Effective theories of the proton: neutrinos, atoms and dark matter

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UK and Fermilab

PIKIO meeting
University of Kentucky

16 September, 2017

many thanks to Susan Gardner, Chris Crawford, Wolfgang Korsch and Michael Kovash for bringing us together !

thanks to many collaborators and colleagues, especially: J.Arrington, M. Betancourt, P. Kammel, G.Lee, W. Marciano, A. Meyer, G. Paz, R. Gran, A. Sirlin, M. Solon

Overview

- Motivation

- Example 1: Muon Capture and neutrinos

- Example 2: universality in WIMP-nucleon scattering

- Example 3: Proton radius puzzle

Focus on 3 numbers

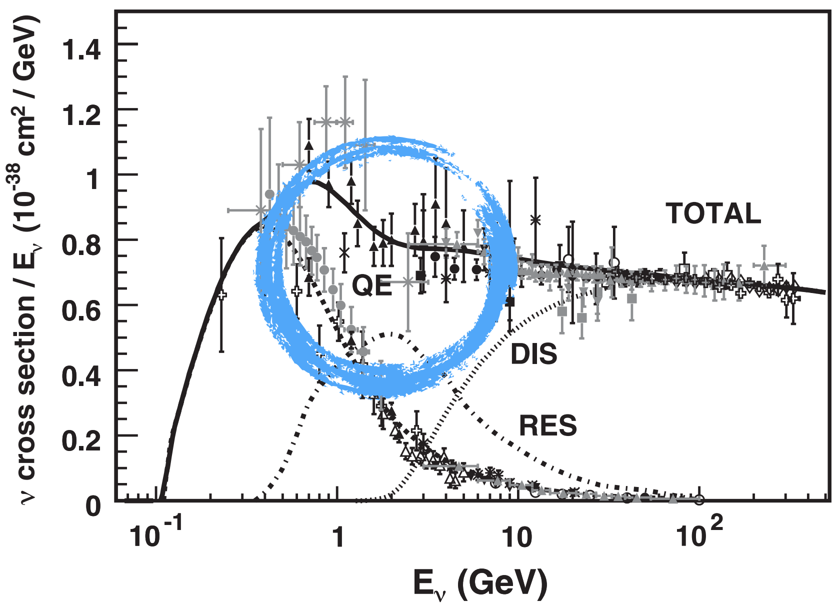
r_A $\sigma_{\chi N}$ r_E

- r_A: *determines signal cross section for LBNE*

- $\sigma_{\chi N}$: *universal cross section for next-gen. DD*

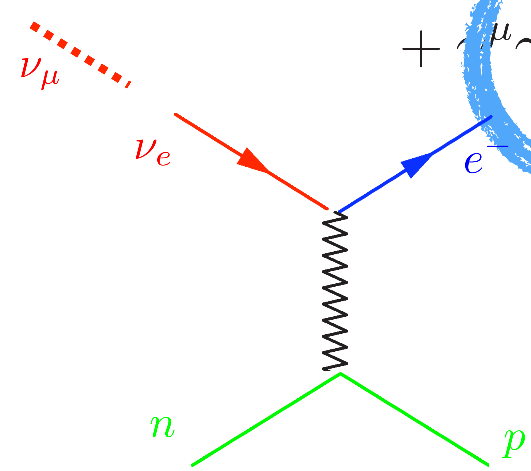
- r_E: *5σ shift in Rydberg (or something even more interesting)*

- <u>neutrinos</u>: must confront large uncertainty in signal process of ν_e appearance at long baseline neutrino experiment

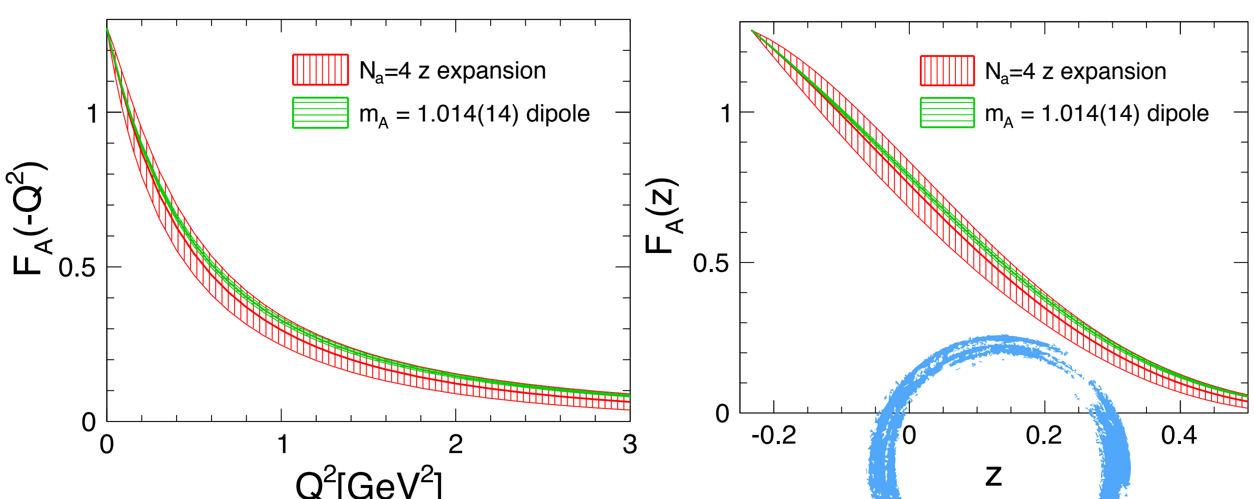


Quasielastic dominance

$$\langle p(p')|J_W^{+\mu}|n(p)\rangle \propto \bar{u}^{(p)}(p')\Big\{\gamma^\mu F_1(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2(q^2) + \gamma^\mu\gamma_5 F_A(q^2) + \frac{1}{m_N}q^\mu\gamma_5 F_P(q^2)\Big\}u^{(n)}(p)$$



Unknown: axial-vector form nucleon form factor

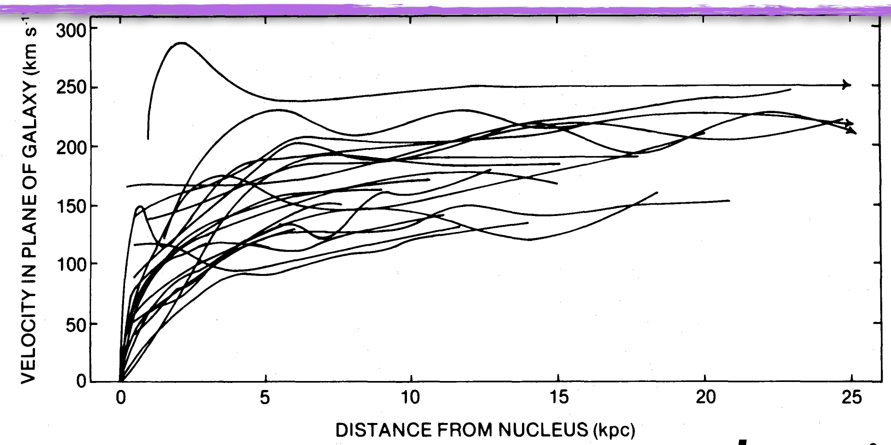


F_A linear in "z" variable

$$\rightarrow \quad r_A^2 \equiv 6\frac{d}{dq^2}F_A(q^2)\Big|_{q^2=0}$$

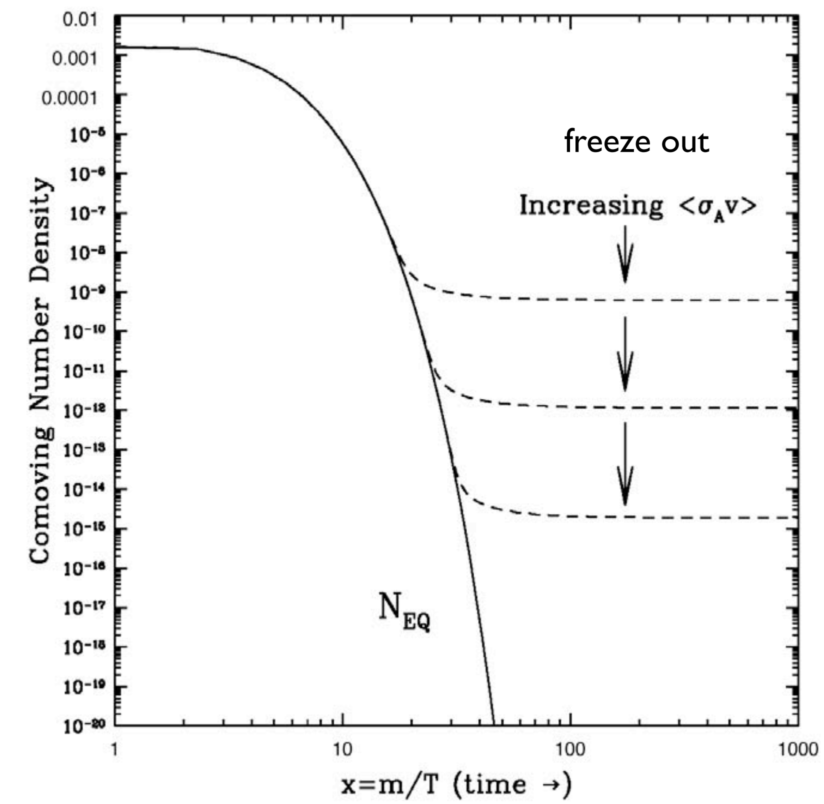
#1 nucleon axial radius

- dark matter:



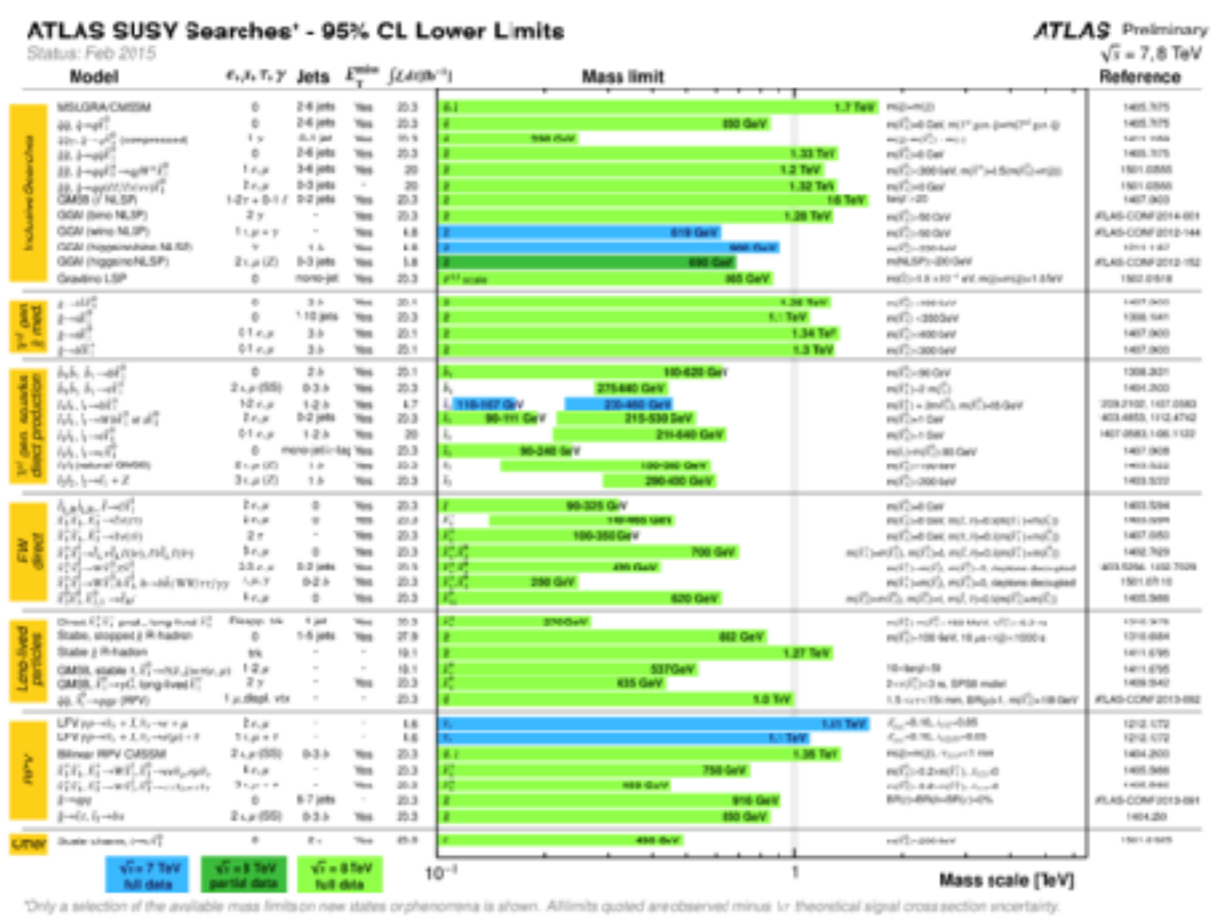
It exists

+



It might be a thermal relic Weakly Interacting Massive Particle (WIMP)

+



WIMP paradigm pushed to larger masses ($\gg m_W$)

→ $\sigma_{\chi N}$

#2 Universal cross section for heavy WIMP-nucleon scattering

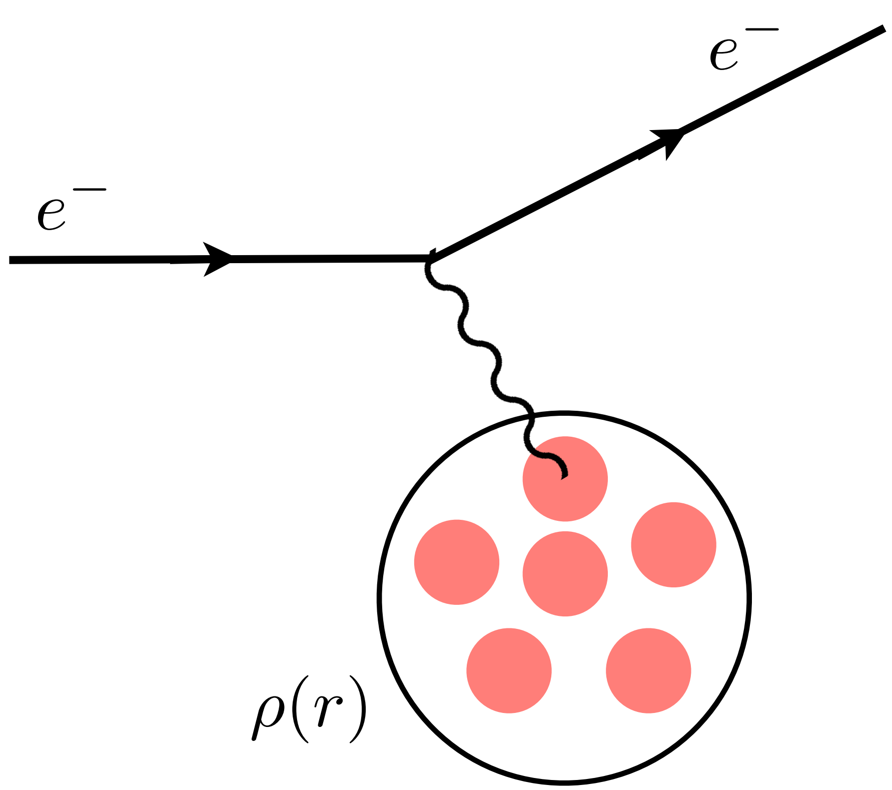
- precision spectroscopy:

Most mundane resolution of the proton radius puzzle:

- *change fundamental Rydberg constant by ~5σ*

- *revise inferences from several decades of both electron scattering and hydrogen spectroscopy*

And the neutrino problem is harder (flux, nuclear effects, statistics). So we need to get this right.

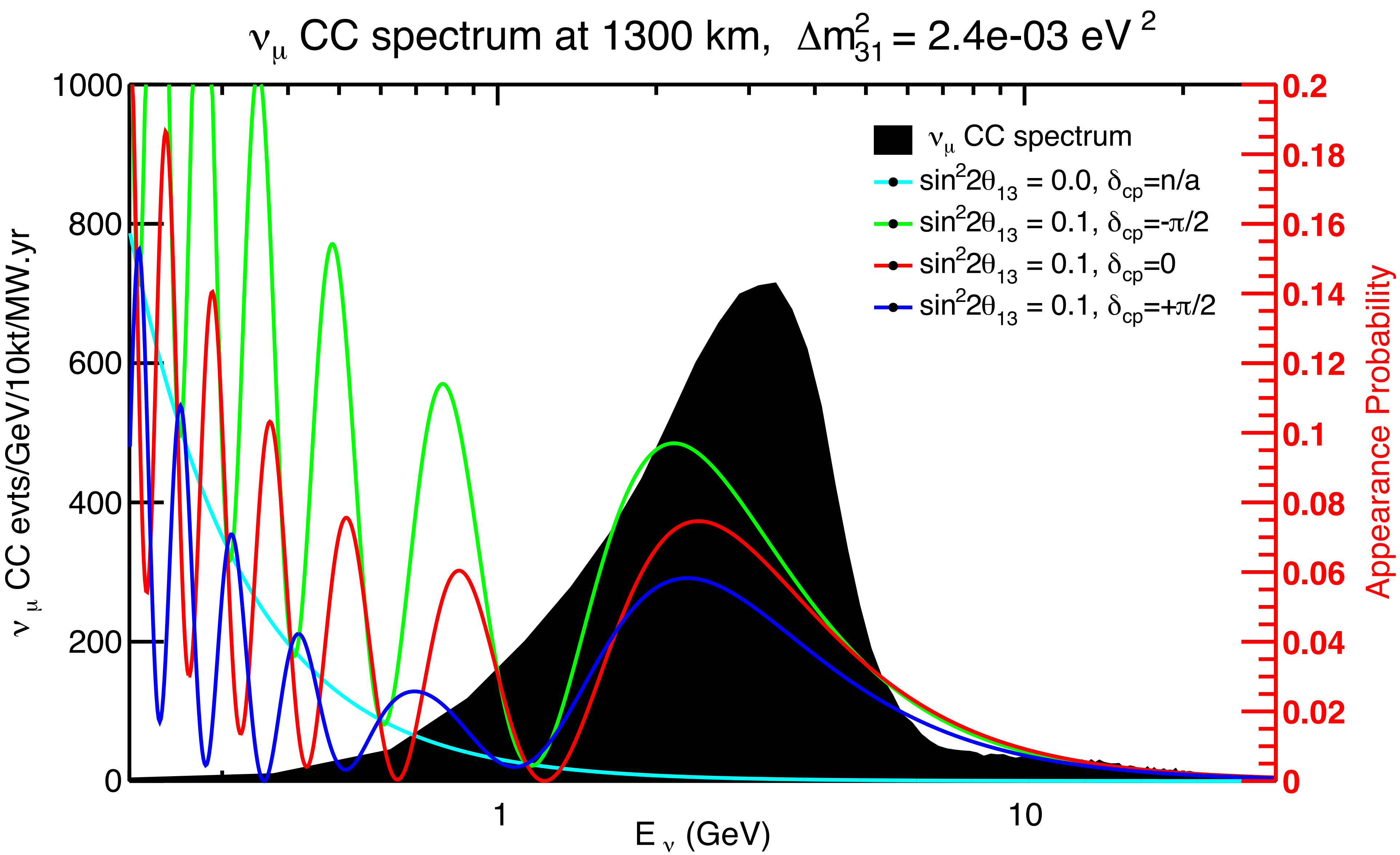


$$E_n \sim \frac{\text{Rydberg}}{n^2} + \frac{r_E^2}{n^3}$$

#3 proton charge radius

1. muon capture and neutrino cross sections

Recent neutrino discoveries (neutrino mixing) have set the stage for yet further discoveries (leptonic CP violation, …)



LBNE, 1307.7335

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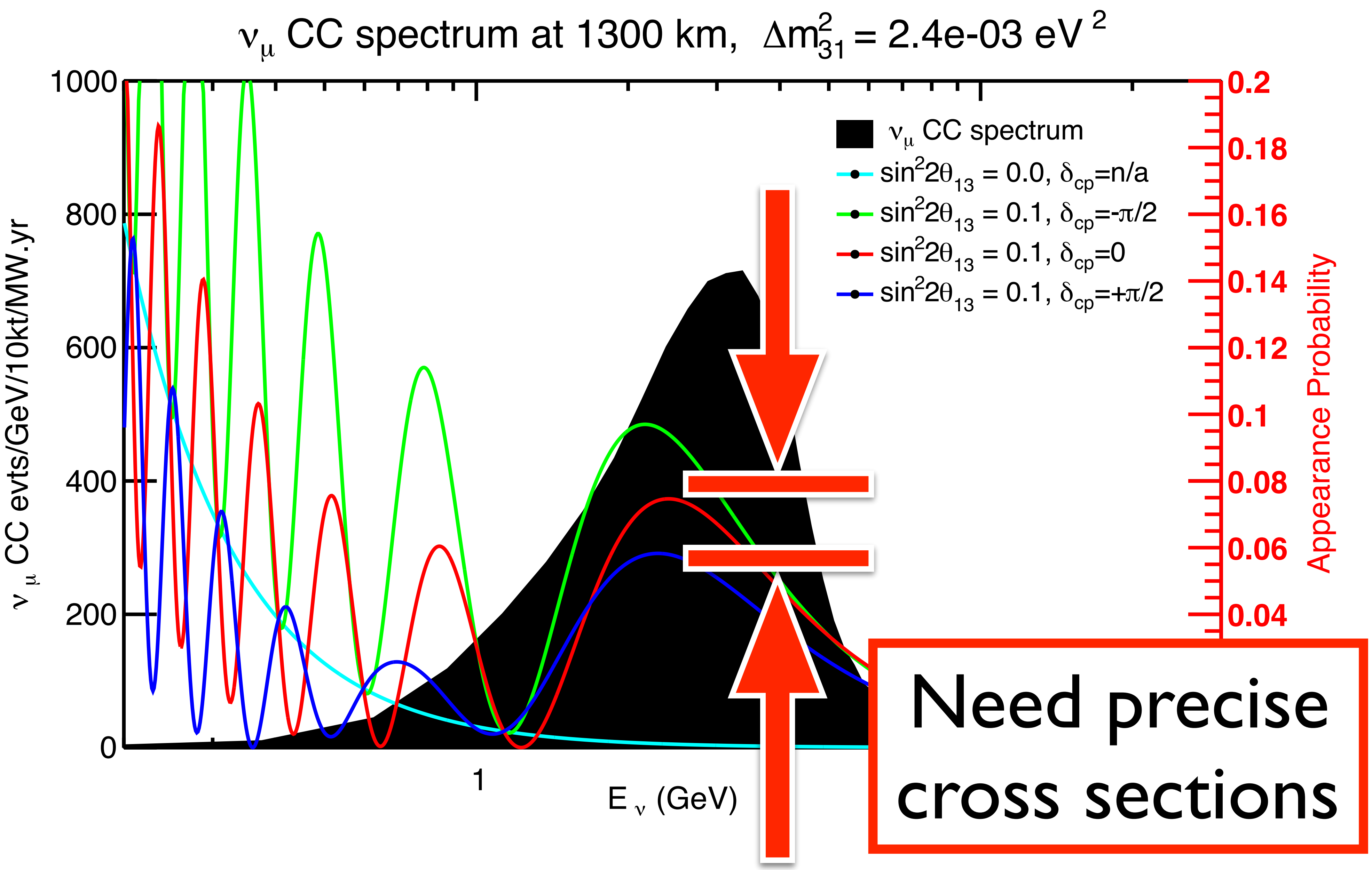

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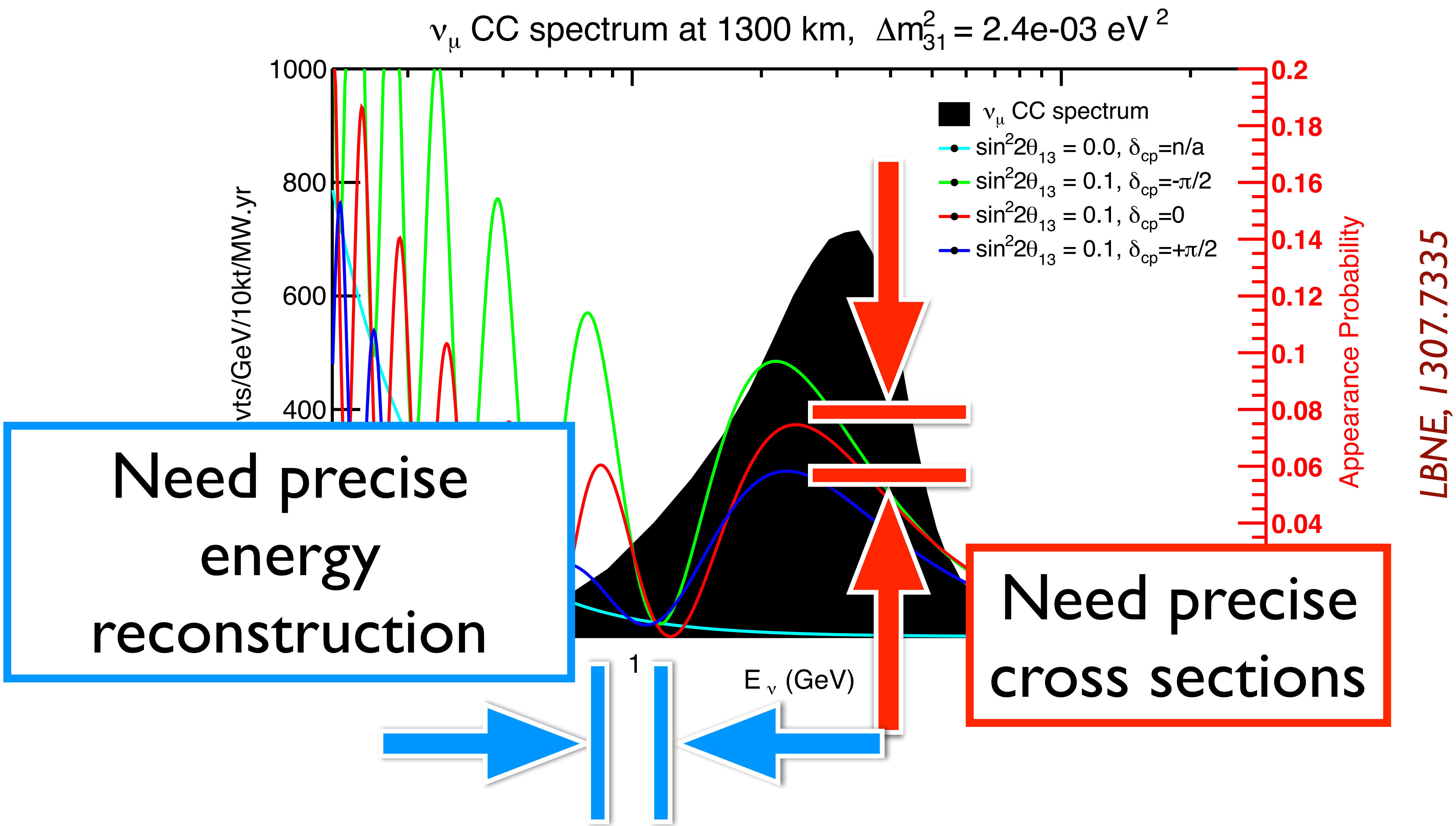
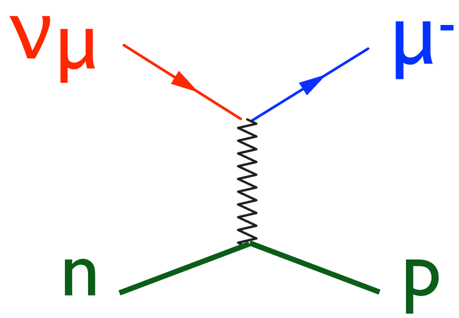


Start with the basic process



$$\sigma(\nu n \to \mu p) = \left| \cdots F_A(q^2) \cdots \right|^2$$

poorly known axial-vector form factor

A common ansatz for F_A has been employed for the last ~40 years:

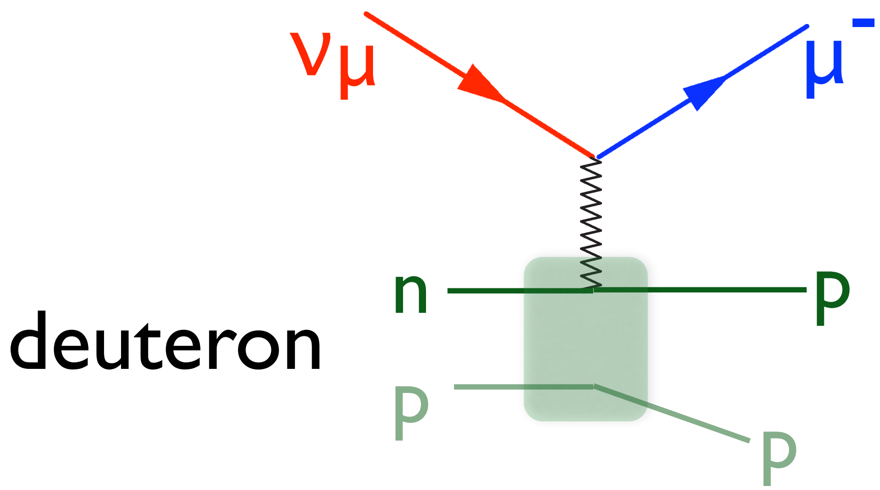
$$F_A^{\mathrm{dipole}}(q^2) = F_A(0) \left(1 - \frac{q^2}{m_A^2}\right)^{-2}$$

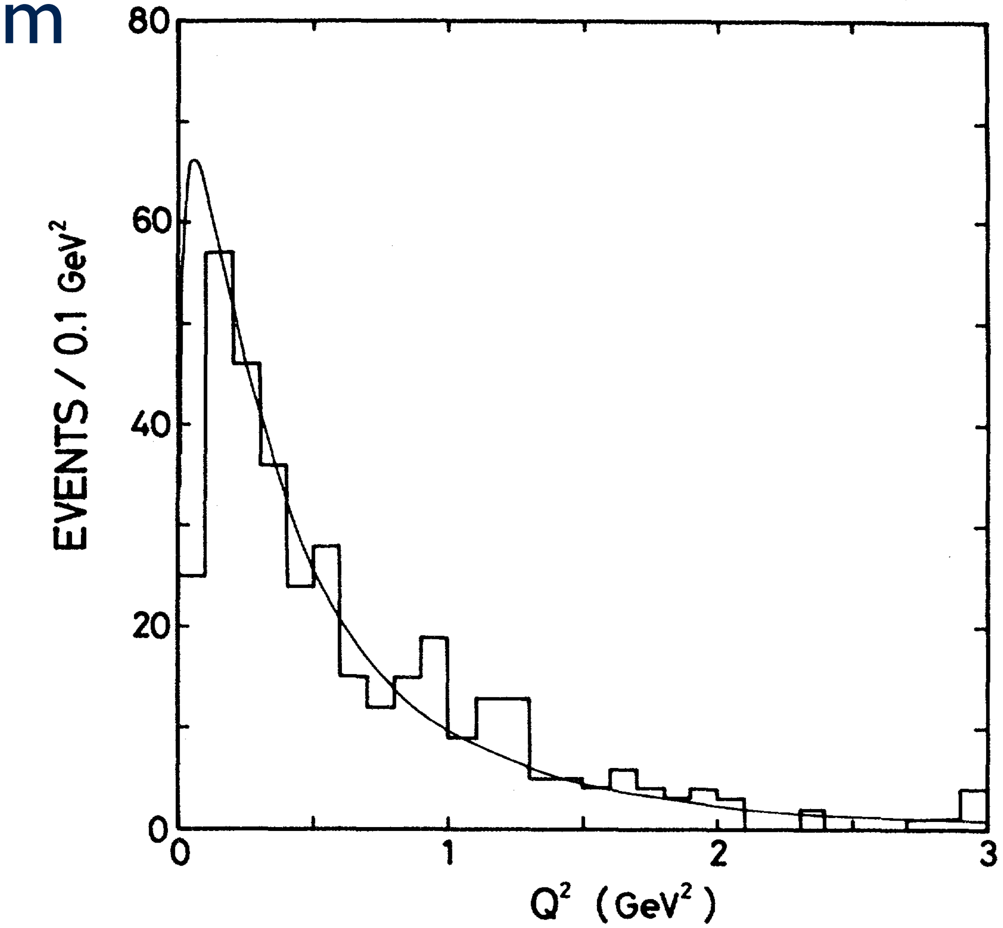
Inconsistent with QCD.

Typically quoted uncertainties are (too) small (e.g. compared to proton charge form factor!)

$$\frac{1}{F_A(0)} \frac{dF_A}{dq^2}\bigg|_{q^2=0} \equiv \frac{1}{6} r_A^2 \qquad r_A = 0.674(9)\ \mathrm{fm}$$

Best source of almost-free neutrons: deuterium





Fermilab 15-foot deuterium bubble chamber, PRD 28, 436 (1983)

Deuterium bubble chamber data

- small(-ish) nuclear effects
- small(-ish) experimental uncertainties
- small statistics, ~3000 events in world data

also:

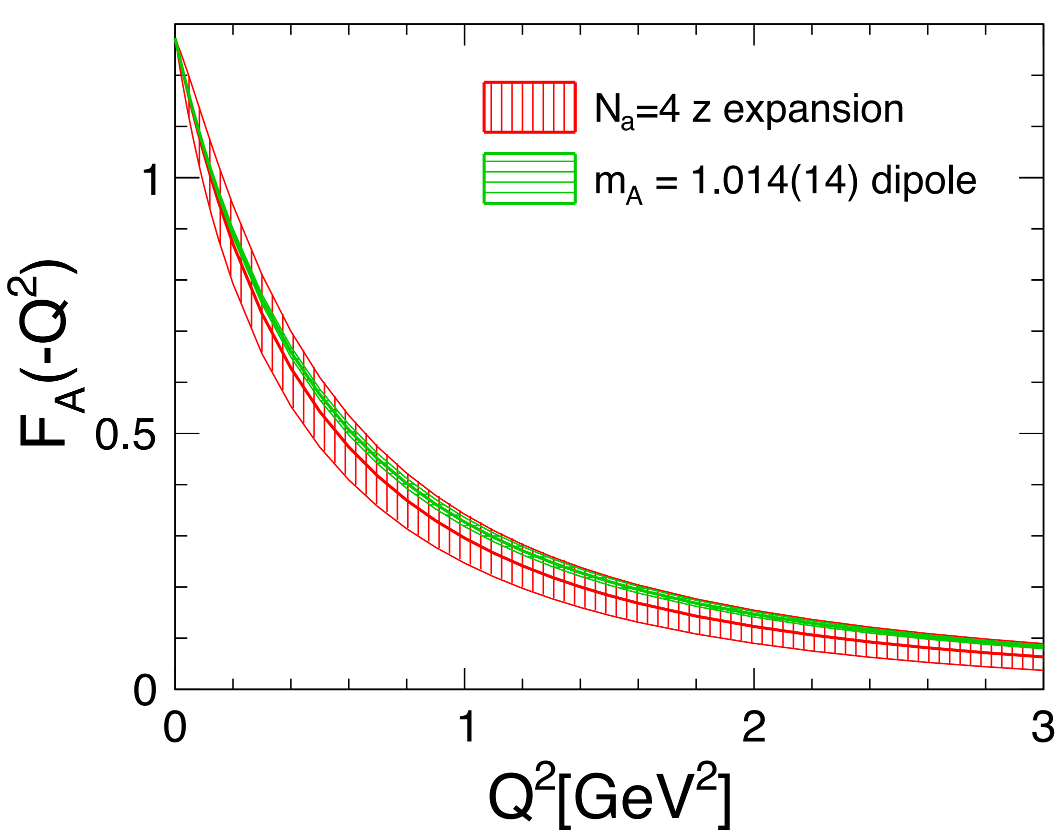
ANL 12-foot deuterium bubble chamber, PRD 26, 537 (1982)

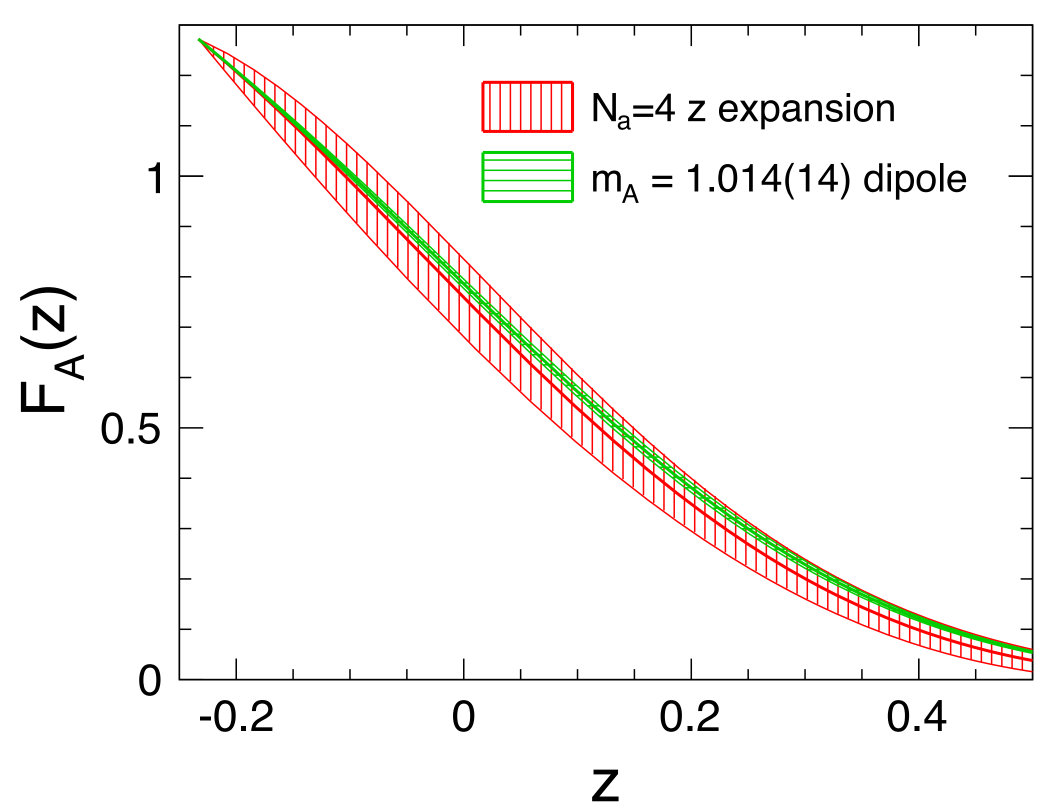
BNL 7-foot deuterium bubble chamber, PRD23, 2499 (1981)

- F_A with complete error budget:

$$[a_1, a_2, a_3, a_4] = [2.30(13), -0.6(1.0), -3.8(2.5), 2.3(2.7)]$$

$$C_{ij} = \begin{pmatrix} 1 & 0.350 & -0.678 & 0.611 \\ 0.350 & 1 & -0.898 & 0.367 \\ -0.678 & -0.898 & 1 & -0.685 \\ 0.611 & 0.367 & -0.685 & 1 \end{pmatrix}$$





Derived observables: 1) axial radius

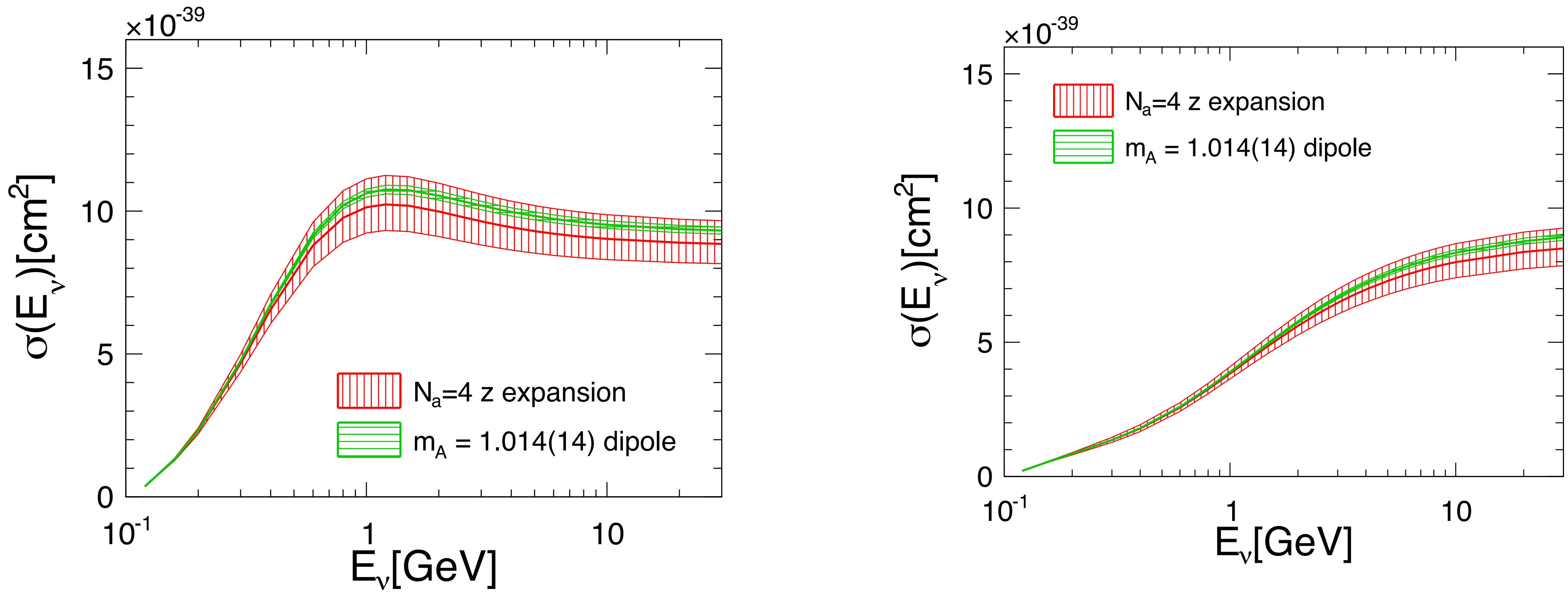
$$\frac{1}{F_A(0)}\frac{dF_A}{dq^2}\bigg|_{q^2=0} \equiv \frac{1}{6}r_A^2$$

$$r_A^2 = 0.46(22)\,\mathrm{fm}^2$$

- order of magnitude larger uncertainty compared to historical dipole fits

- impacts comparison to other data, e.g. pion electroproduction, muon capture

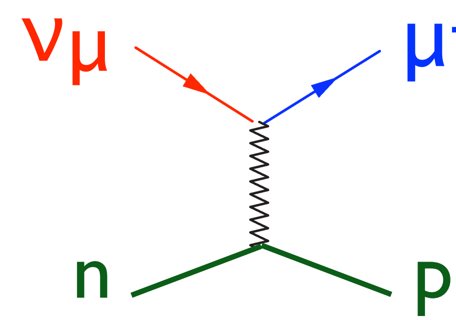
Derived observables: 2) neutrino-nucleon quasi elastic cross sections



$$\sigma_{\nu n \to \mu p}(E_\nu = 1\,\mathrm{GeV}) = 10.1(0.9) \times 10^{-39}\,\mathrm{cm}^2$$

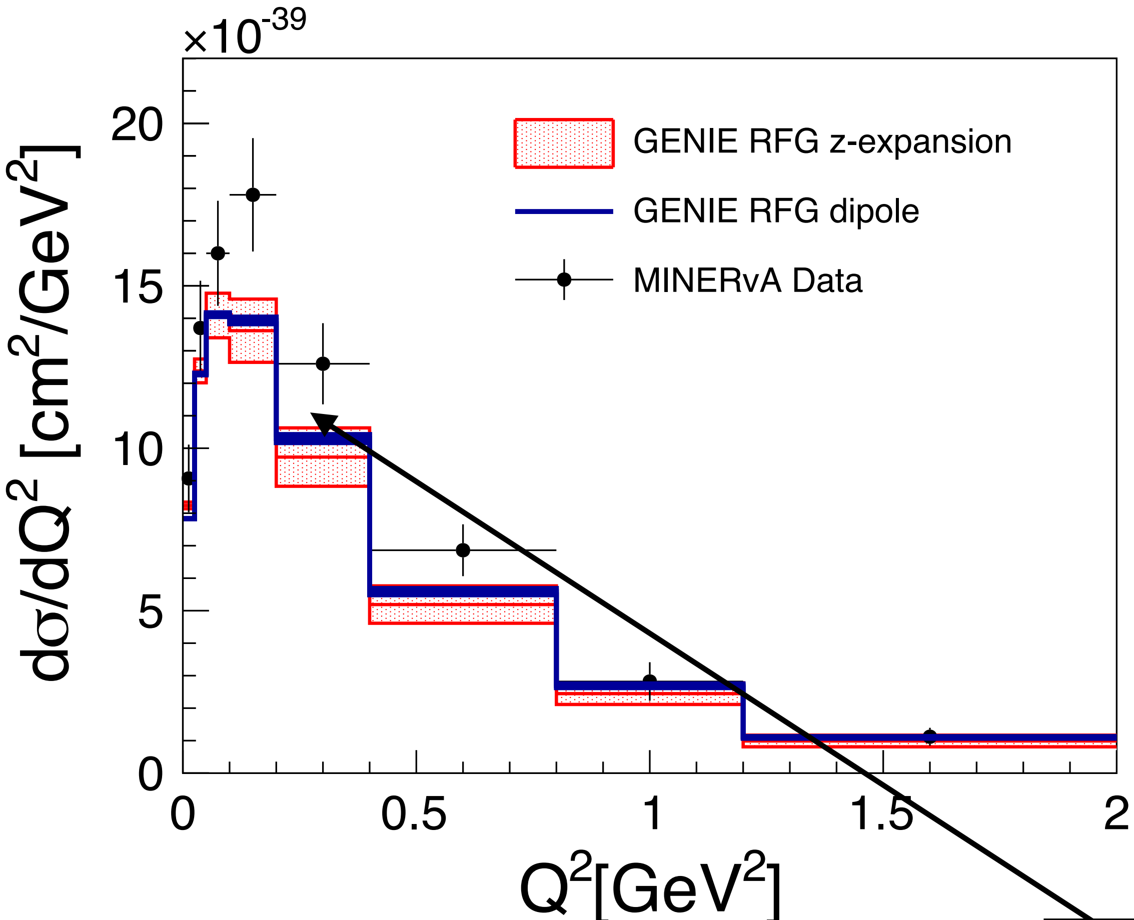
$$\sigma_{\nu n \to \mu p}(E_\nu = 3\,\mathrm{GeV}) = 9.6(0.9) \times 10^{-39}\,\mathrm{cm}^2$$

discriminating nuclear models



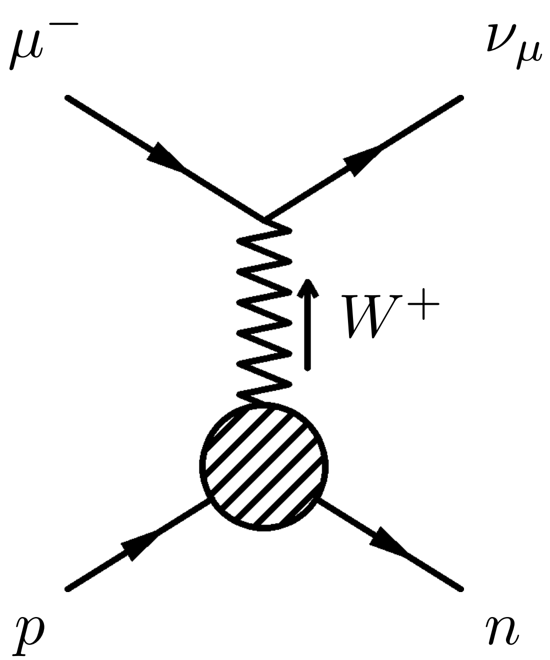
$$\sigma(\nu n \to \mu p) = |\cdots F_A(q^2) \cdots|^2$$

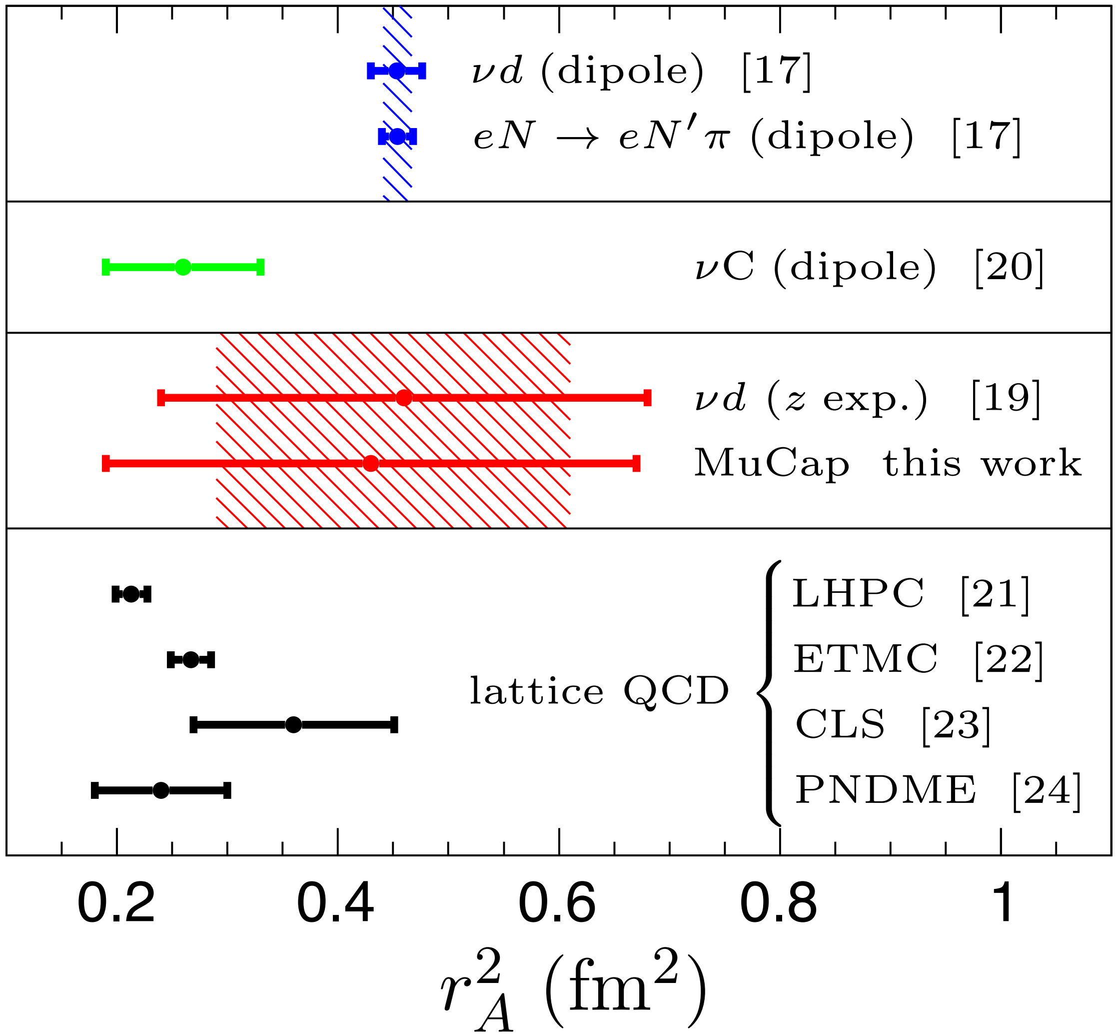
poorly known axial form factor



want to extract nuclear and flux effects from this comparison: but large nucleon level form factor uncertainty

muon capture constraints

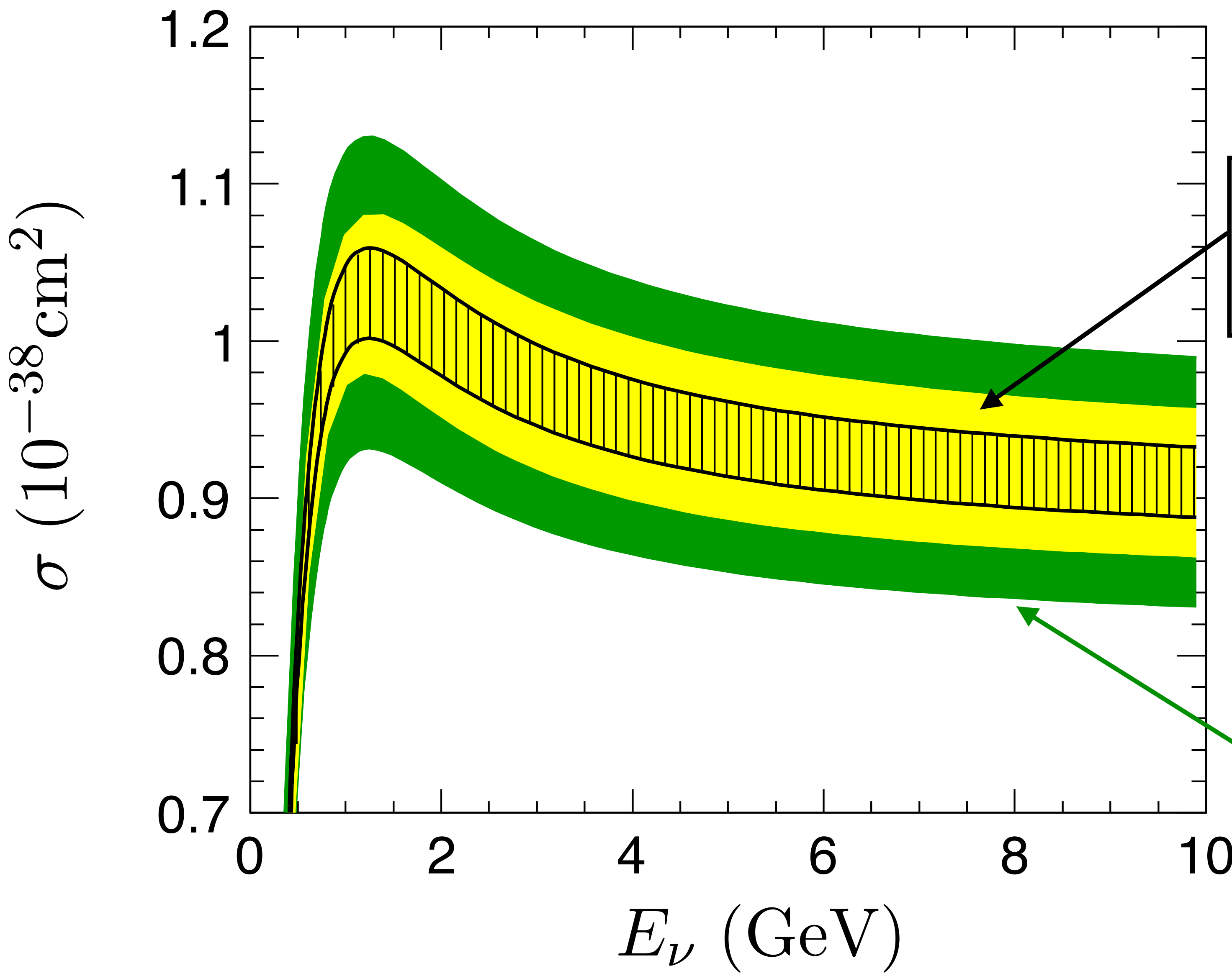




- potential factor ~3 improvement from next generation muon capture experiment

RJH, Kammel, Marciano, Sirlin 1708.08462

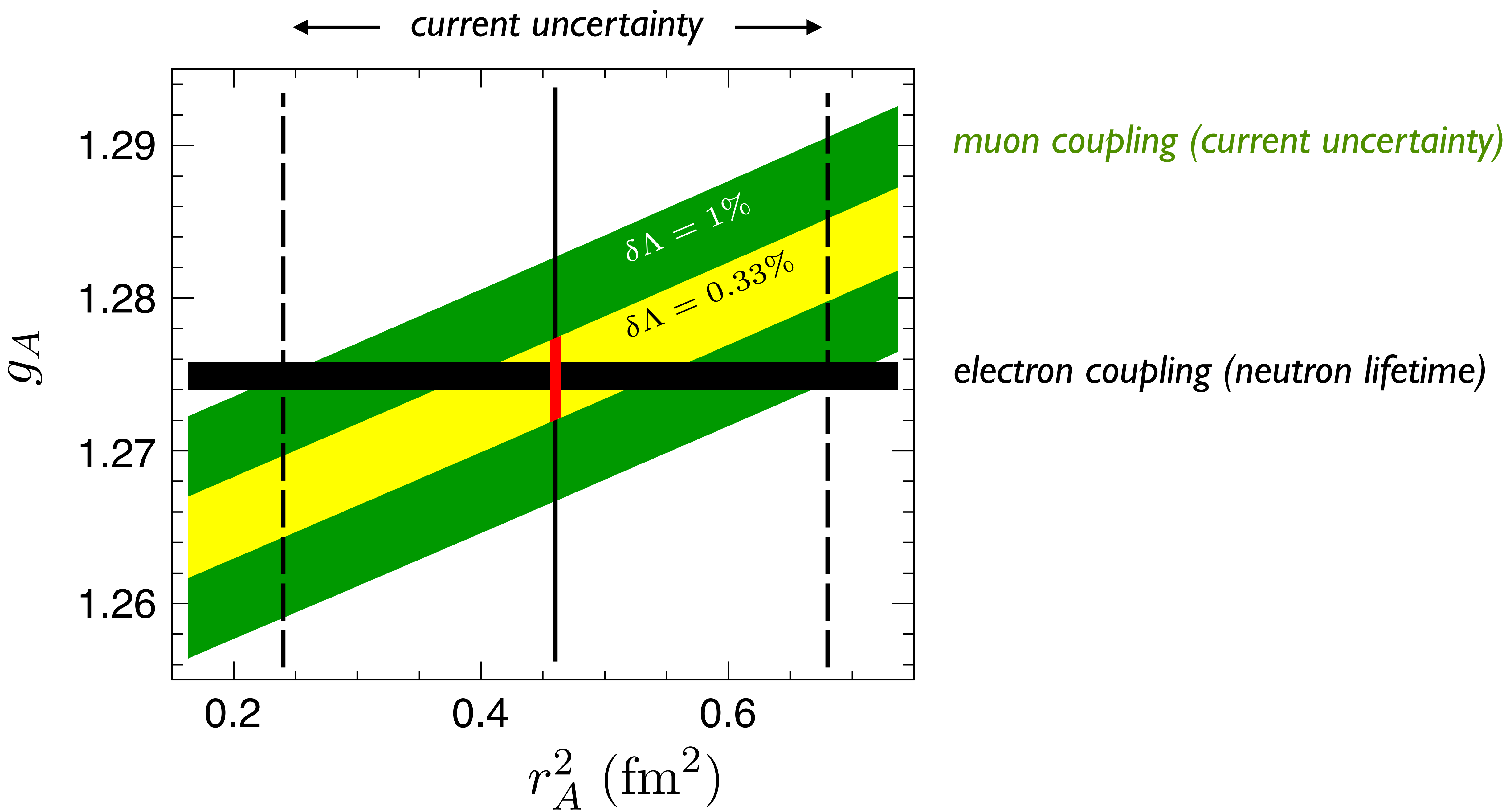
implications for quasielastic neutrino cross sections



with radius constraint: (hatched: external radius error δr_A^2=20%)

existing error (no external radius constraint)

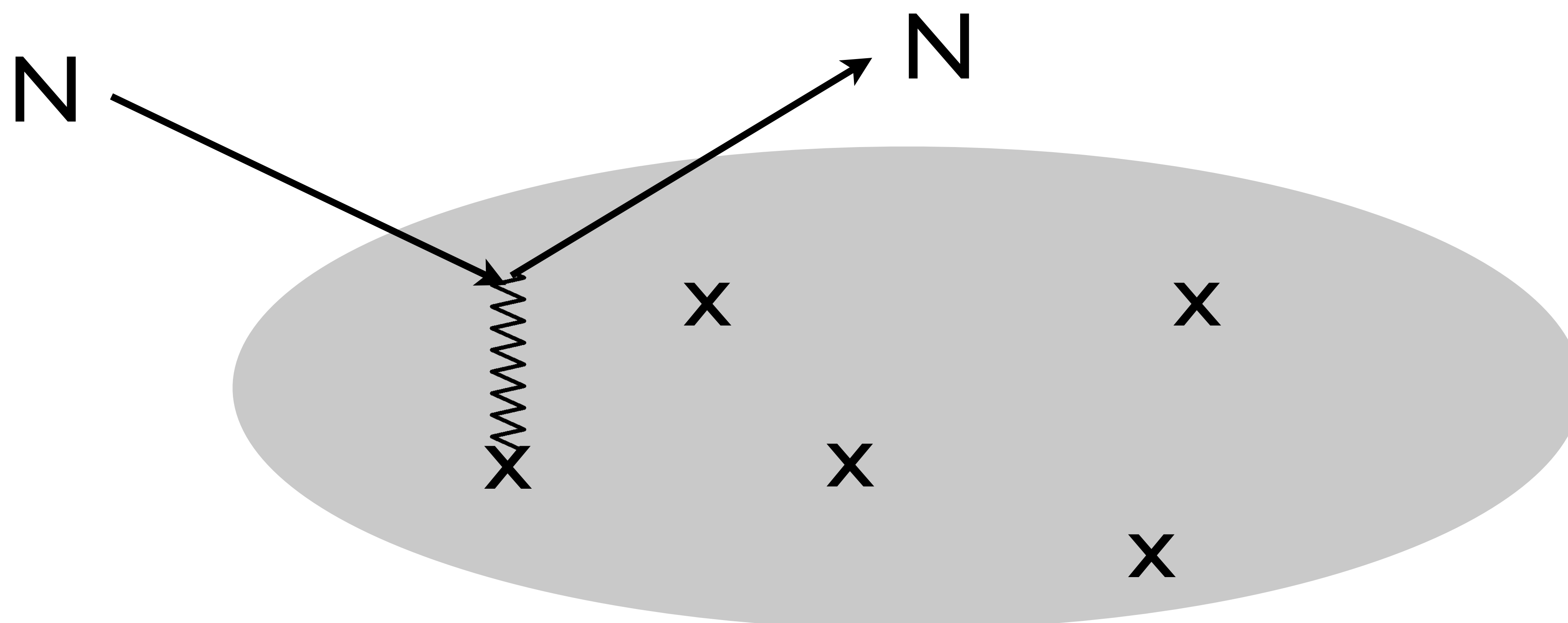
test of electron-muon universality



2. the universal WIMP-nucleon cross section

Mechanisms versus models

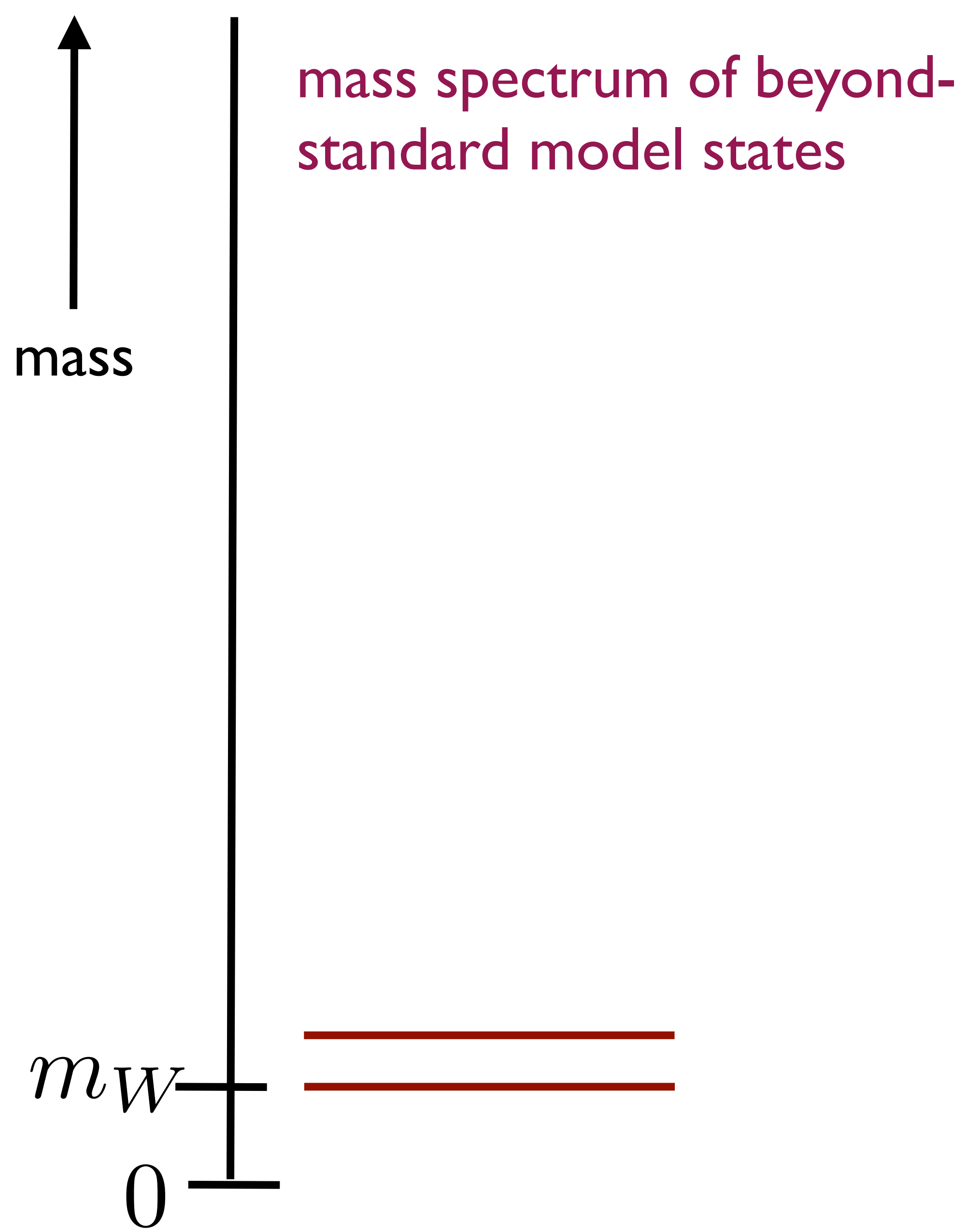
Electroweak charged WIMP Mechanism versus WIMP Model



Focus on self-conjugate SU(2) triplet. Could be:

- SUSY wino
- Weakly Interacting Stable Pion
- Minimal Dark Matter
- ...

Present null results of direct detection and collider searches may indicate large WIMP/New Physics mass scale



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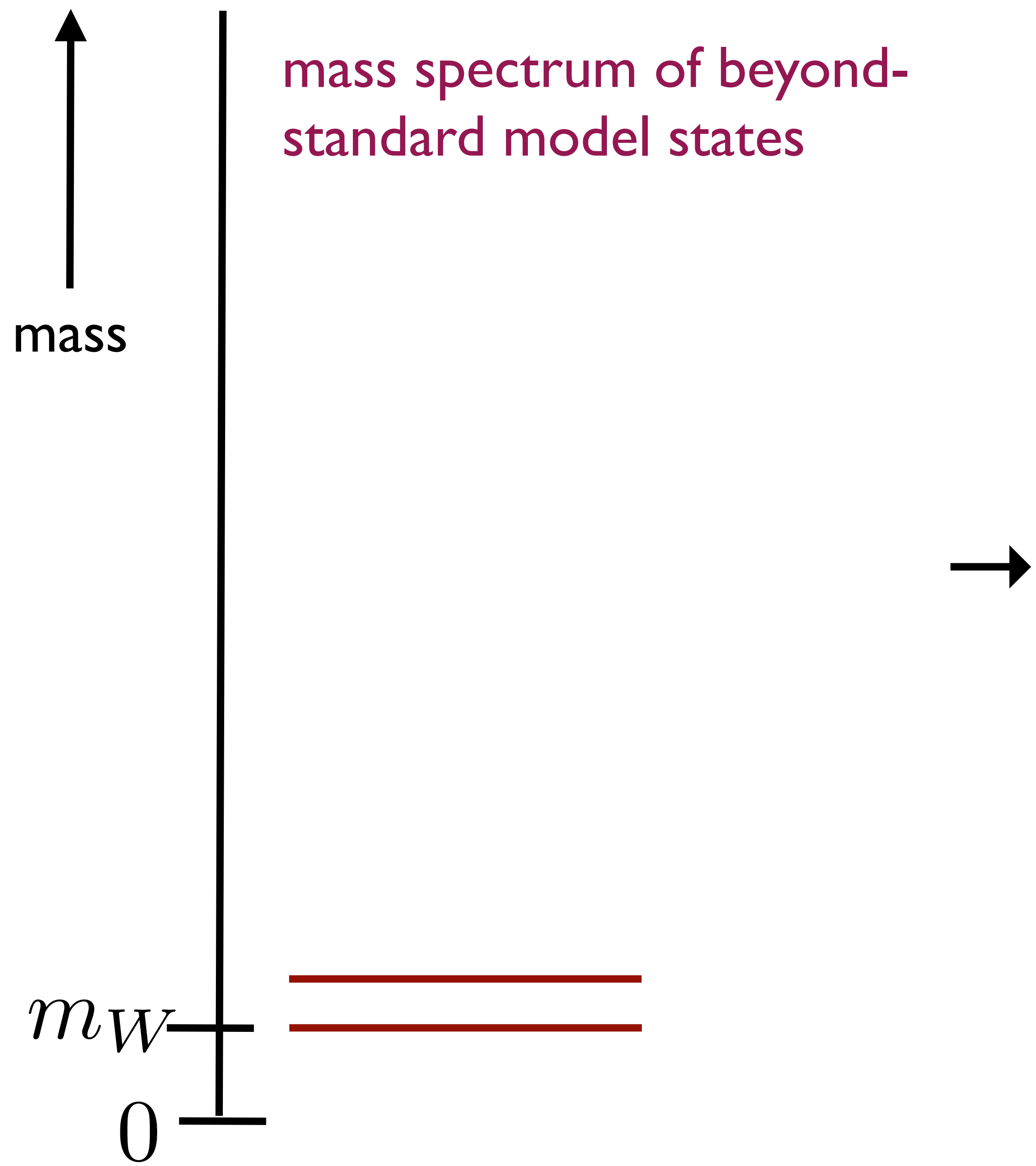
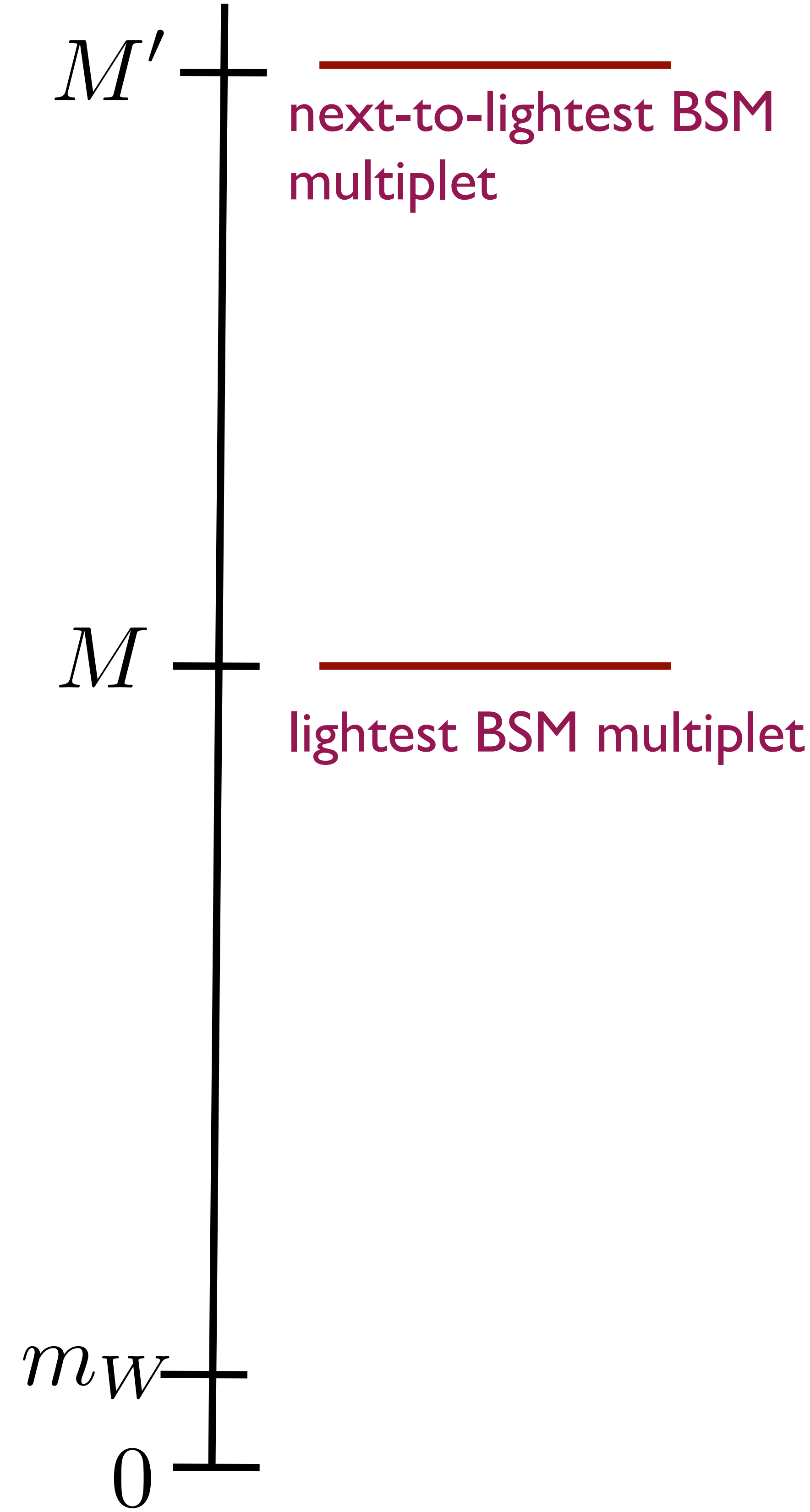
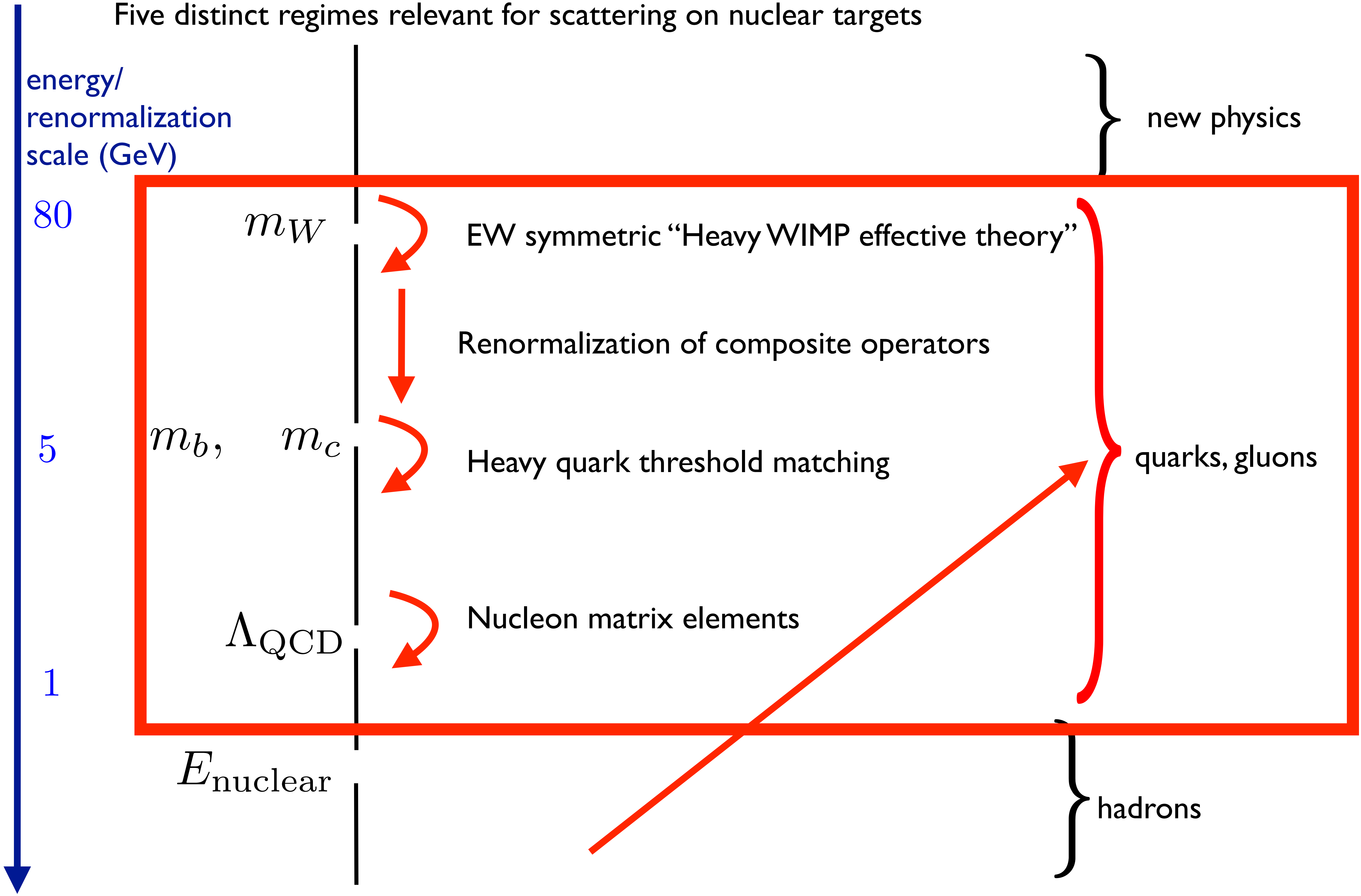




If WIMP mass M >> m_W , isolation (M'-M >> m_W) becomes generic.   Expand in m_W/M, m_W/(M'-M)

Large WIMP mass regime is a focus of future experiments in direct, indirect and collider probes

Five distinct regimes relevant for scattering on nuclear targets



"SM anatomy" of interactions between weak and hadronic scales

Heavy WIMP Effective Theory

- Present null results may point to $\gtrsim$ TeV WIMP mass
- This regime has important challenges and simplifications

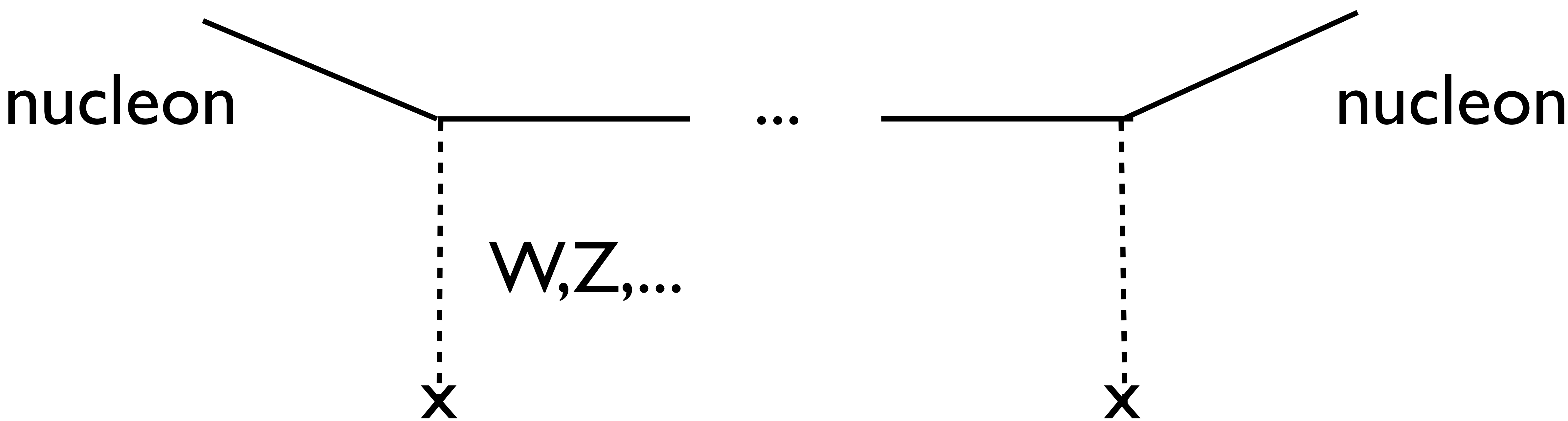
Take as basic WIMP:	SU(3) x SU(2) x U(1)			spin
	1	3	0	s

Many results independent of WIMP spin, and elementary vs. composite nature of WIMP (e.g. wino, composite scalar, …)

Direct detection

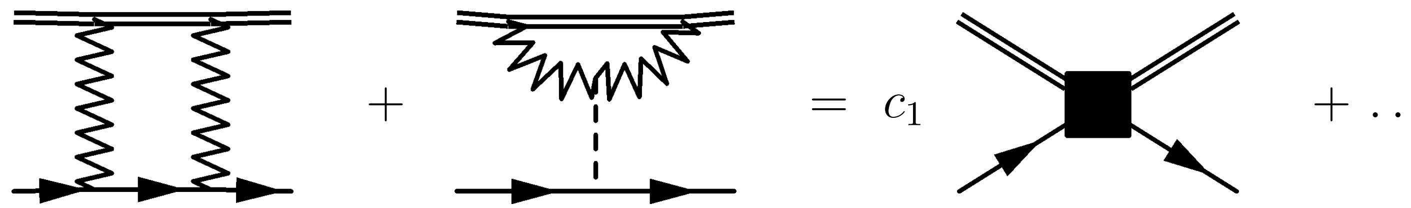
Many manifestations of heavy particle symmetry:

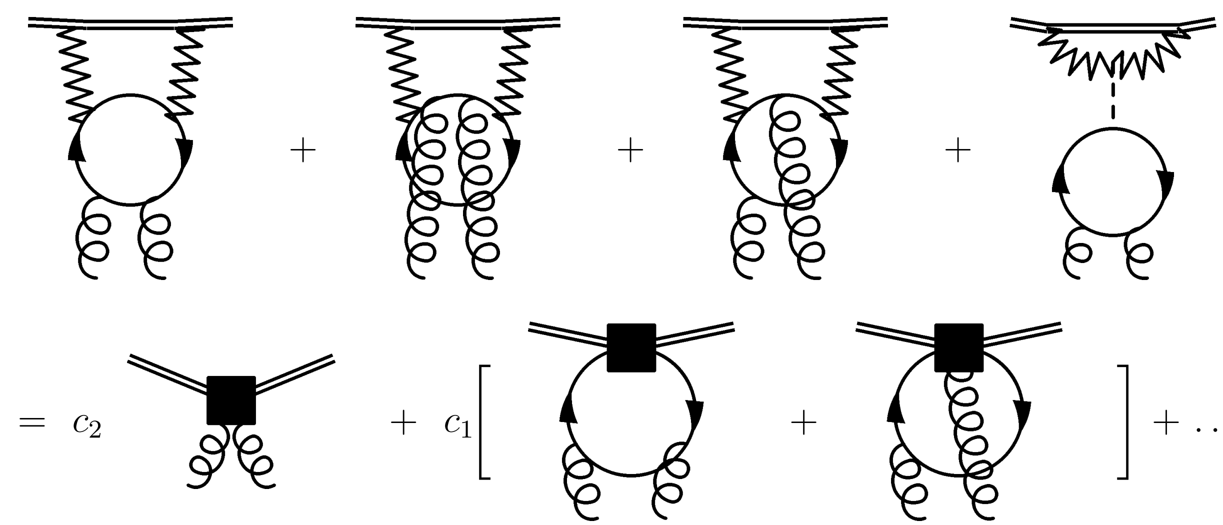
- hydrogen/deuterium spectroscopy $\quad E_n(H) = -\frac{1}{2} m_e (Z\alpha)^2 + \ldots \quad (m_e Z\alpha) \ll m_e$
- heavy meson B/B* transitions $\quad F^{B\to D}(v' = v) = 1 + \ldots \quad \Lambda_{\rm QCD} \ll m_{b,c}$
- DM interactions $\quad \sigma(\chi N \to \chi N) = ? \quad m_W \ll m_\chi$



basic problem in SM physics: scattering of nucleon from SU(2)xU(1) source

Scale separation:	dark sector d.o.f.	SM d.o.f.	# params. (beyond mass)
	$\chi^{(+,-,0)}$	$Q, A^a_\mu, W^i_\mu, B_\mu$	0
M			
	$\chi_v^{(+,-,0)}$	$Q, A^a_\mu, W^i_\mu, B_\mu$	0
m_W			
	$\chi_v^{(0)}$	u, d, s, c, b, A^a_μ	12
m_b, m_c			
	$\chi_v^{(0)}$	u, d, s, A^a_μ	8
Λ_{QCD}			
	$\chi_v^{(0)}$	N, π	3
m_π			
	$\chi_v^{(0)}$	n, p	2
$1/R_{nucleus}$			
	$\chi_v^{(0)}$	$\mathcal{N}$	1





• the heavy lifting is necessary: large gluon matrix element, amplitude cancellations

Scale separation:	dark sector d.o.f.	SM d.o.f.	# params. (beyond mass)
	$\chi^{(+,-,0)}$	$Q, A^a_\mu, W^i_\mu, B_\mu$	0
M			
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m_W			
	$\chi_v^{(0)}$	u, d, s, c, b, A^a_μ	12
m_b, m_c			
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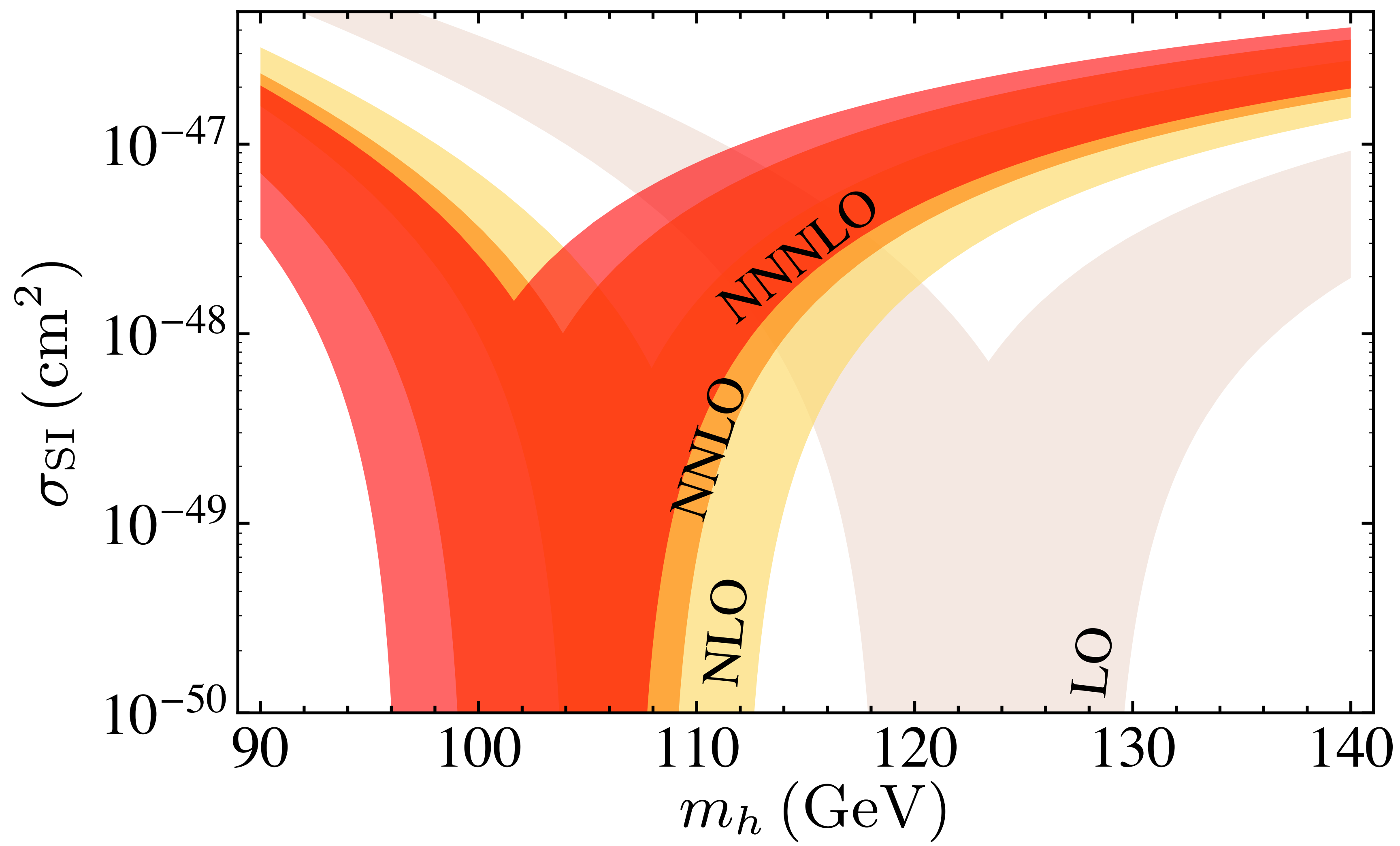
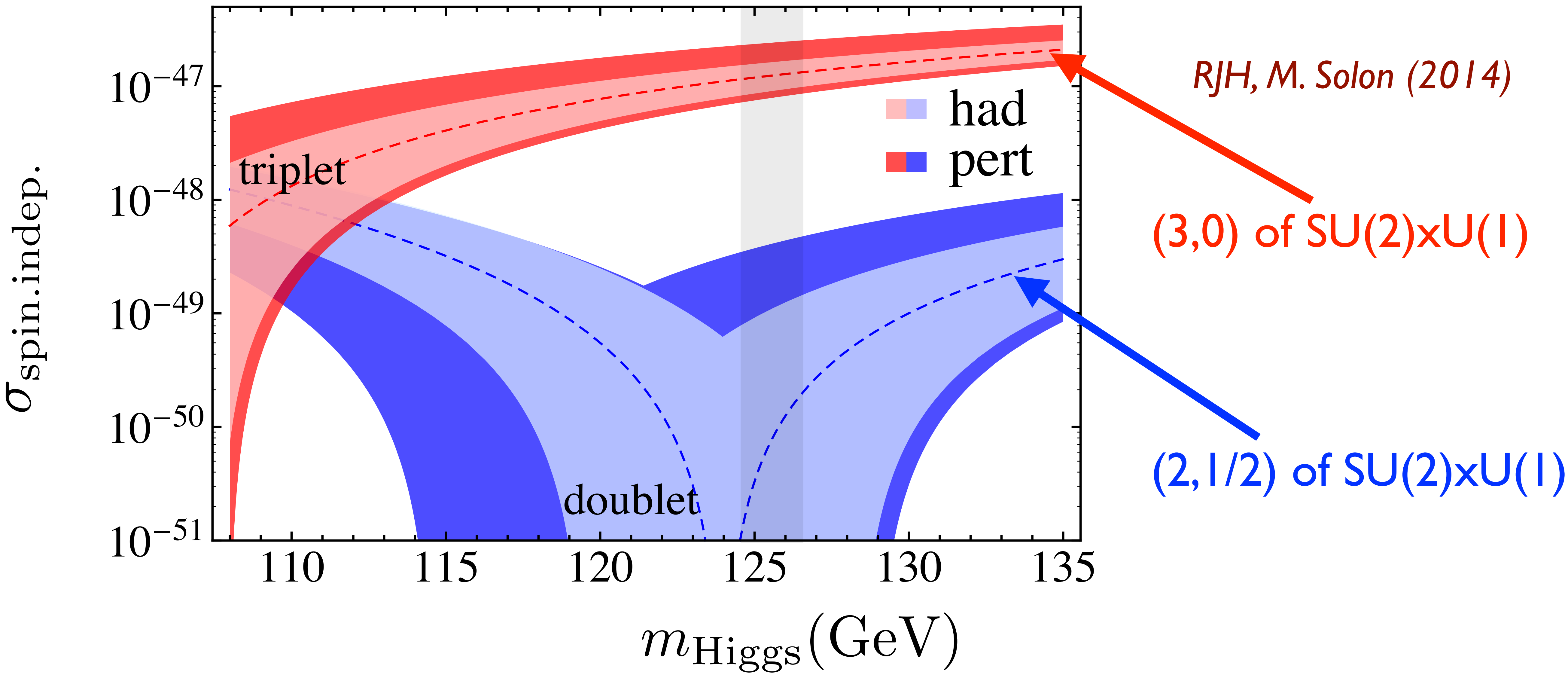
- the heavy lifting is necessary



Benchmarks: large mass, low velocity limit

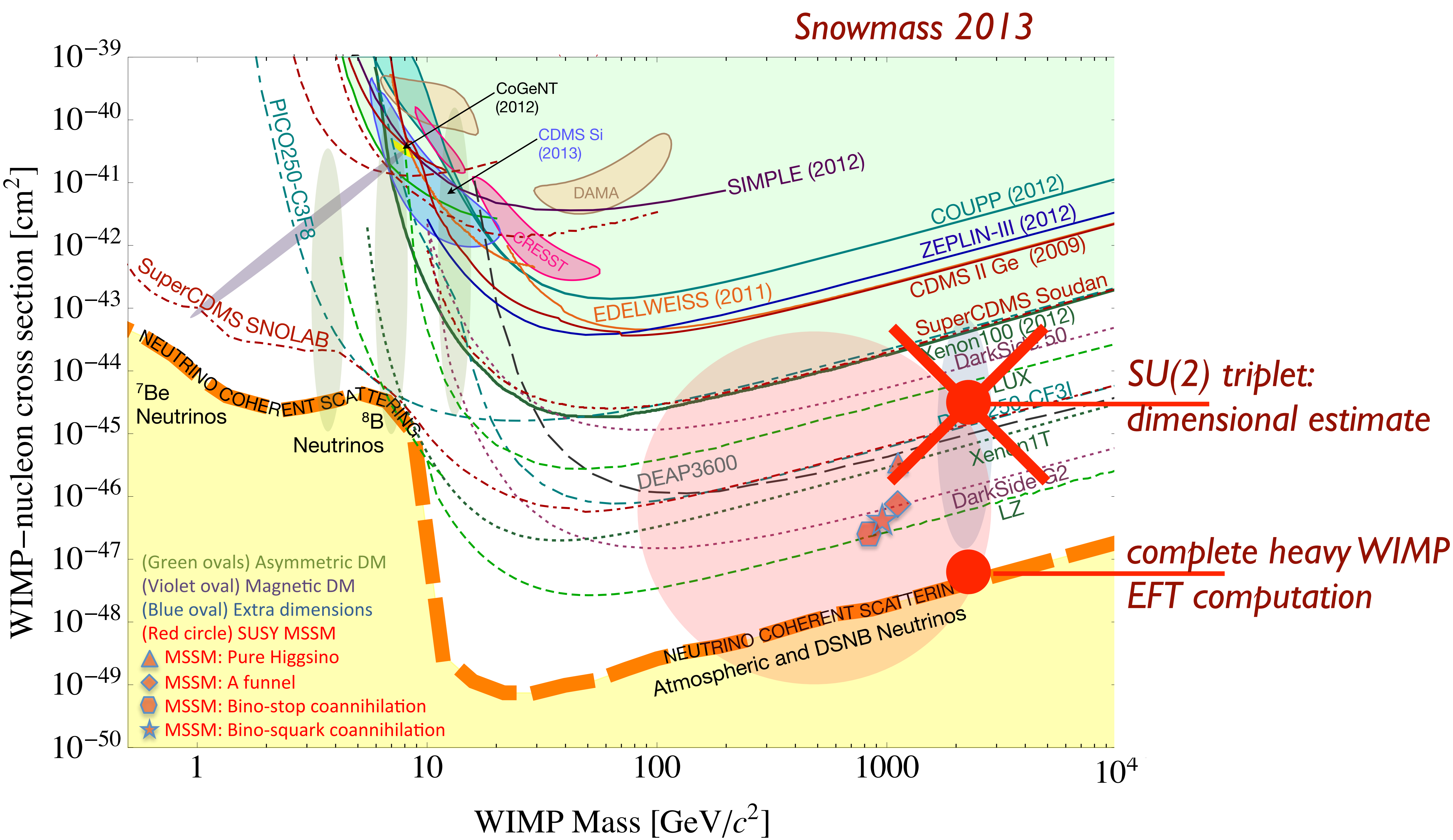


- Suppressed versus dimensional estimate (~10^{-45}cm^2)

- 1/M power corrections under investigation

C.-Y. Chen, RJH, M. Solon, A. Wijangco, to appear

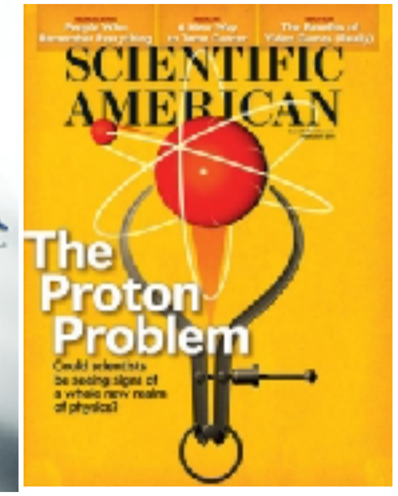
• the heavy lifting is necessary



3. the proton radius puzzle

Some facts about the proton radius puzzle

1) It has generated a lot of attention and controversy



2) The *most mundane* resolution necessitates:

- >5σ shift in fundamental Rydberg constant
- discarding or revising decades of results in e-p scattering and hydrogen spectroscopy

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This is everybody's problem (HEP, NP, AMO,…):

"The good news is that it's not my problem"

3) E.g. systematic effects in electron-proton scattering impact neutrino-nucleus scattering, *at a level large compared to long baseline precision requirements*

the puzzle

Recall hydrogen spectrum:

$$E_n \sim \frac{R_\infty}{n^2} + \frac{r_E^2}{n^3}$$

$$hcR_\infty = \frac{m_e c^2 \alpha^2}{2} \approx 13.6\,\mathrm{eV}$$

proton charge radius

Disentangle 2 unknowns, R_∞ and r_E, using well-measured 1S-2S hydrogen transition *and*

electron-based measurements

muon-based measurements

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muon-based measurements

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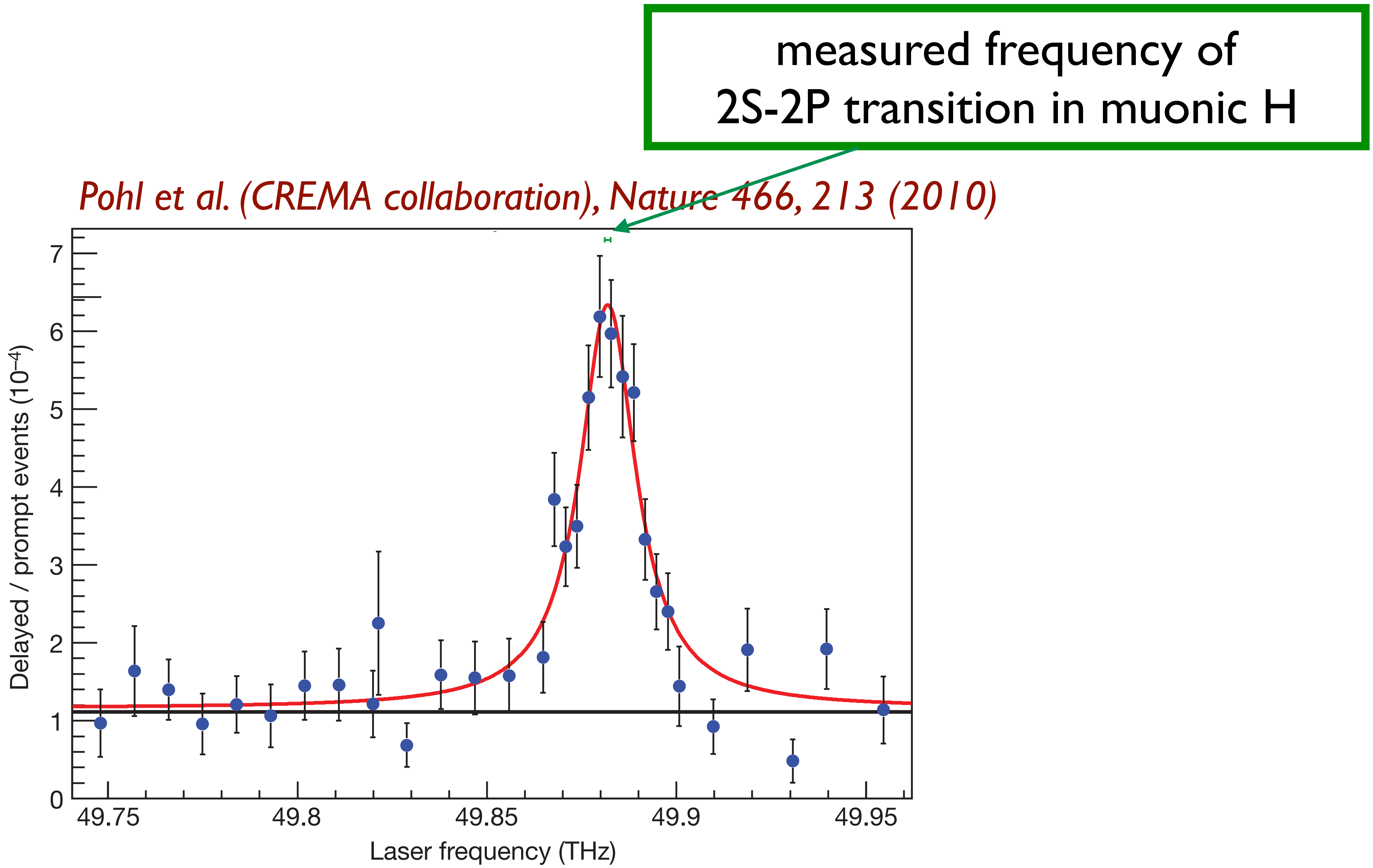
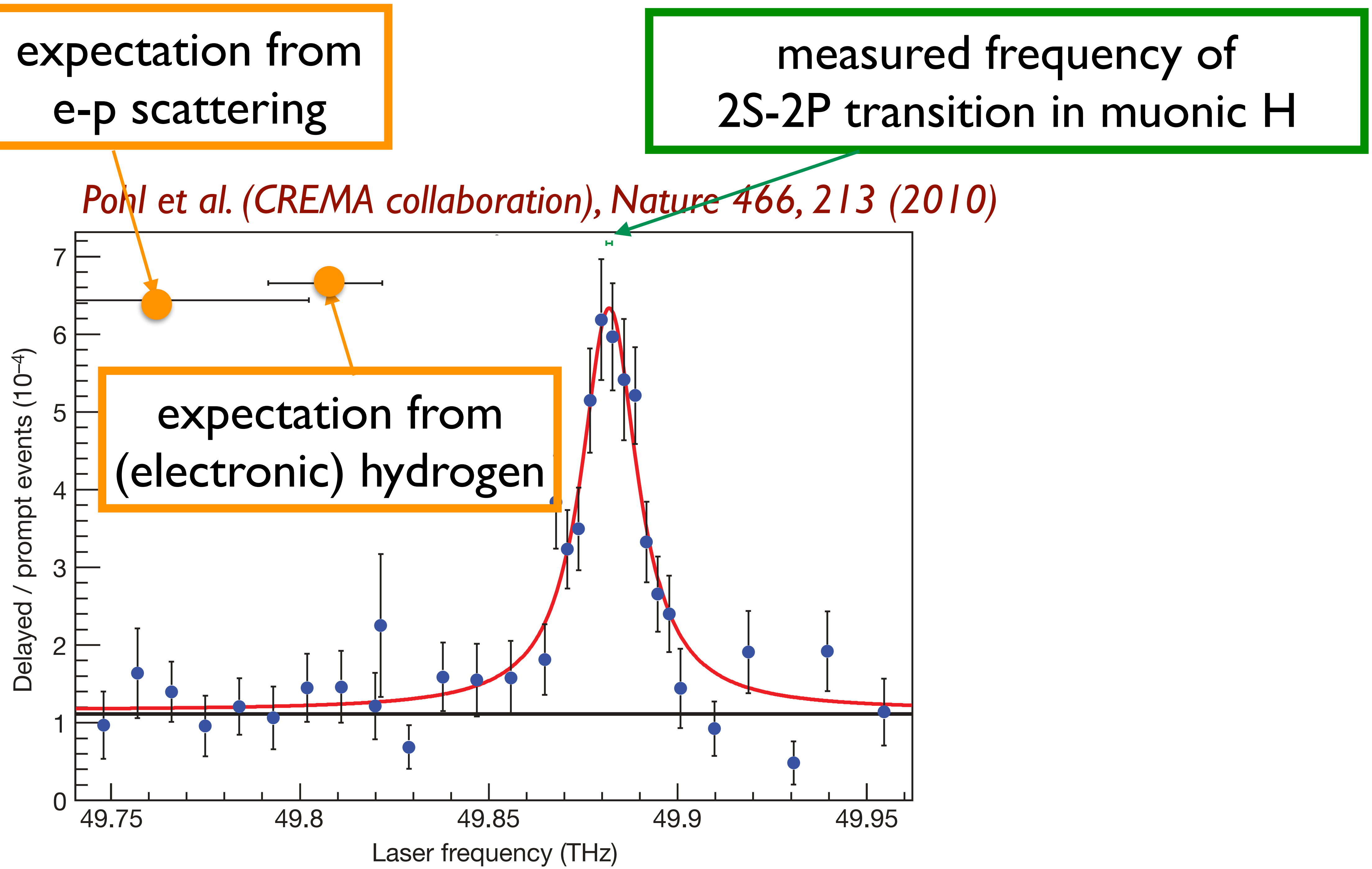
7σ discrepancy between electron-based versus muon-based measurements

muonic hydrogen Lamb shift measurement



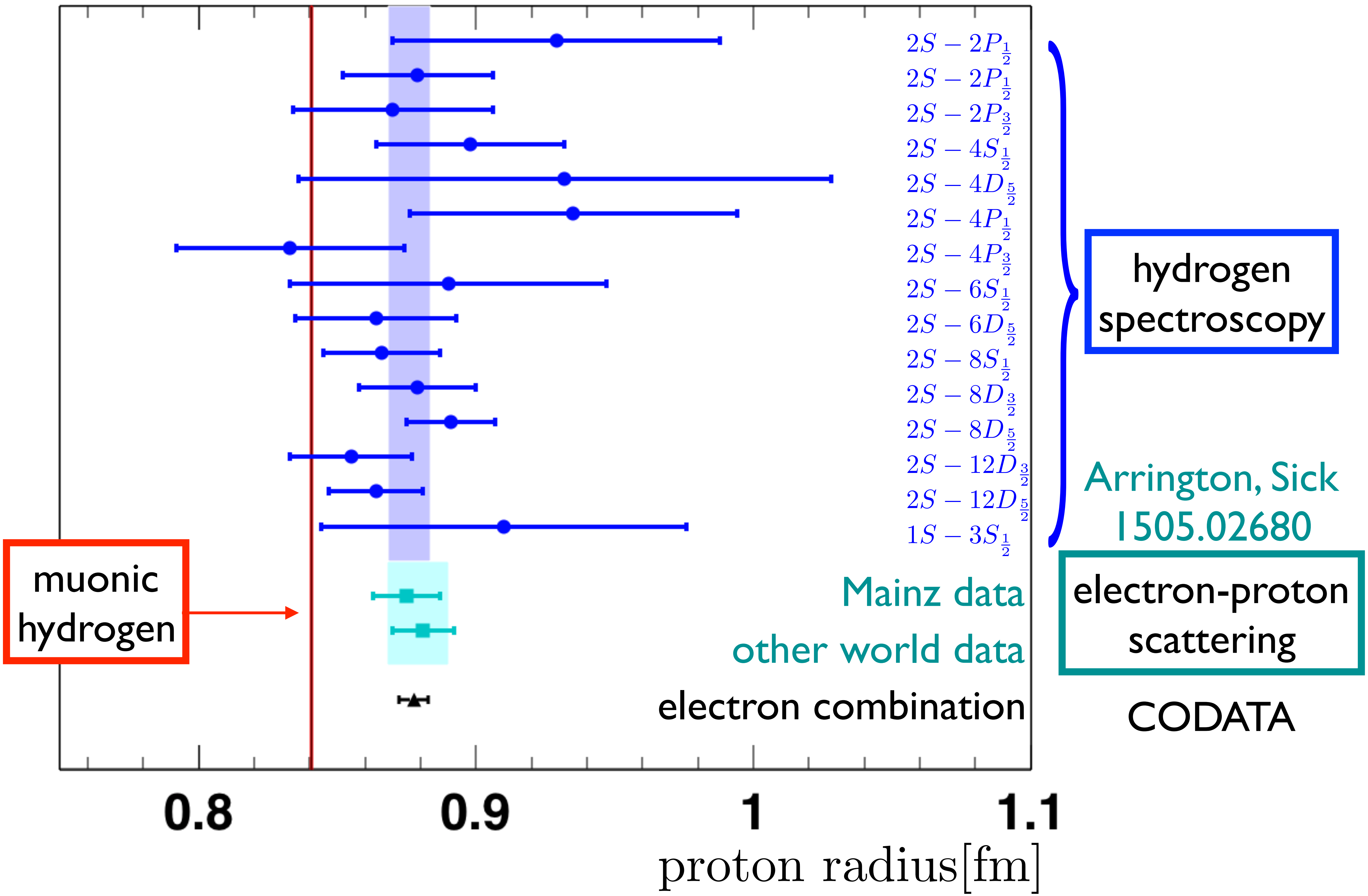
new experimental capabilities: surprises and new insight ?

muonic hydrogen Lamb shift measurement



new experimental capabilities: surprises and new insight ?

summary of electron- and muon- based measurements



status of some theory issues

electron-proton scattering: theory issues

radius is defined as slope of form factor

i) what are the constraints on nonlinearities?

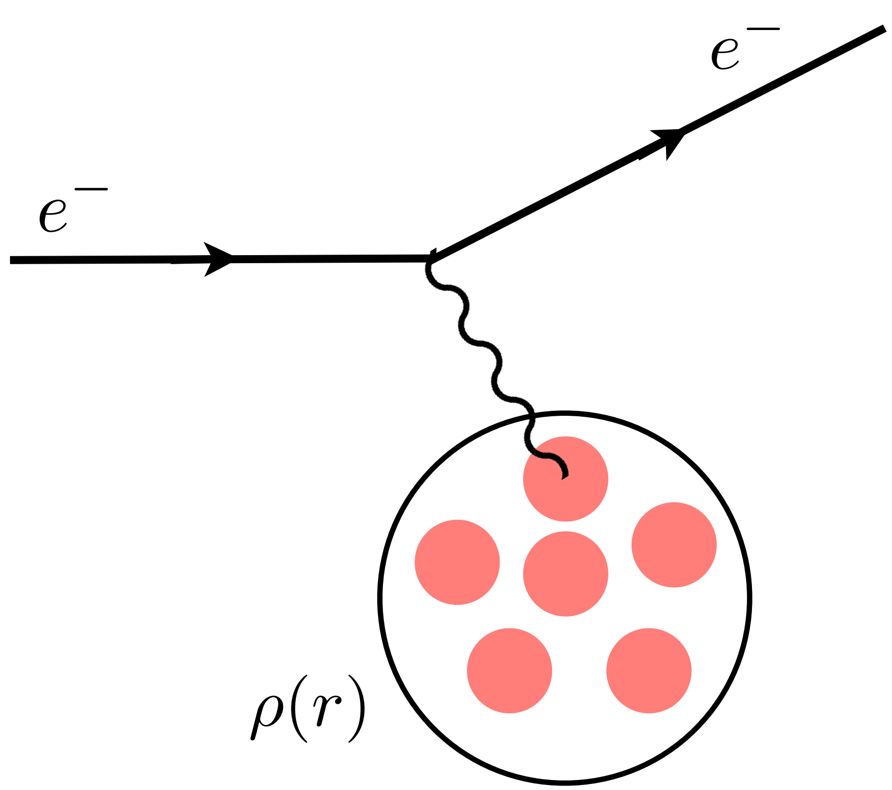
radiative corrections impact radius extraction and can be large (~30%)

ii) are radiative corrections controlled at the sub percent level?

i) what are the constraints on nonlinearities?

recall scattering from extended classical charge distribution:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}} |F(q^2)|^2$$



$$F(q^2) = \int d^3r\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})$$

$$= \int d^3r \left[1 + i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{r})^2 + \dots\right] \rho(r)$$

$$= 1 - \frac{1}{6}\langle r^2\rangle \boldsymbol{q}^2 + \dots$$

for the relativistic, QM, case, *define* radius as slope of form factor

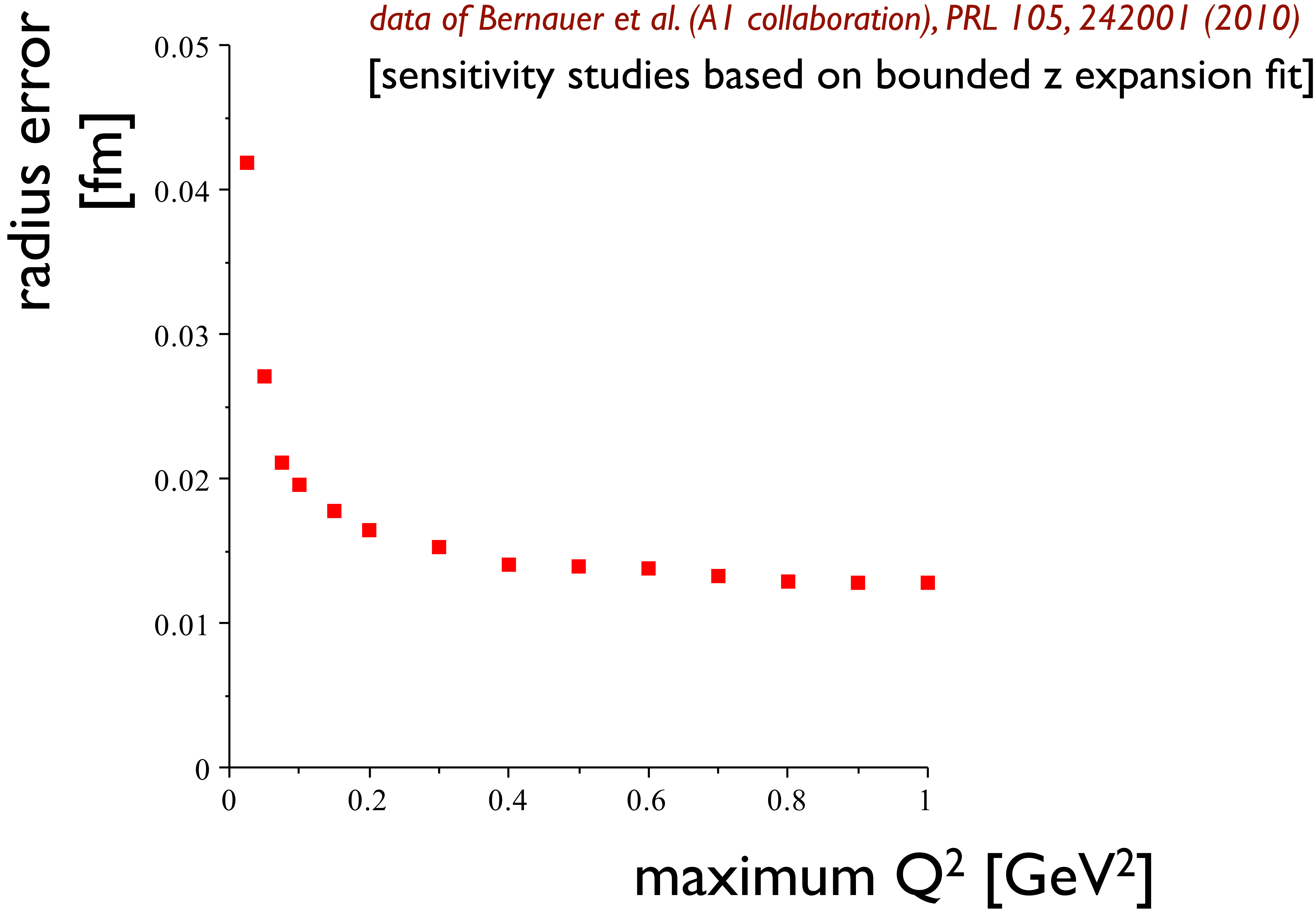
$$\langle J^\mu \rangle = \gamma^\mu F_1 + \frac{i}{2m_p}\sigma^{\mu\nu} q_\nu F_2$$

$$G_E = F_1 + \frac{q^2}{4m_p^2} F_2 \qquad G_M = F_1 + F_2$$

$$r_E^2 \equiv 6 \frac{d}{dq^2} G_E(q^2)\Big|_{q^2=0}$$

(up to radiative corrections)

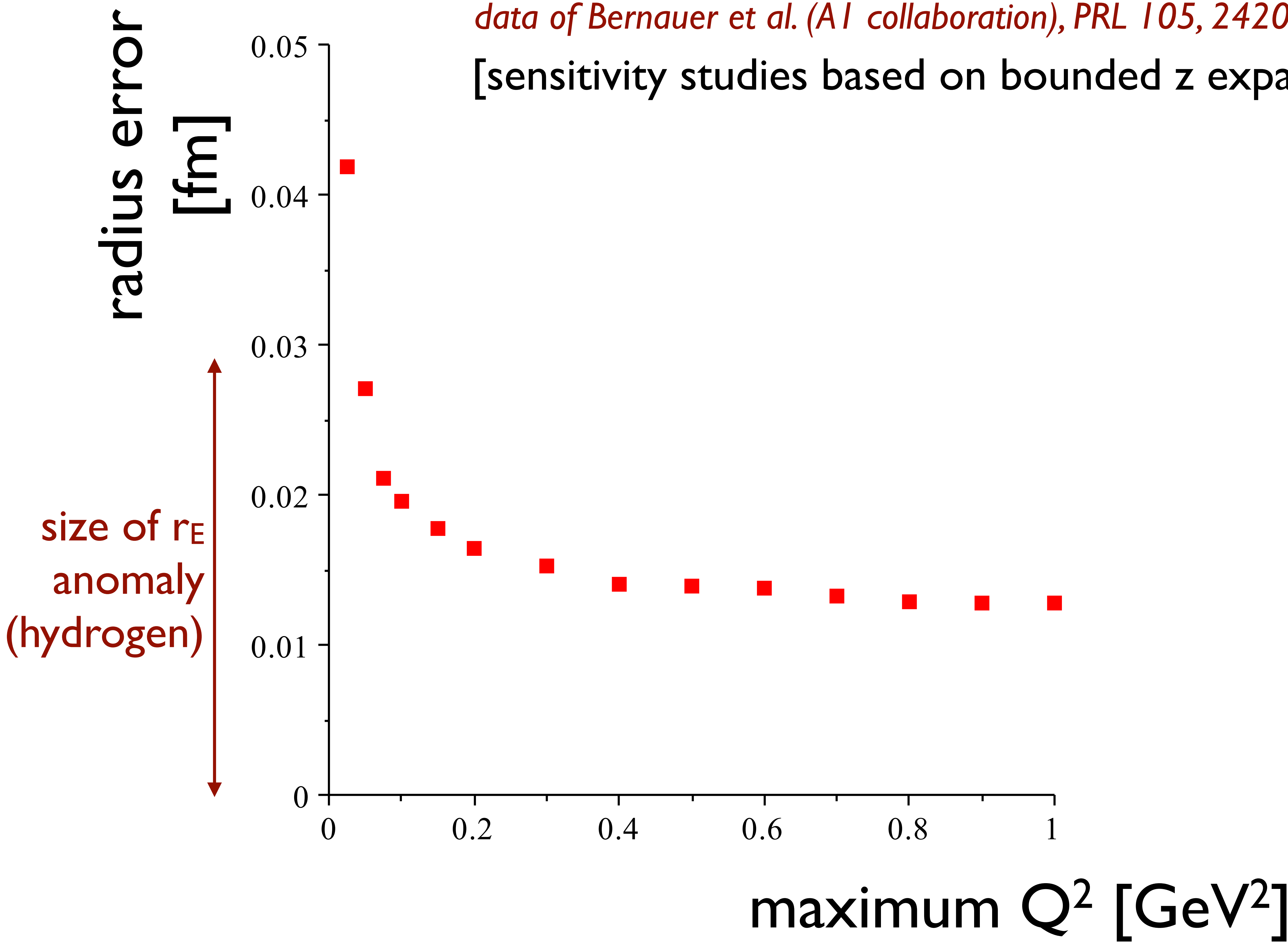
Radius extraction requires data over a Q^2 range where a simple Taylor expansion of the form factor is invalid



Radius extraction requires data over a Q^2 range where a simple Taylor expansion of the form factor is invalid

data of Bernauer et al. (A1 collaboration), PRL 105, 242001 (2010)

[sensitivity studies based on bounded z expansion fit]



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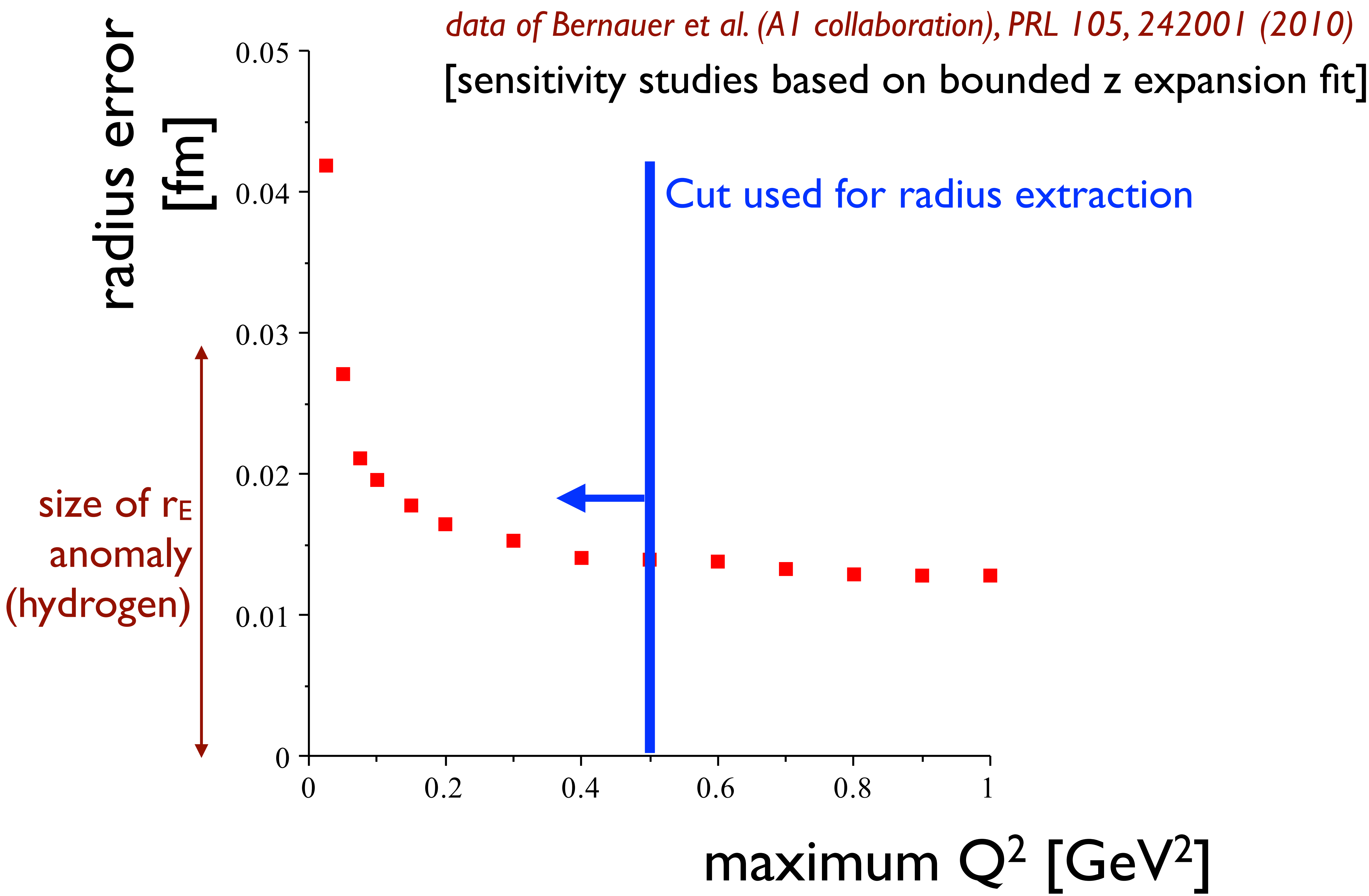

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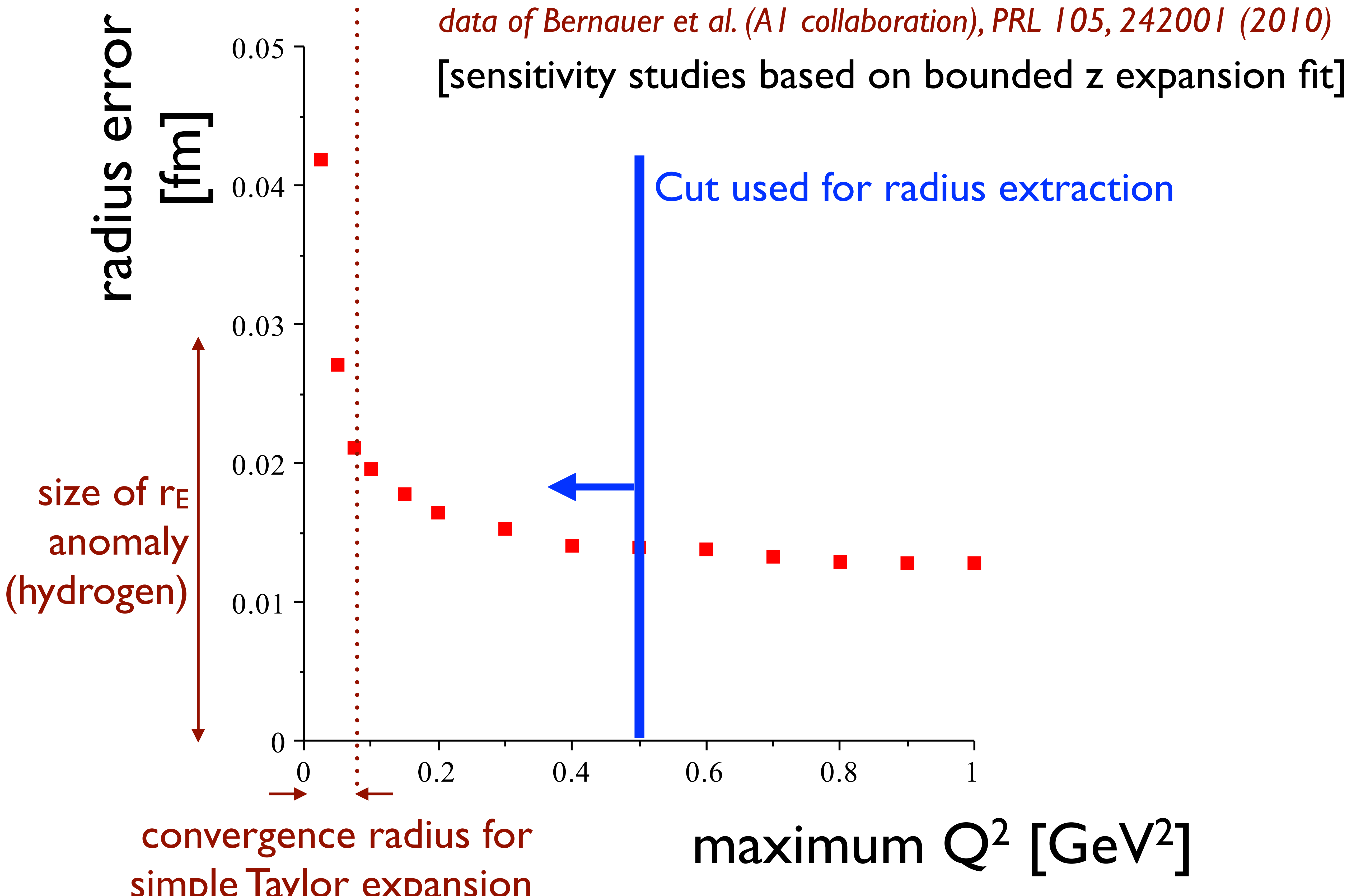
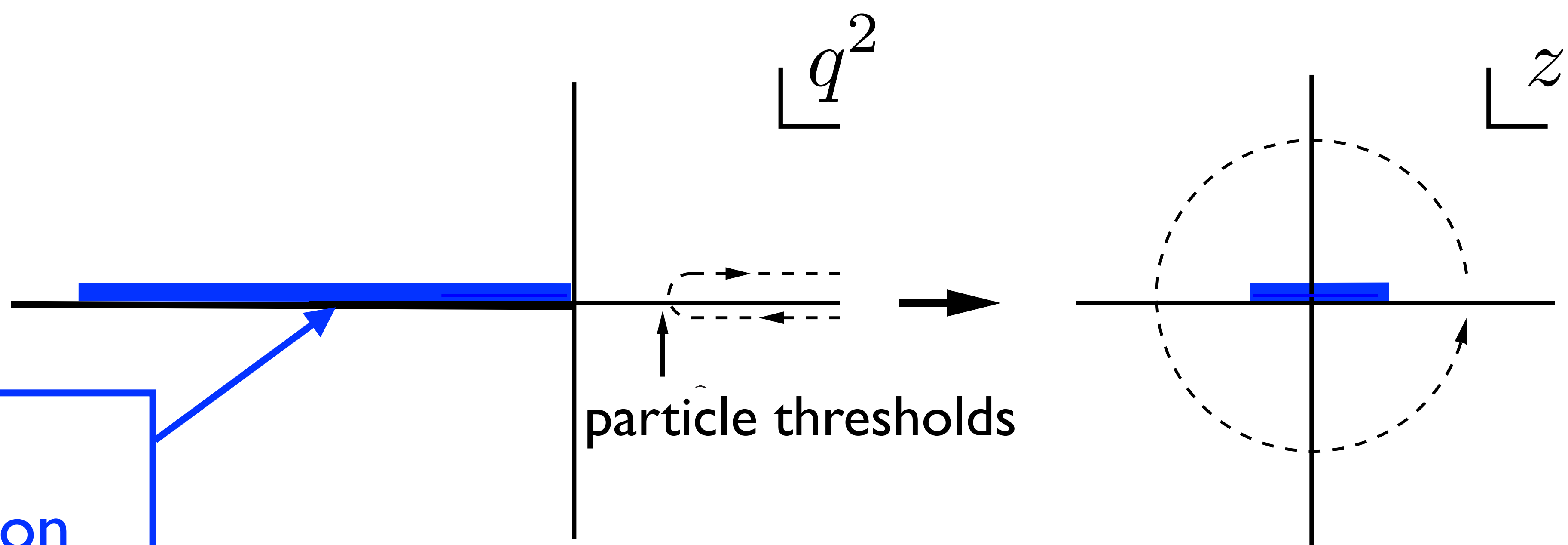


That's ok: underlying QCD tells us that Taylor expansion of form factor in appropriate variable is convergent



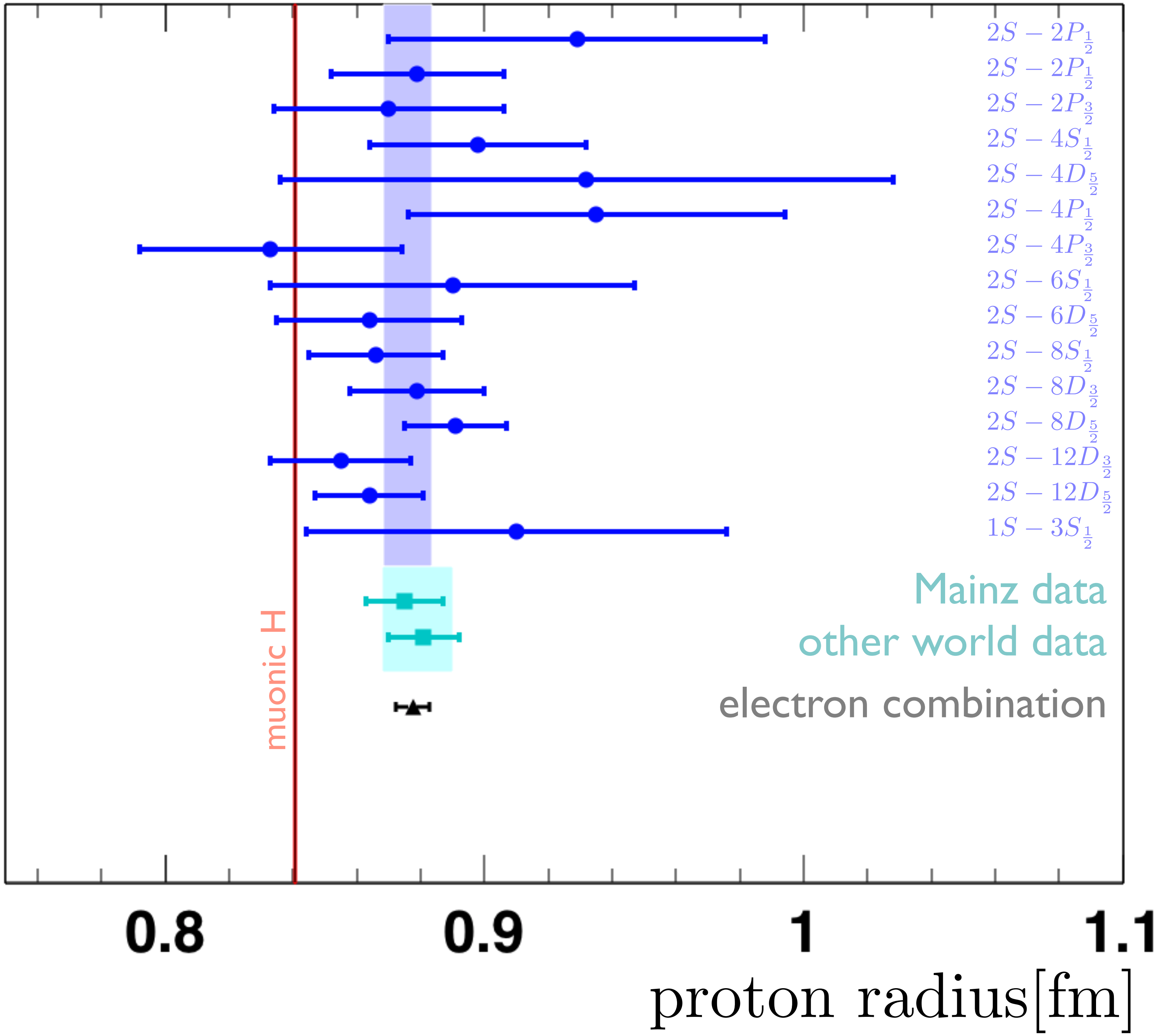
experimental kinematic region

$$z(q^2, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}$$

$$F(q^2) = \sum_k a_k [z(q^2)]^k$$

coefficients in rapidly convergent expansion encode nonperturbative QCD

Reanalysis of scattering data reveals strong influence of shape assumptions



Errors larger, but discrepancy remains

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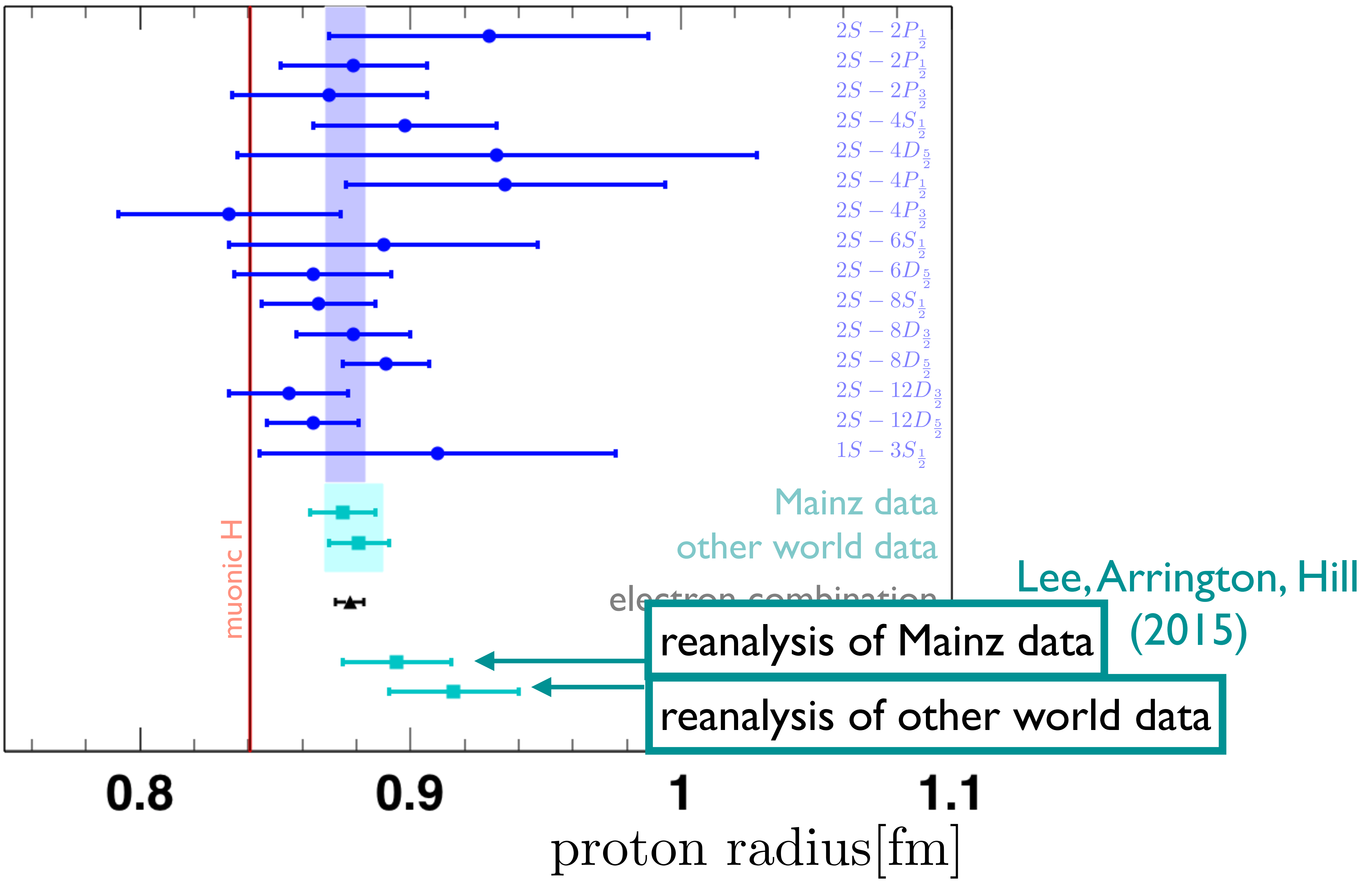
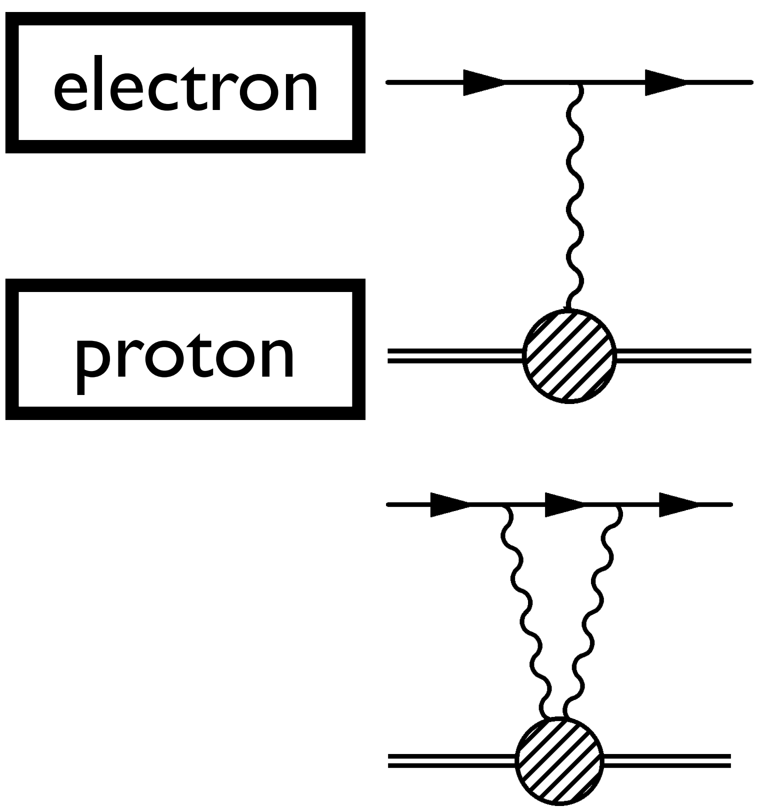


Errors larger, but discrepancy remains

muonic hydrogen spectroscopy: theory issues

muonic atoms more sensitive to radius, but also more sensitive to other proton structure

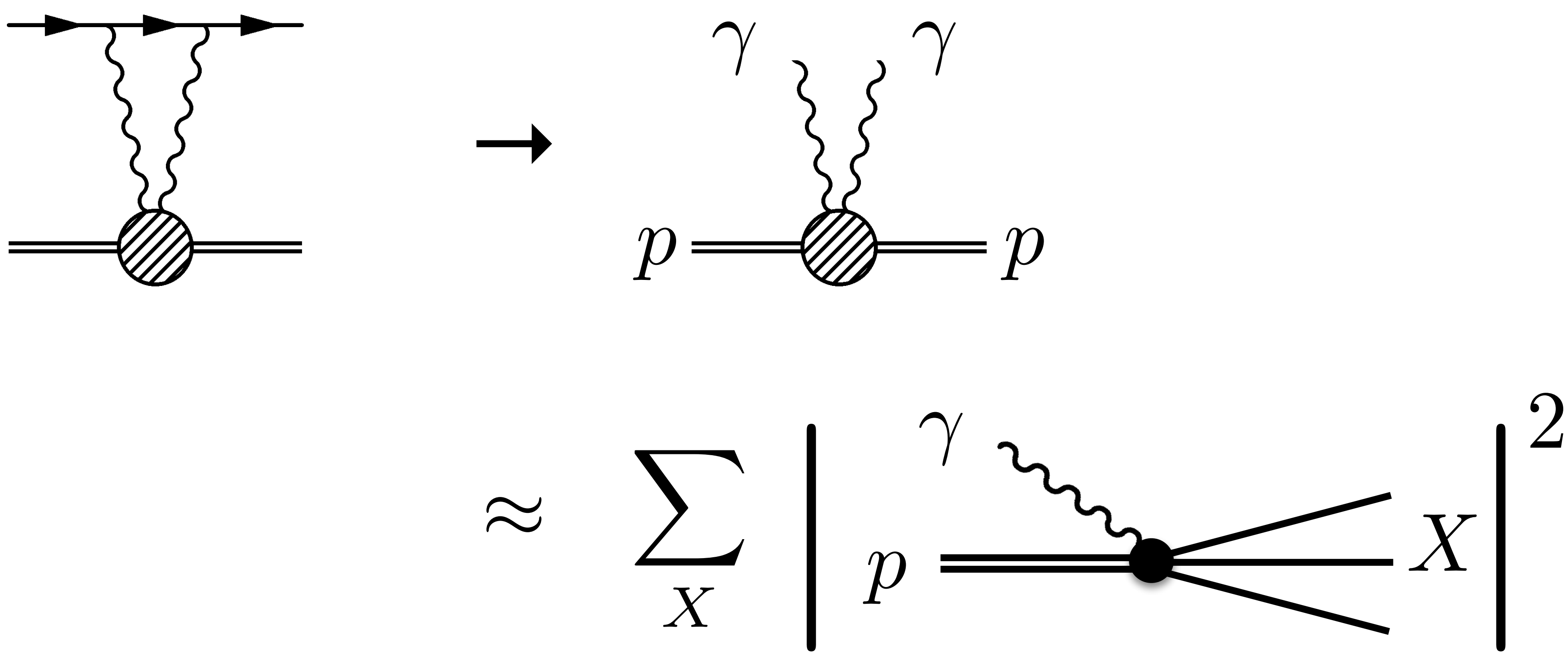


one-photon exchange → radius "signal"

two-photon exchange → "background"

- are subleading proton structure effects under control?

Optical theorem for two-photon exchange in muonic hydrogen



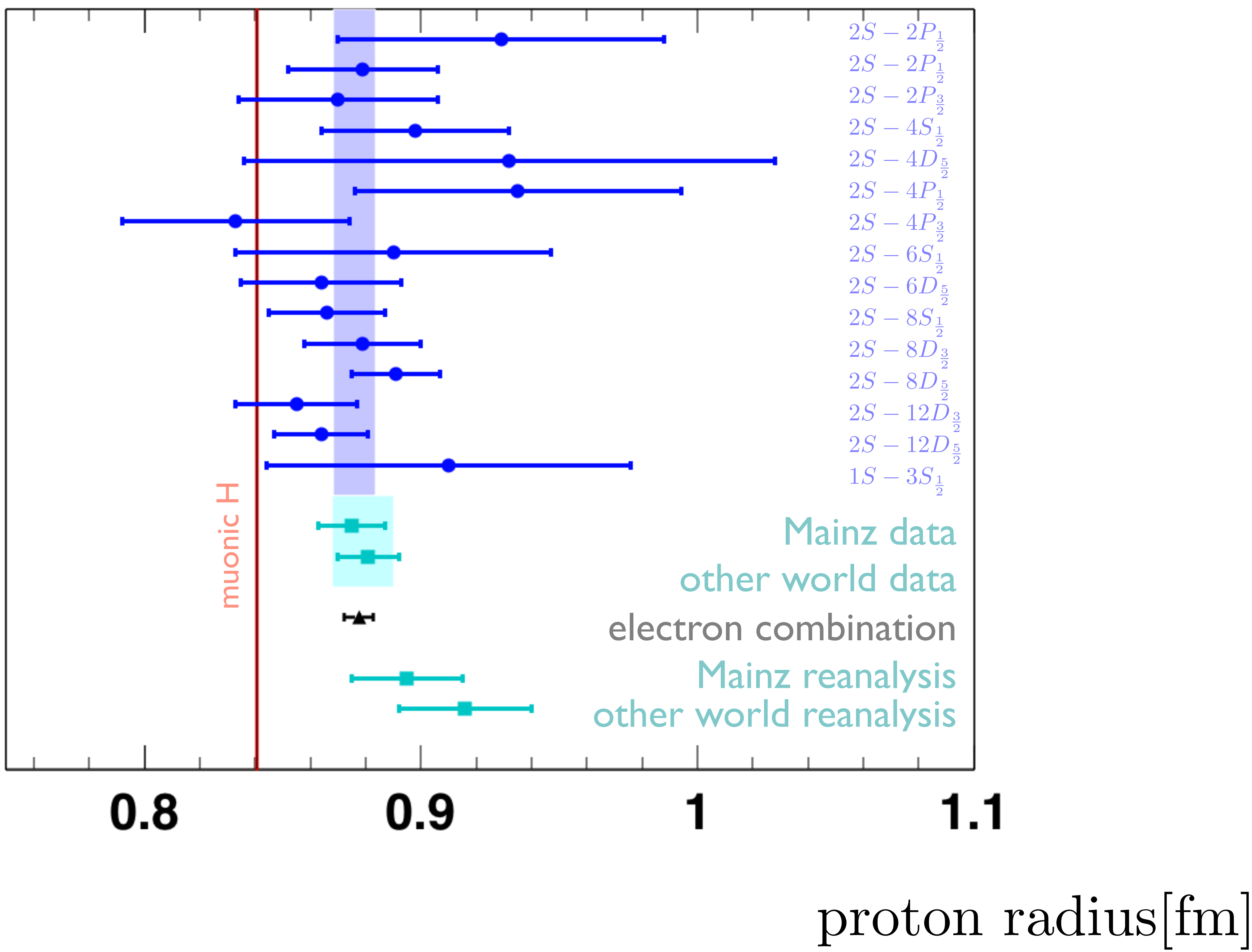
If a dispersion relation is valid, contribution completely determined by measurable quantities in electron-proton scattering. But:

$$W_1(\nu, 0) = -2 + \mathcal{O}(\nu^2) = \frac{1}{\pi}\int d\nu'^2 \frac{ImW_1(\nu', 0)}{\nu'^2 - \nu^2} > 0 \quad ??$$

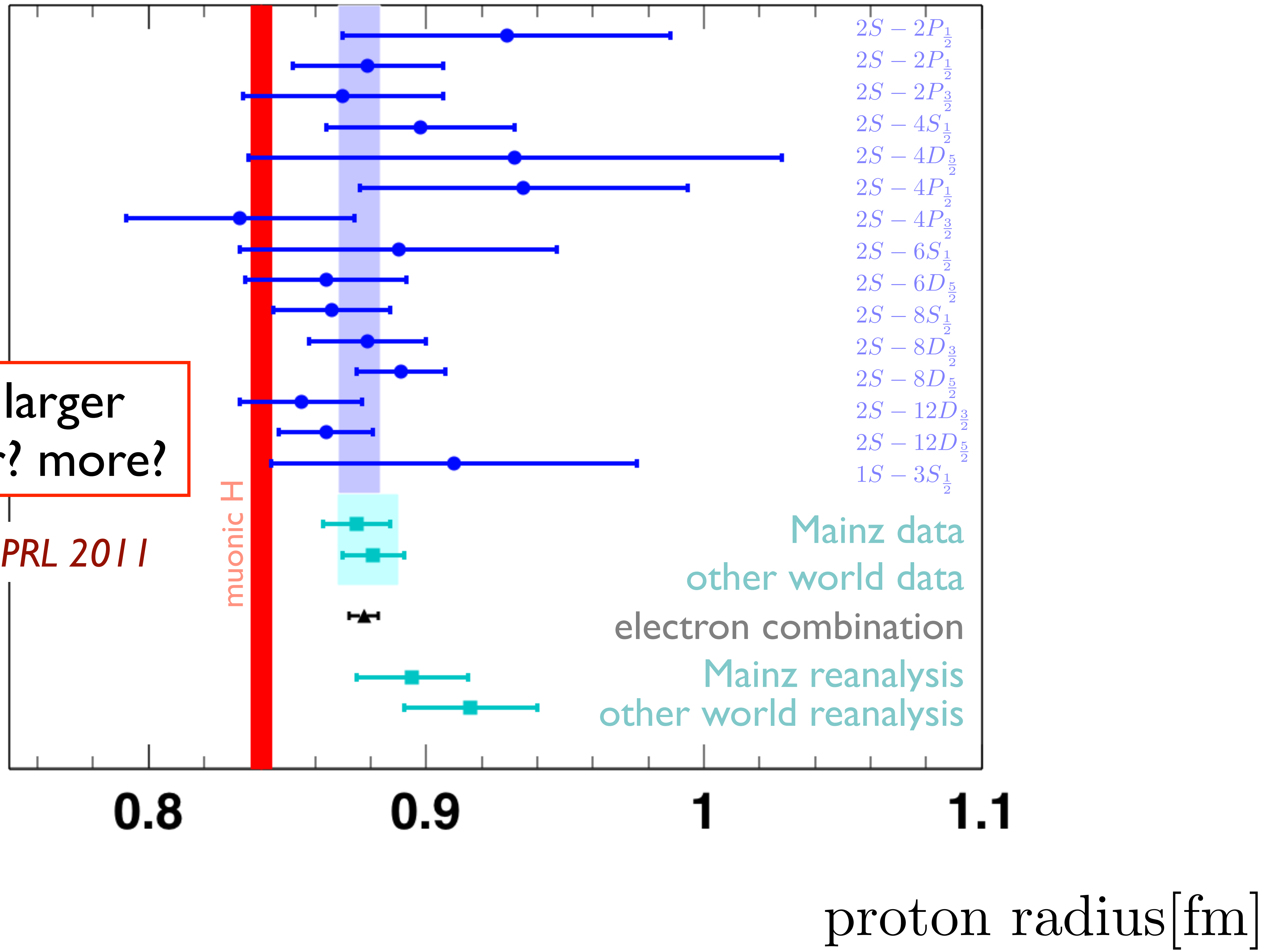
$$\Rightarrow \qquad W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi}\int d\nu'^2 \frac{ImW_1(\nu', 0)}{\nu'^2(\nu'^2 - \nu^2)}$$

new hadronic function

Model-dependent assumptions on two-photon exchange



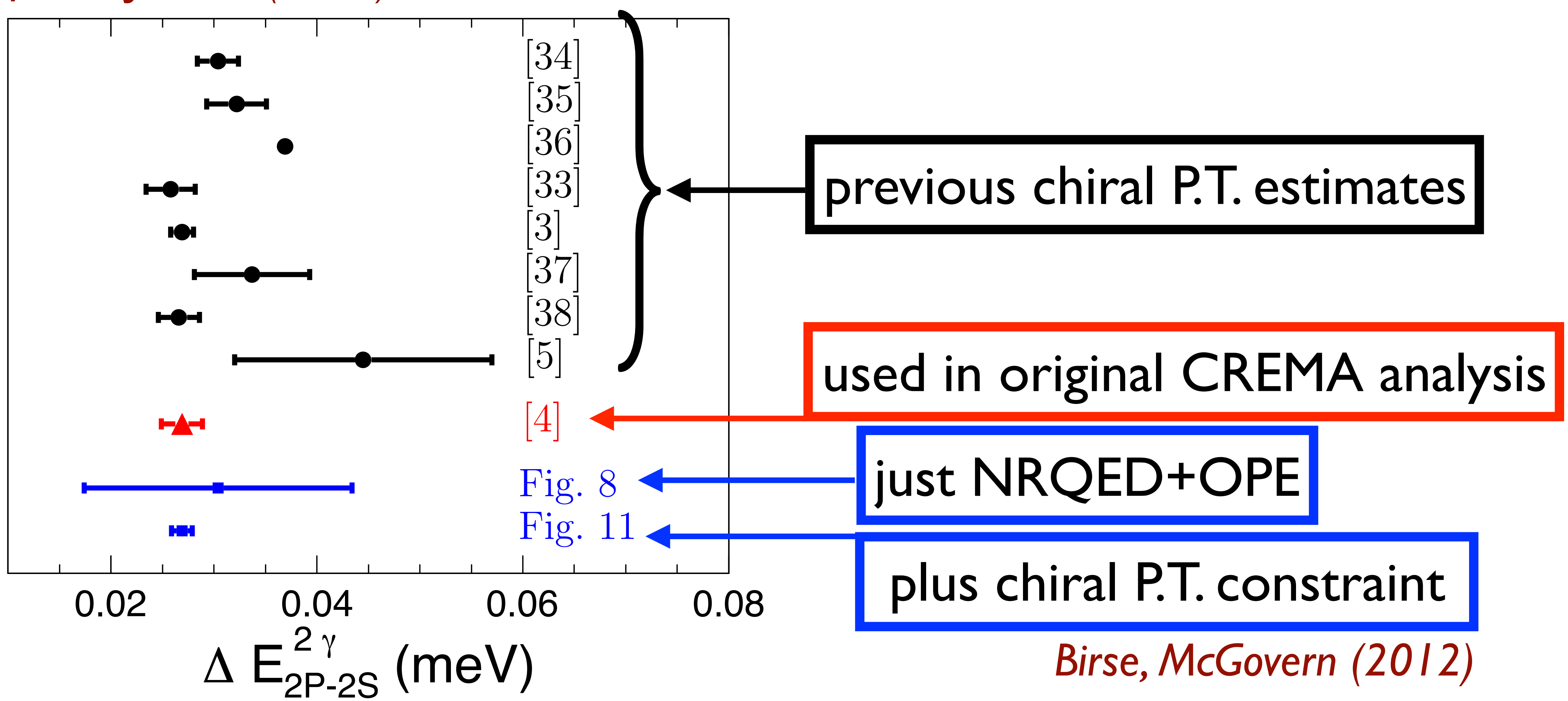
Model-dependent assumptions on two-photon exchange



10 times larger error bar? more?

RJH, Paz, PRL 2011

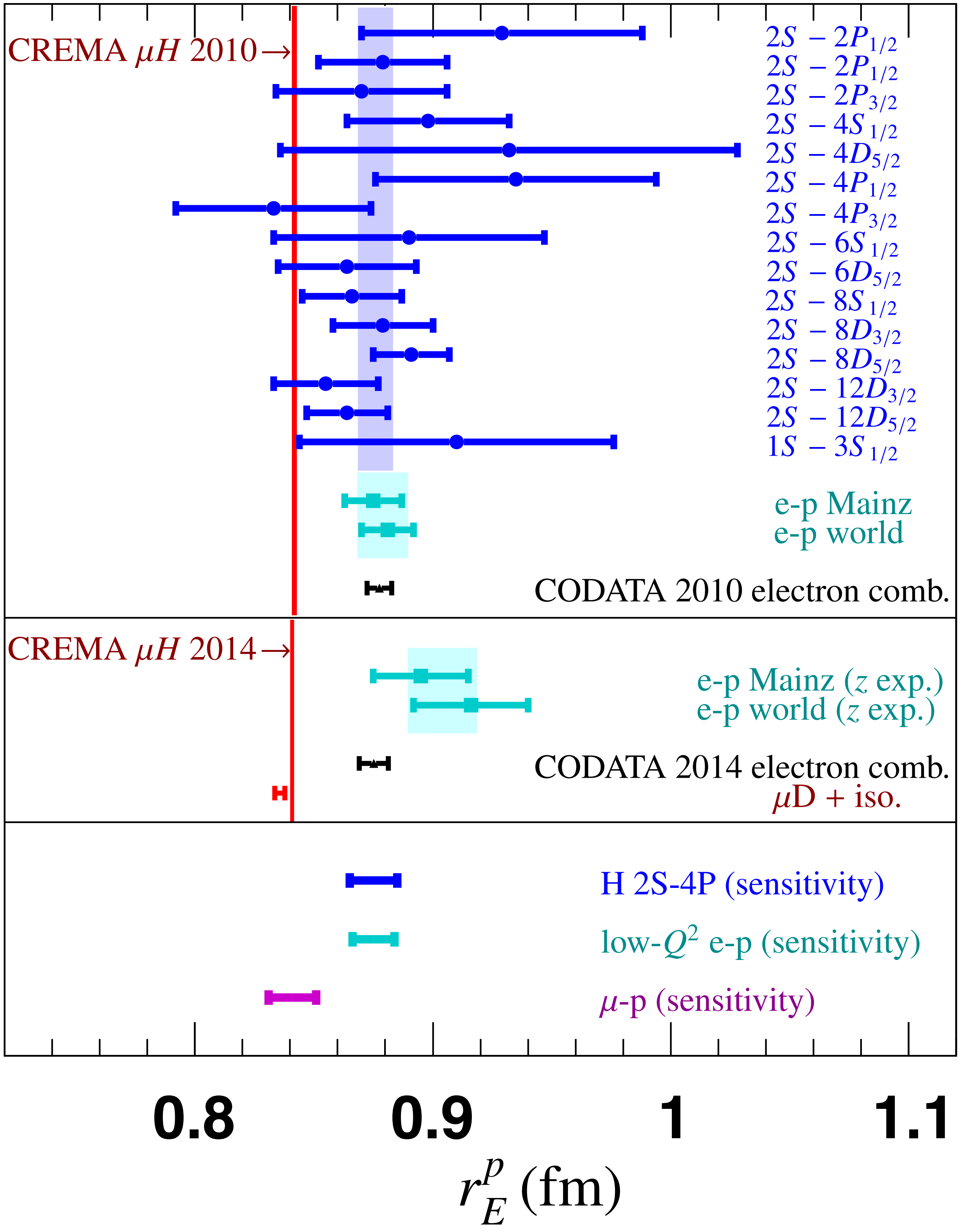
from RJH, Paz (2016)



Birse, McGovern (2012)

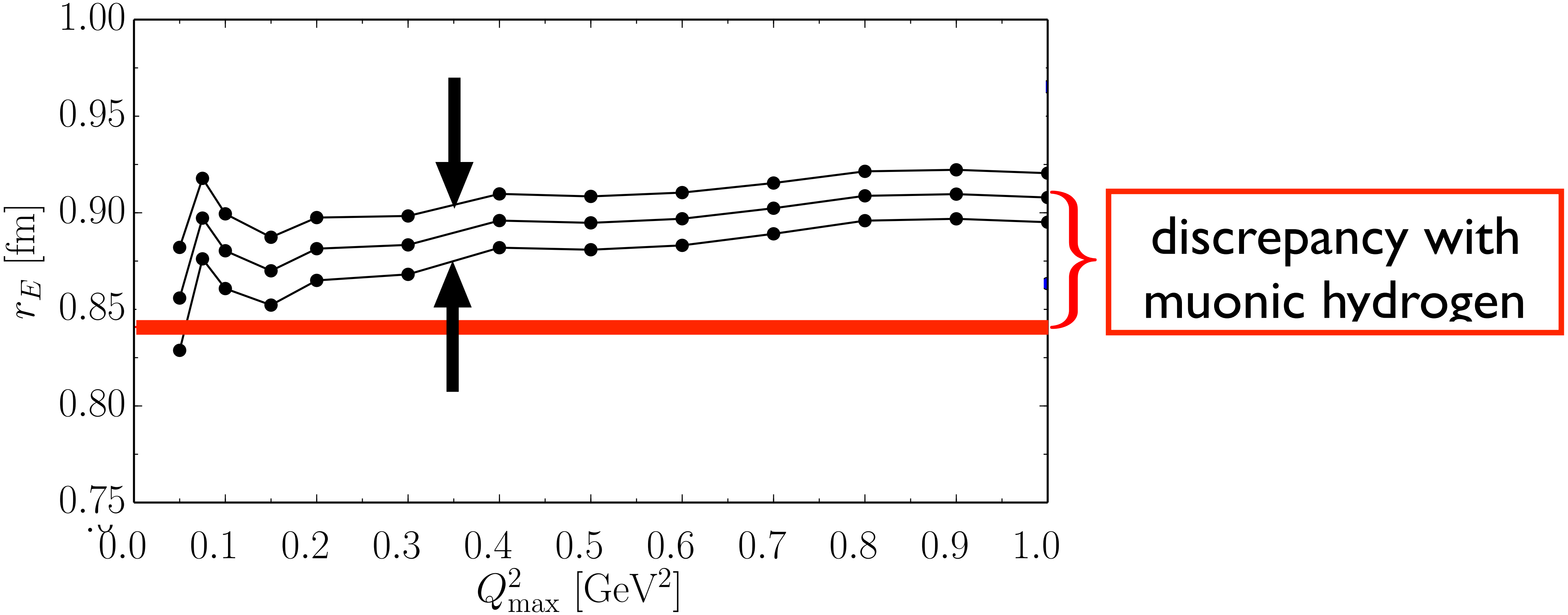
- *TPE remains dominant uncertainty in mu-H Lamb shift, rEp, Rydberg (most precise fundamental constant). But not a solution to proton radius puzzle*

status and prospects


CREMA μH 2010 →
$2S - 2P_{1/2}$
$2S - 2P_{1/2}$
$2S - 2P_{3/2}$
$2S - 4S_{1/2}$
$2S - 4D_{5/2}$
$2S - 4P_{1/2}$
$2S - 4P_{3/2}$
$2S - 6S_{1/2}$
$2S - 6D_{5/2}$
$2S - 8S_{1/2}$
$2S - 8D_{3/2}$
$2S - 8D_{5/2}$
$2S - 12D_{3/2}$
$2S - 12D_{5/2}$
$1S - 3S_{1/2}$
e-p Mainz
e-p world
CODATA 2010 electron comb.
CREMA μH 2014 →
e-p Mainz (z exp.)
e-p world (z exp.)
CODATA 2014 electron comb.
μD + iso.
H 2S-4P (sensitivity)
low-Q^2 e-p (sensitivity)
μ-p (sensitivity)
0.8
0.9
1
1.1
r_E^p (fm)

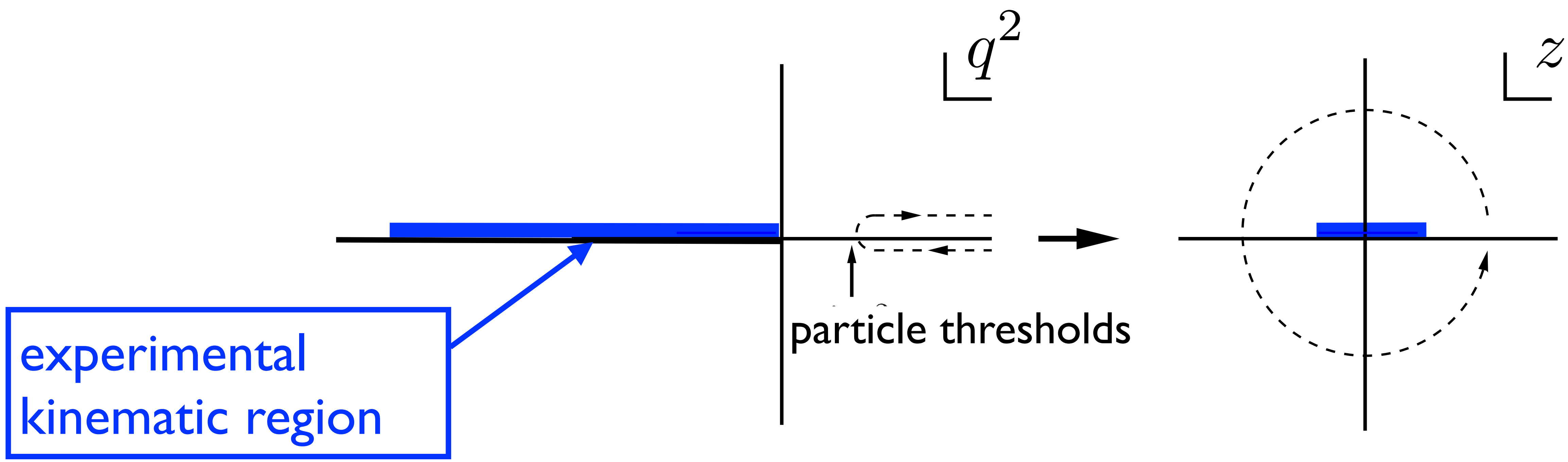
something else?

New physics under our noses?



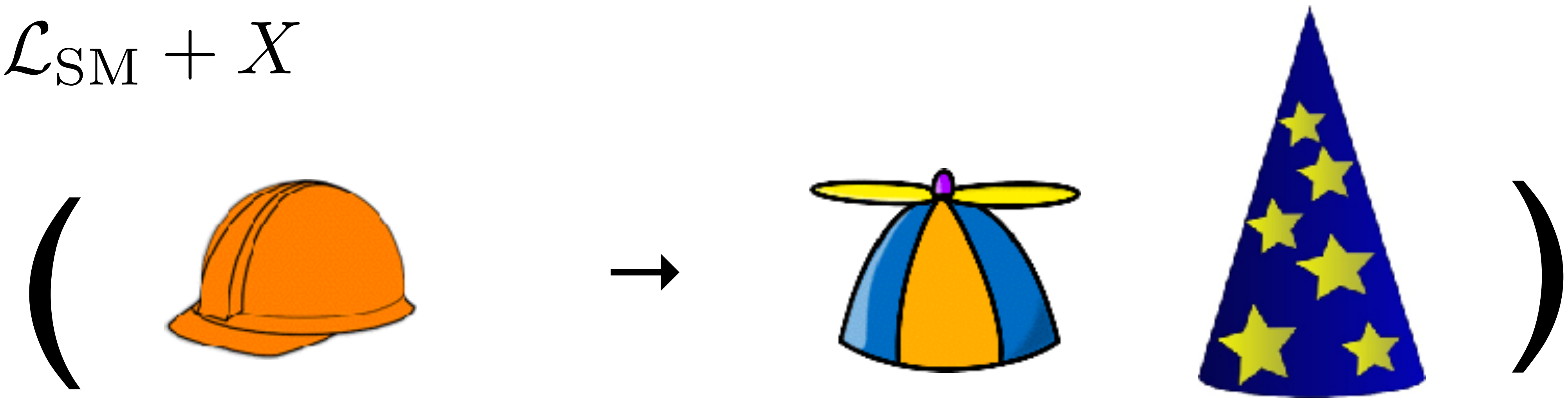
A new particle would violate the assumed analytic structure, and generate momentum-dependent effect

New physics under our noses?



A new particle would violate the assumed analytic structure, and generate momentum-dependent effect

$\mathcal{L} = \mathcal{L}_{\rm SM} + X$

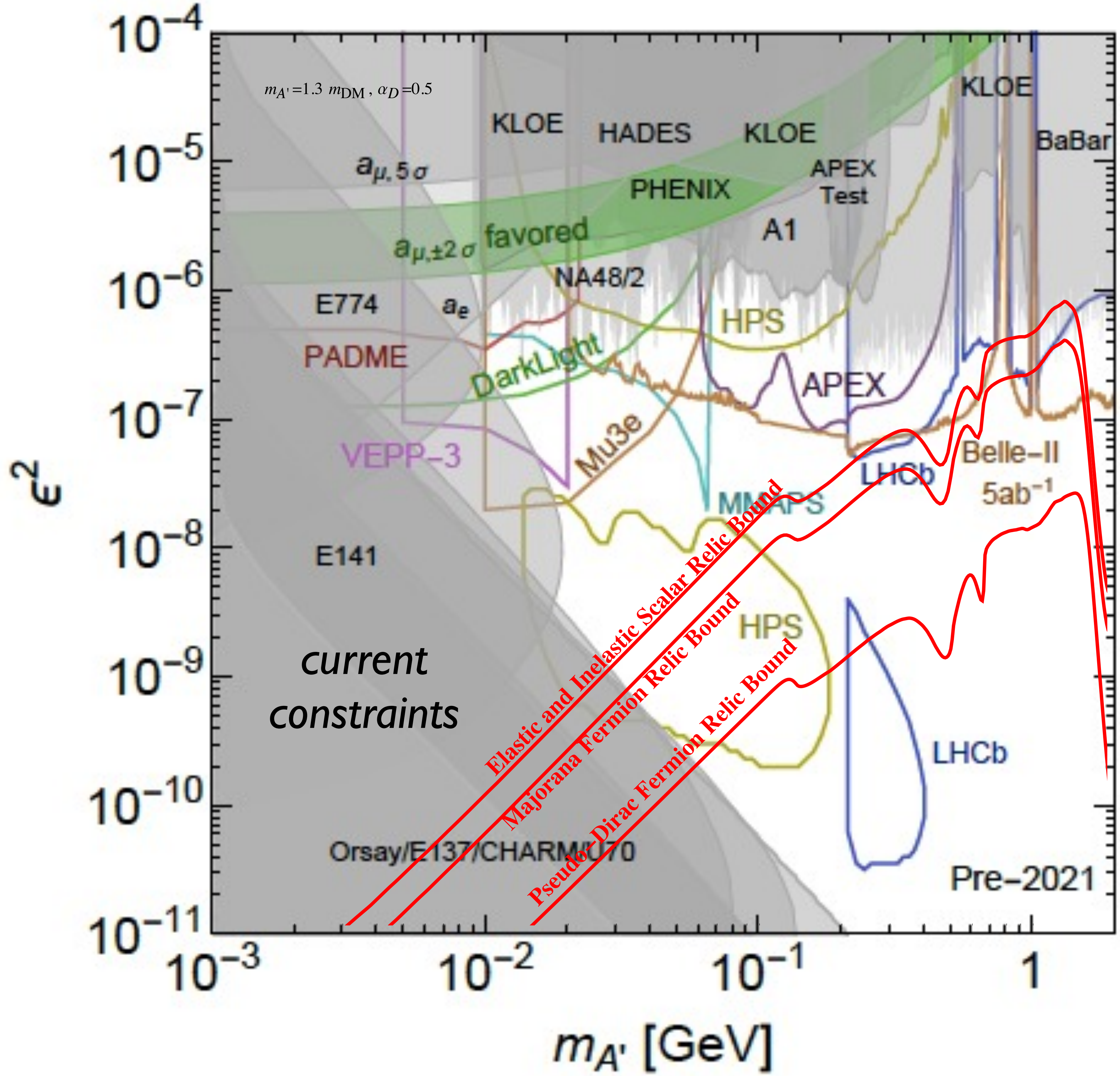


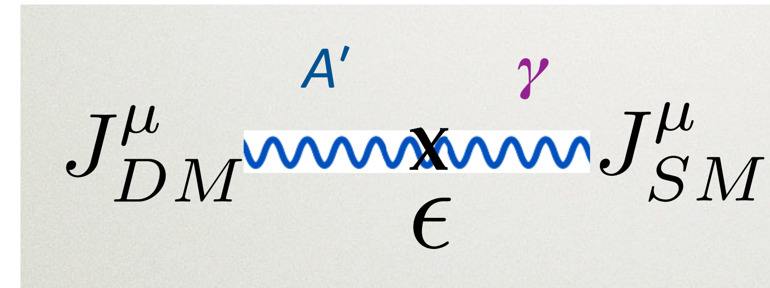
one possibility: X = dark photon

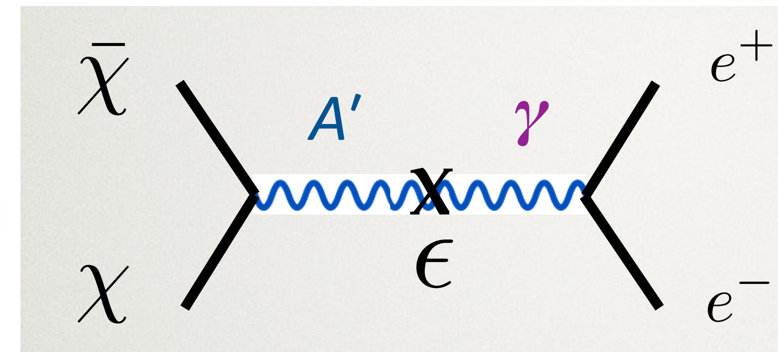
- depending on mass, consistent with $r_{eH} \sim r_{\mu H} < r_{e\text{-}p}$

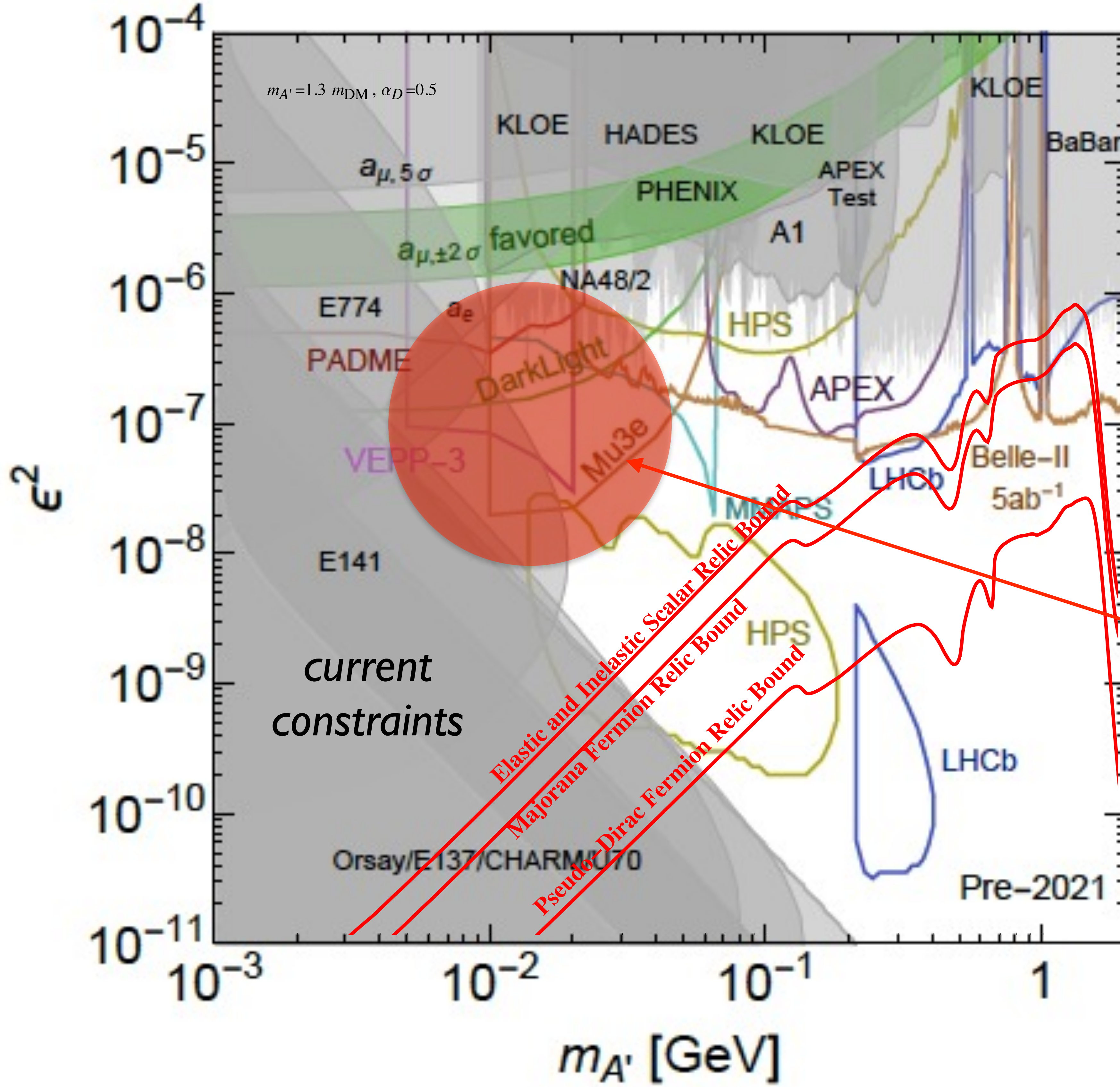
$$-\frac{e^2}{Q^2}F(Q^2) \to -\frac{e^2}{Q^2}F(Q^2) \mp \frac{g^2}{Q^2 + m_V^2}$$

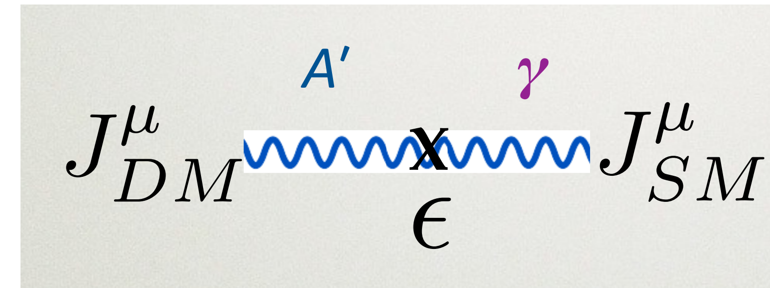
- especially interesting to see new eH results

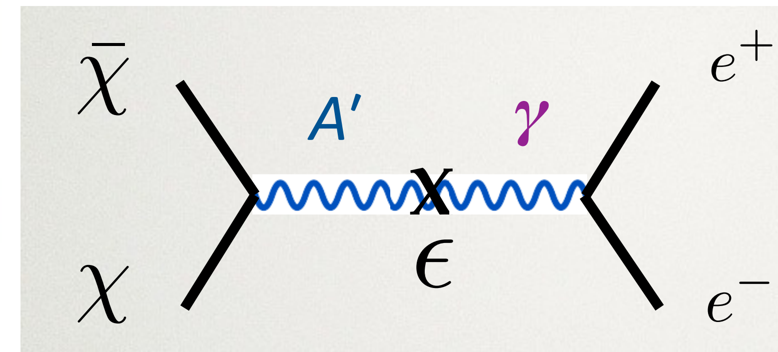

$m_{A'}=1.3\ m_{DM}$, $\alpha_D=0.5$
KLOE
HADES
KLOE
KLOE
BaBar
$a_{\mu,5\sigma}$
PHENIX
APEX Test
A1
$a_{\mu,\pm2\sigma}$ favored
NA48/2
E774
a_e
HPS
PADME
DarkLight
APEX
VEPP-3
Mu3e
LHCb
Belle-II 5ab^{-1}
MMAPS
E141
HPS
Elastic and Inelastic Scalar Relic Bound
Majorana Fermion Relic Bound
Pseudo-Dirac Fermion Relic Bound
current constraints
LHCb
Orsay/E137/CHARM/U70
Pre-2021
ϵ^2
$m_{A'}$ [GeV]
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-3}
10^{-2}
10^{-1}
1


J^μ_{DM}
A'
γ
J^μ_{SM}
ϵ


$\bar{\chi}$
χ
A'
γ
ϵ
e^+
e^-







interesting region for proton radius puzzle

summary

- proton radius puzzle, neutrinos, dark matter: important particle, nuclear, atomic overlap
- impossible not to cross boundaries
  - Operator product expansion: Heavy WIMP direct detection ↔ two-photon effects in Lamb shifts
  - Sudakov resummation: Heavy WIMP annihilation ↔ e-p, ν-N scattering
- need for precision in current and next generation experiments
- opportunity to develop and exploit modern tools and technology

Dark matter - Standard Model interactions

$$\mathcal{L} = \frac{1}{\Lambda^n} O_{\rm DM} \times O_{\rm SM}$$

d	Fermion
3	$\bar{\psi}[1\,,\ i\gamma_5\,,\ \gamma^\mu\gamma_5\,,\ \{\gamma^\mu\,,\ \sigma^{\mu\nu}\}]\psi$
4	$\bar{\psi}[\{1\,,\ i\gamma_5\,,\ \gamma^\mu\gamma_5\}\,,\ \gamma^\mu\,,\ \sigma^{\mu\nu}]i\partial_-^\rho\psi$

d	Scalar
2	$\lvert\phi\rvert^2$
3	$\{\phi^* i\partial_-^\mu\phi\}$

d	Heavy particle
3	$\bar{\chi}_v[1\,,\ \{\sigma_\perp^{\mu\nu}\}]\chi_v$
4	$\bar{\chi}_v[\{1\}\,,\ \sigma_\perp^{\mu\nu}]i\partial_{\perp-}^\rho\chi_v$

d	QCD operator basis
3	$V_q^\mu = \bar{q}\gamma^\mu q$ $A_q^\mu = \bar{q}\gamma^\mu\gamma_5 q$
4	$T_q^{\mu\nu} = i m_q \bar{q}\sigma^{\mu\nu}\gamma_5 q$ $O_q^{(0)} = m_q\bar{q}q\,, \quad O_g^{(0)} = G_{\mu\nu}^A G^{A\mu\nu}$ $O_{5q}^{(0)} = m_q\bar{q}i\gamma_5 q\,, \quad O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}^A G_{\rho\sigma}^A$ $O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}iD_-^{\nu\}} - \frac{g^{\mu\nu}}{4}i\not{D}_-\right)q\,, \quad O_g^{(2)\mu\nu} = -G^{A\mu\lambda}G^{A\nu}{}_\lambda + \frac{g^{\mu\nu}}{4}(G_{\alpha\beta}^A)^2$ $O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu}iD_-^{\nu\}}\gamma_5 q$

complete QCD basis for d≤7

Renormalization and matching (sample):

$$\mathcal{L}_{\phi_0,\mathrm{SM}} = \frac{1}{m_W^3}\phi_v^*\phi_v\left\{\sum_q \left[c_{1q}^{(0)}O_{1q}^{(0)} + c_{1q}^{(2)}v_\mu v_\nu O_{1q}^{(2)\mu\nu}\right] + c_2^{(0)}O_2^{(0)} + c_2^{(2)}v_\mu v_\nu O_2^{(2)\mu\nu}\right\} + \dots$$

$m_q\bar{q}q$ $\qquad$ $G^A_{\mu\nu}G^{A\mu\nu}$

focus on spin-0 (evaluate spin-2 at weak scale)

Renormalization group evolution from weak scale to hadronic scales, with perturbative corrections at heavy quark mass thresholds

$$c_i(\mu_Q) = M_{ij}(\mu_Q)c'_j(\mu_Q)\,.$$

$$M(\mu_Q) = \left(\begin{array}{ccc|c|c} & & & M_{qQ} & M_{qg} \\ & \mathbb{1}(M_{qq}-M_{qq'}) + \mathbb{J}M_{qq'} & & \vdots & \vdots \\ & & & M_{qQ} & M_{qg} \\ \hline M_{gq} & \cdots & M_{gq} & M_{gQ} & M_{gg} \end{array}\right)$$

Can show that:

$$M_{qq} \equiv 1\,, \quad M_{qq'} \equiv 0\,, \quad M_{gq} \equiv 0$$

M_{gQ} and M_{qQ} known through
3 loops:

Chetyrkin et al. (1997)

New results for gluon-induced decoupling relations

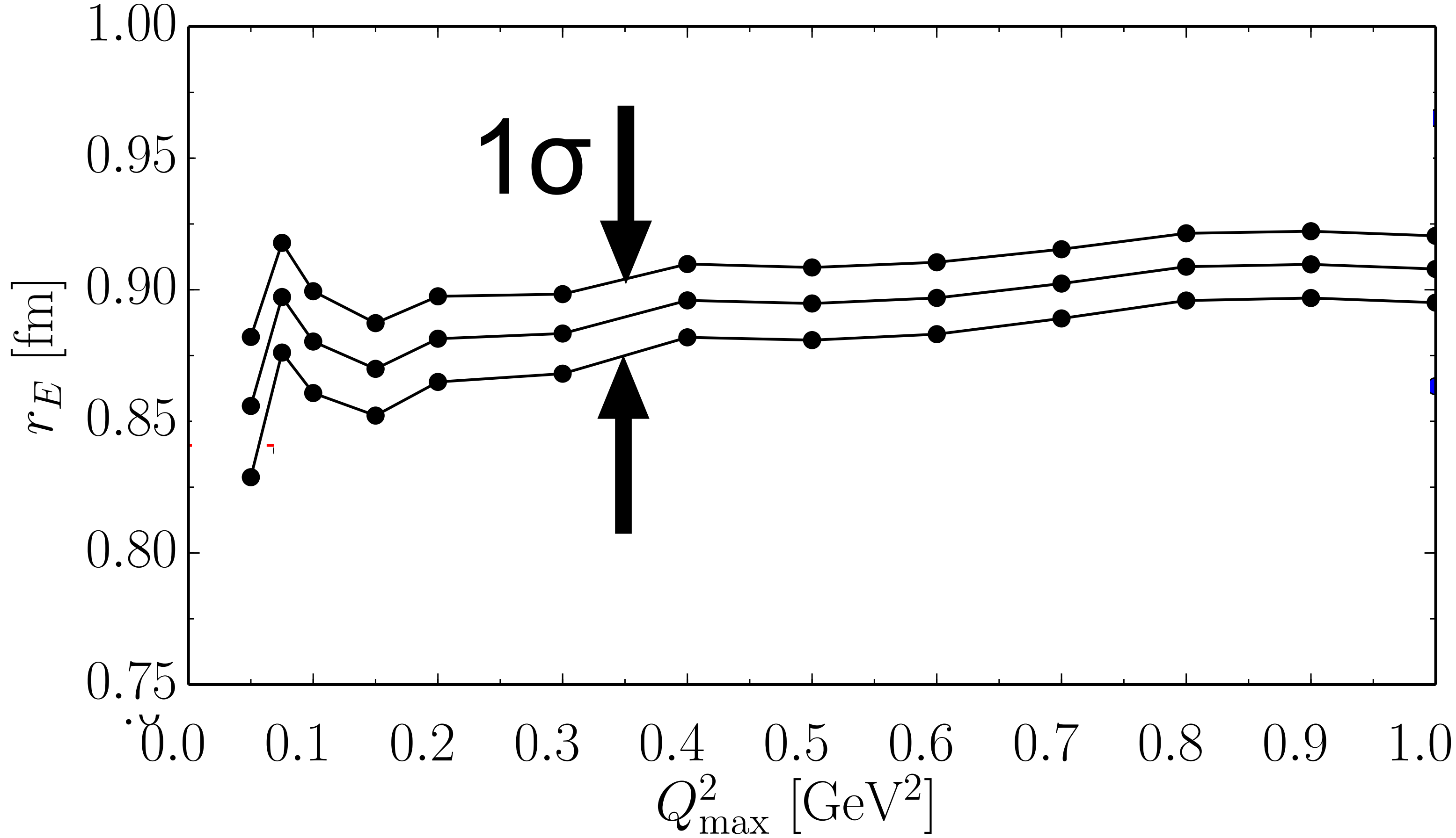
$$M_{gg}^{(2)} = \frac{11}{36} - \frac{11}{6}\log\frac{\mu_Q}{m_Q} + \frac{1}{9}\log^2\frac{\mu_Q}{m_Q}$$

$$M_{gg}^{(3)} = \frac{564731}{41472} - \frac{2821}{288}\log\frac{\mu_Q}{m_Q} + \frac{3}{16}\log^2\frac{\mu_Q}{m_Q} - \frac{1}{27}\log^3\frac{\mu_Q}{m_Q} - \frac{82043}{9216}\zeta(3)$$
$$+ n_f\left[-\frac{2633}{10368} + \frac{67}{96}\log\frac{\mu_Q}{m_Q} - \frac{1}{3}\log^2\frac{\mu_Q}{m_Q}\right],$$

$$M_{qg}^{(2)} = -\frac{89}{54} + \frac{20}{9}\log\frac{\mu_Q}{m_Q} - \frac{8}{3}\log^2\frac{\mu_Q}{m_Q}.$$

Hill, Solon (2014)

Reanalysis of scattering data also reveals potential dependence of radius on chosen Q^2 range



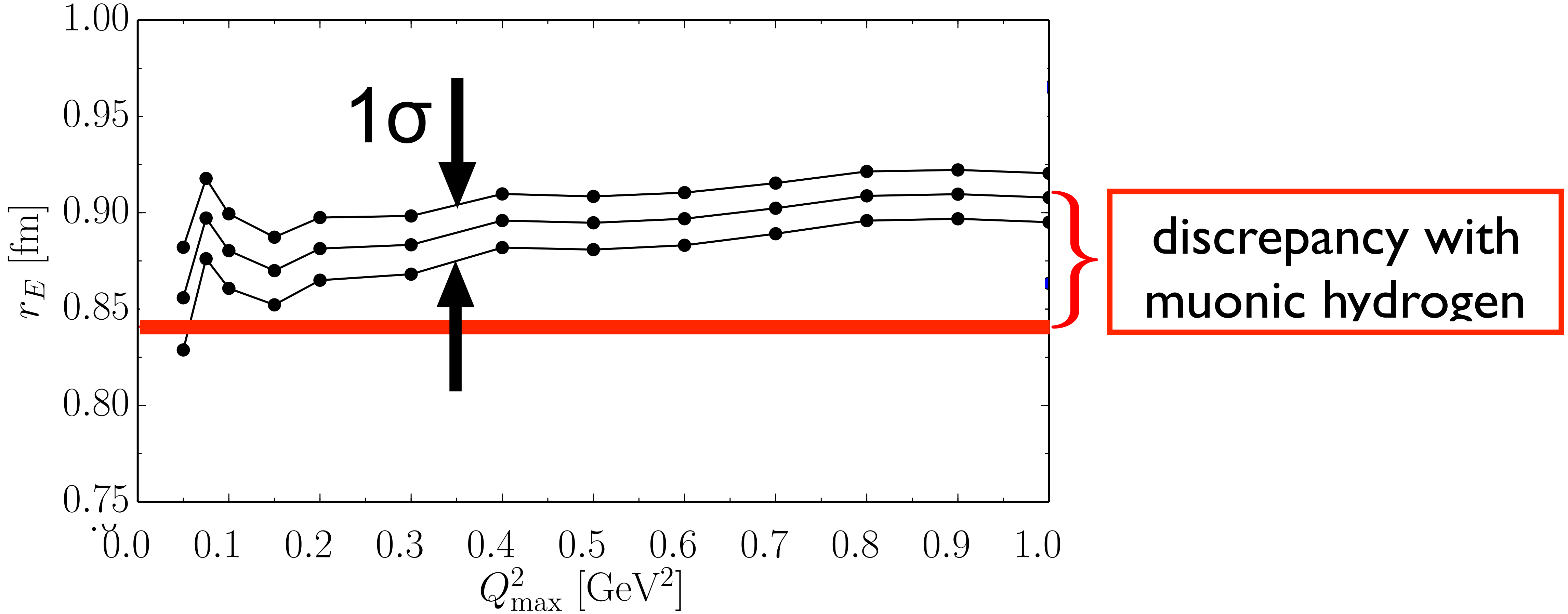
To reconcile e-p scattering with muonic hydrogen, could:

- consider only small Q^2 data (less data ⇒ larger error)
- overrule scattering data with other data or constraints

These options could avoid, but not resolve, the puzzle from electron scattering.

An unaccounted effect impacting especially large Q^2 data?
(radiative corrections: another talk. new physics: another talk)

Reanalysis of scattering data also reveals potential dependence of radius on chosen Q^2 range



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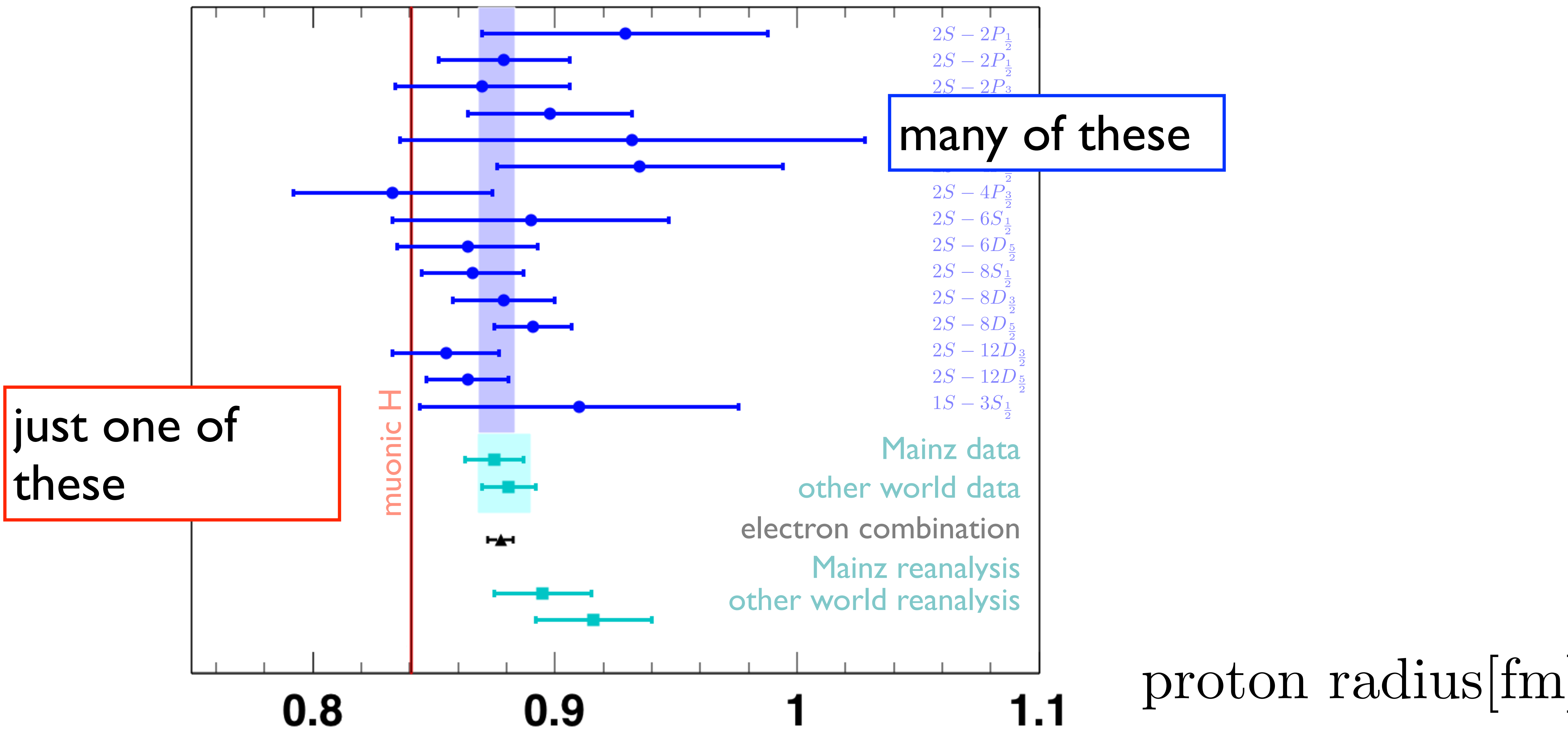
spectroscopy of other light muonic atoms: D, He



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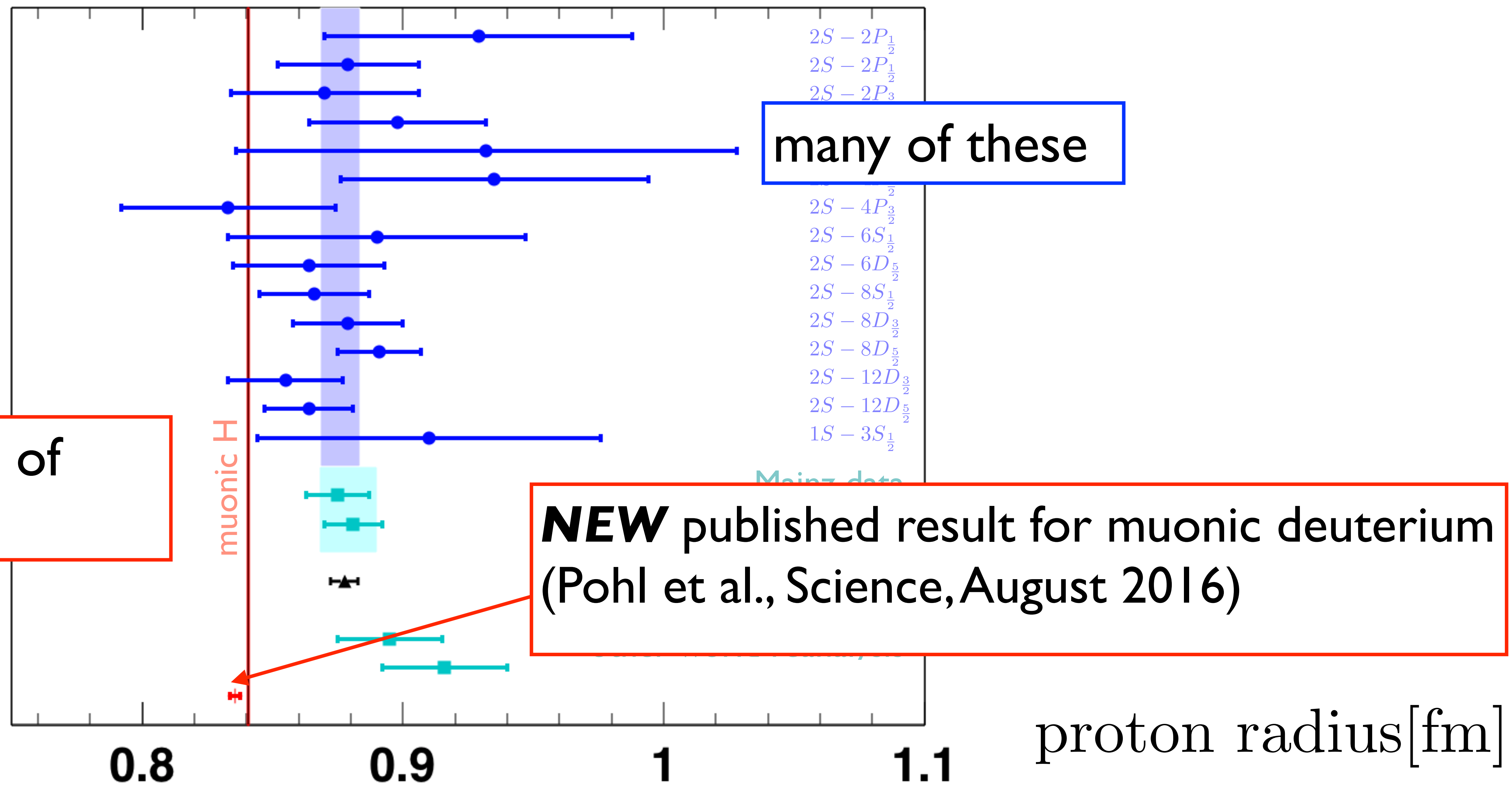


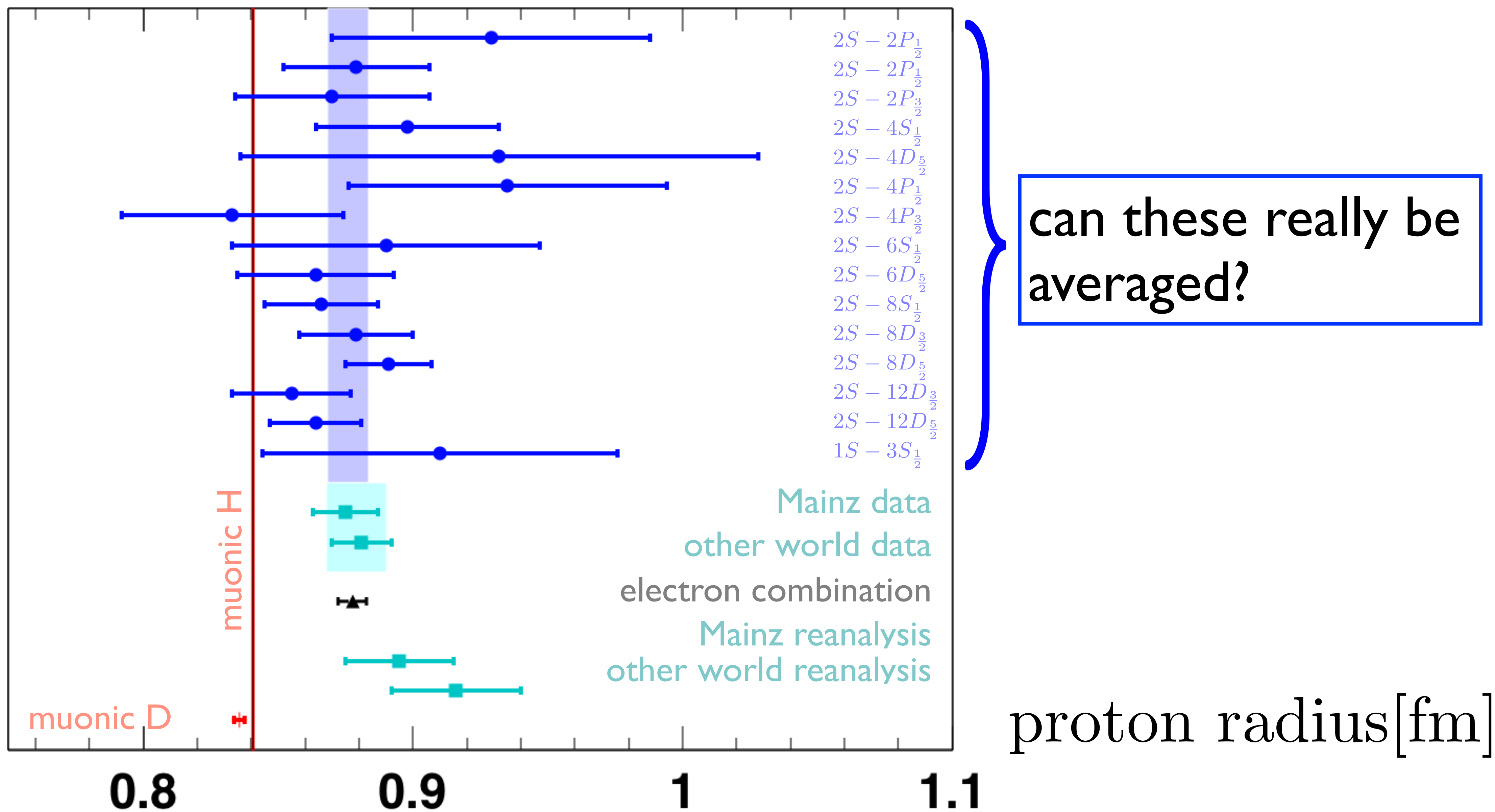
many of these

just one of these

NEW published result for muonic deuterium (Pohl et al., Science, August 2016)

- small discrepancy with muonic hydrogen
- large discrepancy with hydrogen-only radius
- new results also anticipated with muonic helium: *theory improvement needed for nuclear structure corrections*

new (preliminary) hydrogen spectroscopy results



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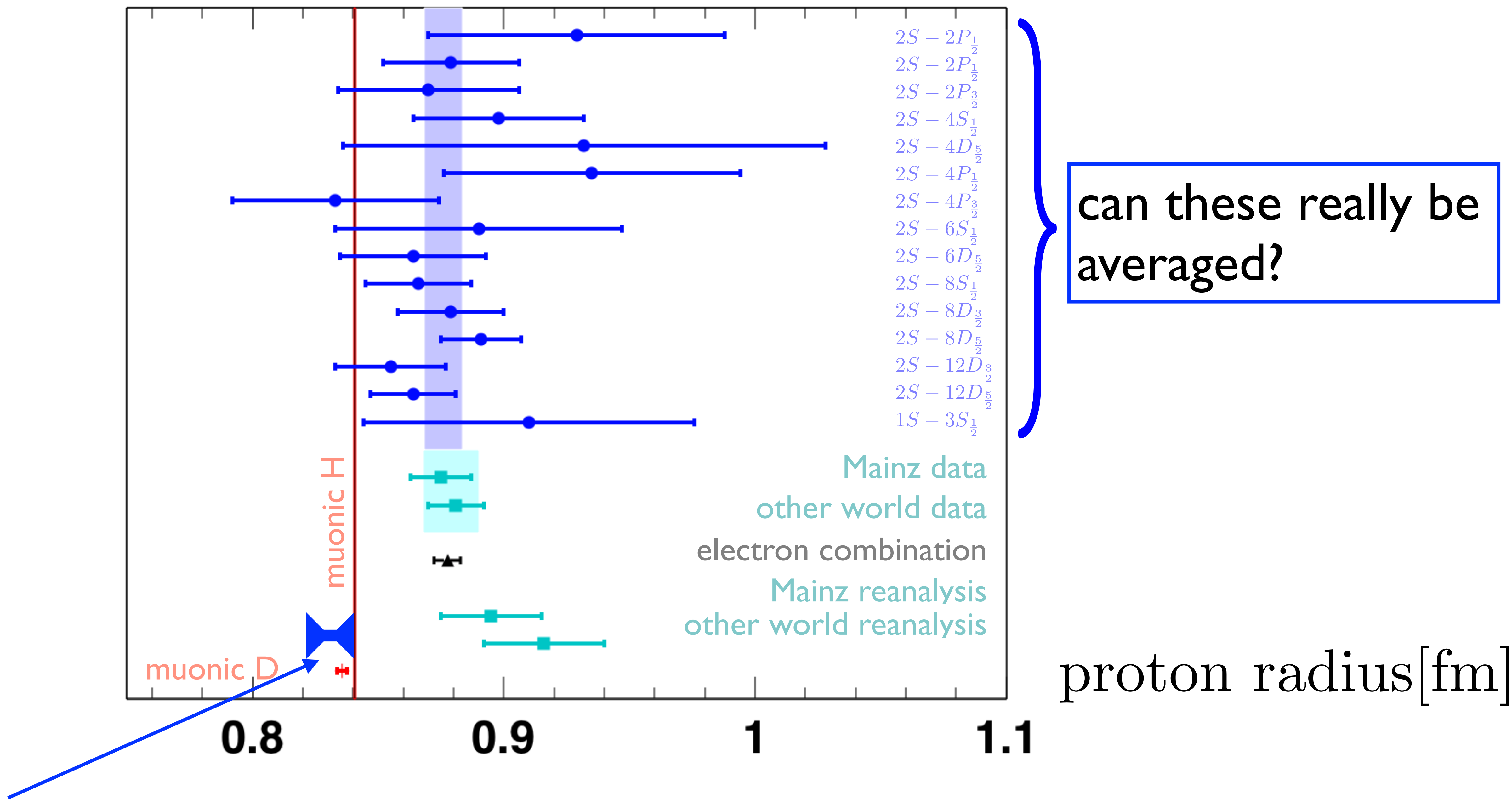
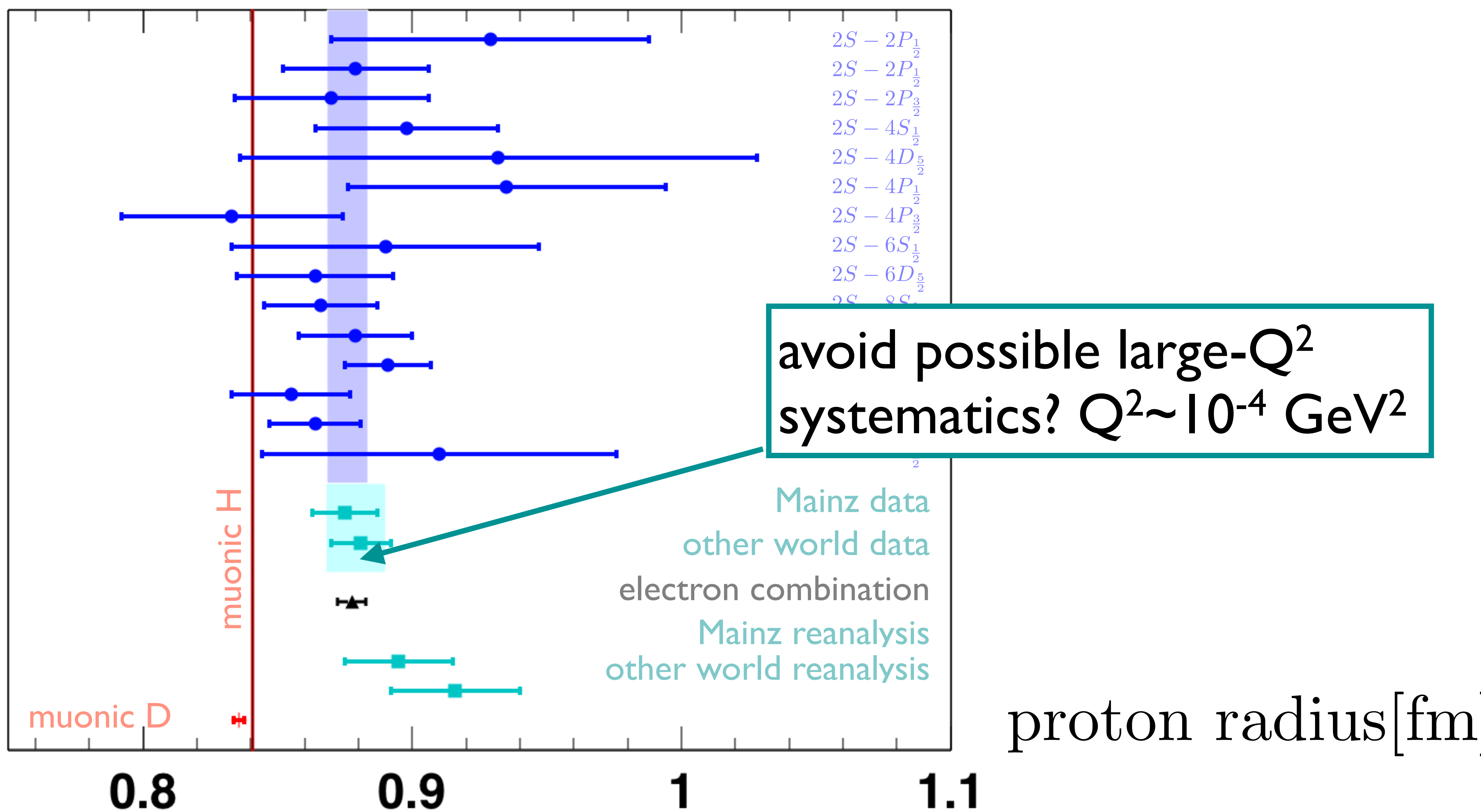


- Beyer, Maisenbacher, Matveev et al. (Garching): result for 2S-4P (submitted). *Error comparable to previous hydrogen average, central value consistent with muonic hydrogen (PRELIMINARY)*

- future new results anticipated from 2S-2P (York), 1S-3S (Paris), others

low-Q^2 electron-proton scattering: PRad at JLab



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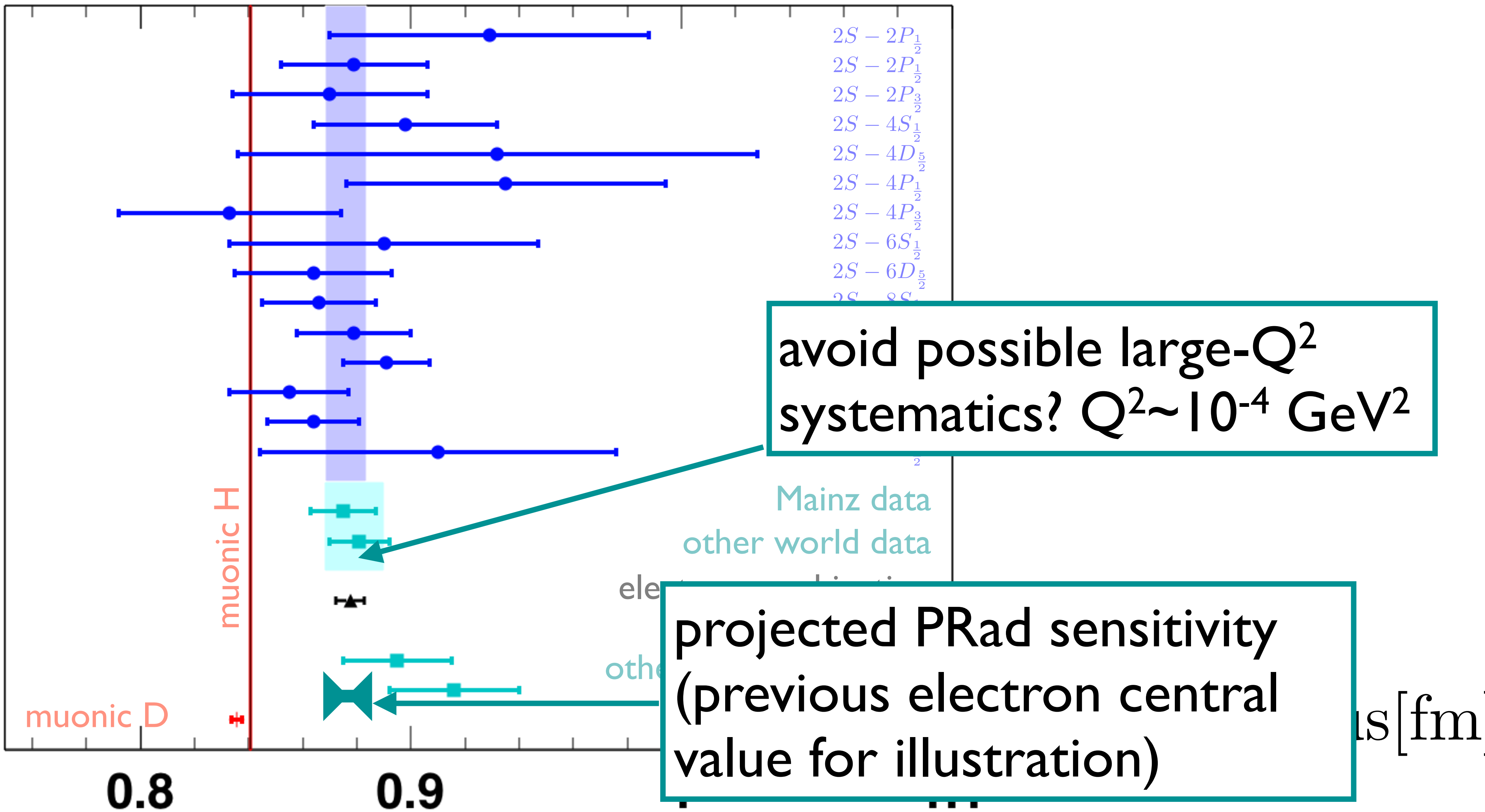
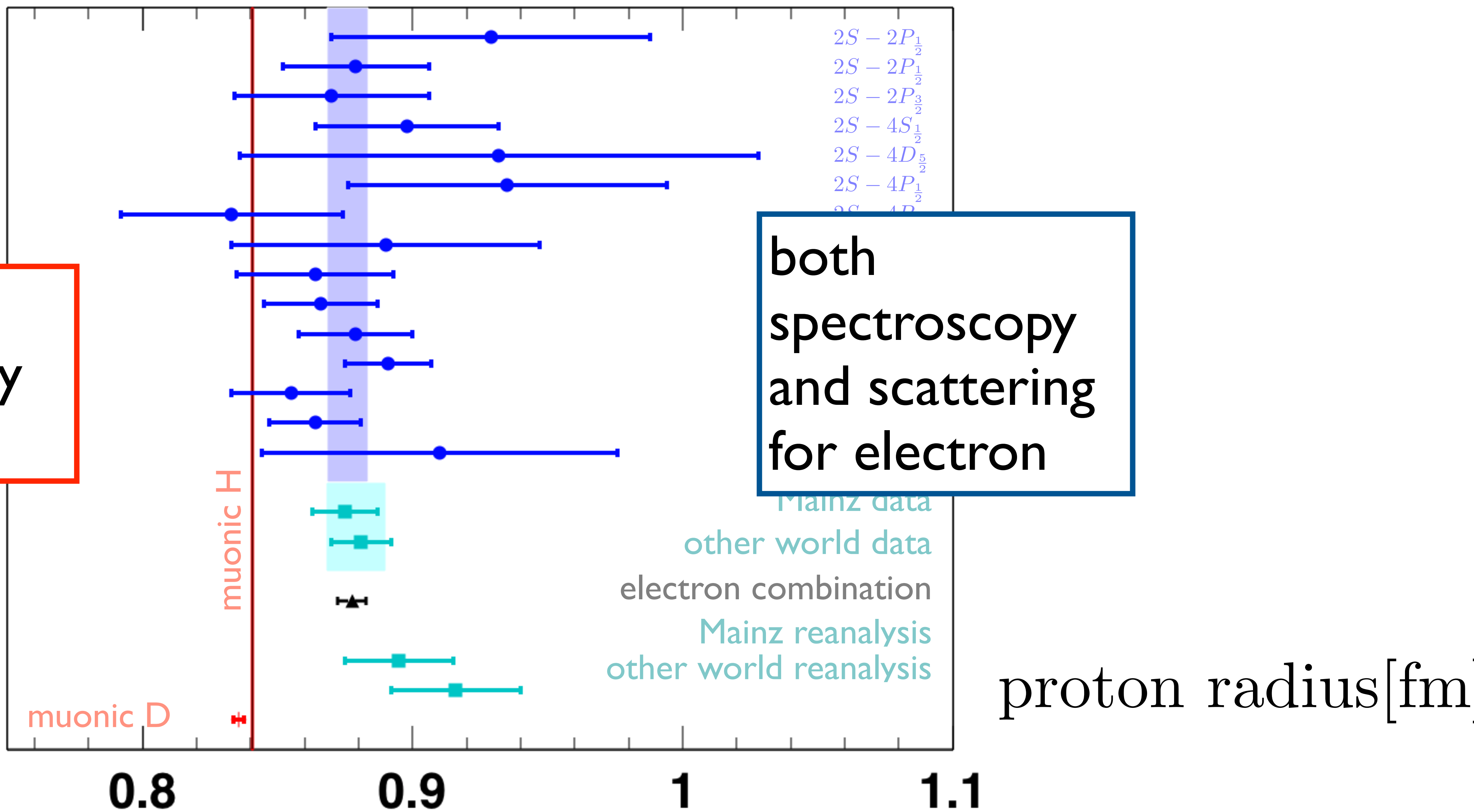


- non-magnetic spectrometer
- simultaneous calibration with $e^- e^-$ (Moller) scattering
- windowless target

data collected in May/June 2016. first analysis

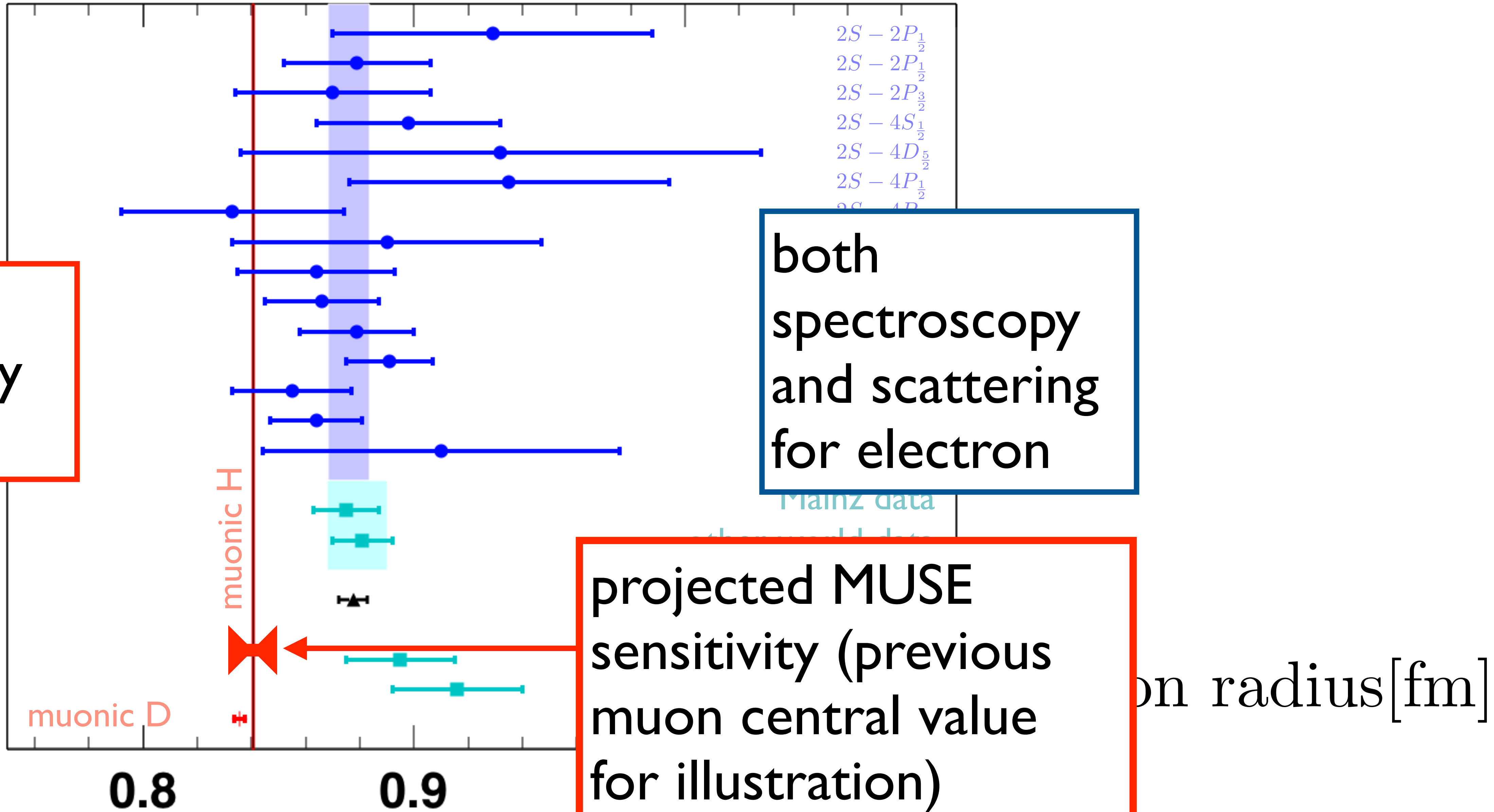
muon-proton scattering: MUSE at PSI

only spectroscopy for muon



both spectroscopy and scattering for electron

muon-proton scattering: MUSE at PSI



only spectroscopy for muon

both spectroscopy and scattering for electron

projected MUSE sensitivity (previous muon central value for illustration)

- measurement of e^+,e^-,μ^+,μ^-
- cancellation of systematics & direct two-photon sensitivity

production data-taking scheduled

- The proton radius puzzle has important implications

    - dramatic shift in fundamental constants, and revising our theoretical understanding of many processes in atomic, nuclear and particle physics

  or

    - new physics?

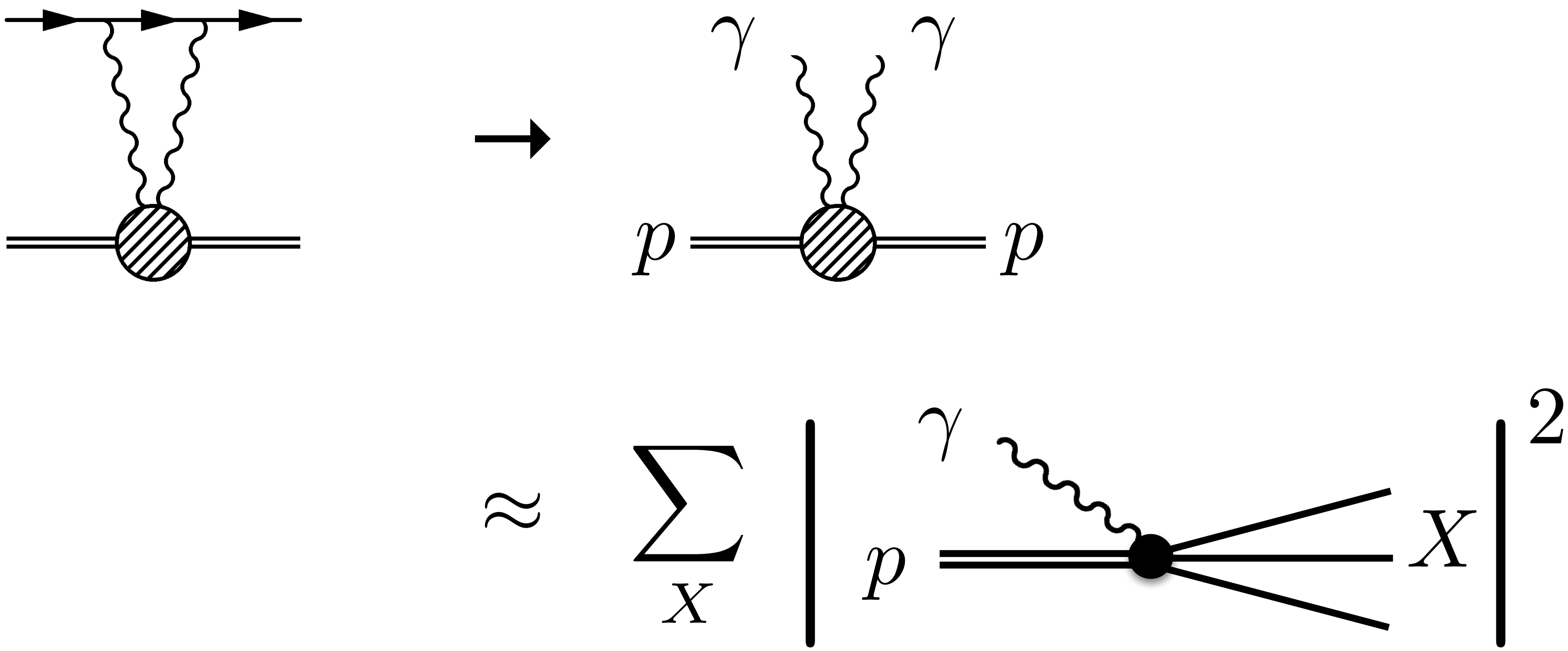
- Improved analysis of muonic hydrogen disfavors enhanced two-photon exchange contribution as explanation

- Radiative corrections reanalyzed so far do not reconcile electron scattering results with muonic hydrogen. Work remains.

- The puzzle is motivating many new experiments

- The puzzle has driven important theoretical developments

Optical theorem for two-photon exchange in muonic hydrogen



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$$W_1(\nu,0) = -2 + \mathcal{O}(\nu^2) = \frac{1}{\pi}\int d\nu'^2 \frac{ImW_1(\nu',0)}{\nu'^2-\nu^2} > 0 \quad ??$$

$$\Rightarrow \qquad W_1(\nu,Q^2) = W_1(0,Q^2) + \frac{\nu^2}{\pi}\int d\nu'^2 \frac{ImW_1(\nu',0)}{\nu'^2(\nu'^2-\nu^2)}$$

new hadronic function

Two theoretical tools to analyze this new hadronic function:

- Heavy particle effective field theory

- Operator product expansion

Adapt techniques developed for dark matter direct detection cross sections.

RJH, M.P. Solon, PRL (2014)

RJH, Paz (2016)

interesting saga: erratum ~40 years after publication

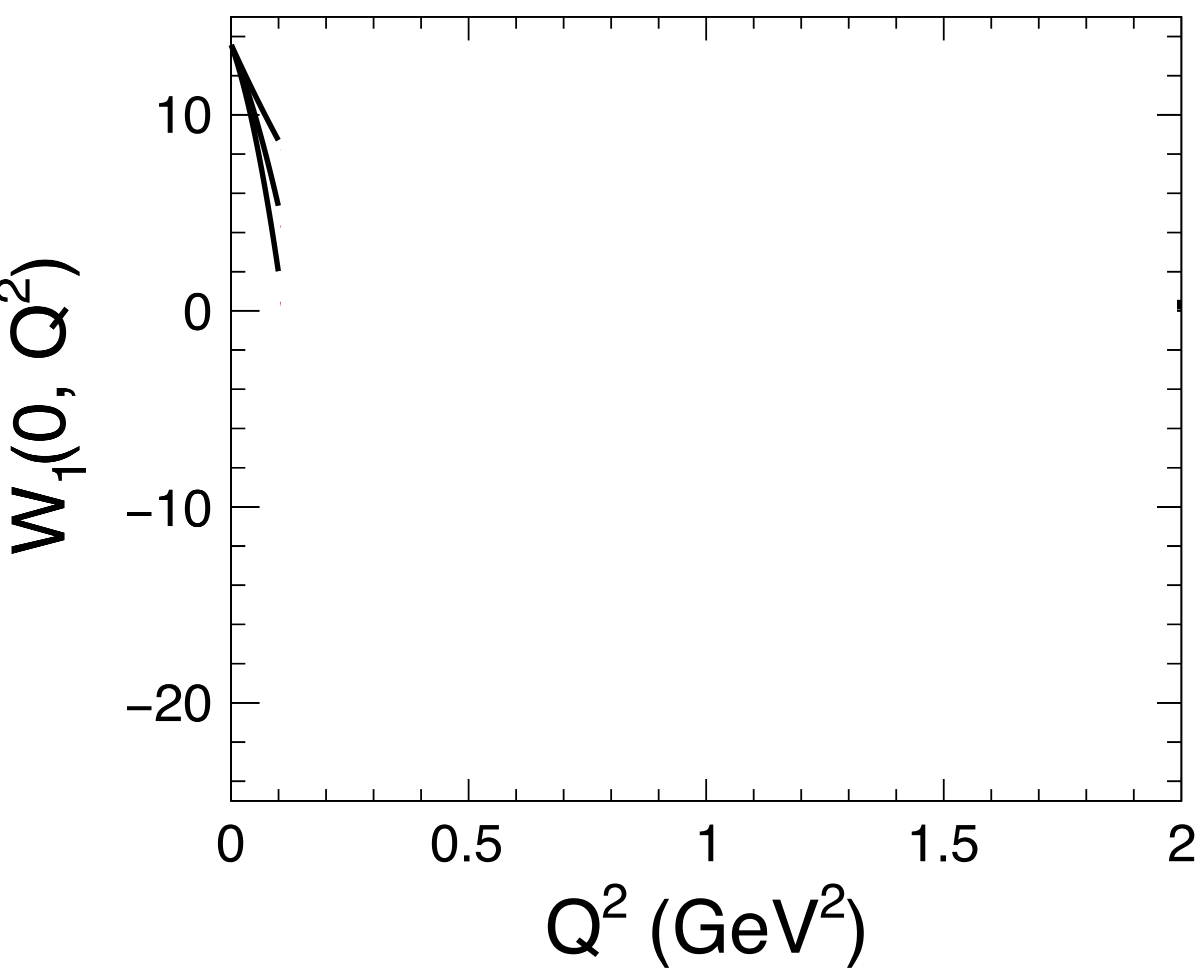
Collins, NPB 149, 90 (1979); erratum ibid 2017

- Heavy particle expansion

Determine interaction coefficients from measurable low-energy processes

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \ldots \right) \psi$$

Apply the thus-determined Lagrangian to predict low-Q^2 expansion of our function

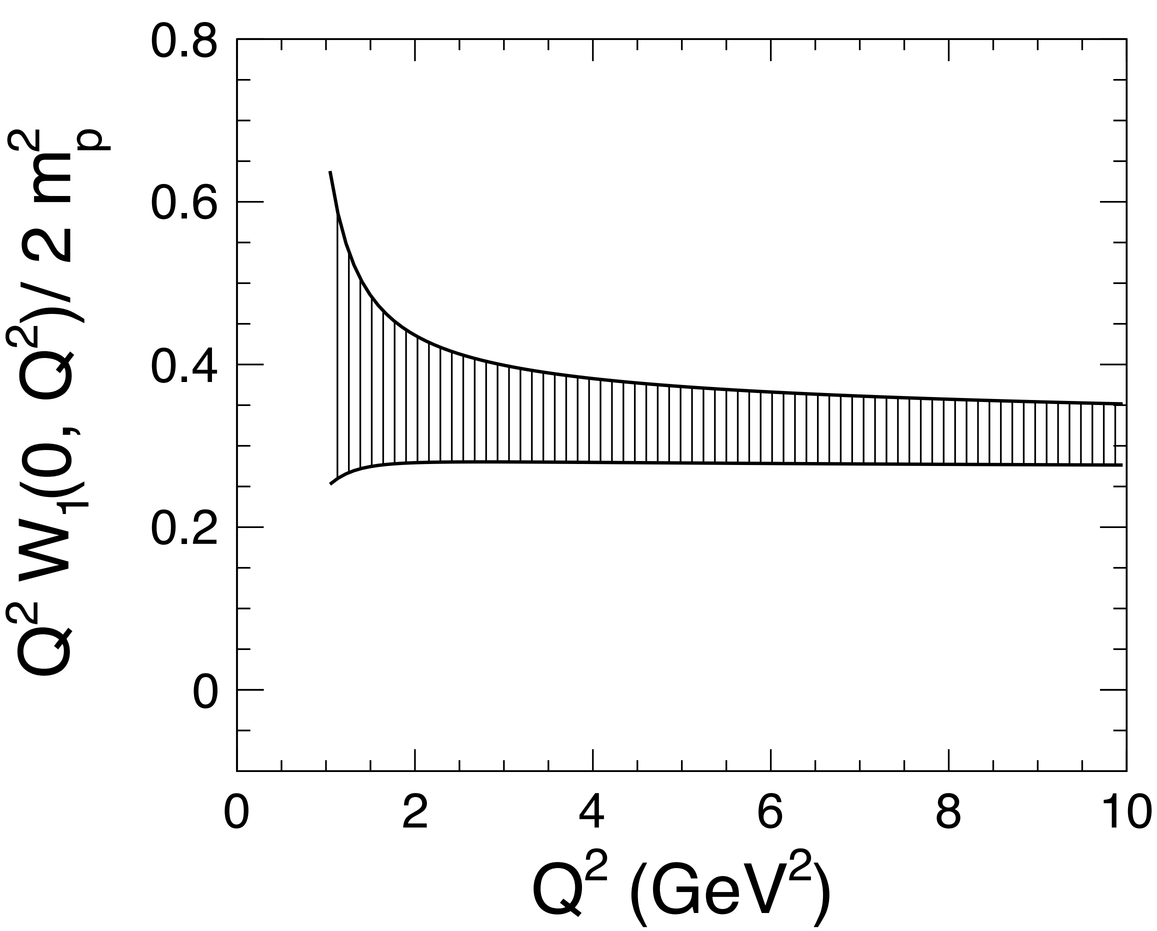


$$W_1(0,Q^2) = 2a_p(2+a_p)$$

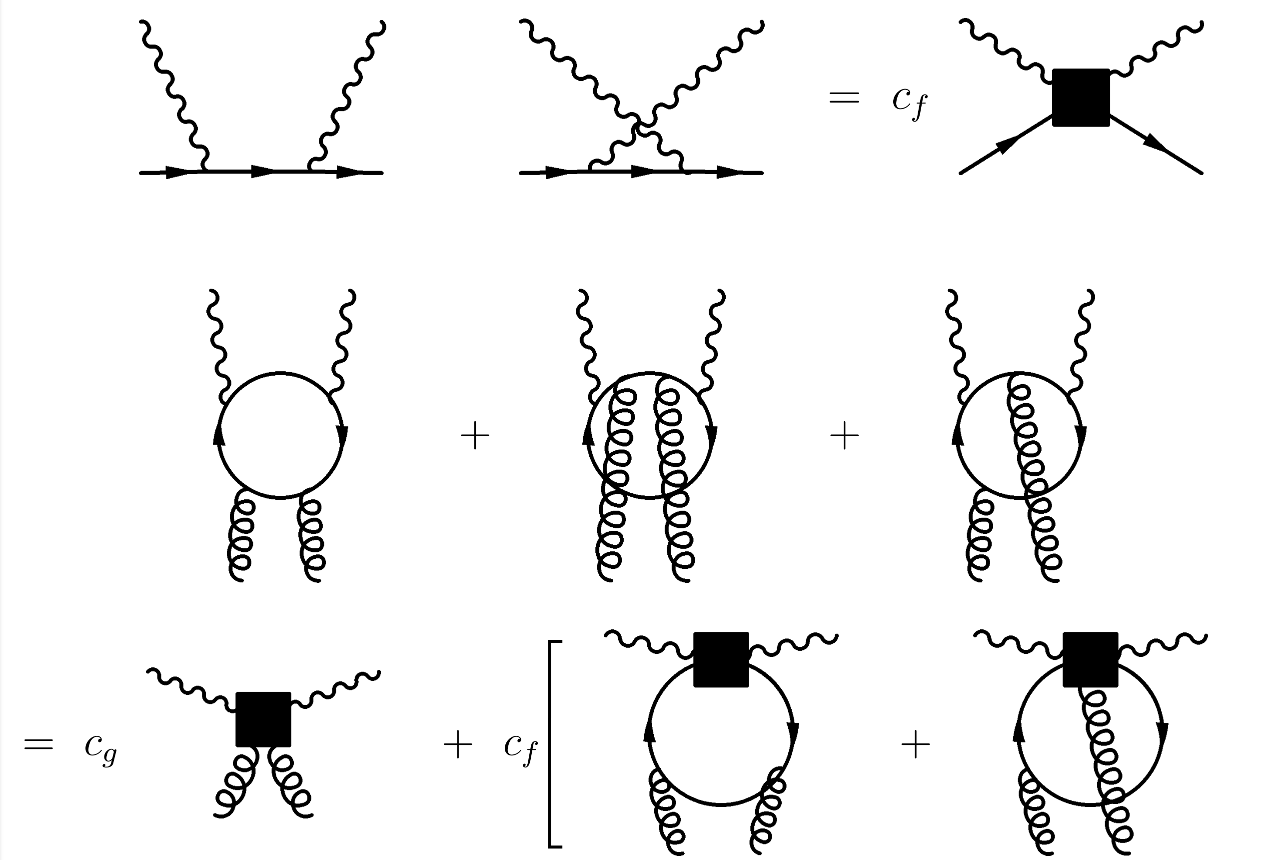
$$+\left\{ \frac{2m_p^3\bar{\beta}}{\alpha} - a_p - \frac{2}{3}\left[(1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2 \right] \right\} \frac{Q^2}{m_p^2} + \ldots$$

- Operator product expansion

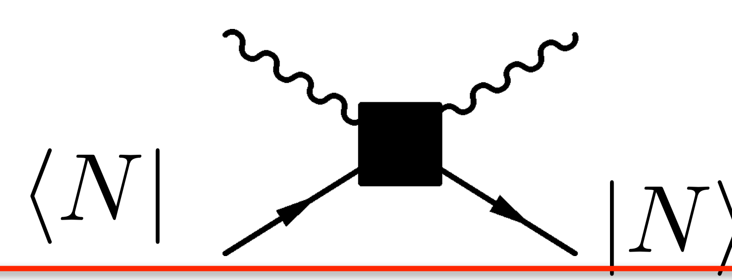
$$W_1(0, Q^2) = \frac{1}{Q^2} \sum_i c_i \langle O_i \rangle$$



c_i short distance matching: perturbative QCD

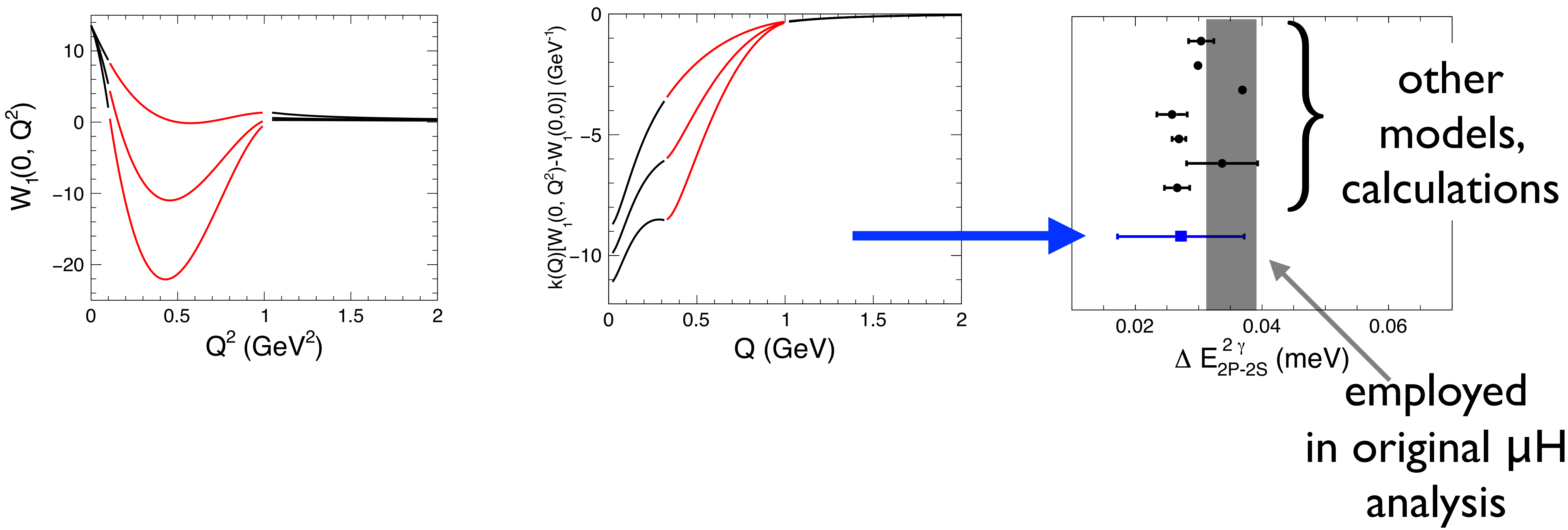


$\langle O_i \rangle$ local operator matrix elements: nonperturbative QCD



• Put these pieces together:

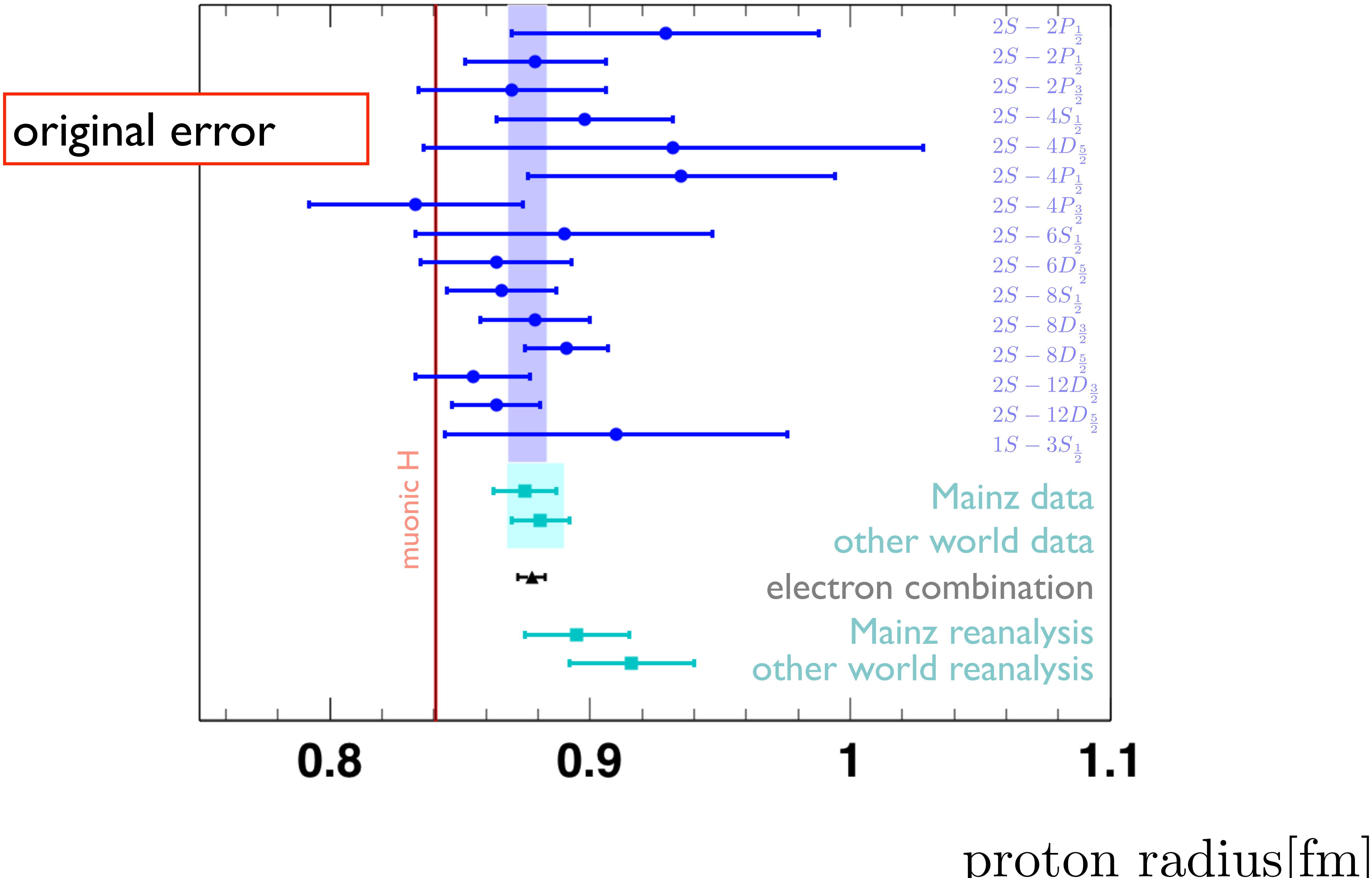
interpolation: | include weighting function: | muonic hydrogen Lamb shift



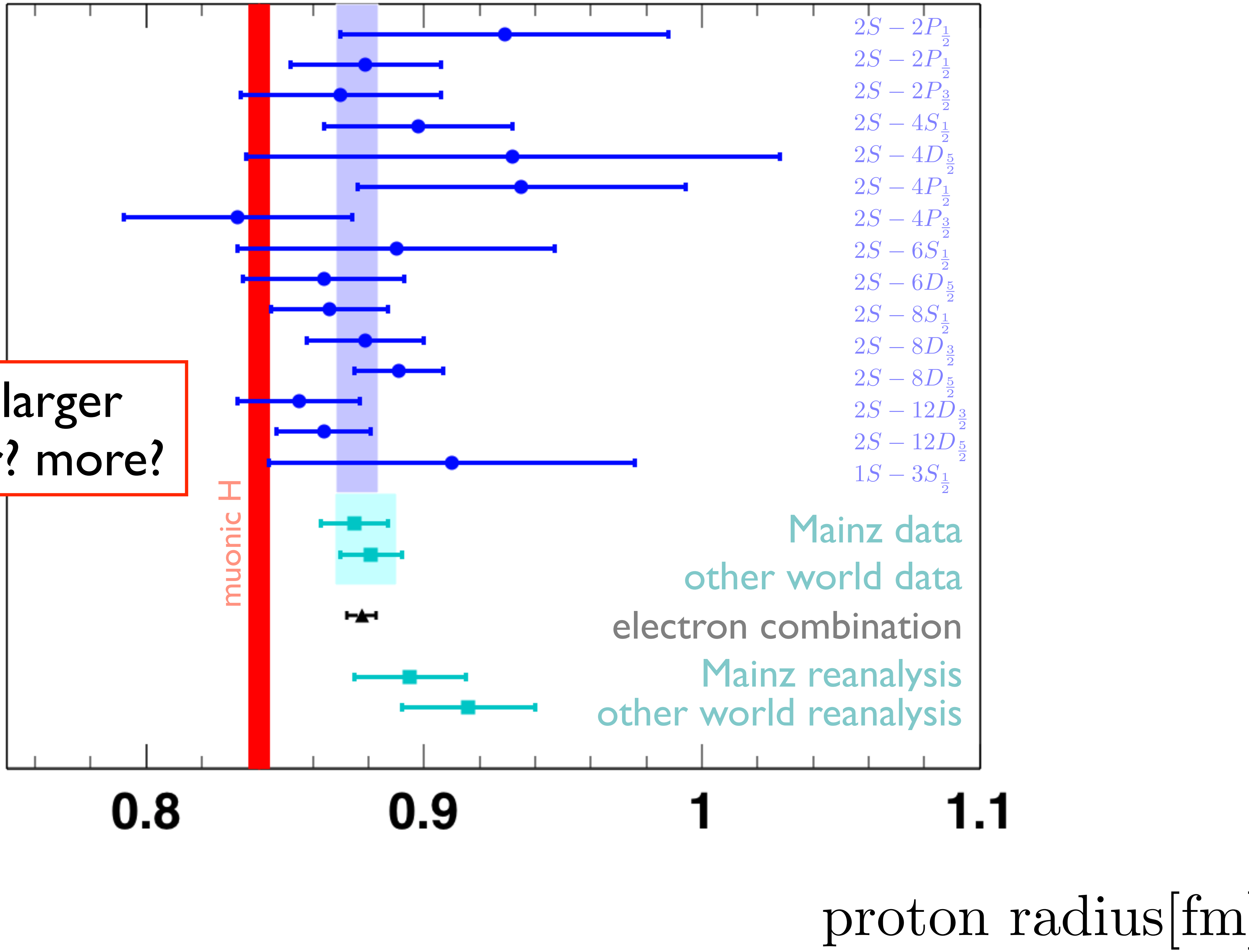
• OPE constraint: turns extrapolation into interpolation

• remains dominant theoretical error for muonic hydrogen

Model-dependent assumptions on two-photon exchange

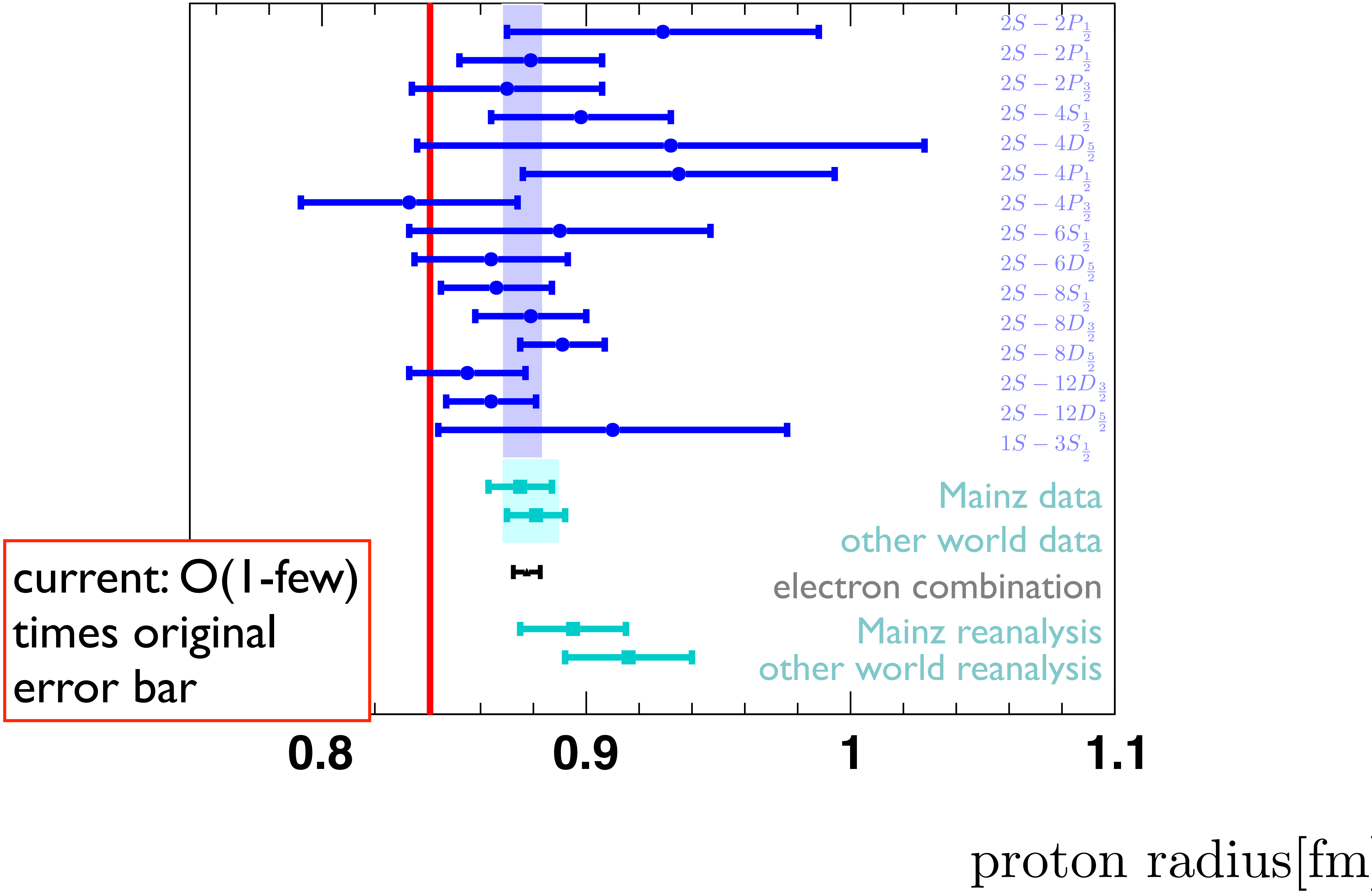


Model-dependent assumptions on two-photon exchange



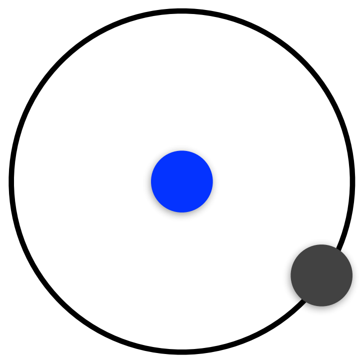
10 times larger error bar? more?

Model-dependent assumptions on two-photon exchange



Many manifestations of heavy particle symmetry:

	prediction:	small parameter:
- hydrogen/deuterium spectroscopy	$E_n(H) = -\frac{1}{2} m_e (Z\alpha)^2 + \dots$	$(m_e Z\alpha) \ll m_e$



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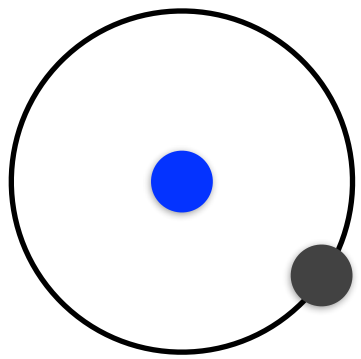

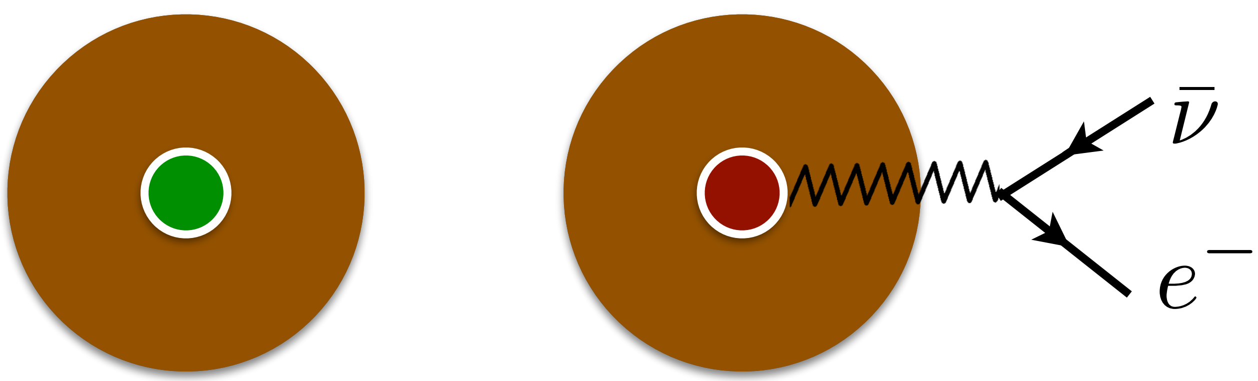

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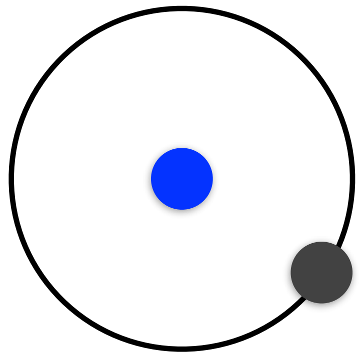
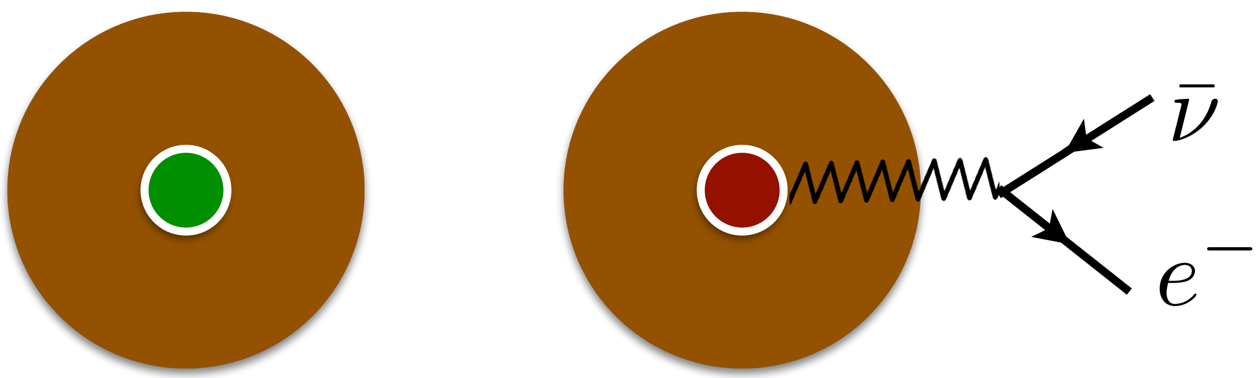
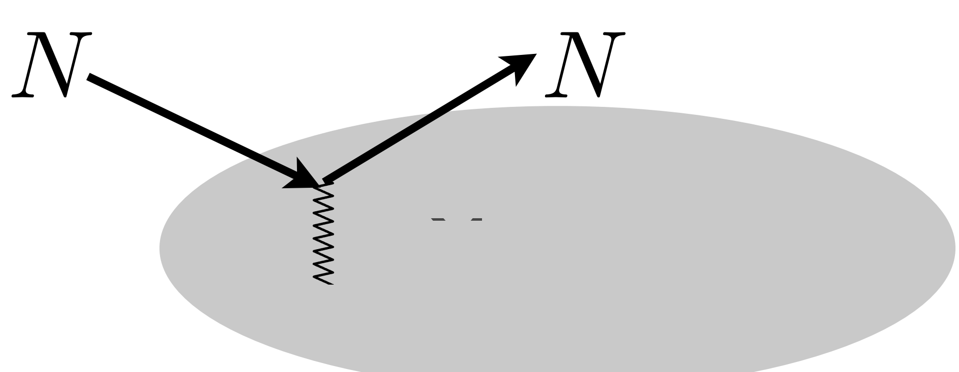
- DM interactions	$\sigma(\chi N \to \chi N) = ?$	$m_W \ll m_\chi$

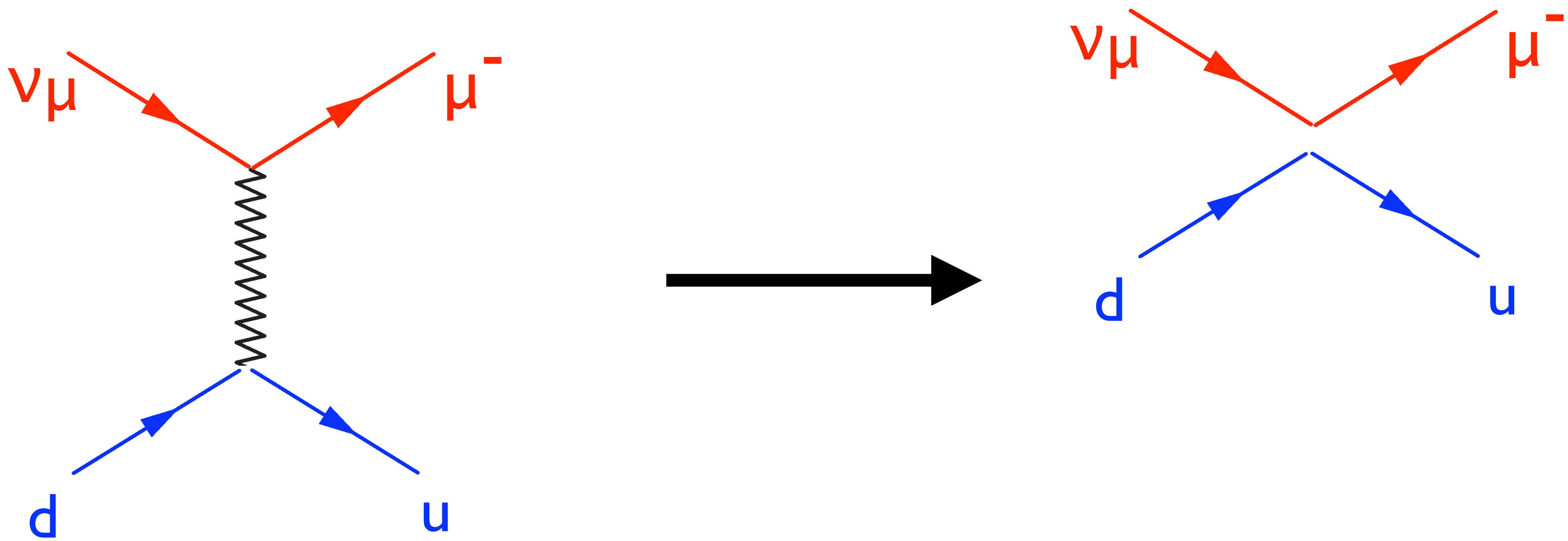






Many applications of operator product expansion:

E.g., Fermi theory of weak interactions:



$$[\bar{u}d]_{V-A}\,\frac{g^2}{\partial^2 - m_W^2}\,[\bar{\mu}\nu]_{V-A}$$

$$G_F\,\underbrace{[\bar{u}d]_{V-A}[\bar{\mu}\nu]_{V-A}} + \ldots$$

short distance coefficient

local operator

Renormalization analysis for log-enhanced radiative corrections

$$d\sigma = H(M) \times \frac{H(\mu)}{H(M)} \times J(\mu) \times S(\mu)$$

hadron structure — $H(M)$

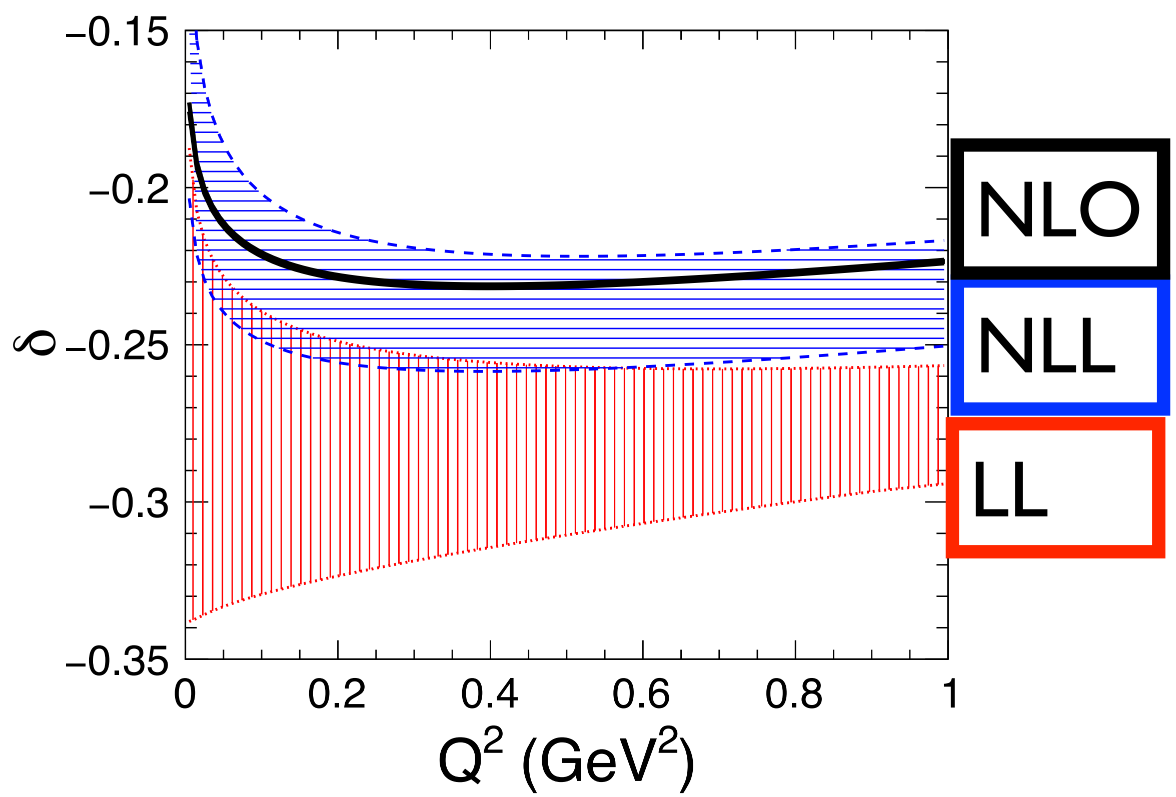
radiative correction — $\frac{H(\mu)}{H(M)} \times J(\mu) \times S(\mu)$

numerically: $\alpha L^2 = \alpha \log^2 \frac{Q^2}{m^2} \sim 1 \quad \Rightarrow \quad \alpha L \sim \alpha^{\frac{1}{2}}$, etc.

electron energy: $E = 1\,\mathrm{GeV}$

electron energy loss cut: $\Delta E = 5\,\mathrm{MeV}$

total radiative correction



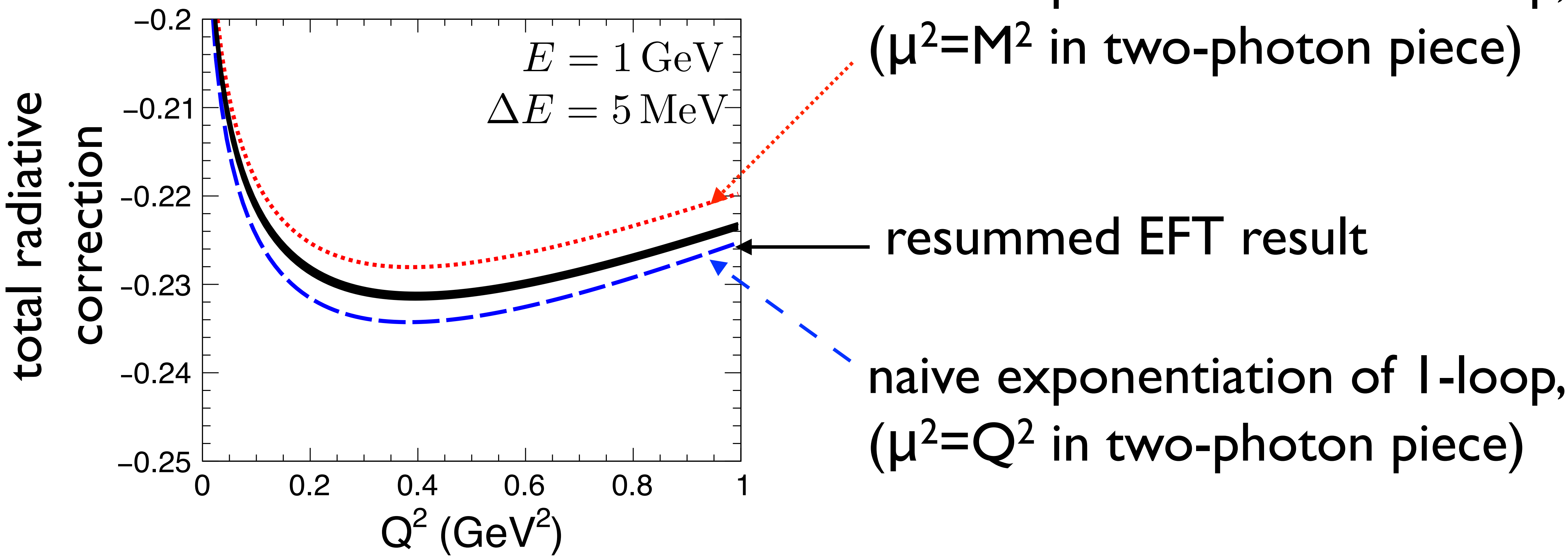
correct through:

NLO: $\mathcal{O}(\alpha)$

NLL: $\mathcal{O}(\alpha^{\frac{1}{2}})$

LL: $\mathcal{O}(1)$

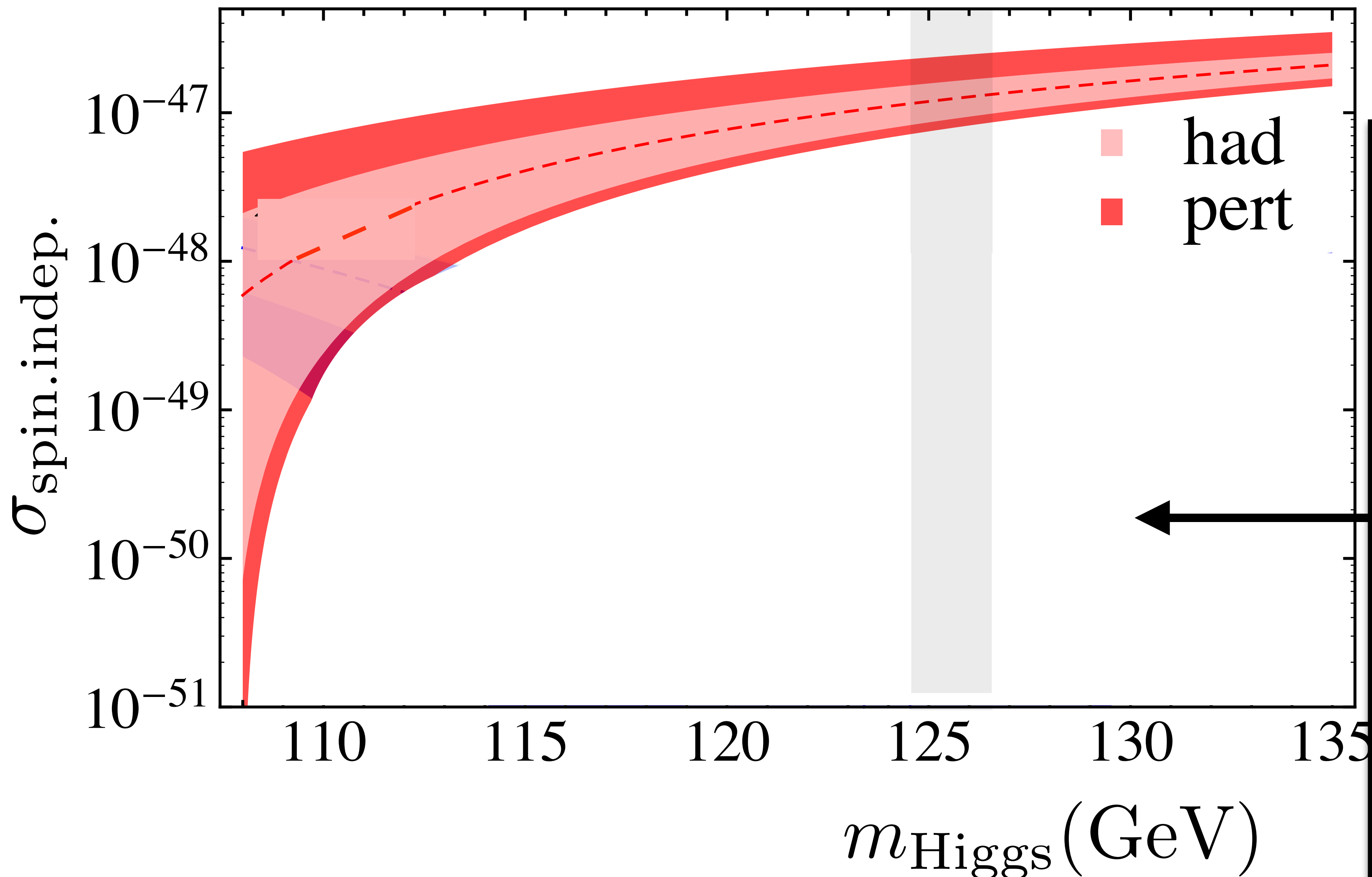
Comparison to previous implementations of radiative corrections, e.g. in A1 analysis of electron-proton scattering data



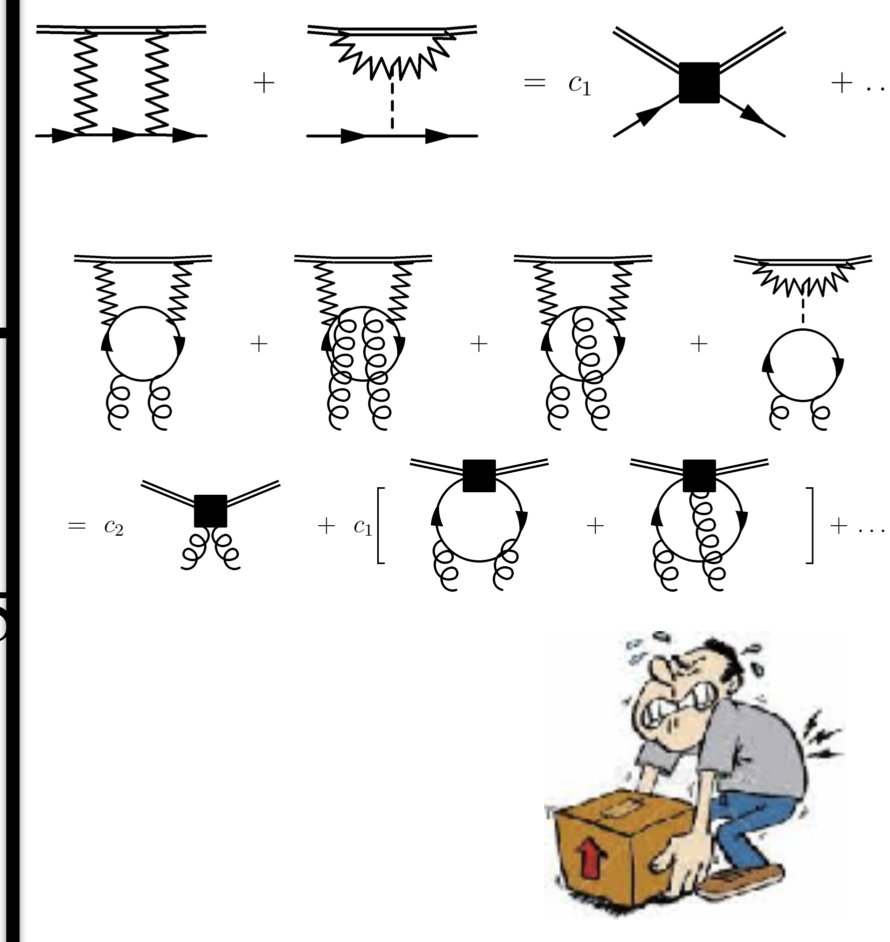
- discrepancies at 0.5-1% compared to currently applied radiative correction models (cf. 0.2-0.5% systematic error budget of A1 experiment)
- should be implemented directly in analysis, but doesn't appear to resolve anomaly (floating normalizations)
- model dependence in hard two-photon exchange remains

Model independent prediction for heavy WIMP scattering

RJH, M.P. Solon, PRL (2014)



Heavy WIMP EFT



- generally, expect WIMP scattering cross section to depend on mass, spin, and electroweak quantum numbers

- heavy WIMP regime: universal prediction for given quantum numbers (here consider electroweak triplet)