CAS on Future Colliders

Case Studies on FCC-ee

... principle attitude:
define the logical path to a feasible design of a state of the art high-energy lepton collider.

TASK:
Design a e+/e- collider with a beam energy of 45 - 180 GeV,
an overall length of 100km and that can provide in parallel
a luminosity of L=10^34 cm^-2 s^-1 to two experiments.

Literature:

Wolski: CAS, CERN-2014-009
Wille: "The Physics of Particle Accelerators"
Teng: Fermilab, TM-1269-0102-000, "Minimising the Emittamnce in Designing the Lattice of an Electron Storage Ring"
Holzer: CAS, CERN-2014-009, "Lattice Design"

Items to discuss:

1.) Linear collider or Circular machine ?
motivate your opinion
We clearly go for a circular machine.
The famous plot luminosity < -- > power for Linear Collider / Ring collider makes it evident.

2.) Luminosity:
Establish the luminosity formula in its basic form and determine the limitations

$$L = \frac{n_b \ f_0 \, N_p^2}{4 \, \pi \sigma_x^* \sigma_y^*}$$

where we assume equal beam size in the two beams.
Limitations:
Single bunch intensity $N_p \leq 2*10^{11}$
Bunch number n_b
ultimate limit = harmonic number; in general the luminosity equation tells us to keep the bunch number as small as possible. It enters linear into the equation, the bunch population however quadratic.
So for a given overall limit in current better push for highest bunch population in few bunches. Example: E-180 GeV, $n_b = 50.$
Beam sizes $\sigma_x^* \sigma_y^*$: determined by emittances and betas. See below.

3.) Determine the limit of the stored beam current
if the overall synchrotron radiation power should not overpass 50MW.
Determine the optimum number of bunches for E_{beam} = 180 GeV.

In medias res: synchrotron radiation power determines everything.

$$P_\gamma = \frac{e^2 c}{6\,\pi \varepsilon_0} * \frac{\gamma^4}{\rho^2}$$

Circumference = 100km → bending radius of dipoles ρ = 12km
(assuming a fill factor of 0.75).
$\gamma = \frac{E_{beam}}{E_0} = 3.5 * 10\text{^}5$
$e = 1.6 * 10^{-19} Cb$
$\varepsilon_0 = 8.8 * 10^{-12} AS/Vm$

$P_\gamma = 4.8 * 10^{-6}$W per particle.

→ maximum number of stored particles:
N_{tot} =1*10^{13}

4.) Number of Bunches:
bunch intensity limit: $N_p \leq$ 2*10^11
→$n_b = \frac{N_{tot}}{N_p} = 50$
we could distribute over more bunches, however for best luminosity we will not do so.

5.) Lower Energy:
How does the situation change if the machine is supposed to run on the Z resonance (E_{beam} = 45 GeV)
(ultimate limit is set by the single bunch intensity on one side and the harmonic number on the other extreme).

$$P_\gamma = \frac{e^2 c}{6\,\pi \varepsilon_0} * \frac{\gamma^4}{\rho^2}$$

The power scales with γ^4 so we can scale the stored current accordingly.
$N_{tot}\,(45\ GeV)$ =1*10^{13} $* \left(\frac{180}{45}\right)^4$=2.5* 10^{15} particles

$n_b = \frac{N_{tot}}{N_p} = 12500$ bunches

Nota bene: The bunch distance for equidistant bunches would be 8m.
Minimum bunch distance, given by a typical 400MHz system would be 0.75m. So we indeed have sufficient space for the design bunch number.

6.) Determine in both cases the rf system.
(frequency, h, installed Voltage, length of the rf sections, where in the ring)
The main issue here is the installed rf Voltage. (Power comes afterwards as second issue).
Energy loss per turn:

$$\Delta E_{turn} = \frac{e^2}{3\pi\varepsilon_0} * \frac{\gamma^4}{\rho} = 7.6\ \text{GeV/turn}$$

for each and every particle. This energy loss has to be compensated by the installed voltage provided by the rf system.

Assuming 20 MV/m acc. Gradient per cavity we need 380 cavities.
Installed in two rf sections it takes 190 m long straight section (one meter per rf cavity is assumed).
The sections should be symmetric to balance the saw tooth effect as much as possible;
They should be dispersion free;
And 4 sections (or more) would even improve the situation.

In a perfect situation each cavity would carry 132 kW power.
Question to discuss:

- Single cavities / multi cells ?
- Input coupler load in case of a multi cell ?
- How does the situation change with energy ?

6.) Choose an adequate arc structure:
(DBA, Doublet, FoDo, etc → dipole fill factor & beam rigidity
here a wide range of options is possible, each leading to some different emittances, the students should discuss these options and motivate their decision)

The structure of highest dipole fill factor is a FODO.
In a DBA e.g. we get fill; factors in the order of 30-40 %. In a FODO we can push up to more than 75%.
As reasonable starting value we choose

Cell length	l_{cell}=50 m
Phase advance per cell:	$\phi_{cell} = 90\ degrees$

The choice of 90 degrees is an optimum for smallest beam sizes and for most effective chromaticity correction (fully non-interleaved sextupole schemes). However this might be a bit beyond the level of the students.

Calculating beta:

$$\hat{\beta} = \frac{(1+\sin\frac{\psi_{cell}}{2})L}{\sin\psi_{cell}} \qquad \beta_{max} = 85\ m$$

$$\breve{\beta} = \frac{(1-\sin\frac{\psi_{cell}}{2})L}{\sin\psi_{cell}} \qquad \beta_{min} = 15\ m$$

Which results in a hor. beam size in the arc of 0.33mm.
Typical aperture needs: 20 σ → 7mm.

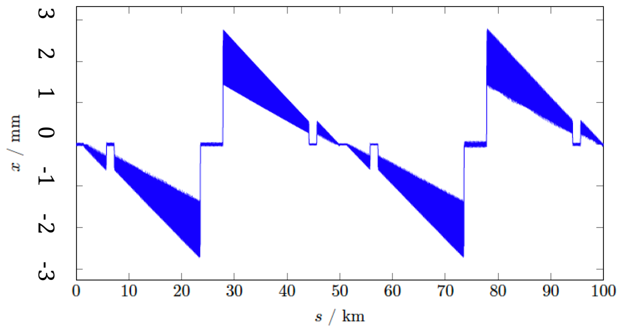
For completeness: Dispersion:

$$\hat{D} = \frac{\ell^2}{\rho} * \frac{\left(1+\frac{1}{2}\sin\frac{\psi_{cell}}{2}\right)}{\sin^2\frac{\psi_{cell}}{2}}, \qquad D_{max} = 11\ cm$$

6a.) Saw tooth effect:

Beam energy loss per turn: $\Delta E_{turn} = 7.6\ GeV$
Assuming 2 symmetricaly placed rf systems, the energy loss from one rf station to the next is 3.8 GeV.

Orbit-offset: $\Delta x = D * \frac{\Delta p}{p} = 11cm * \frac{3.8GeV}{180\ GeV} = 2.3$ mm



******* 7.) Give a first idea about the equilibrium emittance that can be expected***
(-- horizontal & vertical --)
This is clearly a **** question. We refer to Tengs paper on emittance scaling.
In general for a wide band of situations the hor. equilibrium emittance is given by

$\varepsilon_x = \frac{C_q}{J_x} * \gamma^2 \theta^3 F$ with F = 2.5 for a typical FODO

$J_x = 1$

$C_q = 3.8 * 10^{-13}$ m

θ describes the bending angle of the dipoles between the focusing elements.

We design the single cell as L_{cell} =50m, and the length of the dipole chain between the quads as L_B = 36m (75% Dipole fill factor).
100km using 50m long cells makes 2000 cells, each containing 2 dipoles.
→ bending angle per dipole = 1.5 mrad
scaled on 75% dipole fill factor in the ring makes 2mrad bending angle.

→ $\varepsilon_x = 1\ nm.$

… which is indeed on the spot of the FCC-ee design. ;-)
Assumean emittance coupling ration of 10^{-3} whichb is challenging, but for light sources state of the art, we claim that we can get

$\varepsilon_y = 1\ pm.$

These values we can use now to determine finally the luminosity (i.e. the beta * values).

*** 8.) ***Luminosity:***
Given a free space of s= +/- 2m, determine the required beta-function at the IP in x and y, to obtain the required luminosity.
(Hint: assume equal aperture requirements in x & y at the first mini beta quadrupole).

$$L = \frac{n_b\ f_0\, N_p^2}{4\ \pi \sigma_x^* \sigma_y^*}$$

In order to determine the condition for equal aperture requirements we need the scaling of the beta-function.

$$\beta_x(s) = \beta_x^* + \frac{s^2}{\beta_x^*} \quad , \quad \beta_y(s) = \beta_y^* + \frac{s^2}{\beta_y^*}$$

$\sigma_x(s) = \sigma_y(s)$ at s=2m.

So we can determine a condition for β_y^* as a function of β_x^*

$$\beta_y^* = \frac{s^2}{\varepsilon_x / \varepsilon_y * (\beta_x^* + \frac{s^2}{\beta_x^*})}$$

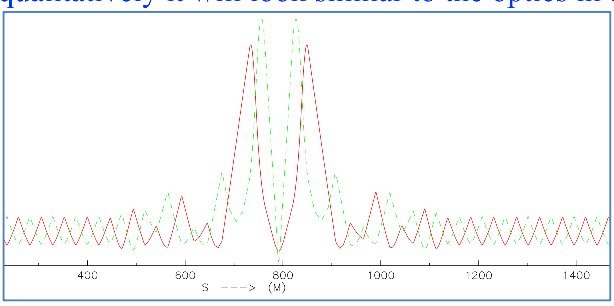
with s = 2m and assuming β_x^*=1m we get
β_y^*=1 mm
a pretty small value.

So for 180 GeV, n_b = 50 bunches, f_0=3003 Hz, β_x^*=1 m, β_y^*=1 mm, $N_p = 2*10^{11}$ particles we get

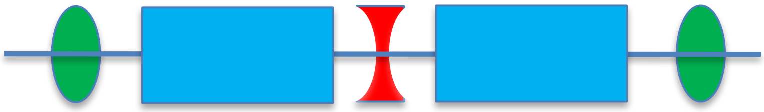
$L = 5*10^{34}\ cm^{-2}\ s^{-1}$

The design parameter list foresees:
β_x^*=1 m, β_y^*=2 mm
→ $\varepsilon_x = 1.5\ nm$, $\varepsilon_y = 2.7\ pm$, and the scaling developed here brings us nicely to the design luminosity of $L = 1.5*10^{34}\ cm^{-2}\ s^{-1}$

qualitatively it will look similar to the optics in the figure.



9.) Dipole Design:
(field, length, critical energy)
basic cell: FODO, L_{cell} = 50m



C=100km, dipole fill factor =75%,
Bending radius ρ = 12000m

→ bending angle per dipole:
$\theta = \frac{L_B}{\rho} = 3\,mrad$
number of dipoles needed:
$$N_B = \frac{2\pi}{\theta} = 2100$$

Dipole field:

Beam rigidity: $B * \rho = \frac{p}{q} = \frac{180\frac{GeV}{c}}{q} = 600Tm$

Bending radius: $\rho = 12000\ m$
→ B=500 Γ

Dipole length = 10m (guess why ?)

**** 10.) Quadrupole design considerations:***
gradient, length
(how do you optimize to limit the synchrotron rad. Energy below the dipole radiation … assume 5 sigma beam size, → magnet length /aperture need / critical energy)

We have first to calculate the k-strength, which determines the ohase advance per cell via …

$\sin(\psi_{cell}/2) = \frac{L_{cell}}{4f}$ With f = focal length = $1/kl_q$

We assume as first guess l_q=1m.
→ $k=5.6*10^{-2}\ /m^2$

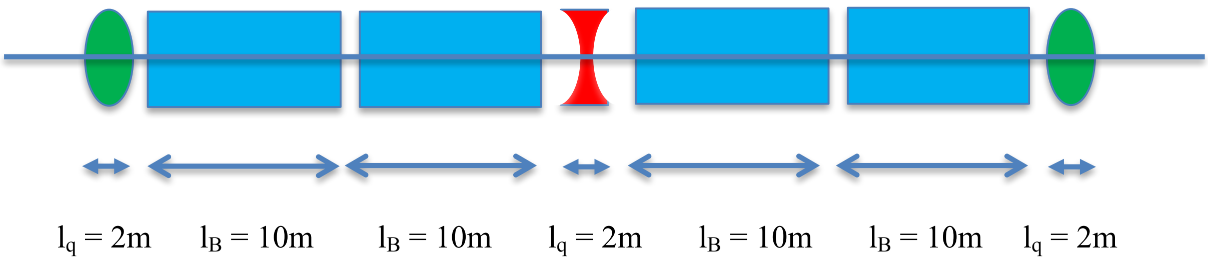
Now the tricky part:

with the beam rigidity $B * \rho = \frac{p}{q} = \frac{180\frac{GeV}{c}}{q} = 600Tm$ we calculate a gradient of g=34 T/m.

The pole tip field at 10 sigma (which includes the beam size but even more also orbit fluctuations) is
$B_{poletip} = g*10*\sigma$ = 1000 Γ (=0.1 T)

This is quite a bitnhogher thanm the dipole field and it might be a wise decision to reduce the quadrupole gradient and thus make the lenses longer ... but softer.
(indeed this was a problem in LEP).
So we choose $l_q = 2m$
$g = 16$ T/m

Finally the cell will look like this:



And we still have some space for other elements like sextupoles, BPMs etc.

******* 11.) design an injection system, including the pre-accelerator & injection mode***
lifetime determined via beam strahlung & radiative bhabha, → 15 minutes,

Basically a steady injection is needed during collision runs.
A ramping of the collider (like in LEP, LHC ertc) is much to inefficient and the duty cycle will be very small.
Thus: top up injection needed, ramp of the collider is not feasible,

Pre-accelerator Choice: A full energy synchrotron is needed.
Due to the same arguments that we discussed in the rf design, the circumference of the booster cannot be much smaller than that of the collider ring.

Try with 50km circumference ;-(

The energy range of the pre-acc has to cover the full energy range of the collider, but as it has to ramp, a minimum energy has to be defined.
Experience tells us that the dynamic range (of the dipole fields, mainly) should be limited to about a factor 10.
→ injection energy proposed: 20 GeV,
dipole field at injection B_{inj} = 55 Gauss, which is already quite challenging to control (remanence, reproducibility etc).

Other challenges: emittance at low field (scales as γ^2, so we will get highly dense bunches.

Alternatively: a full energy linac in the range of 45-180 GeV
... which brings us to the case study Nr. 3. ;-))