Instabilities in High Energy Colliders II

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Contents:

Part 1: Collective forces on beam particles: Wakefields, Impedances and electron clouds

Part 2: Modes of oscillation in bunches, Instabilities and thresholds

Part 3: Landau damping and instability cures
Introduction

- Electromagnetic (EM) interaction of the beam with its surroundings can be described by **impedances or wake fields**. For rough estimates also the interaction with electron cloud can be described by wake functions (see Part 1).

- Impedances/Wakefields cause complex **coherent tune shifts** and (above some threshold intensity) unstable **coherent beam oscillations (Part 2)**.

- **Cures:** Impedance reduction, Landau damping and tune spread (Part 3), feedback systems,..

- **Focus on high energy beams in rings and transverse instabilities in this lecture.**
(Reduced) 3D beam physics model

\[ x'' + \frac{Q_x}{R^2} x + \mathcal{O}(x^2) = \frac{F_x(x, y, z, s)}{\beta_0^2 E_0} \]

(horizontal betatron oscillations)

Nonlinear terms:
- Landau damping

\[ \eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma_0^2} \]

\[ z'' + \frac{Q_s}{R^2} z + \mathcal{O}(z^2) = \eta \frac{qE_s(z, s)}{\beta_0^2 E_0} \]

(synchrotron oscillations)

(frequency slip factor)

3D beam density: \( \rho(x, y, z, s) \approx \rho(x, y, s) \lambda(z, s) \)

Line density: \( \lambda(z, s) = \frac{dN}{dz} \)

Collective forces: \( F_x(z, \bar{x}, s) = q^2 \int_{0}^{z} \lambda(u, s) \langle x(u, s) \rangle W_x(z - u) \, du \)

(Wakes and impedances)

\( E_z(z, s) = q^2 \int_{0}^{z} \lambda(u, s) W_1(z - u) \, du \)

(longitudinal electric field)
Transverse tune shifts (2D or coasting beam)

Betatron oscillations with beam induced forces:

\[
x'' + \frac{Q_x^2}{R^2} x = \frac{F_x(x, \bar{x}, s)}{\beta_0^2 E_0} \quad \text{(only horizontal)}
\]

Assuming small offsets:

\[
x'' + \frac{Q_x^2}{R^2} x = \frac{1}{\beta_0^2 E_0} \left( \frac{\partial F_x}{\partial x} \bigg|_{\tau=0} x + \frac{\partial F_x}{\partial \bar{x}} \bigg|_{x=0} \bar{x} \right)
\]

Incoherent tune shift: \( Q_x = Q_{x0} + \Delta Q_x^{\text{inc}} \)

\[
\Delta Q_x^{\text{inc}} = -\frac{R^2}{2Q_{x0} E_0} \frac{\partial F_x}{\partial x} \bigg|_{\tau=0}
\]

Beam offset oscillations (coherent):

\[
\ddot{x}'' + \frac{Q_x^2}{R^2} \ddot{x} = \frac{1}{\beta_0^2 E_0} \left( \frac{\partial F_x}{\partial x} \bigg|_{\tau=0} \ddot{x} + \frac{\partial F_x}{\partial \bar{x}} \bigg|_{x=0} \ddot{\bar{x}} \right)
\]

Coherent tune shift:

\[
\Delta Q_x^{\text{coh}} \approx -\frac{R^2}{2Q_{x0} E_0} \frac{\partial F_x}{\partial x} \bigg|_{x=0}
\]
Instability growth rate (coasting beams)

\[ \frac{\partial F_x}{\partial \delta x} \bigg|_{x=0} = iqZ_x I \Rightarrow \Delta Q_x^{coh} = -i \frac{qIR}{2E_0} \beta_x Z_x \]

\[ \hat{\beta}_x \approx \frac{R}{Q_x} \quad (\beta \text{-function}) \]

(Thick wall) resistive wall impedance:

\[ Z_\perp(\omega) = (1 - i) \frac{c}{\pi \omega b^3 \delta_s \sigma} \quad \delta_w = \sqrt{\frac{2}{\mu_0 \omega \sigma}} \quad \text{(skin depth)} \]

Instability growth rate:

\[ \ddot{\chi}(t) = \hat{\chi} e^{t/\tau} \]

\[ \tau^{-1} = \omega_0 \Im \Delta Q_x^{coh} \propto \frac{q^2 N \hat{\beta}}{m \gamma_0} \frac{1}{\sigma^{1/2} b^3 \omega} \]

\[ \omega = (n - Q) \omega_0 \]

\[ \omega_0: \text{revolution frequency} \]

Offset oscillations:

\[ \ddot{\chi}(\theta_0, t) = \hat{\chi} e^{\pm i \omega_0 Q t} \]

\[ \ddot{\chi}(\theta, t) = \hat{\chi} e^{i (n \theta - \omega t)} \]

Betatron sideband:

\[ \theta = s / R \]

\[ n = 4 \]
Bunched beams: Transverse head-tail modes

\[ x'' + \frac{Q_x^2}{R^2} x = 0 \]
(betatron oscillations)

\[ z'' + \frac{Q_z^2}{R^2} z = 0 \]
(synchrotron oscillations)

head-tail oscillations

\[ k=0: \quad \ddot{x} = A e^{i\omega_0 Q_x t} \]

\[ k=1: \quad \Delta(t) = \lambda(t) \ddot{x}(t) \]

\[ p_0(t) \times e^{i\omega_0 Q_0 t} \]

Stable (for single bunches)!

\[ |\Delta(\omega)|^2 \quad \omega \]

Sacherer (1974)

Power spectrum of head-tail modes
Head-tail modes: Chromatic phase shift

\[ \Delta Q = \xi \frac{\Delta p}{p} \quad \Delta p = -\frac{z'}{\eta} \]

(Chromatic tune shift)

\[ \chi = \frac{1}{R} \int_{0}^{z_b} \Delta Q ds = \frac{\xi z_b}{\eta R} \]

(head-tail phase shift)

\[ z_b : \text{bunch length} \]

\[ \tilde{x}(z, t) = A e^{i\chi z/z_b} e^{i\omega_0 Q_0 t} \]

Traveling wave pattern!

Unstable head-tail oscillations in the CERN PS

Unstable

\[ |m| = 4 \quad |m| = 5 \quad |m| = 7 \]

Sacherer (1974)

Spectrum:

\[ \omega_x = \frac{c \chi}{z_b} = \frac{\xi}{\eta} \omega_0 \]

\[ |\Delta(\omega)|^2 \rightarrow |\Delta(\omega - \omega_x)|^2 \]
Transverse coupled-bunch instabilities

Coasting beam: \( \Delta Q_x^{coh} = -i \frac{qI}{4\pi E_0} \hat{\beta}_x Z_x \)

\[
\omega_x = \frac{c\chi}{z_b} = \frac{\xi}{\eta} \omega_0
\]

(\text{chromatic frequency shift})

\[
|\Delta(\omega)|^2
\]

Example: Coupled "rigid" bunch mode

Effective impedance:

\[
Z_{x,k}^{\text{eff}} = \sum_p \langle \hat{\beta}_x Z_x(\omega_p) | \Delta_k(\omega_p - \omega_\xi) \rangle^2
\]

Growth rate for mode k:

\[
\frac{1}{\tau_k} = -\frac{1}{1+k} \omega_0 \frac{qMI_b}{4\pi E_0 B_f} \langle \hat{\beta}_x \rangle \Re Z_{x,k}^{\text{eff}}
\]

Sacherer (1974)
Transverse coupled-bunch instabilities II

for a smooth impedance and rigid (k=0) oscillations:

\[
\frac{1}{\tau_0} = -\frac{qMI_b}{2E_0 z_b} \hat{\beta}_x \Re Z_x(\omega_x)
\]

if only one sideband matters:

\[
\frac{1}{\tau_k} = -\frac{1}{1 + k} \omega_0 \frac{qMI_b}{4\pi E_0} \hat{\beta}_x \Re Z_x(\omega_p) F_k^r(\omega_p - \omega_\xi)
\]

(form factor)

Positive/negative chromaticity above/below transition shifts all modes towards the positive frequency side. Mode \( k = 0 \) becomes stable, but mode \( k = 1 \) may be unstable because it samples more negative \( Z \) than positive. **No head-tail instability for compensated chromaticity.**

Sacherer (1974)
Resistive wall instability: Growth rates for LHC and FCC-hh

FCC

Growth rate:

\[ \frac{1}{\tau_k} = -\frac{1}{1 + k_b} \omega_0 q M I_b \hat{\beta}_y R Z_y (\omega_{\text{min}}) F'_k (\omega_{\text{min}} - \omega_\xi) \]

\[ \omega_{\text{min}} = (n - Q_y) \omega_0 \]

growth time at 3.3 TeV:
approx. 200 turns
at 50 TeV:
approx. 800 turns
LHC at 7 TeV:
approx. 2000 turns

Damping by feedback or Landau damping -> Instability thresholds (Part 3)
Two-particle model: Single bunch break up

Assume that you have corrected chromaticity and a (working) feedback system for the "rigid" coupled bunch instability (or you have a linac).

Two-particle coupled betatron oscillations:

\[
x''_1 + \chi x_1 = 0
\]
\[
x''_2 + \chi x_2 = \frac{q^2 N_b W_x(z)}{2LE_0} x_1 \quad \chi = \frac{Q_x^2}{R^2}
\]

New coordinates:

\[
\bar{x}_l = x_l + i \frac{x'_l}{\chi} \quad l = 1, 2
\]

Solution:

\[
\bar{x}_1(s) = \bar{x}_1(0) e^{-i\chi s}
\]
\[
\bar{x}_2(s) = \bar{x}_2(0) e^{-i\chi s} - i \frac{q^2 N_b W_x(z)}{4E_0 L\chi} \bar{x}_1(0) s e^{-i\chi s}
\]

Observation in the CERN PS near transition

R. Cappi et al. (2000)
Two-particle model: Strong head-tail instability

Synchrotron oscillations suppress beam breakup!

Change of position in z every: \[ \frac{s}{c} = \left\{ 0, \frac{T_s}{2}, T_s, \ldots \right\} \]

\[ x''_2 + \kappa x_2 = 0 \]
\[ x''_1 + \kappa x_1 = \frac{q^2 N_b W_x(z)}{2LE_0} x_2 \]

\[
\begin{pmatrix}
\tilde{x}_1 \\
\tilde{x}_2
\end{pmatrix}_{s=cT_s} = e^{i\omega_0 Q_s T_s} \begin{pmatrix} 1 & i\Upsilon \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ i\Upsilon & 1 \end{pmatrix} \begin{pmatrix} \tilde{x}_1 \\
\tilde{x}_2 
\end{pmatrix}_{s=0}
\]

\[ \Upsilon = \frac{\pi q^2 N_b RW_x(z)}{8\pi E_0 Q_s Q_0} \]
\[ \lambda_{\pm} = e^{i\phi} \sin\left(\frac{\phi}{2}\right) = \frac{\Upsilon}{2} \]

A. Chao (p. 181): It is straightforward to show that ....

\[ \Delta(s) \propto \tilde{x}_1(s) + \tilde{x}_2(s) \propto e^{iQ ks/R} \]

\[ Q_k = Q_0 + k Q_s \] (low intensity)
More general: Transverse Mode Coupling Instabilities (TMCI)

B. Salvant et. al. 2008
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Landau damping (LD)

Very simple picture (in beams): $\frac{1}{t} \leq \frac{\delta Q}{\omega_0}$

$\delta Q = \langle \Delta Q^{\text{inc}} \rangle \Rightarrow$ instability thresholds

( rms tune spread due to octupoles, beam-beam.....)

**Remark:** In this short lecture only nonlinearities will be discussed as a source of the tune spread.

Detailed LD analysis requires: Vlasov equation + Poisson equation

**LD of plasma waves:**


Cédric Villani, Fields medal for (nonlinear) Landau damping, 2010

**LD of transverse beam oscillations:**

D. Möhl, F. Ruggiero et al.

reversible (see beam/plasma echoes)

still a very active field of research!
**Nonlinear) tune shifts from octupoles**

Magnet field:

\[ B_x = O_3 (3x^2y - y^3) \]

\[ B_y = O_3 (x^3 - 3xy^2) \]

**O$_3$:** octupole strength

Betatron oscillations (incoherent):

\[ x(s) = \sqrt{2J_x \varepsilon_x \beta_x(s)} \cos[\phi_x(s)] \]

\[ y(s) = \sqrt{2J_y \varepsilon_y \beta_y(s)} \cos[\phi_y(s)] \]

\( \varepsilon_{x,y} \): rms emittances

\( J_{x,y} \): normalized amplitudes

**Tune shifts (integrated):**

\[ \Delta Q_x = a_x J_x - b_{xy} J_y \]

\[ \Delta Q_y = a_y J_y - b_{xy} J_x \]

**Beam distribution:**

\[ \psi_{\perp}(J_x, J_y) = e^{-(J_x + J_y)} \]

**rms tune spread:**

\[ \delta Q_{x,y} = \langle \Delta Q_{x,y}(J_x, J_y) \rangle \]

LHC: 168 dedicated octupoles for Landau damping
Dispersion relation

Driven (nonlinear) betatron oscillations:

\[ x'' + \frac{Q_x^2}{R^2} x + O(x^2) = \frac{2Q_{x0} \Delta Q_x^{coh}}{R^2} \ddot{x} \]

(bare tunes) (octupoles) (impedances)

Ansatz: \[ \ddot{x}(t) = A e^{-i \Omega t} \]

\( \Re \Omega \): real coherent shift

\( \Im \Omega \): growth or damping rate

Remark [1]: 1D beam response

\[ \ddot{x} \propto A e^{-i \Omega t} \int_0^\infty \frac{J \psi'(J)}{\omega(J) - \Omega} dJ \]

2D dispersion relation [2]:

Solve for \( \Omega \)

\[ 1 = \Delta Q^{coh} \int \frac{1}{\Delta Q_{oct} - \Omega/\omega_0} J_x \frac{\partial \psi}{\partial J_x} dJ_x dJ_y \]

Application to LHC: Stability diagram

Tune distribution

\[ \delta Q_{oct} \approx 0.1 \]

\[ \Delta Q_x \left(10^{-3}\right) \]

\[ \Delta Q_y \left(10^{-3}\right) \]

\[ N_{\text{LHC}}^{\text{oct}} = 168 \]

Stability curve

\[ \Im \Delta Q_{coh} \approx 0.1 \]

\[ \Re \Delta Q_{coh} / 10^{-3} \]

\[ \tau^{-1} \approx \delta Q_{inc} \]

\[ U(\Omega) + iV(\Omega) = \int \frac{1}{\Delta Q_{oct} - \Omega/\omega_0} - i\tau^{-1} J_x \frac{\partial \psi_1}{\partial J_x} dJ_x dJ_y \]

\[ \Delta Q_{coh} = \frac{1}{U(\Omega_R) + iV(\Omega_R)} \]

Numerical damping rate: \( \tau^{-1} \ll \Delta Q_{oct} \)
Landau damping at very high energies

The good news: \( \frac{1}{\tau} \propto \frac{1}{E_0} \) (instability growth rate)

The OK news: \( \delta Q_{oct} \approx (\omega_0 \tau)^{-1} \propto \frac{L}{E_0} \) (tune spread required for LD)

The bad news: \( \delta Q_{oct} \approx N_{oct} L_m \frac{\varepsilon}{E_0^2} \) (tune spread provided by octupoles)

\( E_0 = \gamma_0 mc^2 \)

\( L \): circumference

\( L_m \): length of magnet

\( N_{oct} \): # of magnets

\( \varepsilon \): normalized emittance

\( \Rightarrow N_{oct} L_m \propto E_0^2 \)

From LHC to FCC-hh: \( 7^2 \times 168 \) octupoles
Landau damping by electron lenses

Matched transverse beam radii.

Gaussian electron beam provides a nonlinear tune shift.

Example: One e-lens (l=2 m, I_e=1 A) in LHC would provide a tune spread similar to the 168 octupoles.

\[ \Delta Q_x^e = \frac{1 + \beta_e}{\beta_e} \frac{I_e l r_p}{2\pi e c \varepsilon_x} \]

V. Shiltsev et al., PRL (2017)

Similar to the beam-beam force!
Summary and Conclusions: Parts 2+3

- Some aspects of transverse beam instabilities with a focus on hadrons and rings were covered.

- Landau damping due to the nonlinear tune shifts was discussed.

- Besides studies of “passive” Landau damping, active feedback systems are a very active field of R&D and might be able to cure most instabilities in the future.

- The study of the interplay of impedances, electron clouds, beam-beam, ... becomes even more relevant as the machines are operated close to the intensity limits.

- Computer simulations (particle tracking, Vlasov solvers) are essentials tools to study such more complex scenarios.
Backup
Two-particle model: application to e-cloud