CAS – Zürich – 22nd February 2018

Luminosity Goals, Critical Parameters

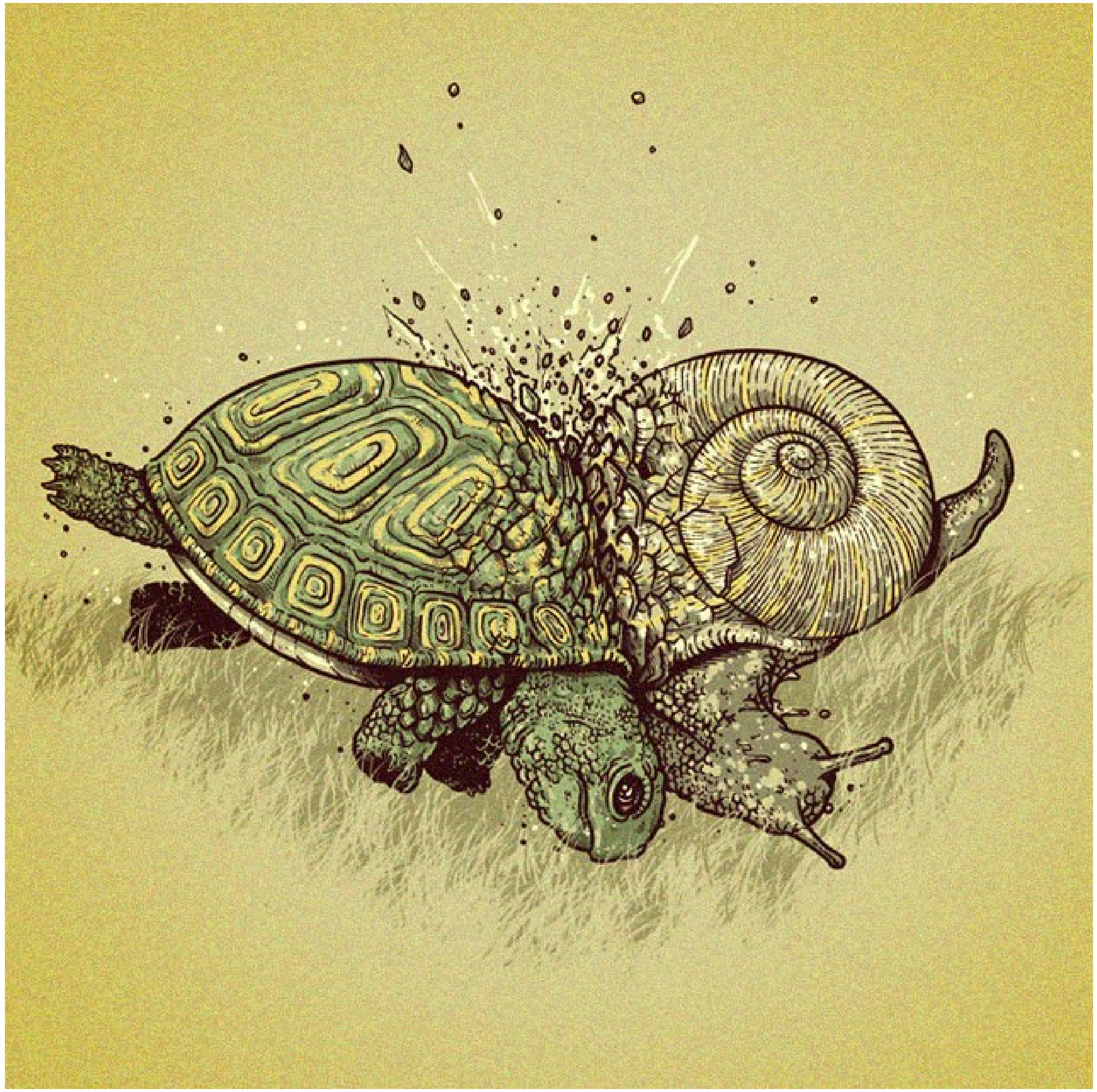
Bruno Muratori,
STFC Daresbury Laboratory & Cockcroft Institute
Werner Herr, CERN

The Cockcroft Institute
of Accelerator Science and Technology

Goals

- At the end of this lecture you should be able to (hopefully) have a rough idea of:
  - What is luminosity for a collider & how to calculate it
  - Get high luminosity but useful at the same time
  - Make the most of the experimental data
  - What happens to luminosity in the case of crossing angles, offsets, hourglass & crab cavities
  - Definition of luminous region & how to calculate it
  - Schemes for luminosity levelling with pros & cons
  - Luminosity measurement

Collisions



- From the side & very slow …

Collisions

- From the back
- Quite fast …
- Still not very efficient!

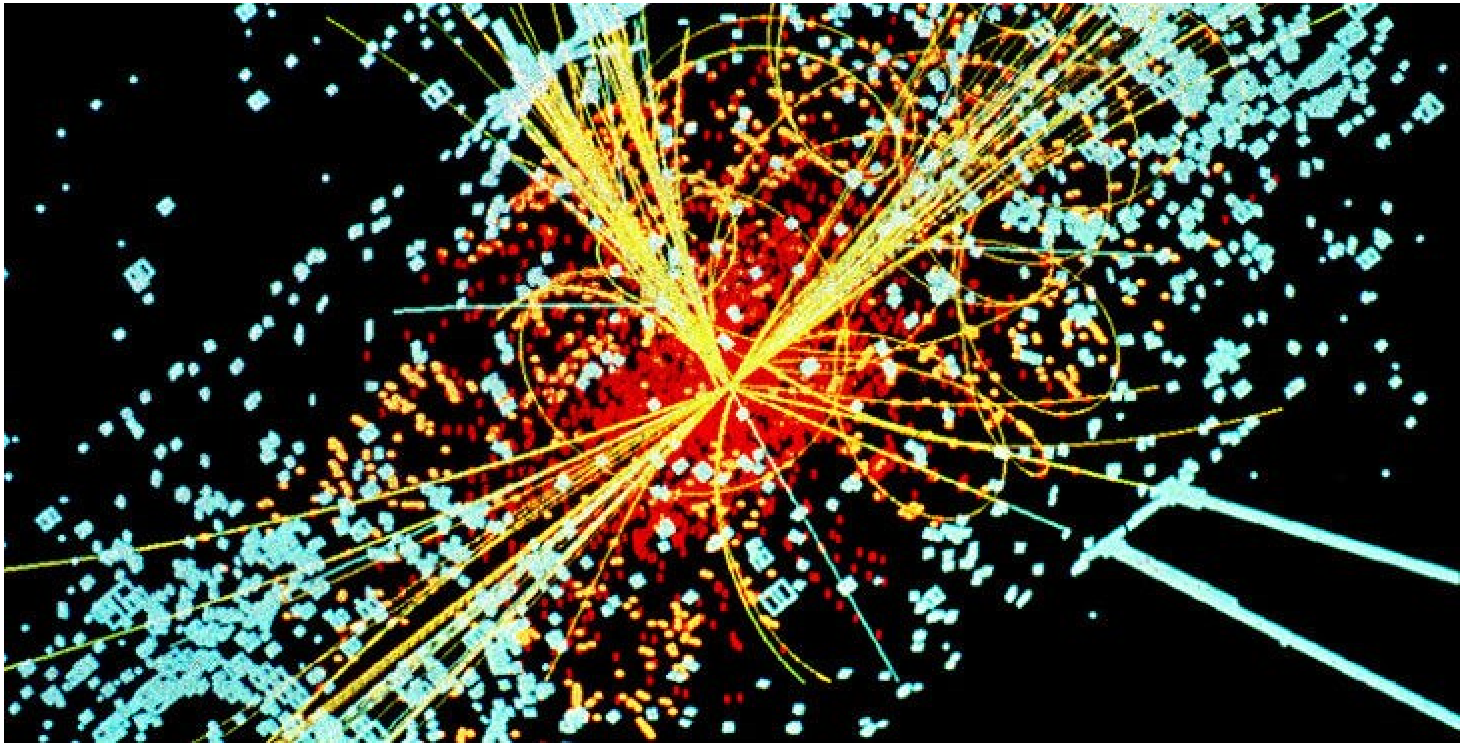
Collisions

- Head-on

Collisions

- Fixed target ☺

Collisions



- What can we do to optimise the performance ?
- Want useful collisions (instead of any collisions)
- Avoid pile-up & background where possible
- What is best for the detectors ?

Performance Issues

- Available energy
- Useful collisions (as opposed to just collisions)
- Maximise total number of interactions
- At the same time, take into account:
  - Time spread of the interactions (when ?) or how often & how many simultaneously ?
  - Spatial spread of the interactions (where ?) or overall size of the interaction region
  - Quality of the interactions (how ?) or dead-time / pile-up / background
  - Pile-up for the LHC is around 20 & upgrade is ~40

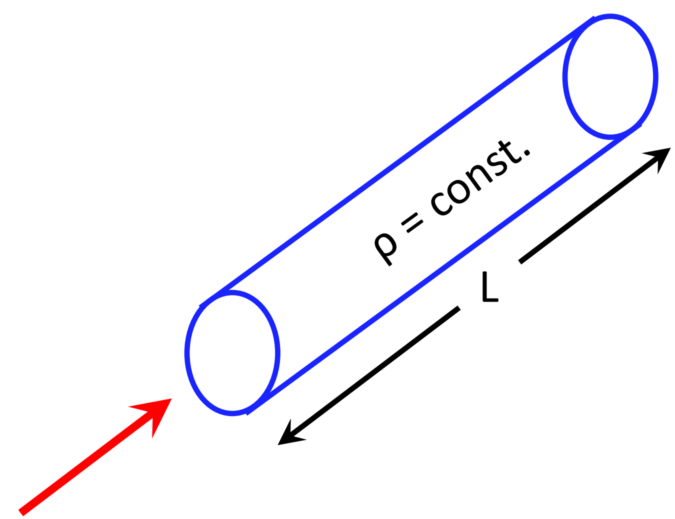
Luminosity

- Proportionality factor between the cross section σ_p at the IP and the no. of interactions / second

$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p$$ units cm^{-2} s^{-1}

- For a fixed target:

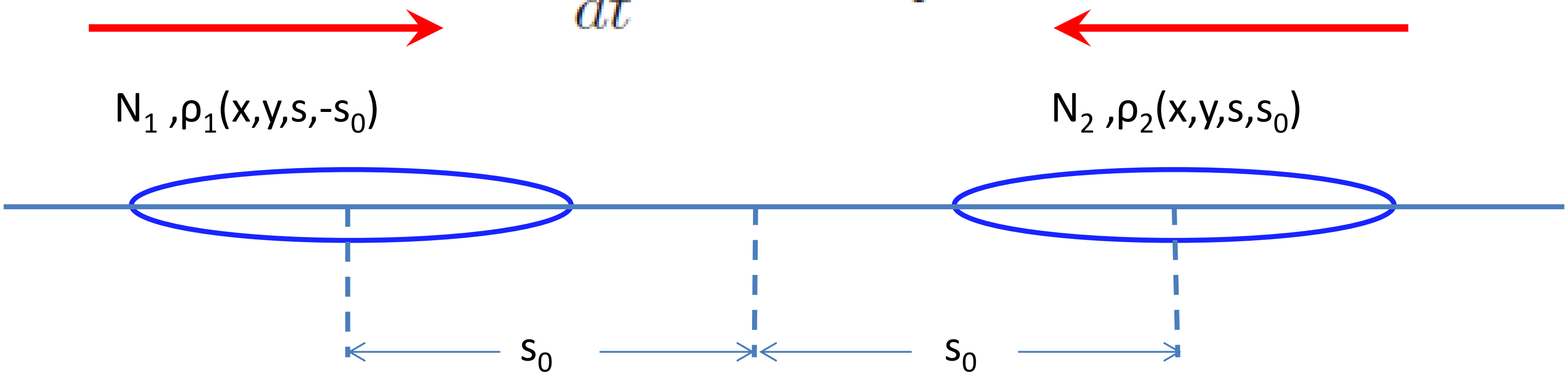
$$\frac{dR}{dt} = \underbrace{\Phi \rho L}_{\mathcal{L}} \times \sigma_p$$



Flux $\Phi = N/s$

Luminosity

- For a collider: $$\frac{dR}{dt} = \mathcal{L} \times \sigma_p$$



- N = particles / bunch, s_0 is time $s_0 = ct$
- ρ = density ≠ const.

$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,s_0)\,dx\,dy\,ds\,ds_0$$

- Kinematic factor: $K = \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - (\vec{v}_1 \times \vec{v}_2)^2/c^2}$

Luminosity

- Assume beams are Gaussian in all directions and independent of each other:

$$\rho^{(i)}(x,y,s,ct) = \rho_x^{(i)}(x)\rho_y^{(i)}(y)\rho_s^{(i)}(s \pm ct)$$

$$\rho_z^{(i)}(z) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma_z^2}\right),$$

$$\rho_s^{(i)}(s \pm ct) = \frac{1}{\sigma_s\sqrt{2\pi}} \exp\left(-\frac{(s \pm ct)^2}{2\sigma_s^2}\right),$$

$$i = 1,2,\ z = x,y,$$

- Look at simplest case first & then introduce the most general crossing angle and offsets

Luminosity

- All the integrals are almost trivial because there is no cross dependence of coordinates
- Repeated application of

$$\int_{-\infty}^{+\infty} e^{-\frac{x^2}{A}} dx = \sqrt{\pi A}$$

- Therefore

$$\mathcal{L} = 2cN_1 N_2 f N_b \cos^2\frac{\phi}{2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho_x^{(1)}(x)\rho_y^{(1)}(y)\rho_s^{(1)}(s-ct)$$

$$\times \rho_x^{(2)}(x)\rho_y^{(2)}(y)\rho_s^{(2)}(s+ct)dxdydsdt$$

- Gives

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y}$$

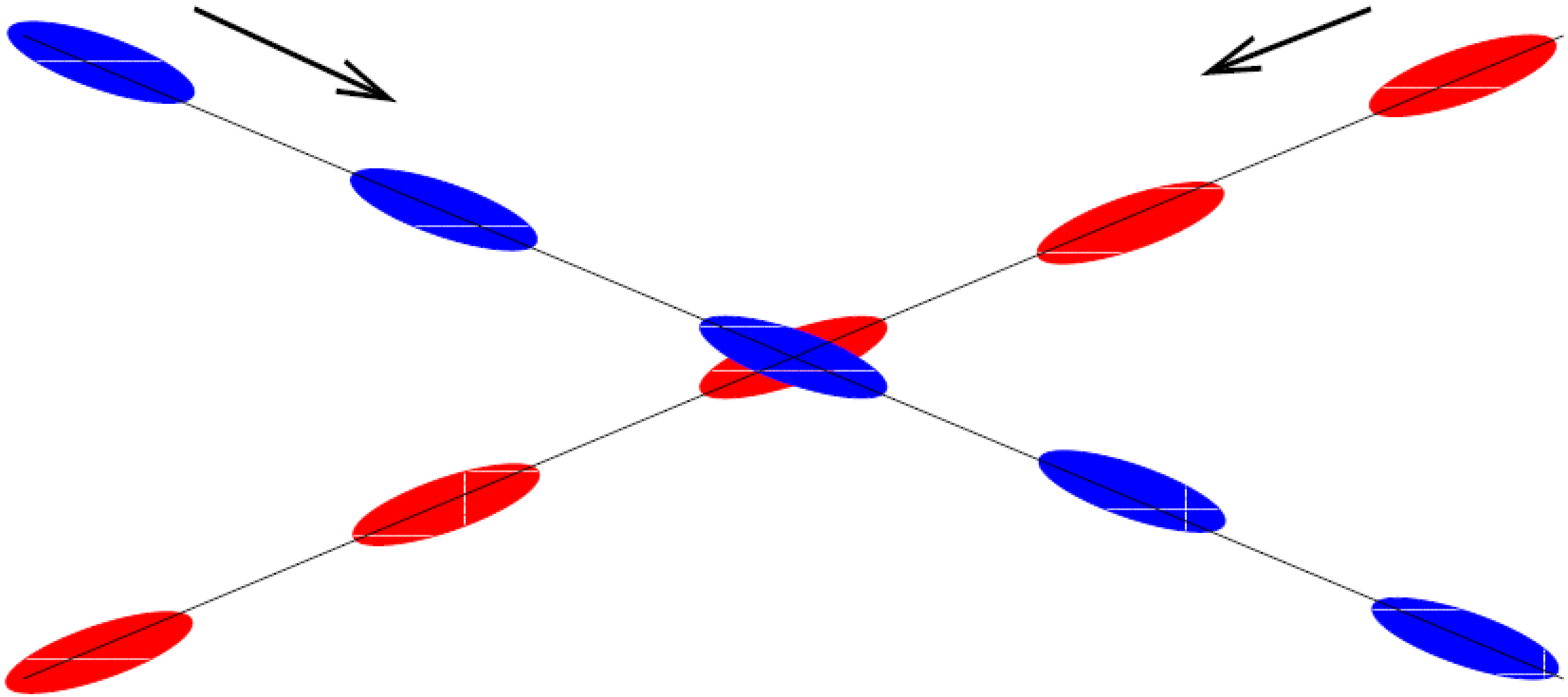
Luminosity

- Nominal luminosity for Gaussian beams is:

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y}$$

- N_1 & N_2 are the number of particles per bunch in beams *1* & *2* respectively
- N_b is the number of colliding bunches per beam
- σ_x & σ_y are the transverse dimensions
- f is the revolution frequency
- Now we can start to complicate things … ☺

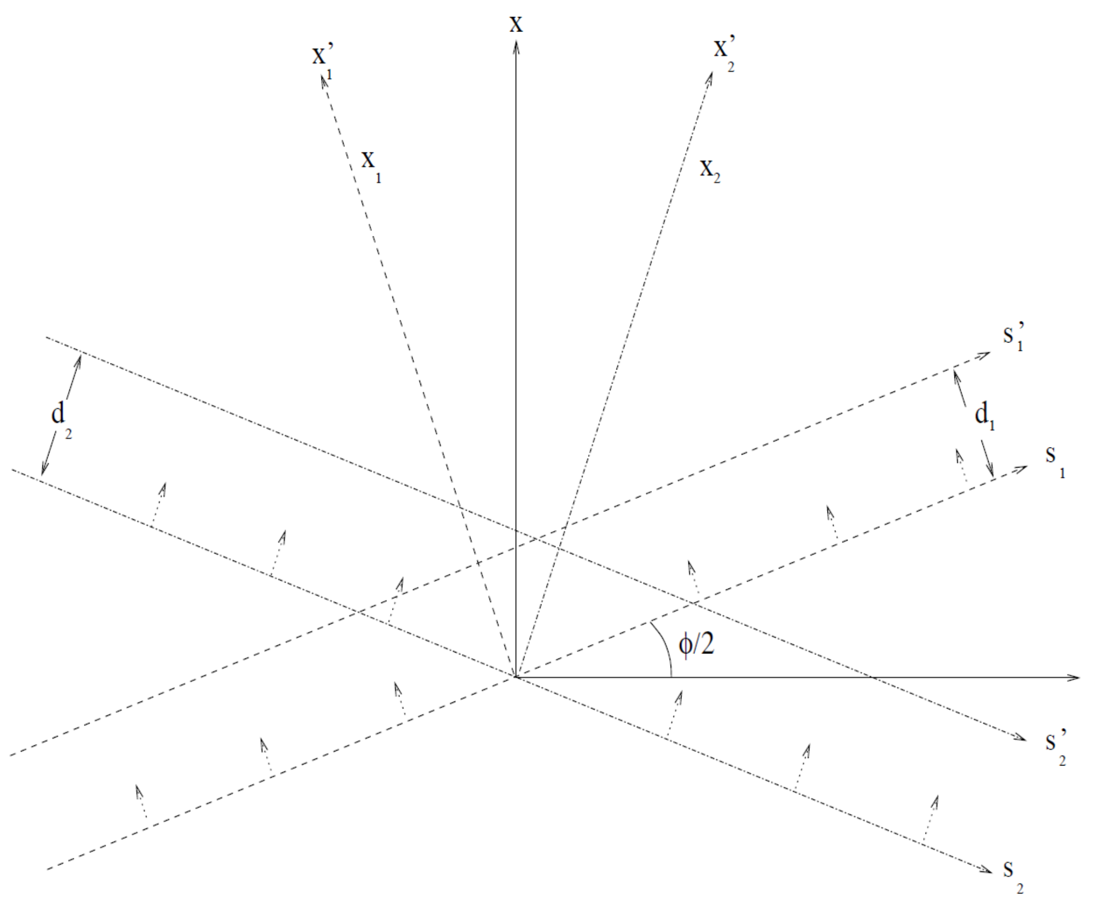
Luminosity (crossing angle & offset)



Crossing angle & offset

- Introduce crossing angle and offsets

$$x_1 = d_1 + x\cos(\phi/2) - s\sin(\phi/2), \quad s_1 = s\cos(\phi/2) + x\sin(\phi/2),$$
$$x_2 = d_2 + x\cos(\phi/2) + s\sin(\phi/2), \quad s_2 = s\cos(\phi/2) - x\sin(\phi/2)$$



Crossing angle & offset

- Beam size is much smaller than the bunch length and the crossing angle ϕ is small (~ *300 μrad*) so

$$s_1 = s_2 = s\cos(\phi/2) \qquad (\sigma_z << \sigma_s)$$

- Calculating all the overlap integrals to get the luminosity:

$$\mathcal{L} = 2cN_1N_2fN_b\cos^2\frac{\phi}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\rho_x^{(1)}(x)\rho_y^{(1)}(y)\rho_s^{(1)}(s-ct)$$

$$\times\rho_x^{(2)}(x)\rho_y^{(2)}(y)\rho_s^{(2)}(s+ct)dxdydsdt$$

- With repeated applications of:

$$\int e^{-(ax^2+2bx)}dx = e^{b^2/a}\frac{1}{2}\sqrt{\frac{\pi}{a}}\mathrm{erf}\left[\frac{b+ax}{\sqrt{a}}\right] + \mathrm{const.}$$

Crossing angle & offset

- Noting: *erf*(-*x*) = - *erf*(*x*), *erf*(*0*) = *0*, *erf*(∞) = *1*
- We obtain:

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi^{\frac{3}{2}}\sigma_s}\cos\frac{\phi}{2}\int_{-\infty}^{+\infty} W\frac{e^{-(As^2+2Bs)}}{\sigma_x\sigma_y}ds.$$

$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2}, \quad B = \frac{(d_2 - d_1)\sin(\phi/2)}{2\sigma_x^2},$$

$$W = e^{-\frac{1}{4\sigma_x^2}(d_2-d_1)^2}.$$

- *W*, σ_x, σ_y are still inside the integral as they may still depend on "*s*", otherwise we have:

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y} W e^{\frac{B^2}{A}} \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}}.$$

Crossing angle & offset

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y} W e^{\frac{B^2}{A}} \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}}.$$

- This shows luminosity is independent of offsets provided $d_1 = d_2$, which makes sense from the crossing angle, however, the interaction could now lie *outside* the detector …
- Also written as: $\mathcal{L} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y} W e^{\frac{B^2}{A}} S,$
- *S* is the luminosity reduction factor $S = \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}},$
- Where we assumed: $\tan(\phi/2) = \phi/2$ valid for a small crossing angle
- *W* is due to the offset & the rest involves both

Crossing angle & offset

- Early LHC parameters were as follows: $N_1 = N_2 = 1.1 \times 10^{11}$, with *2808* bunches per beam & $f = 11.2455\ kHz$, $\gamma = 7461$, $\phi = 300\ \mu rad$, $\beta^* = 0.5\ m$, $\sigma_s = 7.7\ cm$ and $\varepsilon_n = 3.75\ \mu m$, therefore, the luminosity can be calculated as (exercise):

$$\mathcal{L} = 1.21 \times 10^{34} \times 0.809\ \mathrm{cm^{-2}s^{-1}} = 9.79 \times 10^{33}\ \mathrm{cm^{-2}s^{-1}}$$

- First number = nominal luminosity & second = S
- For illustration, if we have offsets $d_1 = 10\ \mu m$, $d_2 = 0$, then (exercise):

$$W = 0.906, \quad e^{\frac{B^2}{A}} = 1.035, \quad S = 0.809$$

$$\mathcal{L} = 1.21 \times 10^{34} \times 0.758\ \mathrm{cm^{-2}s^{-1}} = 9.17 \times 10^{33}\ \mathrm{cm^{-2}s^{-1}}$$

Luminosity

- How does this compare to other colliders ?

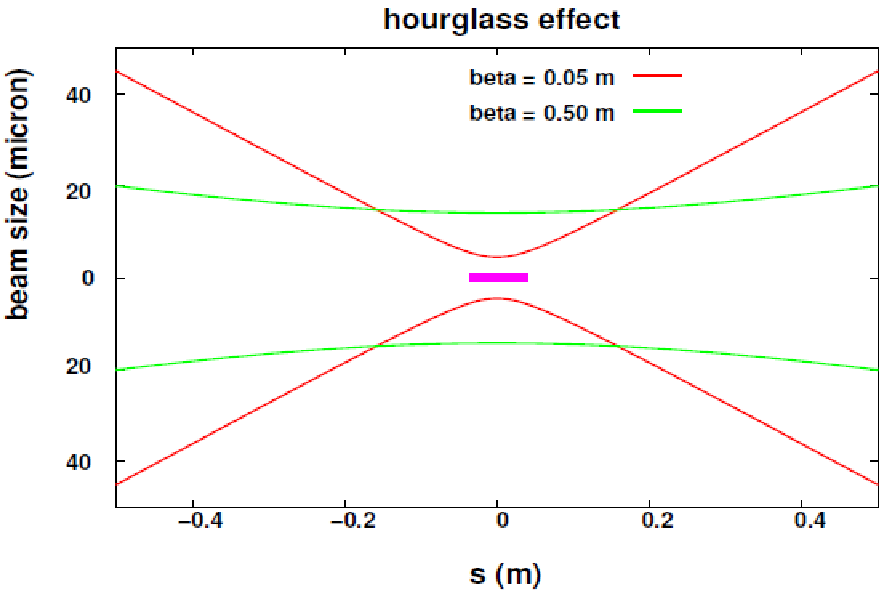
	Energy (GeV)	$\mathcal{L}_{max}$ $cm^{-2}s^{-1}$	rate s^{-1}	σ_x/σ_y $\mu m/\mu m$	Particles per bunch
SPS ($p\bar{p}$)	315x315	$6\ 10^{30}$	$4\ 10^{5}$	60/30	$\approx 10\ 10^{10}$
Tevatron ($p\bar{p}$)	1000x1000	$100\ 10^{30}$	$7\ 10^{6}$	30/30	$\approx 30/8\ 10^{10}$
HERA (e^+p)	30x920	$40\ 10^{30}$	40	250/50	$\approx 3/7\ 10^{10}$
LHC (pp)	7000x7000	$10000\ 10^{30}$	10^{9}	17/17	$\approx 11\ 10^{10}$
LEP (e^+e^-)	105x105	$100\ 10^{30}$	≤ 1	200/2	$\approx 50\ 10^{10}$
PEP (e^+e^-)	9x3	$8000\ 10^{30}$	NA	150/5	$\approx 2/6\ 10^{10}$

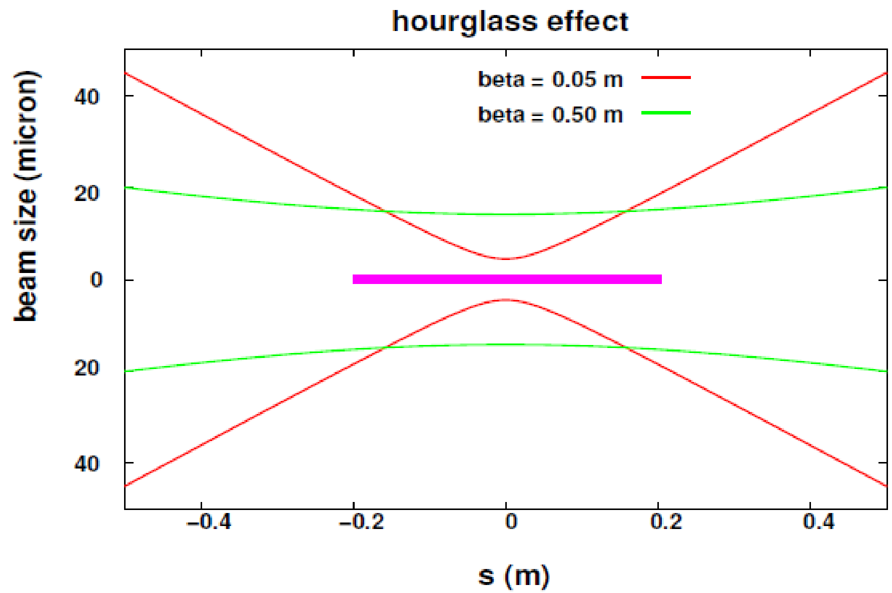
- LEP: 1 event/sec., LHC: 10^9 events/sec.

Luminosity (Hourglass effect)

Hourglass effect

- What if the beam is squeezed at the IP ?





- Hourglass effect leads to a further reduction factor if the bunch length is long enough
- β function either side of the IP behaves as:

$$\beta(s) \approx \beta^*(1 + \left(\frac{s}{\beta^*}\right)^2)$$

Hourglass effect

- So the beam size either side of the IP behaves as:

$$\sigma_z = \sigma_z^* \sqrt{1 + \left(\frac{s}{\beta^*}\right)^2},$$

- For the parameters we had earlier this means:

$$\mathcal{L}_{HG} = \left(\frac{N_1 N_2 f N_b}{4\pi\sigma_x^*\sigma_y^*}\right) \frac{\cos\frac{\phi}{2}}{\sqrt{\pi}\sigma_s} \int_{-\infty}^{+\infty} W \frac{e^{-(As^2+2Bs)}}{1+(\frac{s}{\beta^*})^2} ds,$$

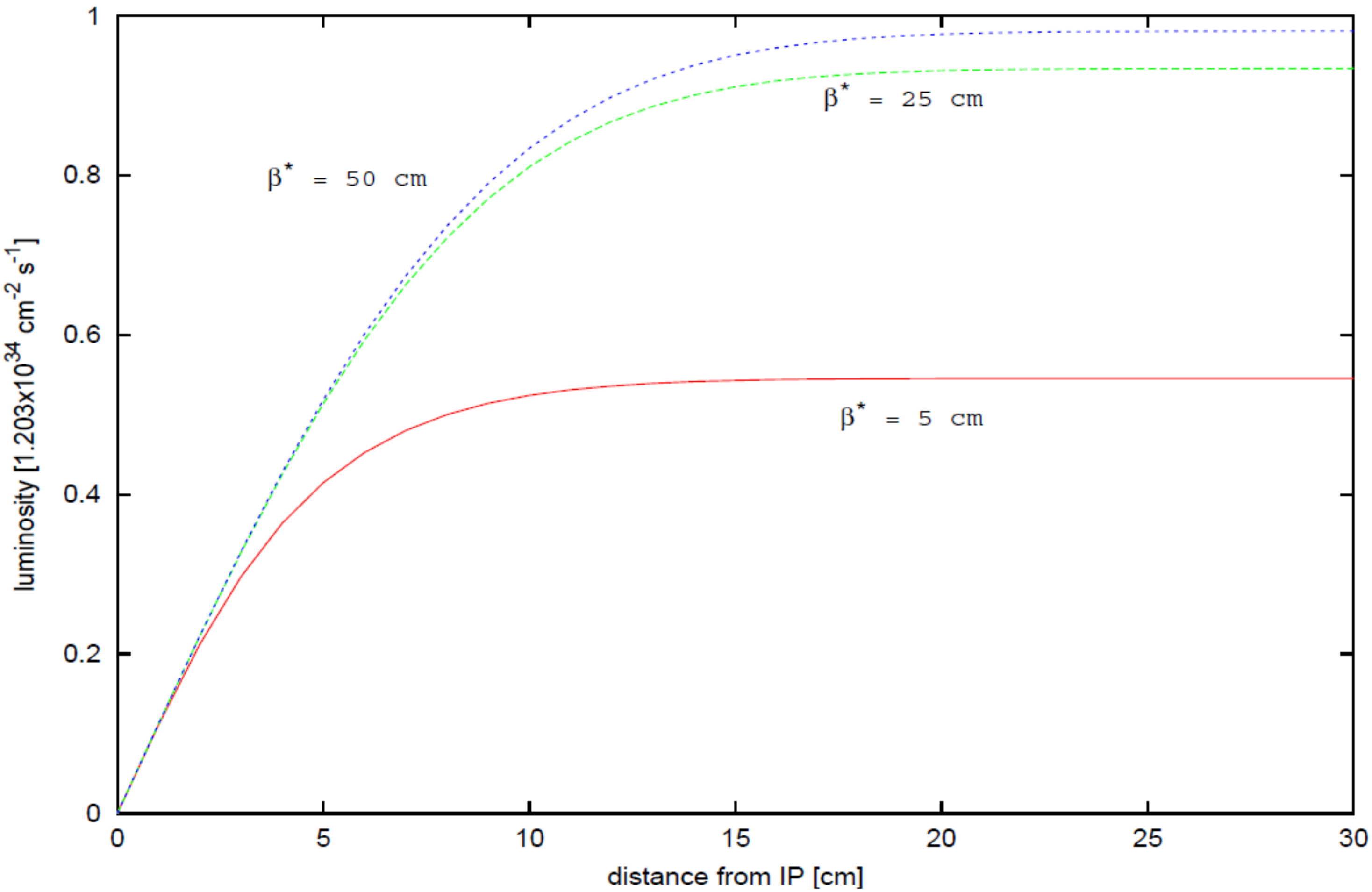
$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2} = \frac{\sigma_s^2\sin^2\frac{\phi}{2} + (\sigma_x^*)^2[1+(\frac{s}{\beta^*})^2]\cos^2\frac{\phi}{2}}{(\sigma_x^*)^2[1+(\frac{s}{\beta^*})^2]\sigma_s^2}.$$

- So, evaluating the integral above numerically:

$$\mathcal{L}_{HG} = 1.21 \times 10^{34} \times 0.755 \text{ cm}^{-2}\text{s}^{-1} = 9.14 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$$

Hourglass effect

- Looking at the effect for various values of β^*:

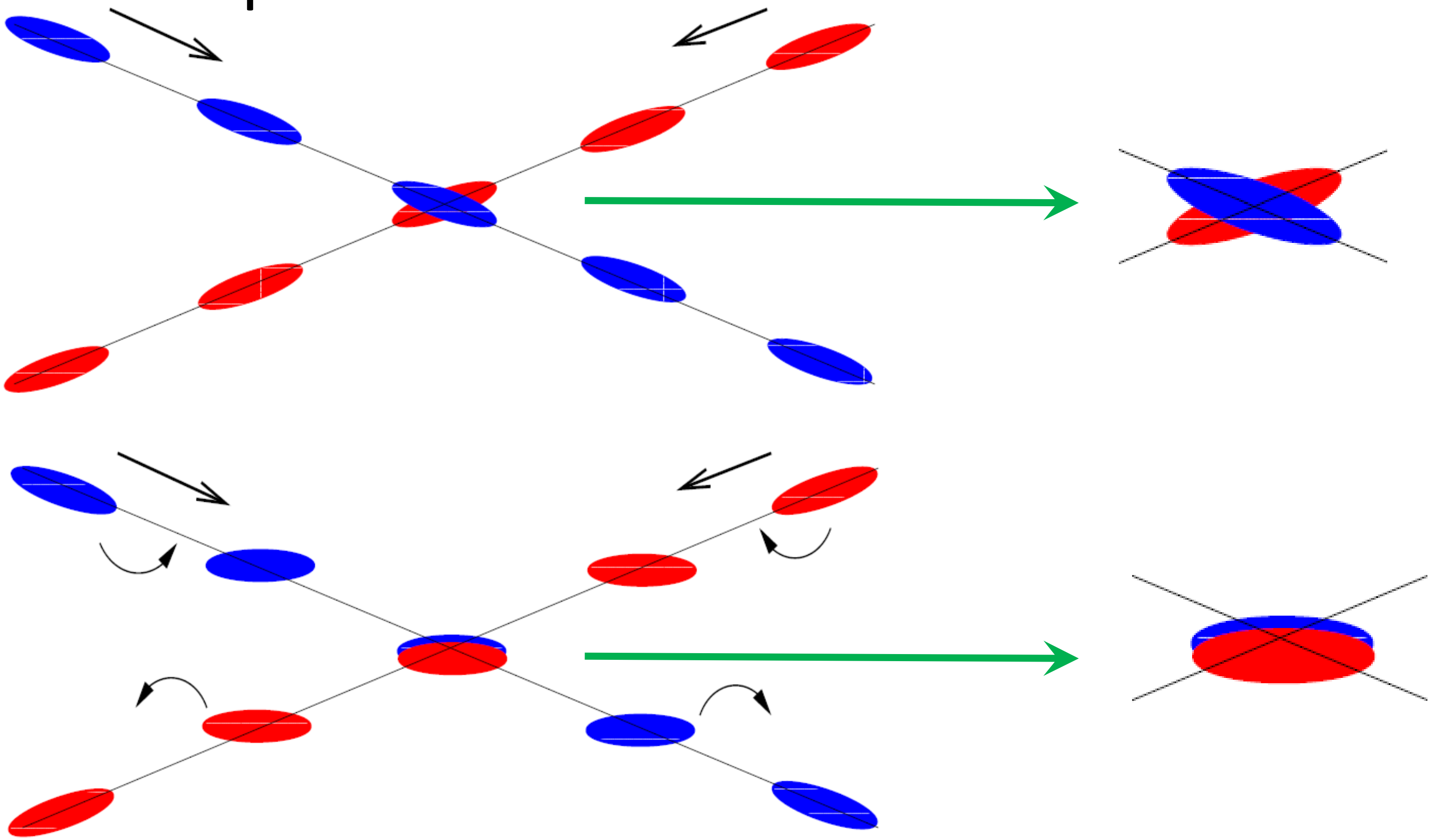


- Together with no crossing angle & b. length *10*cm

Luminosity (Crab crossing)

Crab crossing

- Crab crossing done with crab cavities to give a twist to the colliding bunches to ensure a total overlap at the IP



Luminosity

- How can the best luminosity be achieved ?
- Increase the intensity
- Decrease the beam sizes (small ε_n & β^*)
- Get as many bunches as possible
- Have as small a crossing angle as possible or compensate for it by having crab cavities
- Try to achieve as exact head-on collisions as possible, minimising separation etc.
- Get bunches to be as short as possible
- At the same time – try to minimise beam-beam !

Luminous Region

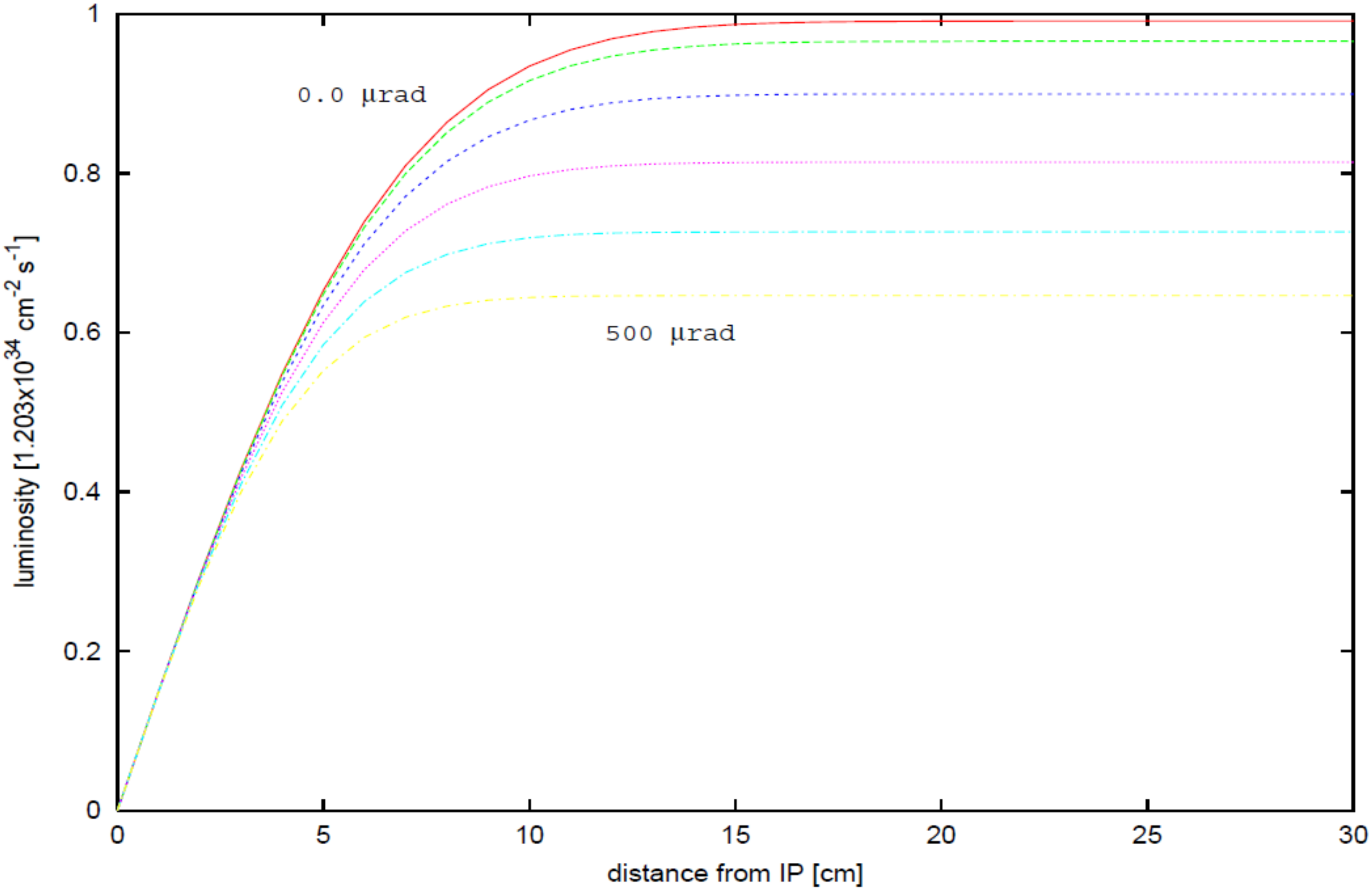
- This is defined as the region where interactions take place within the detector (interaction vertices) and these depend on the beam sizes, the bunch length & the overall geometry
- Perform *y*, *t*, *x* integrations until we are left with just the "s" coordinate dependence and:

$$\mathcal{L} = \int_{-s}^{+s} \mathcal{L}(s')ds'$$

- Instead of the usual: $$\mathcal{L}_0 = \int_{-\infty}^{+\infty} \mathcal{L}(s')ds'$$
- Ratio gives % of luminosity between – *s* & + *s*

Luminous Region

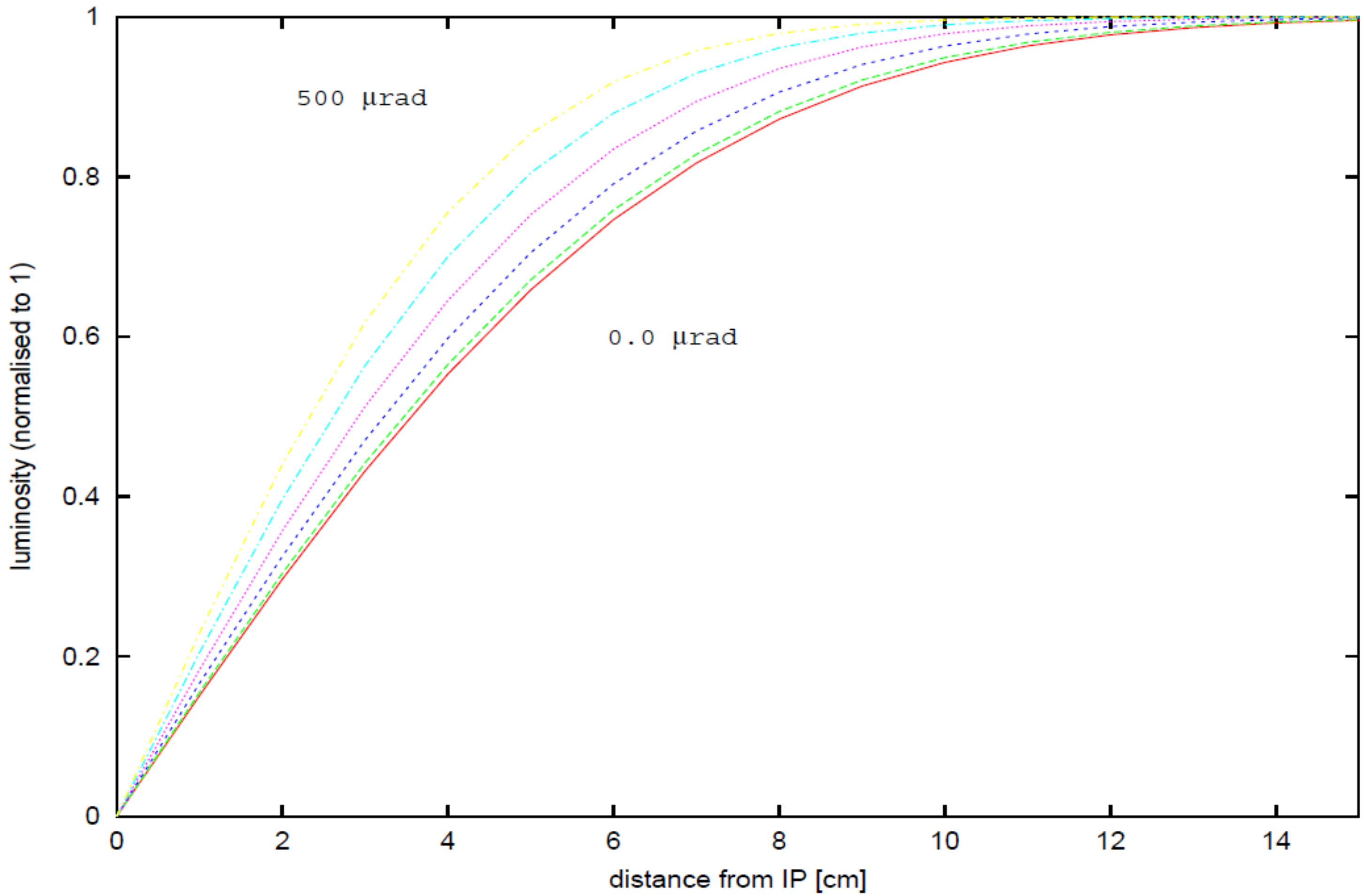
- For a bunch length of *7.5* cm & β^* = *50* cm



- Together with a varying crossing angle

Luminous Region

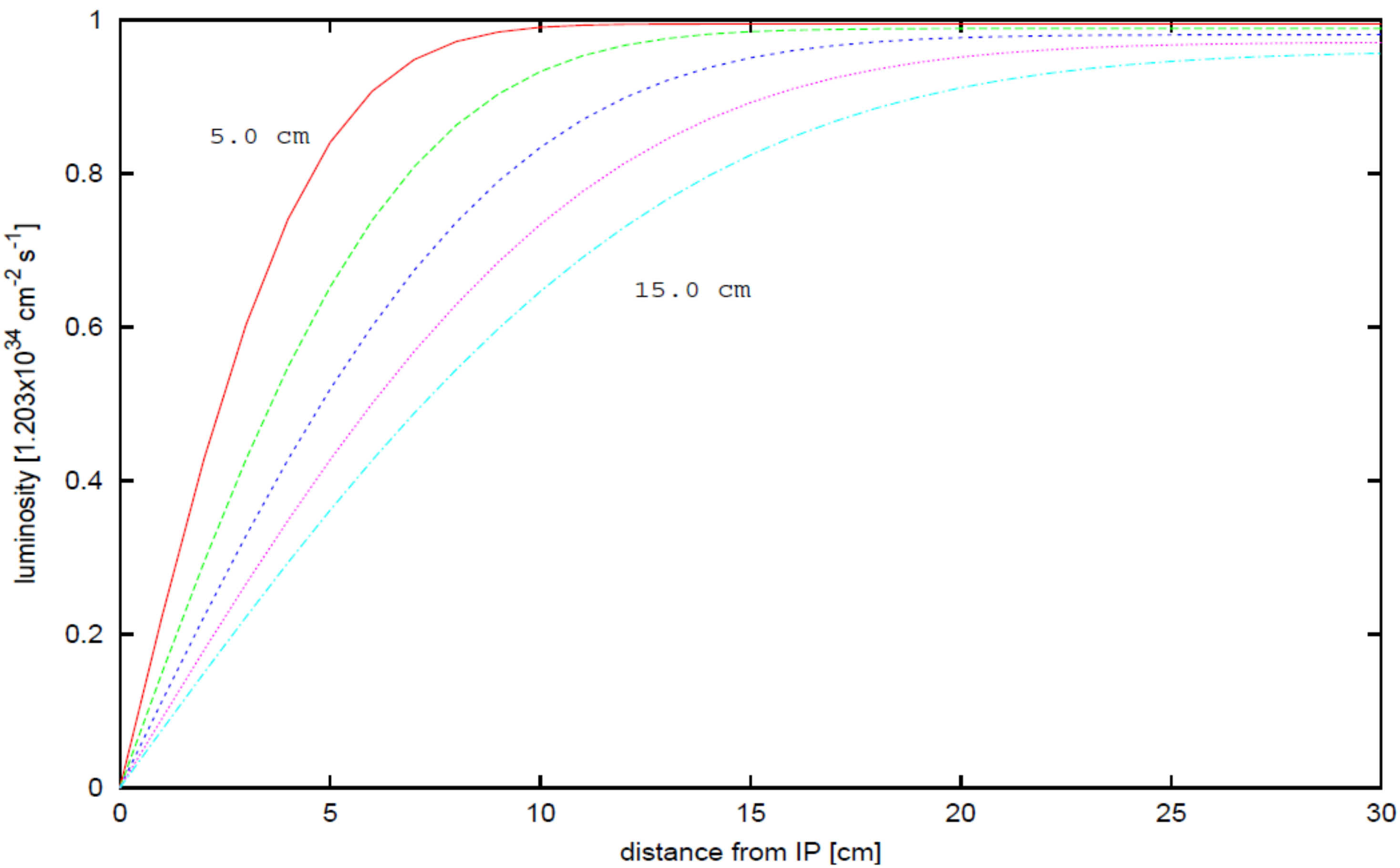
- For a bunch length of *7.5* cm & β^* = *50* cm



- Same as before but normalised w.r.t. maximum

Luminous Region

- For no crossing angle & $\beta^{*} = 50$ cm



- Together with a varying bunch length

Luminous Region

- For a bunch length of *7.5* cm & β^* = *50* cm and a crossing angle of *300* μrad at the LHC, neglecting hourglass:

$$100\% \ \textbf{lumi} \rightarrow \quad s = \pm\ 12\ \text{cm}$$

$$95\% \ \textbf{lumi} \rightarrow \quad s = \pm\ 8\ \text{cm}$$

$$90\% \ \textbf{lumi} \rightarrow \quad s = \pm\ 7\ \text{cm}$$

$$85\% \ \textbf{lumi} \rightarrow \quad s = \pm\ 6\ \text{cm}$$

$$80\% \ \textbf{lumi} \rightarrow \quad s = \pm\ 5.5\ \text{cm}$$

- Probably cannot neglect hourglass for upgrade

Integrated luminosity

- This can be defined straightforwardly, together with the average luminosity as:

$$\mathcal{L}_{int} = \int_0^T \mathcal{L}(t)dt \qquad < \mathcal{L} >= \frac{\int_0^{t_r} \mathcal{L}(t)dt}{t_r + t_p} = \mathcal{L}_0 \times \tau \times \frac{1 - e^{-t_r/\tau}}{t_r + t_p}$$

- Figure of merit: $\mathcal{L}_{int} \times \sigma_p$ = number of events
- Luminosity decays due to decays in intensity and emittance through collisions or other
- Exponential decay is assumed which is realistic:
- E.g.

$$\mathcal{L}(t) \to \mathcal{L}_0 \exp\left(\frac{t}{\tau}\right)$$

Integrated luminosity

- If we know how much preparation time is required then we can optimise $\mathcal{L}_{int}$ easily:

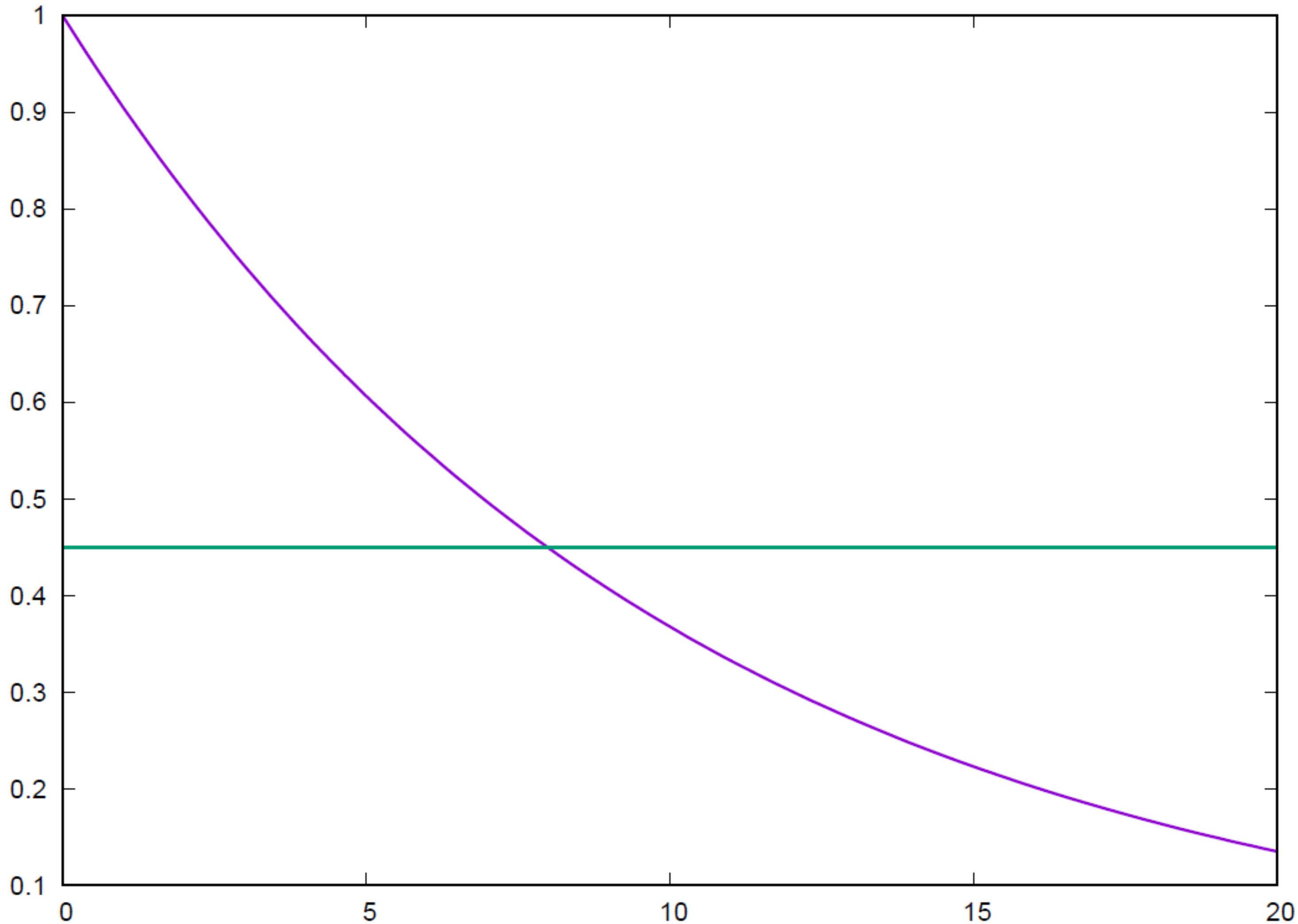
Integrated luminosity

- Typical run times for LEP:
- $t_r \approx 8 - 10$ hours
- For the LHC a long preparation time t_p is usual
- Therefore it is possible to optimise t_r & t_p so as to have the maximum luminosity
- t_r can usually be treated as a free parameter which can be chosen in this optimisation & so we can find a theoretical maximum for t_r:

$$t_r \approx \tau \times \ln\left(1 + \sqrt{2t_p/\tau} + t_p/\tau\right)$$

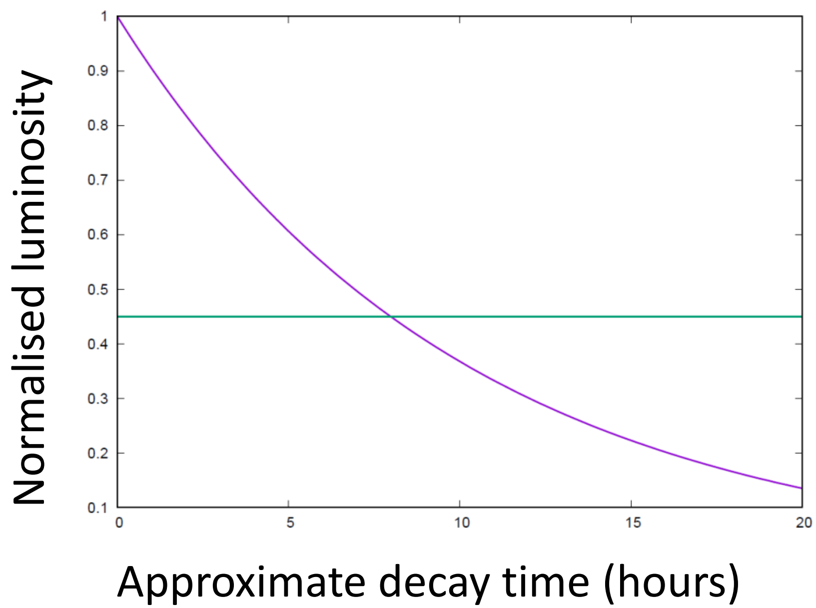
- For the LHC: $t_p \approx 10$ hr, $\tau \approx 15$ hr, → $t_r \approx 15$ hr

Luminosity decay & levelling



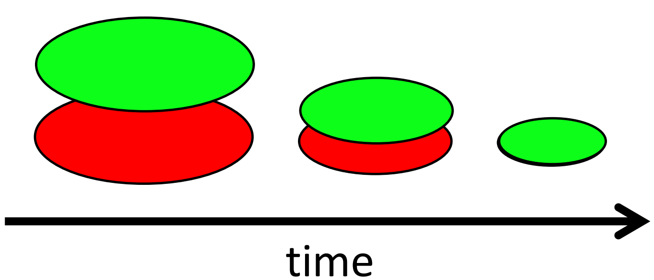
All scales completely arbitrary – this is just to give an idea of the aim

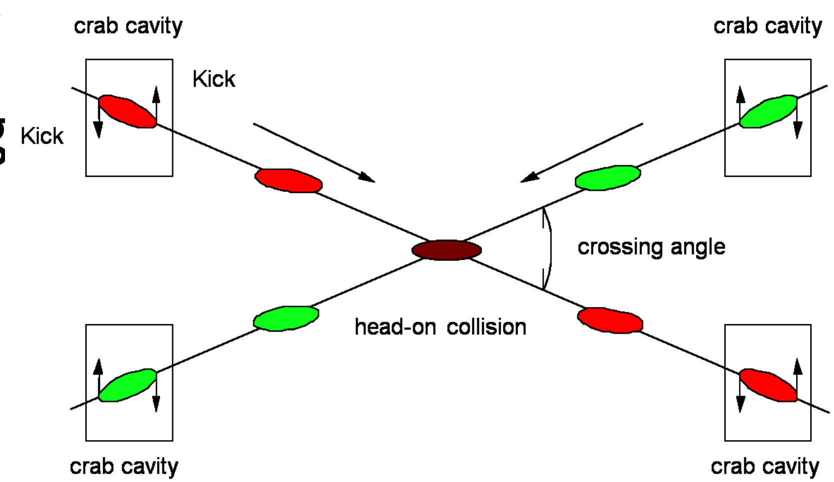
Luminosity decay & levelling



- Luminosity decays exponentially (purple) & can be levelled (green) by spoiling it initially & compensating later (great benefit experimentally)

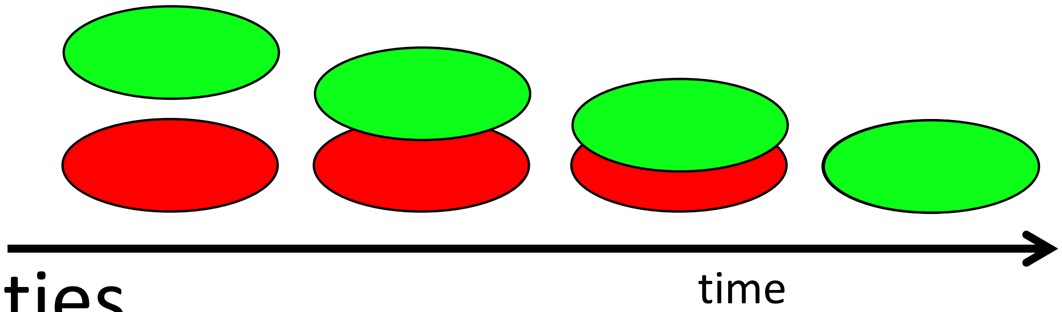
- Various possibilities for levelling
- Offsets, Crab cavities, Squeezing of the beam & combinations



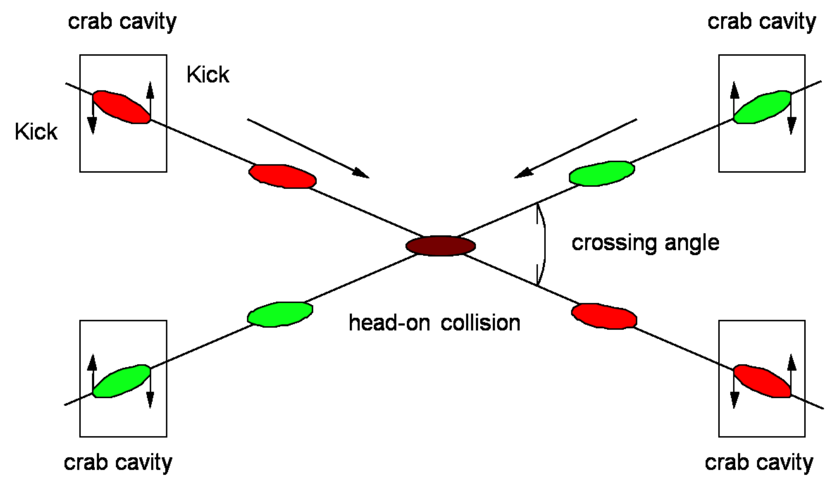


Possibilities for levelling

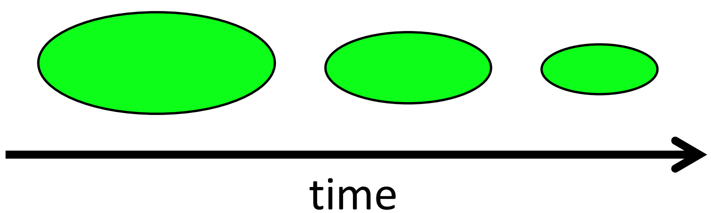
- Offset



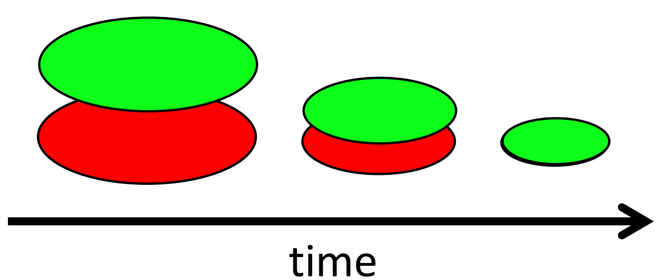
- Crab cavities



- β^* (squeezing the beam)

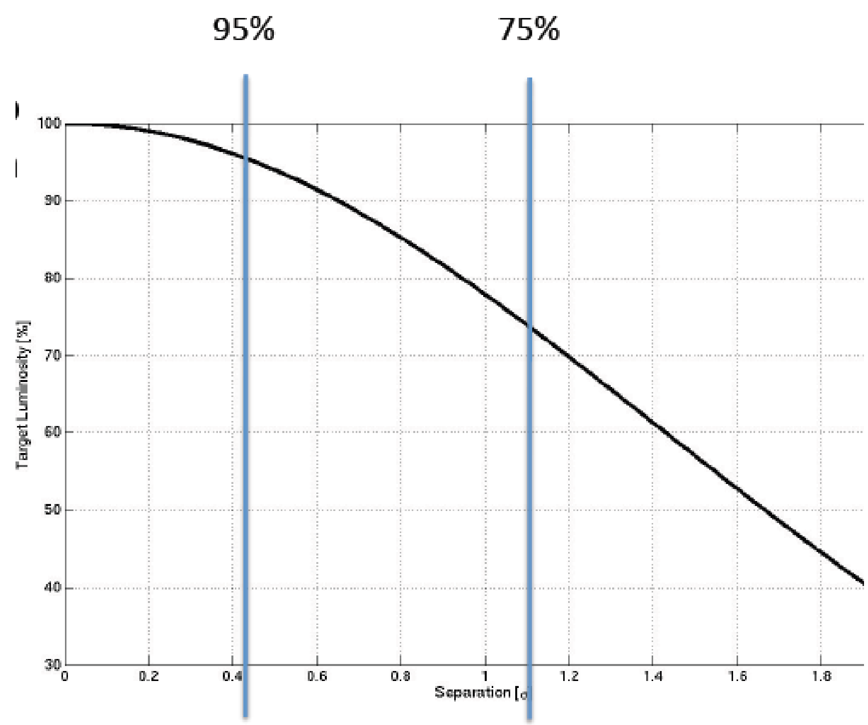


- Combinations & Alternatives & others



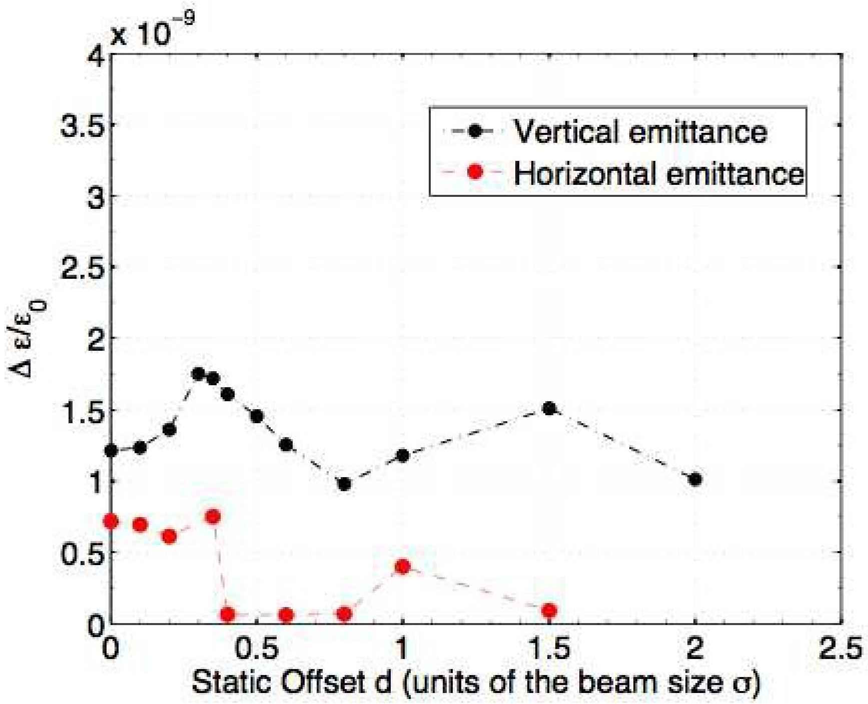
Levelling with offset – pros

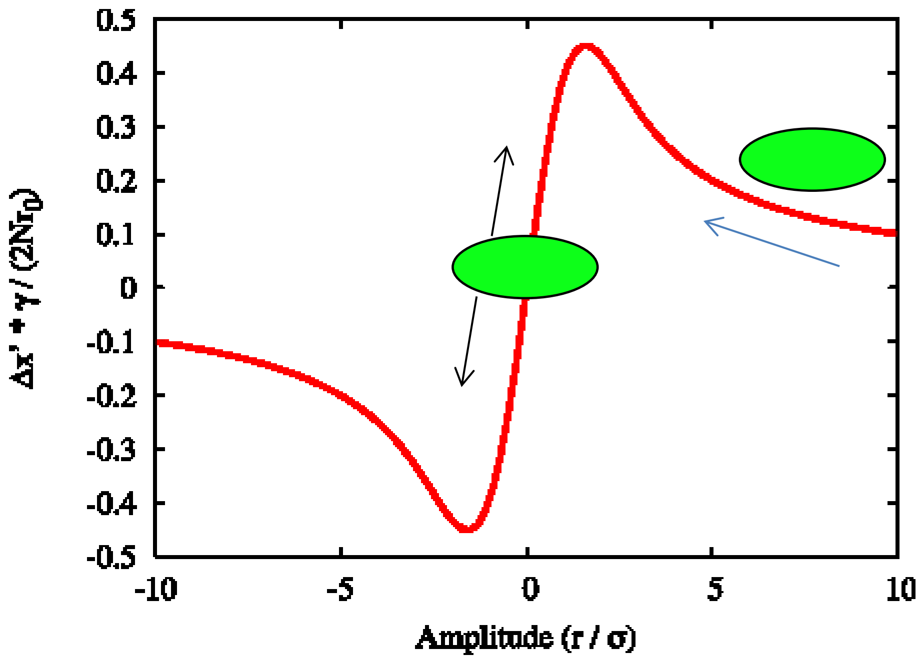
- Simple & easy from operations point of view
- Smaller tune spread → reduced losses
- Constant longitudinal vertex density – great because the average no. of p-p collisions (pile-up) that detectors can handle is limited
- All IPs independent
- Gives a simple & easy option for levelling if required



Levelling with offset – cons (1)

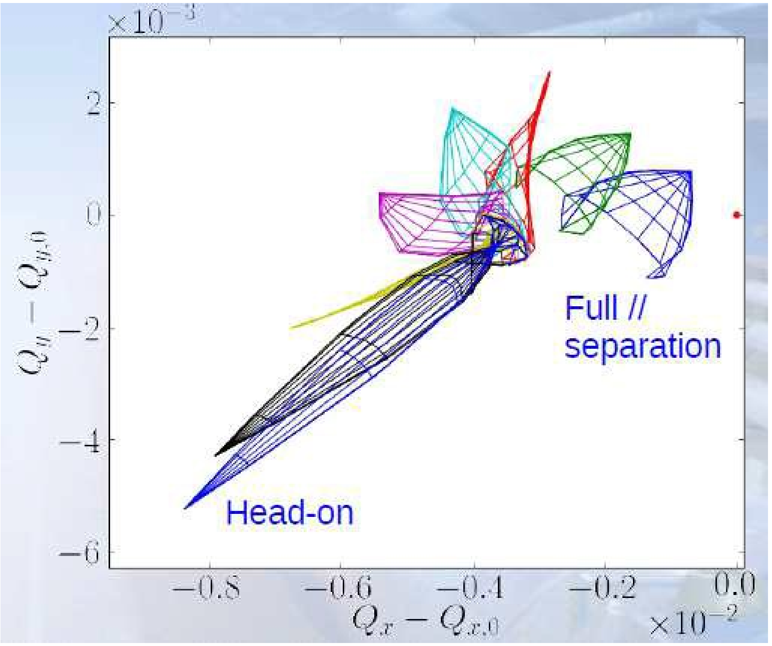
- Different separation → different beam-beam force (focusing / defocusing)
- Emittance growth from offsets

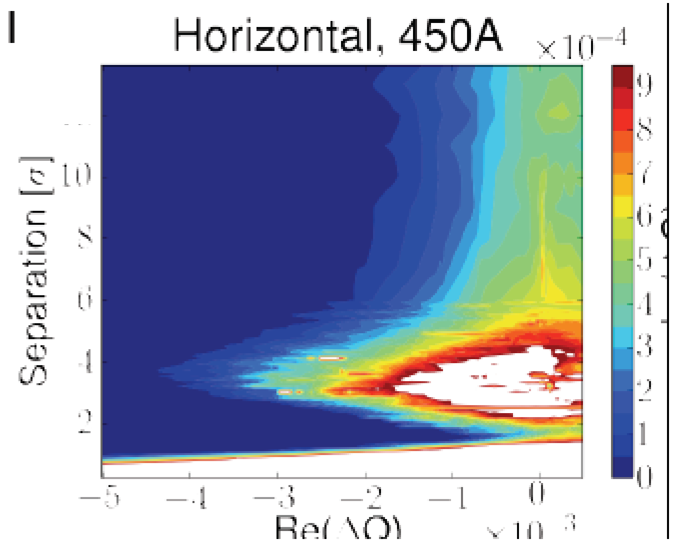




Levelling with offset – cons (2)

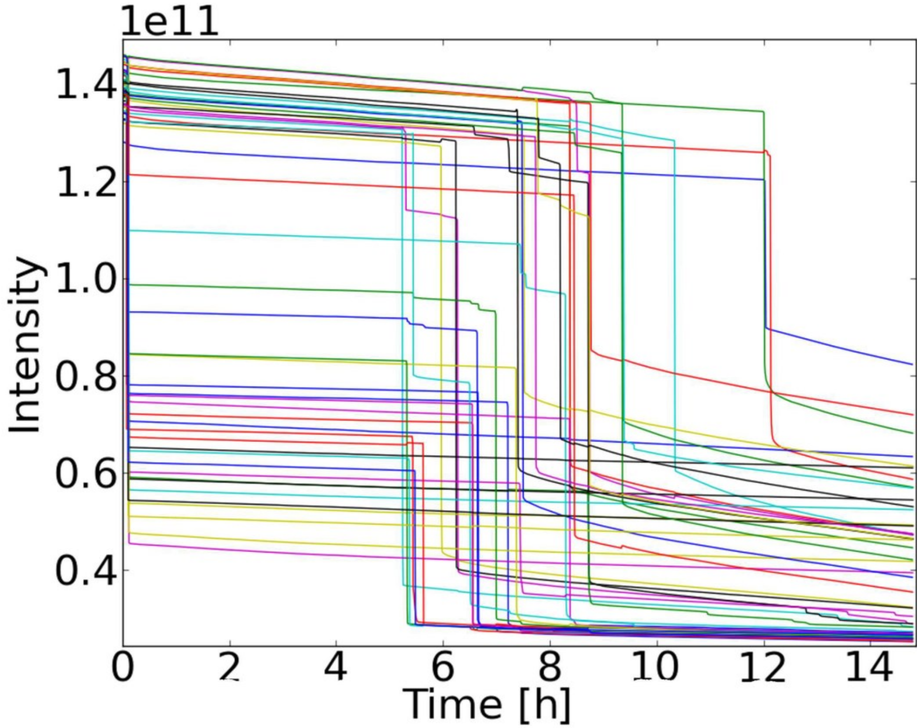
- No head-on collisions → small stability area
- Tune shift keeps changing
- Bunches generally more sensitive to instabilities with respect to head-on

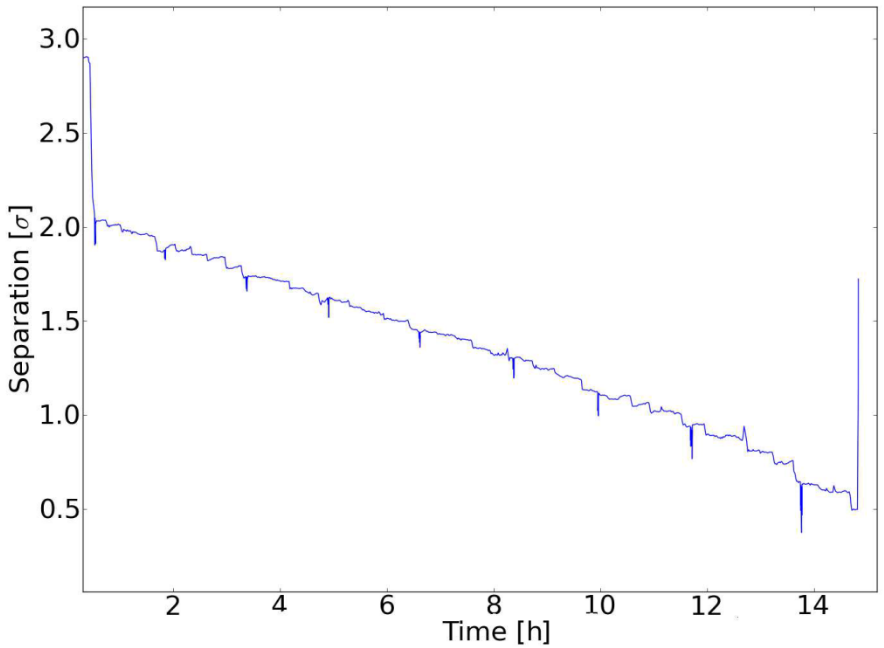




Levelling with offset – cons (3)

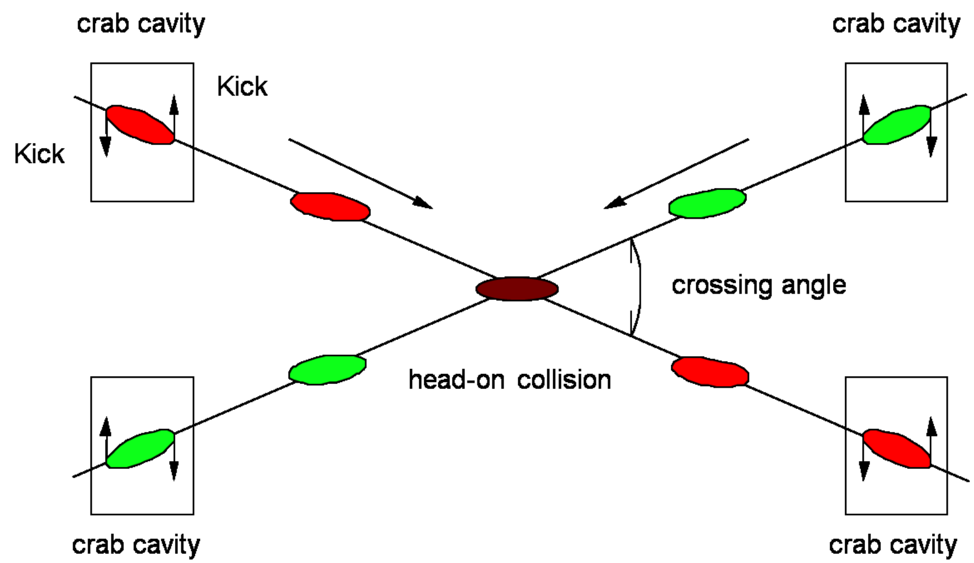
- Stability of bunches affected
- Experiment done in IP8 (so far)
- More IPs would only make it worse ...

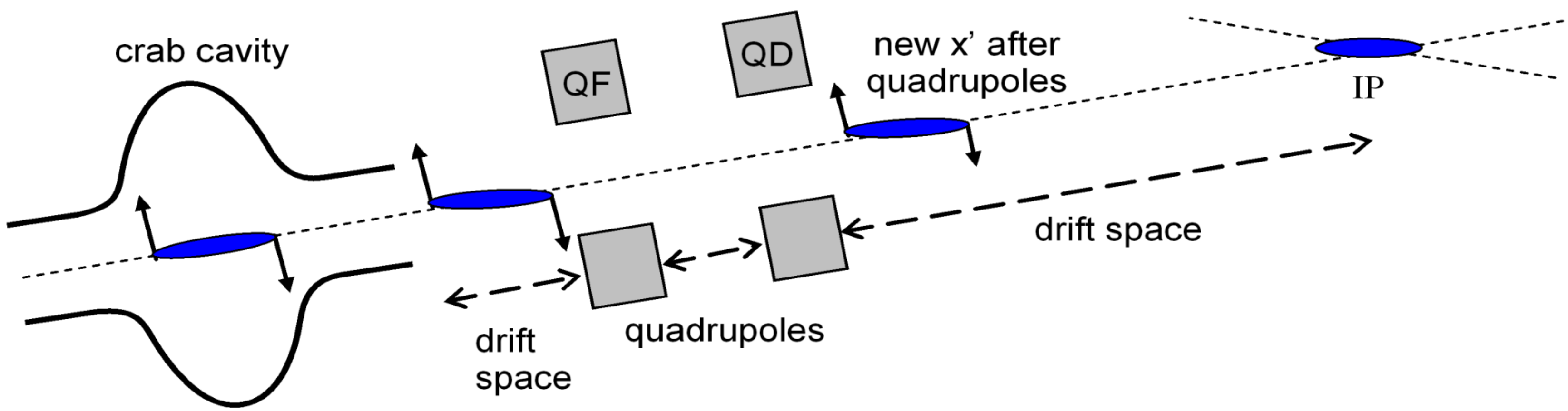




Crab cavity levelling – pros

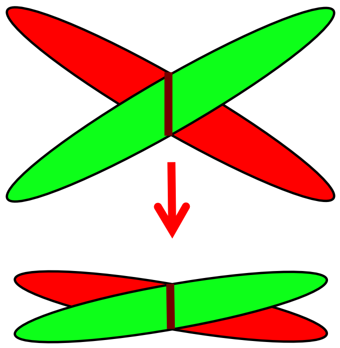
- Reduces the geometrical reduction factor to give a higher luminosity
- All IPs independent
- Can go back and forth (increase & decrease luminosity)





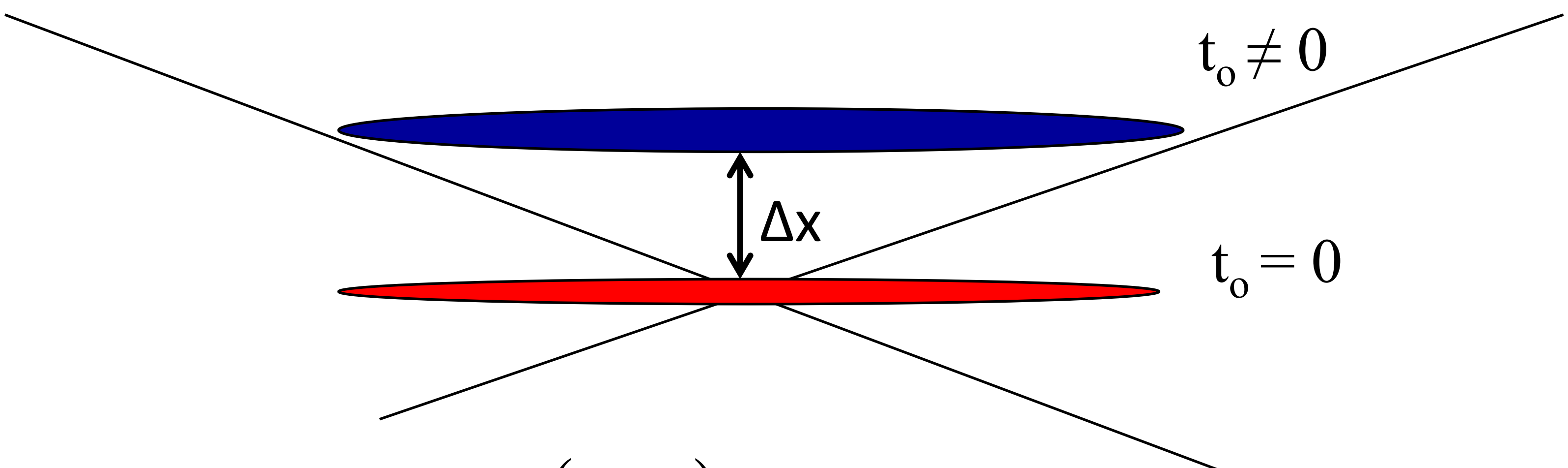
Crab cavity levelling – cons (1)

- Longitudinal vertex density changes with levelled angle
- Tunes change with crossing
- Can reduce reachable beam-beam parameter (ξ_{bb})
- Could introduce noise on colliding beams
- Limited experience with protons so far ...
- Beam-beam & impedance interplay → higher sensitivity to instabilities
- Phase jitter in cavities → reduced luminosity



Crab cavity levelling – cons (2)

- Momentum mismatch
- Differential phase jitter causes the two bunches to have a height mismatch, which can significantly reduce luminosity or cause the bunches to miss.



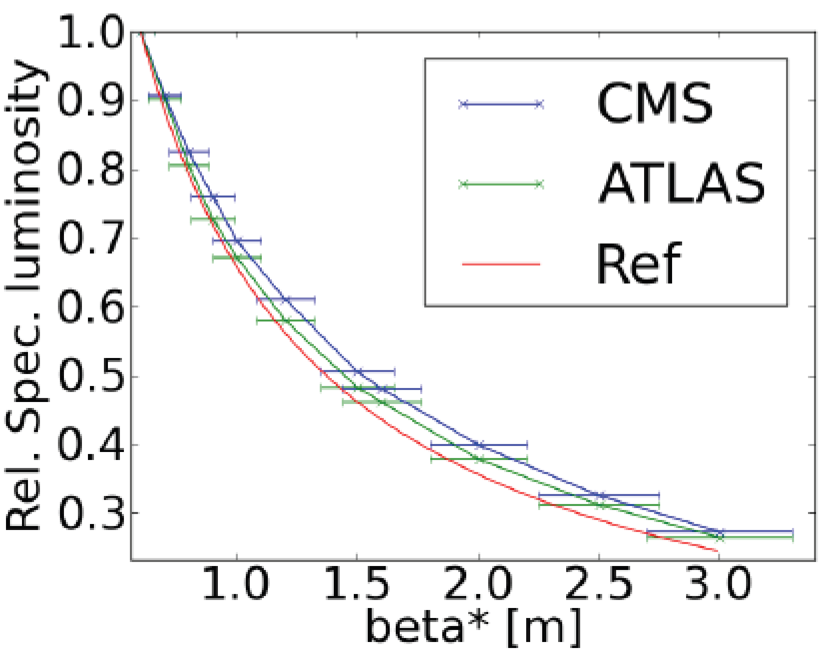
$$\Delta x = \frac{-2\,e\,B\,d\,\sin(\omega t_o)}{m\,\omega}$$

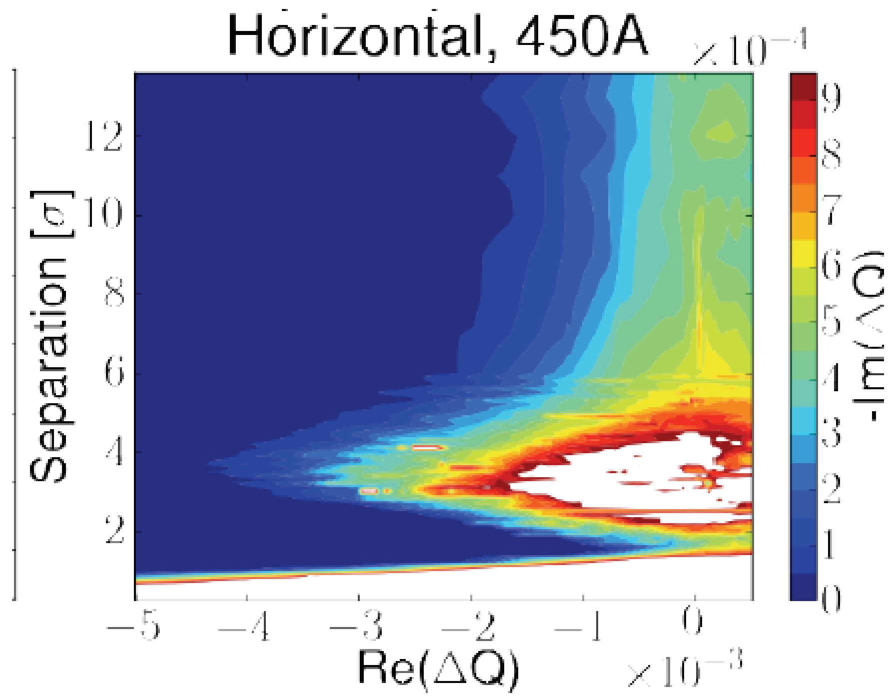
t_o = time bunch enters cavity

d = distance to IP

β^* Levelling – pros

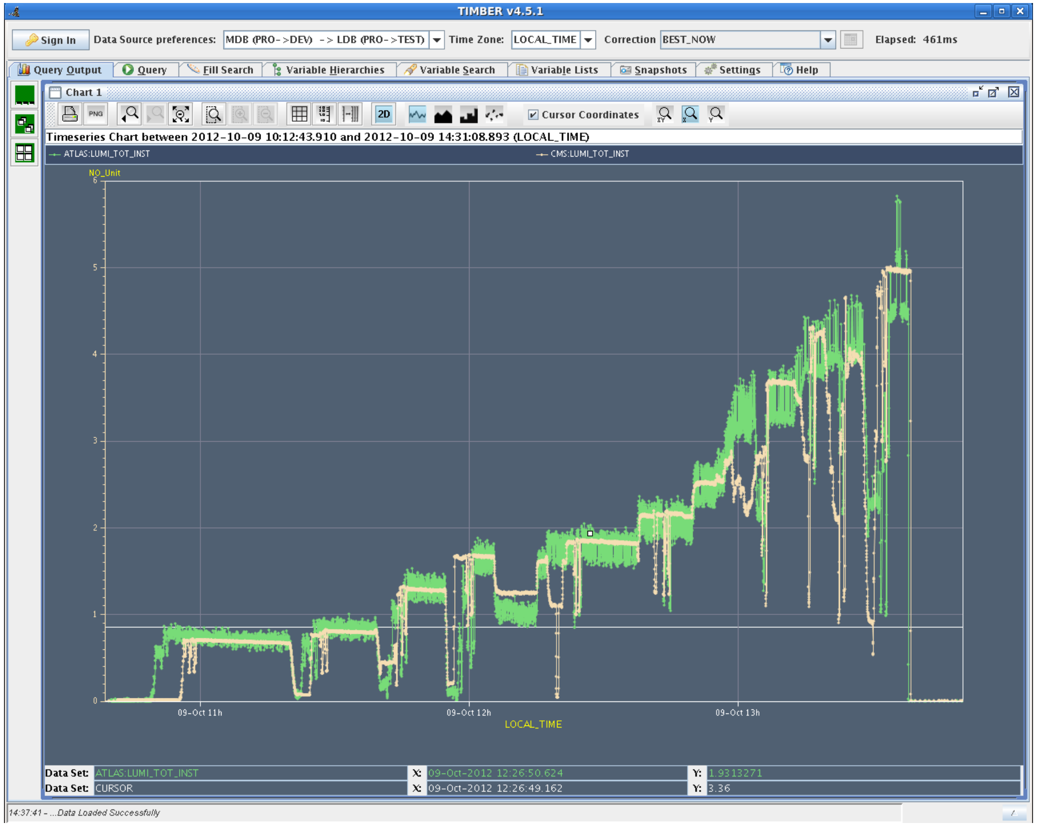
- More stable, largest area for Landau damping
- Tunes do not change & are constant over fill
- Constant longitudinal vertex for experiments





CERN-ATS-Note-2012-071 MD X. Buffat, W. Herr, S. Redaelli J. Wenninger et al.

β^* squeeze levelling experimentally



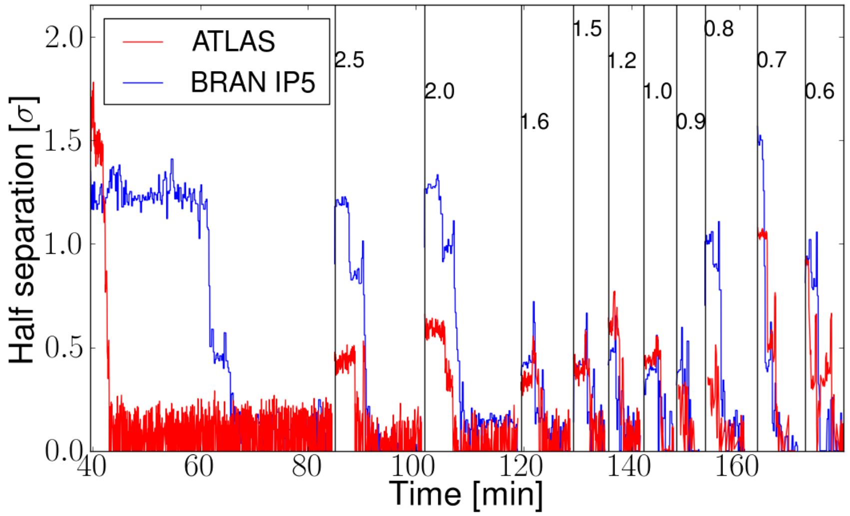
Luminosity vs. Time for the entire shift (~ 6 hours)

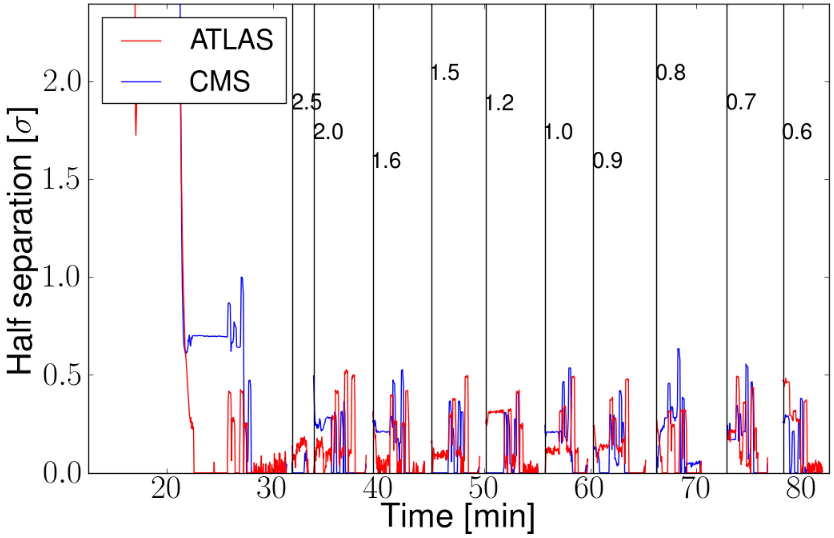
- Conclusion: squeeze done slowly with several steps and everything corrected at every stage doable

- Beams brought into collision at beta* ≈ 9m
- Then tried to squeeze down to beta* ≈ 0.6m
- Orbit Feedback tended to steer beams out of collision so had to go down in small steps while keeping orbit as stable as possible

β^* Levelling – cons

- Feed-forward on orbit required for robustness from an operations point of view
- Need to control orbit during squeeze
- Need several changes from OP point of view





Other levelling possibilities

- Longitudinal cogging:
  - Introducing time delay of couple of RF periods so overlap of colliding bunches is only partial
  - This is done in all IPs at the same time & affects the luminous region
- Large crossing angle:
  - Varying the crossing angle affects the luminosity but also the length of the luminous region
- Flat beam option:
  - Levelling in one plane only -> tune shift const. in other
  - Collimators do not move as much (safety issue)

Luminosity Levelling Techniques

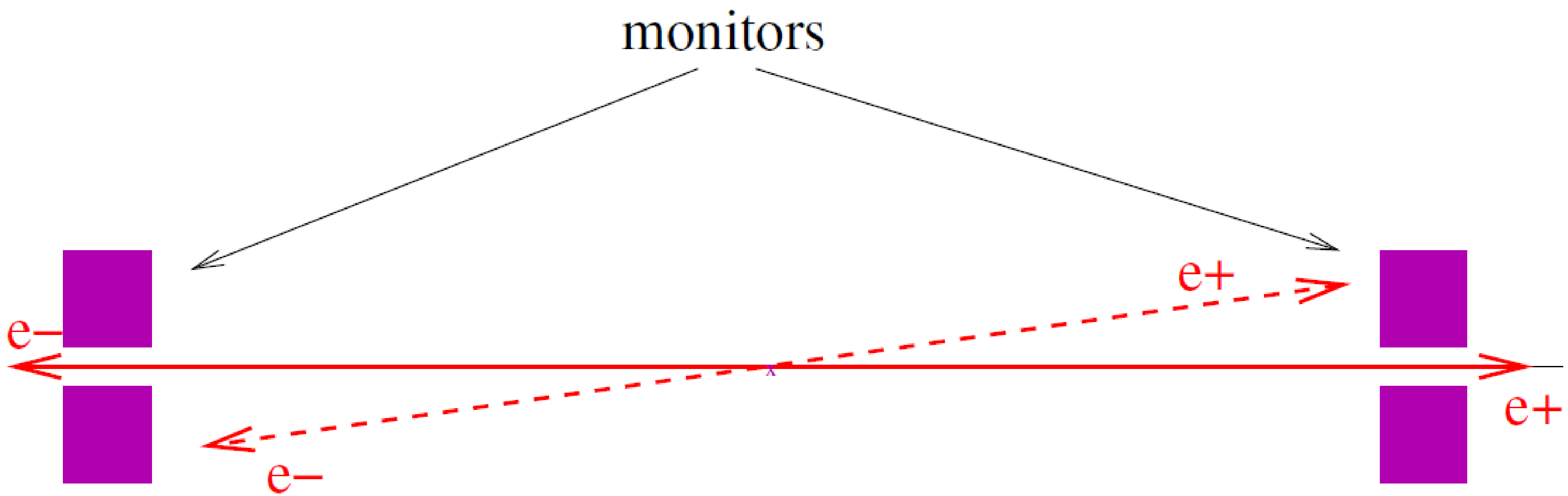
- Benefits of levelling clear:
  - Make events manageable & detectable
  - Make events more evenly spread-out
- All discussed are valid options
  - what is the expected range ?
- Compromise between
  - Experiment requirements and constraints
  - Operational simplicity
  - Beam dynamics issues
    - Landau damping
    - orbit change

Luminosity Measurement

- Luminosity directly proportional to the number of interactions so a good measurement of these is required
- However, these are challenging because they:
  - must cover a wide dynamic range (10^{27}-10^{34} $cm^{-2}s^{-1}$)
  - be very fast – ideally for individual bunches
  - run under different machine conditions
  - reproducible from one run to the next
  - work for different particles (p / ions)
- Once the relative measurement is done, you need to figure out the proportionality const.

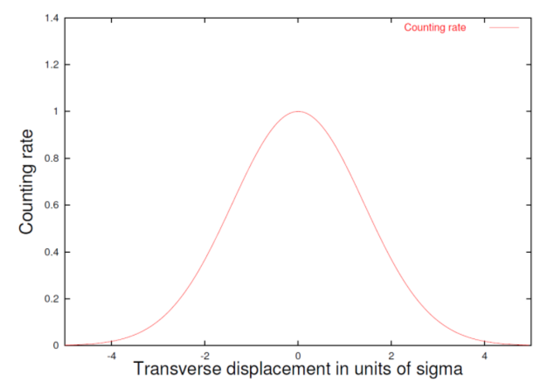
Luminosity Measurement

- Absolute luminosity measurement:
  - Lepton colliders: compare the counting rate to other known processes such as Bhabha scattering for $e^+ e^-$ colliders



Luminosity Measurement

- Absolute luminosity measurement:
  - Hadron colliders: Use similar method to that described before for small angle scattering & also use the scanning of one beam against the other
  - Then using $W = e^{-\frac{d^2}{4\sigma^2}}$ with *d* being the separation between the beams
  - The measurement of the ratio $\mathcal{L}(d)/\mathcal{L}_0$ is a direct measurement of *W*
  - This method was used at CERN on the ISR & is known as a van der Meer scan
  - The expected counting rate is a Gaussian as shown



Not mentioned

- Optical theorem for luminosity measurement
- Coasting beams (e.g. ISR)
- Asymmetric colliders (e.g. PEP, HERA)
- Linear colliders (e.g. TESLA)

Summary

- Looked at the concept of luminosity & how it is important to colliders. Specifically:
  - Luminosity / luminous region are derived / defined
  - How it changes with offsets / crossing angles
  - How the hourglass effect develops for short bunches
  - How crab cavities could be used
  - Luminosity levelling (various types with pros & cons)
  - Measuring luminosity
- Exercise: Go through all the calculations in the lecture – I am here for the next two days & can help you with any problems as can Werner Herr

Further reading

- Luminosity – general concepts:
  - W. Herr & B. Muratori, Concept of luminosity, CERN Accelerator School, Zeuthen 2003, in: CERN 2006-002
- Luminosity – specifics:
  - B. Muratori, “Luminosity and luminous region calculations for the LHC”, LHC Project Note 301, 2002
  - B. Muratori, “Luminosity in the presence of offsets and a crossing angle”, AB-Note-2003-026 (ABP)
  - B. Muratori & T. Pieloni, “Luminosity levelling techniques for the LHC”, CERN beam-beam workshop 2013, in: CERN–2014–004

Thank you ☺