Resummation for top quark pair production at the LHC at NNLO+NNLL′

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Outline

Top quark pairs at the LHC

Top quark physics is now a precision topic. Top quark pair production calculations available at NNLO and with soft gluon resummation.

Total cross section:

[Czakon, Fielder, Mitov: '13, '14]

$$\sigma_{\text{NNLO}}(pp \to \bar{t}t + X) \sim 800 \text{ pb}$$
$$\sigma_{\text{NNLO+NNLL}}(pp \to \bar{t}t + X) \sim 820 \text{ pb}$$

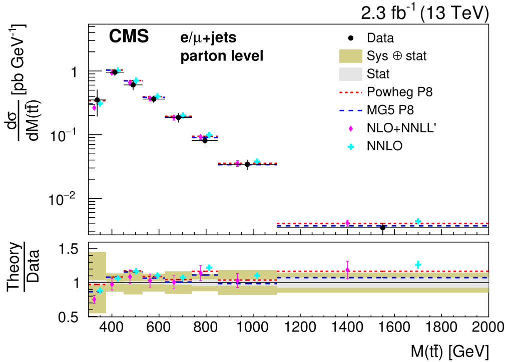
[top++2.0]

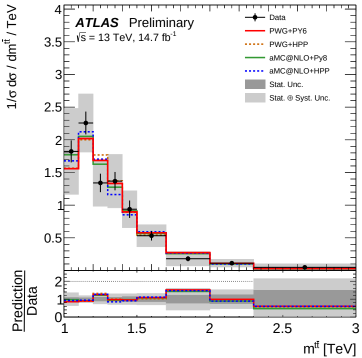
LHC will have produced billions of tops after 3000 fb^{-1}. The tails of distributions may become important.

Top quarks at the LHC

LHC already beginning to probe high energy tails of $t\bar{t}$ distributions

[CMS-TOP-16-008] [ATLAS-CONF-2016-100]





Boosted regime not just a "corner of phase space"

Top quarks and this talk

- The focus of this work is to address the impact of particular higher order (beyond NNLO), logarithmically enhanced contributions to top quark pair production.
- Particular emphasis on the the Pair Invariant Mass (PIM) distribution, M_{tt}, but we also study the top quark p_T.
- We use factorisation theorems derived in Soft Collinear Effective Theory (SCET) to deal with the multitude of scales present in this process.

Fixed Order Calculations

Consider $t\bar{t}$ production at hadron colliders.

$$i(p_1) + j(p_2) \to t(p_3) + \bar{t}(p_4) + X(p_X)$$

With $ij \in \{q\bar{q}, \bar{q}q, gg\}$ at leading order. With QCD factorisation, we can write the differential cross section as:

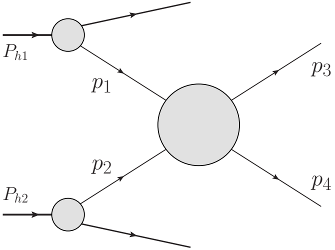
$$\frac{d\sigma_{h_1 h_2 \to \bar{t}tX}(\tau)}{dM} = \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z, \mu_f) \frac{d\hat{\sigma}_{ij}}{dM}(z, \alpha_s(\mu_r), M, m_t, \mu_{f/r})$$

$$\mathcal{L}_{ij}(y) = \int_y^1 \frac{dx}{x} \phi_{h_1/i}(x) \phi_{h_2/j}(y/x)$$

$$s = (P_{h_1} + P_{h_2})^2 \quad \hat{s} = (p_1 + p_2)^2$$

$$M^2 = M_{t\bar{t}}^2 = (p_3 + p_4)^2$$

$$\tau = \frac{M^2}{s} \quad z = \frac{M^2}{\hat{s}}$$



Fixed Order Calculations

Calculating perturbative corrections to the process:

$$d\hat{\sigma} = \alpha_s^2 \left(d\hat{\sigma}^{(0)} + \alpha_s \, d\hat{\sigma}^{(1)} + \alpha_s^2 \, d\hat{\sigma}^{(2)} + \dots \right)$$

Total and differential rates known to α_s^4

[Czakon, Heymes, Fielder, Mitov]

In computing higher order corrections, one encounters various logarithmic corrections. In particular: threshold logs

$$\text{Threshold:} \quad \alpha_s^n \left[\frac{\ln^p(1-z)}{1-z} \right]_+ , \qquad 0 \le p \le 2n-1$$

$z = M/\hat{s}$

Large contributions as $z \to 1$.

Also encounter

$$\text{Small Mass (collinear):} \quad \alpha_s \ln^2 \left(\frac{m_t}{M} \right) ,$$

We expect these to be important for "boosted" tops, $M^2 >> m_t^2$

Factorisation: Goal

$$\frac{d^2\sigma}{dM\ d\cos\theta} = \frac{8\pi\beta_t}{3sM} \sum_{ij=(\bar{q}q,q\bar{q},gg)} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z) C_{ij}(z, M, m_t, ..)$$

The partonic cross section factorises in the $z \to 1$ limit.

$$\hat{s}, M_{tt}^2, m_t^2 >> \hat{s}(1-z)^2$$

In Mellin moment space [Kidonakis, Sterman, 9705234]
Using the SCET framework [Ahrens, Ferroglia, Neubert, Pecjak, Yang: 1003.5827]
Factorisation allows Resummation

$$C_{ij} = \mathrm{Tr}[\mathbf{H}_{ij}^m(M_{t\bar{t}}, m_t, \mu_f, ..)\mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), m_t, \mu_f, ...)] + \mathcal{O}(1-z)$$

$\mathbf{H}^m$, $\mathbf{S}^m$, matrices in colour space

$\mathbf{H}_{ij}$ - Hard Function. Related to virtual corrections
$\mathbf{S}_{ij}$ - Soft Function. Related to real emission of soft gluons.
Contains distributions singular in $(1-z)$.

Factorisation: Booosted Tops

The boosted soft limit, $z \to 1$ and $M >> m_t$,

$$\hat{s}, t_1 >> m_t^2 >> \hat{s}(1-z)^2 >> m_t^2(1-z)^2$$

logs of the form $\ln(M/m_t)$ become important.
Further factorisation in this limit. [Ferroglia, Pecjak, Yang: 1205.3662]

$$C_{ij} = \mathrm{Tr}[\mathbf{H}_{ij}^m(M_{t\bar{t}}, m_t, \mu_f, ..)\mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), m_t, \mu_f, ...)] + \mathcal{O}(1-z)$$

$$M^2 >> m_t^2$$

$$C_{ij} = C_D^2(m_t, \mu_f)\,\mathrm{Tr}\left[\mathbf{H}_{ij}(M, \mu_f, ..)\mathbf{S}_{ij}(\sqrt{\hat{s}}(1-z), \mu_f, ...)\right] \otimes \mathbf{s}_D\left(m_t(1-z), \mu_f\right)$$
$$\otimes\, \mathbf{s}_D\left(m_t(1-z), \mu_f\right) \otimes c_{ij}^t(z, m_t, \mu_f) + \mathcal{O}(1-z) + \mathcal{O}\left(m_t/M\right)$$

Each of these functions known to two-loops

$\mathbf{H}$: [Glover et. al: '00-'01]
$\mathbf{S}$: [Ferroglia, Pecjak, Yang: 1207.4798]
s_D, C_D: [Melnikov, Mitov: 0404143], [Becher, Neubert: 0512208]

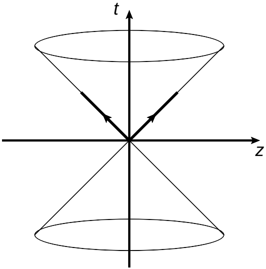
Soft Collinear Effective Theory

Soft Collinear Effective Theory allows one to deal with soft and collinear d.o.f in the presence of a hard interaction.

Can be used to disentangle scales associated with different regimes in collider physics: $\Lambda_{\mathrm{HARD}} >> \Lambda_{\mathrm{JET}} >> \Lambda_{\mathrm{QCD}}$.

Define two light-like vectors in the direction of the incoming partons,

$$n^\mu = (1,0,0,1)$$
$$\bar{n}^\mu = (1,0,0,-1)$$



Use lightcone coordinates

$$p^\mu = (n\cdot p)\frac{\bar{n}^\mu}{2} + (\bar{n}\cdot p)\frac{n^\mu}{2} + p_\perp^\mu$$

In deriving the factorisation theorem, I will focus on the $\bar{q}q$ channel, the gg channel follows similarly

Soft Collinear Effective Theory

In SCET, split fields into separate momentum regions.
$p^\mu = (p^+, p^-, \vec{\mathbf{p}}_\perp) = (n \cdot p, \bar{n} \cdot p, \vec{\mathbf{p}}_\perp)$

$$\psi(x) \to \psi_c(x) + \psi_{\bar{c}}(x) + \psi_s(x)$$

$$\psi_c(x) : p^\mu \sim (\lambda^2, 1, \lambda)Q$$
$$\psi_{\bar{c}}(x) : p^\mu \sim (1, \lambda^2, \lambda)Q$$
$$\psi_s(x) : p^\mu \sim (\lambda^2, \lambda^2, \lambda^2)Q$$

With $\lambda \sim (1-z)$ the expansion parameter of our effective theory.
Hard interactions absorbed in Wilson Coefficients.
We can do the same with the gluon fields: $A^\mu \to A^\mu_c + A^\mu_{\bar{c}} + A^\mu_s$

To leading power we only need $\xi_c(x) = \frac{\not{n}\not{\bar{n}}}{2}\psi_c(x)$, $\xi_{\bar{c}}(x) = \frac{\not{\bar{n}}\not{n}}{2}\psi_{\bar{c}}(x)$

Soft Collinear Effective Theory

It is important to note that the power counting in SCET gives rise to non-local operators. This is due to the fact that certain derivatives of operators are not power suppressed and must be included.

$$\bar{n} \cdot \partial \, \xi_n(x) \sim \lambda^0 \xi_n(x)$$

All such derivatives are typically included via,

$$\xi_n(x + t\bar{n}) = \sum_i \frac{t^i}{n!} (\bar{n} \cdot \partial)^i \, \xi_n(x)$$

And so we will encounter operators of the type,

$$\int dt_1 \, dt_2 \, \tilde{C}(t_1, t_2) \, \bar{\xi}_{\bar{n}}(x + t_2 n) \, \Gamma^\mu \, \xi_n(x + t_1 \bar{n})$$

Soft Collinear Effective Theory

For the purpose of top quark pair production at hadron colliders we need:

- Two sets of collinear fields for the incoming partons: ξ_n, A_n^μ, ..
- Two heavy quark fields (HQET) for the tops: h_{v_3}, h_{v_4}
- Soft fields for the soft interactions amongst the particles: A_s^μ

The lagrangian for interactions between the collinear fermions and the soft gluons is, to leading power,

$$\mathcal{L}_{\text{int}} = \bar{\xi}_n(x)\frac{\not{\bar{n}}}{2}\,g\,n\cdot A_s(x)\xi_n(x) + \bar{\xi}_{\bar{n}}(x)\frac{\not{n}}{2}\,g\,\bar{n}\cdot A_s(x)\xi_{\bar{n}}(x) + \bar{h}_{v_3}(x)\,g\,v_3\cdot A_s(x)\,h_{v_3}(x) + \bar{h}_{v_4}(x)\,g\,v_4\cdot A_s(x)\,h_{v_4}(x)$$

Soft Collinear Effective Theory

The gauge transformations in SCET are somewhat more involved. But we can construct gauge invariant operators to work with using Wilson lines.

$$\chi_n(x) = W_n^\dagger(x)\xi_n(x)\,, \qquad \mathcal{A}^\mu_{n\perp}(x) = W_n^\dagger(x)\left[iD_\perp^\mu W_n(x)\right]\,,$$

where the Wilson line is defined using the collinear gluon field

$$W_n(x) = \mathcal{P}\exp\left\{ig\int_{-\infty}^0 ds\;\bar{n}\cdot A_n(x+s\bar{n})\right\}$$

It is also possible to decouple the collinear and soft interactions by means of a similar field redefinition. For the collinear quark fields:

$$\chi_n \to [S_n(x)]\,\chi_n^{(0)}(x)$$

$$S_n(x) = \mathcal{P}\exp\left\{ig\int_{-\infty}^0 dt\;n\cdot A_s(x+tn)\right\}$$

Additional Operators

We will also need additional operators to describe our process, $q\bar{q}(gg) \to \bar{t}t$,

$$\mathcal{H}_{\text{eff}} = \sum_{I,m} \int dt_1\, dt_2\, e^{im_t(v_3+v_4)\cdot x} \left[\tilde{C}^{\bar{q}q}_{Im}(t_1, t_2) \mathcal{O}^{\bar{q}q}_{Im} + (gg) \right]$$

$$\mathcal{O}^{\bar{q}q}_{Im} = \left(c^{\bar{q}q}_I\right) \bar{\xi}_{\bar{n}}(x + t_2 n) \Gamma'_m \xi_n(x + t_1 \bar{n}) \bar{h}_{v_3}(x) \Gamma''_m h_{v_4}(x)$$

m - labels Dirac structures, Γ_m
I - labels colour structures. E.g. $(c^{\bar{q}q}_2)_{\{a_1 a_2 a_3 a_4\}} = t^m_{a_2 a_1} t^m_{a_3 a_4}$

Additional Operators

We can apply the decoupling relations from earlier to our effective operators to arrive at the operator in a factorised form

$$\mathcal{O}^{\bar{q}q}_{Im}(x,t_1,t_2) = c^{\bar{q}q}_I \left[\mathcal{O}^h_m(x)\right]\left[\mathcal{O}^c_m(x,t_1,t_2)\right]\left[\mathcal{O}^s(x)\right]$$

$$\begin{aligned}\left[\mathcal{O}^h_m(x)\right] &= \bar{h}_{v_3}(x)\,\Gamma''_m\,h_{v_4}(x)\\ \left[\mathcal{O}^c_m(x,t_1,t_2)\right] &= \bar{\xi}_{\bar{n}}(x+t_2 n)\,\Gamma'_m\,\xi_n(x+t_1\bar{n})\\ \left[\mathcal{O}^s(x)\right] &= S^\dagger_{v_3}(x)S_{v_4}(x)S^\dagger_{\bar{n}}(x)S_n(x)\end{aligned}$$

Our squared matrix element:

$$\sim \left|\sum_m \langle t(p_3)\bar{t}(p_4)X_s(p_s)|\,\mathcal{O}^{q\bar{q}}_m(0)\,|q(p_1)\bar{q}(p_2)\rangle\,|C_m\rangle\right|^2$$

Note: The colour indices for the soft Wilson lines are in general contracted between the fermion fields and the colour structure terms. I am omitting discussion regarding the colour structure.
$|C_m\rangle$ - Wilson Coefficient as a vector in colour space

Factorised Result

$$\sim \left| \sum_m \langle t(p_3)\bar{t}(p_4)X_s(p_s)| \, \mathcal{O}_m^{q\bar{q}}(0) \, |q(p_1)\bar{q}(p_2)\rangle \, |C_m\rangle \right|^2$$

Because the different fields in our operator no longer interact, the result factorises..

$$d\hat{\sigma} \sim \sum_{m,m'} \left[\langle\langle \mathcal{O}_m \rangle\rangle^\dagger_{\text{tree}} \, \langle\langle \mathcal{O}_{m'} \rangle\rangle_{\text{tree}} \, \langle C_m| \, \langle 0| \, \overline{\mathbf{T}}[\mathcal{O}^{s\dagger}(x)]\mathbf{T}[\mathcal{O}^s(x)] \, |0\rangle \, |C_{m'}\rangle \right]$$

With $\langle\langle \mathcal{O}_m \rangle\rangle_{\text{tree}} = \langle t(p_3)\bar{t}(p_4)| \, \mathcal{O}_m^h(0)\mathcal{O}_m^c(0) \, |q(p_1)\bar{q}(p_2)\rangle_{\text{tree}}$

We obtain our hard function and *position space* soft function

$$\mathbf{H}(M, m_t, \cos\theta, \mu) \sim \sum_{m,m'} \langle\langle \mathcal{O}_{m'} \rangle\rangle_{\text{tree}} \, |C_{m'}\rangle \, \langle C_m| \, \langle\langle \mathcal{O}_m \rangle\rangle^\dagger_{\text{tree}}$$

$$\mathbf{W}(x, \mu) \sim \langle 0| \, \overline{\mathbf{T}}[\mathcal{O}^{s\dagger}(x)]\mathbf{T}[\mathcal{O}^s(x)] \, |0\rangle$$

With $\mathbf{W}$ related to the momentum space soft function, $\mathbf{S}$, after utilising Fourier transforms in conjunction with the phase space integrals.

Boosted-Soft Factorisation

In order to derive our second factorisation formula, it is easier to start from the cross section.

$$\frac{d^2\sigma}{dM\ d\cos\theta} = \frac{8\pi\beta_t}{3sM} \sum_{ij=(\bar{q}q,q\bar{q},gg)} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z) C_{ij}(z, M, m_t, ..)$$

We employ a factorisation for a single top quark [Mele,Nason: Nucl.Phys. B361 626-644],

$$\frac{d\sigma_t}{dz}(z, m_t, \mu) = \sum_a \int_z^1 \frac{dx}{x}\ \frac{d\hat{\sigma}_a}{dx}(x, m_t, \mu) D_{a/t}^{(n_l+n_h)}\left(\frac{z}{x}, m_t, \mu\right)$$

- $d\hat{\sigma}_a/dx$ - production of *massless* parton a
- $D_{a/t}^{(n_f)}$ - Heavy-Quark fragmentation function (α_s with n_f flavours)

Boosted-Soft Factorisation

Utilising this twice, we obtain

$$C_{ij}(z, M, m_t, \mu) = \sum_{a,b} C_{ij}^{ab}(z, M, \mu_f) \otimes D_{a/t}(z, m_t, \mu_f) \otimes D_{b/\bar{t}}(z, m_t, \mu_f) \otimes c_{ij}^{t}(z, m_t, \mu_f) + \mathcal{O}(m_t/M)$$

We now take the soft limit of this.
C_{ij}^{ab} factorises as it did for the threshold case, but is now massless. So the resulting hard and soft functions are independent of the top mass.

$$C_{ij}^{t\bar{t}} = \mathrm{Tr}\left[\mathbf{H}_{ij}(M, t_1, \mu_f)\mathbf{S}_{ij}(M(1-z), t_1, \mu_f)\right] + \mathcal{O}(1-z)$$

The fragmentation functions also factorise in the $z \to 1$ limit.

$$D_{t/t}(z, m_t, \mu_f) = C_D(m_t, \mu_f) S_D(m_t(1-z), \mu_f) + \mathcal{O}(1-z)$$

[Korchemsky, Marchesini:9210281, Cacciari, Catani: 0107138, Gardi: 0501257, Neubert: 0706.2136]

Mellin Space

For this talk, we are going to work in Mellin space. Convolutions become products

$$d\tilde{\sigma}(N) = \tilde{\mathcal{L}}(N)\tilde{C}(N)$$

where

$$\tilde{f}(N) = \int_0^1 dx\, x^{N-1} f(x) \qquad f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N} \tilde{f(N)}$$

In Mellin space, the $z \to 1$ limit corresponds to $N \to \infty$

$$P_n(z) = \left[\frac{\ln^n(1-z)}{1-z}\right]_+ \qquad \bar{N} = N e^{\gamma_E}$$

$$\mathcal{M}[P_0] = -\ln \bar{N} + \mathcal{O}(1/N)$$
$$\mathcal{M}[P_1] = \frac{1}{2}\left(\ln^2 \bar{N} + \frac{\pi^2}{6}\right) + \mathcal{O}(1/N)$$
$$\mathcal{M}[P_2] = -\frac{1}{3}\left(\ln^3 \bar{N} + \frac{\pi^2}{2}\ln \bar{N} + 2\zeta(3)\right) + \mathcal{O}(1/N)$$

Aside: Parton Luminosity in Mellin Space

Our calculation requires parton luminosities in Mellin space. Normally given in momentum space.

$$\mathcal{L}(z)_{ij} = \int_z^1 \frac{dx}{x} \phi_{i/h_1}(x) \phi_{j/h_2}(z/x)$$

We approximate the luminosity in terms of Chebyshev polynomials
[Bonvini: 1212.0480] [Furmanski, Petronzio :164978]

$$\mathcal{L}(z) = \frac{1}{z} \sum_{i=0}^{n} (-2)^i \ln^i(z) \frac{1}{w_{min}^i} \sum_{k=i}^{n} \binom{i}{k} \tilde{c}_k$$

The Mellin transform gives

$$\mathcal{L}(N) = \int_0^1 \mathrm{d}z \; z^{N-1} \mathcal{L}(z) = \sum_{p=0}^{n} \frac{\bar{c}_p}{(N-1)^{p+1}}$$

where

$$\bar{c}_p = \frac{2^p}{w_{min}^p} \sum_{k=p}^{n} \frac{k!}{(k-p)!} \tilde{c}_k$$

Mellin Space

Our factorisation formula becomes

$$C(N) = C_D^2(m_t, \mu_f)\,\mathrm{Tr}\left[\mathbf{H}(M, \mu_f, ..)\tilde{\mathbf{S}}\left(\ln\frac{M^2}{\bar{N}^2\mu_f^2}, \mu_f, ...\right)\right]$$
$$\times\, \tilde{\mathbf{s}}_D^2\left(\ln\frac{m_t}{\bar{N}\mu_f}, \mu_f\right)\tilde{c}^t(\ln\bar{N}, m_t, \mu_f) + \mathcal{O}\left(1/N\right) + \mathcal{O}\left(m_t/M\right)$$

We now have single scale functions.

Aside: Heavy flavour matching coefficient, $\tilde{c}_{ij}^t$, introduces additional $\ln m_t$ dependence which is not resummed. We add such contributions in fixed order.

Renormalisation Group Equations

Resummation is achieved by deriving and solving the RG equations.

Evaluate the hard and soft function at their *natural* scales, where the perturbative expansion is well behaved.

Use RG equations to run both to a common scale where the cross-section is evaluated.

Hard Function RG

Hard function RG equation: $\frac{d}{d\ln\mu}\mathbf{H} = \Gamma_H \mathbf{H} + \mathbf{H}\Gamma_H^\dagger$

$$\Gamma_{q\bar{q}} = \Big[C_F \gamma_{\text{cusp}}(\alpha_s) \left(\ln \frac{M^2}{\mu^2} - i\pi \right) + C_F \gamma_{\text{cusp}}(\beta_{34}, \alpha_s) + 2\gamma^q(\alpha_s) + 2\gamma^Q(\alpha_s) \Big] \mathbb{1} + \frac{N}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Big[\gamma_{\text{cusp}}(\alpha_s) ... \Big] + ...$$

γ_{cusp} - cusp anomalous dimension

β_{34} - cusp angle.

γ^q, γ^Q - incoming & outgoing quark anomalous dimensions.

Hard Function RG

$$\frac{d}{d\ln\mu}\mathbf{H} = \Gamma_H \mathbf{H} + \mathbf{H}\Gamma_H^\dagger$$

The solution can be written as

$$\mathbf{H}(\mu) = \mathbf{U}(\mu_h,\mu)\mathbf{H}(\mu_h)\mathbf{U}^\dagger(\mu_h,\mu)$$

which implies

$$\frac{d}{d\ln\mu}\mathbf{U}(\mu_h,\mu) = \Gamma_H(\mu)\mathbf{U}(\mu_h,\mu)$$

This has the formal solution

$$\mathbf{U}(\mu_h,\mu) = \mathcal{P}\exp\left\{\int_{\mu_h}^{\mu}\frac{d\mu'}{\mu'}\Gamma_H\right\}$$

Hard Function RG

Pulling the piece proportional to the identity matrix out front gives

$$\mathbf{U} = \exp\left\{2S(\mu_h,\mu) - a_\Gamma(\mu_h,\mu)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right\}\boldsymbol{u}(\mu_h,\mu)$$

Where $\boldsymbol{u}(\mu_h,\mu)$ is the path ordered piece of the exponential.

$$S(\mu_h,\mu) = -\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}\Gamma_{\mathrm{cusp}}(\alpha)\int_{\alpha_s(\mu_h)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')}$$

$$a_\Gamma(\mu_h,\mu) = -\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}\Gamma_{\mathrm{cusp}}(\alpha)$$

Since $\frac{d\alpha_s}{\beta(\alpha)} = d\ln\mu$, the above suggests $S(\mu_h,\mu)$ resums double logs while $a_\Gamma(\mu_h,\mu)$ resums single logs.

Soft Function RG

$$d\sigma \sim \mathrm{Tr}\left[\mathbf{H}_{ij}(M_{t\bar{t}},\mu_f,..)\mathbf{S}_{ij}\left(\ln\frac{M_{t\bar{t}}^2}{\bar{N}^2\mu_f^2},\mu_f,...\right)\right]\mathcal{L}(N)+\mathcal{O}(1-z)$$

Since the hadronic cross-section should be scale independent, the RG equations for the soft function can be derived from the knowledge of the hard RG and DGLAP equations which govern PDF evolution.

$$\frac{d}{d\ln\mu}\mathbf{S} = -\left[\Gamma_{\text{cusp}}\ln\frac{M^2}{\bar{N}^2\mu^2}+\gamma^{s\dagger}\right]\mathbf{S}-\mathbf{S}\left[\Gamma_{\text{cusp}}\ln\frac{M^2}{\bar{N}^2\mu^2}+\gamma^{s}\right]$$

$\gamma^s = \gamma^h + 2\gamma^\phi \mathbb{1}$

γ^ϕ - PDF anomalous dimension.

Solved in the same way as the hard function

Resumed Results

Putting together our results form the RG equations we arrive at our resummed cross section.
The result can be written as,

$$C(N) = \exp\left\{\frac{4\pi}{\alpha_s(\mu_h)}(g_1(\lambda_s,\lambda_f)+g_1^D(\lambda_{dh},\lambda_{ds},\lambda_f)) + (g_2(\lambda_s,\lambda_f)+g_2^D(\lambda_{dh},\lambda_{ds},\lambda_f)) + ...\right\}$$
$$\times \mathrm{Tr}\left[\mathbf{u}(M,\cos\theta,\mu_h,\mu_s)\mathbf{H}(M,\cos\theta,\mu_h)\mathbf{u}^\dagger(M,\cos\theta,\mu_h,\mu_s) \times \tilde{\mathbf{S}}\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2},M,\cos\theta,\mu_s\right)\right]C_D^2(m_t,\mu_{dh})\tilde{s}_D^2\left(\ln\frac{m_t^2}{\bar{N}^2\mu_{ds}^2},\mu_{ds}\right)$$

Where,

$$\lambda_i = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0\ln\left(\frac{\mu_h}{\mu_i}\right) \qquad \mathbf{u}(M,\cos\theta,\mu_h,\mu_s) = \mathcal{P}\exp\left\{\left\{\int_{\mu_h}^{\mu_s}\frac{d\mu'}{\mu'}\gamma^h(M,\cos\theta,\alpha(\mu'))\right\}\right.$$

We can pick the scale for each function to free it of large logs.

$\mu_h \sim M$, $\mu_s \sim M/\bar{N}$, $\mu_{dh} \sim m_t$ and $\mu_{ds} \sim m_t/\bar{N}$

Resummation accuracy

Schematically,

Boosted soft:

$$C(N) = \exp\left\{\frac{4\pi}{\alpha_s} g_1 + g_2 + \frac{\alpha_s}{4\pi} g_3 + ...\right\} \mathrm{Tr}\left[\mathbf{u}\,\mathbf{H}(\mu_h)\,\mathbf{u}^\dagger\,\tilde{\mathbf{S}}(\mu_s)\right] C_D^2(m_t, \mu_{dh}) \tilde{s}_D^2\,(\mu_{ds})$$

Soft:

$$C(N) = \exp\left\{\frac{4\pi}{\alpha_s} g_1^m + g_2^m + \frac{\alpha_s}{4\pi} g_3^m + ...\right\} \mathrm{Tr}\left[\mathbf{u}\,\mathbf{H}^m(\mu_h)\,\mathbf{u}^\dagger\,\tilde{\mathbf{S}}^m(\mu_s)\right]$$

To achieve a given resummation accuracy

	g_i	γ_h	$\mathbf{H}^{(m)}, \widetilde{\mathbf{s}}^{(m)}, c_D, \widetilde{s}_D$
NLL	$g_1,\ g_2$	LO	LO
NNLL	$g_1,\ g_2,\ g_3$	NLO	NLO
NNLL′	$g_1,\ g_2,\ g_3$	NLO	NNLO

In this work we work to NNLL accuracy for the soft resummation and NNLL′ for the boosted soft resummation.

Mellin Inversion

To obtain results in momentum space, we need to invert the Mellin transform

$$\frac{d\sigma(\tau)}{dM\ d\cos\theta} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\ \tau^{-N}\,\frac{d\tilde{\sigma}(N)}{dM\ d\cos\theta}$$

With c to the right of all singularities. But our resummed coefficient function contains (exponentiated)

$$g_1(\lambda_s,\lambda_f) = \frac{\Gamma_0}{4\beta_0^2}\left[\lambda+(1-\lambda_s\ln(1-\lambda_s)+\lambda_s\ln(1-\lambda_f))\right] \quad \lambda_s = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0\ln\left(\frac{\mu_h}{\mu_s}\right)$$

Since we pick $\mu_s \sim M/N$, pole at $\lambda_s = 1$

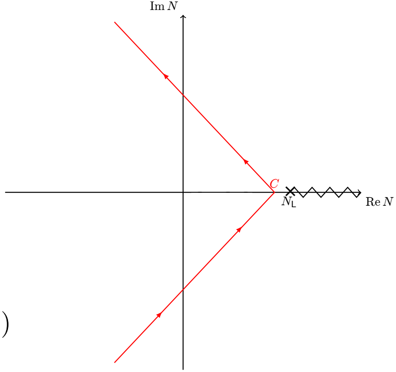
$$N_L = \exp\left\{\frac{2\pi}{\alpha_s\beta_0}\right\}$$

Minimal Prescription

- We need to select a method to deal with the Landau pole.
- We use the *Minimal Prescription*: Select our point on the real axis to be to the *left* of the Landau pole, but to the right of all other singularities in the integrand.

  [Catani, Mangano, Nason, Trentadue '96]

$$\frac{d\sigma(\tau)}{dM\ d\cos\theta} = \frac{1}{2\pi i}\int_{\mathrm{MP}_C} dN\ \tau^{-N}\mathcal{L}(N)C(N)$$



Combining and Matching with NNLO results

We wish to combine the results from the two separate resummations and match these with recent NNLO calculations

[Czakon, Fiedler , Heymes, Mitov]

Matching:

$d\sigma_b \sim$ boosted soft factorisation

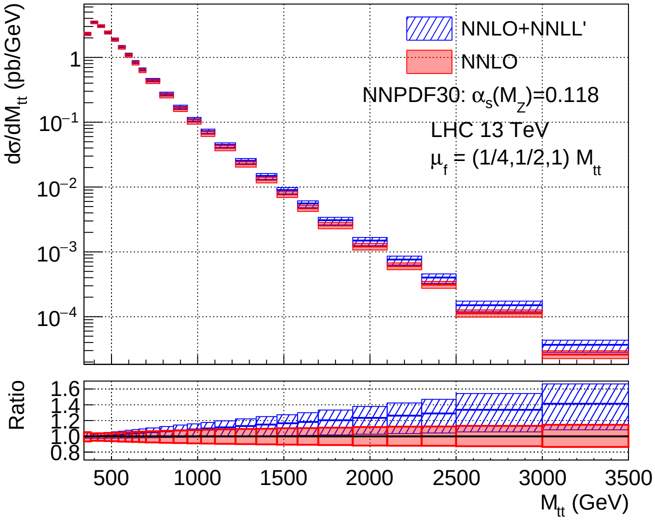
$d\sigma_{\text{Threshold}} \sim$ threshold factorisation

$$d\sigma^{\text{NNLO+NNLL'}} = \underbrace{d\sigma_b^{\text{NNLL'}}}_{\substack{\text{Missing parts subleading} \\ \text{in } m_t/M \text{ and } 1/N}} + \overbrace{\left(\underbrace{d\sigma_{\text{Threshold}}^{\text{NNLL}}}_{\substack{\text{Missing parts} \\ \text{subleading in } 1/N}} \underbrace{- d\sigma_b^{\text{NNLL}} \Big|_{\substack{\mu_{dh}=\mu_h \\ \mu_{ds}=\mu_s}}}_{\substack{\text{Removes double} \\ \text{counting}}} \right)}^{\substack{\text{Adds in parts subleading in } m_t/M \\ \text{but enhanced by } \ln N}}$$

$$+ \underbrace{\left(d\sigma^{\text{NNLO}} - d\sigma^{\text{NNLL}}_{\text{“top line”}} \Big|_{\substack{\text{NNLO} \\ \text{expansion}}} \right)}_{\substack{\text{Adds exact NNLO results,} \\ \text{avoiding double counting}}}$$

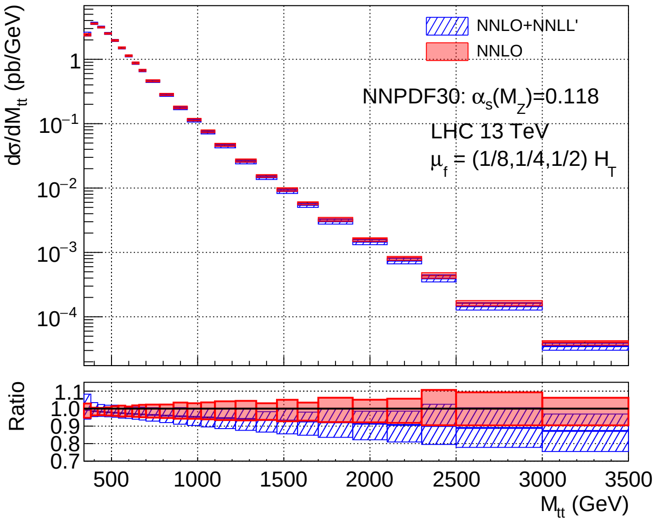
Distributions: M_{tt}

$\mu_f = M_{tt}/2$

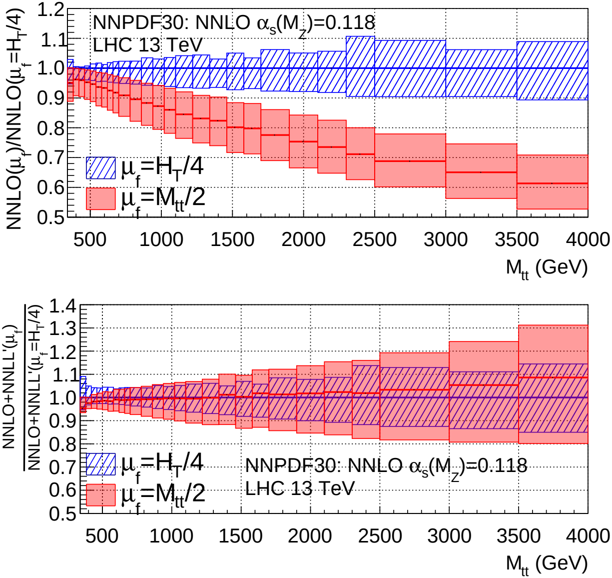


Distributions: M_{tt}

$$\mu_f = H_T/4, \left(H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2}\right)$$



Distributions: M_{tt}



Comparison with Threshold Resummation

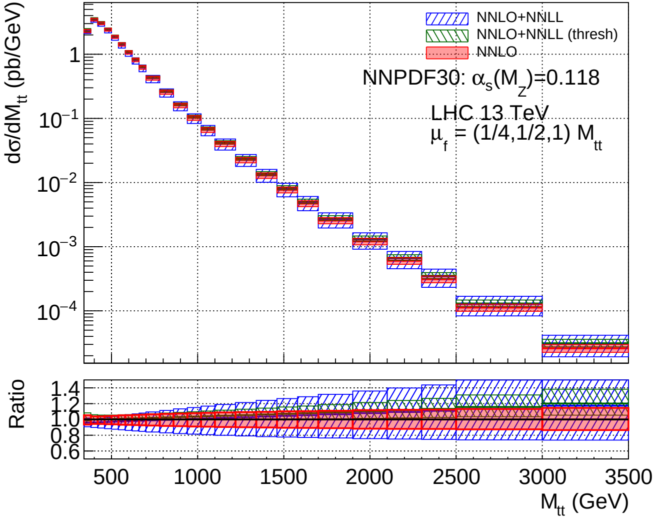
- So far we have only looked at the results of the combined resummed result matched with standard threshold resummation.
- We can compare these results with what one gets from performing just threshold resummation.

$$d\sigma^{\text{NNLO+NNLL}}_{\text{Soft Res}} = d\sigma^{\text{NNLL}}_{\text{Threshold}} + \left(d\sigma^{\text{NNLO}} - d\sigma^{\text{NNLL}}_{\text{Threshold}}\Big|_{\mu_h=\mu_s=\mu_f} \right)$$

$$d\sigma^{\text{NNLO+NNLL}}_{\text{Join Res}} = d\sigma^{\text{NNLL}}_{b} + \left(d\sigma^{\text{NNLL}}_{\text{Threshold}} - d\sigma^{\text{NNLL}}_{b}\Big|_{\substack{\mu_{dh}=\mu_h \\ \mu_{ds}=\mu_s}} \right) + \left(d\sigma^{\text{NNLO}} - d\sigma^{\text{NNLL}}_{\text{``top line''}}\Big|_{\substack{\text{NNLO} \\ \text{expansion}}} \right)$$

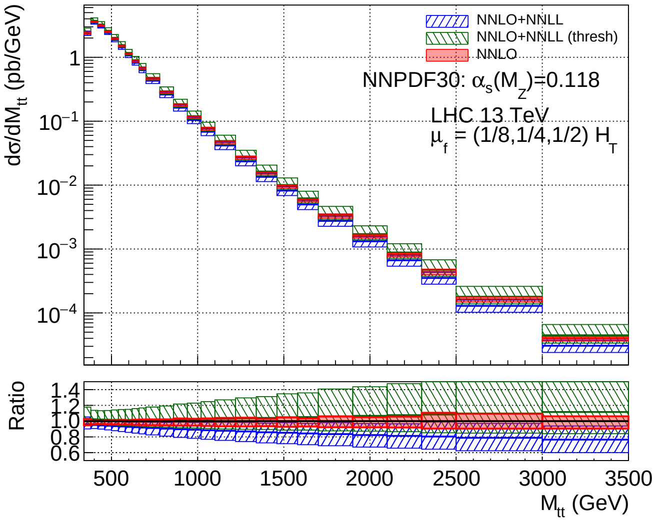
Note: We compute only to NNLL here for both cases for a fair comparison.

Comparison with Threshold Resummation



[PRELIMINARY]

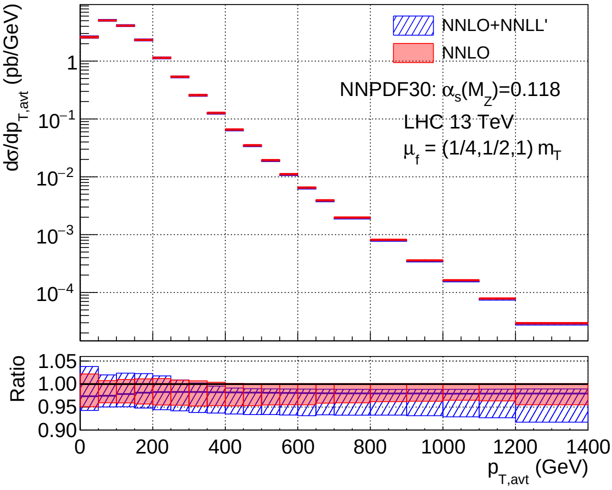
Comparison with Threshold Resummation



[PRELIMINARY]

Distributions: p_T

$$\mu_f = m_T/2, \left(m_T = \sqrt{m_t^2 + p_{T,t}^2}\right)$$



Conclusions & Outlook

- Presented factorised differential cross sections:
  - Threshold resummation ($z \to 1$)
  - Boosted Soft resummation ($z \to 1$, $M_{tt} >> m_t$)
- Combined these and matched with fixed order NNLO results, NNLO+NNLL$'$
- Results for M_{tt} and p_T distributions at 13 TeV LHC
- Resummed results for the M_{tt} distributions are less sensitive to the scale choice

BACKUP SLIDES

Total Cross Section

We can also look at the effect on the total cross section

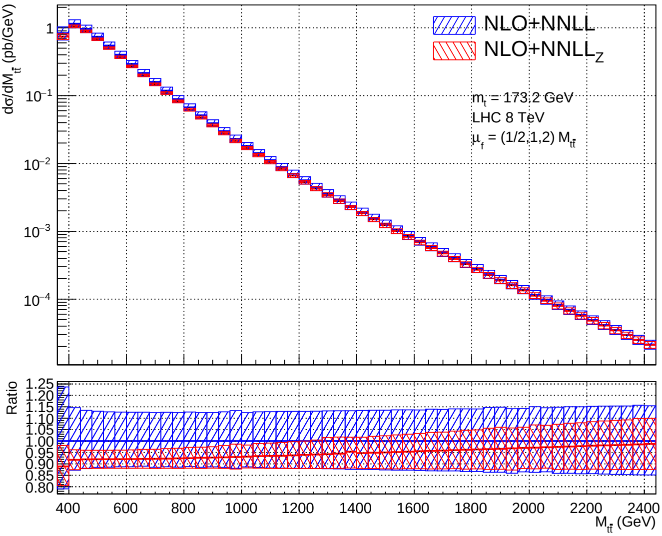
LHC 13 TeV	NNLO	NNLO+NNLL′
$\sigma(\mu_f = m_T)$	$791.8\ ^{+35.7}_{-49.0}$	$787.8\ 1^{+21.1}_{-0.00}$
$\sigma(\mu_f = m_T/2)$	$827.5\ ^{+9.28}_{-35.7}$	$808.9\ ^{+37.2}_{-21.1}$
$\sigma(\mu_f = M_{tt}/2)$	$779.4\ ^{+38.6}_{-50.4}$	$793.8\ ^{+24.4}_{-0.00}$
$\sigma(\mu_f = H_T/4)$	$828.0\ ^{+11.9}_{-36.6}$	$809.3\ ^{+39.8}_{-21.9}$
$\sigma(\mu_f = m_t)$	$802.7\ ^{+28.1}_{-45.30}$	—
$\sigma(\mu_f = m_t/2)$	$830.8\ ^{+0.00}_{-28.1}$	—

top++ can perform NNLL threshold resummation.

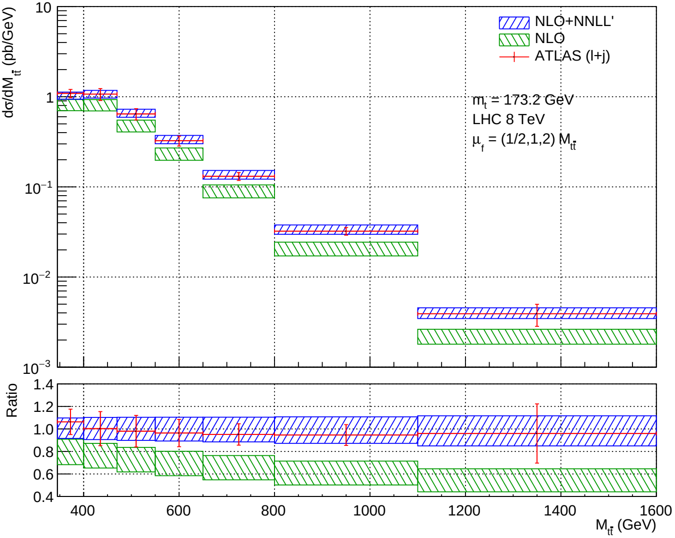
$$\sigma^{\text{NNLO+NNLL}}(\mu_f = m_t/2) = 827.7^{+0.0}_{-6.4}$$
$$\sigma^{\text{NNLO+NNLL}}(\mu_f = m_t) = 821.3^{+9.6}_{-0.0}$$

Momentum v Mellin Space: (old result)

MSWT2008 PDFs



Comparison with experimental data: (old result)



[ATLAS: 1511.04716]