebel, François Nez, Paul Rabinowitz, Joaquim M. F. dos Santos, Lukas A. Schaller, Karsten Schuhmann, Catherine Schwob, David Taqqu, João F. C. A. Veloso, RP

Thanks a lot for your attention

My new Mainz group:

Jan Haack, Merten Heppener, Rishi Horn, Ahmed Ouf, Stefan Schmidt, Gregor Schwendler, Lukas Schumacher, Andreas Wieltsch, Marcel Willig

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Group at JGU Mainz

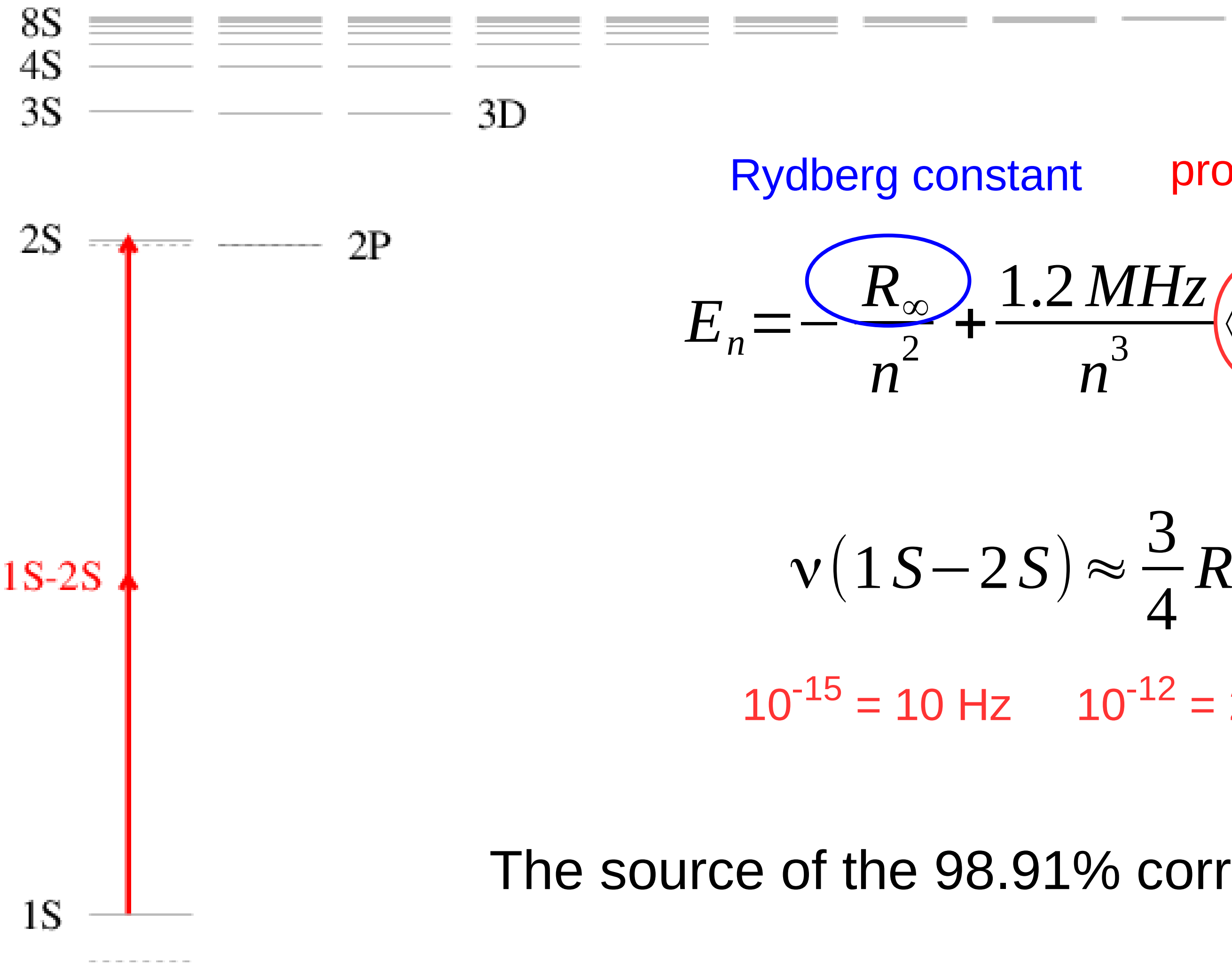
Group at JGU Mainz

Open Positions!

pohl @ uni-mainz.de

. . .

Correlation between R_∞ and R_p / R_d



Rydberg constant

proton radius

$$E_n = -\frac{R_\infty}{n^2} + \frac{1.2\,MHz}{n^3}\langle r^2\rangle\delta_{l0} + \Delta(n,l,j)$$

$$\nu(1S-2S) \approx \frac{3}{4}R_\infty - \frac{7}{8}E_{NS}$$

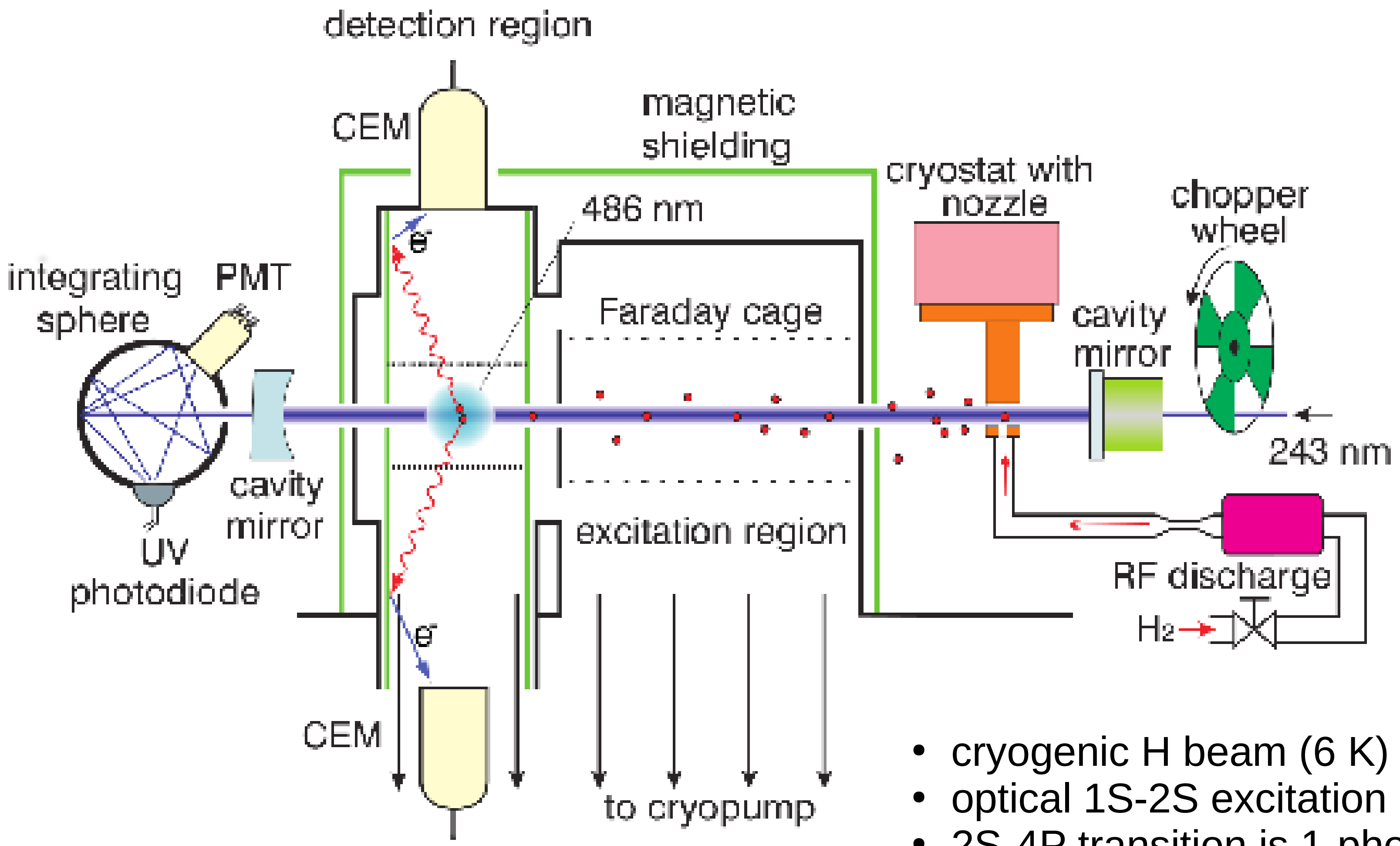
10^{-15} = 10 Hz

10^{-12} = 20 kHz

The source of the 98.91% correlation of R_∞ and R_p

1S-2S: Parthey, RP et al., PRL 107, 203001 (2011)

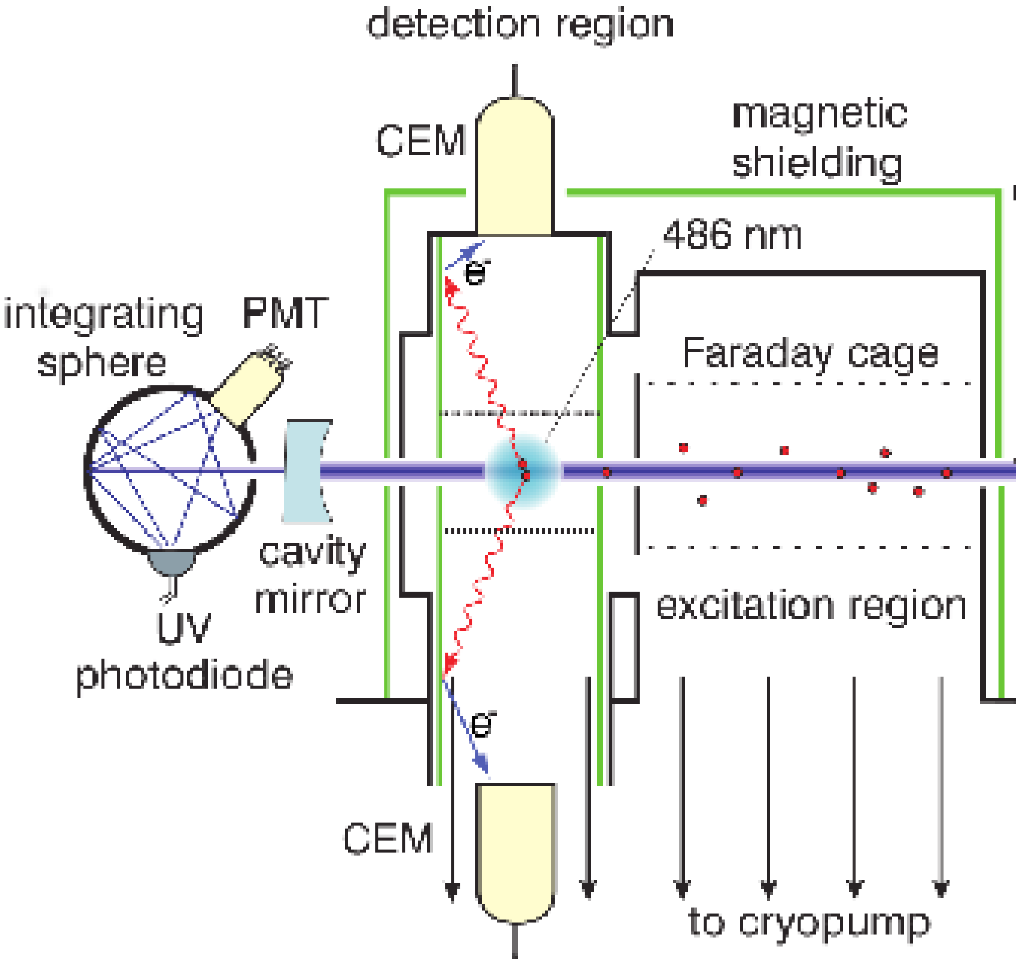
Garching H(2S-4P)



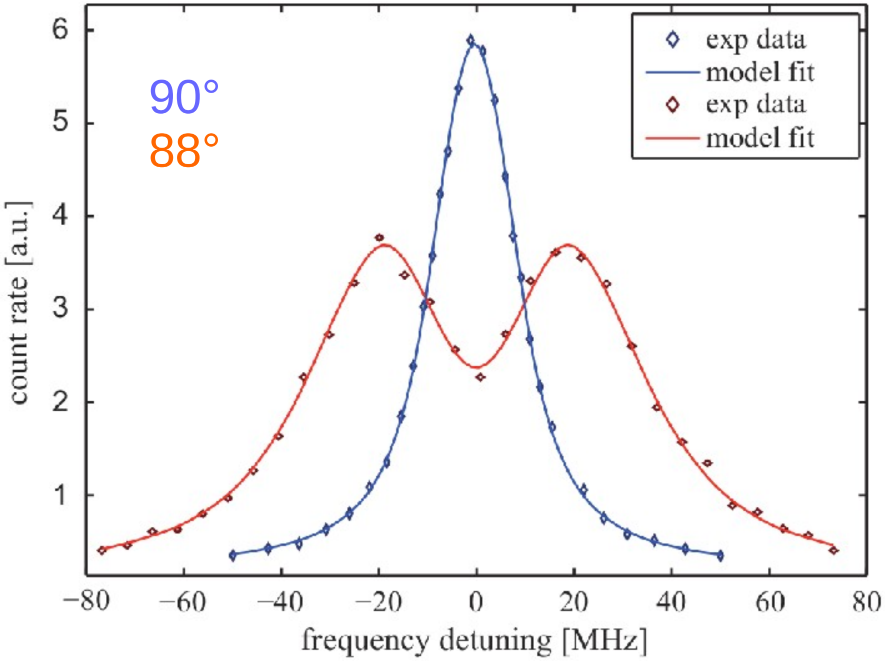
- cryogenic H beam (6 K)
- optical 1S-2S excitation (2S, F=0)
- 2S-4P transition is 1-photon: retroreflector
- split line to 10^{-4} !!!
- 2.3 kHz vs. 9 kHz PRP
- large systematics

Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Garching H(2S-4P)



1st order Doppler cancellation



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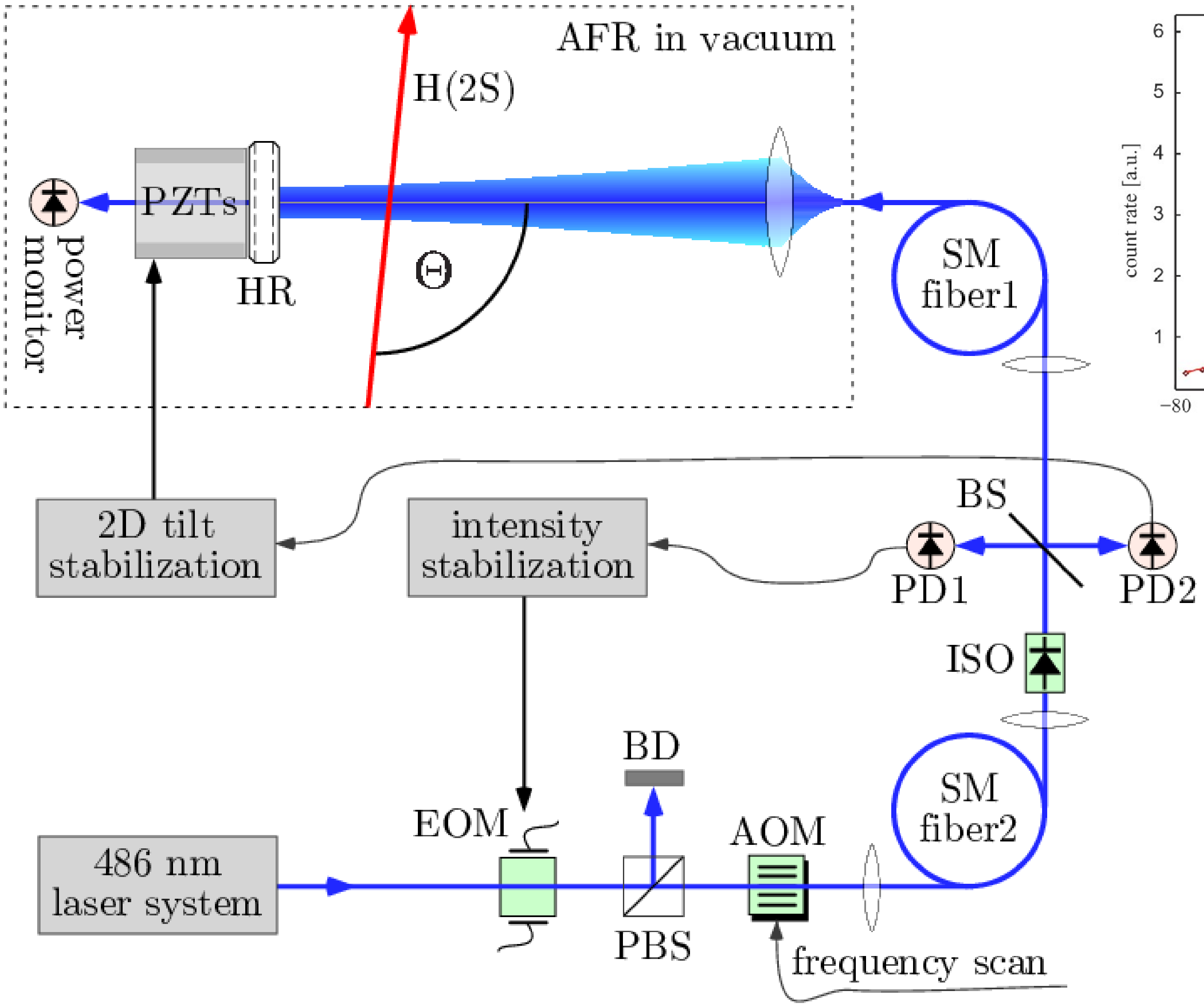
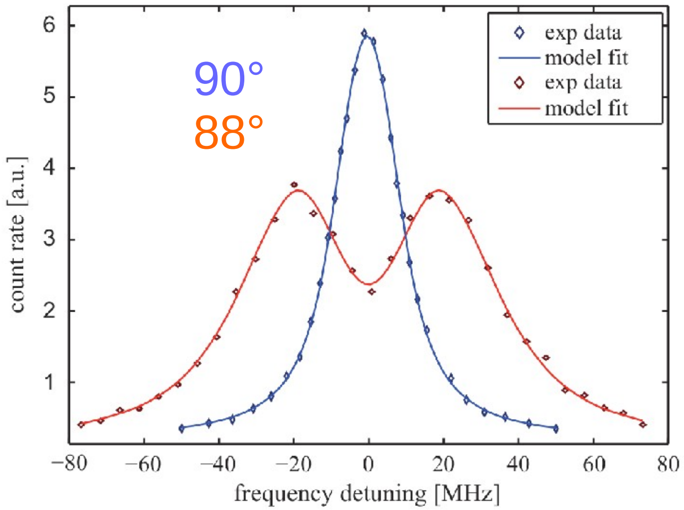
Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

1^{st} order Doppler shift

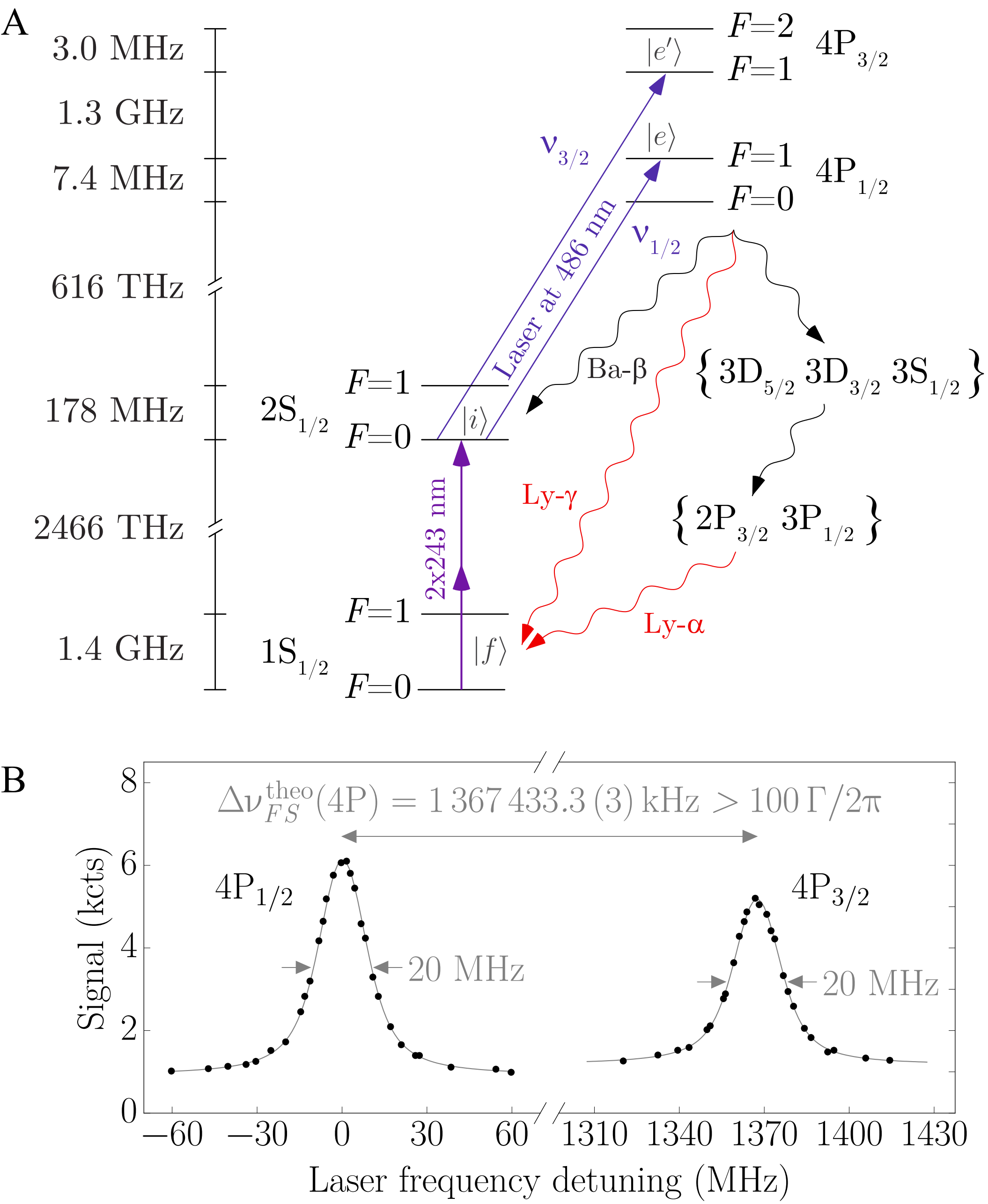
AFR: Active Fiber-based Retroreflector

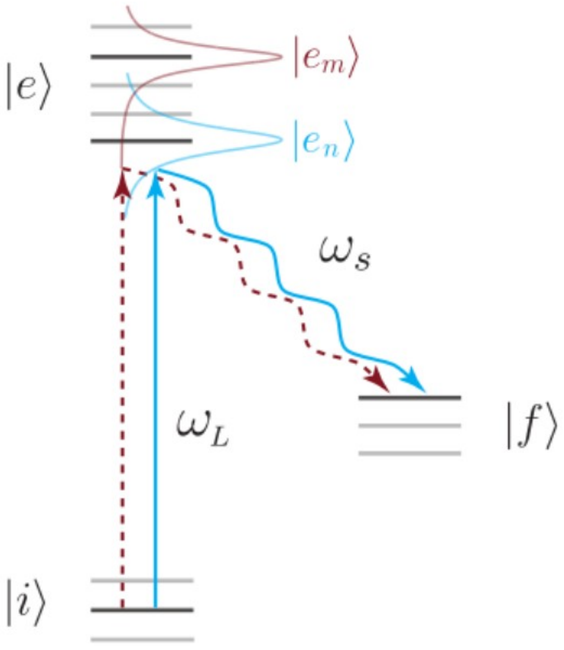




Beyer, RP et al, Opt. Expr. 24, 17470 (2016)

Quantum interference shifts





$$P(\omega)\propto\left|\frac{(\vec{d}_1\vec{E}_0)\vec{d}_1}{\omega_1-\omega_L+i\gamma_1/2}+\frac{(\vec{d}_2\vec{E}_0)\vec{d}_2 e^{i\Delta\Phi}}{\omega_2-\omega_L+i\gamma_2/2}\right|^2$$

= Lorentzian(1) + Lorentzian(2)

+ cross-term (QI)

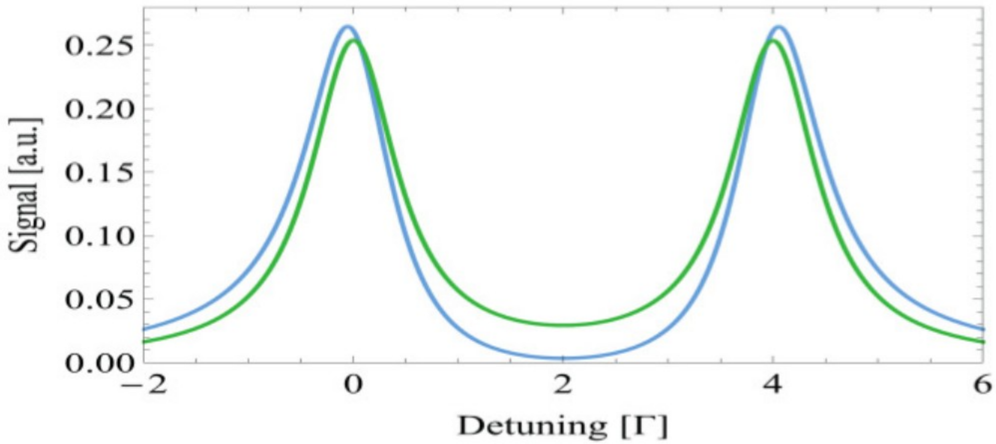
see

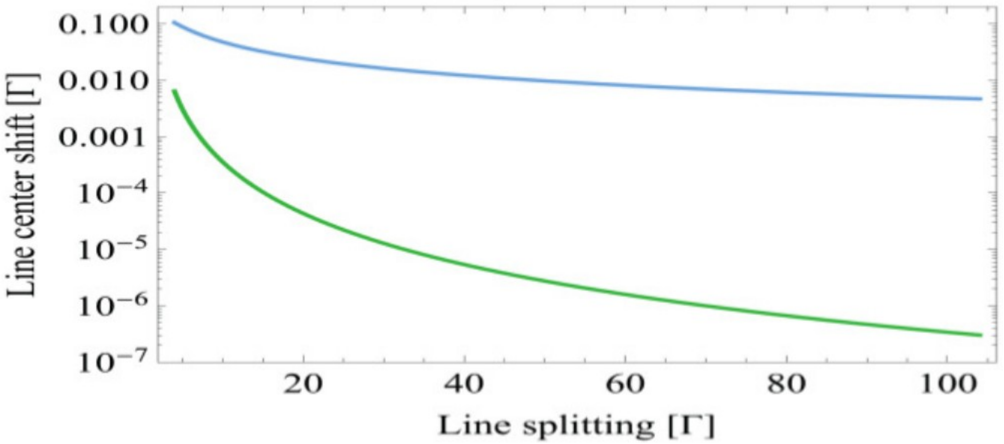
Horbatsch, Hessels, PRA 82, 052519 (2010); PRA 84, 032508 (2011); PRA 86 040501 (2012)

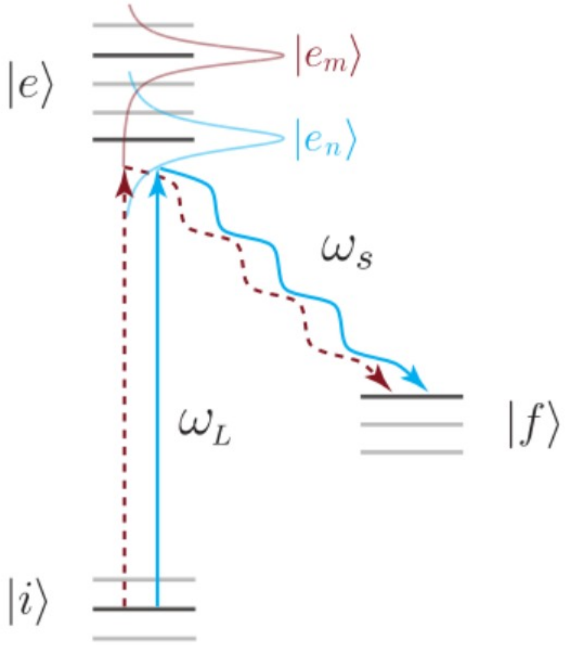
Sansonetti et al., PRL 107, 021001 (2011)

Brown et al., PRA 87, 032504 (2013)

Quantum interference shifts







$$P(\omega) \propto \left| \frac{(\vec{d}_1 \vec{E}_0)\vec{d}_1}{\omega_1 - \omega_L + i\gamma_1/2} + \frac{(\vec{d}_2 \vec{E}_0)\vec{d}_2 e^{i\Delta\Phi}}{\omega_2 - \omega_L + i\gamma_2/2} \right|^2$$

= Lorentzian(1) + Lorentzian(2)

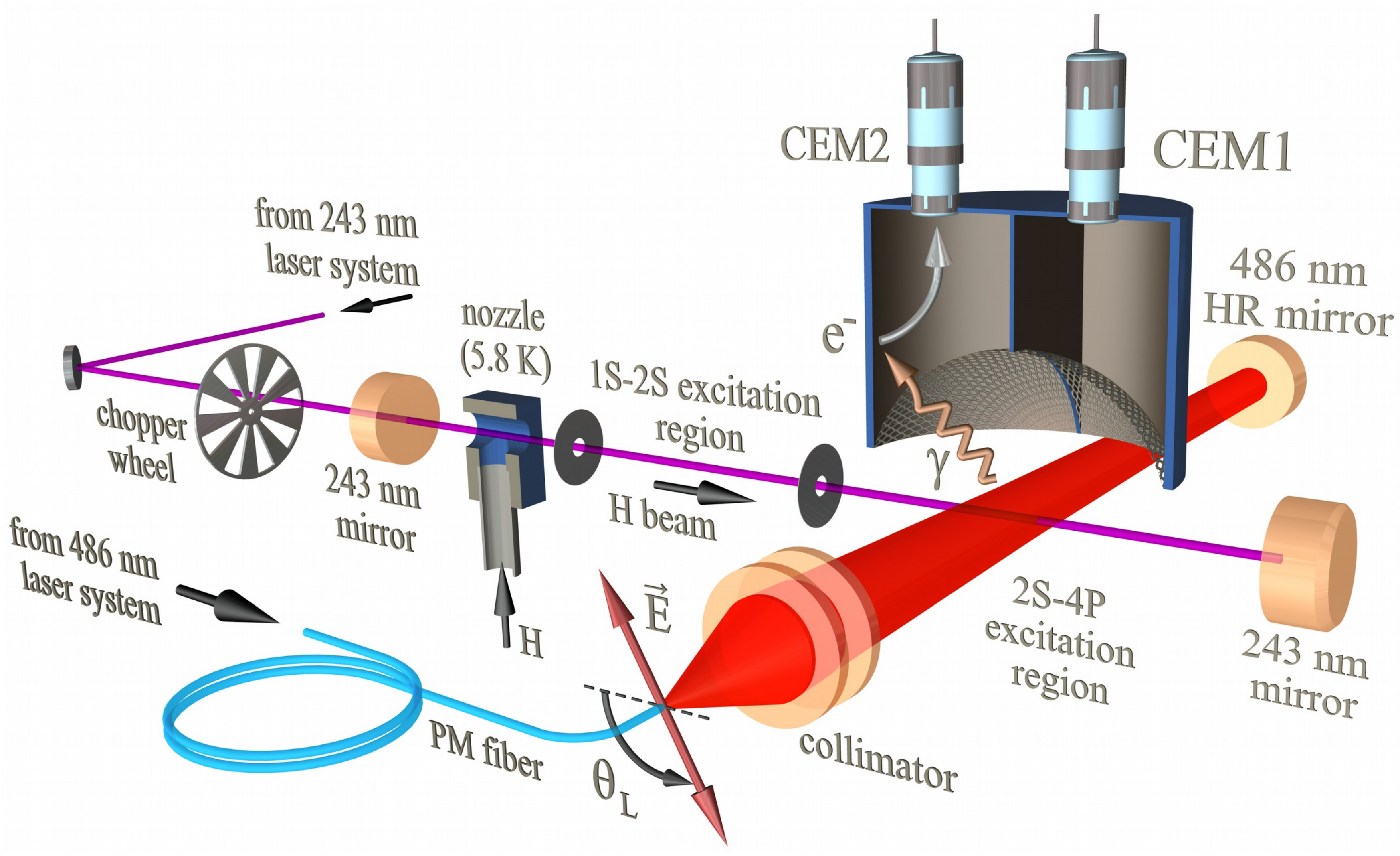
+ cross-term (QI)

Fitting this with 2 Lorentzians creates

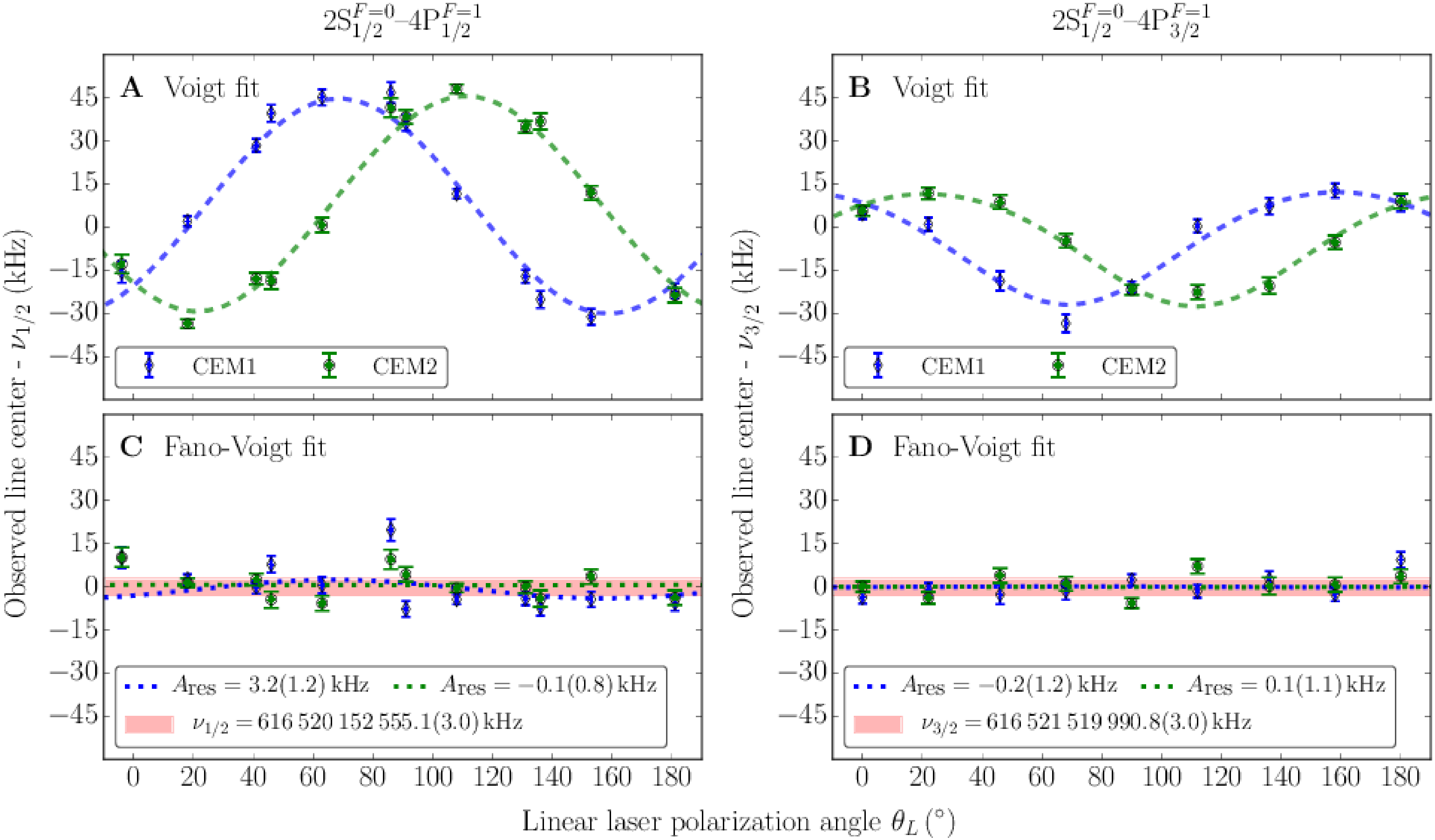
line shifts

see
Horbatsch, Hessels, PRA 82, 052519 (2010); PRA 84, 032508 (2011); PRA 86 040501 (2012)
Sansonetti et al., PRL 107, 021001 (2011)
Brown et al., PRA 87, 032504 (2013)

Studying QI in 2S-4P

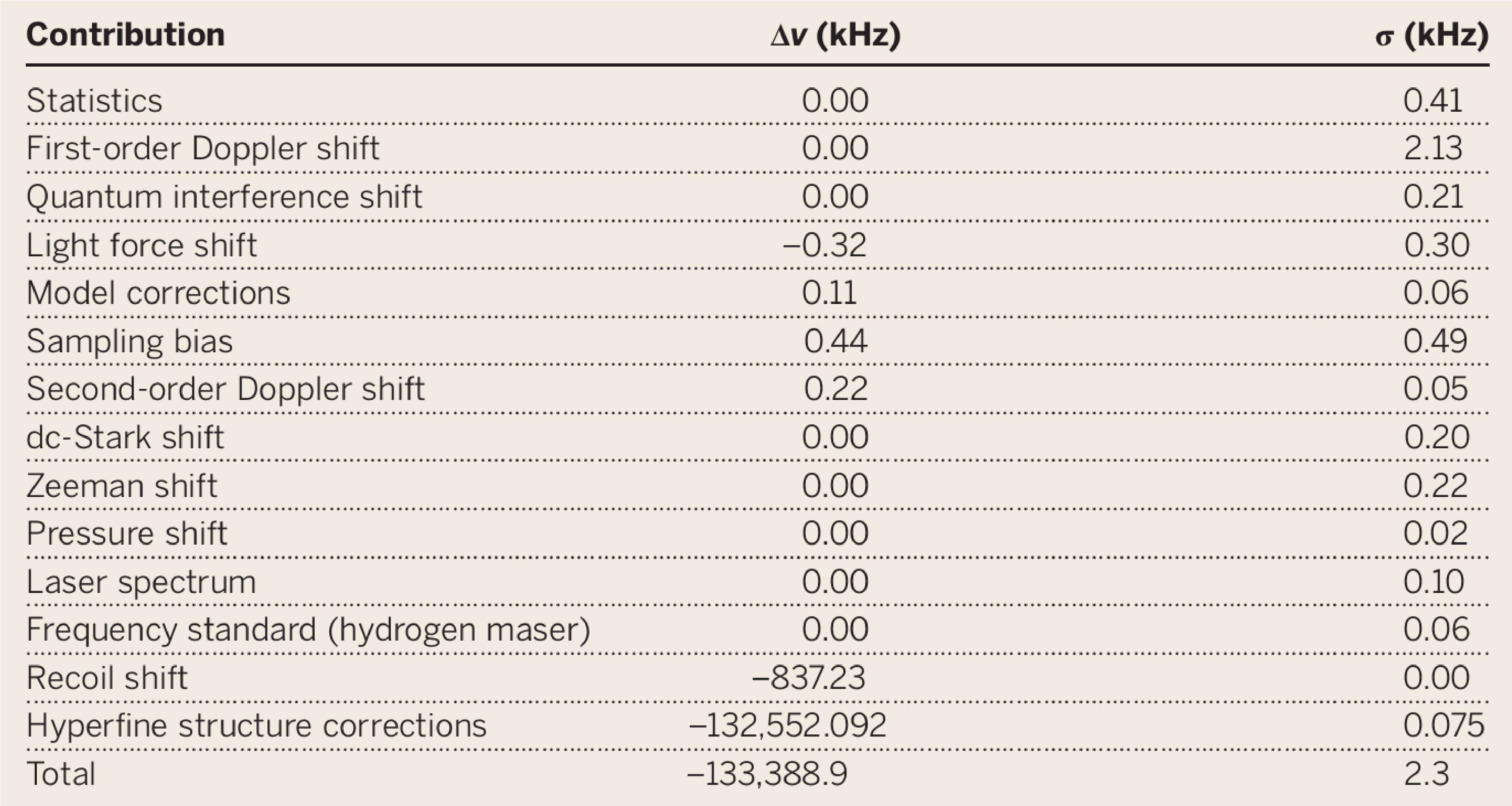


QI in hydrogen   (Δ = 100 Γ)



Beyer, Maisenbacher, RP et al, Science 358, 79 (2017)

Systematics

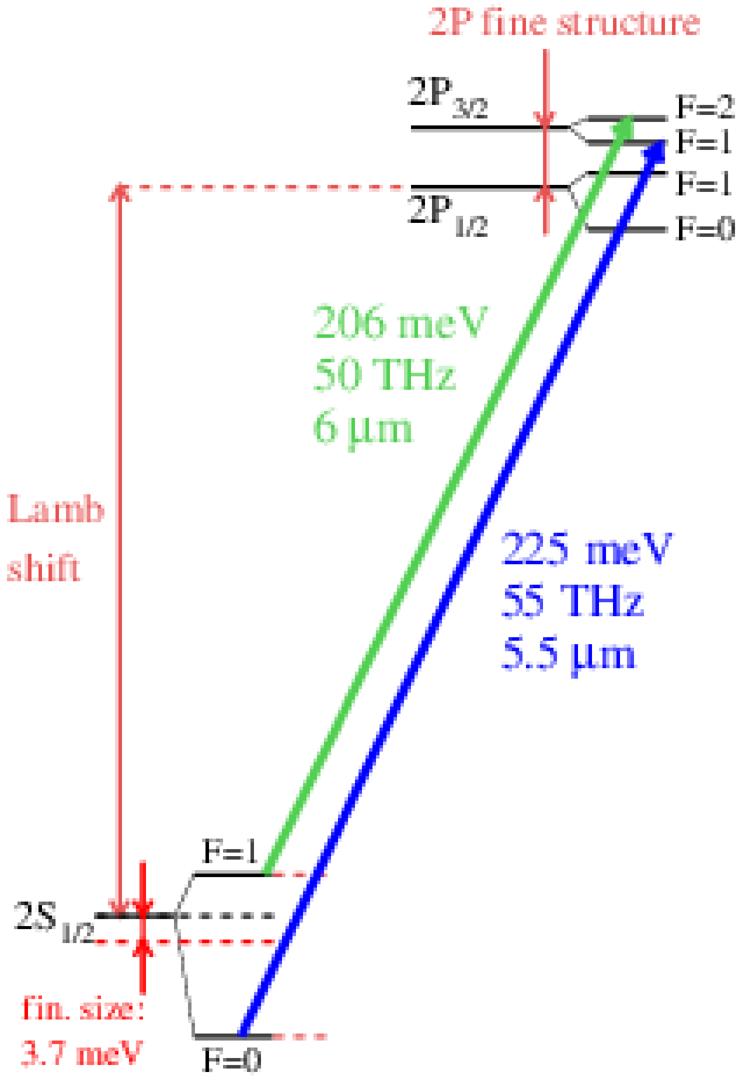


Contribution	$\Delta\nu$ (kHz)	σ (kHz)
Statistics	0.00	0.41
First-order Doppler shift	0.00	2.13
Quantum interference shift	0.00	0.21
Light force shift	–0.32	0.30
Model corrections	0.11	0.06
Sampling bias	0.44	0.49
Second-order Doppler shift	0.22	0.05
dc-Stark shift	0.00	0.20
Zeeman shift	0.00	0.22
Pressure shift	0.00	0.02
Laser spectrum	0.00	0.10
Frequency standard (hydrogen maser)	0.00	0.06
Recoil shift	–837.23	0.00
Hyperfine structure corrections	–132,552.092	0.075
Total	–133,388.9	2.3

Theory in muonic H

$$\Delta E_{\text{Lamb}} = 206.0336\,(15)\ \text{meV}_{\text{QED}} + 0.0332\,(20)\ \text{meV}_{\text{TPE}} - 5.2275\,(10)\ \text{meV/fm}^2 * \mathbf{R_p}^{\mathbf{2}}$$

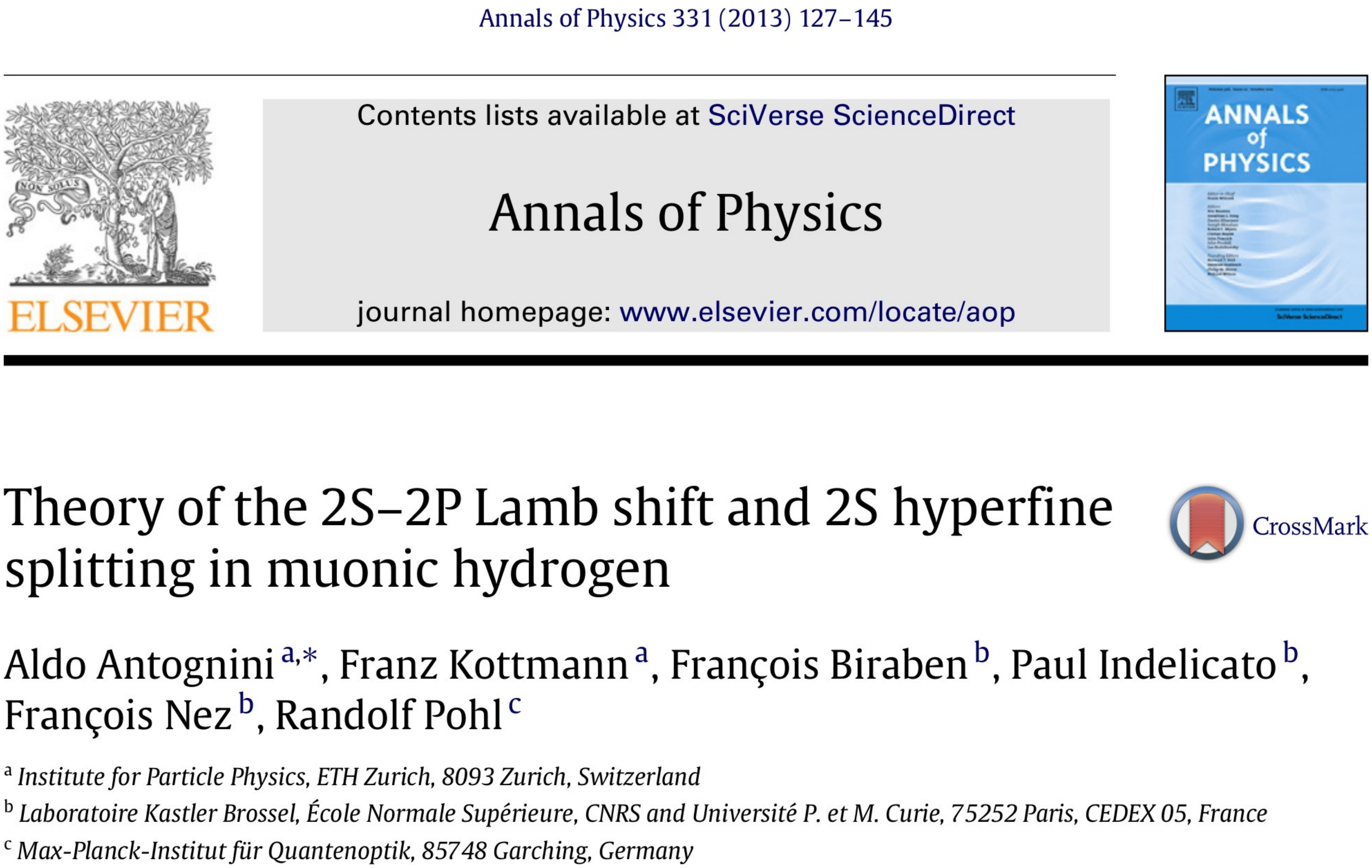
Simple-looking formula
based on decades of work by
E. Borie, M.C. Birse, P. Blunden, C.E. Carlson,
M.I. Eides, R. Faustov, J.L. Friar, G. Paz,
A. Pineda, J. McGovern, K. Griffioen, H. Grotch,
F. Hagelstein, H.-W. Hammer, R.J Hill, P.Indelicato,
U.D. Jentschura, S.G. Karshenboim, E.Y. Korzinin,
V.G. Ivanov, I.T. Lorenz, A.P. Martynenko,
G.A. Miller, U.-G. Meissner, P.J. Mohr,
K. Pachucki, V. Pascalutsa, J. Rafelski,
V.A. Shelyuto, I. Sick, A.W. Thomas,
M. Vanderhaeghen, V. Yerokhin,
…..
(shout if I missed your name!)



Antognini, RP at al., Ann. Phys. (N.Y.) 331, 127 (2013)

Theory in muonic H

$$\Delta E_{Lamb} = 206.0336\ (15)\ \mathrm{meV}_{QED} + 0.0332\ (20)\ \mathrm{meV}_{TPE} - 5.2275\ (10)\ \mathrm{meV/fm}^2 * \mathbf{R_p}^{\mathbf{2}}$$



Annals of Physics 331 (2013) 127–145

Contents lists available at SciVerse ScienceDirect

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journal homepage: www.elsevier.com/locate/aop

Theory of the 2S–2P Lamb shift and 2S hyperfine splitting in muonic hydrogen

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[a] Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland
[b] Laboratoire Kastler Brossel, École Normale Supérieure, CNRS and Université P. et M. Curie, 75252 Paris, CEDEX 05, France
[c] Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany

Our attempt to summarize all the original work by many theorists….

Theory I: “pure” QED

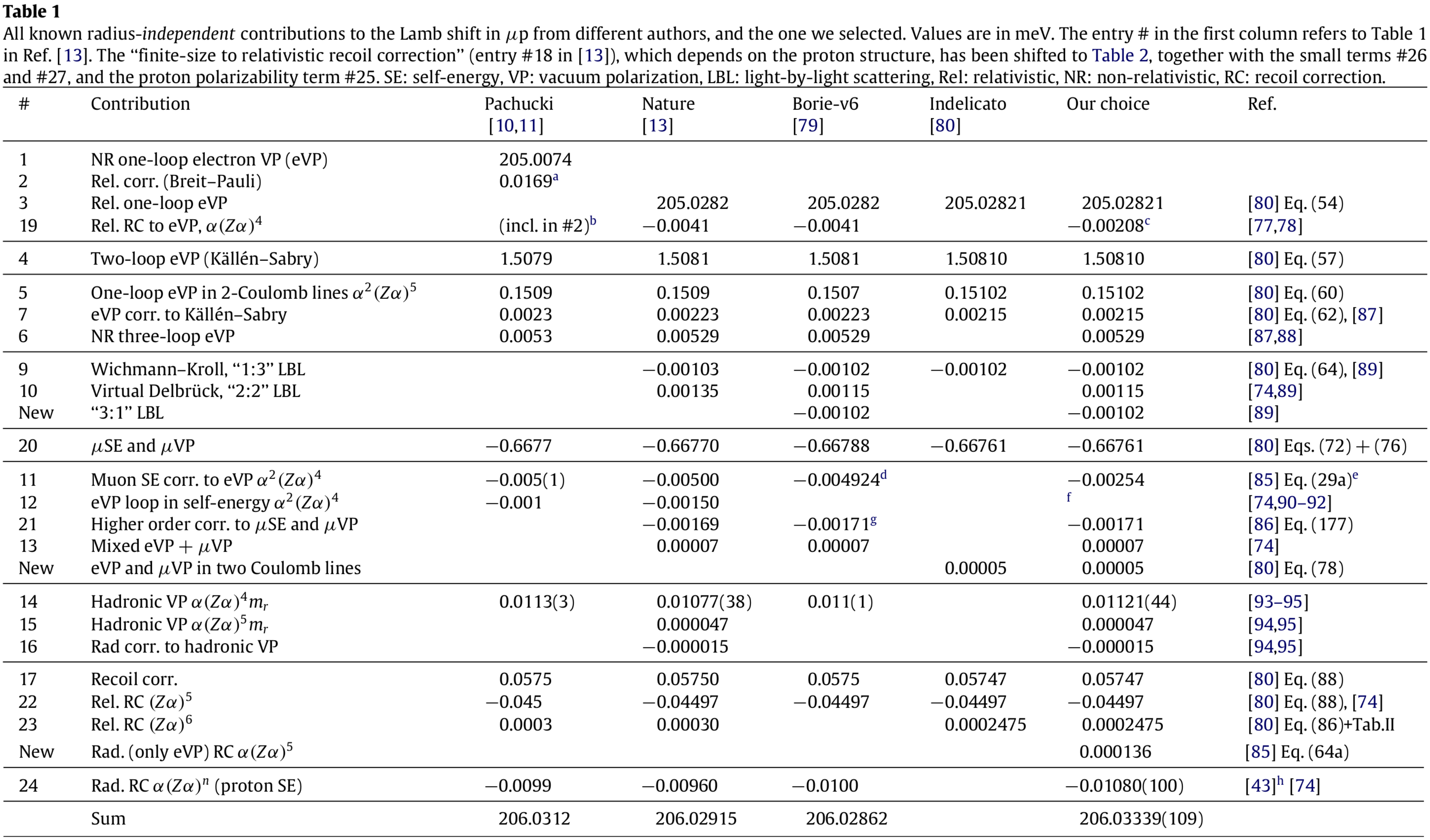


Table 1

All known radius-*independent* contributions to the Lamb shift in μp from different authors, and the one we selected. Values are in meV. The entry # in the first column refers to Table 1 in Ref. [13]. The “finite-size to relativistic recoil correction” (entry #18 in [13]), which depends on the proton structure, has been shifted to Table 2, together with the small terms #26 and #27, and the proton polarizability term #25. SE: self-energy, VP: vacuum polarization, LBL: light-by-light scattering, Rel: relativistic, NR: non-relativistic, RC: recoil correction.

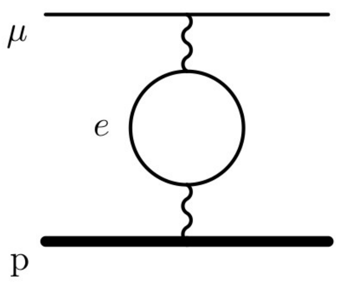
#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
1	NR one-loop electron VP (eVP)	205.0074					
2	Rel. corr. (Breit–Pauli)	0.0169[a]					
3	Rel. one-loop eVP		205.0282	205.0282	205.02821	205.02821	[80] Eq. (54)
19	Rel. RC to eVP, $\alpha(Z\alpha)^4$	(incl. in #2)[b]	−0.0041	−0.0041		−0.00208[c]	[77,78]
4	Two-loop eVP (Källén–Sabry)	1.5079	1.5081	1.5081	1.50810	1.50810	[80] Eq. (57)
5	One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$	0.1509	0.1509	0.1507	0.15102	0.15102	[80] Eq. (60)
7	eVP corr. to Källén–Sabry	0.0023	0.00223	0.00223	0.00215	0.00215	[80] Eq. (62), [87]
6	NR three-loop eVP	0.0053	0.00529	0.00529		0.00529	[87,88]
9	Wichmann–Kroll, “1:3” LBL		−0.00103	−0.00102	−0.00102	−0.00102	[80] Eq. (64), [89]
10	Virtual Delbrück, “2:2” LBL		0.00135	0.00115		0.00115	[74,89]
New	“3:1” LBL			−0.00102		−0.00102	[89]
20	μSE and μVP	−0.6677	−0.66770	−0.66788	−0.66761	−0.66761	[80] Eqs. (72) + (76)
11	Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$	−0.005(1)	−0.00500	−0.004924[d]		−0.00254	[85] Eq. (29a)[e]
12	eVP loop in self-energy $\alpha^2(Z\alpha)^4$	−0.001	−0.00150			[f]	[74,90–92]
21	Higher order corr. to μSE and μVP		−0.00169	−0.00171[g]		−0.00171	[86] Eq. (177)
13	Mixed eVP + μVP		0.00007	0.00007		0.00007	[74]
New	eVP and μVP in two Coulomb lines				0.00005	0.00005	[80] Eq. (78)
14	Hadronic VP $\alpha(Z\alpha)^4 m_r$	0.0113(3)	0.01077(38)	0.011(1)		0.01121(44)	[93–95]
15	Hadronic VP $\alpha(Z\alpha)^5 m_r$		0.000047			0.000047	[94,95]
16	Rad corr. to hadronic VP		−0.000015			−0.000015	[94,95]
17	Recoil corr.	0.0575	0.05750	0.0575	0.05747	0.05747	[80] Eq. (88)
22	Rel. RC $(Z\alpha)^5$	−0.045	−0.04497	−0.04497	−0.04497	−0.04497	[80] Eq. (88), [74]
23	Rel. RC $(Z\alpha)^6$	0.0003	0.00030		0.0002475	0.0002475	[80] Eq. (86)+Tab.II
New	Rad. (only eVP) RC $\alpha(Z\alpha)^5$					0.000136	[85] Eq. (64a)
24	Rad. RC $\alpha(Z\alpha)^n$ (proton SE)	−0.0099	−0.00960	−0.0100		−0.01080(100)	[43][h] [74]
	Sum	206.0312	206.02915	206.02862		206.03339(109)	

Antognini, RP at al., Ann. Phys. (N.Y.) 331, 127 (2013)

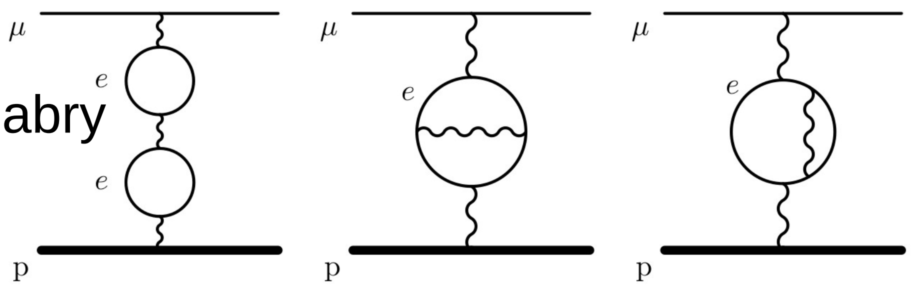
Theory in muonic H

$$\Delta E_{Lamb} = 206.0336\ (15)\ \mathrm{meV}_{QED} + 0.0332\ (20)\ \mathrm{meV}_{TPE} - 5.2275\ (10)\ \mathrm{meV/fm}^2 * \mathbf{R_p}^{\mathbf{2}}$$

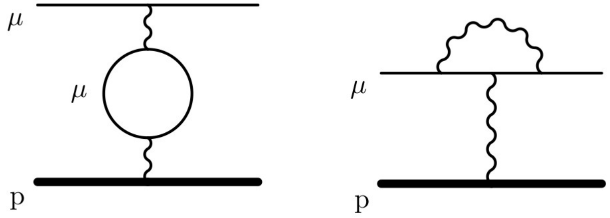
Uehling



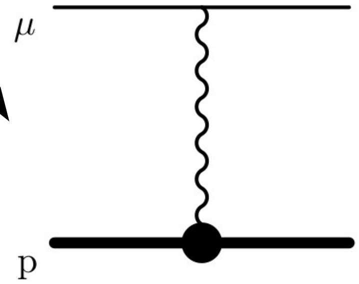
Källen-Sabry



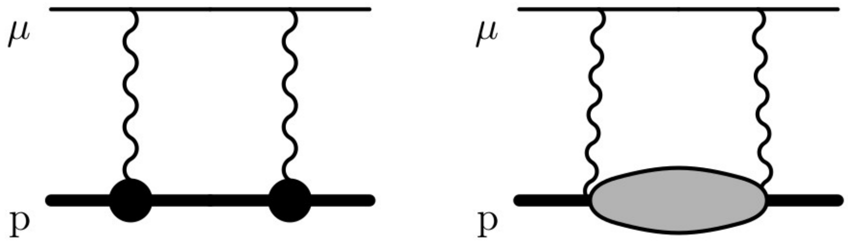
Muon SE+VP



and 20+ more….



Proton form factor

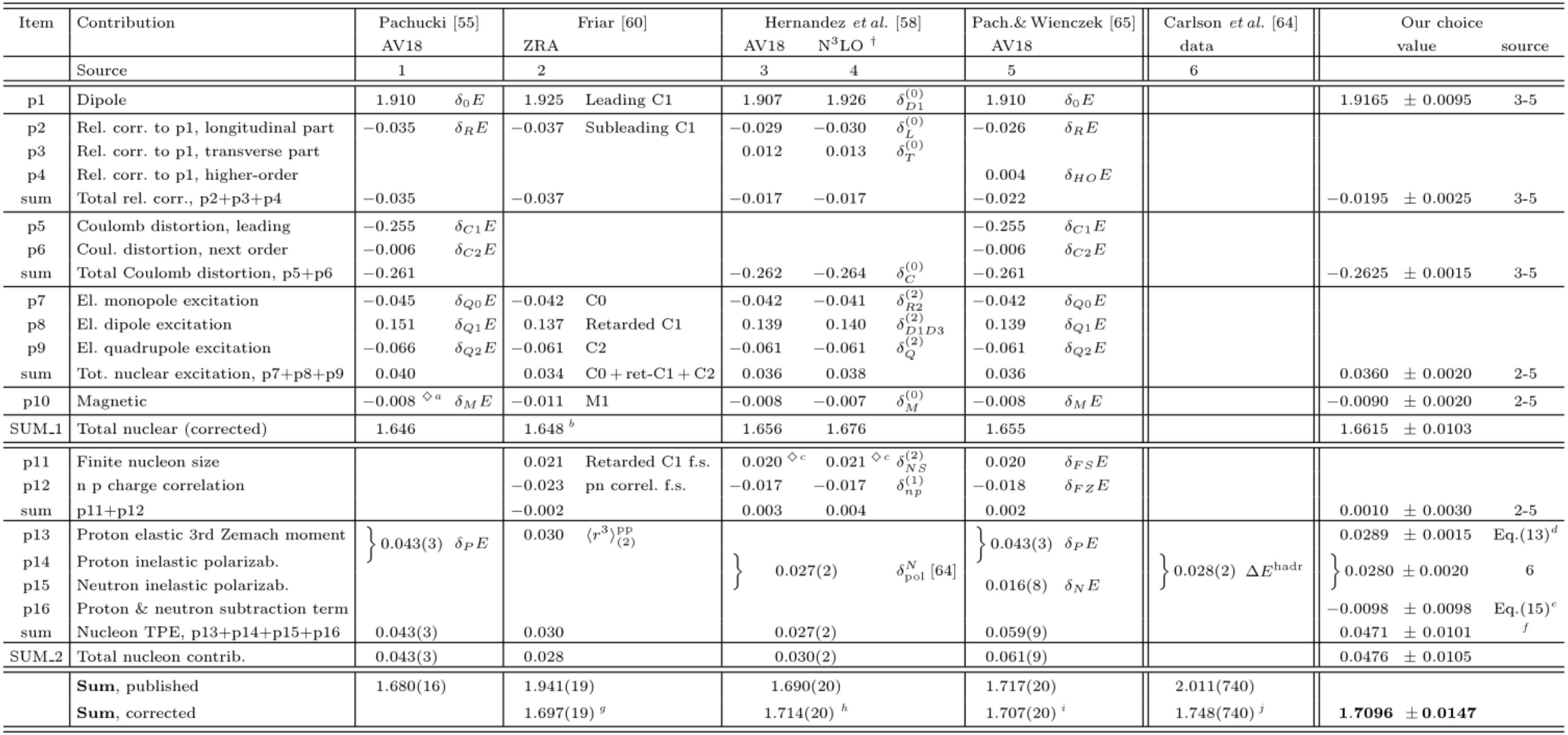


elastic and inelastic two-photon exchange
(Friar moment and polarizability)

Theory in muonic D

$$\Delta E^{\mu D}_{\mathrm{Lamb}} = 228.7854\ (13)\ \mathrm{meV}_{\mathrm{QED}} + 1.7150\ (\mathbf{230})\ \mathrm{meV}_{\mathrm{TPE}} - 6.1103\ (3)\ \mathrm{meV/fm}^2 * \mathbf{R_d^2}$$

Nuclear structure contributions to the Lamb shift in muonic deuterium.

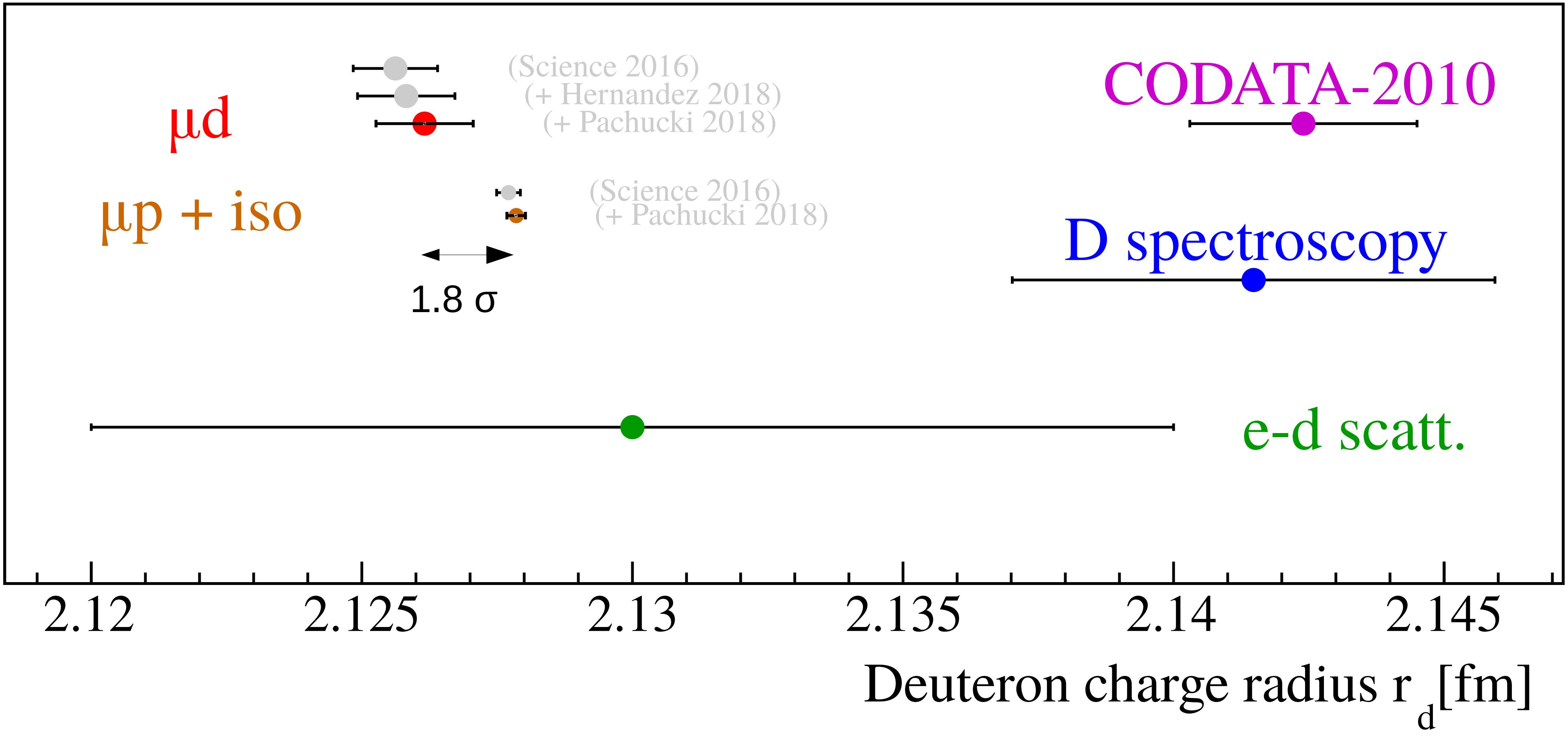


Item	Contribution	Pachucki [55]		Friar [60]		Hernandez *et al.* [58]			Pach.& Wienczek [65]		Carlson *et al.* [64]		Our choice		
		AV18		ZRA		AV18	N^{3}LO †		AV18		data		value		source
	Source	1		2		3	4		5		6				
p1	Dipole	1.910	$\delta_0 E$	1.925	Leading C1	1.907	1.926	$\delta^{(0)}_{D1}$	1.910	$\delta_0 E$			1.9165	± 0.0095	3-5
p2	Rel. corr. to p1, longitudinal part	−0.035	$\delta_R E$	−0.037	Subleading C1	−0.029	−0.030	$\delta^{(0)}_{L}$	−0.026	$\delta_R E$					
p3	Rel. corr. to p1, transverse part					0.012	0.013	$\delta^{(0)}_{T}$							
p4	Rel. corr. to p1, higher-order								0.004	$\delta_{HO} E$					
sum	Total rel. corr., p2+p3+p4	−0.035		−0.037		−0.017	−0.017		−0.022				−0.0195	± 0.0025	3-5
p5	Coulomb distortion, leading	−0.255	$\delta_{C1} E$						−0.255	$\delta_{C1} E$					
p6	Coul. distortion, next order	−0.006	$\delta_{C2} E$						−0.006	$\delta_{C2} E$					
sum	Total Coulomb distortion, p5+p6	−0.261				−0.262	−0.264	$\delta^{(0)}_{C}$	−0.261				−0.2625	± 0.0015	3-5
p7	El. monopole excitation	−0.045	$\delta_{Q0} E$	−0.042	C0	−0.042	−0.041	$\delta^{(2)}_{R2}$	−0.042	$\delta_{Q0} E$					
p8	El. dipole excitation	0.151	$\delta_{Q1} E$	0.137	Retarded C1	0.139	0.140	$\delta^{(2)}_{D1D3}$	0.139	$\delta_{Q1} E$					
p9	El. quadrupole excitation	−0.066	$\delta_{Q2} E$	−0.061	C2	−0.061	−0.061	$\delta^{(2)}_{Q}$	−0.061	$\delta_{Q2} E$					
sum	Tot. nuclear excitation, p7+p8+p9	0.040		0.034	C0 + ret-C1 + C2	0.036	0.038		0.036				0.0360	± 0.0020	2-5
p10	Magnetic	−0.008 $^{\diamond a}$	$\delta_M E$	−0.011	M1	−0.008	−0.007	$\delta^{(0)}_{M}$	−0.008	$\delta_M E$			−0.0090	± 0.0020	2-5
SUM_1	Total nuclear (corrected)	1.646		1.648 ^{b}		1.656	1.676		1.655				1.6615	± 0.0103	
p11	Finite nucleon size			0.021	Retarded C1 f.s.	0.020 $^{\diamond c}$	0.021 $^{\diamond c}$	$\delta^{(2)}_{NS}$	0.020	$\delta_{FS} E$					
p12	n p charge correlation			−0.023	pn correl. f.s.	−0.017	−0.017	$\delta^{(1)}_{np}$	−0.018	$\delta_{FZ} E$					
sum	p11+p12			−0.002		0.003	0.004		0.002				0.0010	± 0.0030	2-5
p13	Proton elastic 3rd Zemach moment	} 0.043(3)	$\delta_P E$	0.030	$\langle r^3 \rangle^{\mathrm{pp}}_{(2)}$				} 0.043(3)	$\delta_P E$			0.0289	± 0.0015	Eq.(13)^{d}
p14	Proton inelastic polarizab.					} 0.027(2)		$\delta^{N}_{\mathrm{pol}}$ [64]			} 0.028(2)	ΔE^{hadr}	} 0.0280	± 0.0020	6
p15	Neutron inelastic polarizab.								0.016(8)	$\delta_N E$					
p16	Proton & neutron subtraction term												−0.0098	± 0.0098	Eq.(15)^{e}
sum	Nucleon TPE, p13+p14+p15+p16	0.043(3)		0.030		0.027(2)			0.059(9)				0.0471	± 0.0101	^{f}
SUM_2	Total nucleon contrib.	0.043(3)		0.028		0.030(2)			0.061(9)				0.0476	± 0.0105	
	Sum, published	1.680(16)		1.941(19)		1.690(20)			1.717(20)		2.011(740)				
	Sum, corrected			1.697(19) ^{g}		1.714(20) ^{h}			1.707(20) ^{i}		1.748(740) ^{j}		**1.7096**	**± 0.0147**	

Krauth, RP at al., Ann. Phys. (N.Y.) 366, 168 (2016)

+ Pachucki et al., PRA 97, 062511 (2018)
+ Hernandez et al., PLB 778, 377 (2018)

Deuteron radius



Hernandez et al, Phys. Lett. B 778, 377 (2018)
Pachucki et al., PRA 97, 062511 (2018)