Linear Collider Beam Dynamics

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Introduction

Stepping Stones

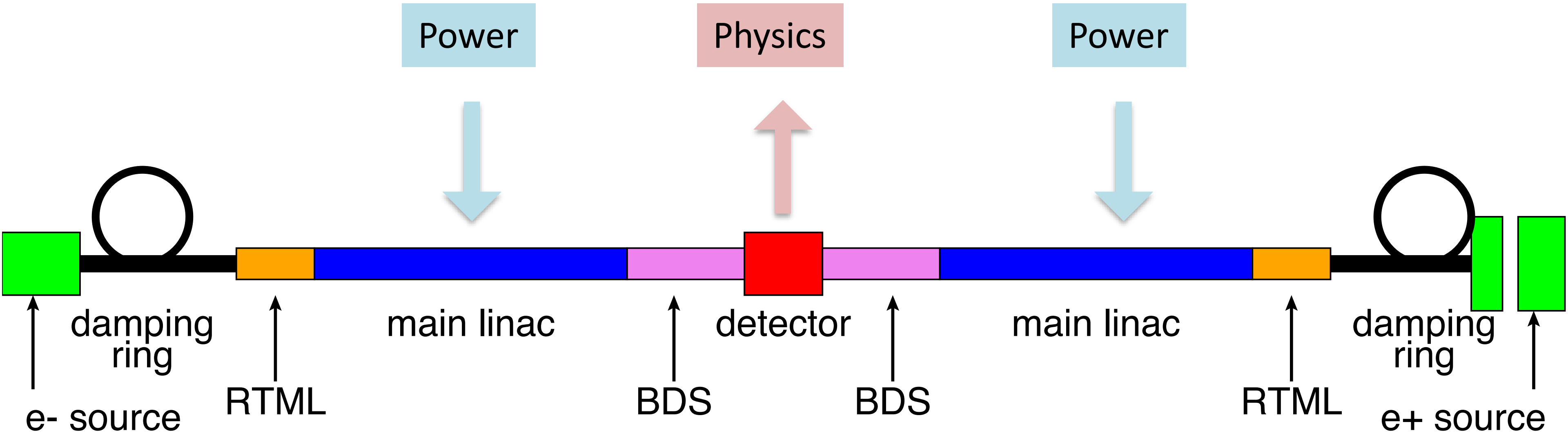
- Beam parameters along the collider and system functionalities
- Power efficiency in the main linac

- Wakefields and single bunch energy spread
- Single bunch beam break-up
- Multi-bunch beam break-up

- Static imperfections
- Dynamic imperfections

Overall Design and Parameters

Main Linear Collider Overview



Need to reach energy and luminosity goal

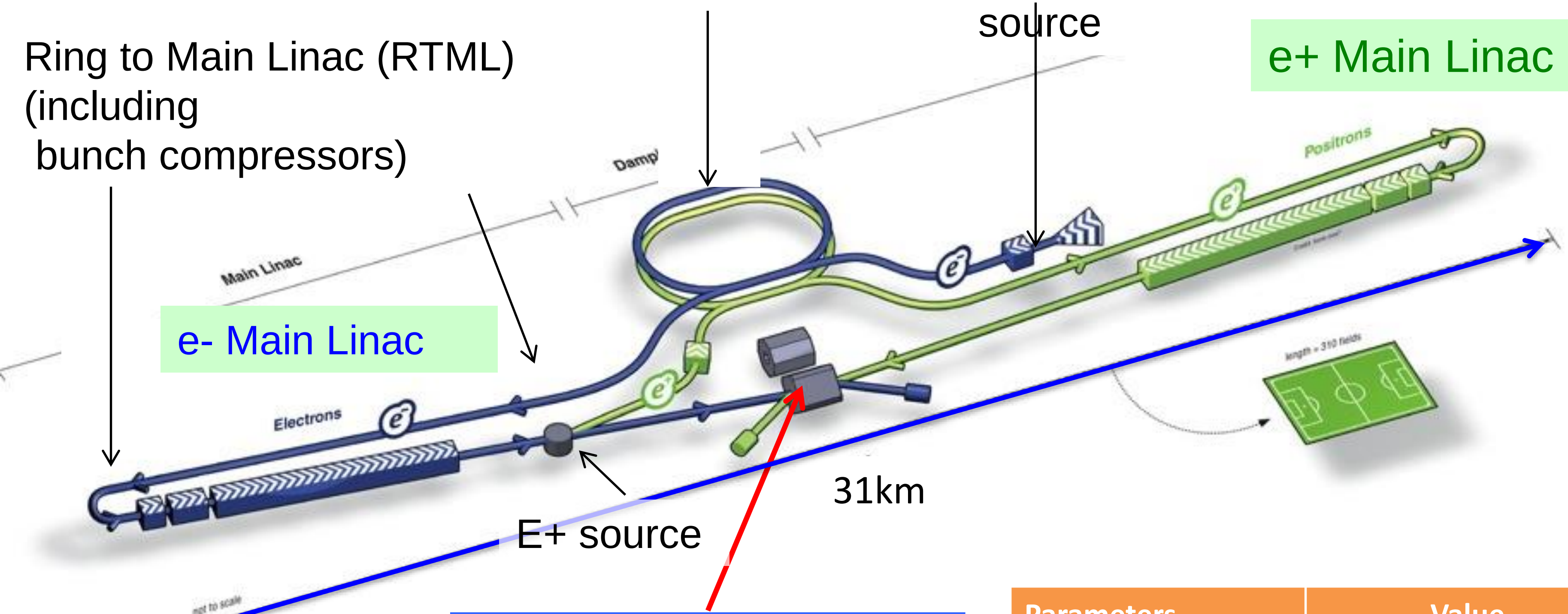
Single pass in linac to reach energy

⇒ Technology challenge

Single pass to reach luminosity

⇒ Technology and beam dynamics

ILC



Damping Rings

Polarised electron source

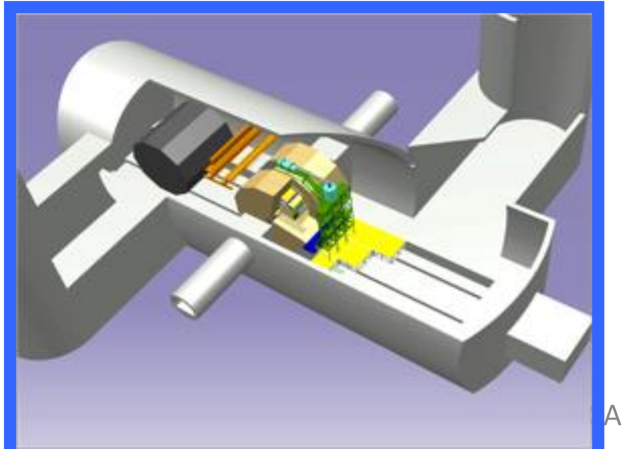
Ring to Main Linac (RTML) (including bunch compressors)

e+ Main Linac

e- Main Linac

31km

E+ source



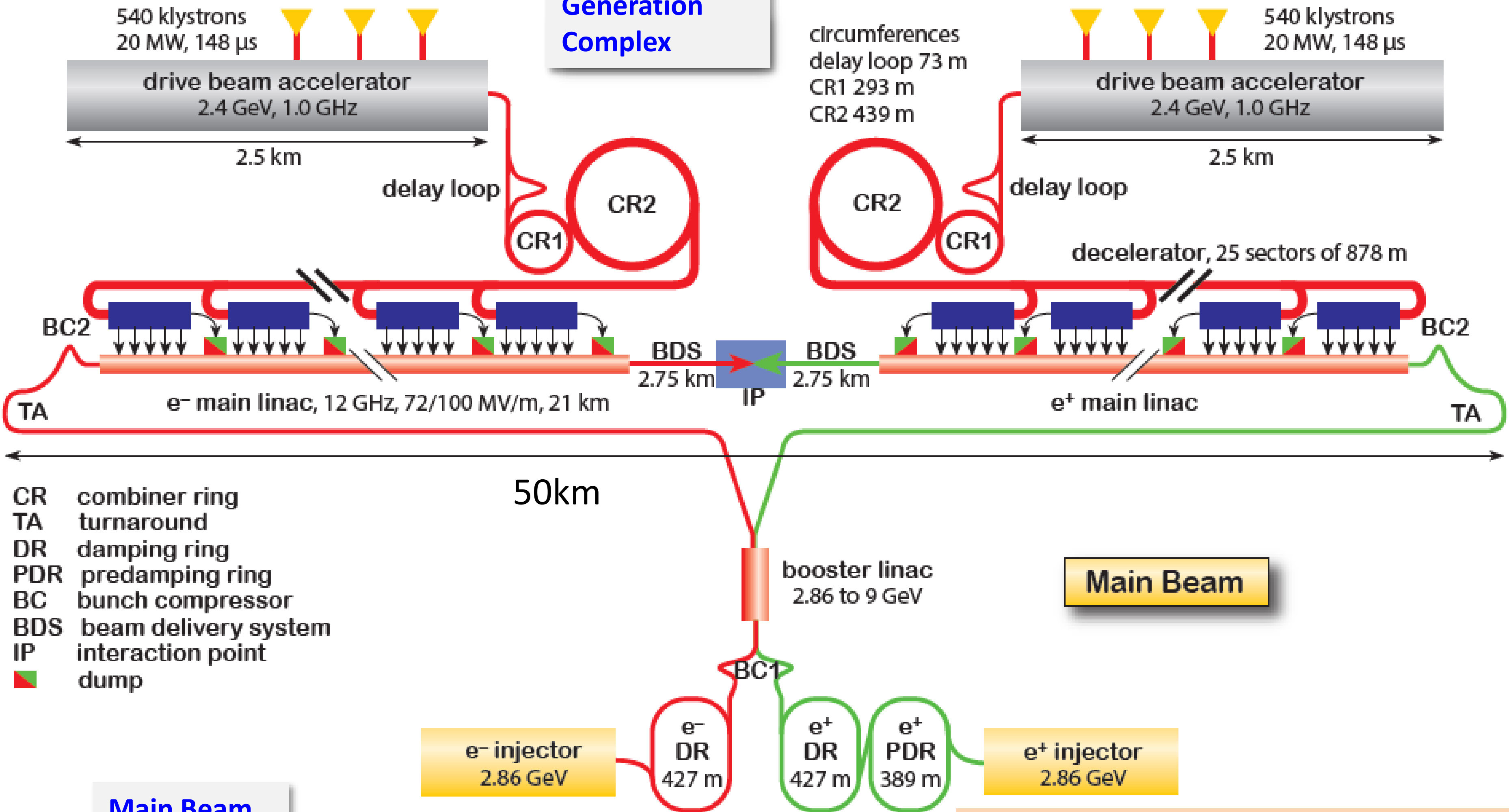
ILC Scheme | © www.form-one.de

Parameters	Value
C.M. Energy	500 GeV
Peak luminosity	1.8 x10^{34} cm^{-2}s^{-1}
Beam power	10.5 MW
Beam Rep. rate	5 Hz
E gradient	31.5 MV/m +/-20%

CLIC (3 TeV)

Goal: Lepton energy frontier

CLIC at 3TeV shown



Drive Beam Generation Complex

Main Beam

Main Beam Generation Complex

CR	combiner ring
TA	turnaround
DR	damping ring
PDR	predamping ring
BC	bunch compressor
BDS	beam delivery system
IP	interaction point
🟥	dump

Stages at E_{cms}=0.38, 1.5 and 3TeV

L=6×10^{34}cm^{-2}s^{-1} at 3TeV

Beam power 30MW at 3TeV

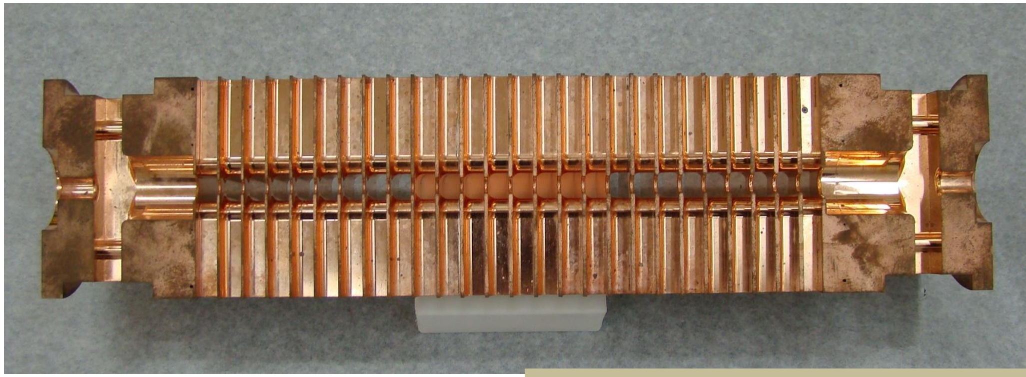
Examples of ILC and CLIC Main Parameters

Parameter	Symbol [unit]	SLC	ILC	CLIC	CLIC
Centre of mass energy	E_{cm} [GeV]	92	500	380	3000
Luminosity	L [10^{34}cm^{-2}s^{-1}]	0.0003	1.8	1.5	6
Luminosity in peak	$L_{0.01}$ [10^{34}cm^{-2}s^{-1}]	0.0003	1	0.9	2
Gradient	G [MV/m]	20	31.5	72	100
Particles per bunch	N [10^9]	37	20	5.2	3.72
Bunch length	σ_z [µm]	1000	300	70	44
Collision beam size	$\sigma_{x,y}$ [nm/nm]	1700/600	474/5.9	143/2.9	40/1
Emittance	$\varepsilon_{x,y}$ [µm/nm]	~3/3000	10/35	0.95/30	0.66/20
Bunches per pulse	n_b	1	1312	352	312
Bunch distance	Δz [mm]	-	554	0.5	0.5
Repetition rate	f_r [Hz]	120	5	50	50

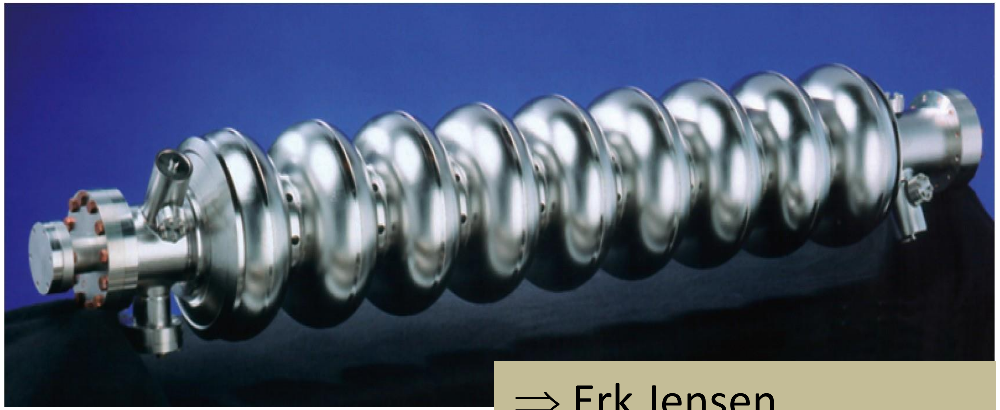
Energy Drivers

Energy is largely determined by main linac

Gradient in the accelerating structure or cavity
Key technology challenge



⇒ Walter Wuensch,
Sunday/Monday 4/5.3.



⇒ Erk Jensen,
Thursday 1.3.

Length of the linacs
Cost of technology

Affordability of the project is key

Luminosity Drivers

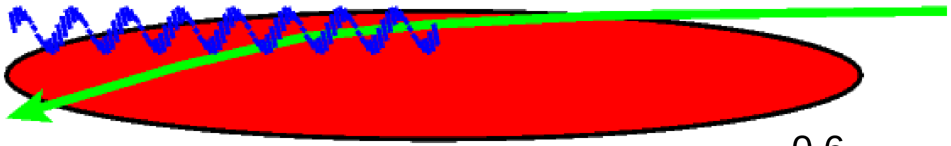
Can re-write normal luminosity formula

$$\mathcal{L} = H_D \frac{N^2}{4\pi\sigma_x\sigma_y} n_b f_r$$

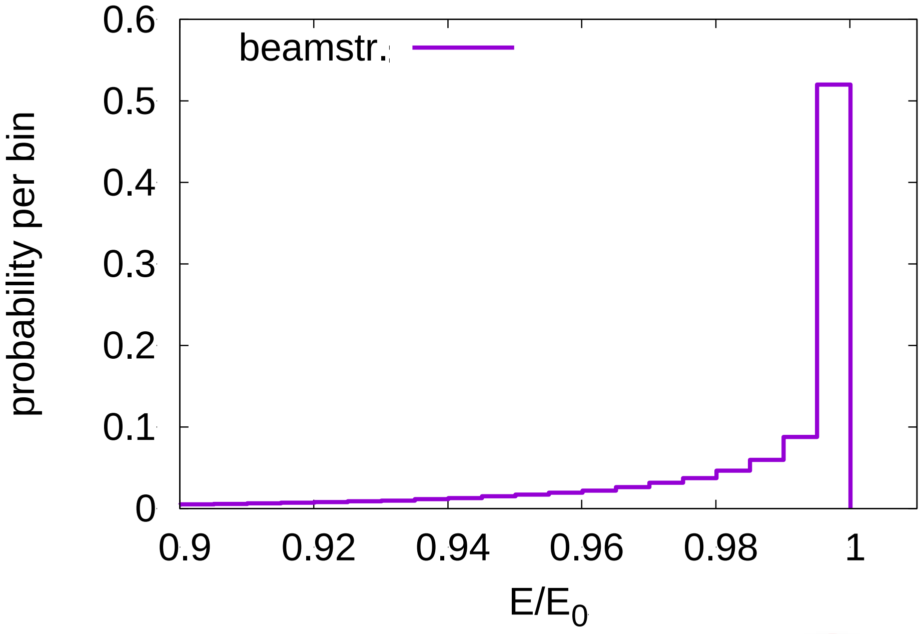
$$\mathcal{L} \propto H_D \underset{\substack{\uparrow\\ \text{Luminosity spectrum}}}{\frac{N}{\sigma_x}} \quad \underset{\substack{\uparrow\\ \text{Beam current}}}{N n_b f_r} \quad \underset{\substack{\uparrow\\ \text{Beam Quality (+bunch length)}}}{\frac{1}{\sigma_y}}$$

H_D : pinch enhancement, typically 1-2
N : number of particles per bunch
n_b : number of bunches per train
f_r : number of trains per second
$\sigma_{x,y}$: transverse beamsizes

Beamstrahlung



⇒ Werner Herr
Monday 26.2.



Number of photons dominates $L_{0.01}/L$

$$n_\gamma \propto \frac{N}{\sigma_x + \sigma_y}$$

+

$$\mathcal{L} \propto \frac{N}{\sigma_x \sigma_y}$$

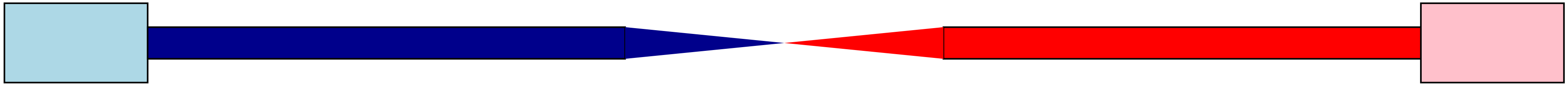
$$\sigma_x \gg \sigma_y$$

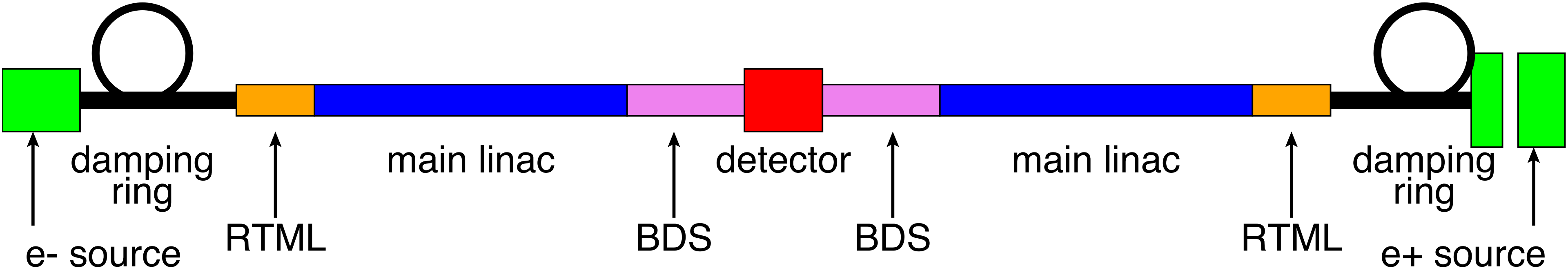
$$\sigma_x + \sigma_y \approx \sigma_x$$

$$\propto n_\gamma$$

$$\mathcal{L} \propto H_D \frac{N}{\sigma_x} N n_b f_r \frac{1}{\sigma_y}$$

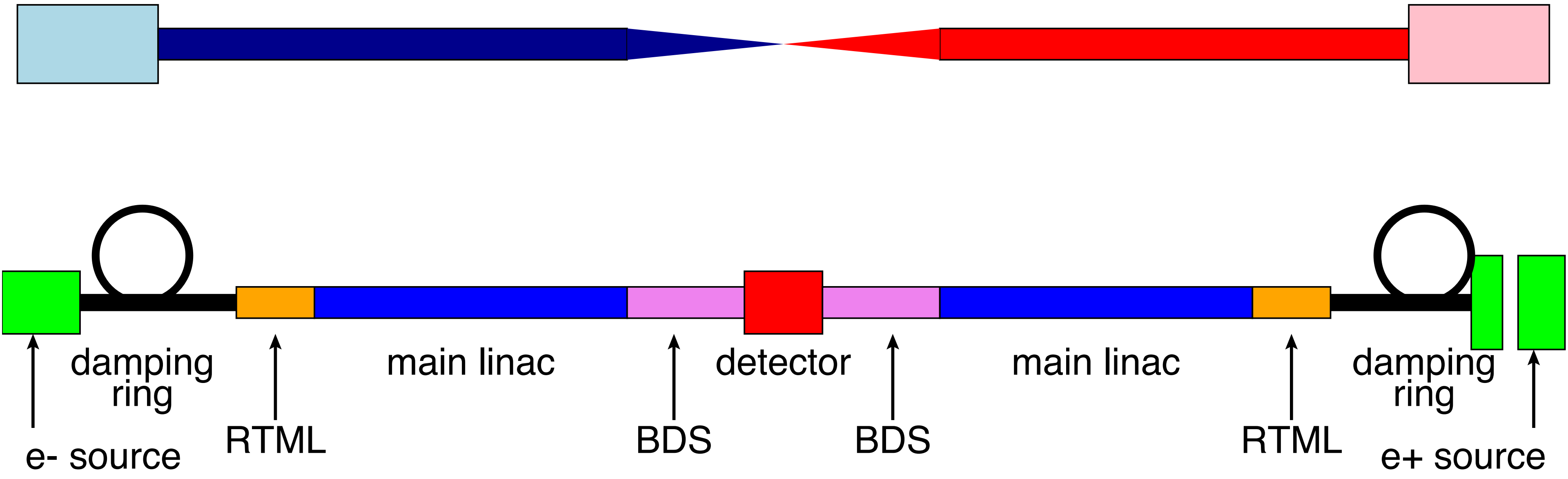
Beam Parameters Along the Collider (CLIC 380)





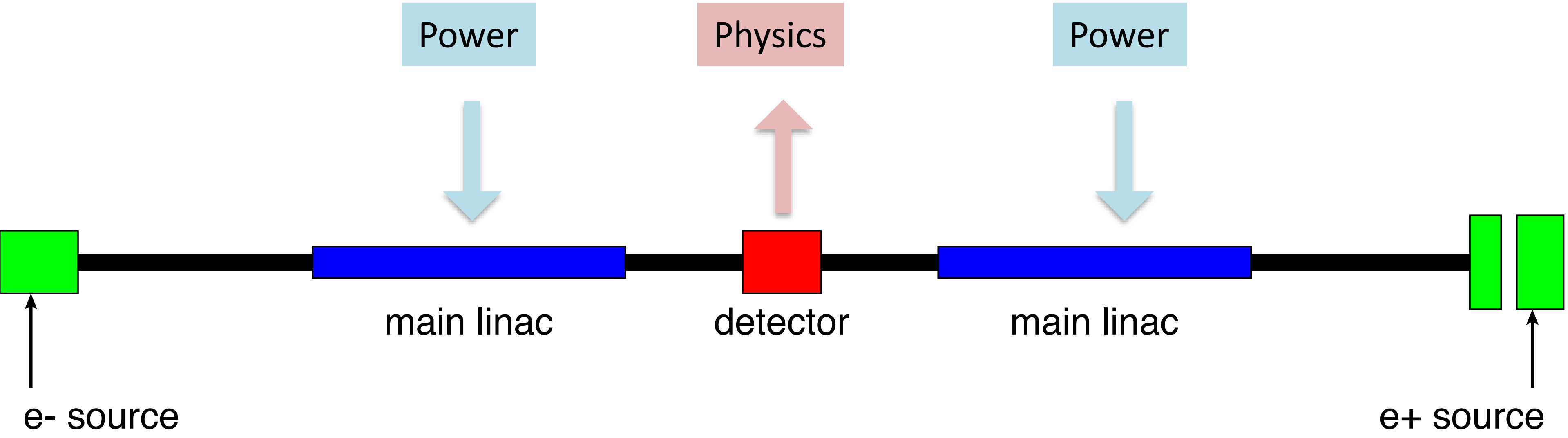
	ε_x [nm]	ε_y [nm]	σ_z [μm]	E [GeV]
Damping ring exit	700	5	1600	2.86
End of RTML	850	10	70	9.0
End of main linac	900	20	70	190.0
Interaction point	950	30	70	190.0

Beam Parameters Along the Collider (CLIC 380)



	Design limits $\Delta\varepsilon_y$ [nm]	Static imperfections $\Delta\varepsilon_y$ [nm]	Dynamic imperfections $\Delta\varepsilon_y$ [nm]
Damping ring exit	5	0	0
End of RTML	1	2	2
End of main linac	0	5	5
Interaction point	0	5	5
sum	6	12	12

Main Linac



- The key for higher energies

- The main cost driver
- Main power consumer

- A main ingredient for site choice

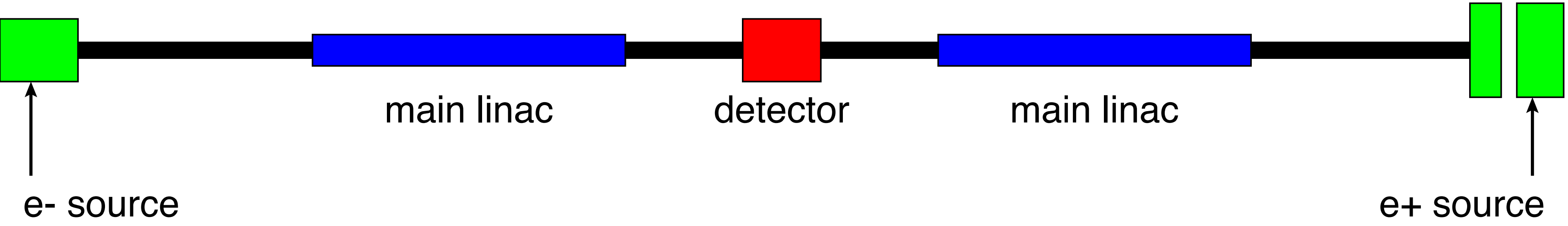
- The key design driver for other systems

Centre of this lecture

Note: 12 hours of main linac lecture in linear collider school only scratches the surface

Sources

⇒ Masao Kuriki
Thursday 1.3.



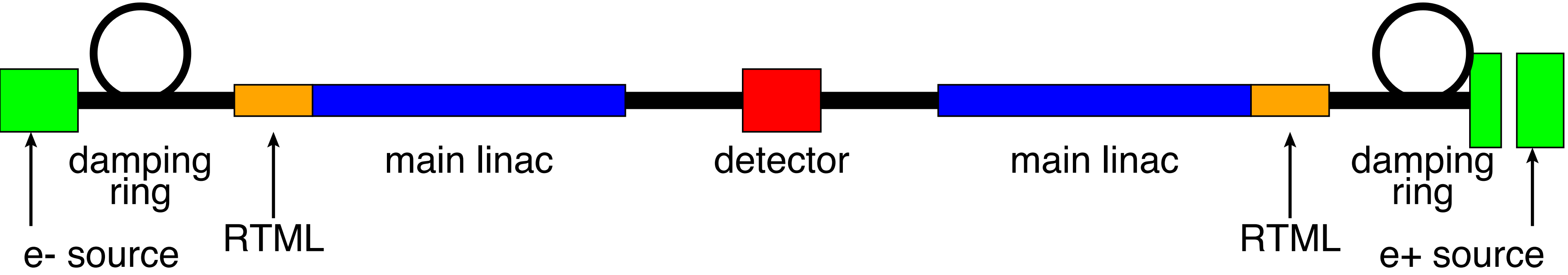
Produce the electron beam

- use a laser to kick electrons out of a cathode

Produce the positron beam

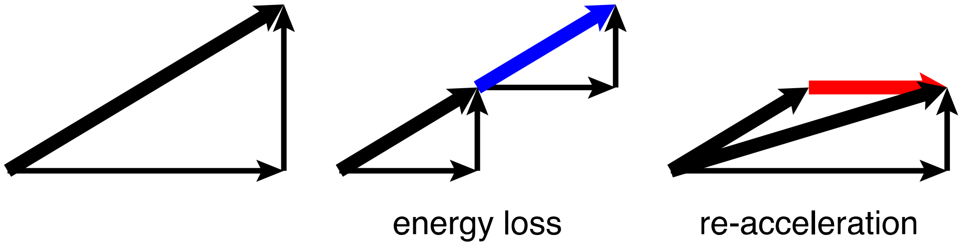
- use an electron beam to produce photons
  - In CLIC in a crystal
  - In ILC in a wiggler
- the photons produce showers in matter
  - harvest the positrons

Damping Rings



Cool the beams

- in particular positron beam
- make particles emit synchrotron radiation (bends and wigglers)
- and reaccelerate



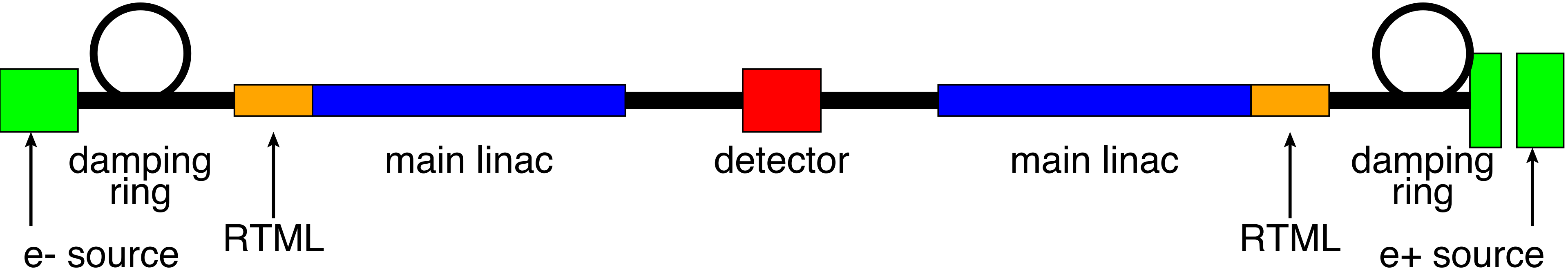
Main limit for horizontal emittance and contributes to vertical emittance

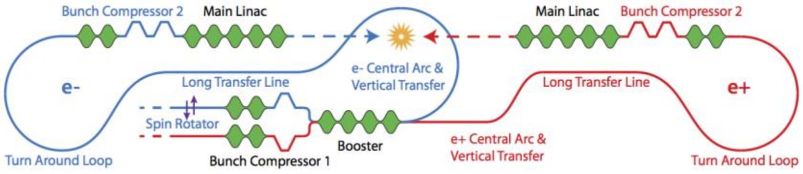
Important single particle and collective beam dynamics

⇒ Hermann Schmickler
Friday 23.2. ☺

⇒ Katsunobu Oide
Monday 26.2., Tuesday 27.2.

Ring To Main Linac





Transports beam from damping ring to main linac

- Compresses bunches from damping ring to main linac (e.g. from 1.6 mm to 70 µm in CLIC)
- Increase the beam energy to be high enough for transport and main linac
- Manipulate the spin

⇒ Frank Tecker
Saturday 24.2.

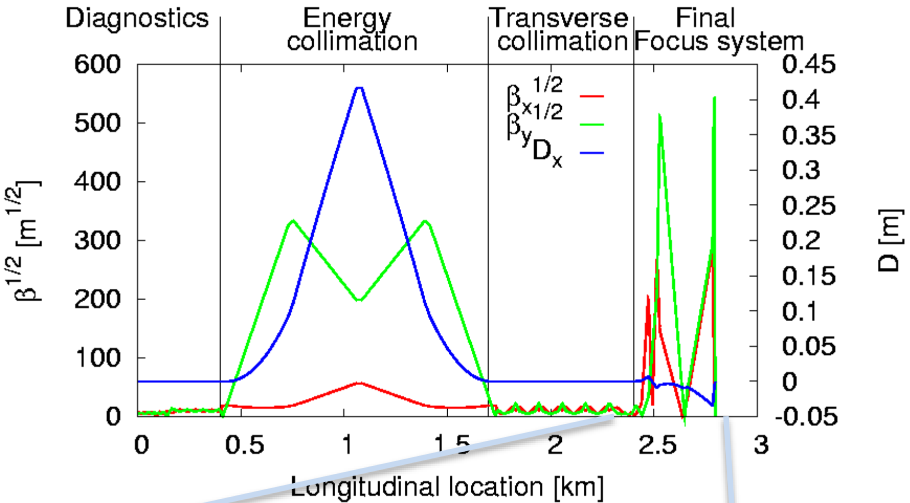
Beam Delivery System

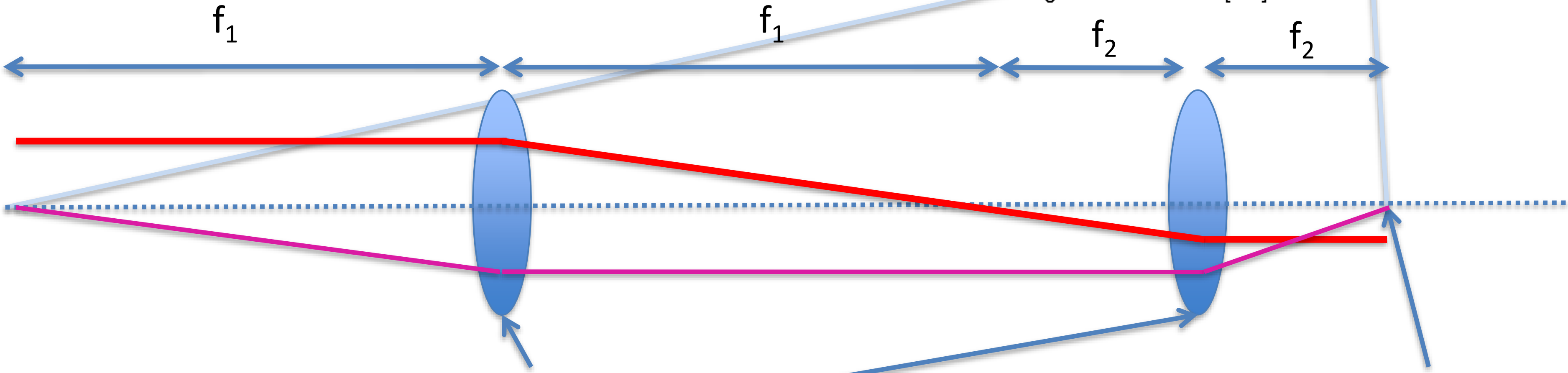
⇒ Andrei Seryi
Sunday 4.3

Final focus system: Telescope to squeeze the beam to small size
i.e. small beta-function

Chromaticity is a problem
(similar to camera lens)

Correction with nonlinear elements,
but has limited capabilities
⇒ Have to limit beam energy spread



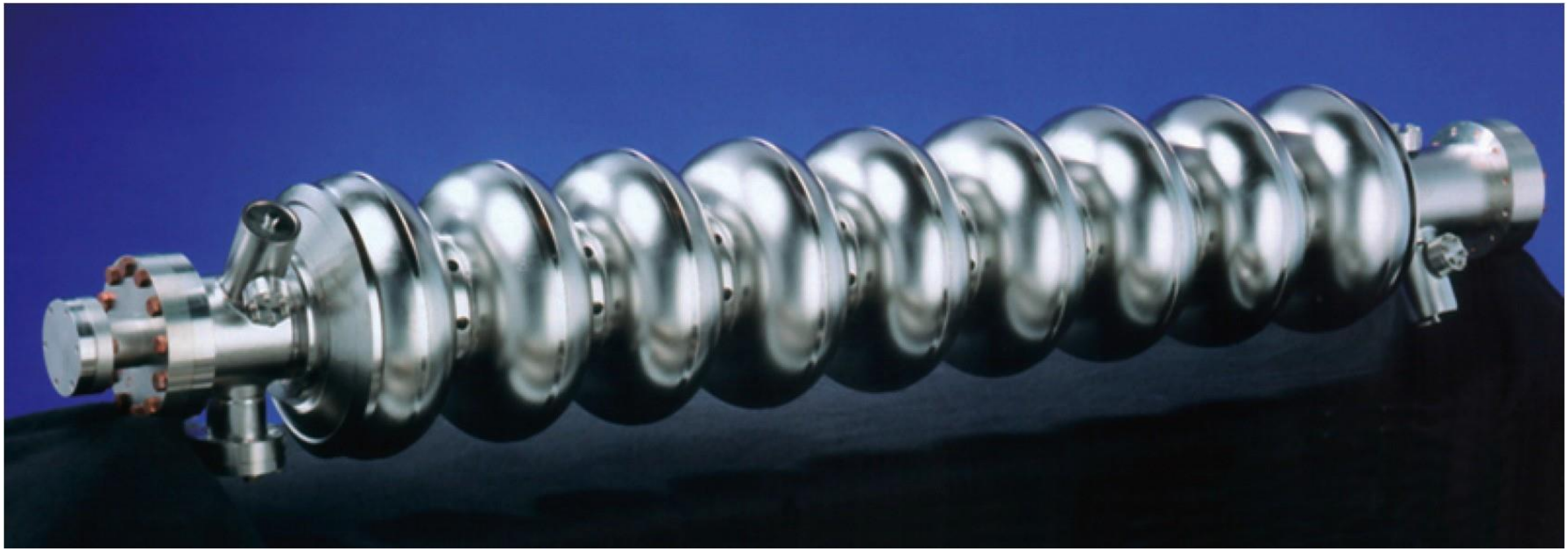


Focusing lens
Actually a doublet of quadrupoles

IP, i.e. focal point

Main Linac and Energy

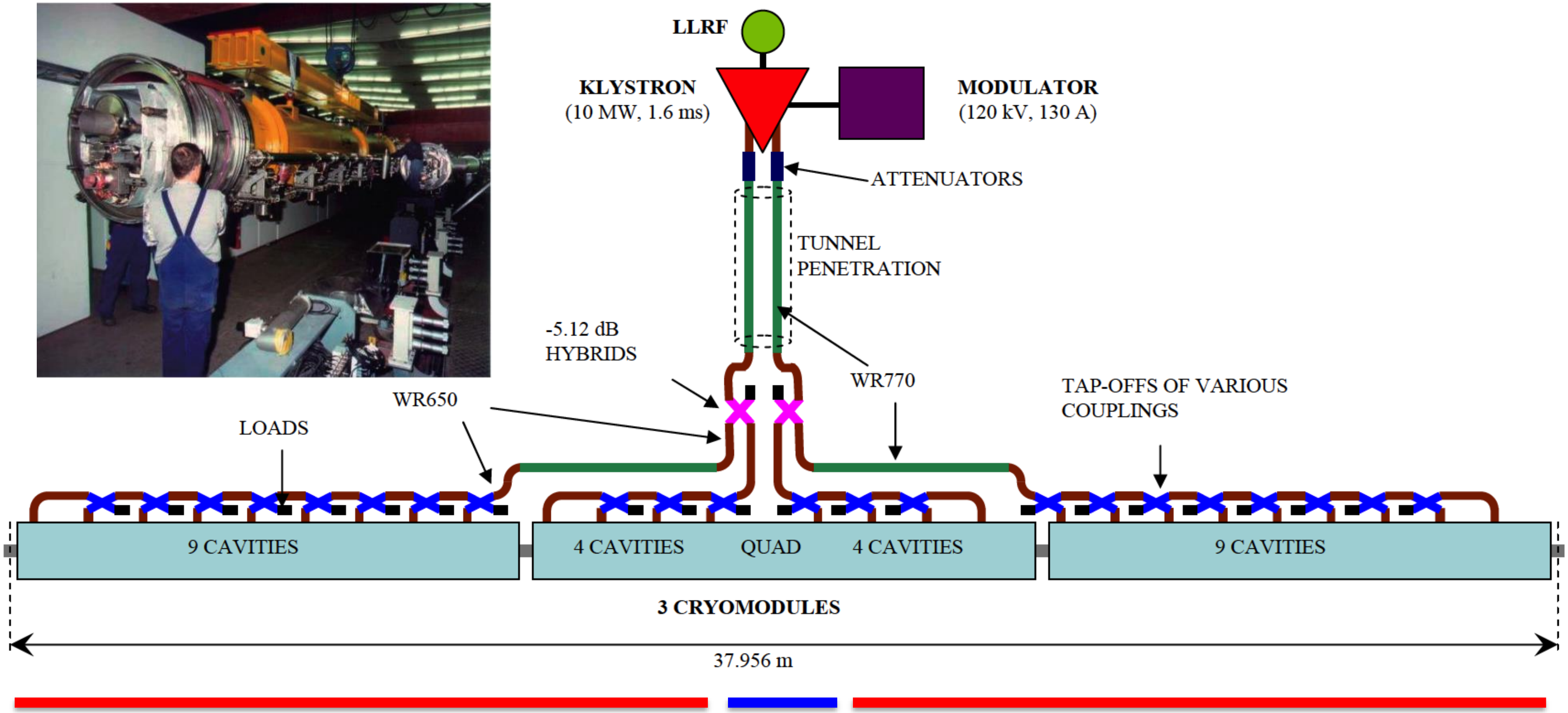
ILC Accelerating Cavity



- About 1 m long cavity with 31.5 MV/m,
  - super-conducting
  - 1.3 GHz
  - standing wave
  - constant impedance

$\Rightarrow$ Erk Jensen,
$\Rightarrow$Thursday 1.3.

Main Linac Unit



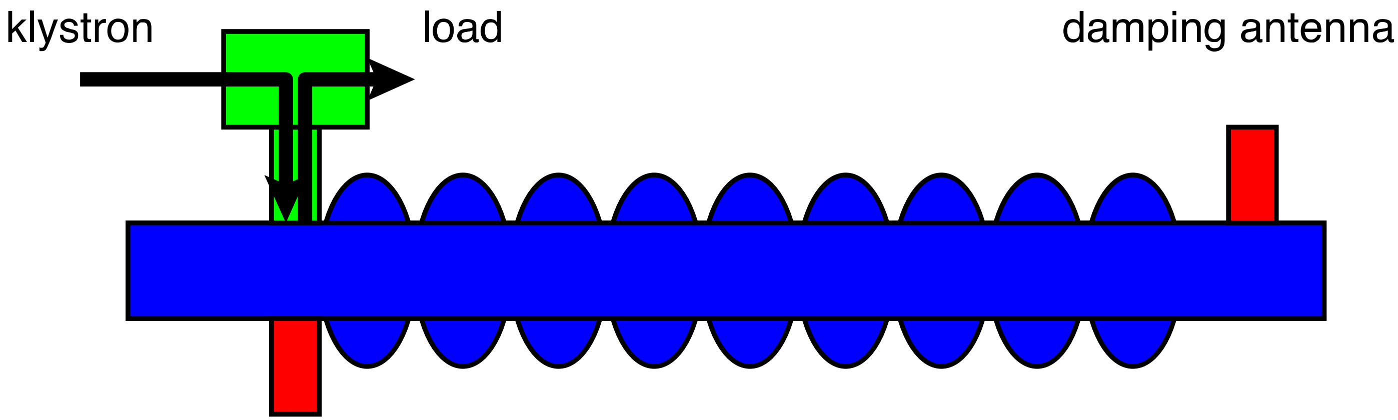
Accelerating cavities
O(65%) of linac length

Beam guiding quadrupole
Beam position monitor
Corrector kicker

Accelerating cavities

Total length for 500 GeV cms 31 km, some length for beam cleaning and focusing

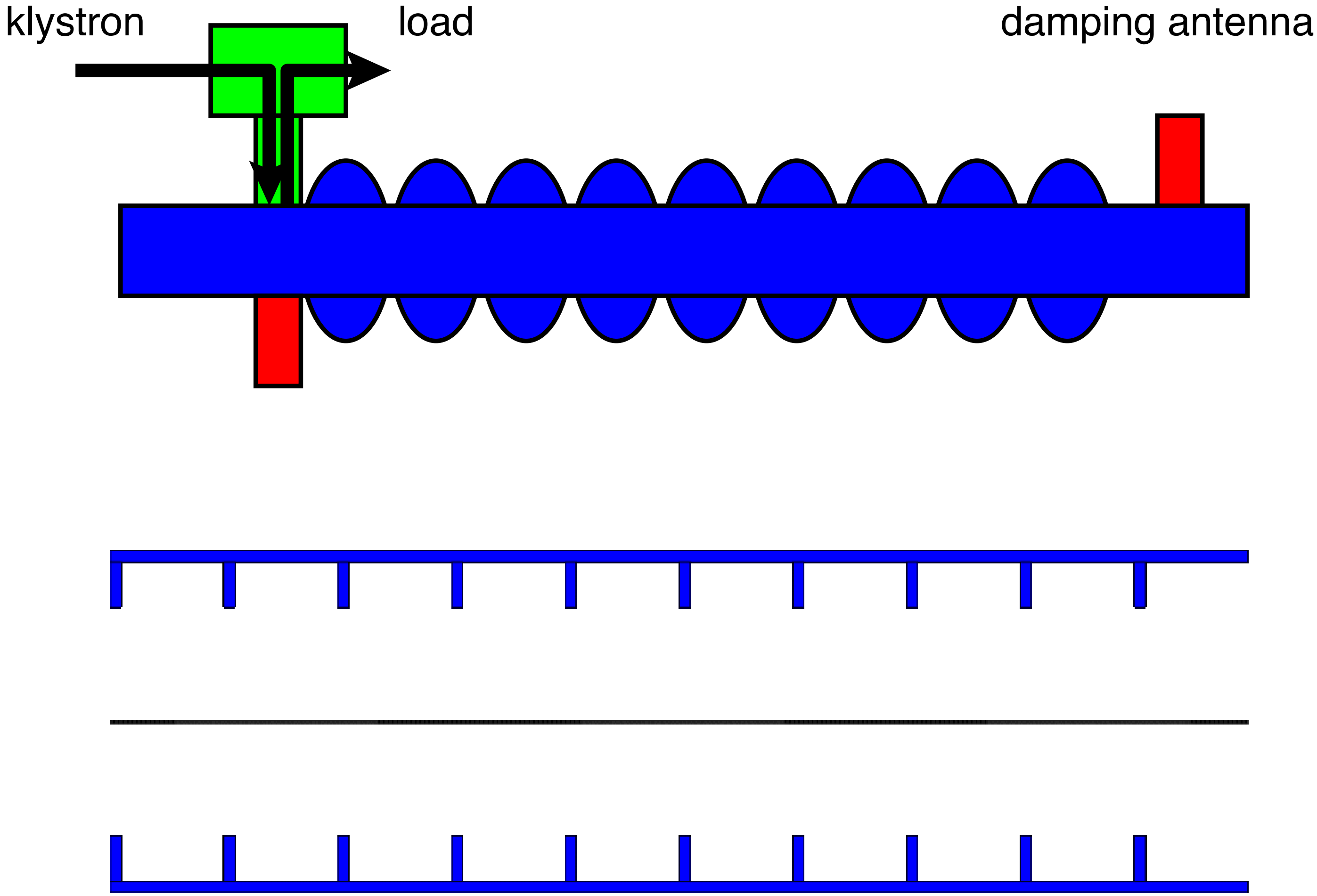
Standing Wave Cavity



- The power is feed into one end
  - the power is reflected at the coupler
  - as the power in the cavity is increasing, the reflection is reduced
- there is a level when there is no reflection

⇒ now switch on the beam

Standing Wave Cavity

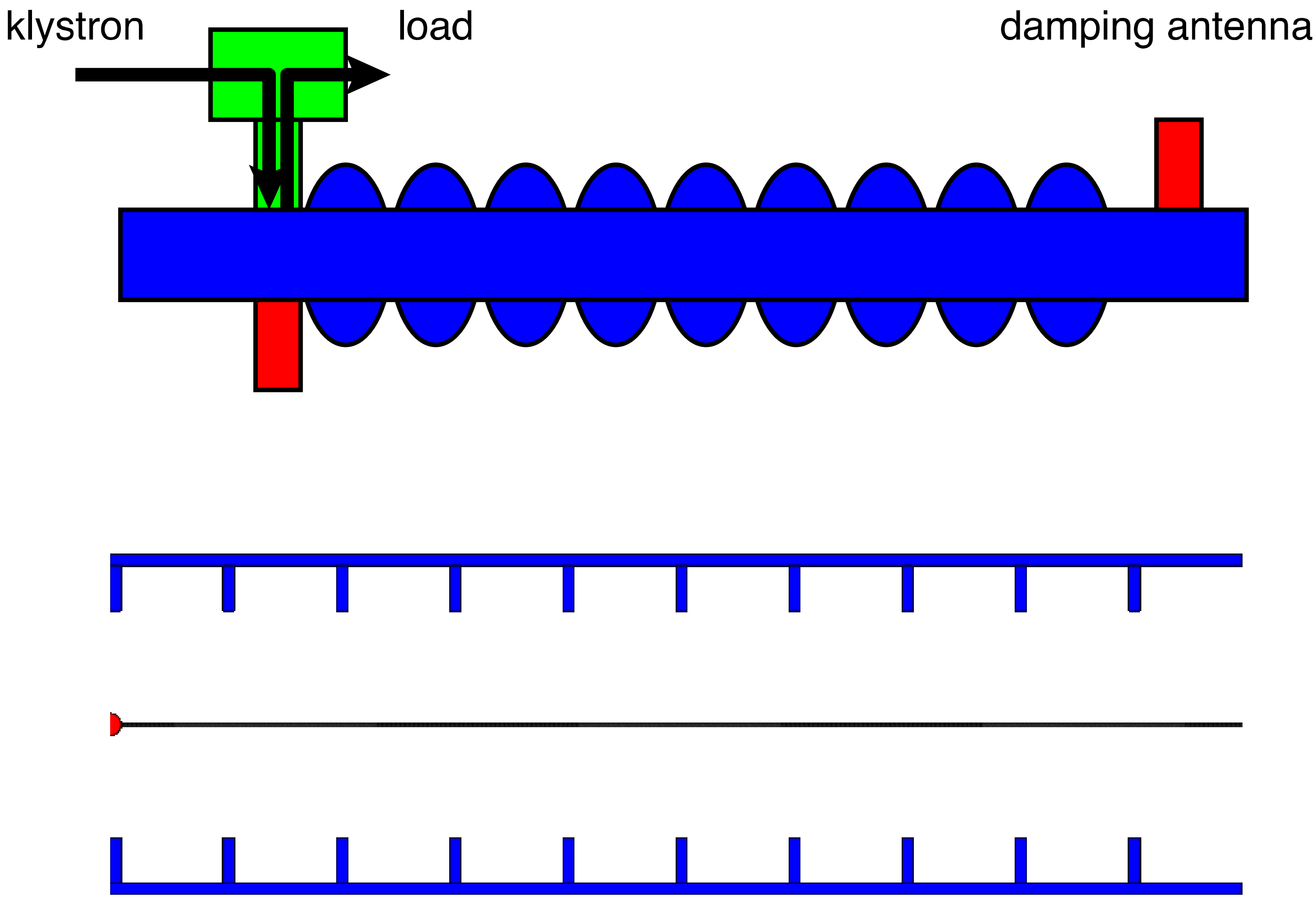


klystron

load

damping antenna

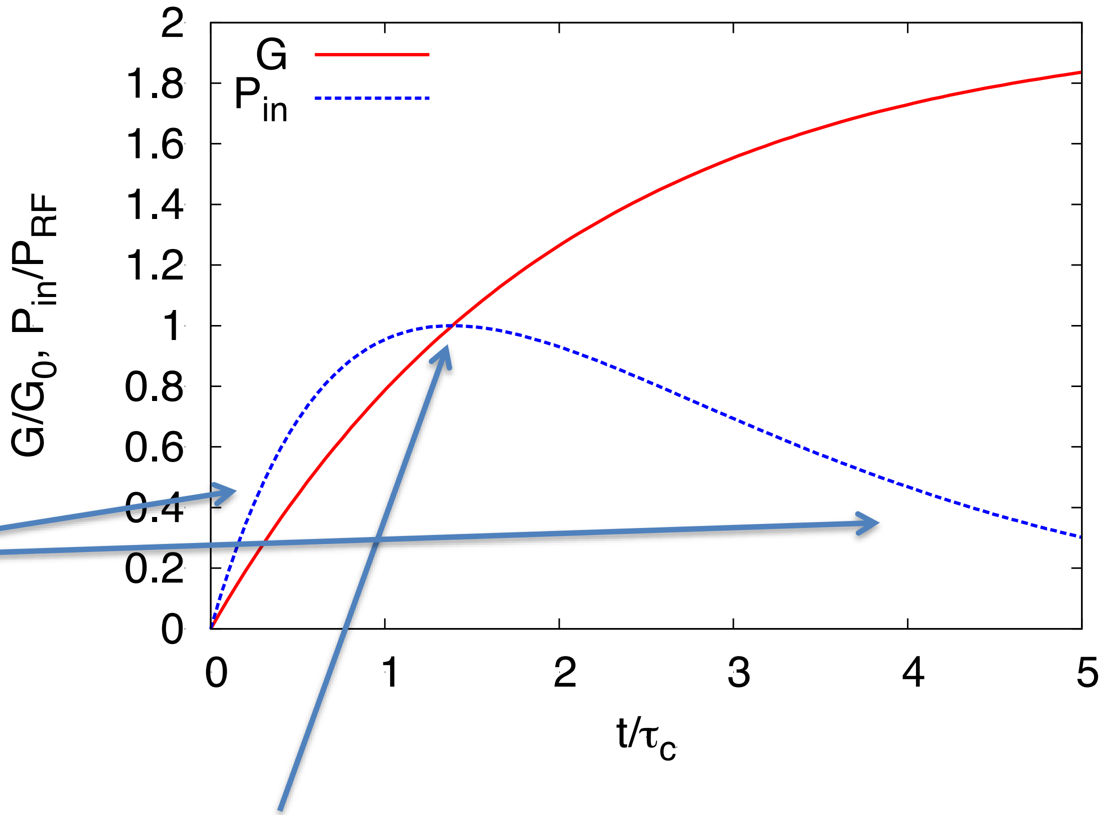
Standing Wave Cavity



Filling a Standing-wave Cavity

Select the target gradient G_0

Adjust the coupling of the cavity to the RF “external Q”



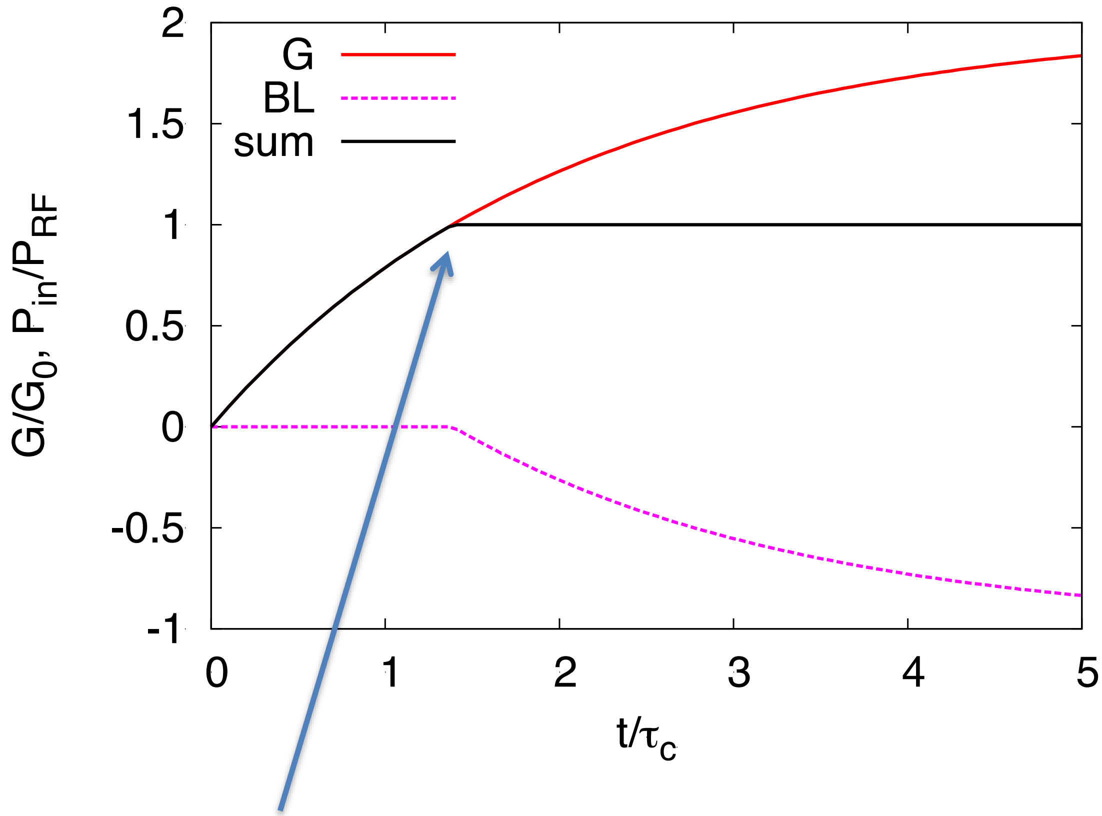
Only part of RF power flows into cavity

All the RF power flows into cavity

In ILC
Filling time is 900 μs
Beam time is 720 μs

⇒ Erk Jensen,
⇒Thursday 1.3.

Filling a Standing-wave Cavity



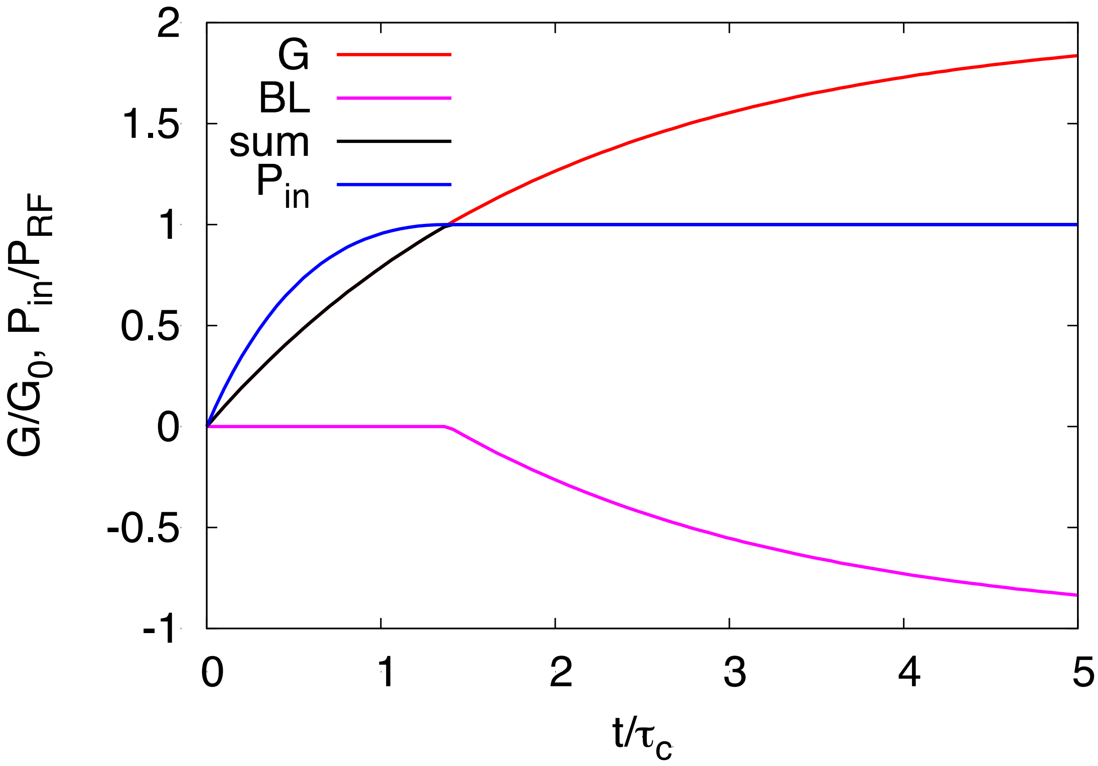
Switch the beam on

Takes as much power from cavity as flows in

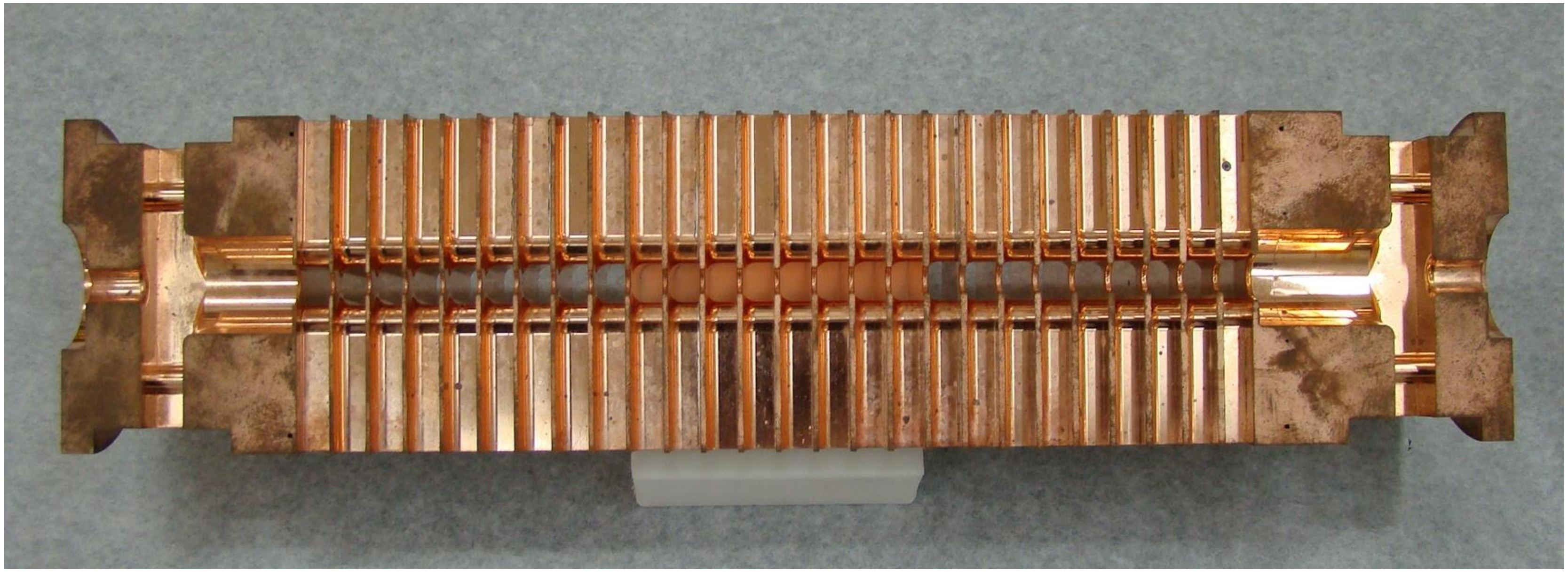
Gradient remains constant

All RF power continues to flow in

Filling a Standing-wave Cavity



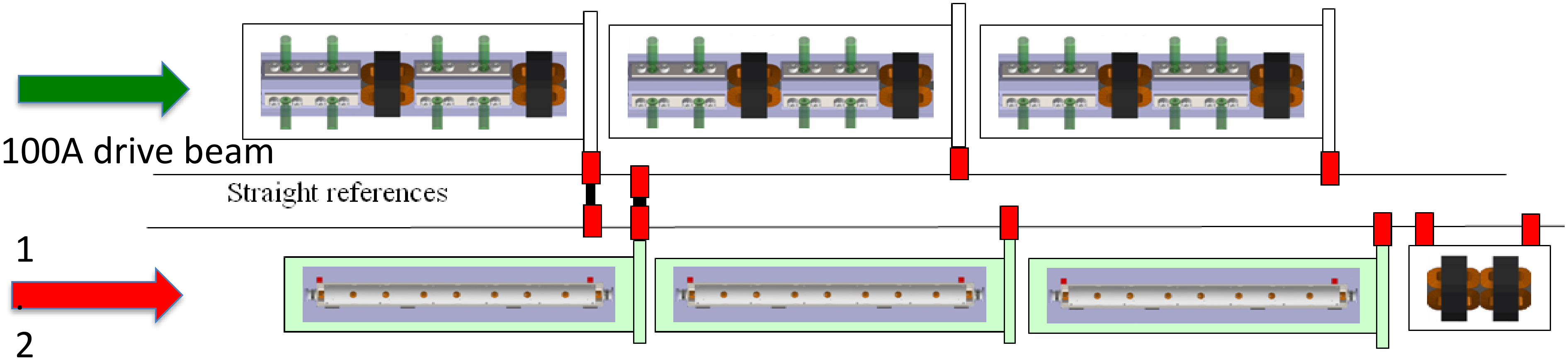
CLIC Accelerating Structure

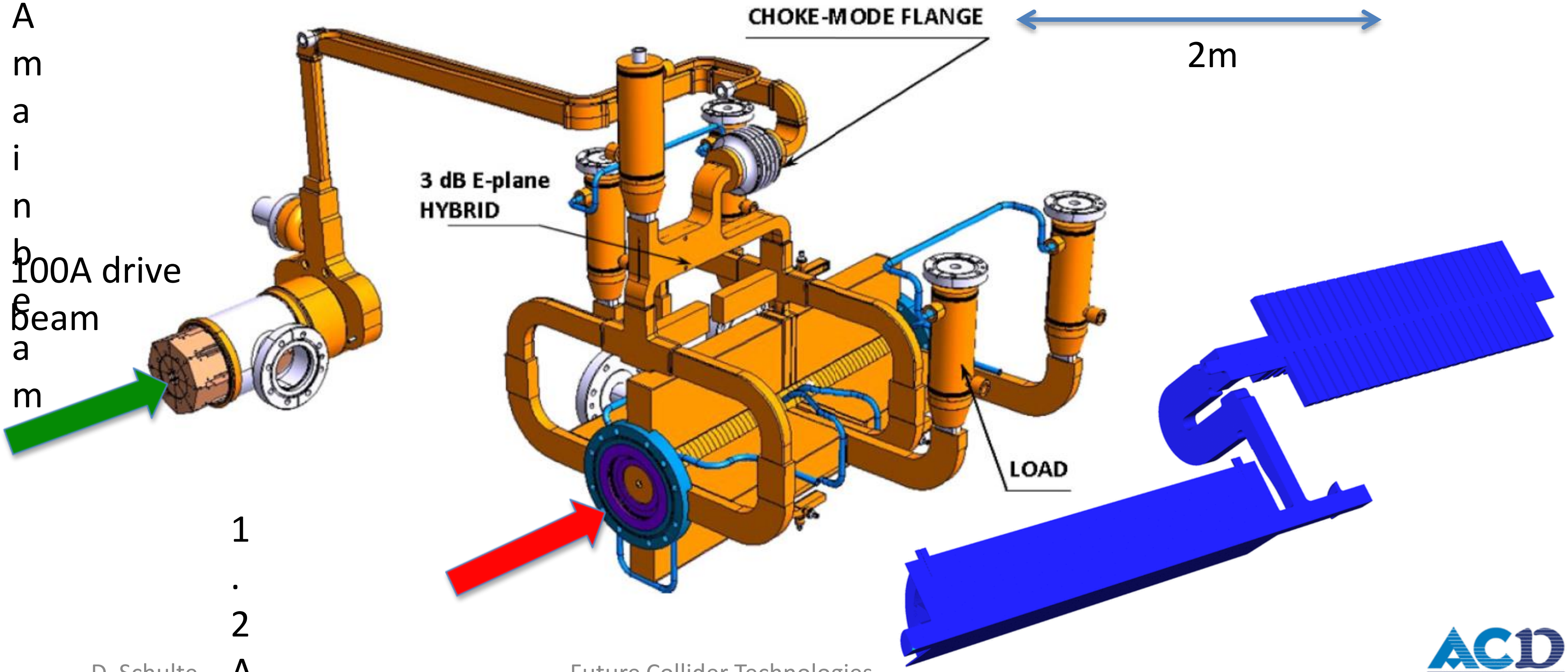


- About 23 cm long structure with G = 100 MV/m
  - normal-conducting
  - 12 GHz
  - travelling wave
  - constant gradient (almost)

$\Rightarrow$ Walter Wuensch,
Sunday/Monday 4/5.3.

CLIC Two-beam Concept



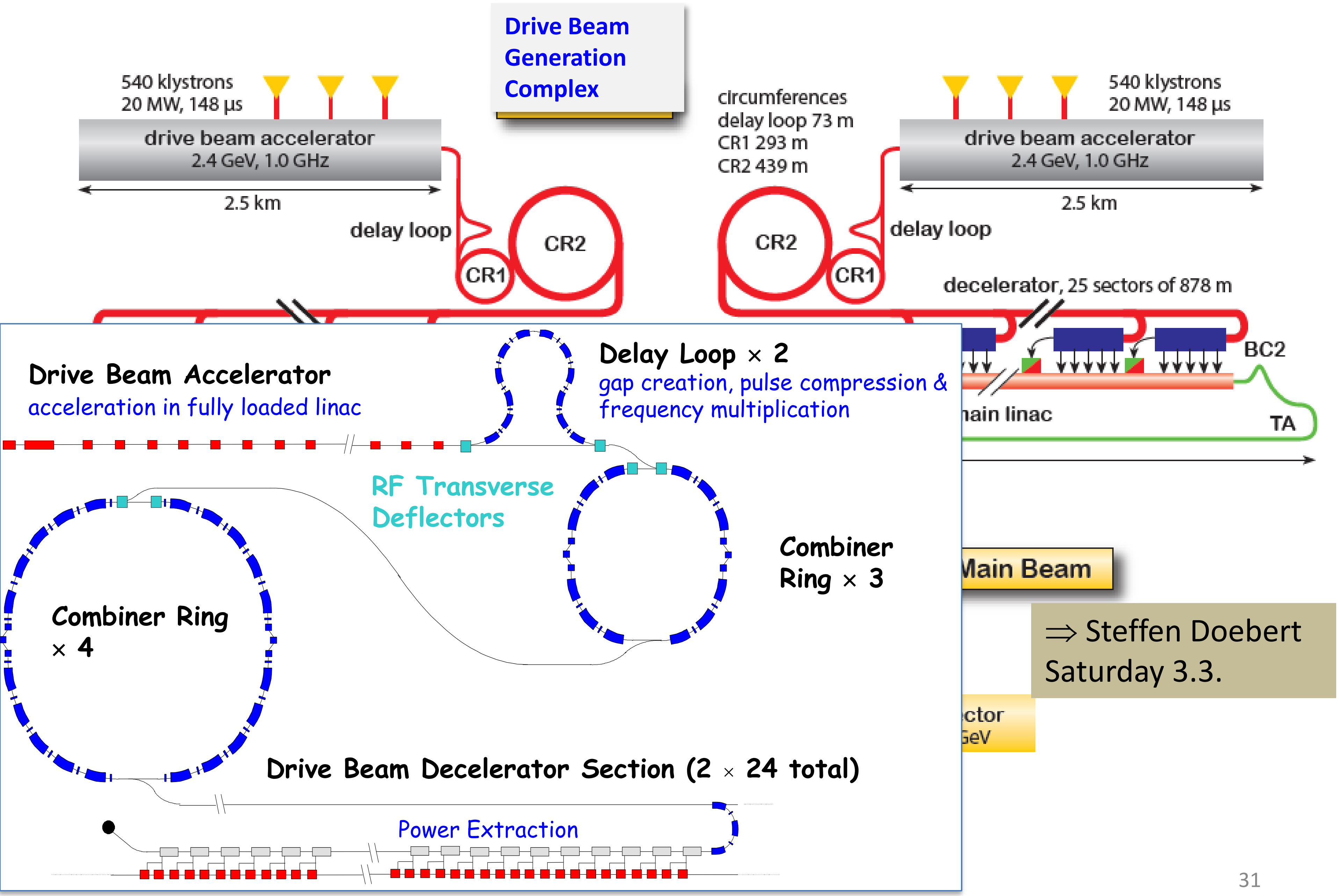


CLIC Two-beam Module

1st module

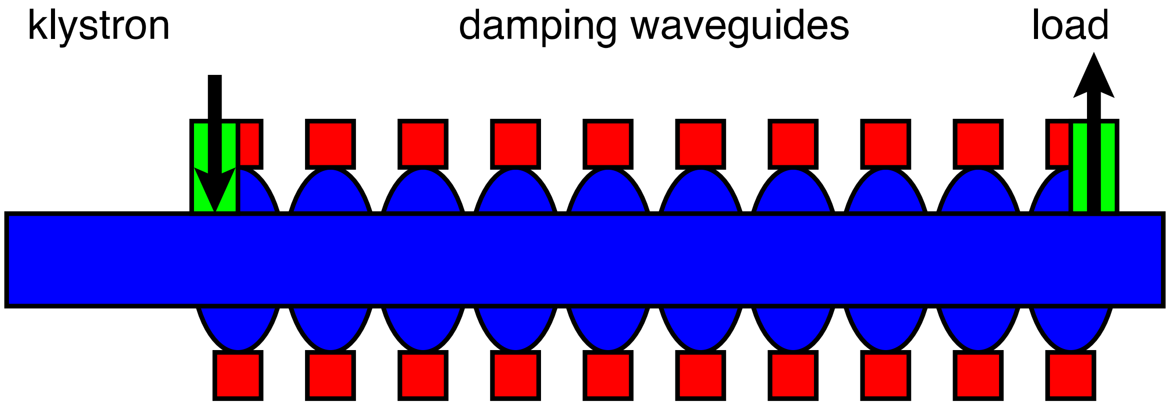
80 % filling with accelerating structures
11 km for 380 GeV cms
50 km for 3 TeV

CLIC: Drive Beam



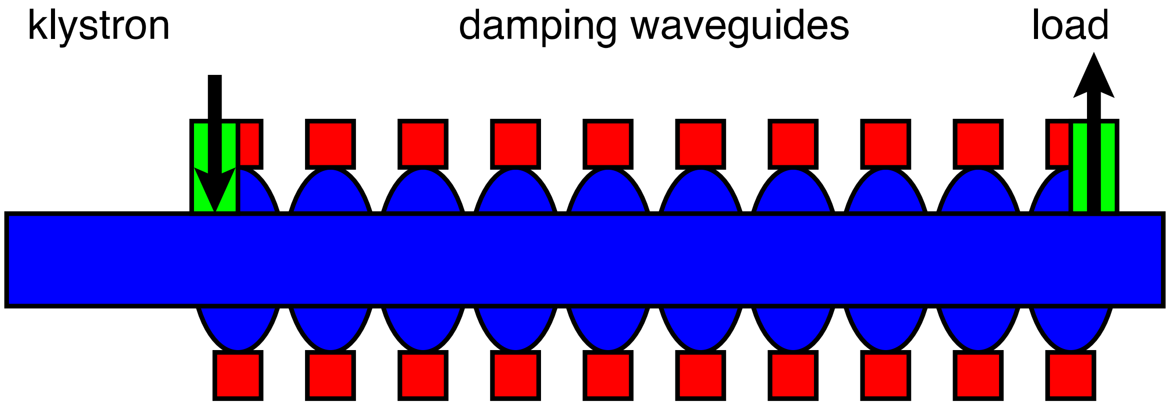
⇒ Steffen Doebert Saturday 3.3.

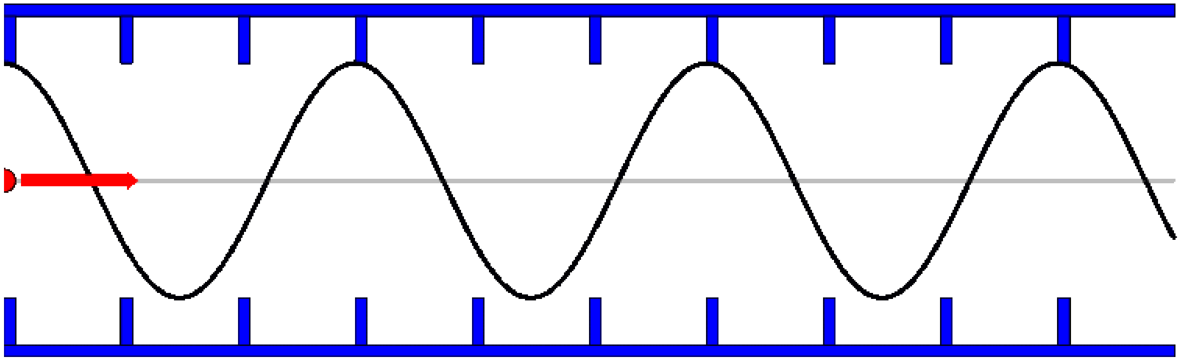
Travelling Wave Structure



- The power is feed into one end
  - no reflection if designed properly
- It slowly moves through the structure
  - group velocity is typically a few percent of the speed of light

Travelling Wave Structure





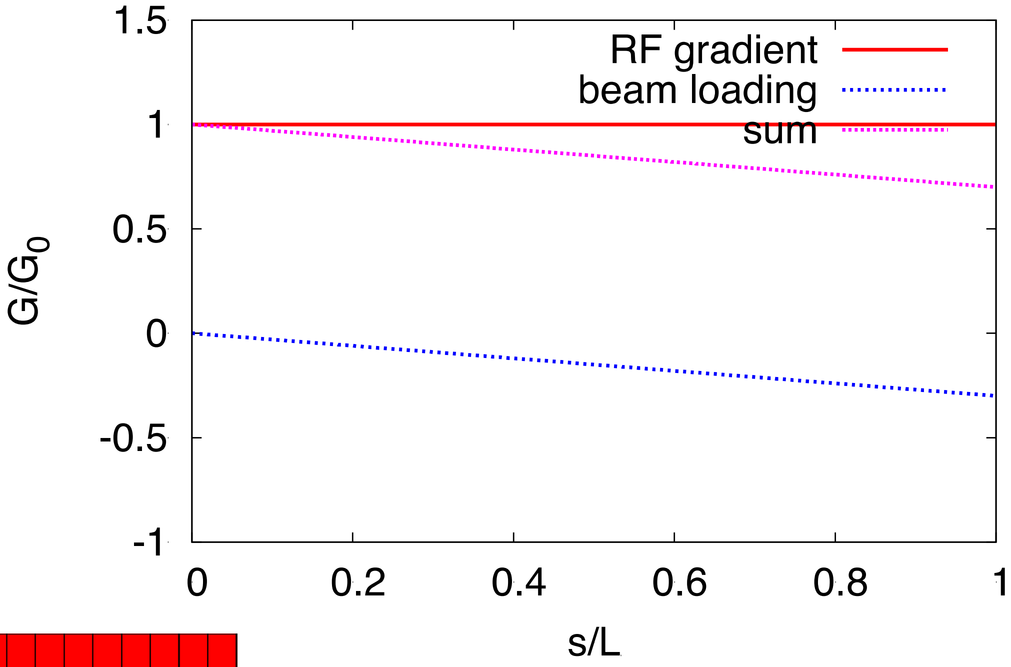
Note: Field should also vary with position, but that exceeds my graphic competences

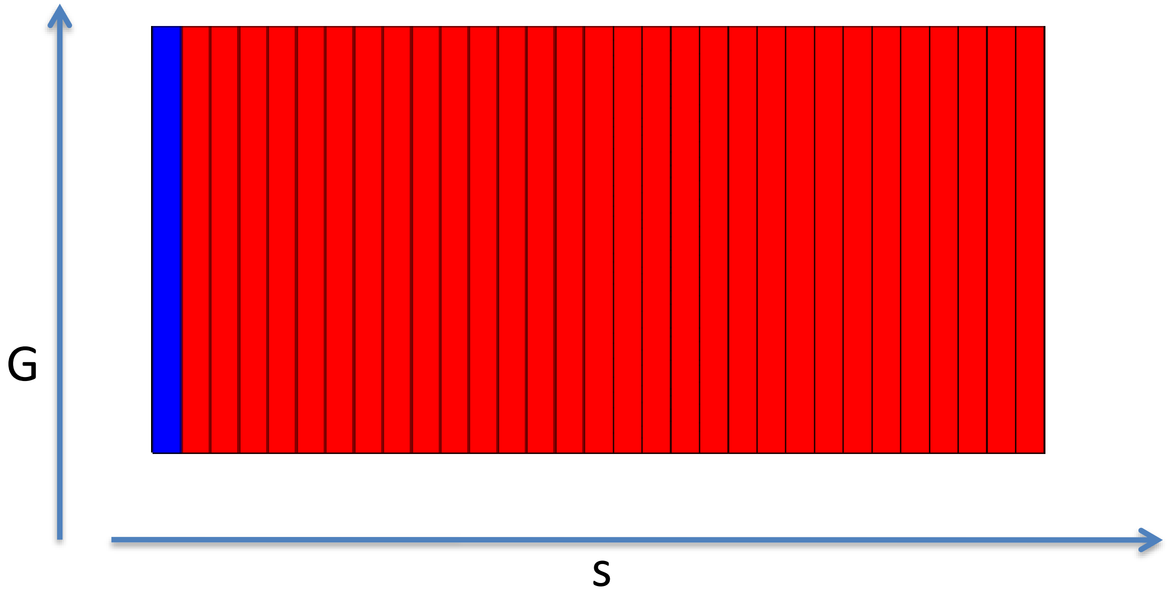
Filling a Travelling Wave Structure

The RF energy is flowing along the structure

Some is given to the beam, some is lost in the wall

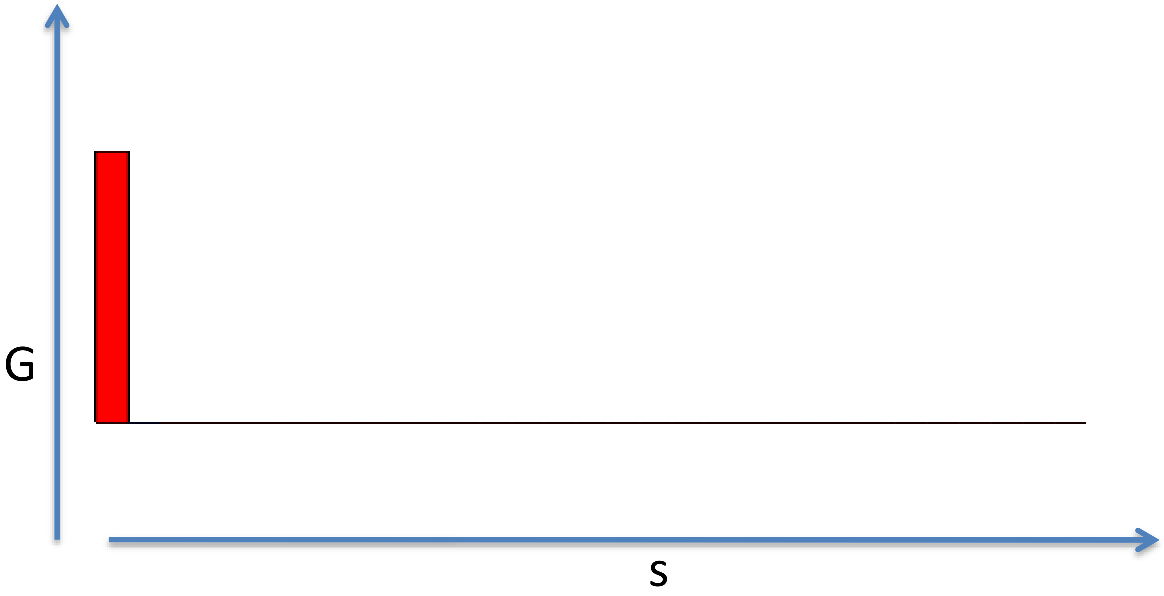
Gradient profile developes





Some power is leaking out at the end

In CLIC filling time is O(80 ns) and beam time (O 160 ns)
Slightly different for different structures



Choice of Material

- The material is the most fundamental design choice

- Super-conducting structures
  - allow a small beam current
- ⇒ low background per unit time in IP
- ⇒ intra-pulse feedback is possible everywhere

- Normal conducting structures
  - allow for high gradient
- ⇒ high centre-of-mass energy
  - need high beam current
- ⇒ significant wakefield effects
  - use short pulses
- ⇒ smaller damping ring

Efficiency

RF Power to Beam Efficiency

$$\mathcal{L} \propto H_D \, n_\gamma \, \eta_{RF->beam} \frac{P_{RF}}{E_{cm}} \frac{1}{\sigma_y}$$

$$\eta_{RF\rightarrow beam} = \frac{\text{Energy taken by one beam pulse}}{\text{Energy in each RF pulse}}$$

For constant RF pulse power

Power into beam
During beam passage

$$\eta_{RF\rightarrow beam} = \frac{\tau_{beam}}{\tau_{RF}} \cdot \frac{P_{beam}}{P_{RF}}$$

RF power during pulse

Simplified

$$\eta_{RF\rightarrow beam} = \frac{\tau_{beam}}{\tau_{beam} + \tau_{fill}} \cdot \frac{P_{beam}}{P_{beam} + P_{loss} + P_{out}}$$

Note: what I call τ_{fill} contains several components of which the fill time is the most important; RF experts will learn more

RF to Beam Power Efficiency

$$\eta_{RF\rightarrow beam} = \frac{\tau_{beam}}{\tau_{beam}+\tau_{fill}} \cdot \frac{P_{beam}}{P_{beam}+P_{loss}+P_{out}}$$

- In a super-conducting cavity
  - \- little RF power is lost in the walls during the pulse
  - \- but the cooling requires some significant overhead
  - \- some cooling is also needed against heating from the environnement

$$\eta_{RF\rightarrow beam} = \frac{\tau_{beam}}{\tau_{beam}+\tau_{fill}}$$

- In normal conducting structures
  - \- A significant fraction of the RF power is lost into the walls
  - \- some power will be draining out of the travelling wave structure (usually)

$$\eta_{RF\rightarrow beam} = \frac{\tau_{beam}}{\tau_{beam}+\tau_{fill}} \cdot \frac{P_{beam}}{P_{beam}+P_{loss}+P_{out}}$$

Impedance

Energy in the cavity or accelerating cell

Voltage along cavity or cell

$$U = \frac{(GL)^2}{(R/Q)\omega}$$

Impedance, depends on shape of cavity/cell, does not depend on frequency

RF frequency

High R/Q means high wakefields
A bunch extracts the same amount of energy for higher R/Q
But field in cavity must change more since less energy is stored

Important example: smaller apertures yield higher R/Q

Impedance

Energy in the cavity or accelerating cell

Voltage along cavity or cell

$$U = \frac{(GL)^2}{(R/Q)\omega}$$

Impedance, depends on shape of cavity/cell, does not depend on frequency

RF frequency

High R/Q mea
A bunch extra /Q
But field in ca tored

Important exa

Warning

This definition is in “Linac Ohms”

People also use “Circuit Ohms”

2 “Linac Ohms” = 1 “Circuit Ohm”

Power Lost in the Structure

Voltage

Power loss

$$P_{loss} = \frac{U}{Q}\omega = \frac{(GL)^2}{R/Q}\frac{1}{Q}$$

Cavity design

Cavity material

$$Q = \frac{\text{Stored energy}}{\text{Ohmic energy loss per radian of RF cycle}} = \frac{U}{P_{loss}}\omega$$

Examples: $Q=O(10^{10})$ for superconducting and $O(10^4)$ for normal conducting

But frequency dependent

Examples

parameter	CLIC	ILC (RDR)
R'/Q	$\approx 11\,\mathrm{k\Omega/m}$	$1.036\,\mathrm{k\Omega/m}$
Q	≈ 6000	$\approx 10^{10}$
R'	$\approx 66\,\mathrm{M\Omega/m}$	$\approx 10^{7}\,\mathrm{M\Omega/m}$

- ILC: $I \approx 5.8\,\mathrm{mA}$

$\Rightarrow$

$$\frac{P'_{beam}}{P'_{wall}} \approx 1650$$

- CLIC: $I \approx 1.2\,\mathrm{A}$

$\Rightarrow$

$$\frac{P'_{beam}}{P'_{wall}} \approx 0.8$$

- Efficiency is

$$\eta = \frac{\tau_{beam}}{\tau_{beam} + \tau_{fill}} \frac{P_{beam}}{P_{beam} + P_{loss} + P_{out}}$$

- Plugging in numbers for ILC

$$\eta \approx \frac{730\,\mu\mathrm{s}}{730\,\mu\mathrm{s} + 900\,\mu\mathrm{s}} \approx 0.45$$

- Plugging in (slightly older) numbers for CLIC

$$\eta = \frac{156\,\mathrm{ns}}{156\,\mathrm{ns} + 83\,\mathrm{ns}} \cdot \frac{27\,\mathrm{MW}}{27\,\mathrm{MW} + 25\,\mathrm{MW} + 12\,\mathrm{MW}} \approx 0.65 \cdot 0.42 \approx 0.277$$

Cryogenics Power (ILC)

Cavities have small losses

$$P_{loss} = const \frac{1}{Q_0} \times G^2$$

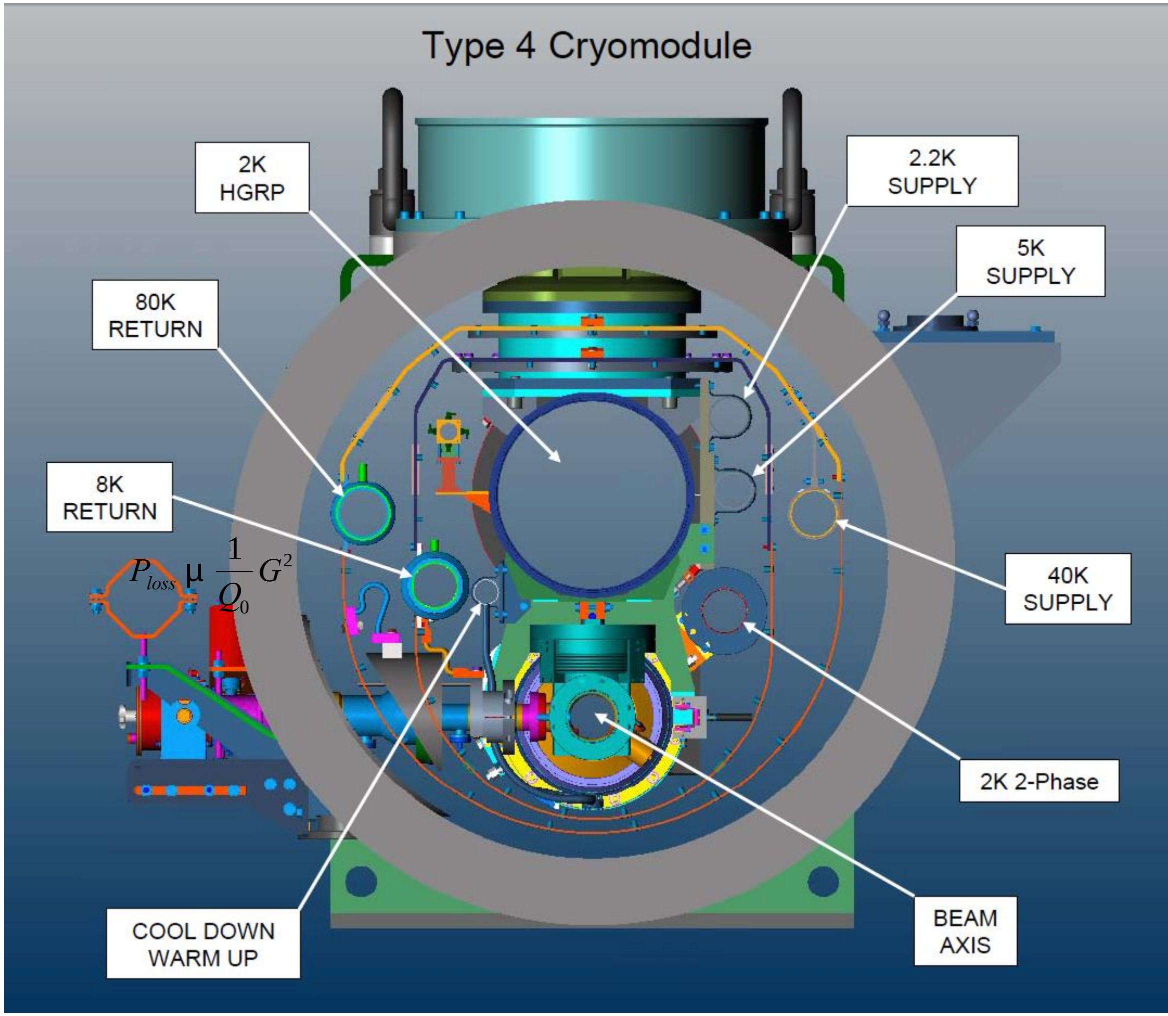
About 1W/m

But cooling costly at low temperatures

Remember Carnot:

$$P_{cryo} = \frac{1}{\eta} \frac{T_{room} - T_{source}}{T_{source}} \times P_{loss}$$

$$P_{cryo} \approx 700 \times P_{loss}$$



$$P_{loss} \propto \frac{1}{Q_0} G^2$$

The typical heat load of 1 W/m
⇒ about 1 kW/m for cryogenics

Average RF power: 1.6kW/m (3kW/m)
Power into beam about 0.7kW/m

ILC Main Linac Pulse Optimisation

Disclaimer: there have been heated discussions on how that should be optimised…

More RF peak power

- Higher beam current
- Shorter cavity fill time
- Higher cost (klystrons and modulators)
- Either higher bunch charge or more bunches

Longer RF pulses

- Higher average beam current
- Cavity fill time is smaller fraction of pulse
- Higher cost (modulators and klystrons)
- More cooling required
- Either higher bunch charge or more bunches

Higher pulse rate

- Higher average beam current
- Higher cost (modulators and klystrons)
- More cooling required
- Faster damping in damping rings

Higher bunch charge makes beam dynamics more challenging
More bunches makes damping ring more challenging

CLIC Main Linac Pulse Optimisation

Power to beam $P'_{beam} = IG$

Power lost in structure $P'_{loss} = \frac{G^2}{R'}$

Maximise

$$\frac{P'_{beam}}{P'_{loss}} = R' \frac{I}{G}$$

Maximise current as R' allows

Maximise bunch charge
Minimise distance between bunches
Got to the limit!

R' is impacts beam stability

Low gradient make machine expensive

$$\mathcal{L} \propto H_D \, n_\gamma \, \eta_{RF->beam} \frac{P_{RF}}{E_{cm}} \frac{1}{\sigma_y}$$

Beam Quality

Note: CLIC Optimisation

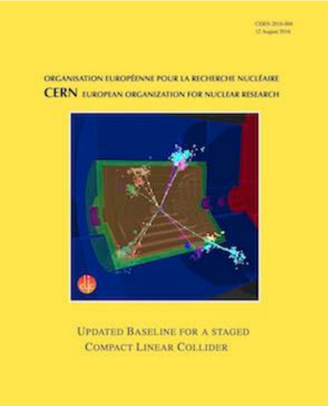
Scan 1.7 billion cases:

Fix structure design parameters: $a_1, a_2, d_1, d_2, N_c, \phi, G$

Determine main linac beam parameters

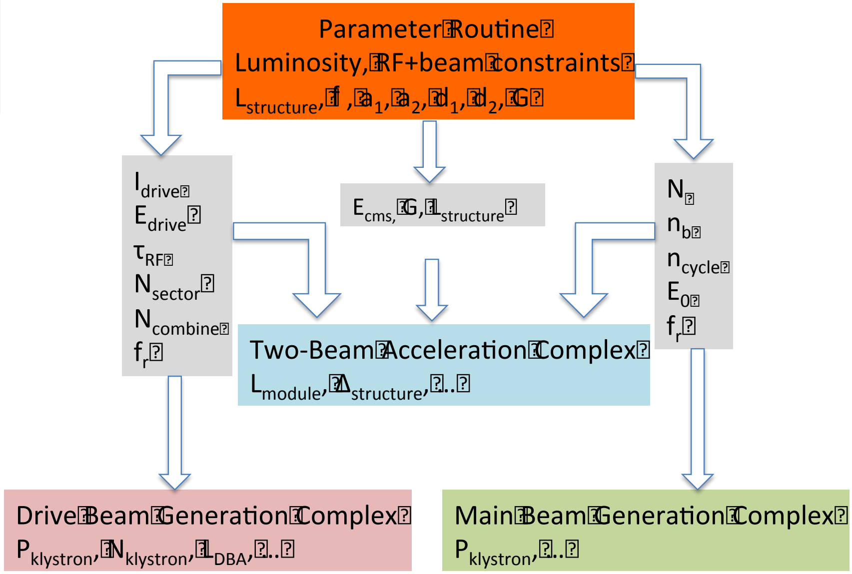
Calculate luminosity (including performance of other systems)

Calculate cost and power


ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

UPDATED BASELINE FOR A STAGED COMPACT LINEAR COLLIDER

CERN-2016-004


Parameter Routine
Luminosity, RF+beam constraints
$L_{structure}$, f, a_1, a_2, d_1, d_2, G

I_{drive}
E_{drive}
τ_{RF}
N_{sector}
$N_{combine}$
f_r

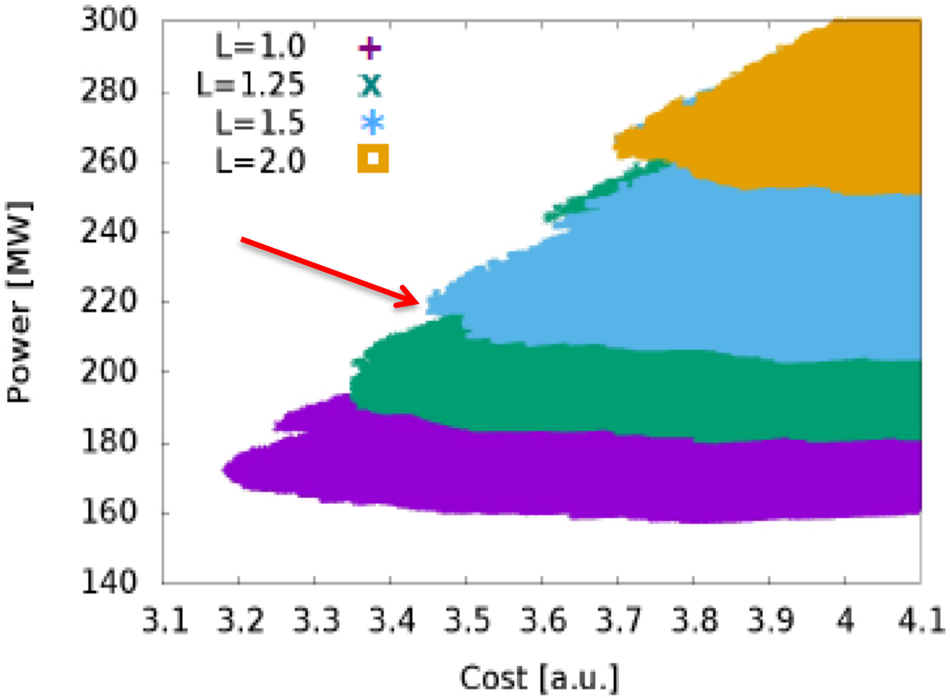
E_{cms}, G, $L_{structure}$

N
n_b
n_{cycle}
E_0
f_r

Two-Beam Acceleration Complex
L_{module}, $\Delta_{structure}$, ...

Drive Beam Generation Complex
$P_{klystron}$, $N_{klystron}$, L_{DBA}, ...

Main Beam Generation Complex
$P_{klystron}$, ...



Note: Coordinate System

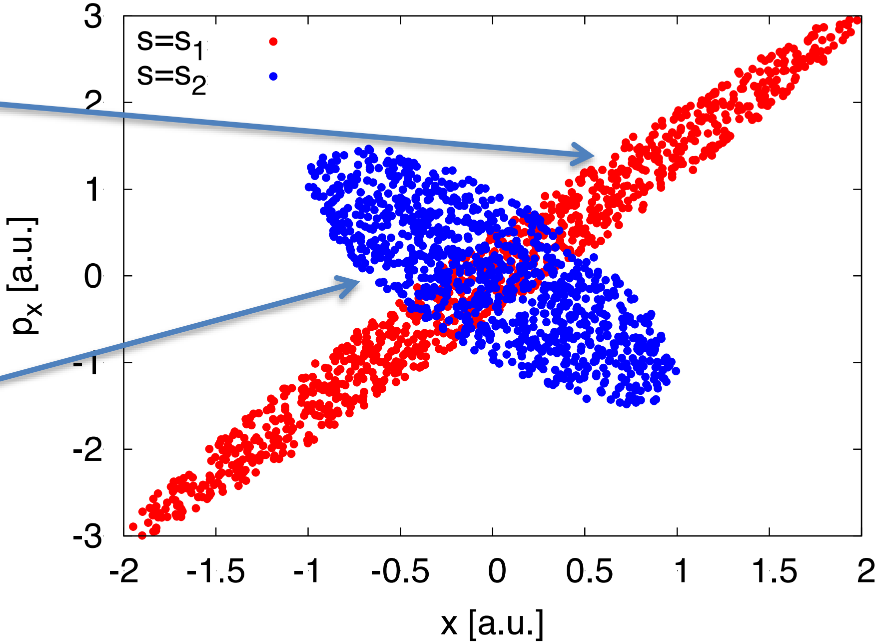
- We use two frames, the laboratory frame and the beam frame
- The nominal direction of motion of the beam is called *s* in the laboratory frame, the beam moves toward increasing *s*
- The longitudinal direction is called *z* in the beam frame, with particles at smaller *z* moving ahead of particles with larger *z*
- A particle preserves its longitudinal position within the beam
- The transverse dimensions are *x* in the horizontal and *y* in the vertical plane, in both coordinate systems
- People use different systems so find out what they talk about

Emittance

The beam particles have different coordinates; they occupy some phase space

Liouville theorem (from the Liouville equation): the density in phase space around a trajectory remains constant in an unperturbed system, i.e. “the phase space is preserved”

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \sum_{i=1}^{N}\left[\frac{\partial\rho}{\partial q_i}\dot{q}_i + \frac{\partial\rho}{\partial p_i}\dot{p}_i\right] = 0$$



Particle coordinates at one location

Particle coordinates at other location

Area does not change

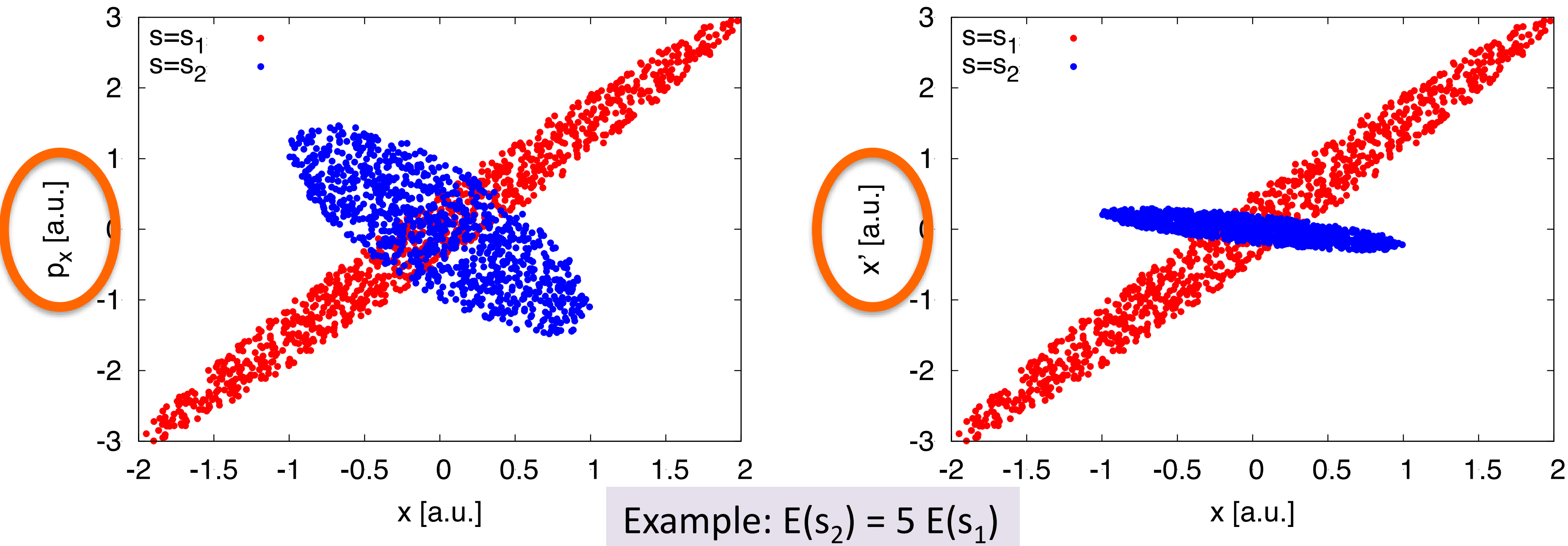
Emittance and Acceleration

When accelerating the phase space remains constant in canonical coordinates

$$(x, y, z, p_x, p_y, p_z)$$

But with the definition used in accelerators emittance shrinks

$$(x, y, z, x', y', E)$$



Example: $E(s_2) = 5\ E(s_1)$

To avoid this linac experts use **normalised emittance** $\varepsilon_N = \gamma\varepsilon$ that does not change
I will always do that here but not use the index N

Emittance Definition

Use projected emittance

Offset

If the beam jitters luminosity is lost

If the beam emittance grows due to decoherence, the luminosity loss remains the same (on average)

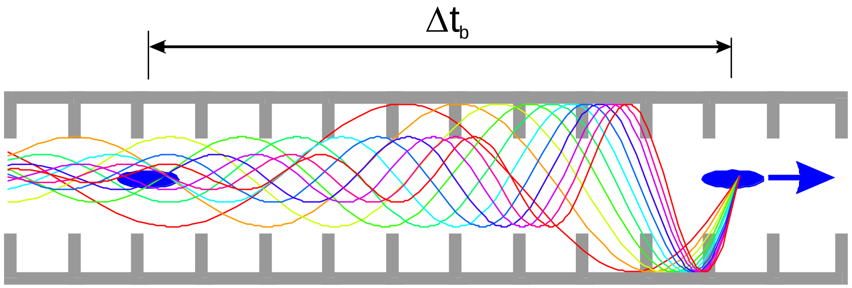
Decoherence/emittance growth means:

This luminosity loss cannot be corrected downstream

Single Bunch Energy Spread

Wakefields

Particle leaves fields behind that affect subsequent particles: “Wakefields”



Use wakefields to describe the effect of first particle on second one, here relevant are

$$P_z c = N e W_L(z) L e$$

$$P_x c = N e W_\perp(z) L e \Delta x$$

$$P_y c = N e W_\perp(z) L e \Delta y$$

Charge of driving particle

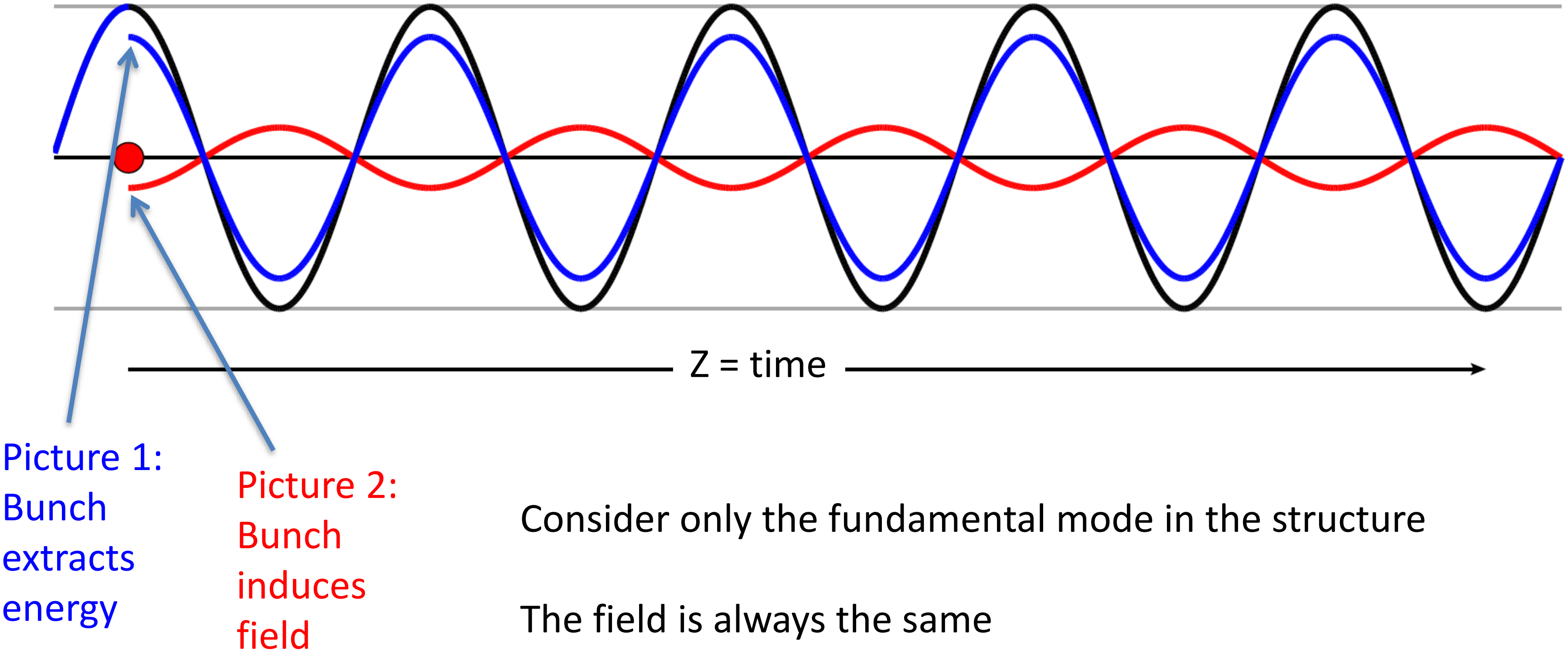
Longitudinal/transverse wakefield

Structure length

Witness particle charge

Transverse offset

Longitudinal Wakefield and Energy



Picture 1: Bunch extracts energy

Picture 2: Bunch induces field

Consider only the fundamental mode in the structure

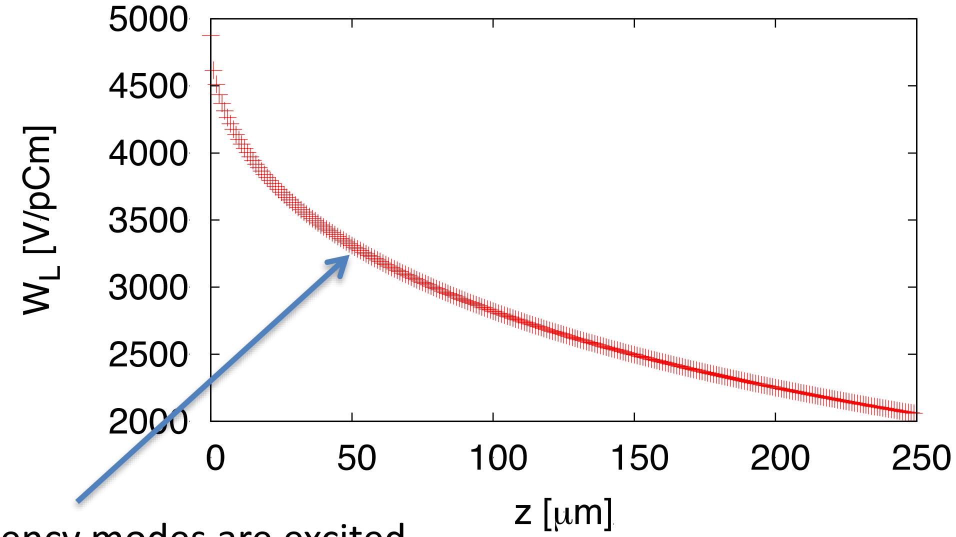
The field is always the same
The extracted energy is automatically correct

If a bunch extracts a large fraction of the energy in the structure the tail will gain much less energy

Longitudinal Wakefield (CLIC)

The particle does not change z-position

⇒ Sees the same wakefield in each accelerating structure



Many high-frequency modes are excited
They add because they are cosine-like
But they decohere rapidly

- We will use wakefields based on fits derived by Karl Bane

  l length of the cell

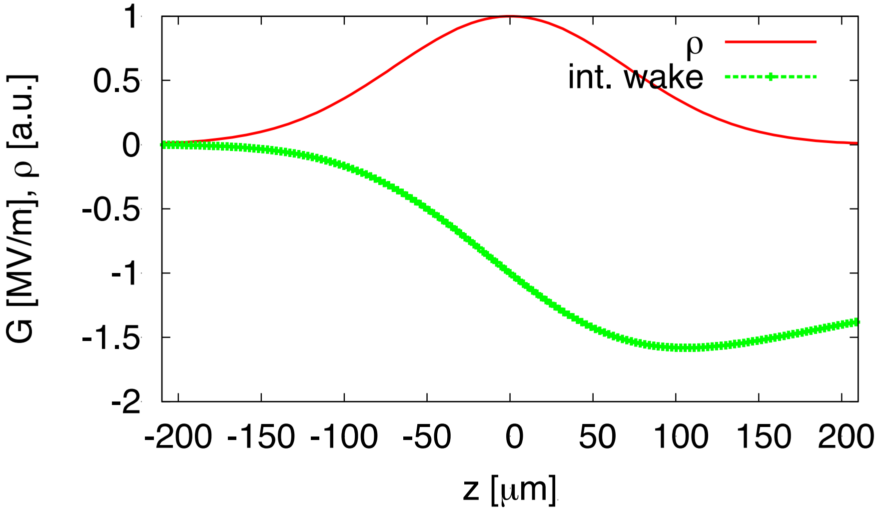
  a radius of the iris aperture

  g length between irises

$$z_0 = 0.41 a^{1.8} g^{1.6} \left(\frac{1}{l}\right)^{2.4}$$

$$W_L(z) = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{\frac{z}{z_0}}\right)$$

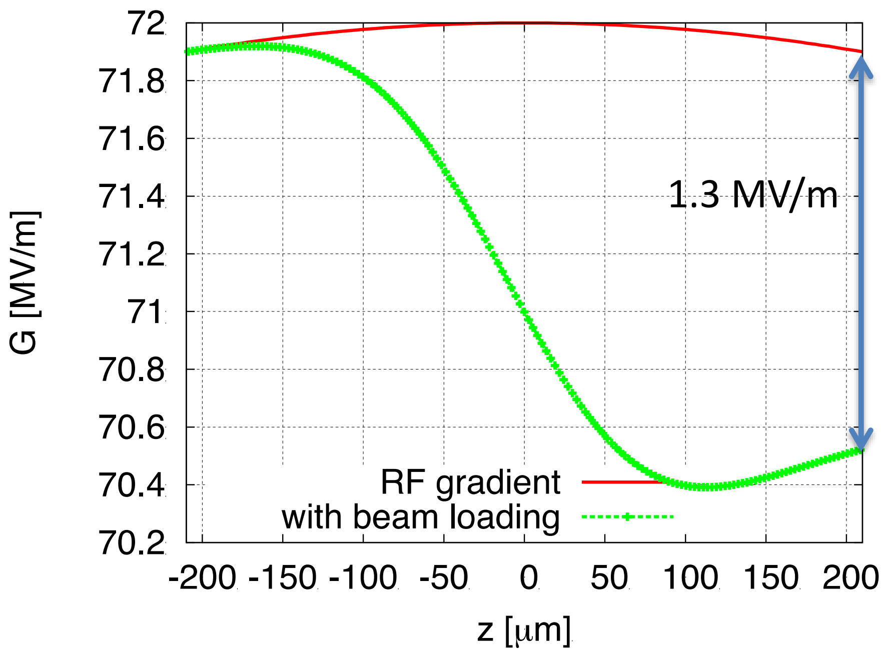
- Use CLIC structure parameters

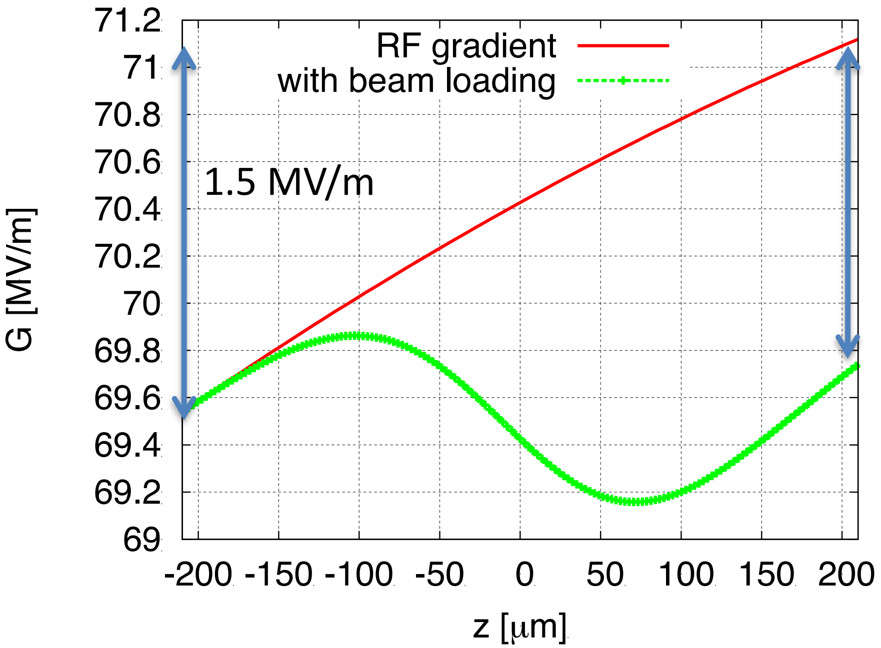


Longitudinal Wakefields and Energy Spread

Loaded gradient seen by a single bunch for on-crest acceleration:

- more than 2% full gradient spread
- 0.7% RMS energy spread





1.3 MV/m

Loaded gradient seen by a single bunch for off-crest acceleration (12°):

- 1% full gradient spread
- 0.35% RMS gradient spread
- Loose about 2% in gradient

Note: Energy Spread along Linac

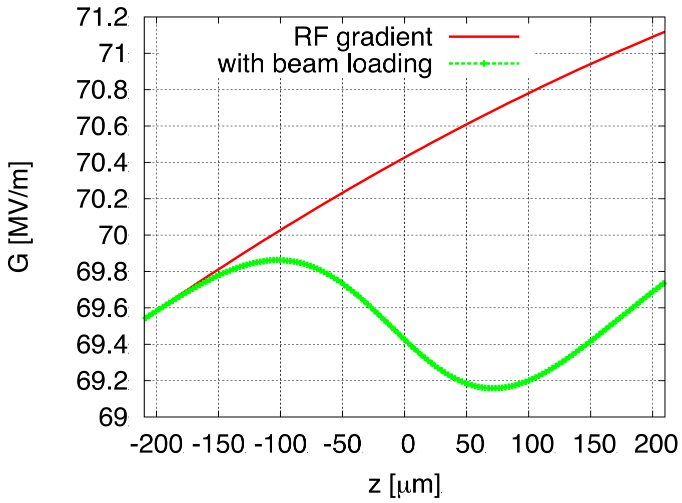
Can chose different RF phases along the linac

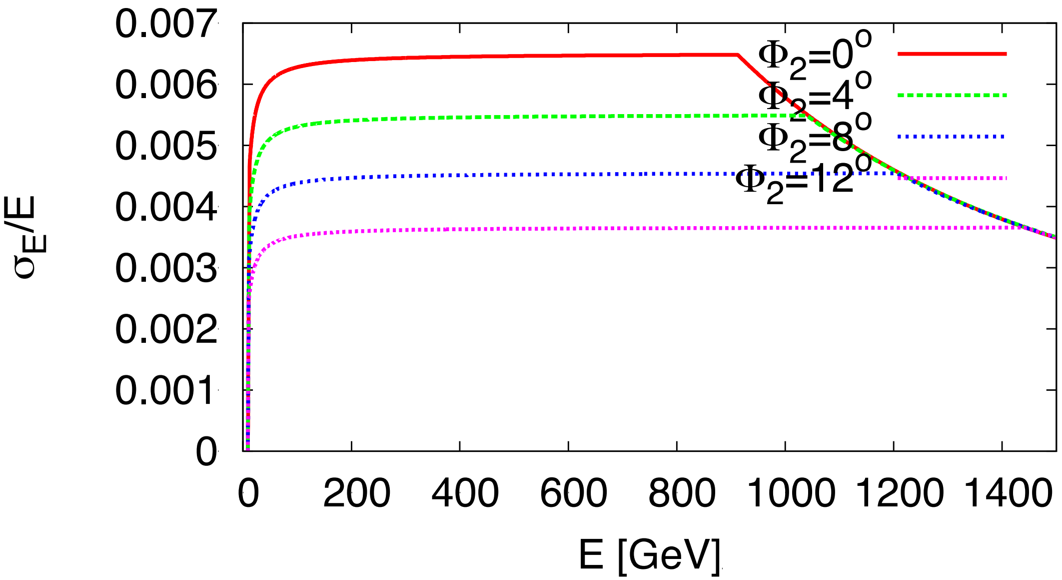
- Small phase in the main part
- and 30° at the end
- To have **average** phase of 12°

Allows to have larger energy spread in the main part of the linac than at the end

This can help beam stability

⇒ See next section

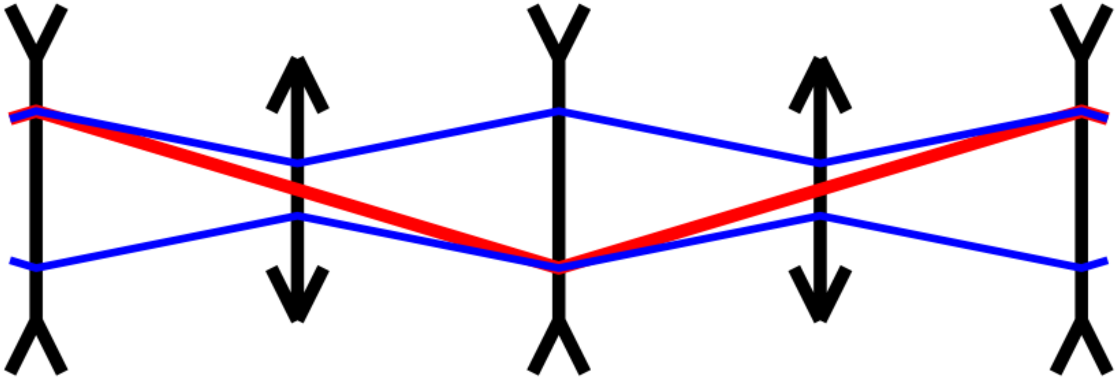




Single-bunch Stability

Lattice Concept

FODO lattice is used in main linacs



Guiding quadrupoles act like a spring

Particle is comparable to harmonic oscillator (driven with wakes)

$$x_1''(s) + \frac{1}{\beta^2} x_1(s) = 0$$

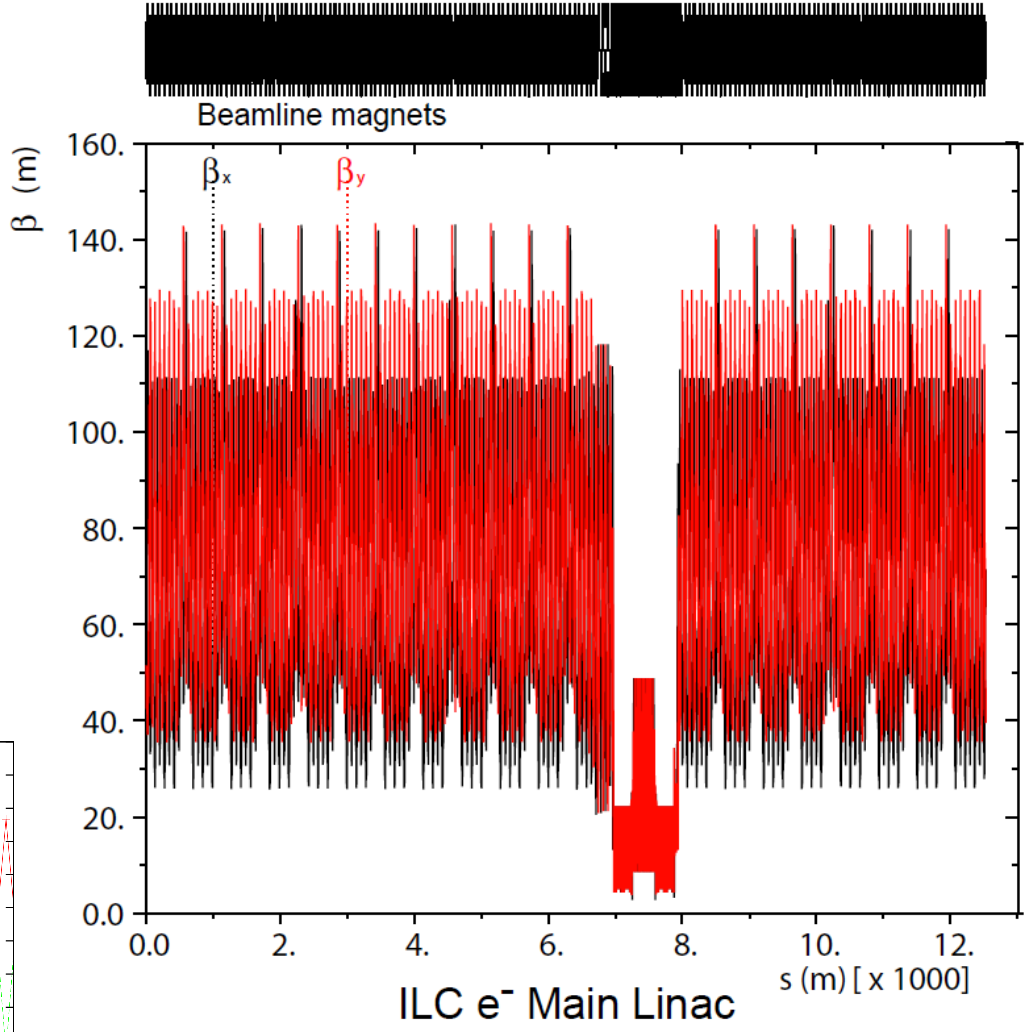
Local wavelength is β
Strong focusing means smaller β

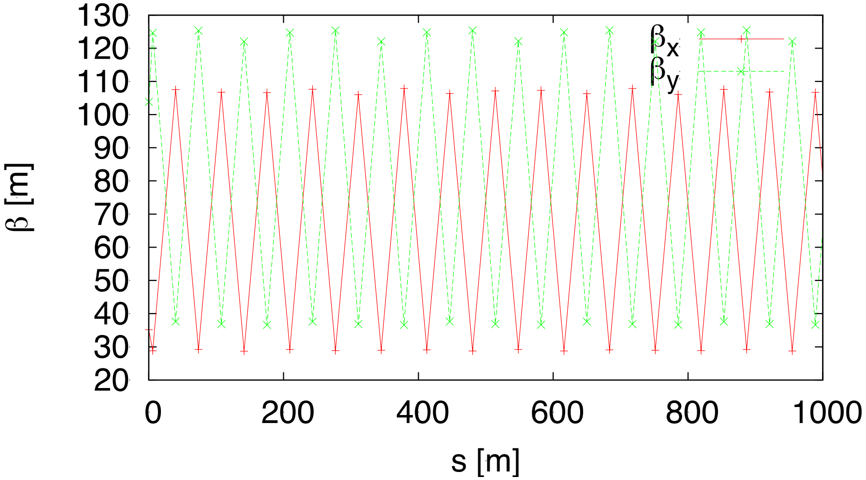
A function of longitudinal position s
But equivalent to time dependence t

ILC Lattice Design

Constant quadrupole spacing
Constant phase advance

Different phase advance in horizontal and vertical to decouple planes against wakefield effects

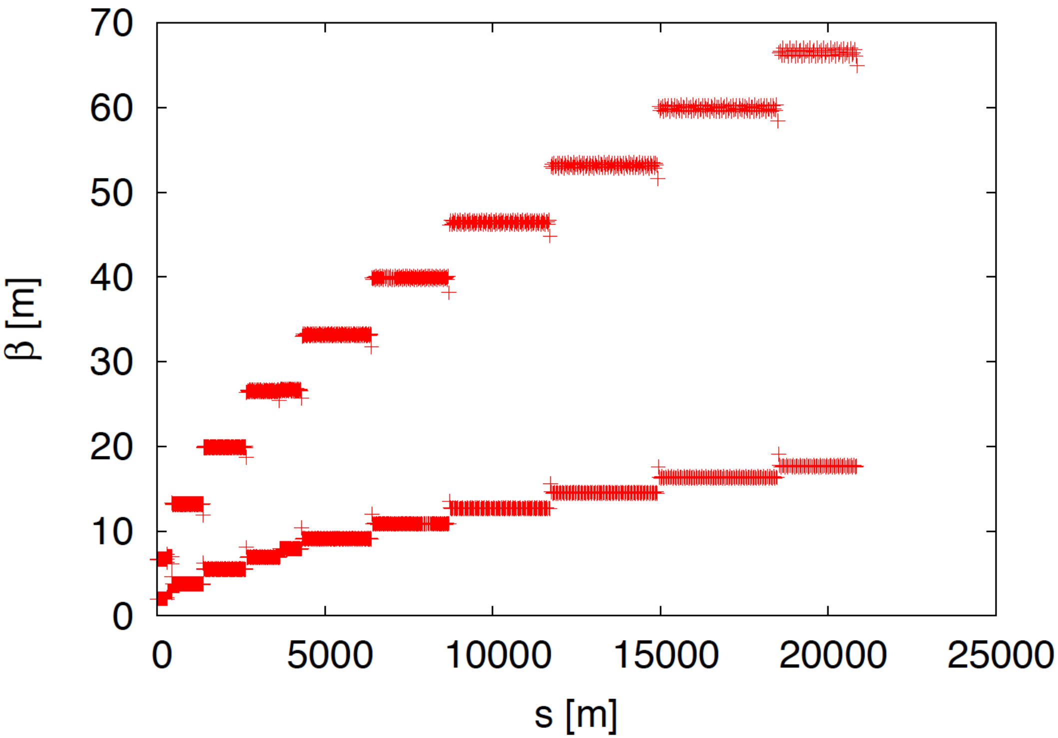




CLIC Lattice Design (3 TeV)

- Use strong focusing (small ®) to stabilise beam
  - 10% of linac are quadrupoles
- Used ® $\propto E^{1/2}$, □√ = const
  - Quadrupole spacing and length scale as $E^{1/2}$

  ⇒ roughly constant fill factor
  - phase advance is chosen to balance between wakefield and ground motion effects
- Total length 20867.6m
  - fill factor 78.6%



- 12 different sectors used
- Matching between sectors using 7 quadrupoles to allow for some energy bandwidth

Note: fill factor = active length/total length

Passage Through the Linac

For simplicity consider constant beta-function
Replacing FODO lattice with permanent focusing
Great approximation to understand physics

$$x_1''(s) + \frac{1}{\beta^2} x_1(s) = 0$$

$$x_1(0) = x_0 \quad x_1'(0) = 0$$

Solution is well-known

$$x_1(s) = x_0 \cos\left(\frac{s}{\beta}\right)$$

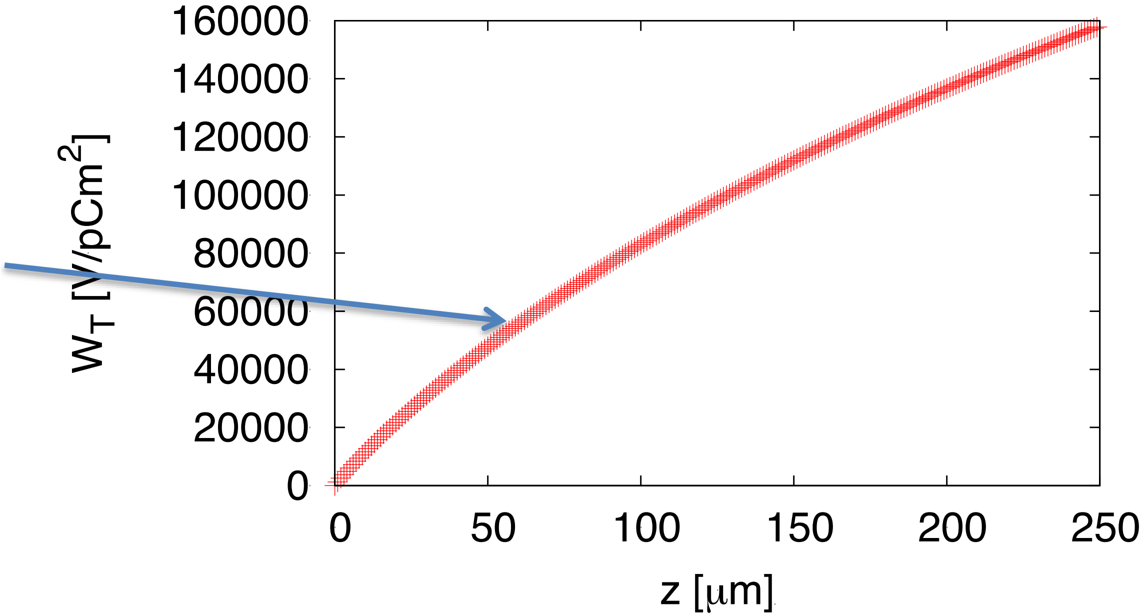
Transverse Wakefields

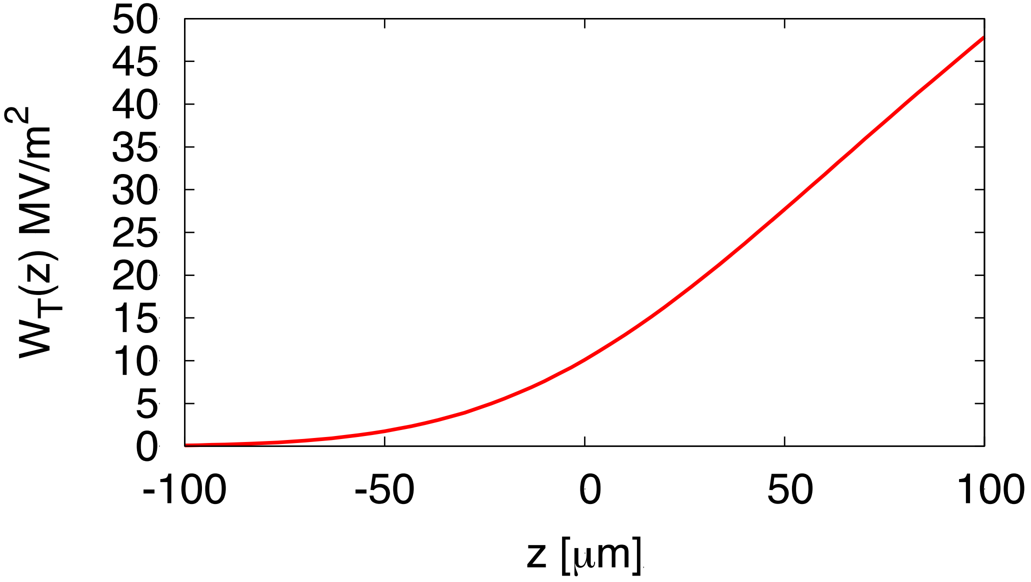
For short distances the wake-field rises linear

Summation of an infinite number of sine-like modes with different frequencies

Coherent offset of bunch (worst case)

The tail is deflected to the outside





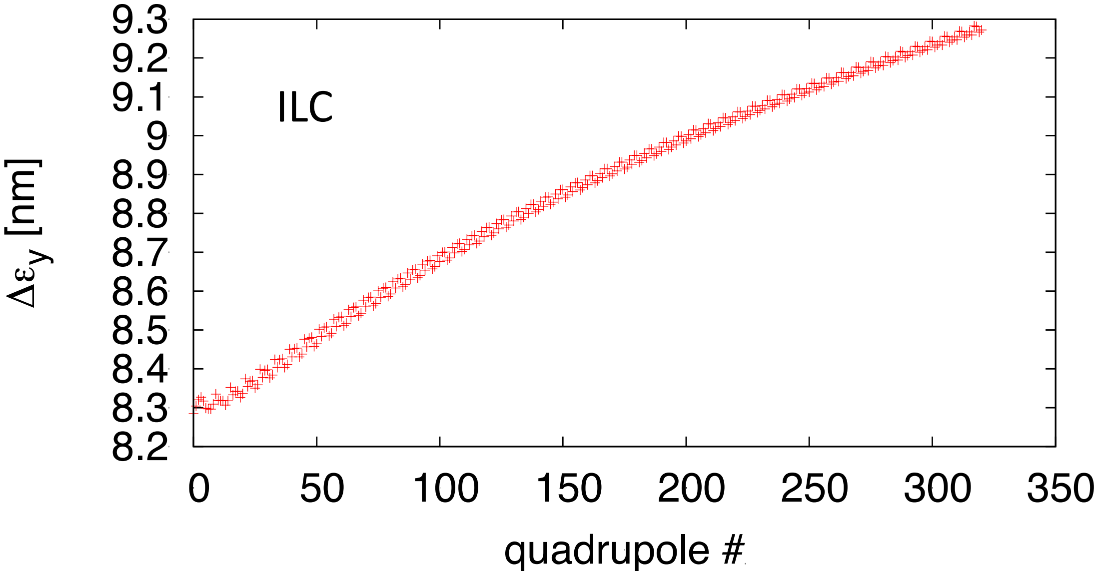
Emittance in Linac

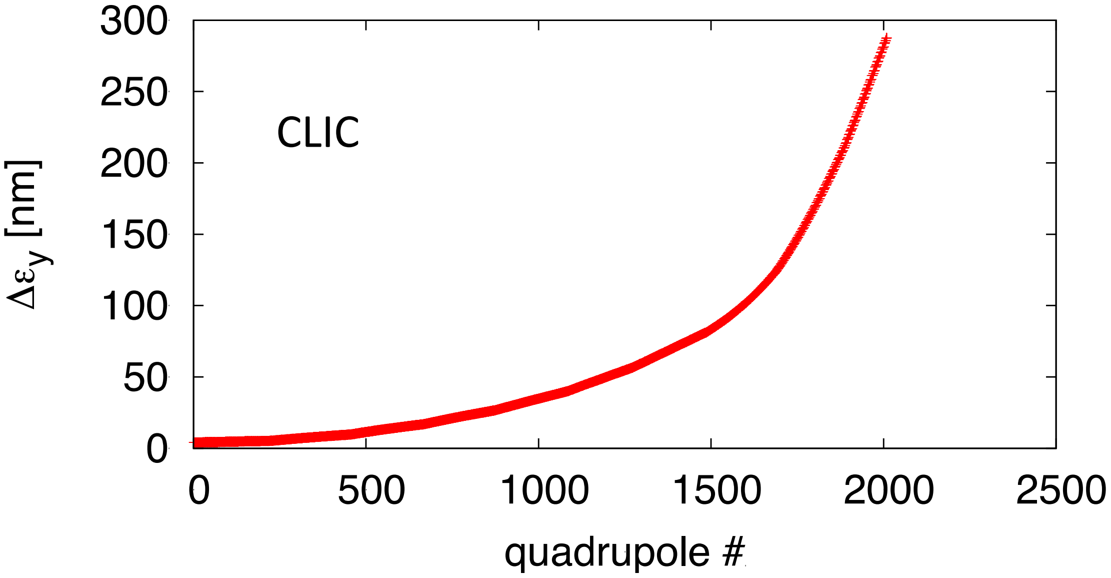
Transverse stability of beam with initial offset σ_y

- No energy spread
- Emittance with respect to beam axis shown

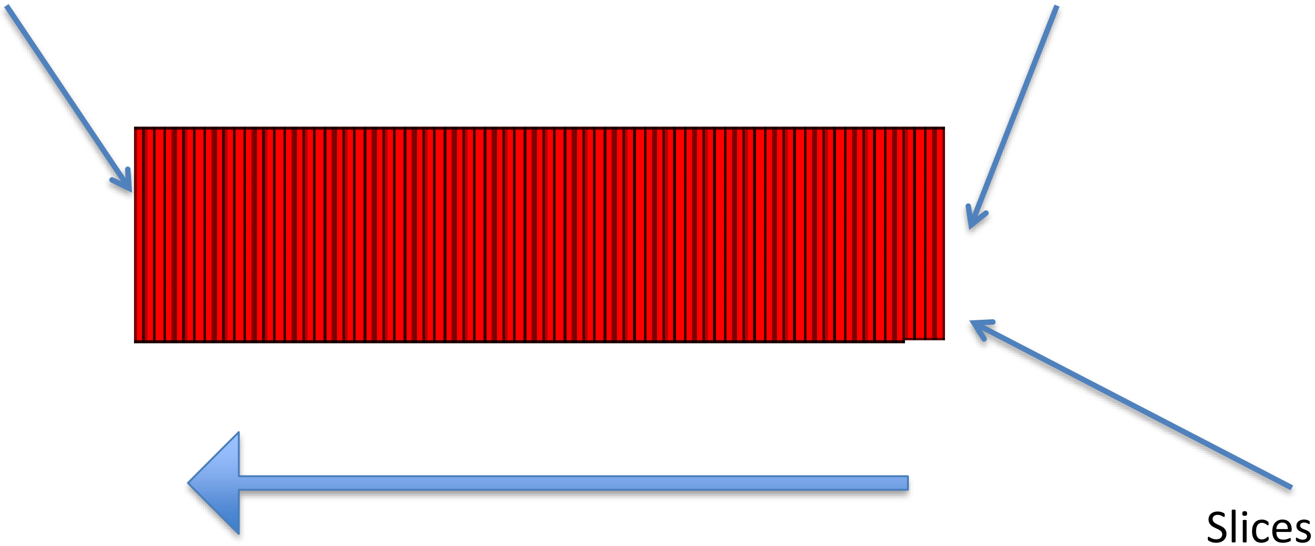
$\Rightarrow$ Acceptable for ILC

$\Rightarrow$ Not acceptable for CLIC





Bunch Transverse Motion (CLIC)



Head performs simple betatron oscillation

Tail starts to flap around

Direction of motion

Slices will shrink due to energy increase

Wakefield Model

Assume bunch can be represented by two particles and constant *K*(*s*) = 1/*β*²

- Second particle is kicked by transverse wakefield

$$x_2''(s)+\frac{1}{\beta^2}x_2(s)=\frac{Ne^2W_\perp(\Delta z)}{\boxed{P_Lc}}x_1(s)$$

$$=E$$

$$x_2(0)=x_0 \quad x_2'(0)=0$$

$$x_1(s)=x_0\cos\left(\frac{s}{\beta}\right)$$

Solution is simple with an ansatz

$$x_2(s)=x_0\cos\left(\frac{s}{\beta}\right)+x_0\left(Ne^2W_\perp(\Delta z)\right)\left(\frac{\beta s}{2E}\right)\sin\left(\frac{s}{\beta}\right)$$

⇒ Amplitude of second particle oscillation is growing linearly with *s*

Discussion

$$x_2(s) = x_0 \cos\left(\frac{s}{\beta}\right) + x_0 \left(Ne^2 W_\perp(\Delta z)\right) \left(\frac{\beta s}{2E}\right) \sin\left(\frac{s}{\beta}\right)$$

With proper calculation one finds $$\frac{\beta s}{E} \Rightarrow \int \frac{\beta(s)}{E(s)} ds$$

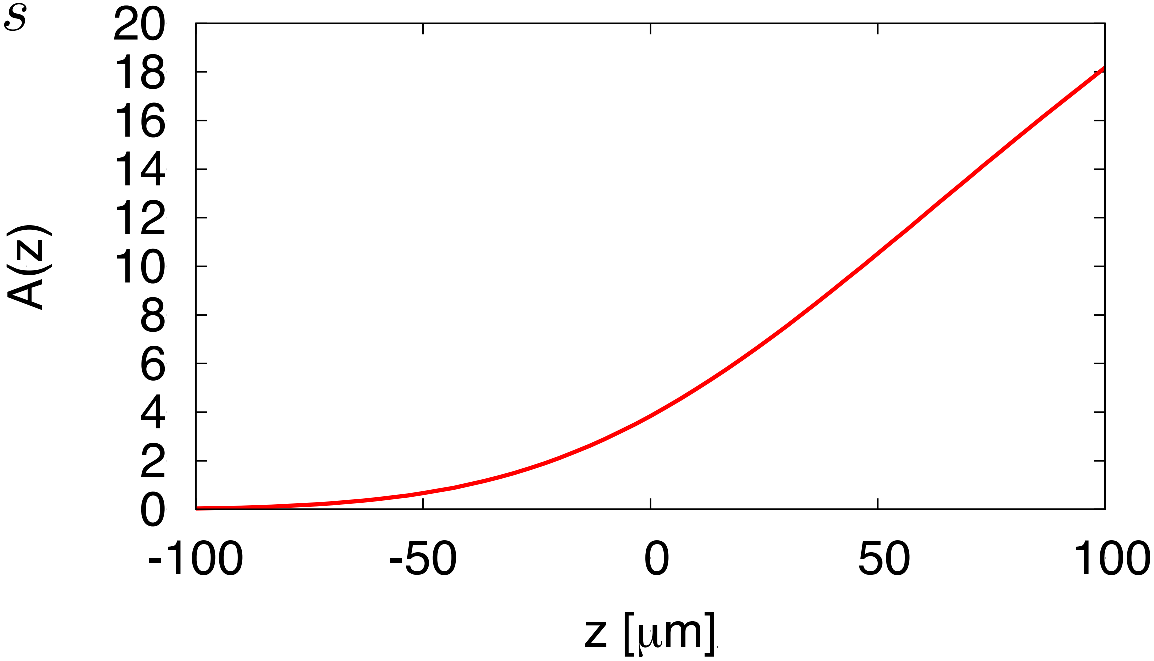
- Factors for the amplitude growth of the second particle
  - β: small beta-function (strong focusing) helps
  - 1/*E*: high energy helps
  - $W_\perp$: small wakefield helps
  - Shorter bunches
  - *N* : small bunch charge helps
  - *s*: shorter linac helps (i.e. higher gradient G)

Discussion

$$x_2(s) = x_0 \cos\left(\frac{s}{\beta}\right) + x_0 \left(Ne^2 W_\perp(\Delta z)\right) \left(\frac{\beta s}{2E}\right) \sin\left(\frac{s}{\beta}\right)$$

$$\tilde{W}_\perp(z_0) = \int_{-\infty}^{z_0} \rho(z) W_\perp(z_0 - z) Ne dz$$

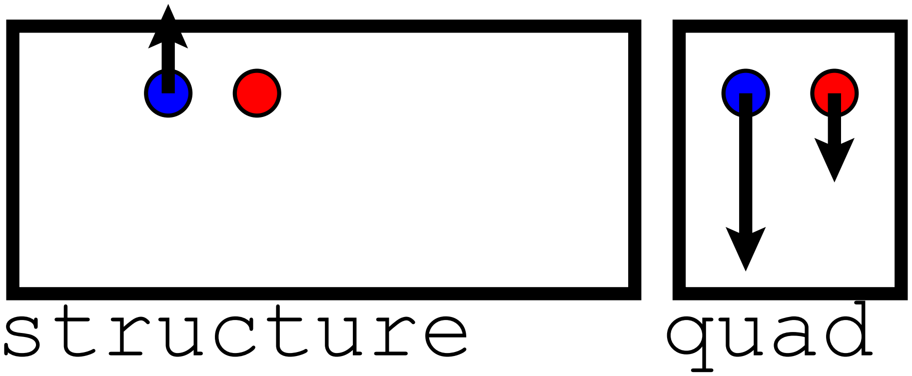
$$A(z_0) = \tilde{W}_\perp(z_0) e \int \frac{\beta(s)}{2E} ds$$



BNS Damping Concept

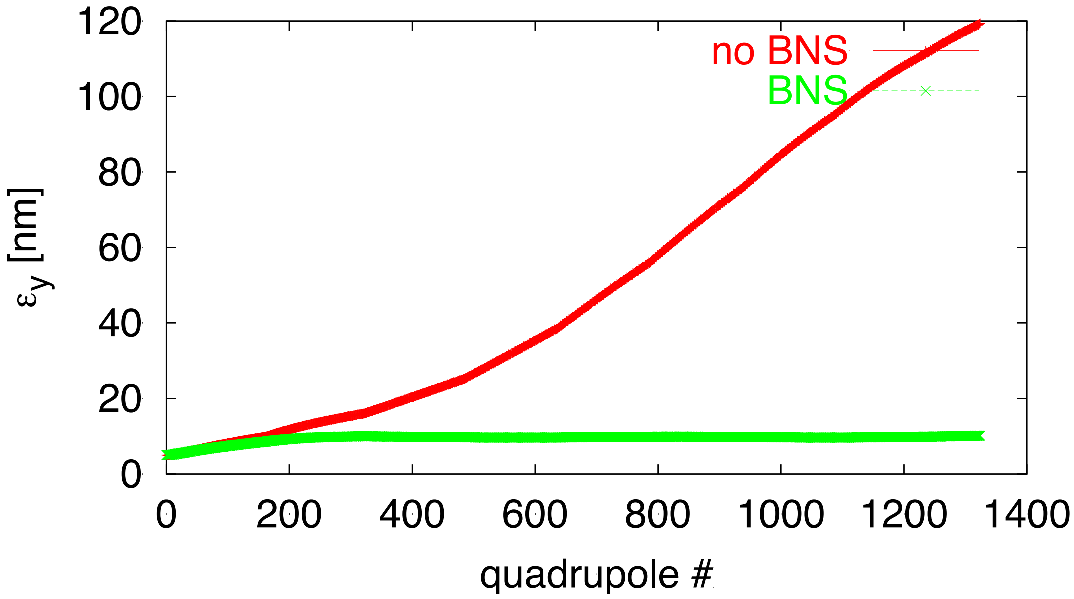
Transverse wakes act as defocusing force on tail

⇒ beam jitter is exponentially amplified



BNS damping (Balakin, Novokhatsky and Smirnov) prevents this growth

- manipulate RF phases to have energy spread
- take spread out at end



BNS Damping

For simplicity assume initial offset but no angle

First particle performs a harmonic oscillation

$$x_1(s) = x_0 \cos\left(\frac{s}{\beta}\right)$$

We **want** the second particle to perform the **same oscillation**, i.e.

$$x_2(s) = x_0 \cos\left(\frac{s}{\beta}\right)$$

Change particle energy for this purpose

$$x_2''(s) + \frac{1}{\underbrace{(1+\delta)\beta^2}}x_2(s) = \frac{Ne^2W_\perp(\Delta z)}{E}\overbrace{x_0 \cos\left(\frac{s}{\beta}\right)}^{x_1(s)}$$

Same as changing mass in
harmonic oscillator

BNS Damping

$$x_2''(s) + \frac{1}{(1+\delta)\beta^2} x_2(s) = \frac{Ne^2 W_\perp(\Delta z)}{E} x_0 \cos\left(\frac{s}{\beta}\right)$$

Plugging in our **wanted** solution for *x*$_2$(*s*)

$$x_2(s) = x_0 \cos\left(\frac{s}{\beta}\right) = x_1(s)$$

we find

$$-\frac{1}{\beta^2} x_0 \cos\left(\frac{s}{\beta}\right) + \frac{1}{(1+\delta)\beta^2} x_0 \cos\left(\frac{s}{\beta}\right) = x_0 \frac{Ne^2 W_\perp(\Delta z)}{E} \cos\left(\frac{s}{\beta}\right)$$

BNS Damping

$$-\frac{1}{\beta^2}\cancel{x_0}\cancel{\cos\left(\frac{s}{\beta}\right)}+\frac{1}{(1+\delta)\beta^2}\cancel{x_0}\cancel{\cos\left(\frac{s}{\beta}\right)}=\cancel{x_0}\frac{Ne^2W_\perp(\Delta z)}{E}\cancel{\cos\left(\frac{s}{\beta}\right)}$$

which is fulfilled for

$$\frac{1}{(1+\delta)\beta^2}-\frac{1}{\beta^2}=\frac{Ne^2W_\perp(\Delta z)}{P_Lc}$$

$$\delta\approx-\frac{\beta^2}{E}Ne^2W_\perp(\Delta z)$$

Small beta-function

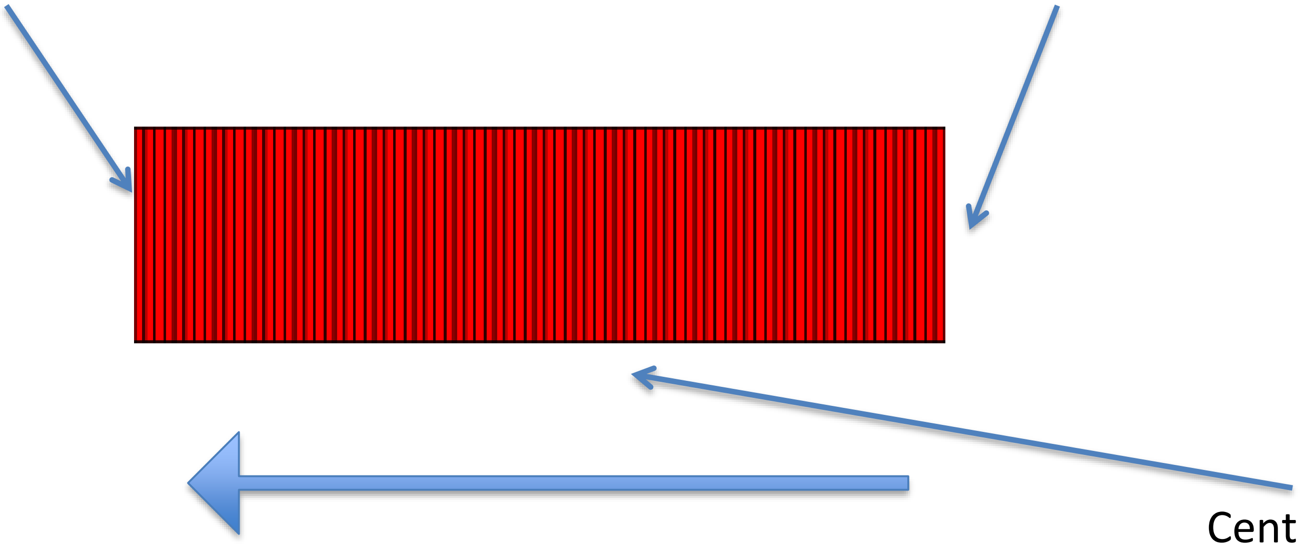
Small bunch charge

Small wakefields

CLIC choice $\beta(s)\propto\sqrt{E(s)}$

Allows $\delta=\mathrm{const}$

Bunch in Main Linac



Head still performs simple betatron oscillation

Tail still flaps a little bit

Direction of motion

Centre of bunch is much more stable

Energy Spread in the Linac

Cannot exactly match energy profile and wakefield

⇒ Shapes of energy spread and integrated wake differ

Only cure coherent offset

⇒ Slope along bunch still has an effect

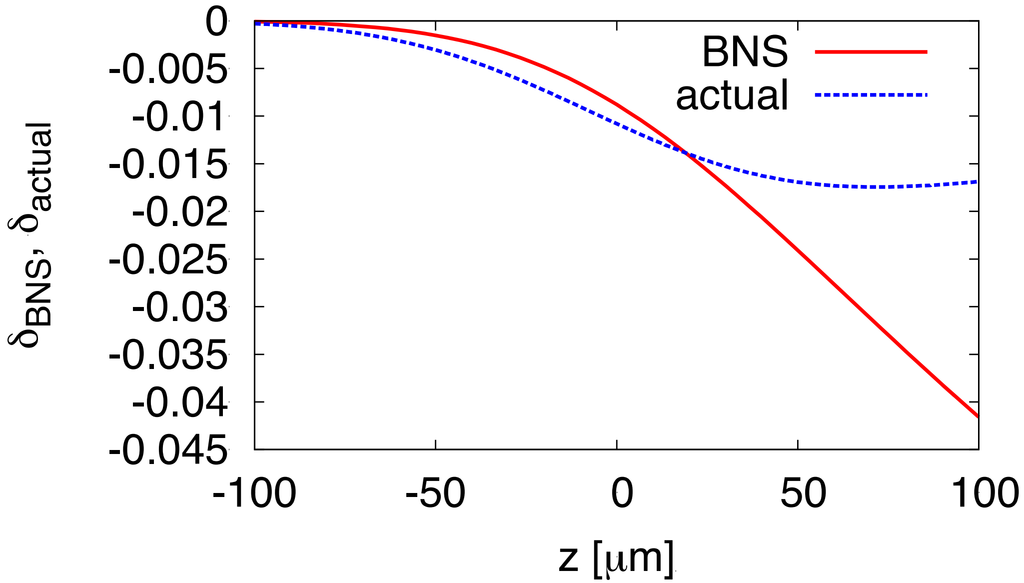
Energy spread also helps detuning

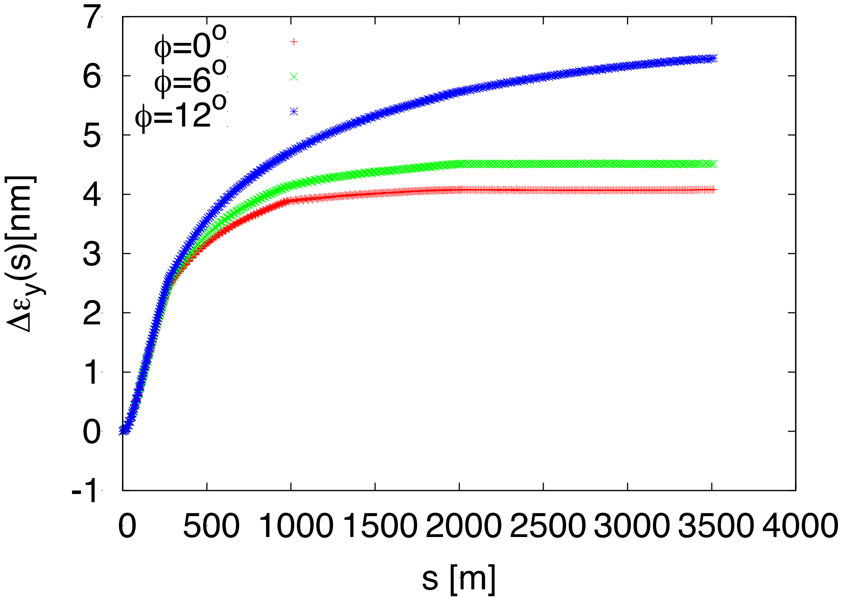
In summary

⇒ Can only obtain some correction

⇒ Broad acceptable range

⇒ Different RF phases in linac are OK

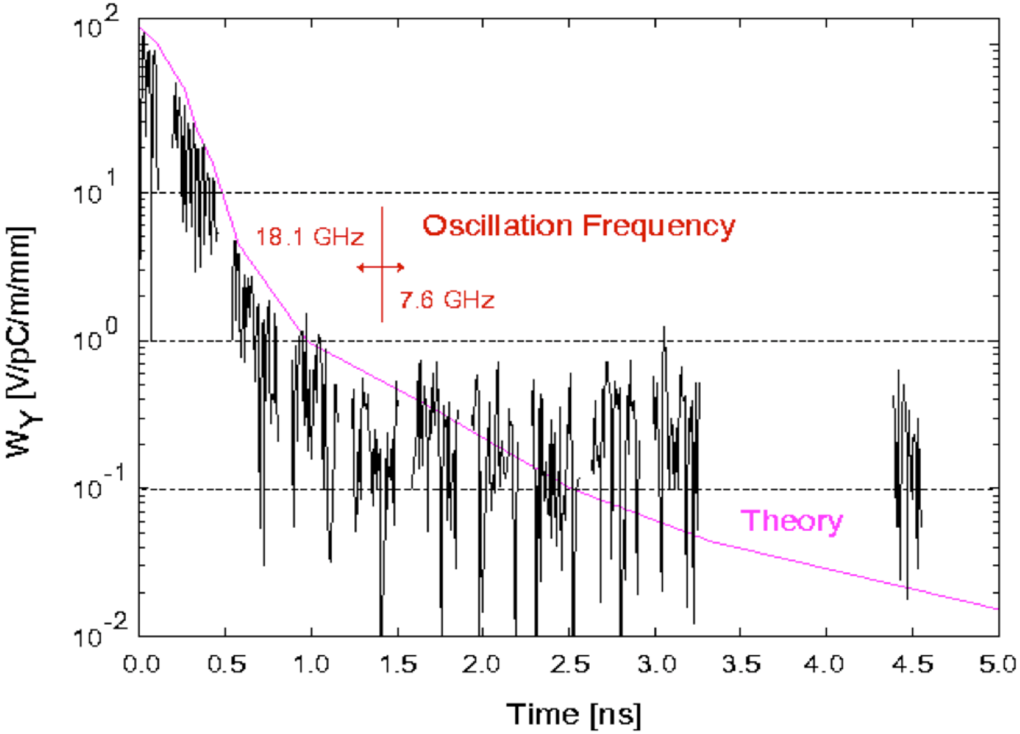




Multi-bunch Stability

Multi-bunch Wakefields

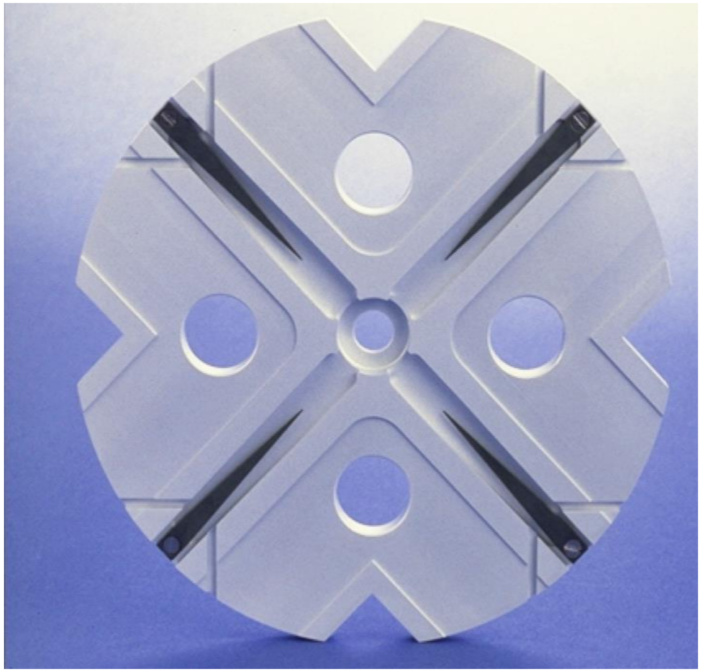
- Long-range transverse wakefield determines how close one can put the bunches in the linac
- Longrange transverse wakefields are sine-like
- They can be reduced by
  - Damping
  - Detuning

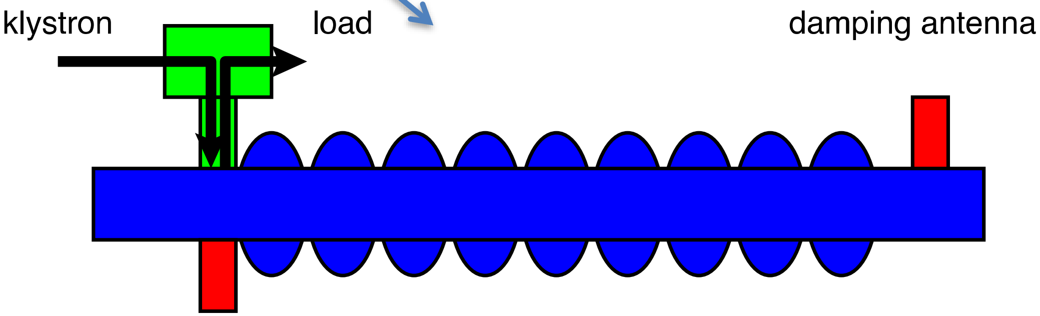


$$W_{\perp}(z) = \sum_{i}^{\infty} 2k_i \sin\left(2\pi\frac{z}{\lambda_i}\right)\exp\left(-\frac{\pi z}{\lambda_i Q_i}\right)$$

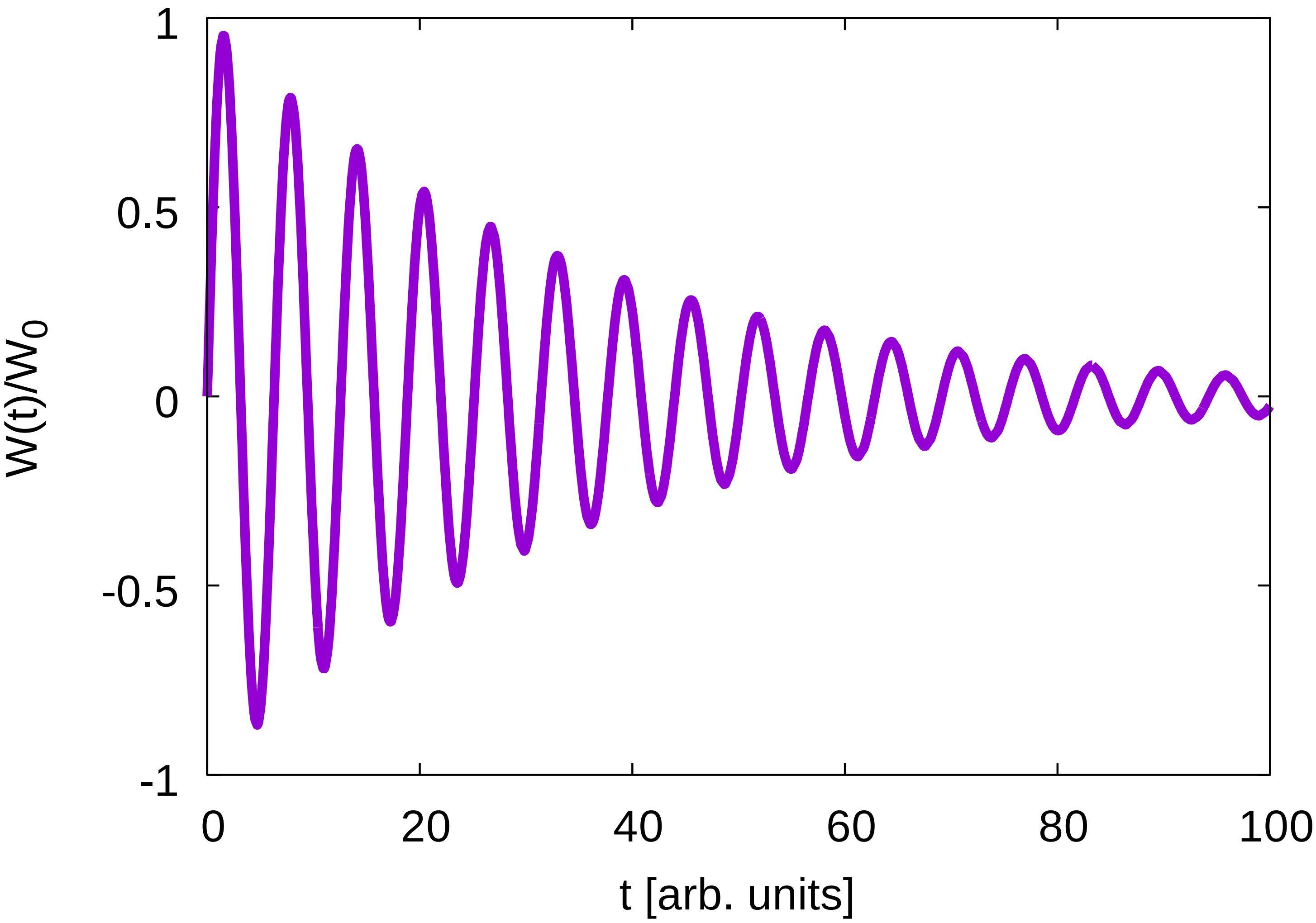
Damping

- Damping = extract power of transverse modes
- In CLIC, each cell has waveguides
  - Fundamental mode cannot escape
  - Strong damping, Q=O(10)
- ILC has antennas at the end
  - Weaker damping, Q=O(10^4)





Effect of Damping



Detuning

Introducing a spread in wakefield frequencies helps:

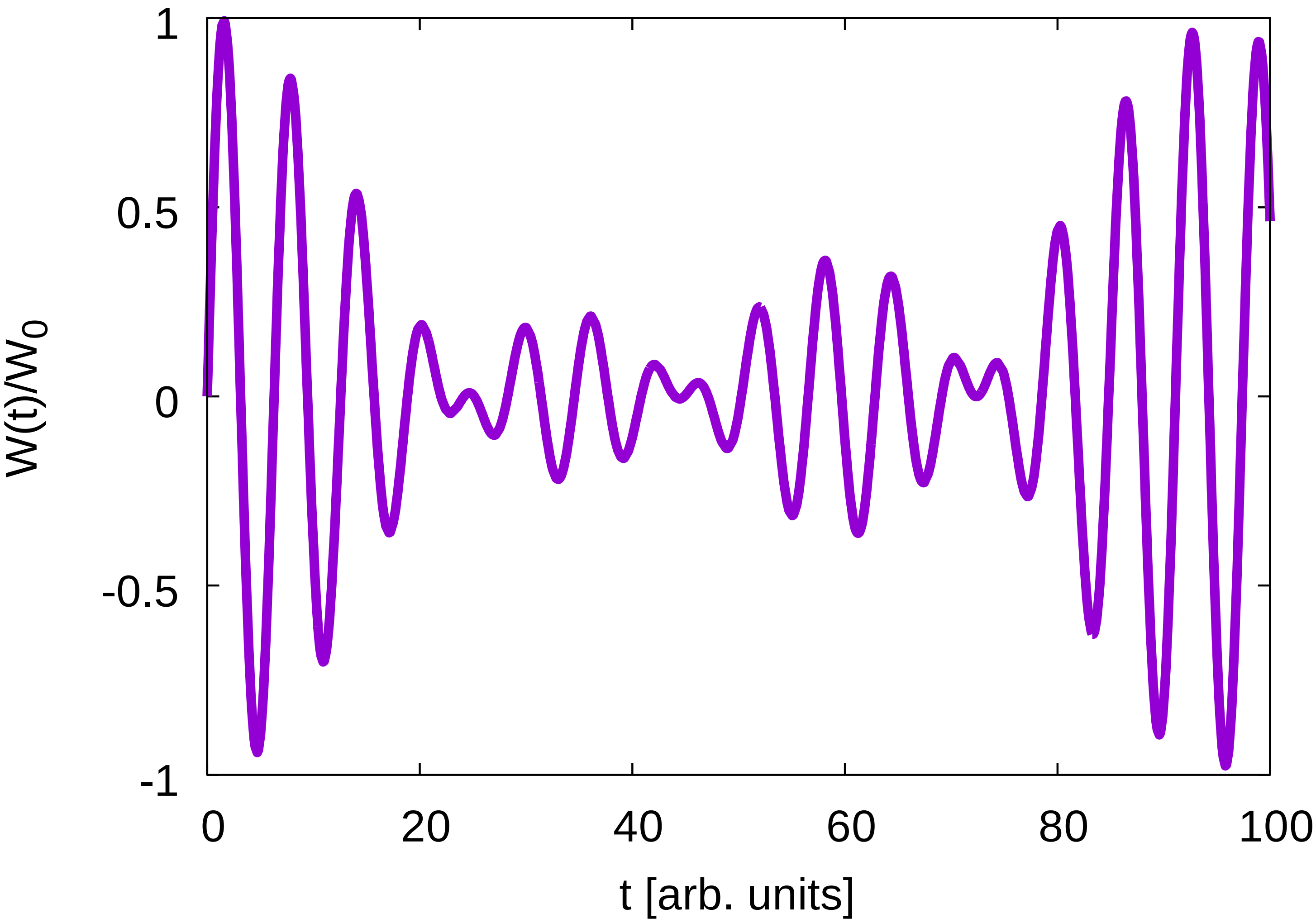
Example for two modes

$$W(z) = W_0 \frac{\sin((k+\Delta)z) + \sin((k-\Delta)z)}{2}$$

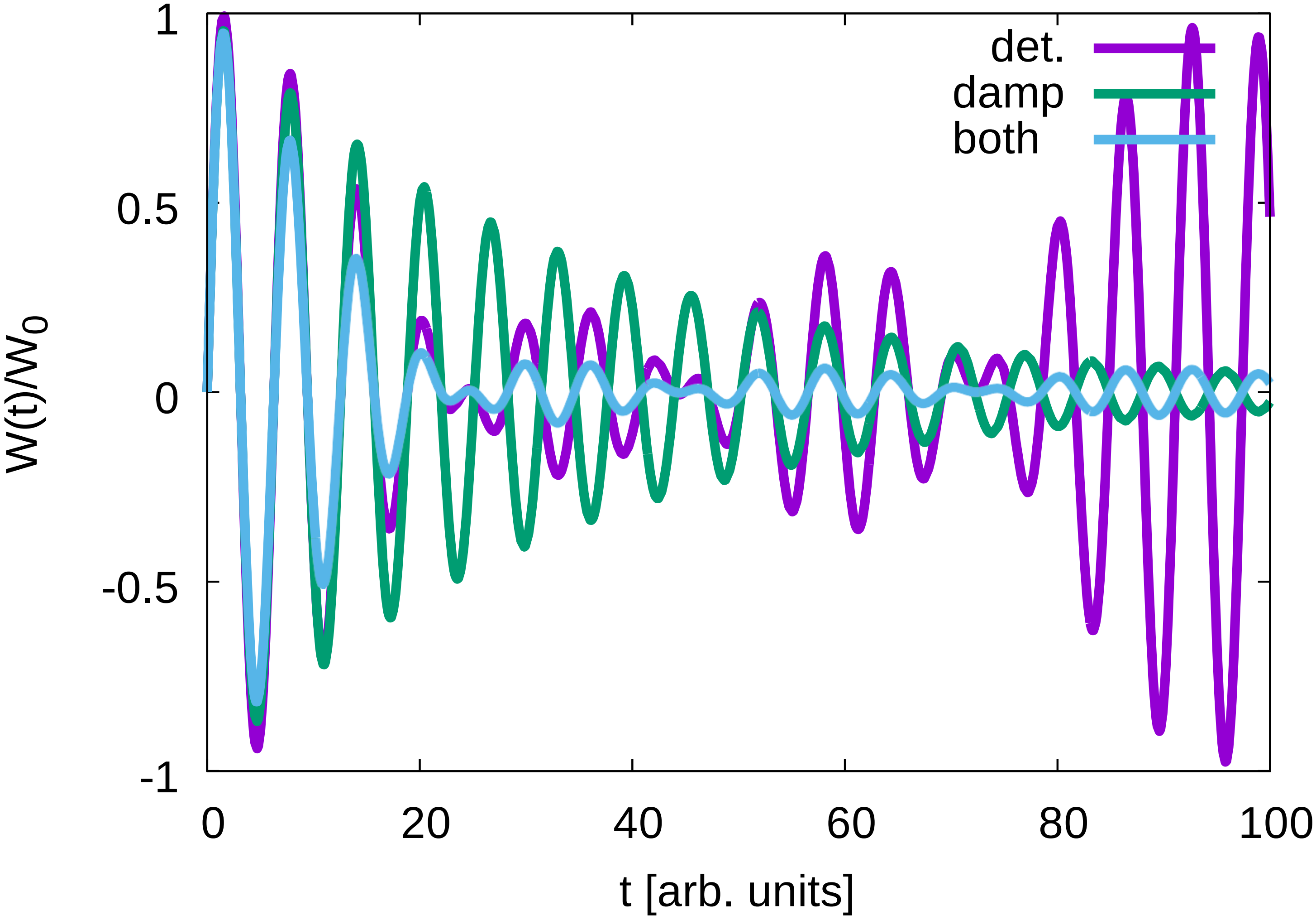
$$W(z) = W_0 \sin(kz) \cos(\Delta z)$$

In CLIC structure each cell is different, has a different transverse mode

Illustration of Detuning



Combined Effect



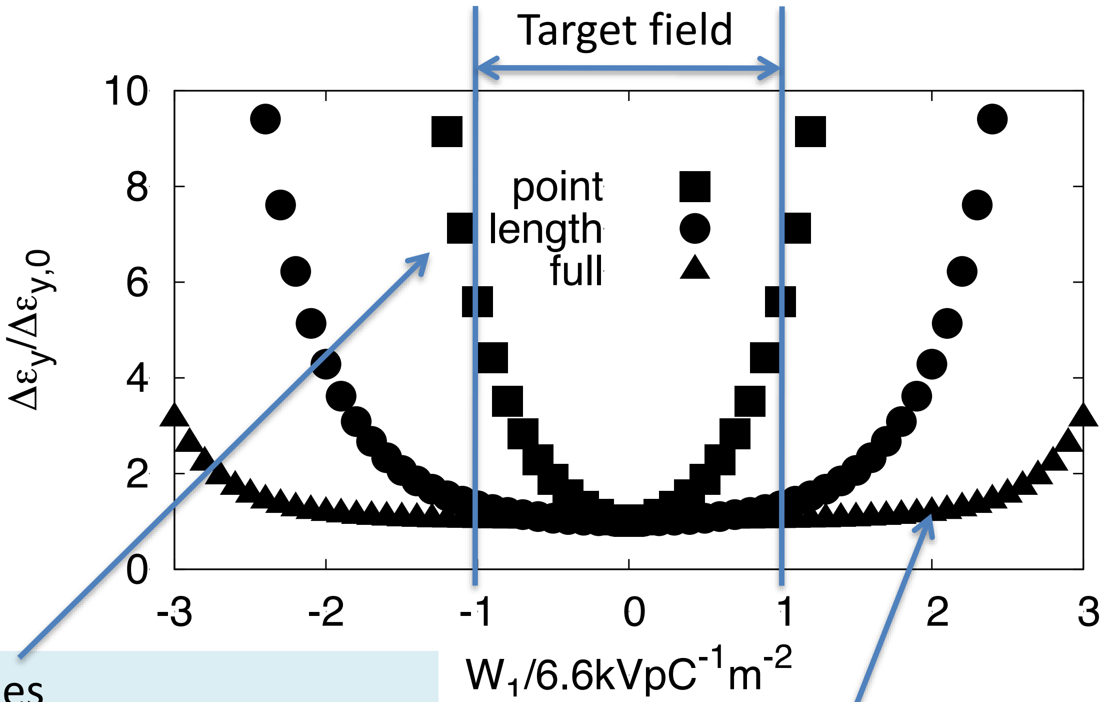
Multi-bunch Effect in CLIC

Wakefield amplitudes are large

Strong damping (Q=O(10))

Detuning (each cell is different)

Each bunch mainly kicks the immediately following one



Analytic estimate: point-like bunches

Using model similar to two-particle model but for many

https://cds.cern.ch/record/1227215/files/fr5rfp055.pdf

Luminosity loss is amplified by factor 4.9, acceptable

Fully real simulation:

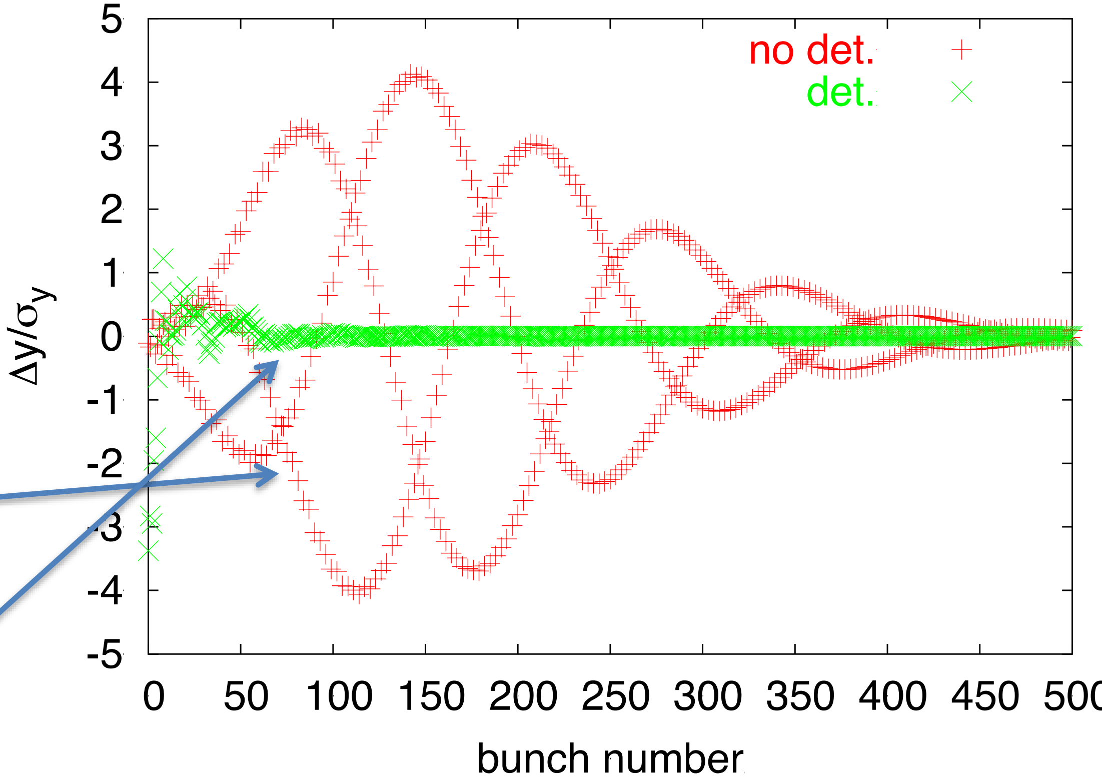
Energy spread stabilises, very acceptable

Multi-bunch effect in ILC

Small wakefield amplitudes

- little damping (Q=O(10^4))
- random detuning cavity to cavity (O(10^{-3}))

Cavity misalignment simulated



No detuning is not acceptable

Residual bunch-to-bunch offsets with detuning
But should be acceptable

Imperfections

Static Imperfections

- Pre-alignment errors are most important
- Pre-Alignment imperfections can be roughly categorised into short-distance and long- distance errors

- To first order, the imperfections can be treated as independent
  - as long as a linear main linac model is sufficient

- The short-distance misalignments give largest emittance contribution
  - misalignment of elements is largely independent
  - simulated by scattering elements around a straight line
  - or slightly more complex local model

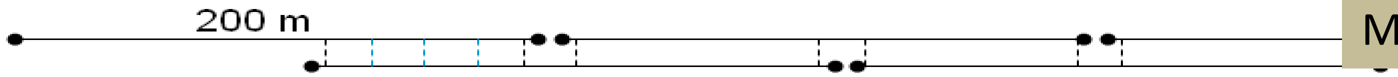
- The long-distance misalignments are dominated by the wire system

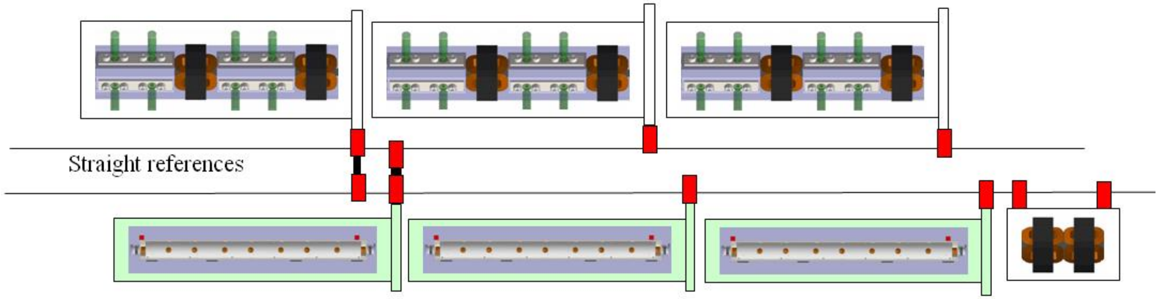
⇒ ignore short-distance misalignments and simulate wire errors only

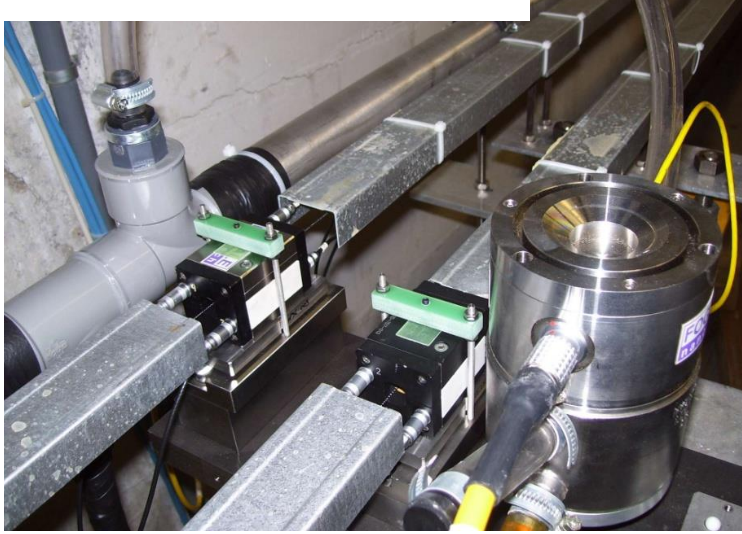
- Combined studies are mainly for completeness

CLIC Pre-alignment Procedures

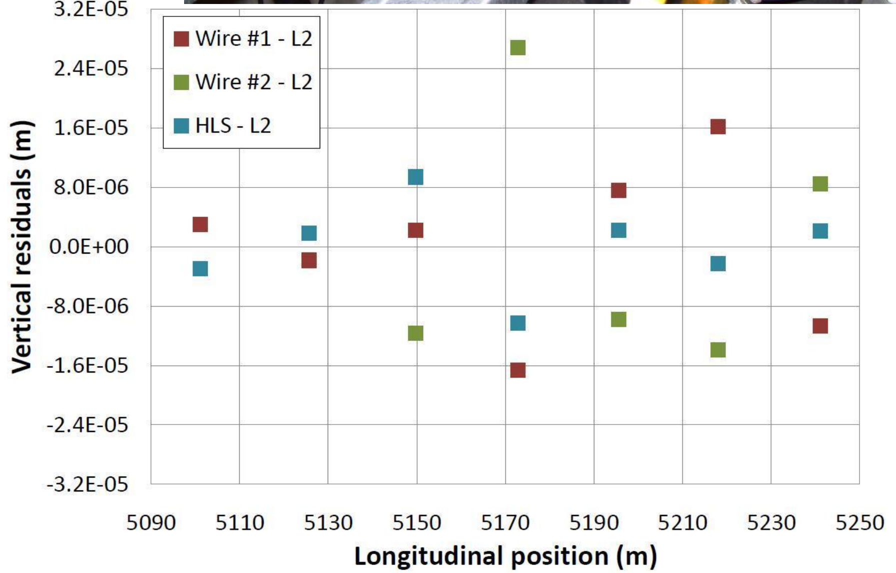
⇒ Dominique Missiaen
Monday 5.3







- Required accuracy of reference points is 10µm



- Test of prototype shows
    - vertical RMS error of 11µm
    - i.e. accuracy is approx. 13.5µm
- Improvement path identified

Pre-alignment Performance

Element	error	with respect to	alignment	
			ILC	CLIC
Structure	offset	Girder	$300\,\mu m$	$5\,\mu m$
Structure	tilts	Girder	$300\,\mu radian$	$200(*)\,\mu m$
Girder Girder	offset	survey line	$200\,\mu m$	$9.4\,\mu m$
Quadrupole	tilt	survey line	$20\,\mu radian$	$9.4\,\mu radian$
	offset	girder/survey line	$300\,\mu m$	$17\,\mu m$
Quadrupole	roll	survey line	$300\,\mu radian$	$\leq 100\,\mu radian$
BPM	offset	girder/survey line	$300\,\mu m$	$14\,\mu m$
BPM	resolution	BPM	$\approx 1\,\mu m$	$0.1\,\mu m$
Wakefield mon.	offset	center wake center	—	$3.5\,\mu m$

Difficult to pre-align components in superconducting module

Important R&D development has been carried out for CLIC

* This is mainly bookshelfing

Emittance Growth (ILC)

Error	with respect to	value	$\Delta\varepsilon_y$ [nm]
Cavity offset	module	300 μm	3.5
Cavity tilt	module	300 μradian	2600
BPM offset	module	300 μm	0
Quadrupole offset	module	300 μm	700000
Quadrupole roll	module	300 μradian	2.2
Module offset	perfect line	200 μm	250000
Module tilt	perfect line	20 μradian	880

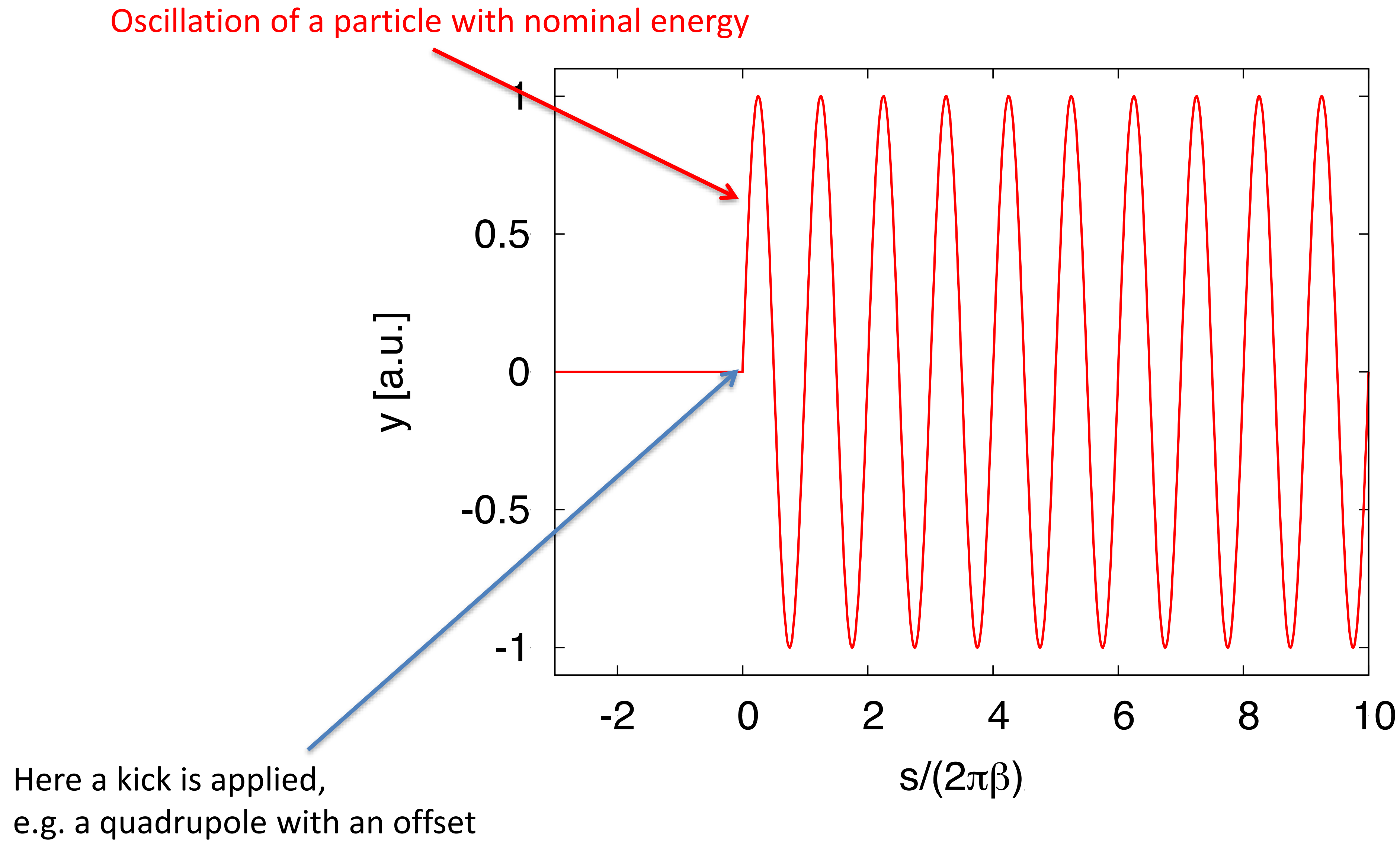
Largest problem
quadrupole
offsets

Cavity tilts are important
Beam is kicked by accelerating field

Module angles lead to cavity tilts

Module offset
offsets
quadrupole

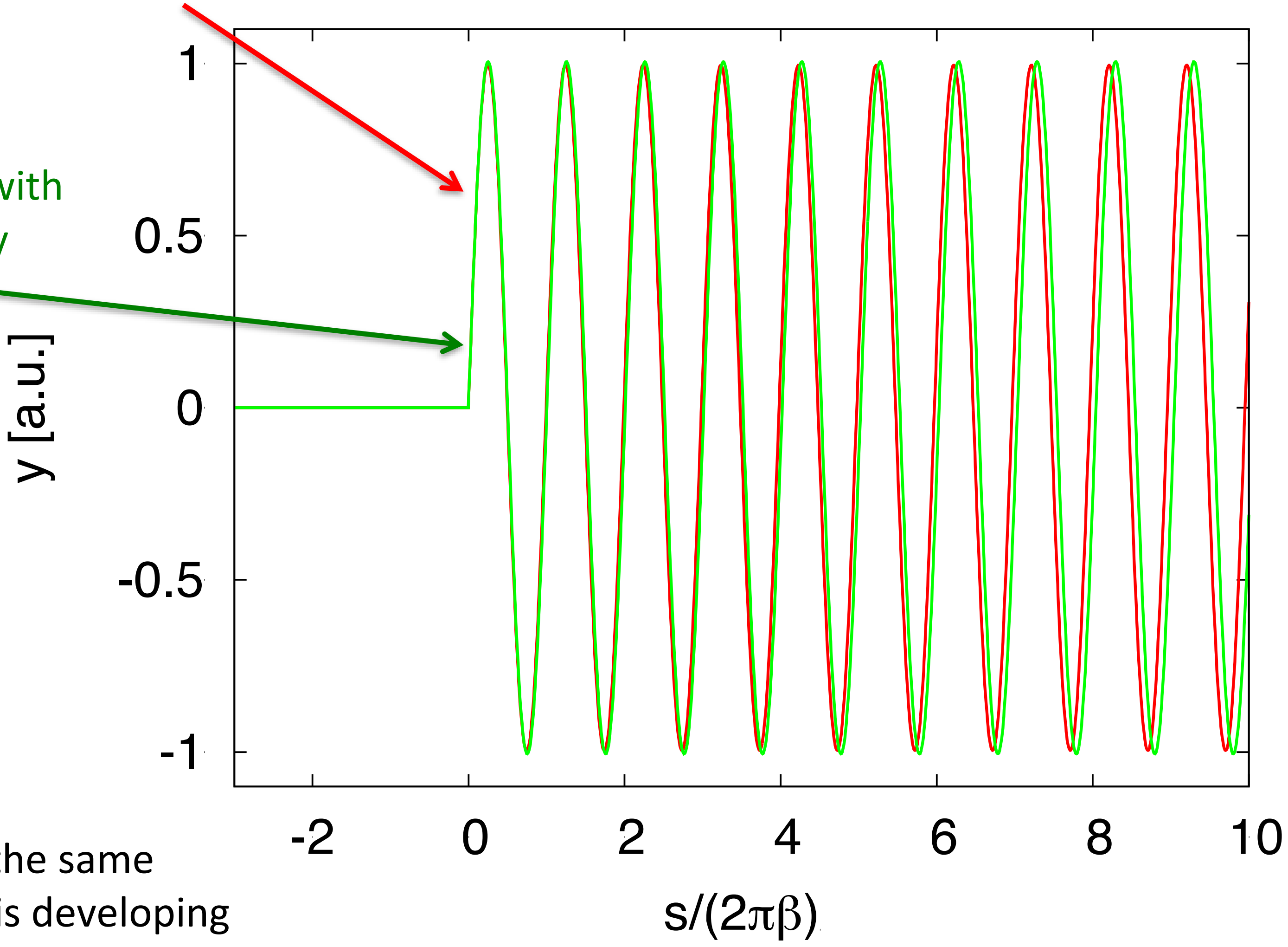
Dispersion and Emittance Growth



Oscillation of a particle with nominal energy

Here a kick is applied,
e.g. a quadrupole with an offset

Dispersion and Emittance Growth

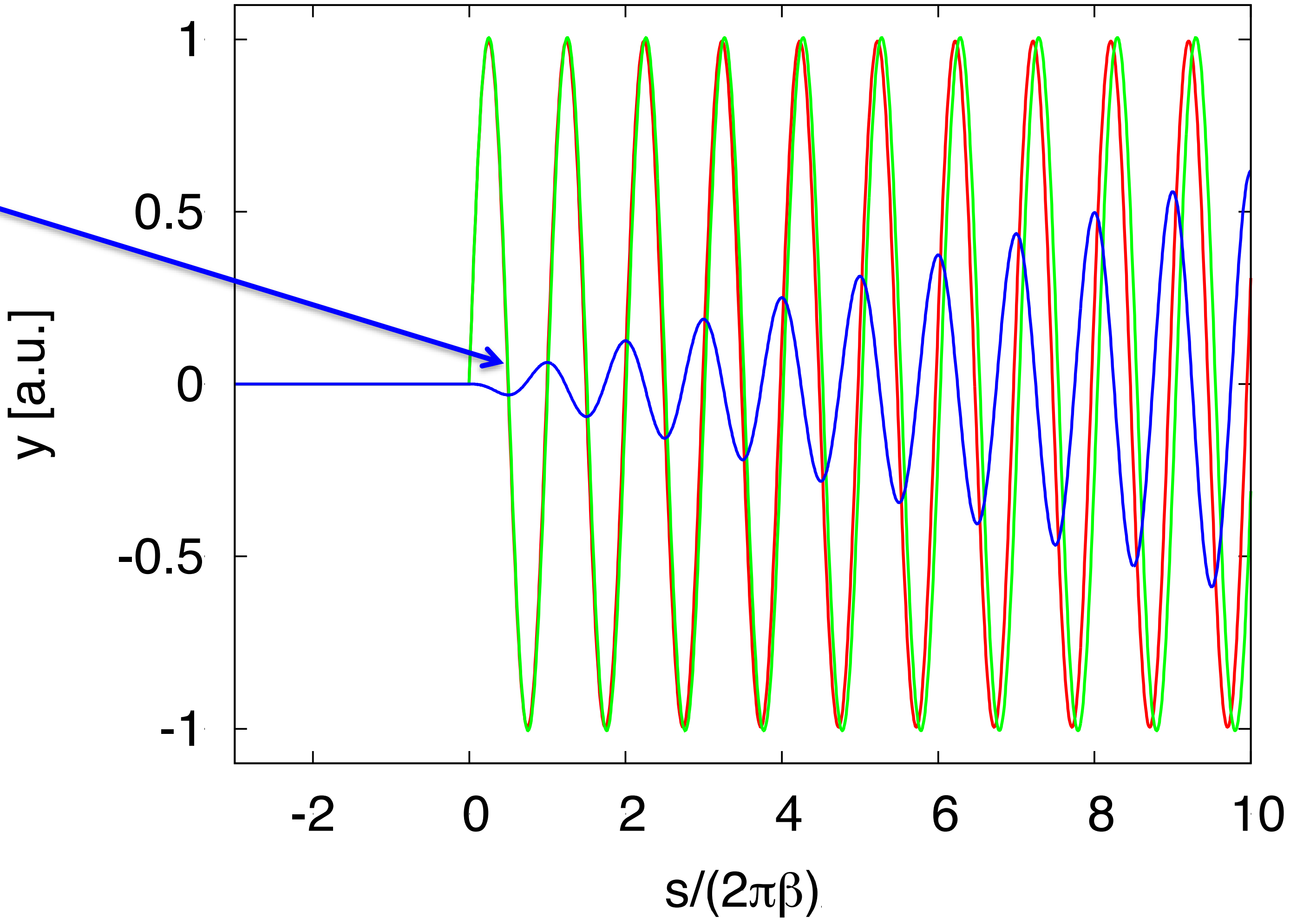


Oscillation of a particle with 100.5 % of nominal energy

Oscillation of a particle with 99.5% of nominal energy

Amplitude is almost the same
But slight dephasing is developing

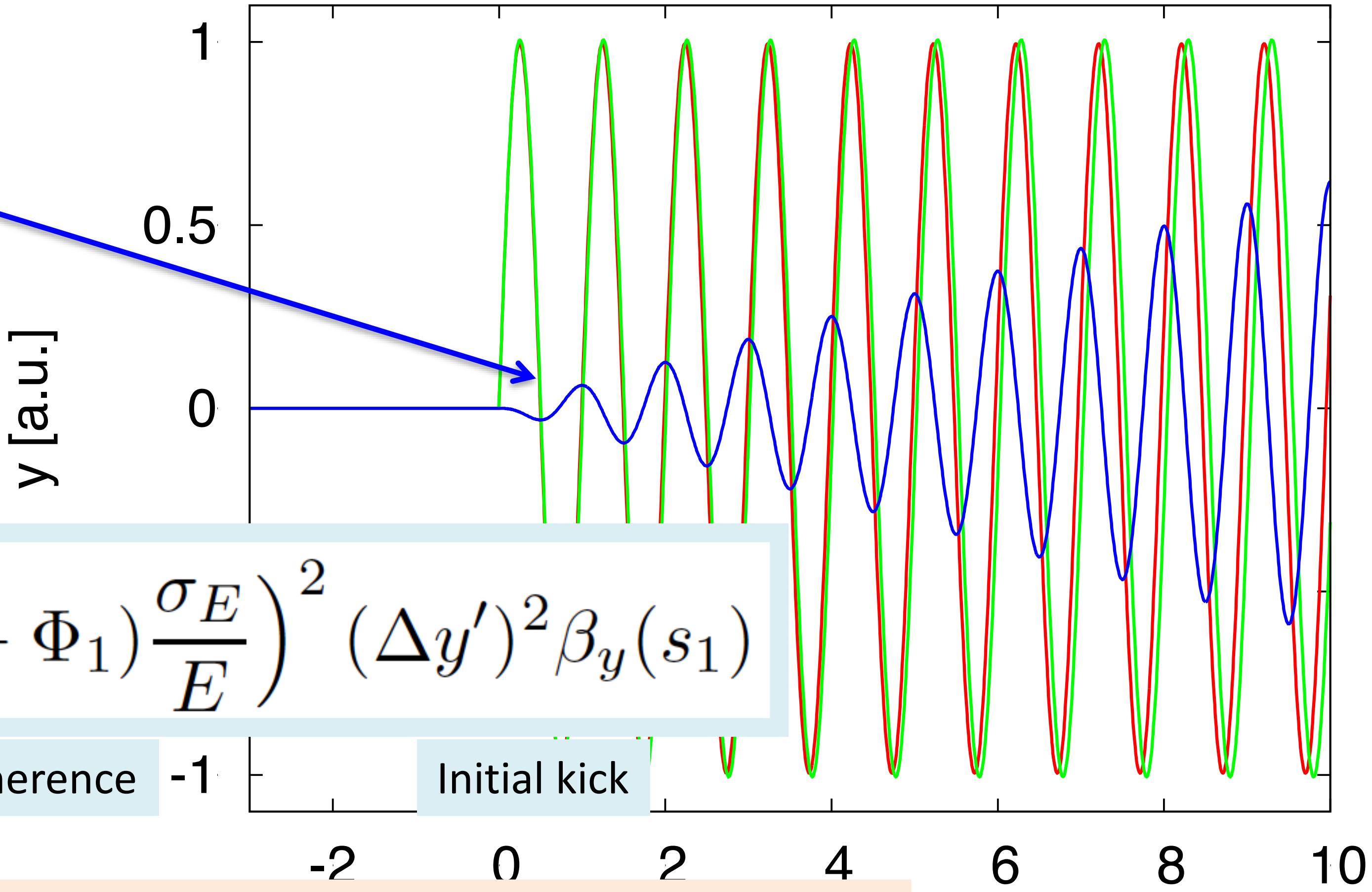
Dispersion and Emittance Growth



Difference between trajectories grows along accelerator

Dispersion and Emittance Growth

Difference between trajectories grows along accelerator



Emittance grows as

$$\Delta\epsilon_1 \propto \left((\Phi_e - \Phi_1)\frac{\sigma_E}{E}\right)^2 (\Delta y')^2 \beta_y(s_1)$$

Decoherence

Initial kick

For long linacs

$$\Delta\epsilon_1 \propto (\Delta y')^2 \beta_y(s_1)$$

Beam-based Alignment and Tuning

- Make beam pass linac by aligning quadrupoles
  - one-to-one correction

- Remove dispersion by aligning BPMs and quadrupoles
  - dispersion free steering
  - Ballistic alignment
  - kick minimisation

- Remove wakefields locally (CLIC only)
  - RF alignment

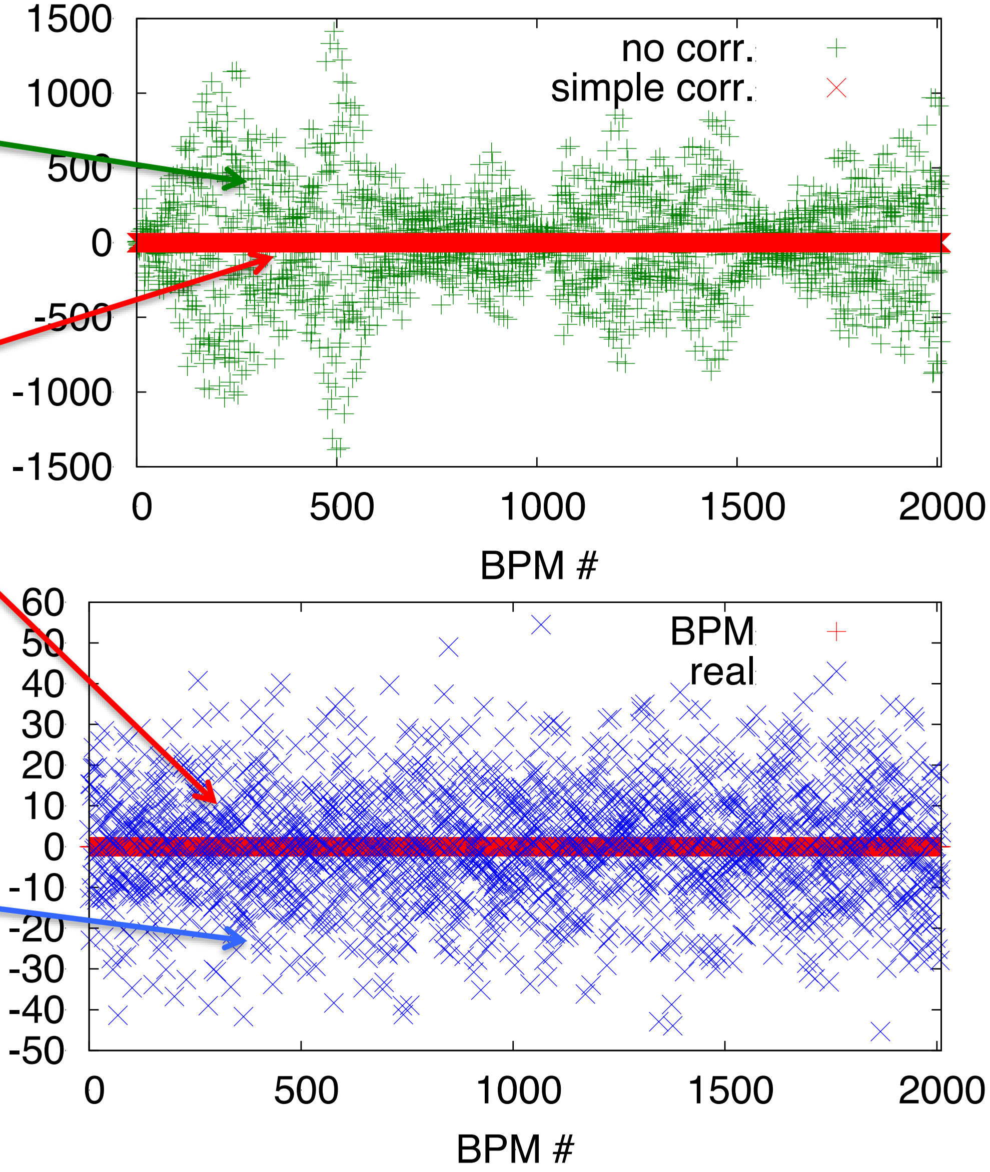
- Remove dispersive and wakefield effects globally
  - Emittance tuning bumps
  - Luminosity tuning bumps

Trajectory with Simple Correction

BPM readings if no beam-based correction is applied

After one-to-one correction all BPMs read zero

But beam still is offset, because BPMs have offsets

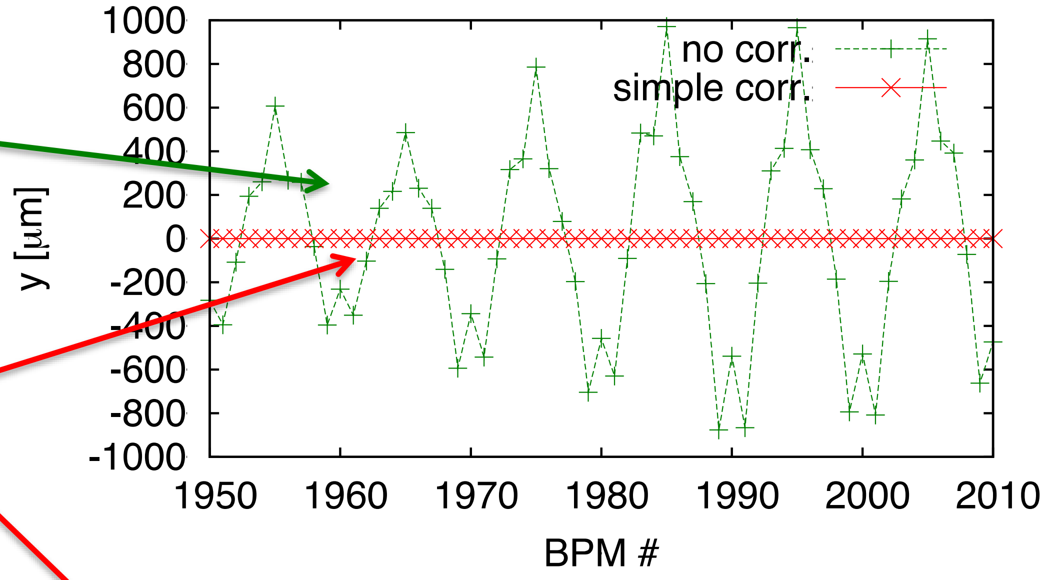


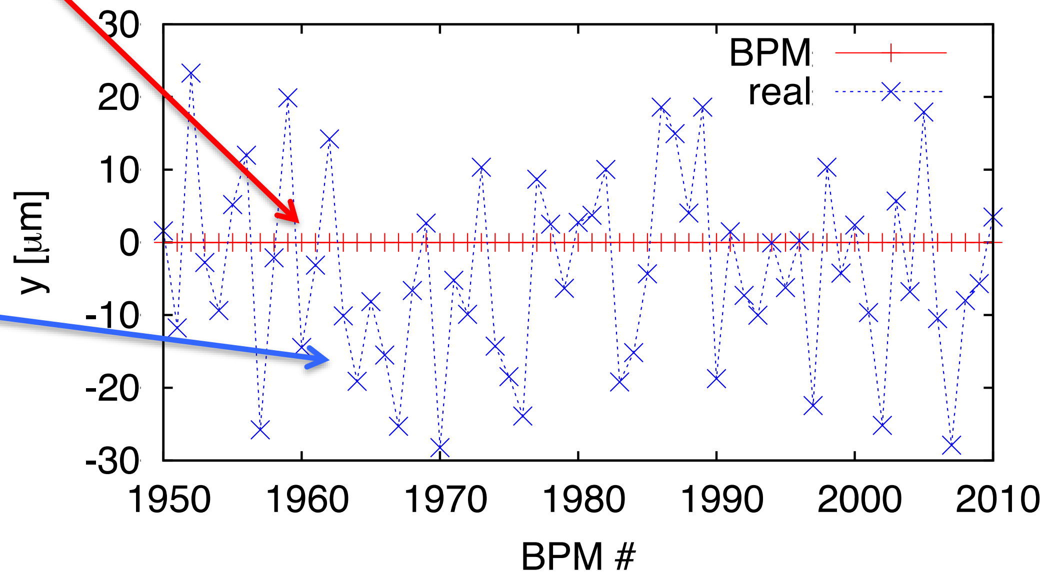
Trajectory at the End of the Linac

With no correction, at the end of the linac beam performs betatron oscillation

After one-to-one correction all BPMs read zero

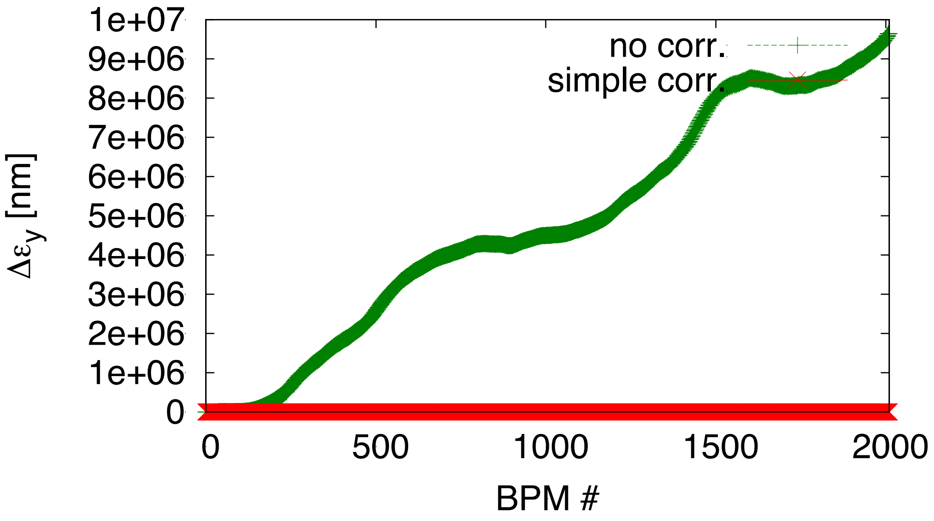
No betatron oscillation has been build-up if we use one-to-one correction





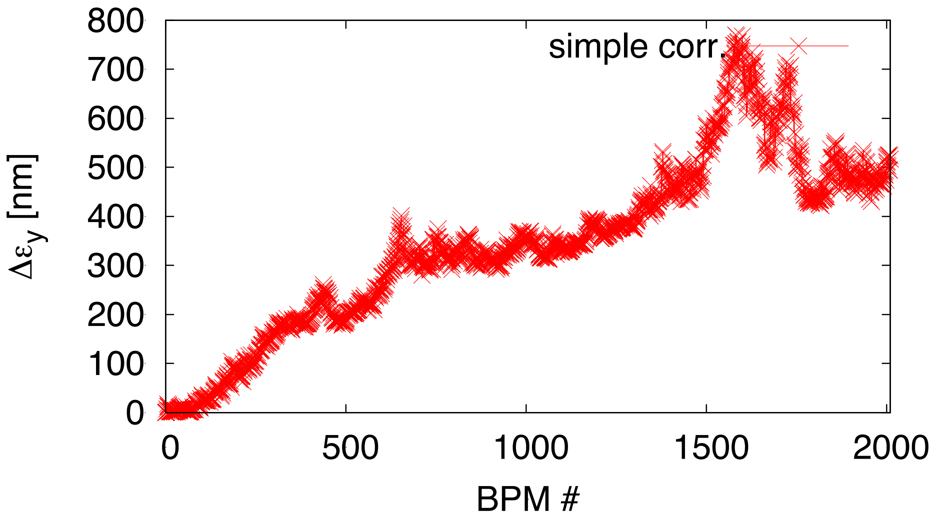
Emittance Growth

The emittance growth with no correction is very large

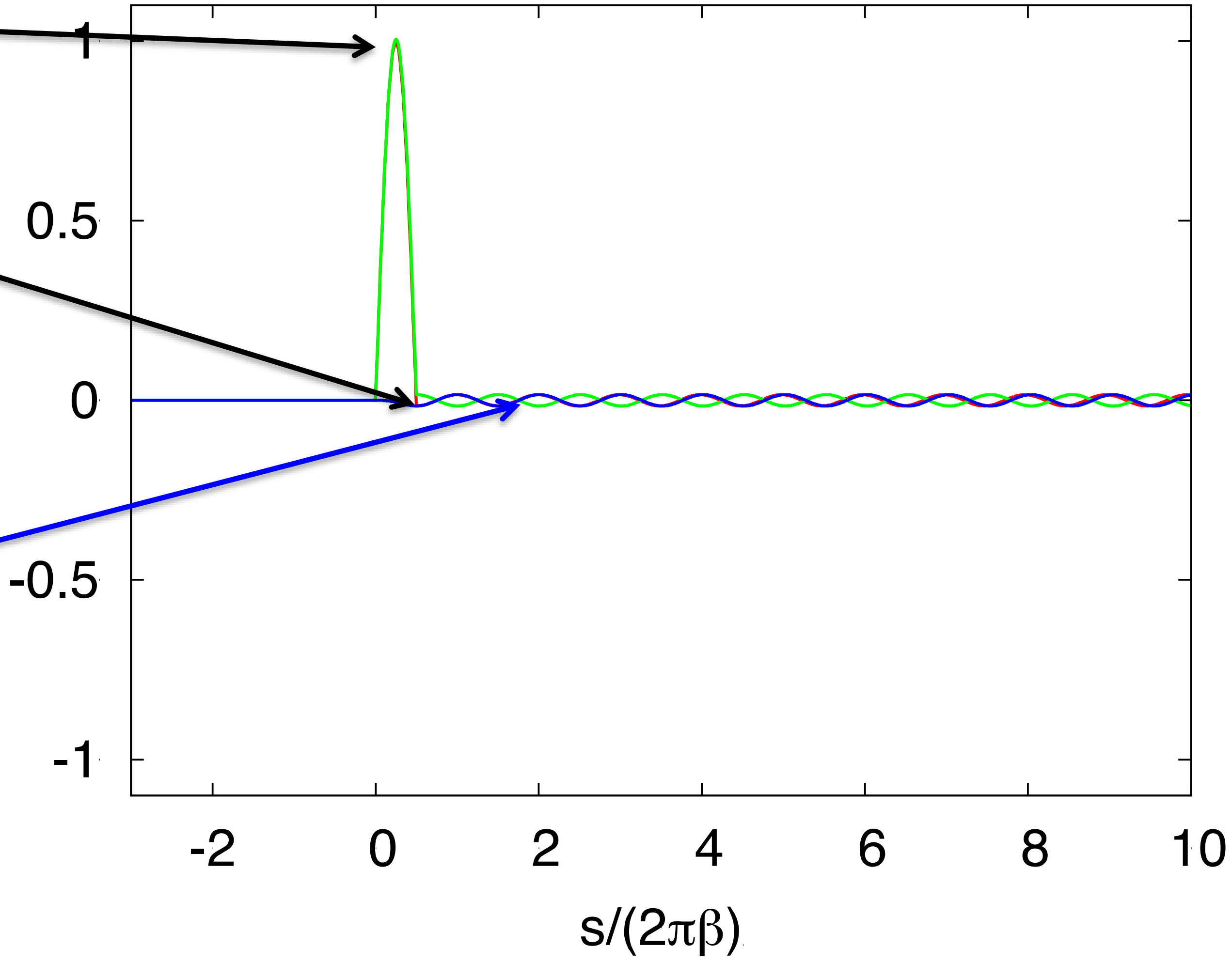


The simple steering yields many orders of magnitude improvement

But still the emittance growth is far above the target



Dispersion and Emittance Growth

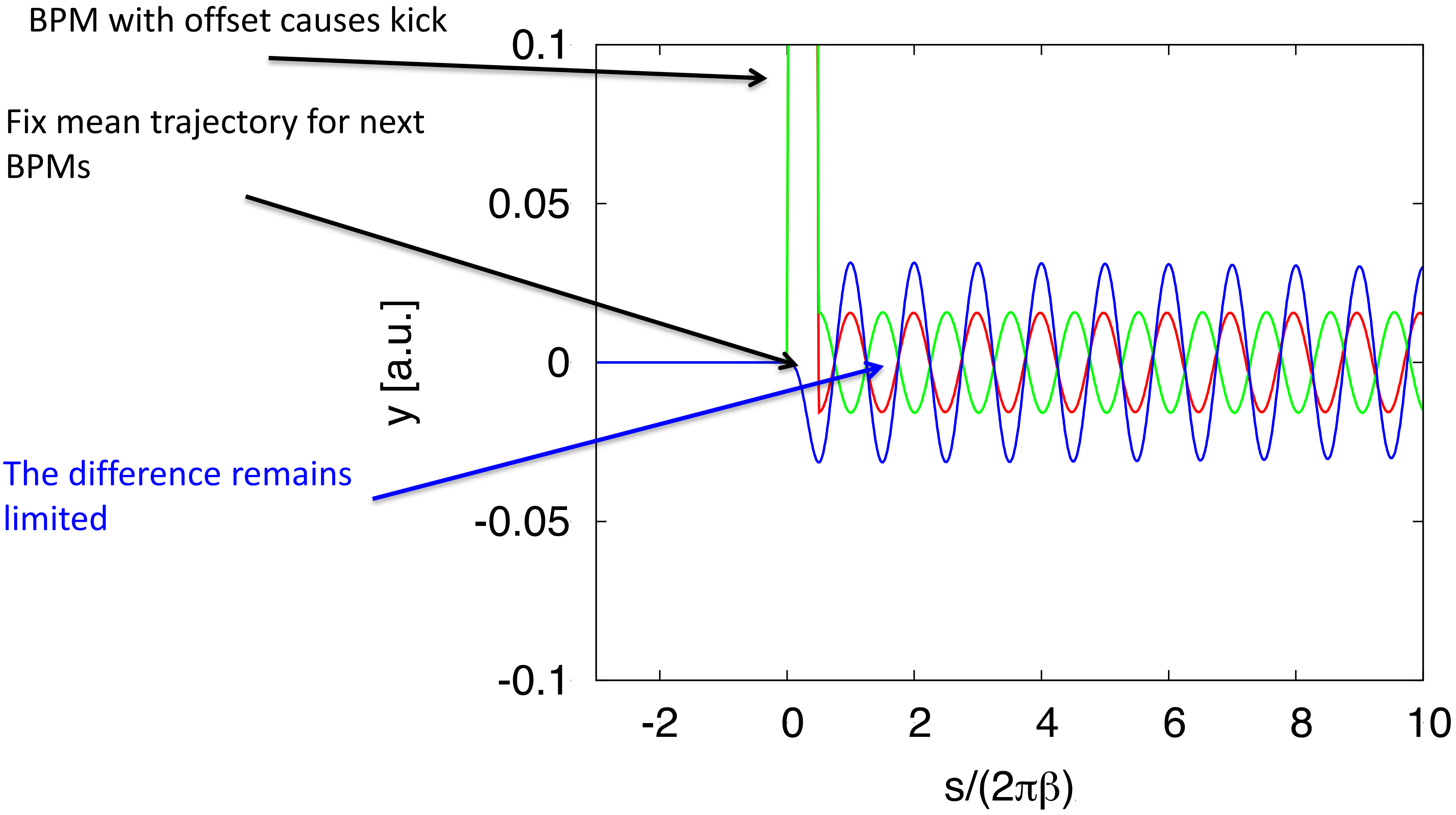


BPM with offset causes kick

Fix mean trajectory for next BPMs

The difference remains limited

Dispersion and Emittance Growth



BPM with offset causes kick

Fix mean trajectory for next BPMs

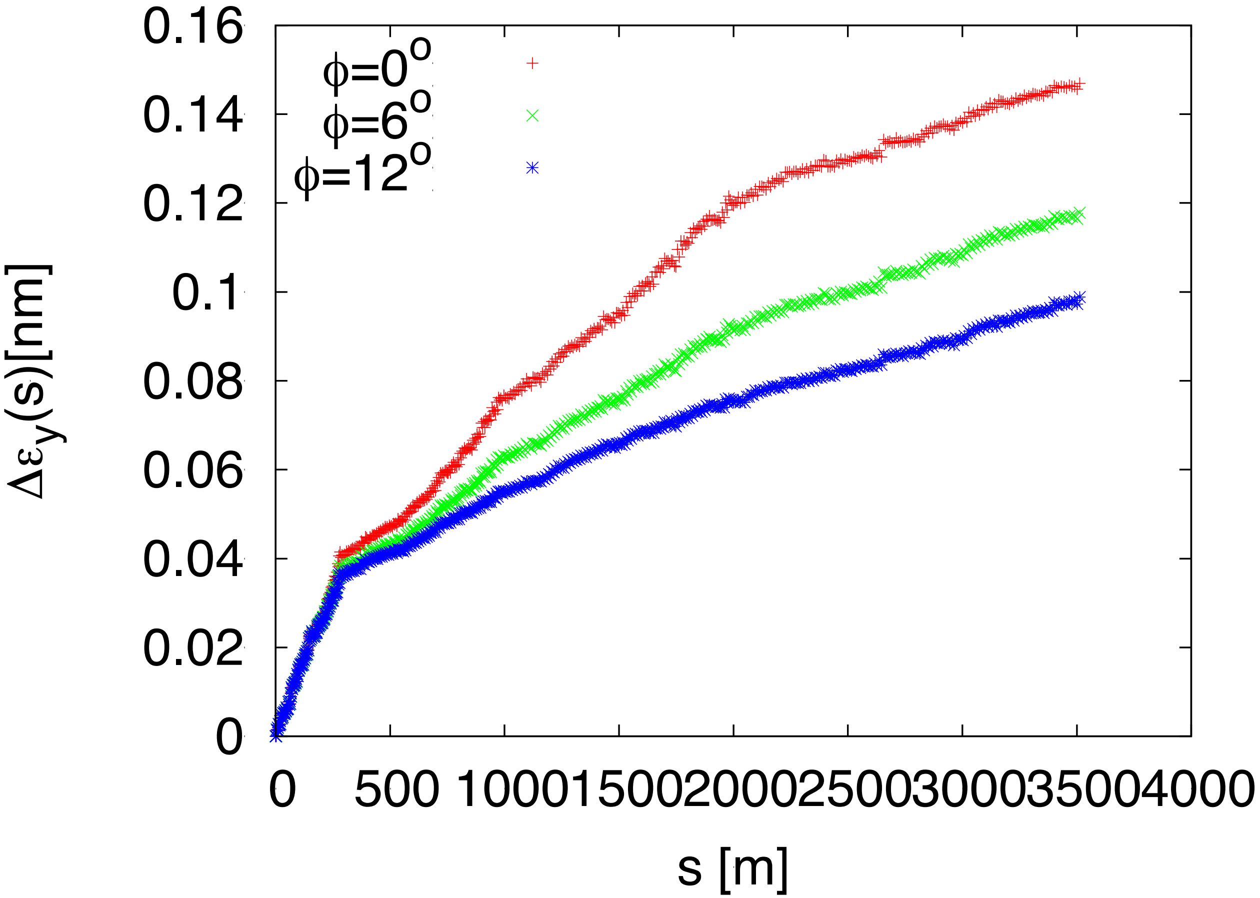
The difference remains limited

Example: BPM Misalignment in CLIC 380 GeV

Larger energy spread makes us more sensitive to BPM misalignments

Values for 0.4μm scatter and one-to-one correction are shown

About 6 times less emittance growth than in CLIC at 3TeV



Emittance Growth after One-to-one Steering (ILC)

Error	with respect to	value	$\Delta\varepsilon_y$ [nm]	$\Delta\varepsilon_{y,121}$ [nm]
Cavity offset	module	300 μm	3.5	0.2
Cavity tilt	module	300 μradian	2600	< 0.1
BPM offset	module	300 μm	0	360
Quadrupole offset	module	300 μm	700000	0
Quadrupole roll	module	300 μradian	2.2	2.2
Module offset	perfect line	200 μm	250000	155
Module tilt	perfect line	20 μradian	880	1.7

Quadrupole issue solved

BPM issue created
Module offset leads to BPM offset

Still much better than before

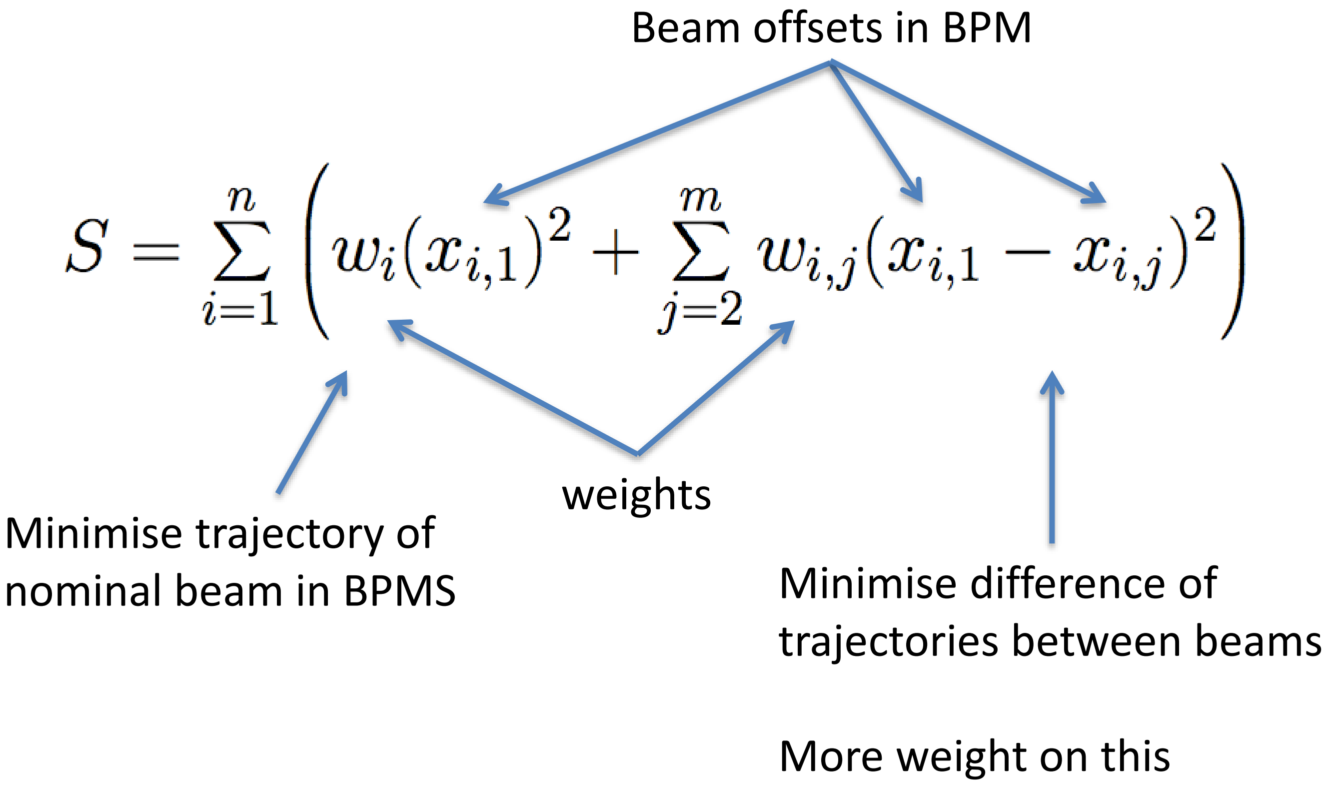
Note:
Emittance should scale as

$$\Delta\epsilon \propto (\Delta y)^2$$

✓

Dispersion Free Steering

- Basic idea: use different beam energies
- Accelerate beams with different gradient and initial energy
- Optimise trajectories for different energies together



Beam offsets in BPM

$$S = \sum_{i=1}^{n} \left(w_i (x_{i,1})^2 + \sum_{j=2}^{m} w_{i,j} (x_{i,1} - x_{i,j})^2 \right)$$

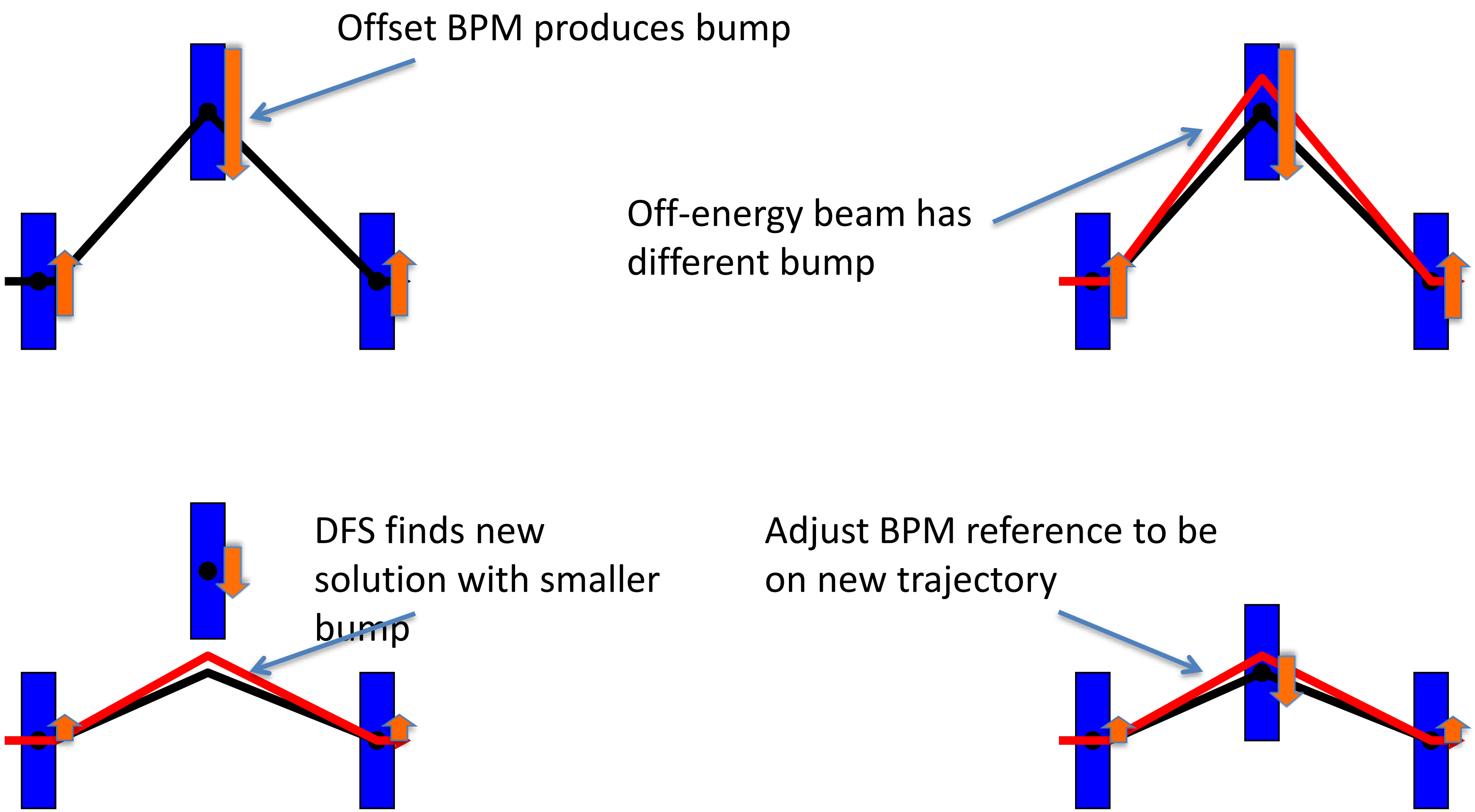
weights

Minimise trajectory of
nominal beam in BPMS

Minimise difference of
trajectories between beams

More weight on this

Dispersion Free Illustration



Offset BPM produces bump

Off-energy beam has different bump

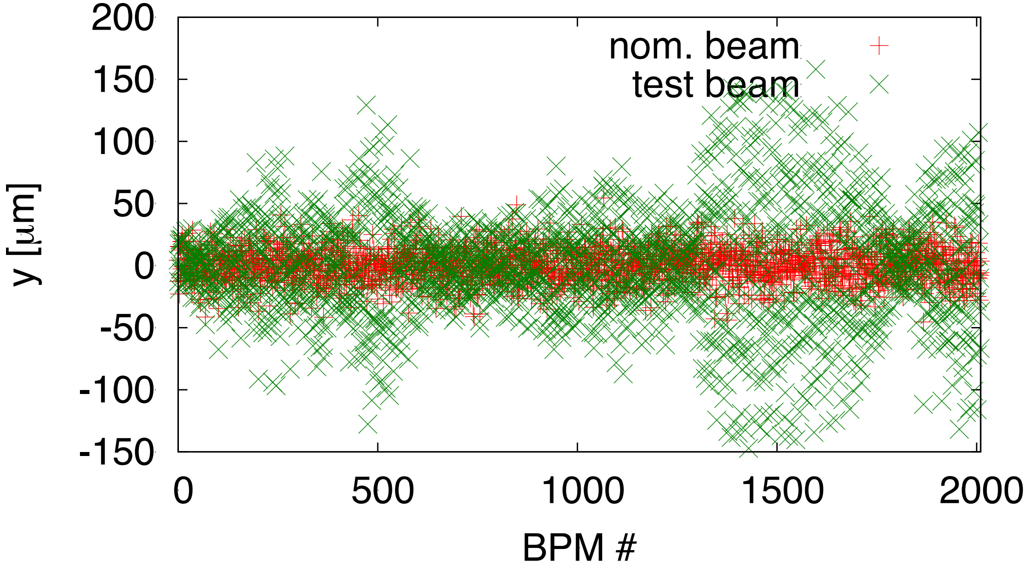
DFS finds new solution with smaller bump

Adjust BPM reference to be on new trajectory

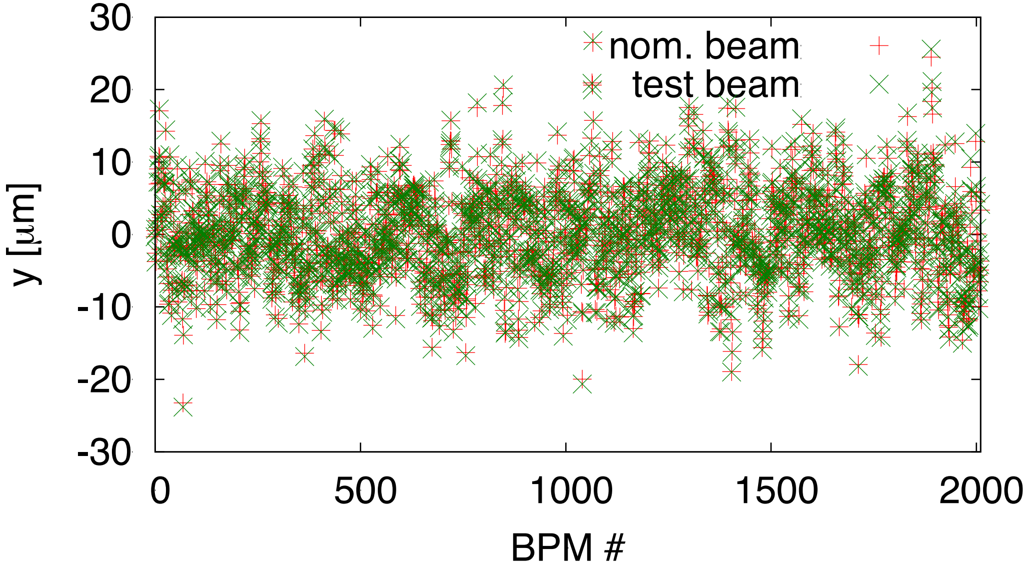
Dispersion Free Steering BPM Readings

A beam that has a different energy has a bad trajectory

The cancellation of different corrector kicks does not work very well because the phase advance is different for different energies

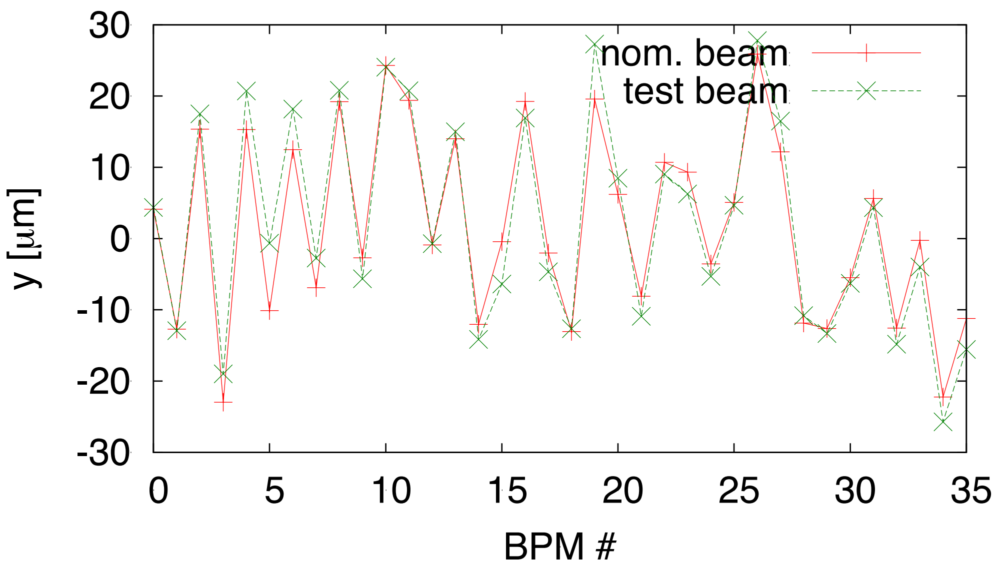


After dispersion free steering both beams take almost the same path

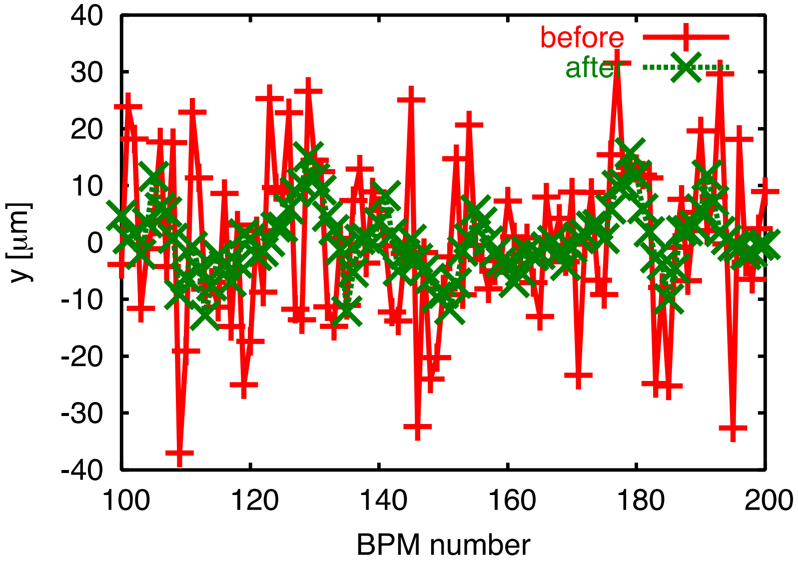


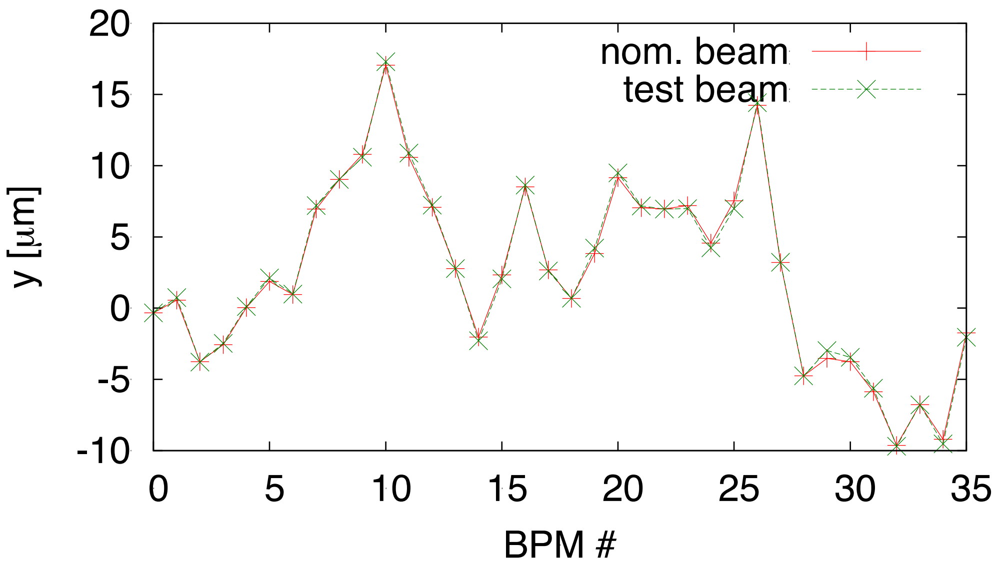
At the Beginning of the Linac

A small difference in trajectories starts between the two beams



The dispersion free steering almost completely removes this difference





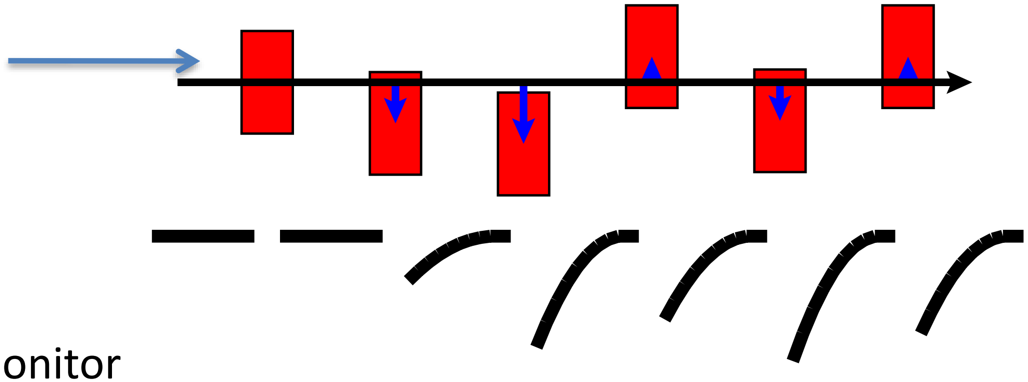
Resulting Emittance Growth (ILC)

Error	with respect to	value	$\Delta\varepsilon_y$ [nm]	$\Delta\varepsilon_{y,121}$ [nm]	$\Delta\varepsilon_{y,dfs}$ [nm]
Cavity offset	module	300 μm	3.5	0.2	0.2(0.2)
Cavity tilt	module	300 μradian	2600	< 0.1	1.8(8)
BPM offset	module	300 μm	0	360	4(2)
Quadrupole offset	module	300 μm	700000	0	0(0)
Quadrupole roll	module	300 μradian	2.2	2.2	2.2(2.2)
Module offset	perfect line	200 μm	250000	155	2(1.2)
Module tilt	perfect line	20 μradian	880	1.7	—

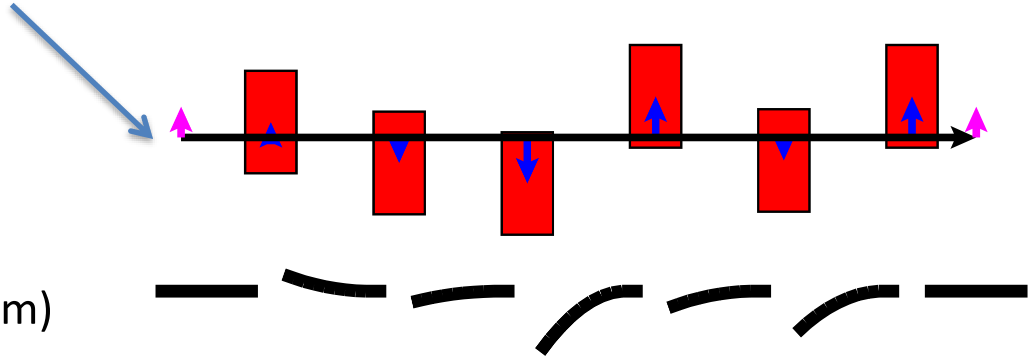
Dispersion free steering largely cures the BPM offset issue

RF Structure Alignment

Structures are scattered on the girder
⇒ Wakefield kick



Measure beam offset with wakefield monitor
Move girder to remove mean offset
⇒ No net wakefield kick



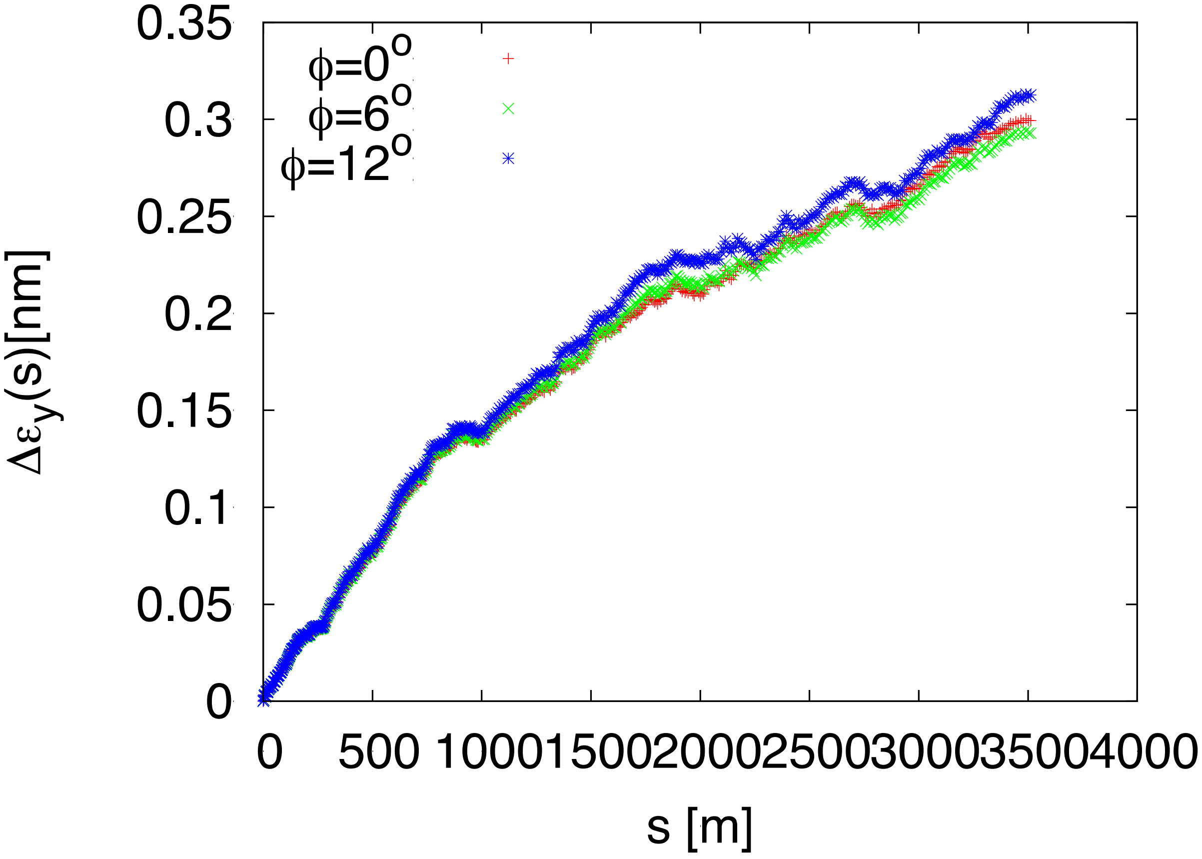
Limit mainly from

- accuracy of wakefield monitors (3.5 µm)
- reproducibility of wakefield
- tiny variation of betatron phase along girder

Example: Structure Misalignment in CLIC 380 GeV

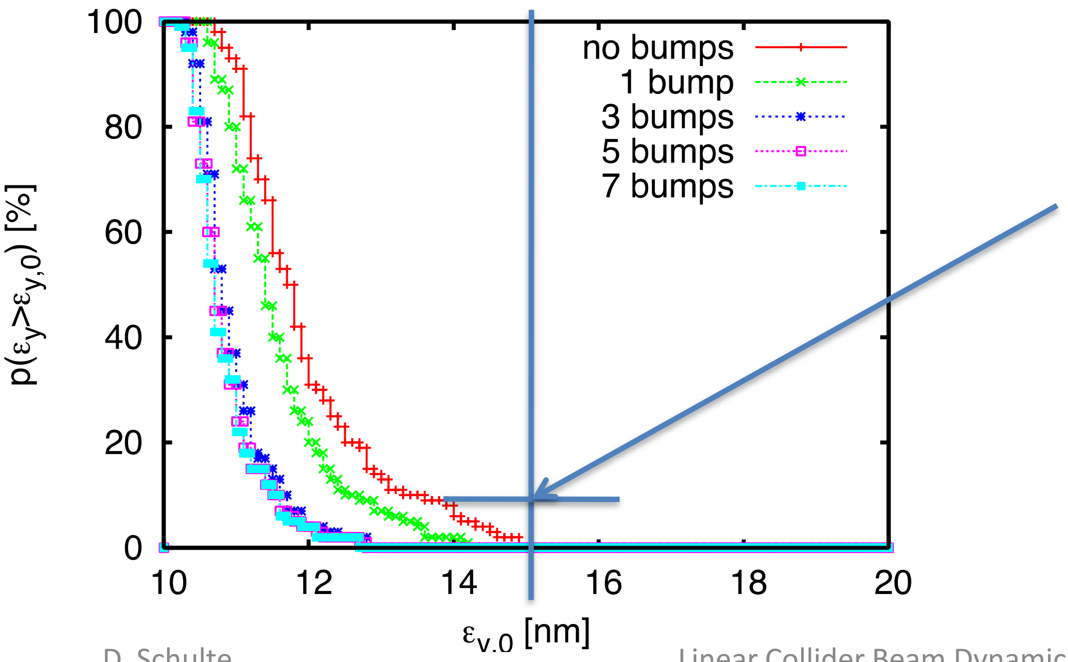
Average emittance growth in CDR for 3TeV is Δε=0.54nm

So gain about a factor 2



Final Emittance Growth (CLIC)

imperfection	with respect to	symbol	value	emitt. growth
BPM offset	wire reference	σ_{BPM}	14 μm	0.367 nm
BPM resolution		σ_{res}	0.1 μm	0.04 nm
accelerating structure offset	girder axis	σ_4	10 μm	0.03 nm
accelerating structure tilt	girder axis	σ_t	200 μradian	0.38 nm
articulation point offset	wire reference	σ_5	12 μm	0.1 nm
girder end point	articulation point	σ_6	5 μm	0.02 nm
wake monitor	structure centre	σ_7	3.5 μm	0.54 nm
quadrupole roll	longitudinal axis	σ_r	100 μradian	$\approx$ 0.12 nm



Goal: less than 10% above 15 nm ✓

Further improvement using tuning bumps

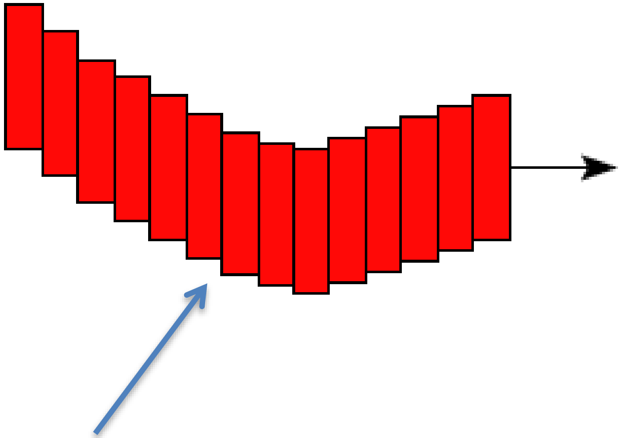
Tuning Bumps

Compensate an effect globally

Minimise beam size/emittance
or maximise luminosity

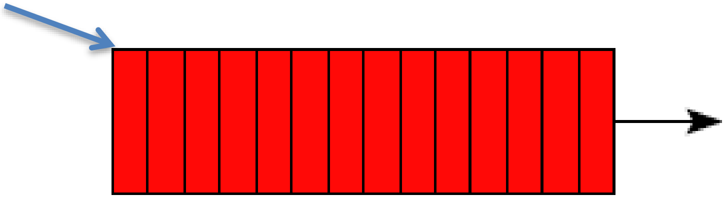
Remove a correlation between particles
e.g. average wakefield kick can be
compensated in one location

Energy spread and phase advance give limits



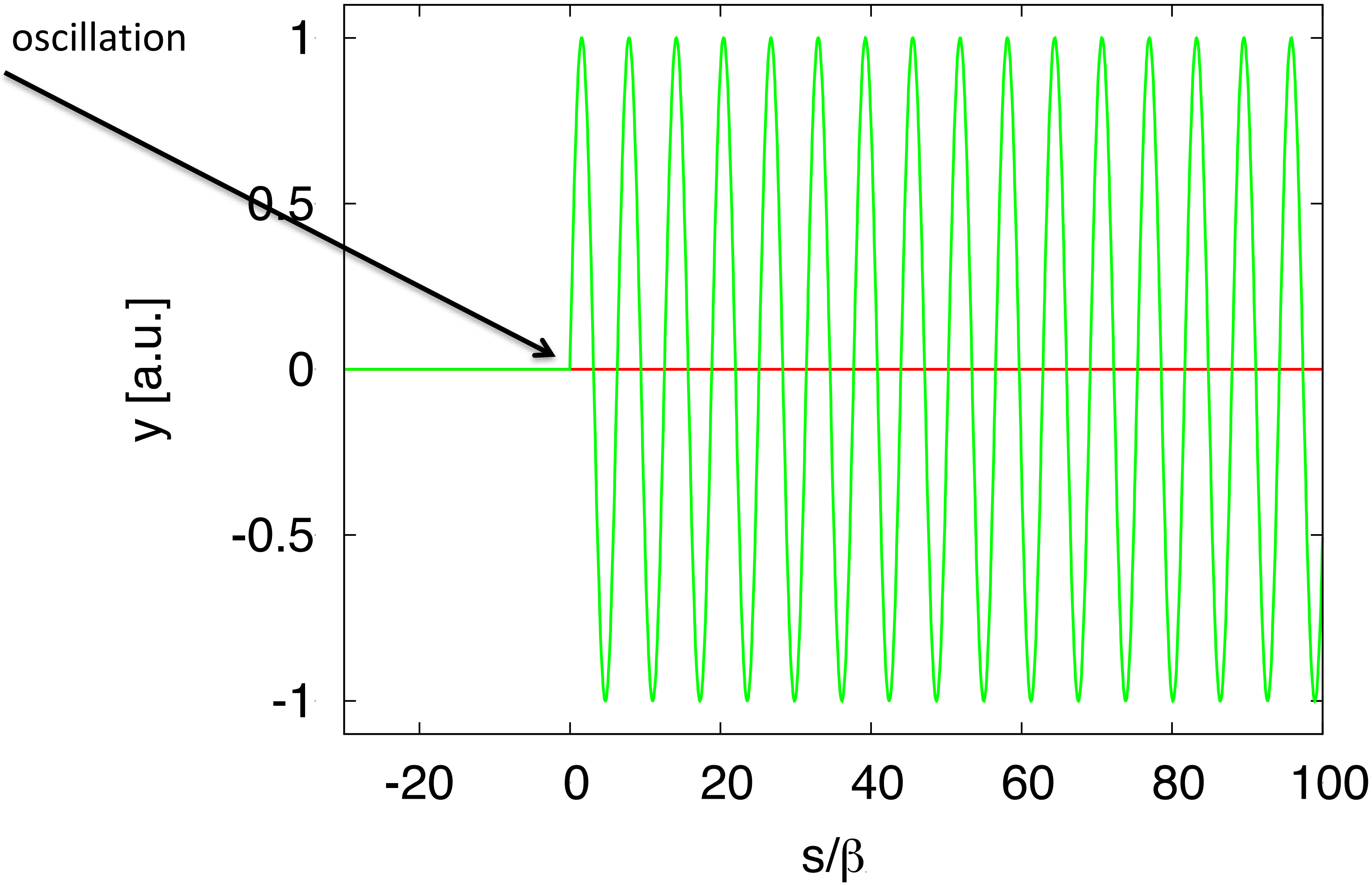
Shape due to wakefields

Apply wakefield kick to
make bunch straight again



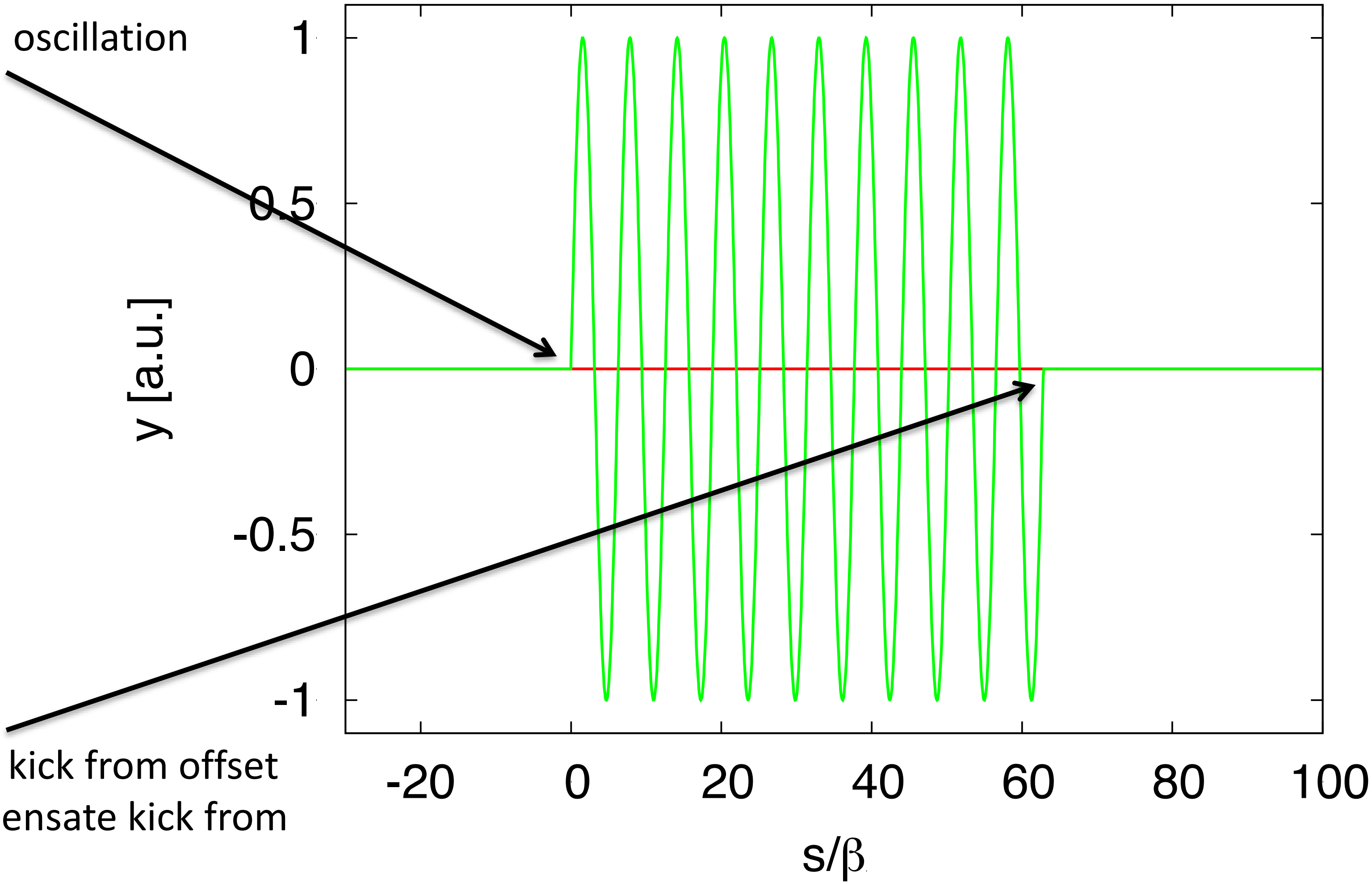
Simple Wakefield Model

Wakefield kick from offset structure
First particle is not kicked
Second starts and oscillation



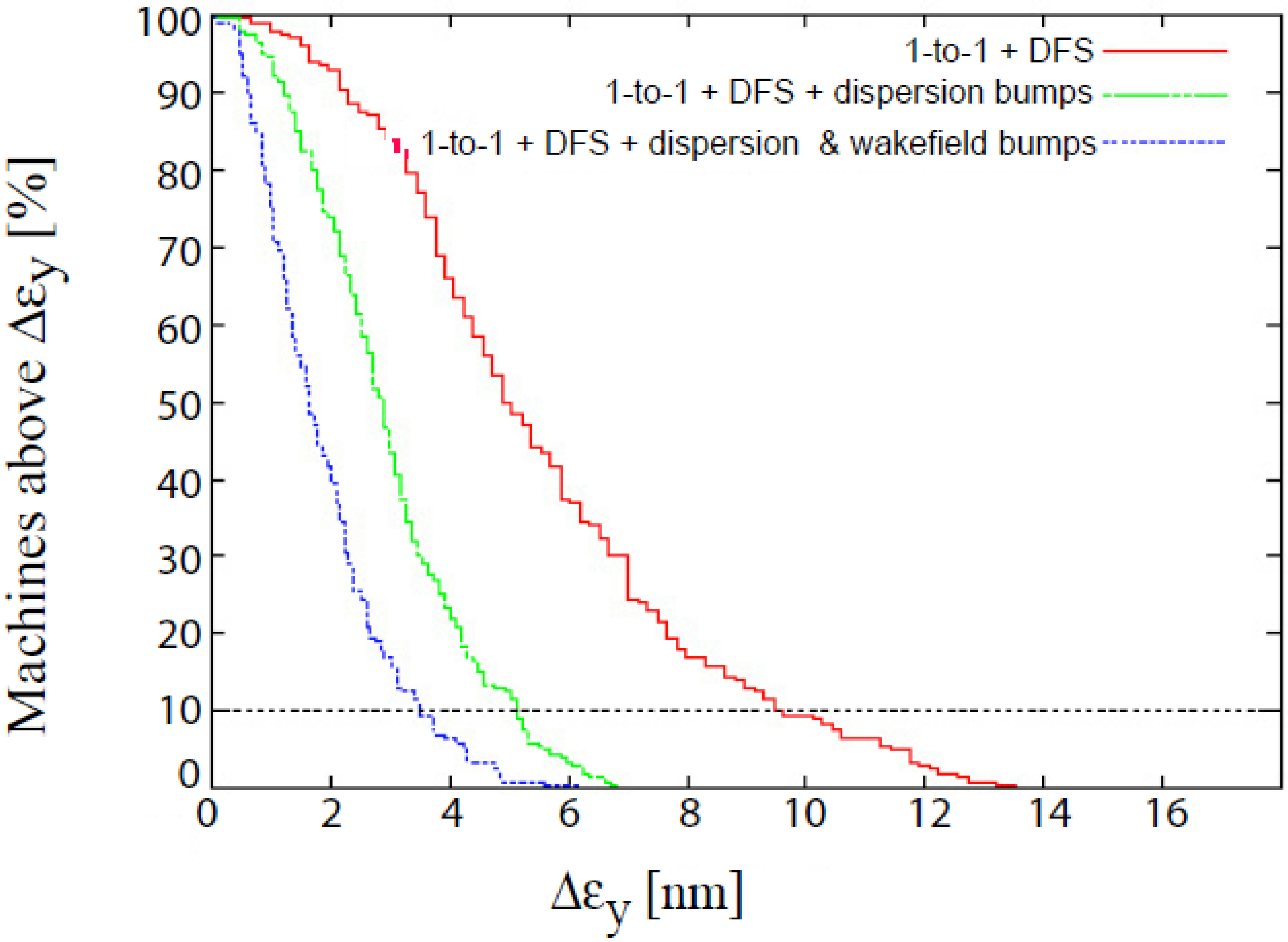
Simple Wakefield Bump Model

Wakefield kick from offset structure
First particle is not kicked
Second starts and oscillation



Second wakefield kick from offset
Happens to compensate kick from
first one

Some Old Example for ILC

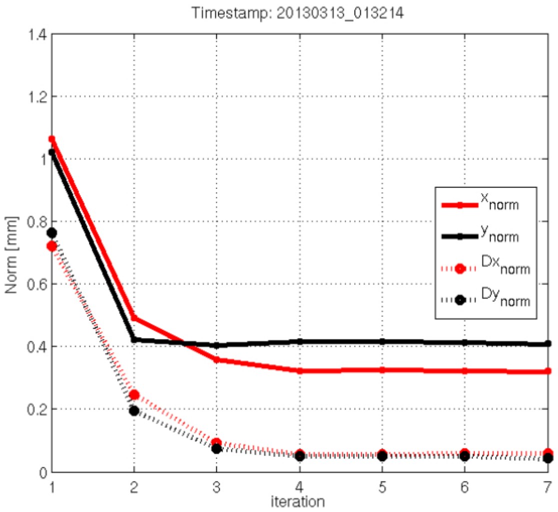


CLIC Beam-Based Alignment Tests at FACET

Dispersion-free Steering (DFS) proof of principle – March 2013

A. Latina,
J. Pfingstner,
E. Adli,
D. Schulte

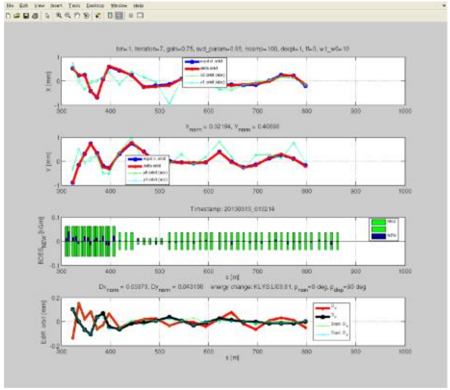
Orbit/Dispersion



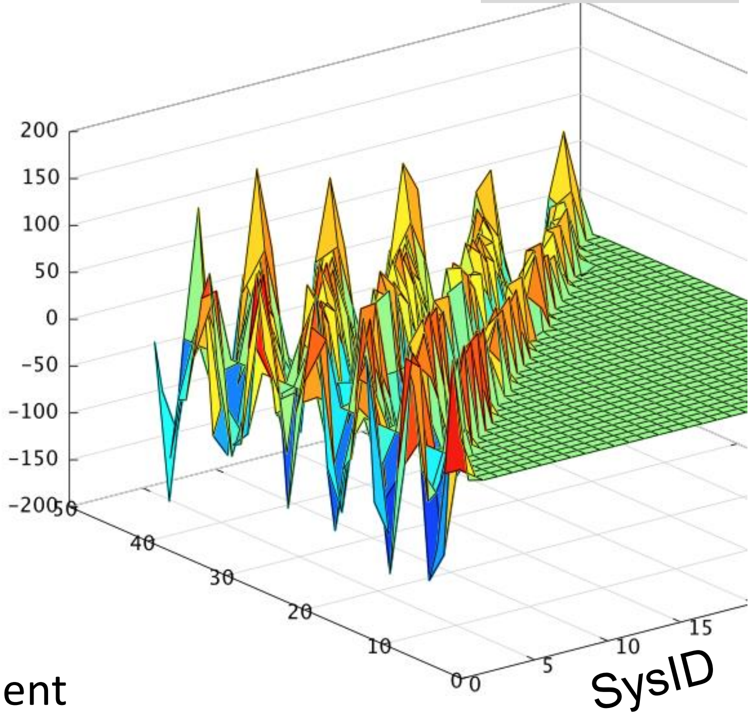
DFS correction applied to 500 meters of the SLC linac

- SysID algorithms for model reconstruction
- DFS correction with GUI
- Emittance growth is measured

Graphic User Interface:

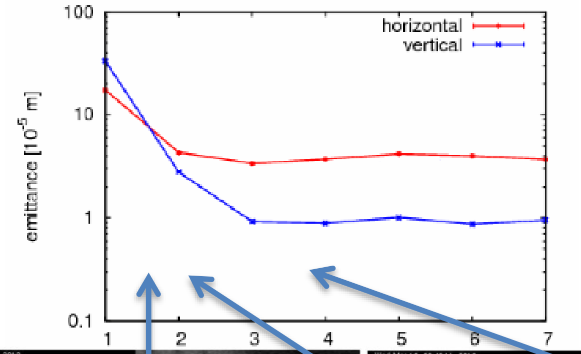


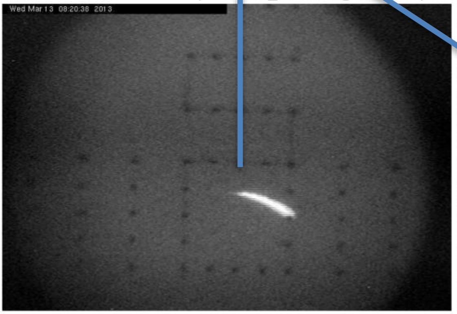
Beam profile measurement



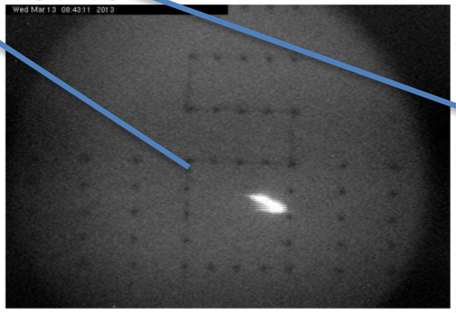
SysID

Emittance

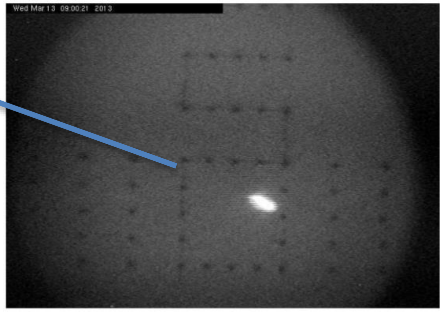




Before correction



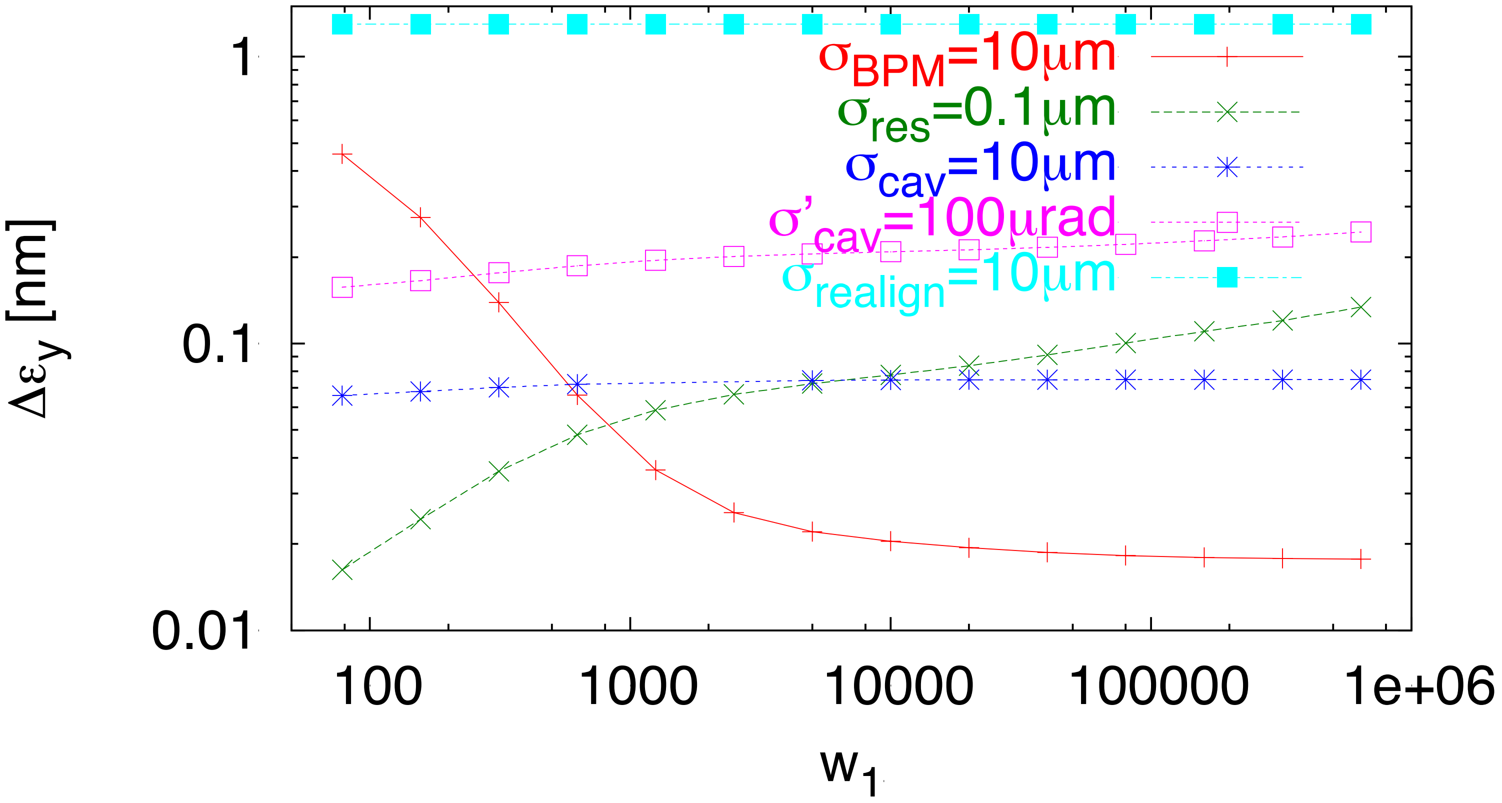
After 1 iteration



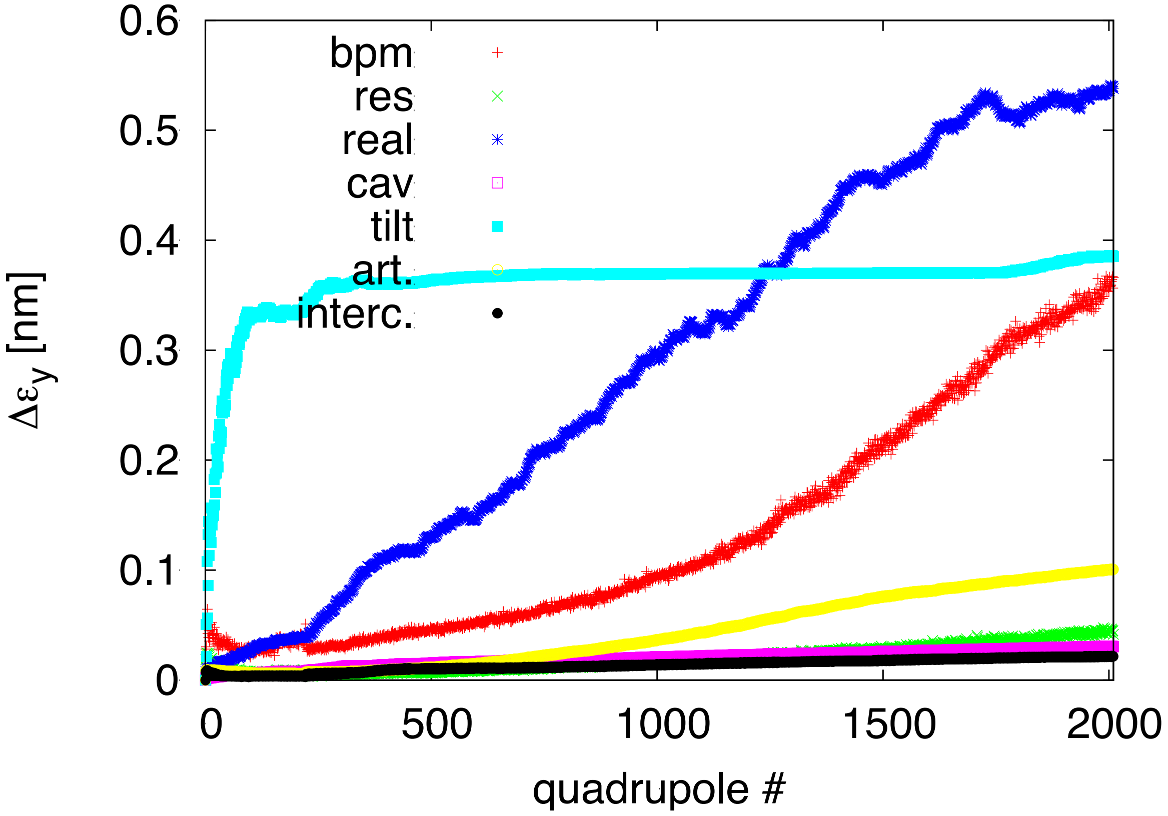
After 3 iterations

Incoming oscillation/dispersion is taken out and flattened; emittance in LI11 and emittance growth significantly reduced.

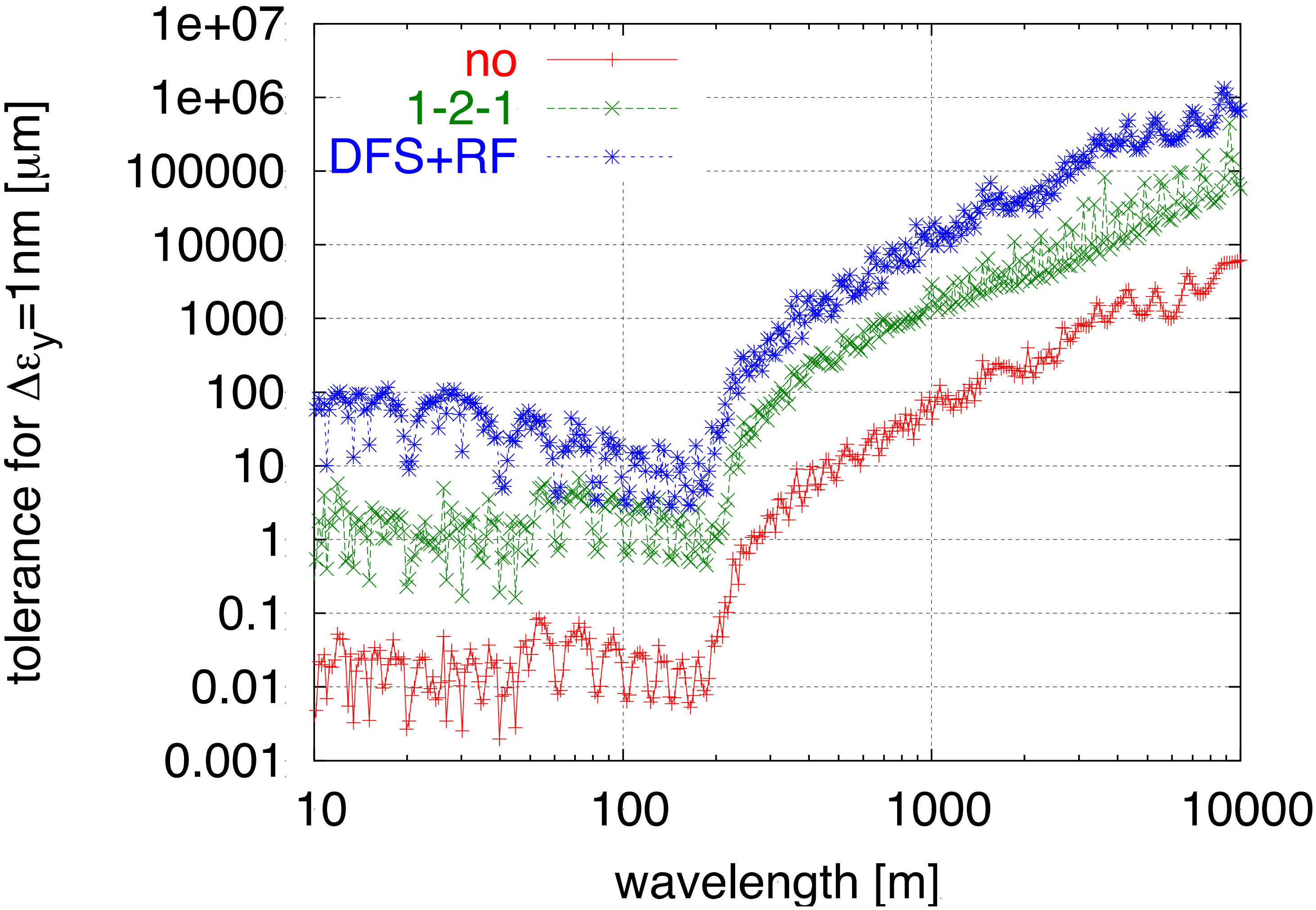
Note: Choice of Weights



Note: Emittance Along the Linac



Note: Emittance and Wavelength



Dynamic Imperfections

Dynamic Imperfections

Many sources exist, e.g.
- Ground motion
- Cooling water induced mechanical element vibration
- RF amplitude and phase jitter
- Magnet field jitter
- External magnetic field jitter
- …

They can compromise the luminosity
- Direct loss (trajectory jitter, emittance growth)
- Luminosity fluctuations can impact tuning
- Trajectory jitter can impact beam-based alignment

Need to mitigate them
- Beam-based feedback
- Stable hardware
- Specific systems

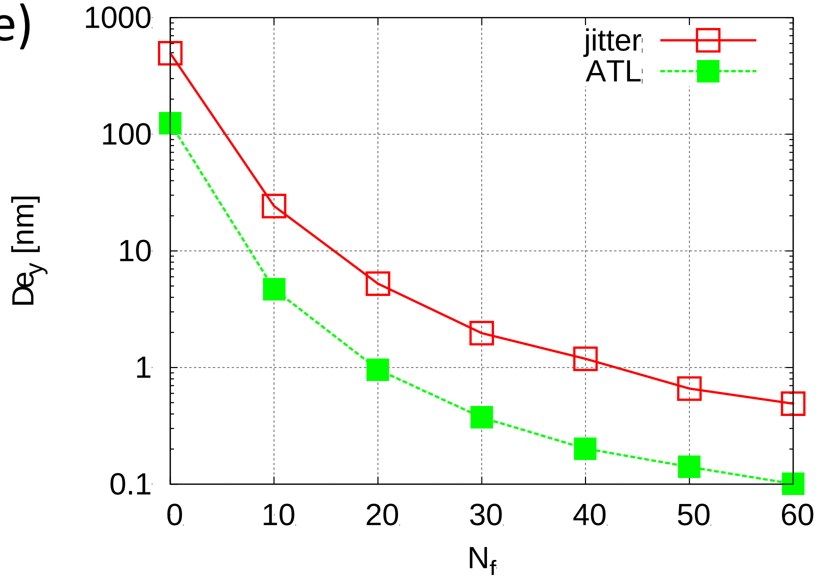
Need to consider the machine as a whole

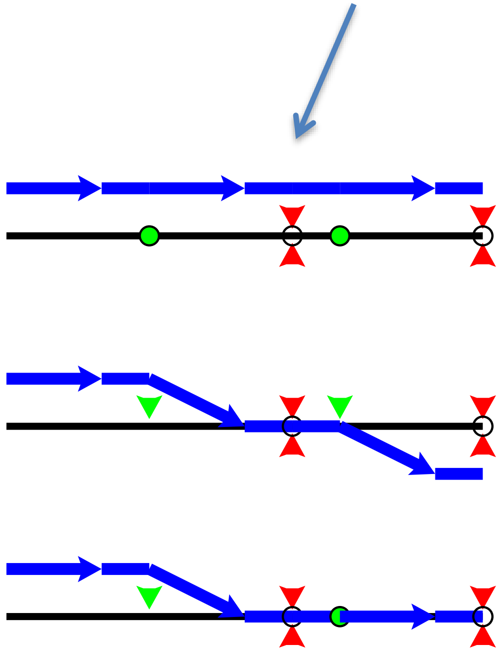
Feedback Design

Can use local feedback (fix trajectory in one place)

- But does not fix emittance growth

Several trajectory feedback points help
But lead to overcorrection if independent





Can use MIMO (Multiple Input Multiple Output)

- Take all information and correct globally

Feedback Design and Speed

Local feedback within a pulse

- Marginal for CLIC (e.g. beam-beam feedback)
- Possible for ILC, but bunch-to-bunch noise will be amplified along the machine

⇒(Very) Few loops

MIMO feedback within one beam pulse

- Need to communicate along machine, limited by speed of light
- Impossible for CLIC (170 ns beam pulse)
- Marginal for ILC (720 μs beam pulse, 60 μs roundtrip for linac)

⇒ Not really used

MIMO from pulse to pulse

- Possible in both machines

⇒ Important basis of the feedback systems, e.g. trajectory feedback
⇒ But cannot correct faster than 20 μs (CLIC) and 200 μs (ILC)

⇒ Use additional feedback systems, independent of the beam

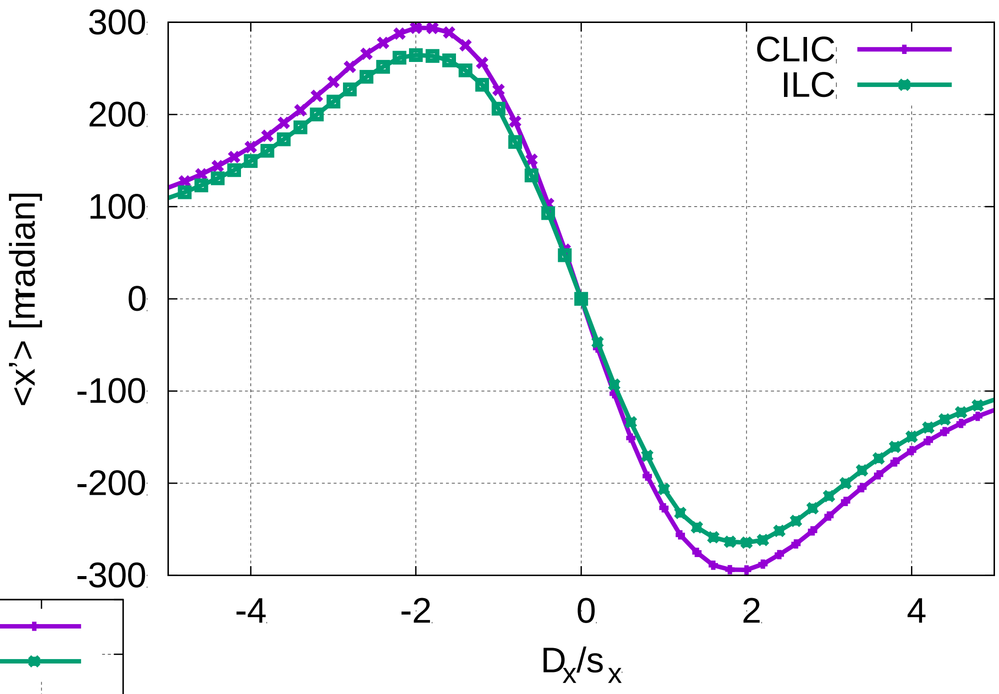
CLIC Beam-beam Feedback System

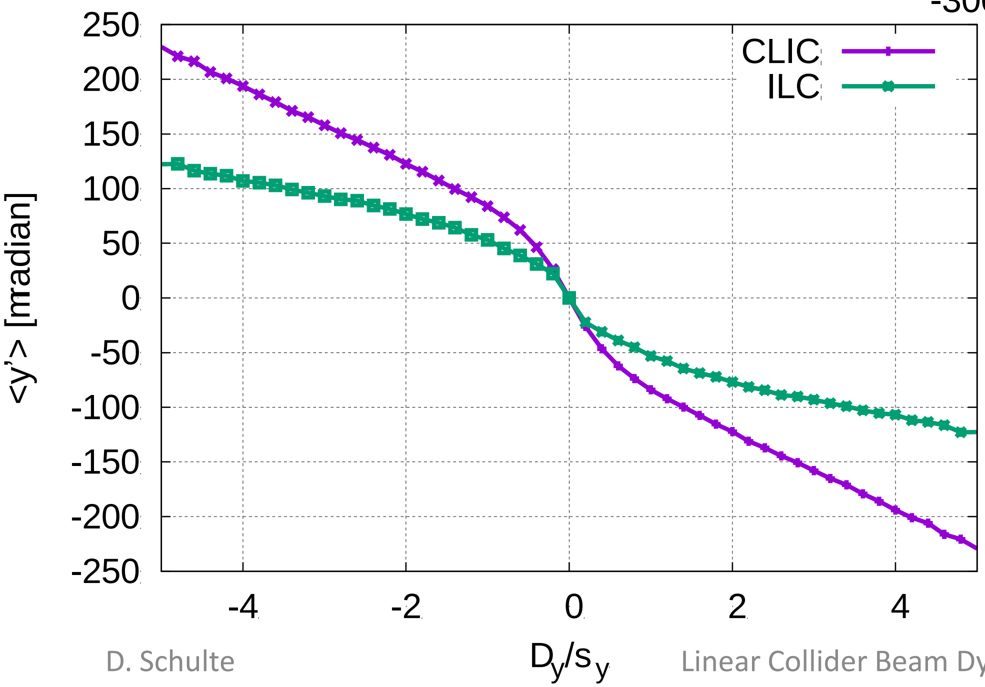
Strong deflection allows to easily measure and correct offset

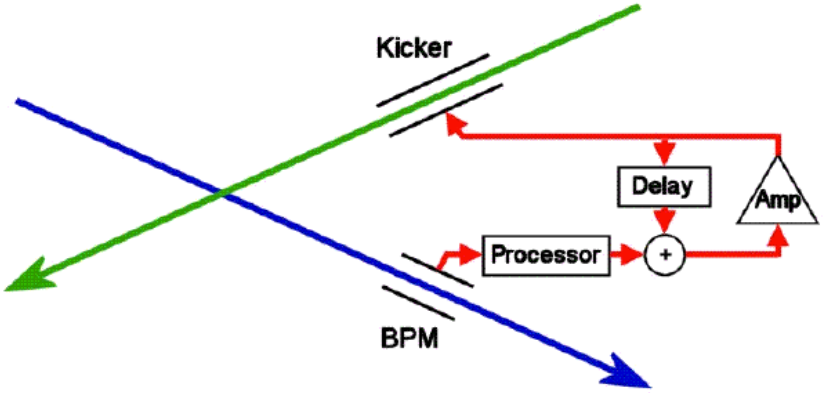
In CLIC an offset $\Delta_y = 0.1\sigma_y = 0.1\text{nm}$

$\Rightarrow$ 3m downstream of IP 40μm beam offset

Get great signals for the BPMs







FONT system (Oxford): 13 ns

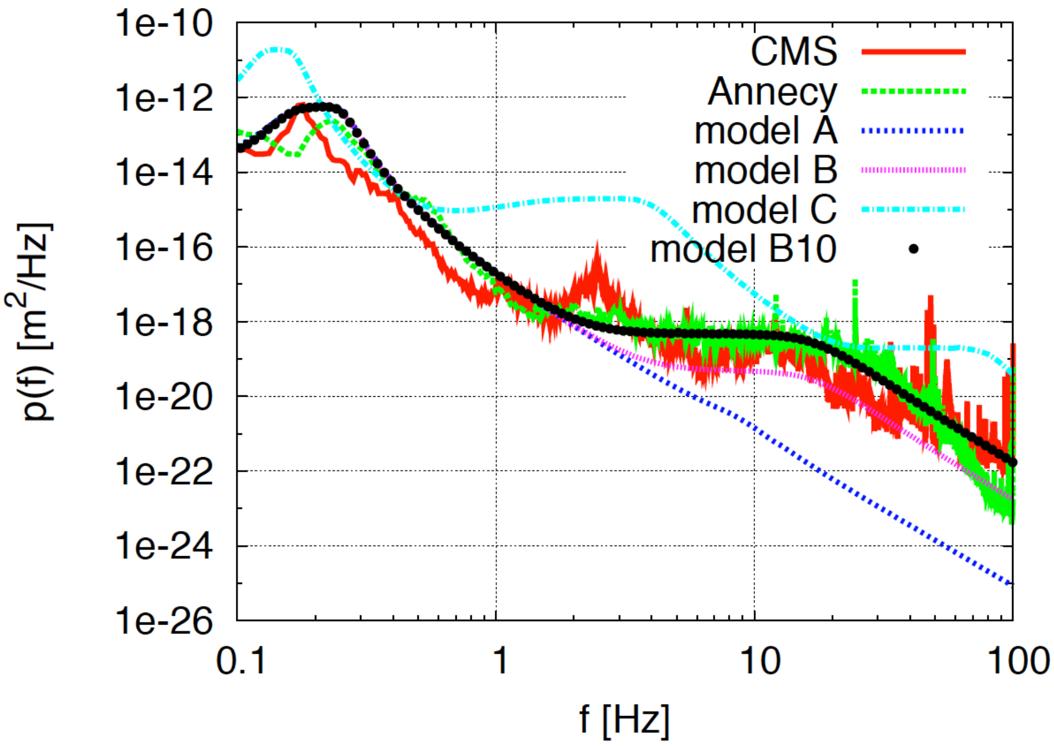
Example: Ground Motion

In CLIC can reduce dynamic effects at frequencies lower than a few Hz

$\Rightarrow$ Andrei Seryi
Friday 2.3.

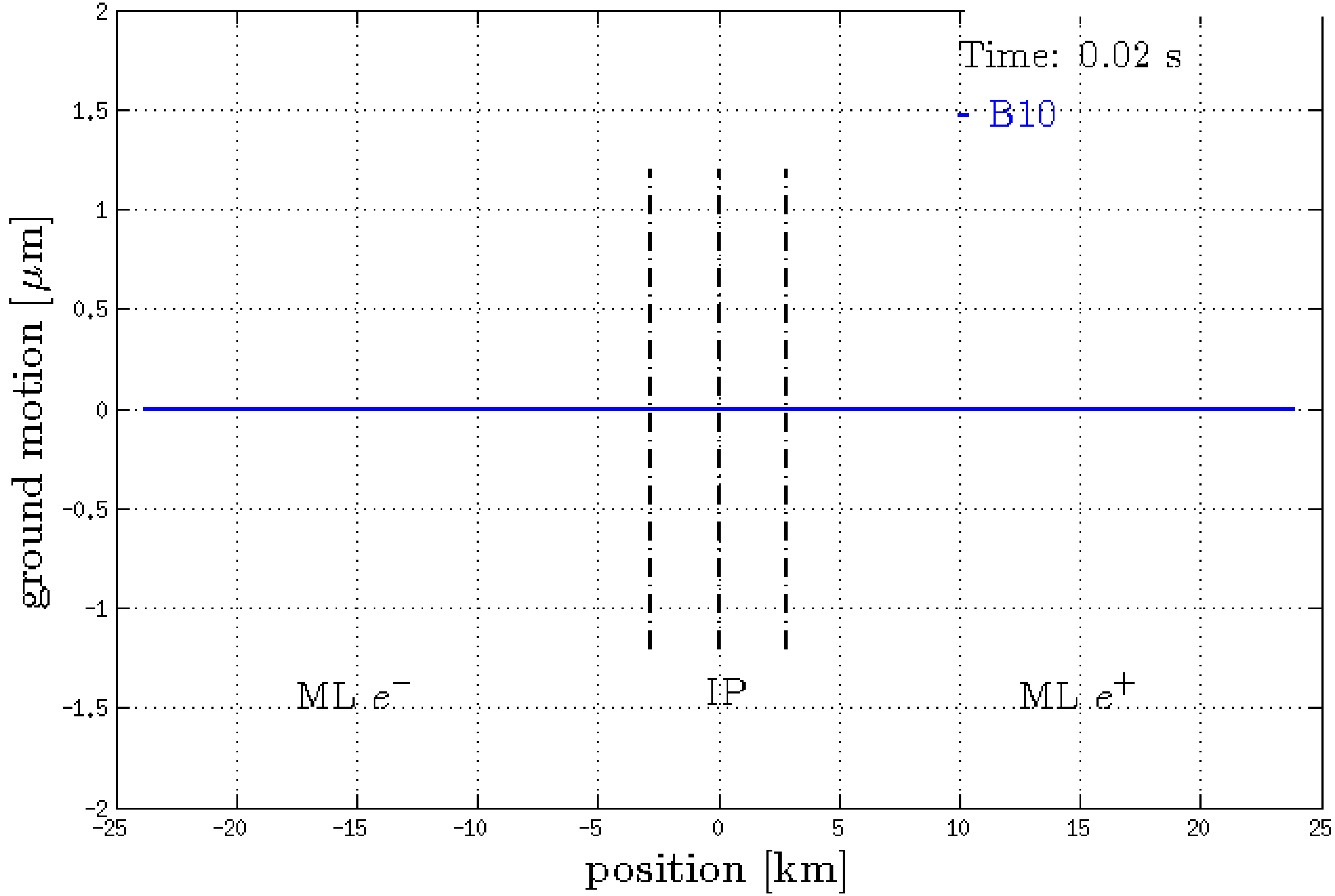
In ILC can use a bunch-bunch feedback system

- But be careful, bunch-to-bunch noise will be amplified
- e.g. the damping ring extraction kicker kicks each bunch separately, so it will induce noise



Example Issue: Ground Motion at CLIC

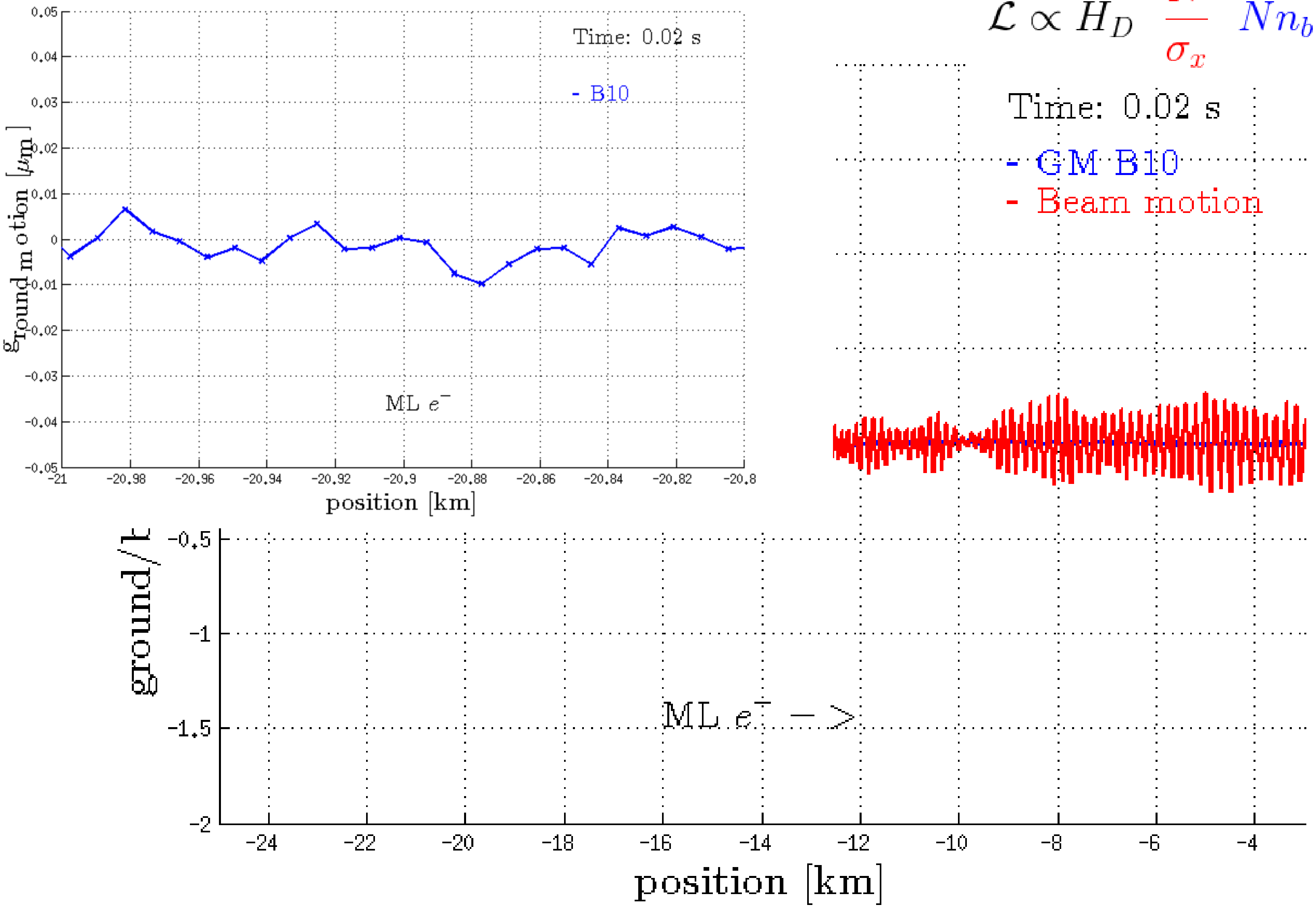
$$\mathcal{L} \propto H_D \frac{N}{\sigma_x} N n_b f_r \frac{1}{\sigma_y}$$



J. Pfingstner

Resulting Beam Jitter

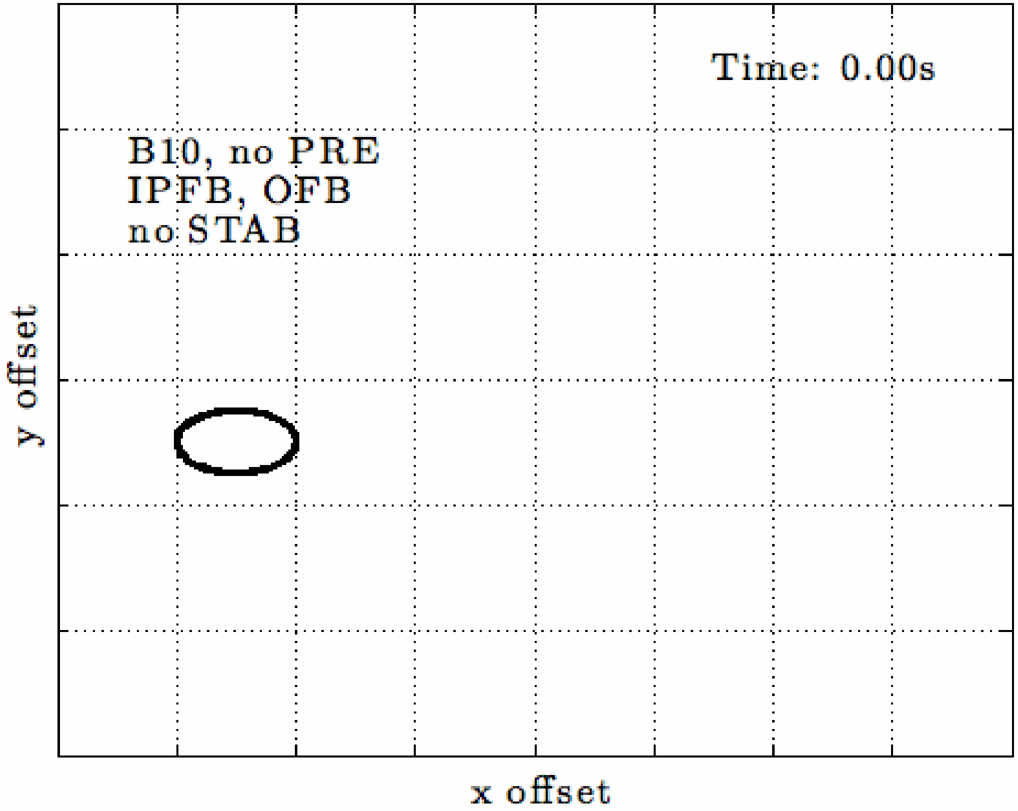
$$\mathcal{L} \propto H_D \frac{N}{\sigma_x} N n_b f_r \boxed{\frac{1}{\sigma_y}}$$



J. Pfingstner

Beams at Collision

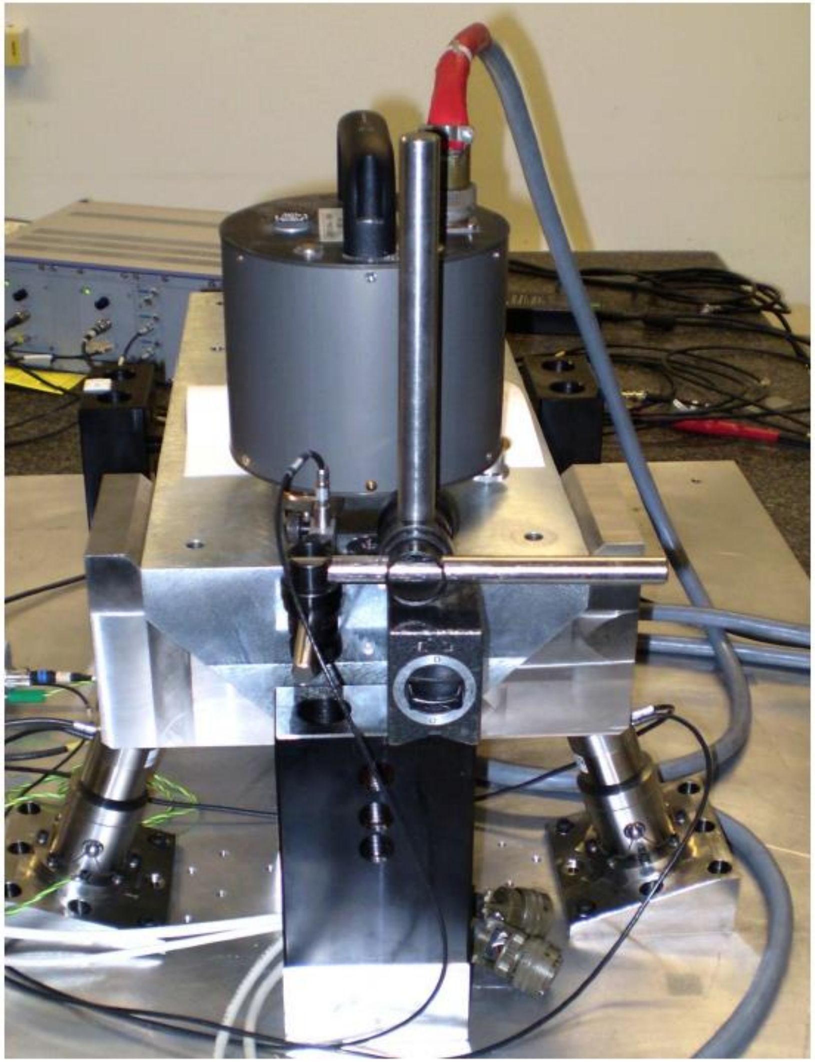
$$\mathcal{L} \propto H_D \frac{N}{\sigma_x} N n_b f_r \boxed{\frac{1}{\sigma_y}}$$



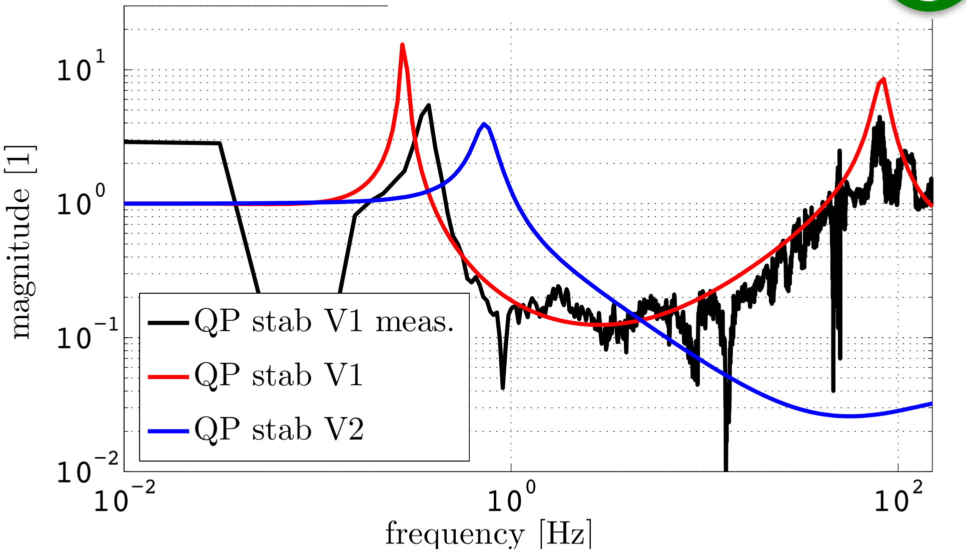
J. Pfingstner

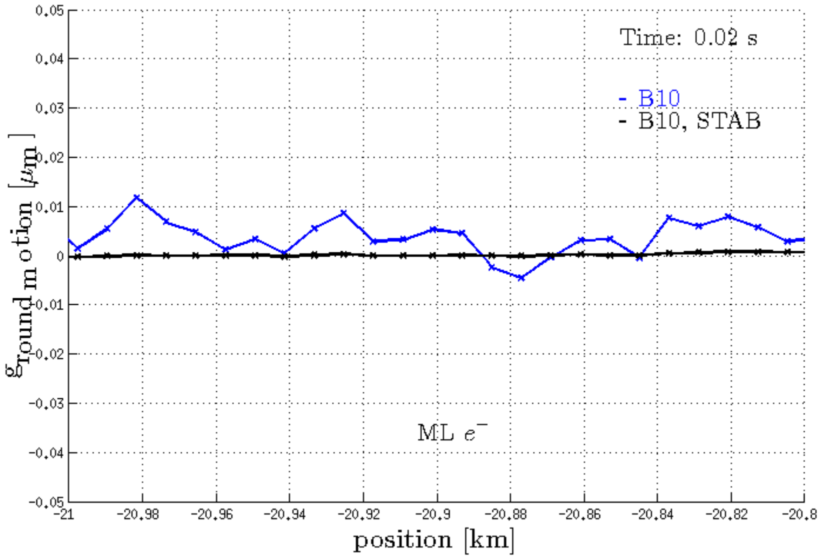
Stabilisation System

$$\mathcal{L} \propto H_D \frac{N}{\sigma_x} N n_b f_r \boxed{\frac{1}{\sigma_y}}$$



K. Artoos et al.

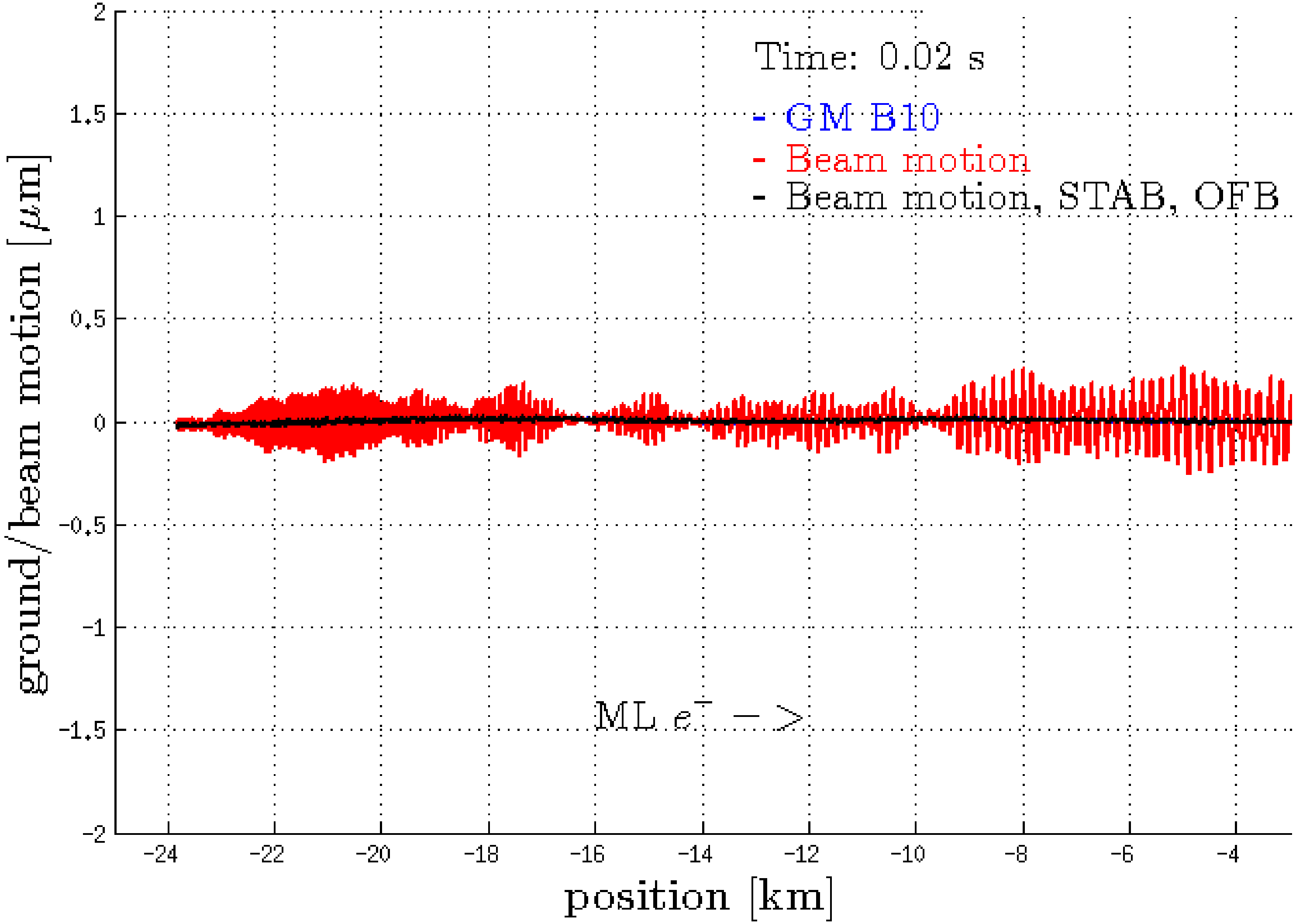




J. Snuverink, et al.

Impact of Stabilisation on Beam

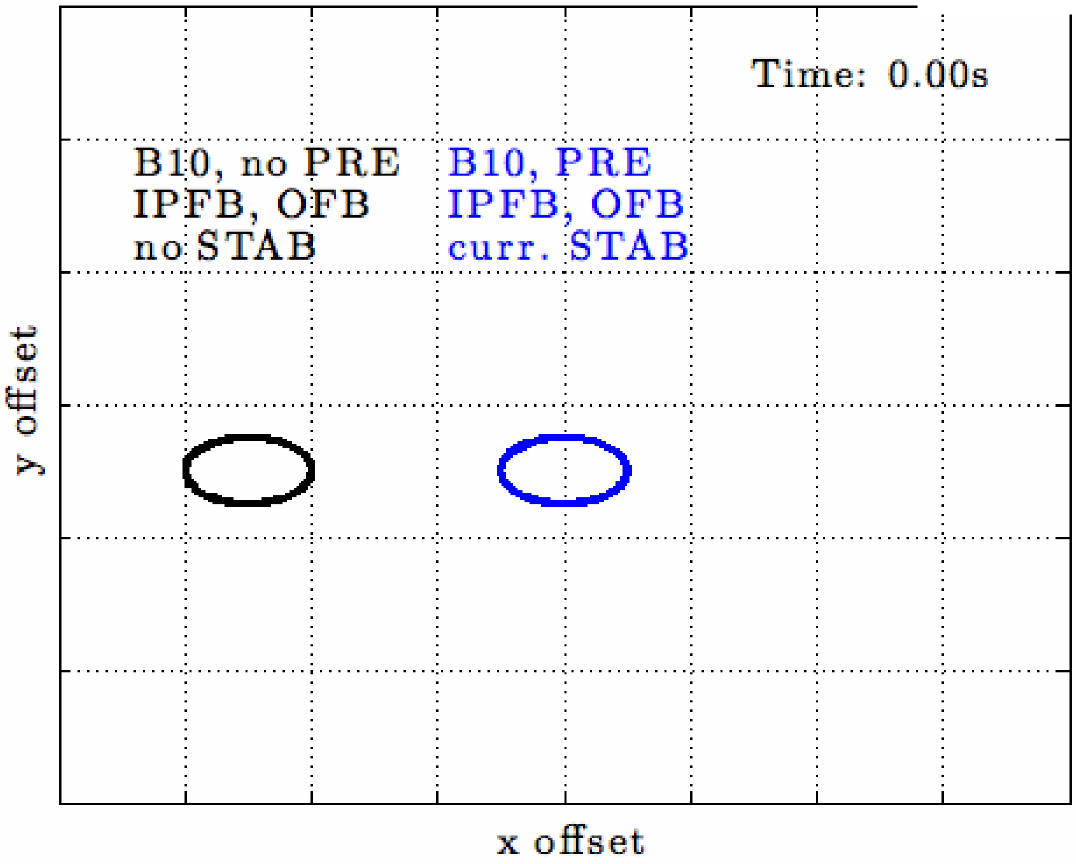
$$\mathcal{L} \propto H_D \frac{N}{\sigma_x} N n_b f_r \frac{1}{\sigma_y}$$



J. Pfingstner

Beam at Collision

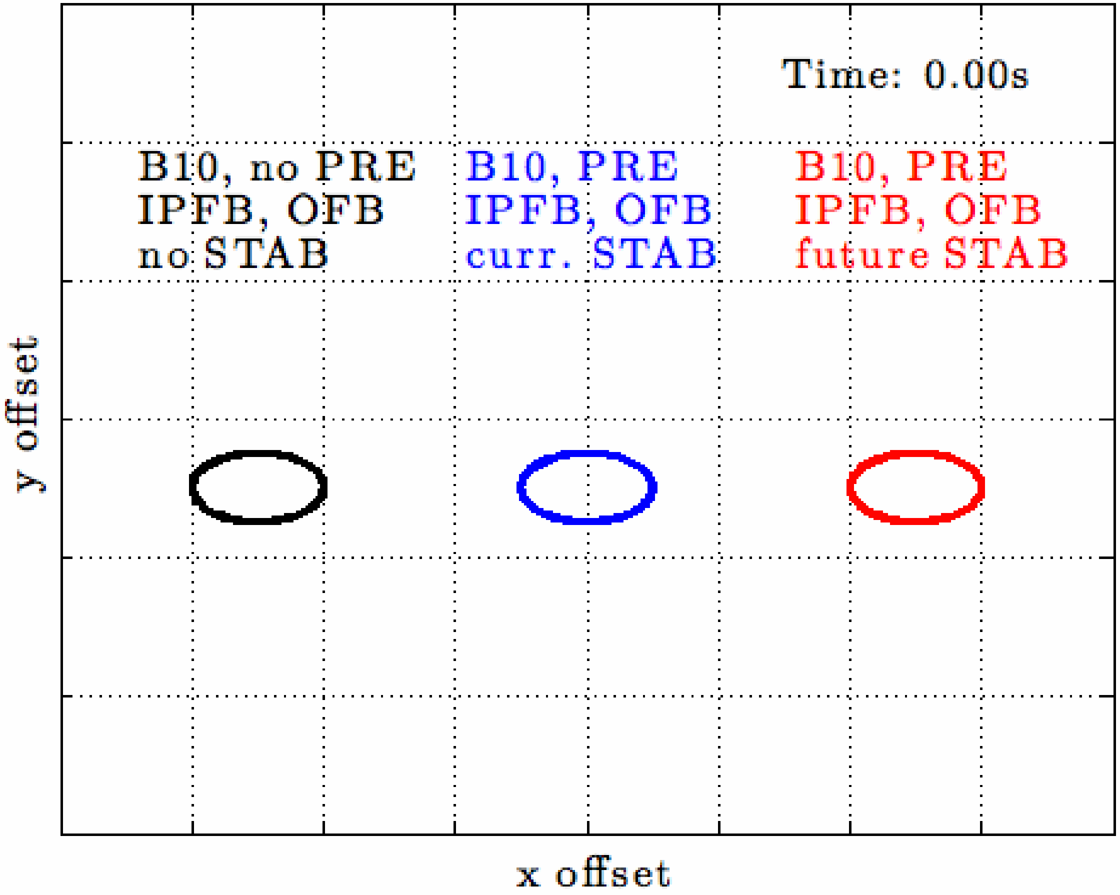
$$\mathcal{L} \propto H_D \frac{N}{\sigma_x} N n_b f_r \boxed{\frac{1}{\sigma_y}}$$



J. Pfingstner

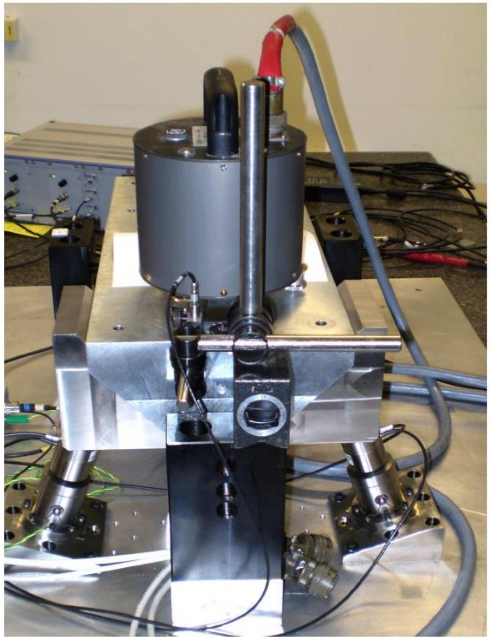
Beam at Collision

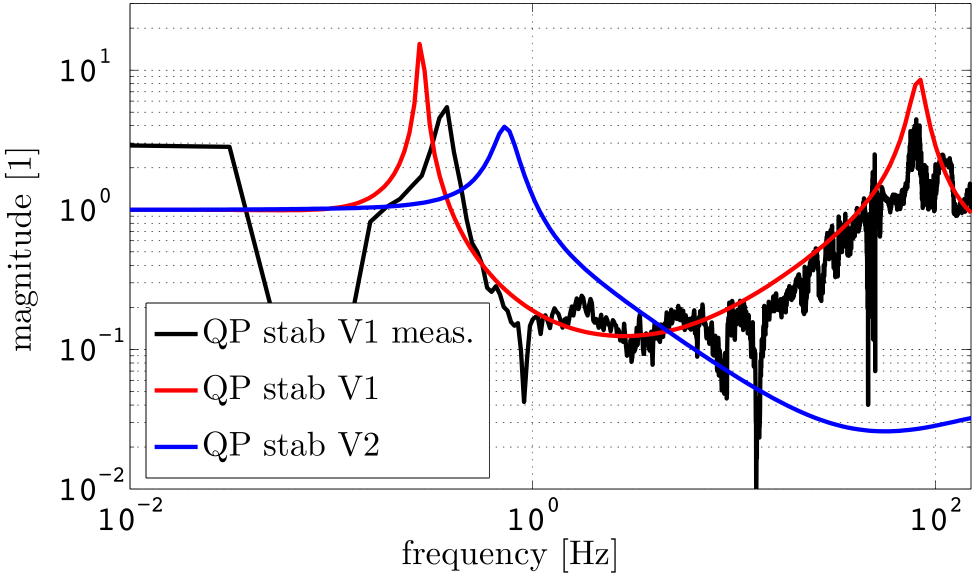
$$\mathcal{L} \propto H_D \frac{N}{\sigma_x} N n_b f_r \boxed{\frac{1}{\sigma_y}}$$

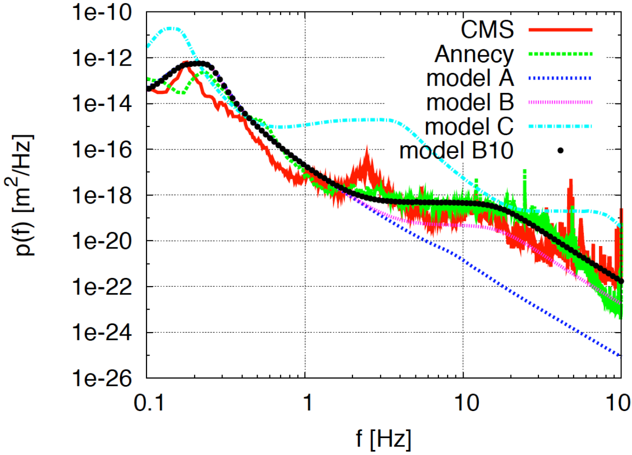


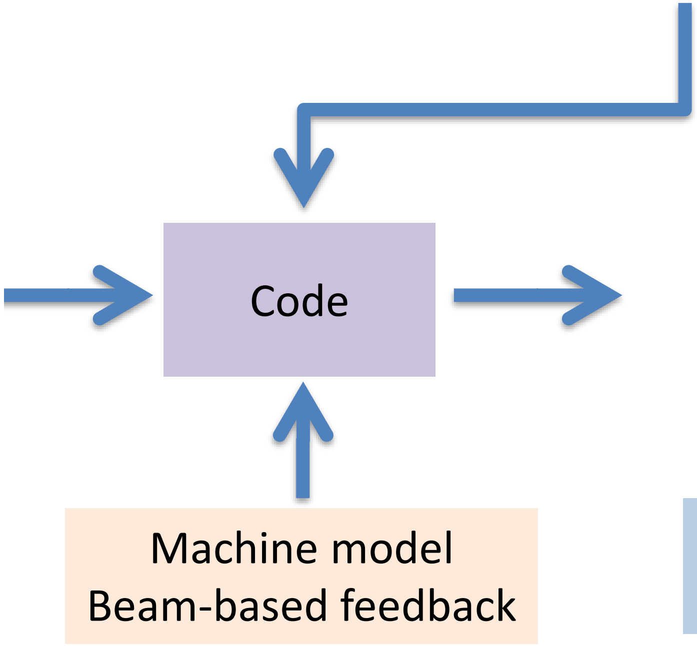
J. Pfingstner

Active Stabilisation Results









Code

Machine model
Beam-based feedback

Luminosity achieved/lost [%]	
	B10
No stab.	53%/68%
Current stab.	108%/13%
Future stab.	118%/3%

Close to/better than target

Note: The Banana Effect

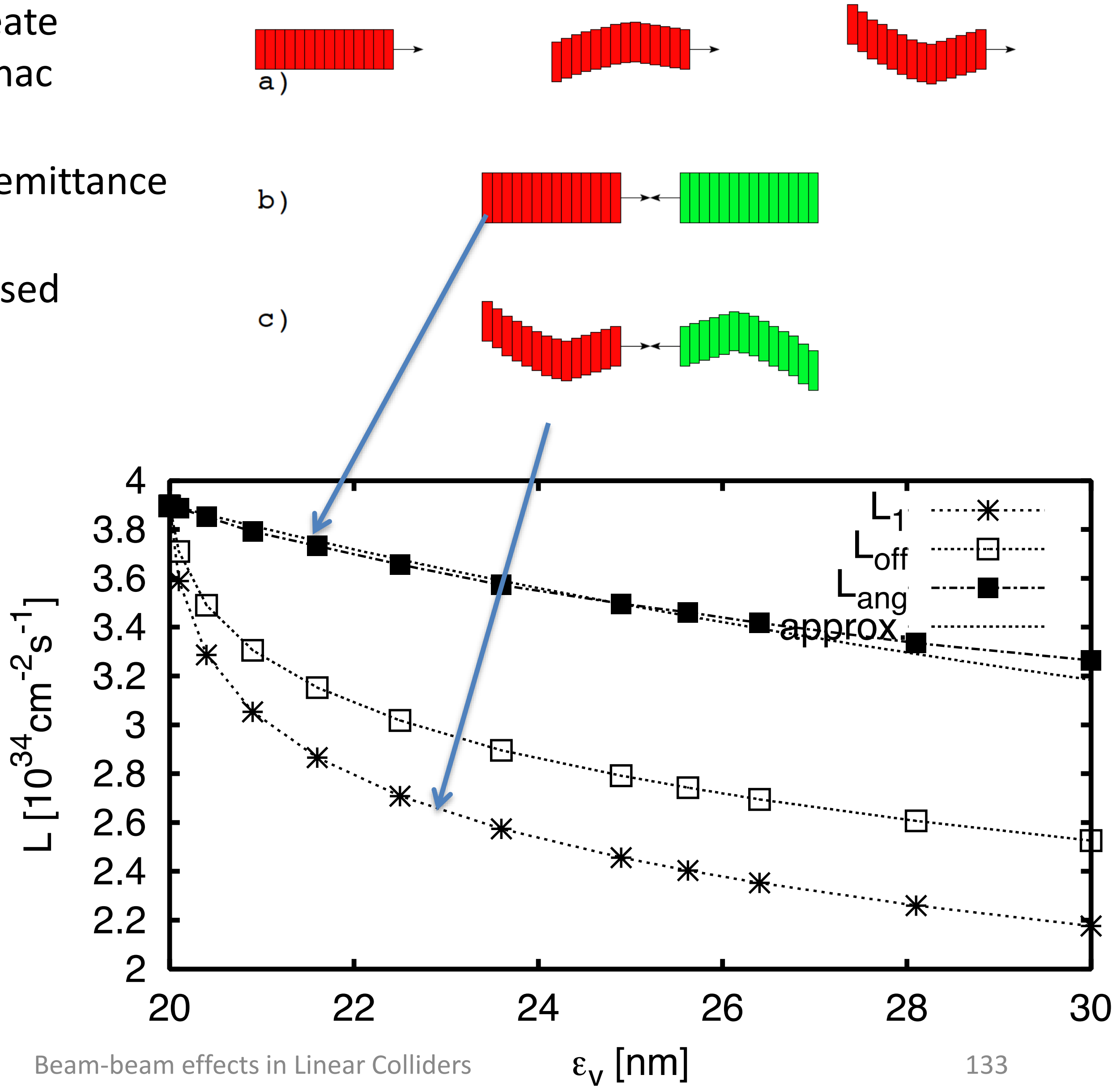
a) Wakefields+dispersion can create banana-shaped bunch in main linac

b) Do not model with projected emittance

c) The correct shape should be used

For large disruption (ILC) banana can reduce luminosity

Study done for TESLA
Similar disruption as ILC

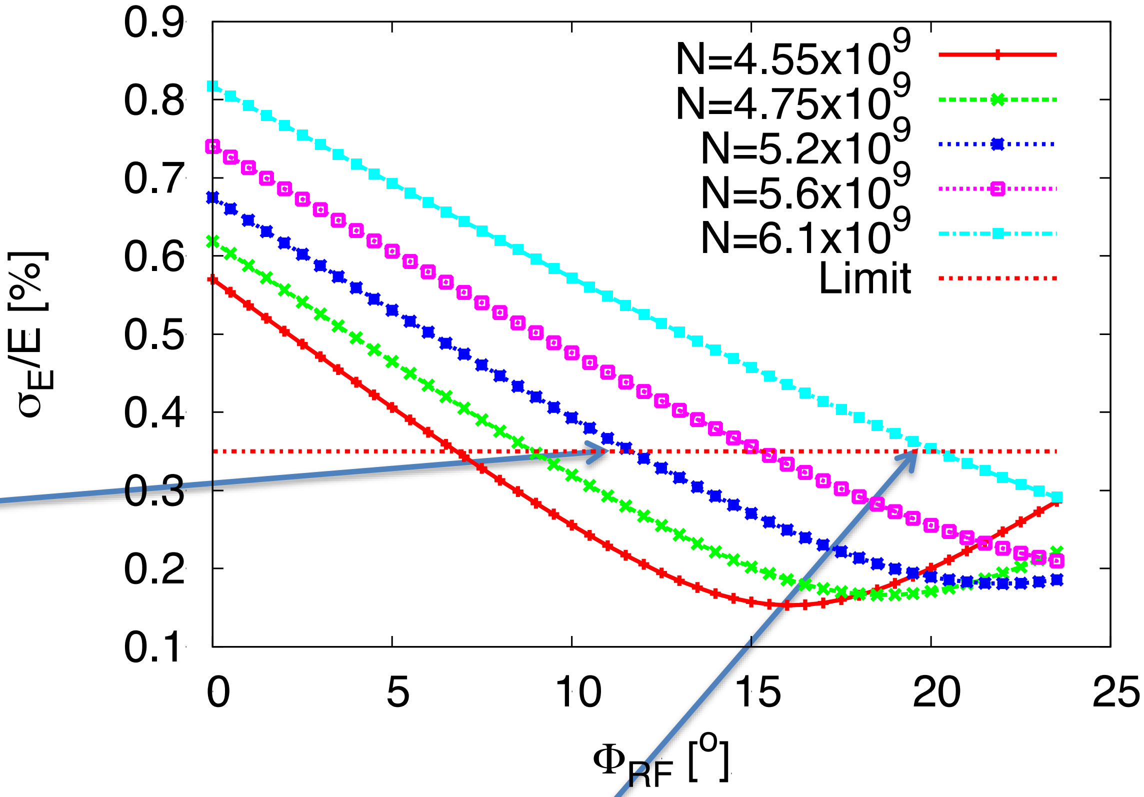


Conclusion

Reserve

Note: Choice of RF Phase

Examples of beams with the same transverse wakefield effects
i.e. larger N means shorter bunch



CLIC bunch at 380 GeV
Running at 12°
2% gradient loss

12° is a good compromise

17% more charge requires 20°
6% gradient loss
More sensitive to phase jitter