Radiative lepton flavor violating B, D, & K decays

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Outline

- Introduction
  - Motivation
  - Effective Lagrangian
- Three-body B, D, & K decay diagrams
- General P to $\gamma \ell_1 \ell_2$ calculation
- Form-factors
- Vector operator contribution to B_q decays
- Summary

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- New physics model independence: Any NP scenario involving LFV can be matched to L_{eff}.
- Experiments such as Belle 2 and LHCb could provide currently unmeasured decay constraints

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- Pseudo-scalar mesons: Bd (db), Bs (sb), D (cu), K (ds)
- Calculate decays to $\tau\mu\gamma, \tau e\gamma, \mu e\gamma$

Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_D + \mathcal{L}_{\ell q} + \ldots$$ *

*A. Celis, V. Cirigliano and E. Passemar, Phys. Rev. D 89, no. 9, 095014 (2014)

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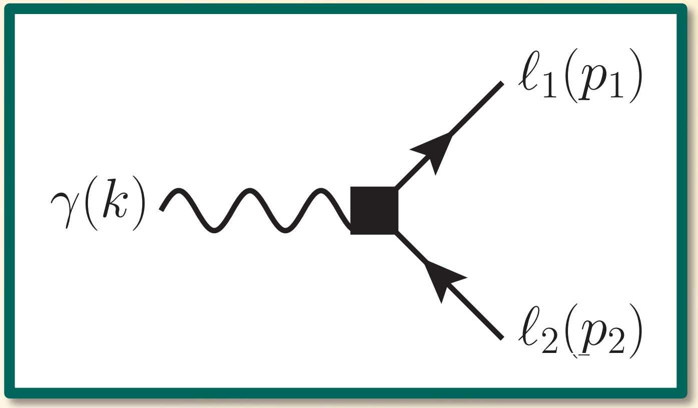

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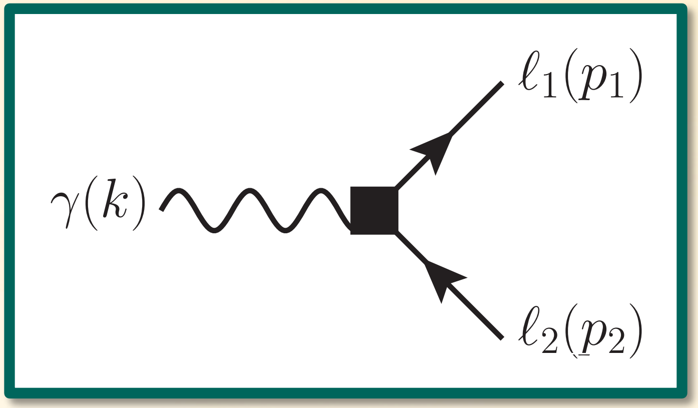

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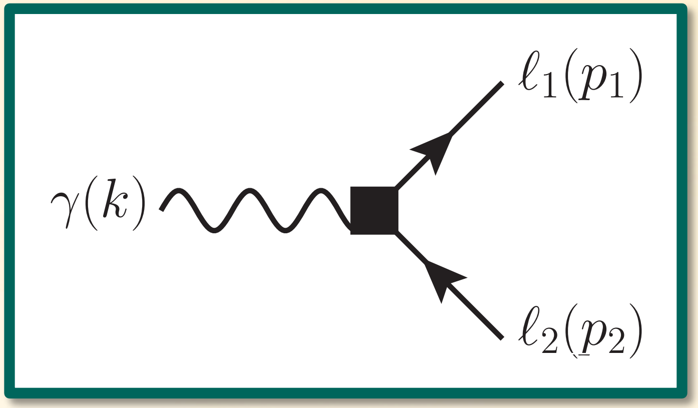

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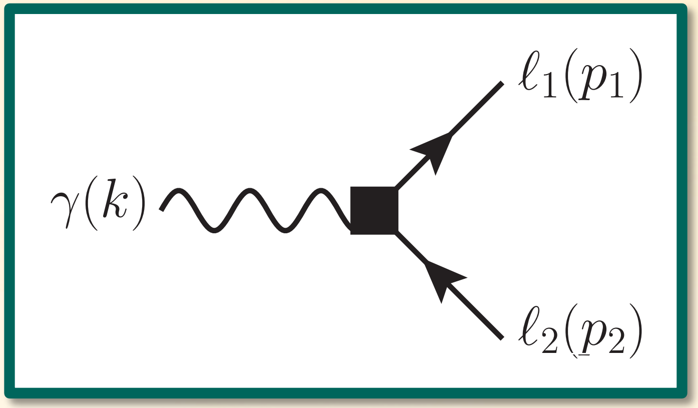

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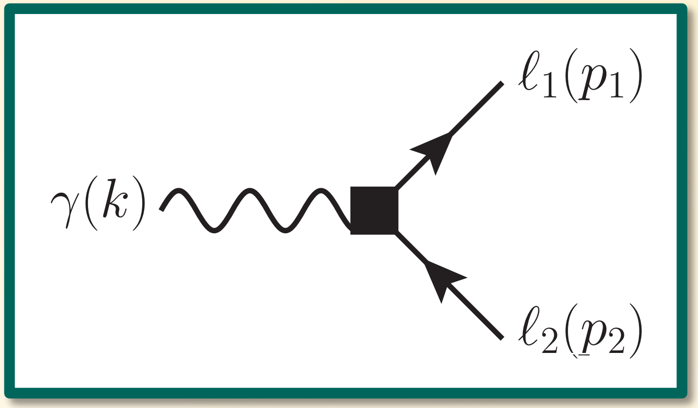
$P_{R(L)} = (1 \pm \gamma^5)/2$



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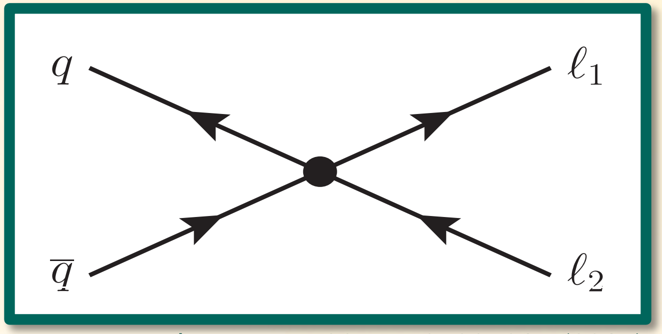

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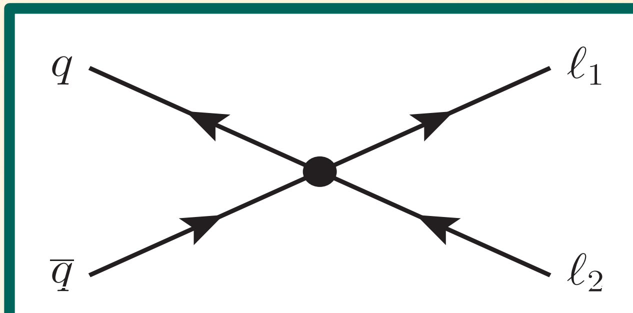
Vector Wilson coefficients



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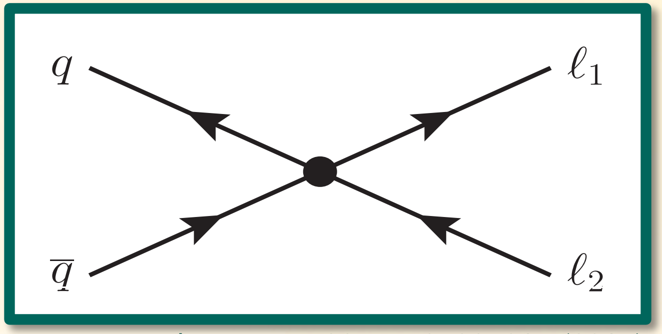
Axial Wilson coefficients



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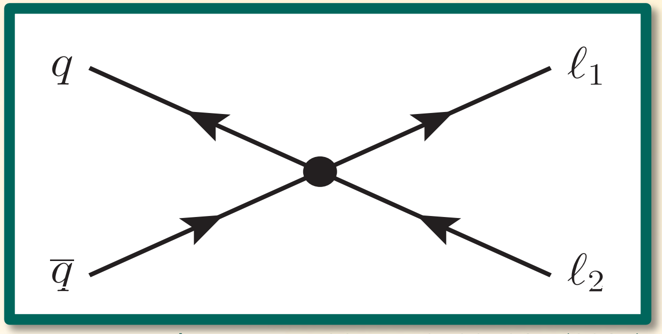
Scalar Wilson coefficients



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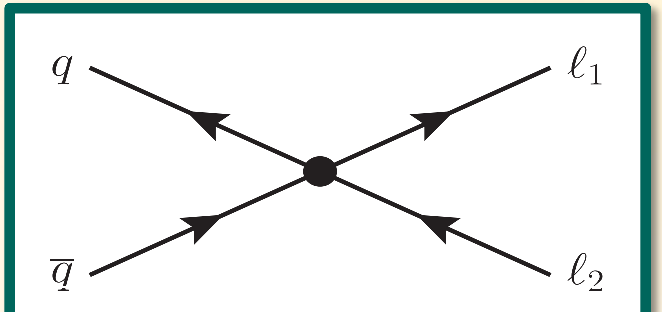
Pseudoscalar Wilson coefficients



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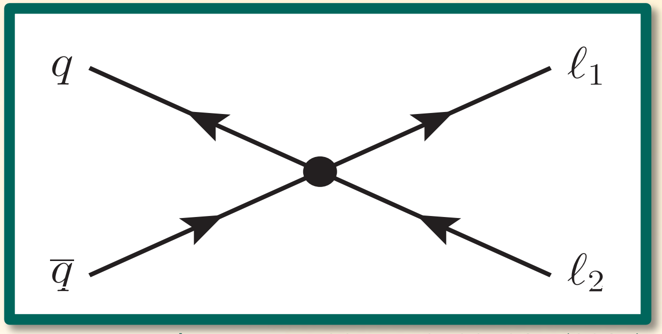
Tensor Wilson coefficients



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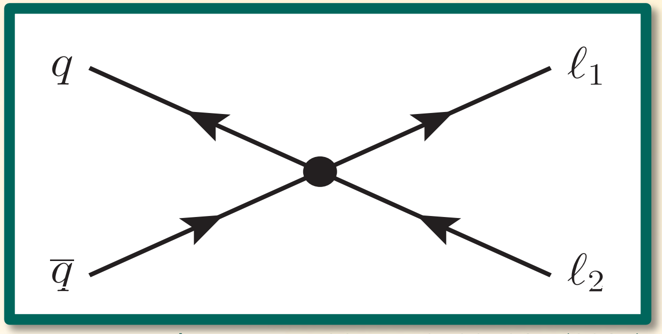
m_{q2} = MS bar quark mass



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$$\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2} \sum_{q_1,q_2} \Big[\left(C_{VR}^{q_1 q_2 \ell_1 \ell_2}\, \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{VL}^{q_1 q_2 \ell_1 \ell_2}\, \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q}_1 \gamma_\mu q_2$$

$$+ \left(C_{AR}^{q_1 q_2 \ell_1 \ell_2}\, \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q_1 q_2 \ell_1 \ell_2}\, \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q}_1 \gamma_\mu \gamma_5 q_2$$

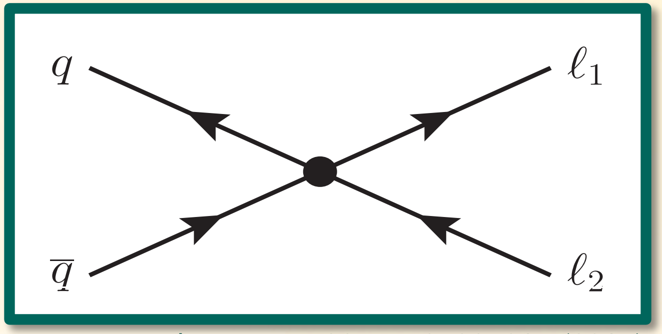
$$+ m_2 m_{q_2} G_F \left(C_{SR}^{q_1 q_2 \ell_1 \ell_2}\, \bar{\ell}_1 P_L \ell_2 + C_{SL}^{q_1 q_2 \ell_1 \ell_2}\, \bar{\ell}_1 P_R \ell_2 \right) \bar{q}_1 q_2$$

$$+ m_2 m_{q_2} G_F \left(C_{PR}^{q_1 q_2 \ell_1 \ell_2}\, \bar{\ell}_1 P_L \ell_2 + C_{PL}^{q_1 q_2 \ell_1 \ell_2}\, \bar{\ell}_1 P_R \ell_2 \right) \bar{q}_1 \gamma_5 q_2$$

$$+ m_2 m_{q_2} G_F \left(C_{TR}^{q_1 q_2 \ell_1 \ell_2}\, \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q_1 q_2 \ell_1 \ell_2}\, \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right) \bar{q}_1 \sigma_{\mu\nu} q_2 \; + \; h.c. \Big]^*$$

m_{q2} = MS bar quark mass

G_F = Fermi constant



*A. Celis, V. Cirigliano and E. Passemar, Phys. Rev. D 89, no. 9, 095014 (2014)

Effective Lagrangian

Four fermion Lagrangian

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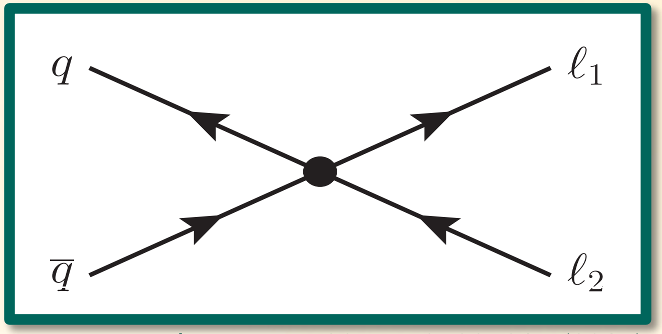
\end{aligned}
$$

All of the operators ***<u>except</u>*** the scalar operators will contribute to the RLFV decays.



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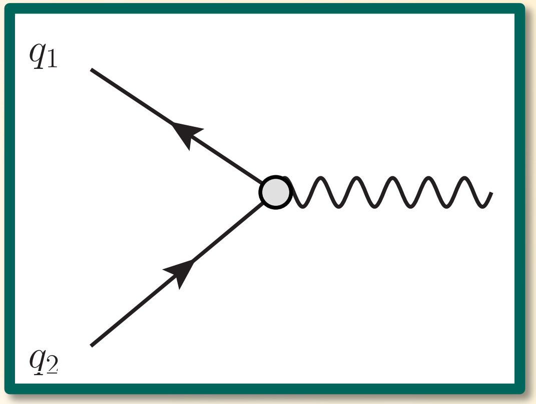
SM Dipole Penguin Lagrangian

$$\mathcal{L}_{\text{peng}} = \frac{G_F}{\sqrt{2}} \sum_q \lambda_q C_{7\gamma} \frac{\sqrt{\pi\alpha}}{\pi^2} m_{q_2} \overline{q}_1 \sigma_{\mu\nu} \left(1 - \gamma_5\right) F^{\mu\nu} q_2 + h.c. + ...$$



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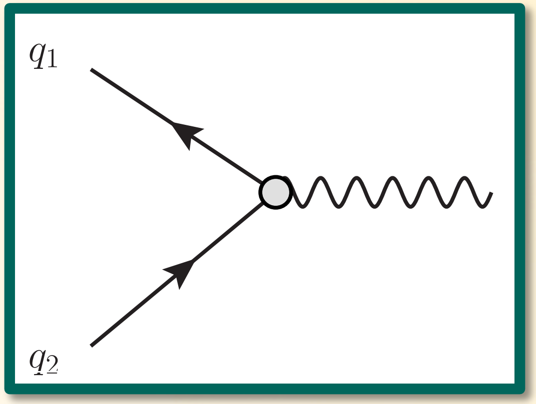

$\lambda_q = V_{qq1} V^*_{qq2}$ (appropriate CKM matrix elements)

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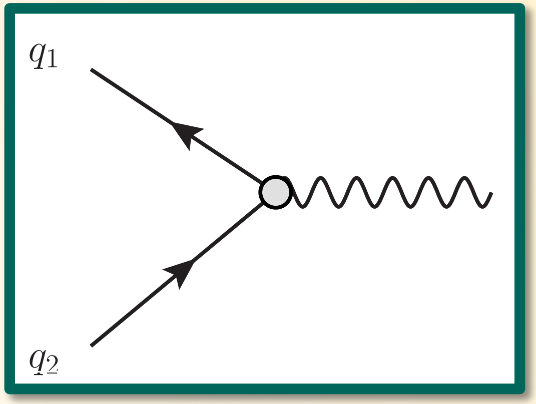

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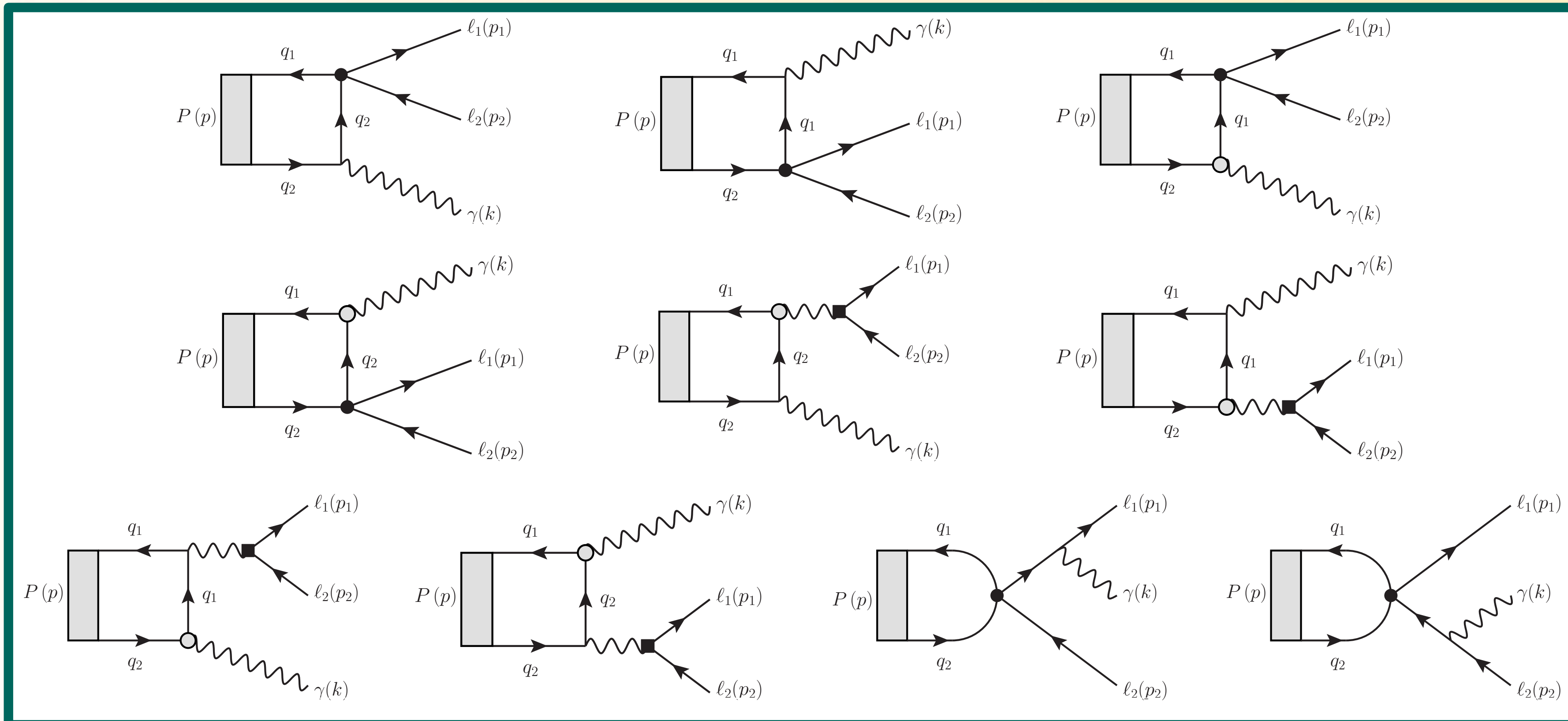
$C_{7\gamma}$= 0.299 dipole penguin operator Wilson coef.

Feynman diagrams B,D,K to $\gamma\ell_1\ell_2$



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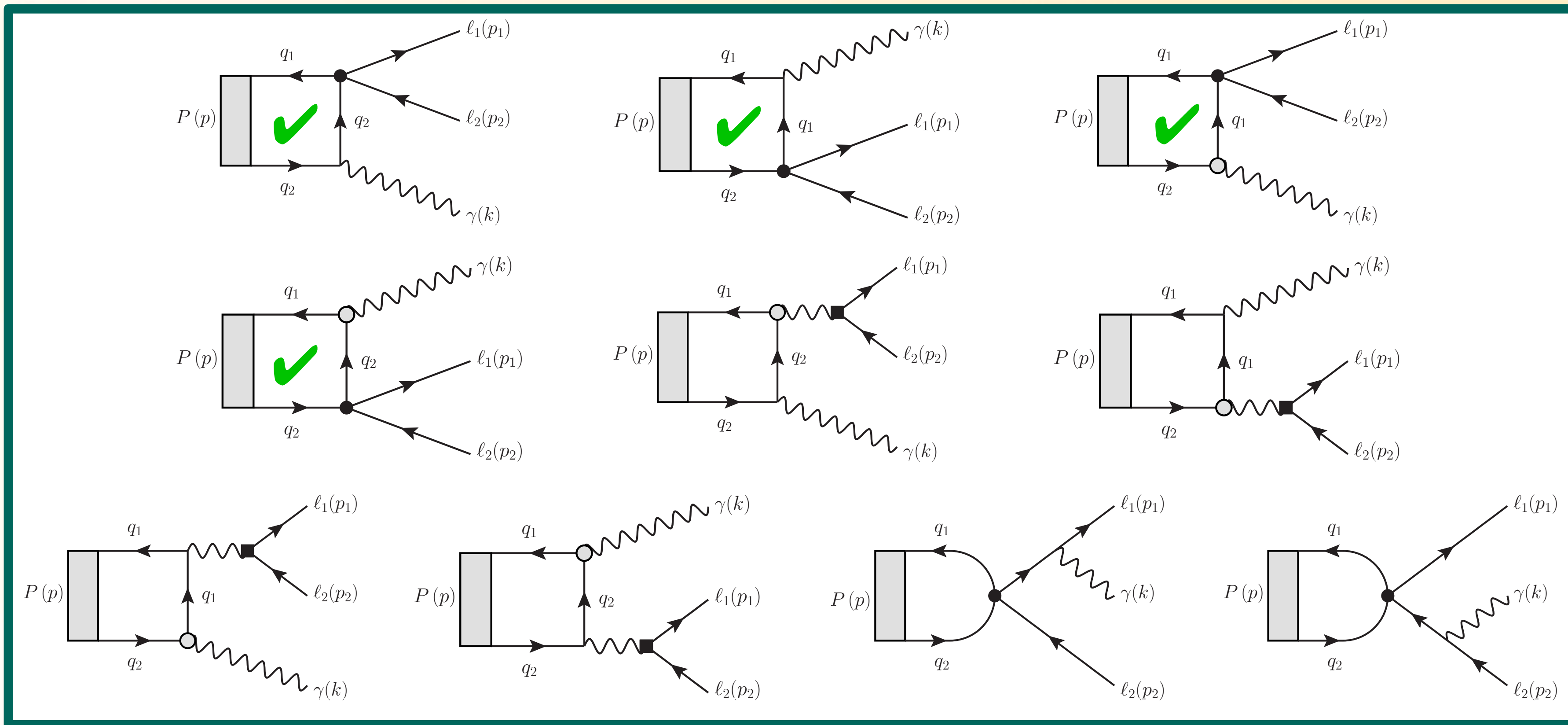

- Vector, axial, tensor operators of 4 fermion Lagrangian

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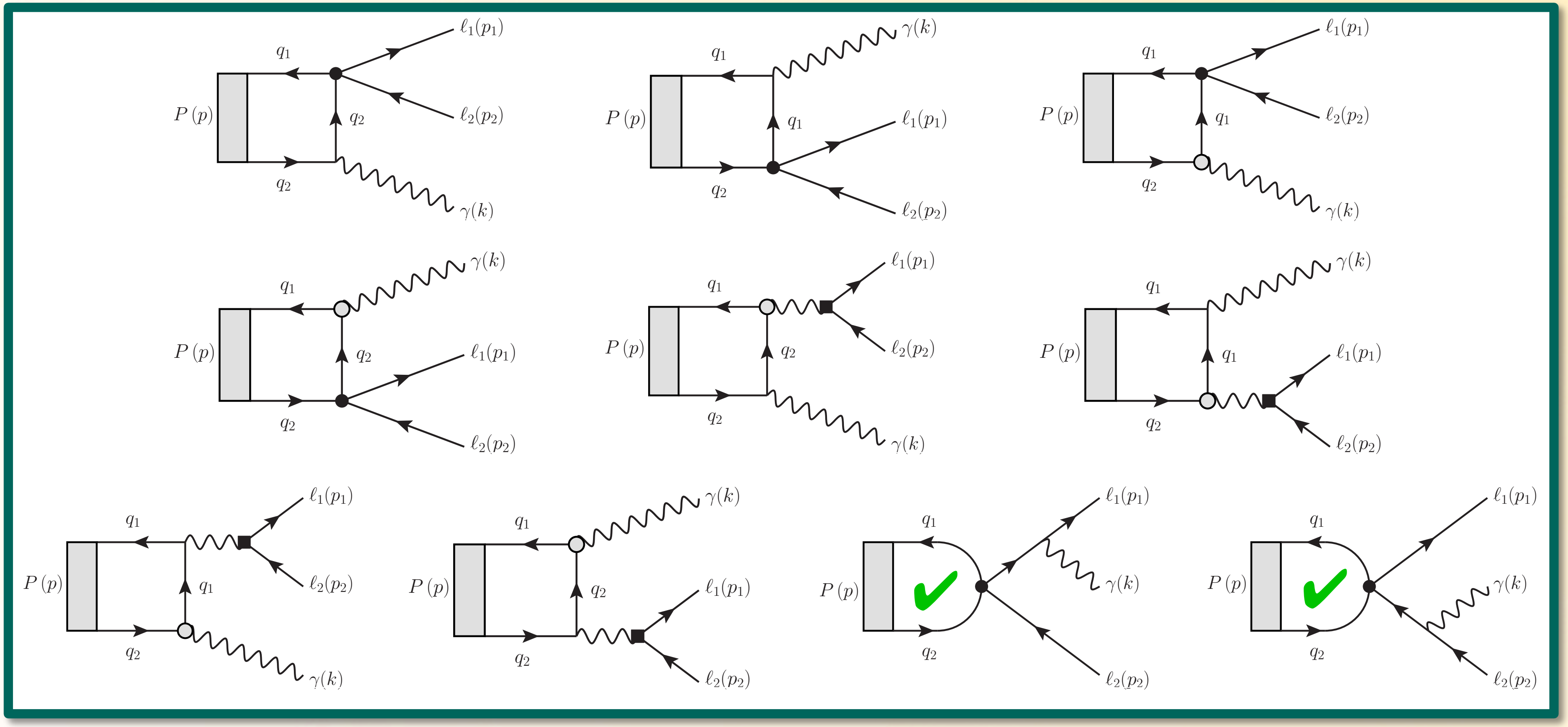
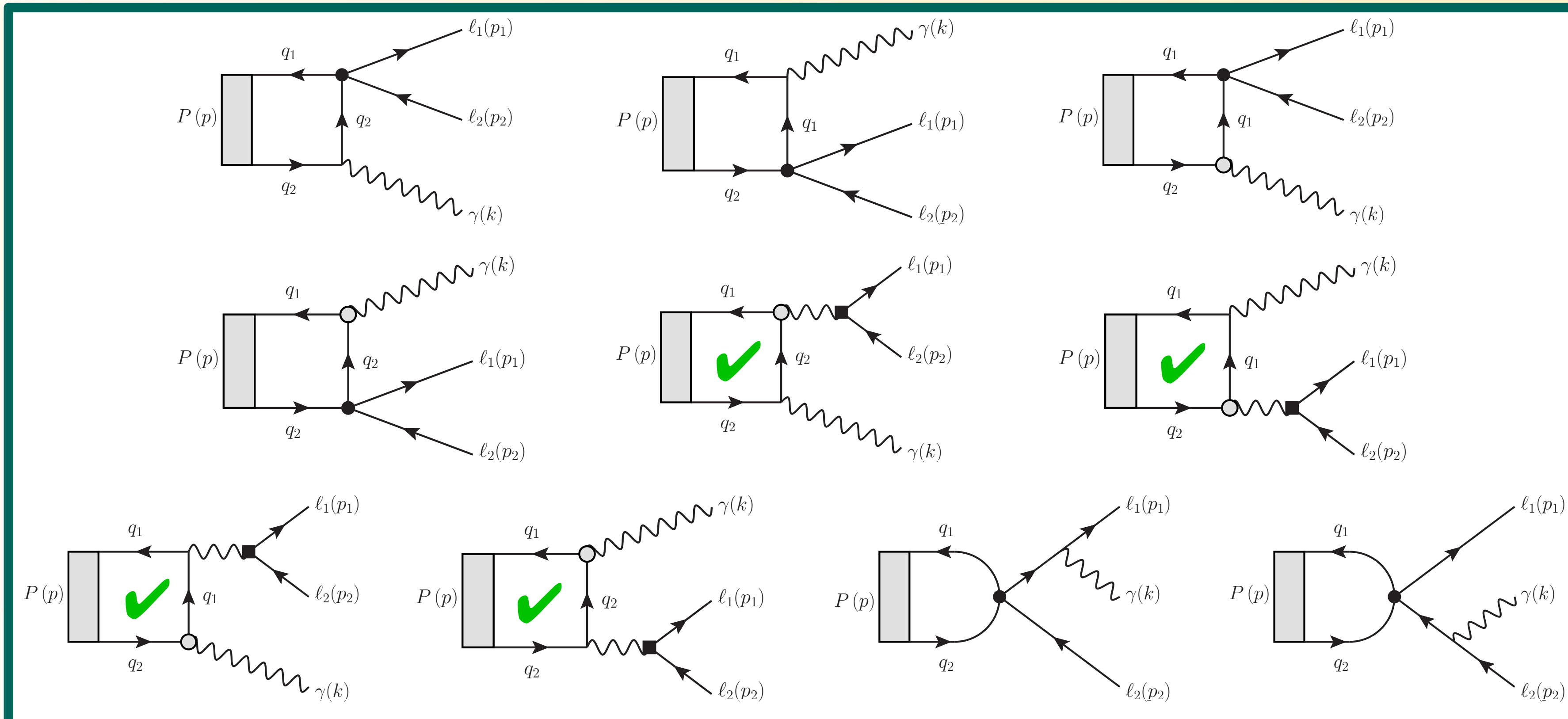


- Axial and pseudo-scalar operators of 4 fermion Lagrangian

Feynman diagrams B,D,K to $\gamma \ell_1 \ell_2$



■ Dipole Lagrangian operators

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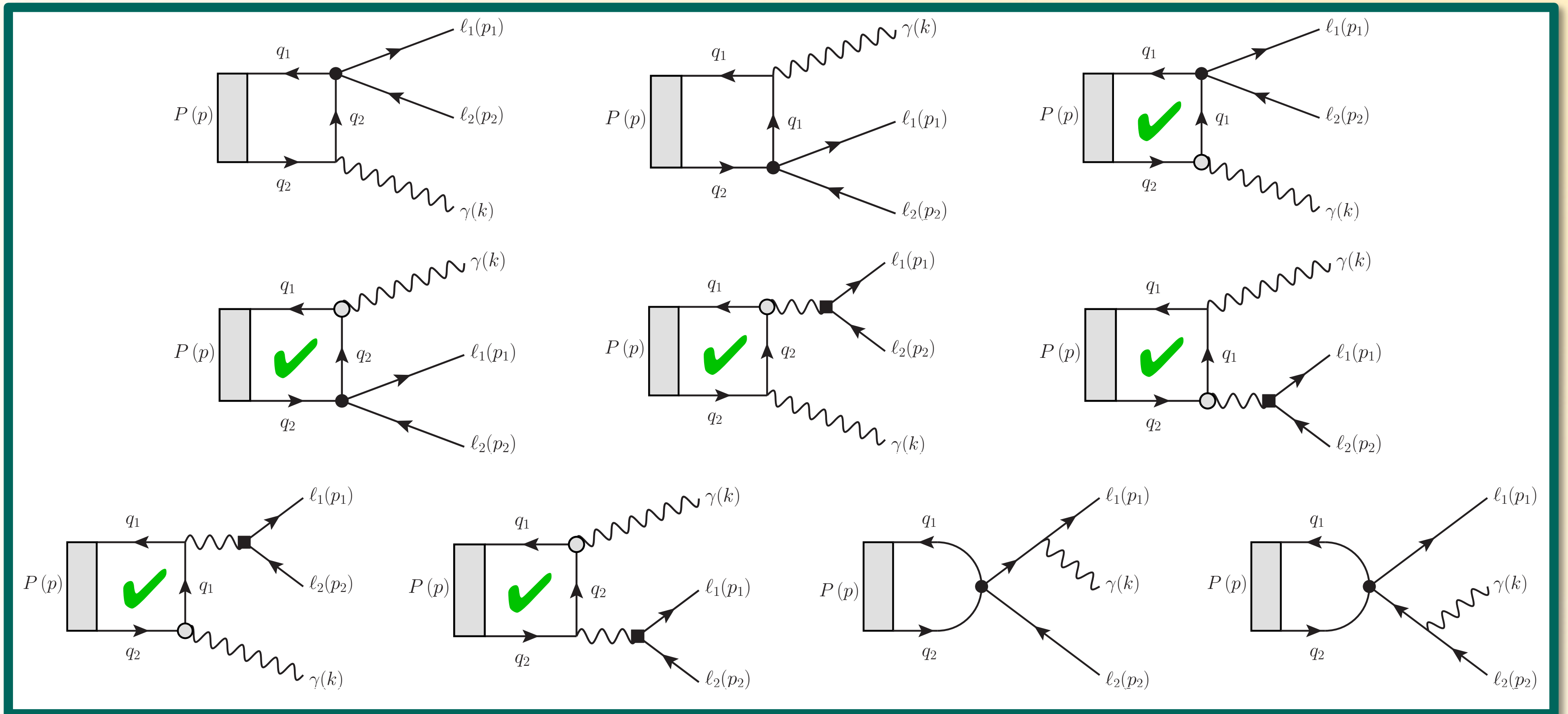

- SM dipole penguin operators in the penguin Lagrangian

General Amplitude

Method*

$$A(P(p) \to \gamma(k)\ell_1(p_1)\overline{\ell}_2(p_2)) = \overline{u}(p_1, s_1)\ M^{\mu}(p, k, q)\ v(p_2, s_2)\ \epsilon^{*}_{\mu}(k)$$

*W. A. Bardeen and W. K. Tung, Phys. Rev. 173, 1423 (1968)

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- $M^\mu(p,k,q)$

$$M^\mu(p,k,q) = \sum_i \ell_i^\mu(p,q,k) M_i^{q\ell_1\ell_2}(p^2,\ldots)$$

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  - $\ell_i^\mu(p,q,k)$ must be gauge invariant structure without kinematic singularities

$$M^\mu(p,k,q) = \sum_i \ell_i^\mu(p,q,k) M_i^{q\ell_1\ell_2}(p^2, ...)$$

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General Amplitude

- The most general parameterization contains many form factors which can be reduced to 12:

$$\begin{aligned} M^\mu(p,k,q) &= \gamma^\mu\left(M_1^{P\ell_1\ell_2} + \not{k}M_2^{P\ell_1\ell_2}\right) + i\gamma_5\gamma^\mu\left(M_3^{P\ell_1\ell_2} + \not{k}M_4^{P\ell_1\ell_2}\right) \\ &+ q^\mu\left(M_5^{P\ell_1\ell_2} + \not{k}M_6^{P\ell_1\ell_2}\right) + i\gamma_5 q^\mu\left(M_7^{P\ell_1\ell_2} + \not{k}M_8^{P\ell_1\ell_2}\right) \\ &+ p^\mu\left(M_9^{q\ell_1\ell_2} + \not{k}M_{10}^{q\ell_1\ell_2}\right) + i\gamma_5 p^\mu\left(M_{11}^{P\ell_1\ell_2} + \not{k}M_{12}^{P\ell_1\ell_2}\right) \end{aligned}$$

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- The minimal set can be found using the projection operator $P^{\mu\nu}$, which projects out the gauge invariant structures in $M^\mu(p,k,q)$.

$$P^{\mu\nu} = g^{\mu\nu} - \frac{p^{\mu}k^{\nu}}{(p \cdot k)}$$

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$$P^{\mu\nu} = g^{\mu\nu} - \frac{p^\mu k^\nu}{(p\cdot k)}$$

- We learn that p^μ terms don't belong to minimal set.

General Amplitude

- After removing the kinematical singularities from the gauge invariant structures projected out by $P^{\mu\nu}$, we define $M^{\mu}(p,k,q)$:

$$M^{\mu}(p,k,q) = \sum_i L_i^{\mu}(p,q,k) A_i^{P\ell_1\ell_2}(p^2,\ldots)$$

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- Here each L_i^{μ} is individually gauge invariant ($k_{\mu}L_i^{\mu}(p,q,k)=0$):

$$\begin{aligned} L_1^{\mu} &= \gamma^{\mu}\not{k}, \quad L_2^{\mu} = i\gamma_5\gamma^{\mu}\not{k}, \quad L_3^{\mu} = (p\cdot k)\,q^{\mu} - (k\cdot q)\,p^{\mu}, \\ L_4^{\mu} &= i\gamma_5\left[(p\cdot k)\,q^{\mu} - (k\cdot q)\,p^{\mu}\right], \quad L_5^{\mu} = (p\cdot k)\,\gamma^{\mu} - p^{\mu}\not{k}, \quad L_6^{\mu} = i\gamma_5\left[(p\cdot k)\,\gamma^{\mu} - p^{\mu}\not{k}\right], \\ L_7^{\mu} &= q^{\mu}\not{k} - (k\cdot q)\,\gamma^{\mu}, \quad L_8^{\mu} = i\gamma_5\left[q^{\mu}\not{k} - (k\cdot q)\,\gamma^{\mu}\right] \end{aligned}$$

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$$L_1^\mu = \gamma^\mu \not{k}, \quad L_2^\mu = i\gamma_5 \gamma^\mu \not{k}, \quad L_3^\mu = (p\cdot k)\, q^\mu - (k\cdot q)\, p^\mu,$$

$$L_4^\mu = i\gamma_5 \left[(p\cdot k)\, q^\mu - (k\cdot q)\, p^\mu\right], \quad L_5^\mu = (p\cdot k)\,\gamma^\mu - p^\mu \not{k}, \quad L_6^\mu = i\gamma_5 \left[(p\cdot k)\,\gamma^\mu - p^\mu \not{k}\right],$$

$$L_7^\mu = q^\mu \not{k} - (k\cdot q)\,\gamma^\mu, \quad L_8^\mu = i\gamma_5 \left[q^\mu \not{k} - (k\cdot q)\,\gamma^\mu\right]$$

- $A_i^{P\ell_1\ell_2}(p^2, \ldots)$ are the new scalar form-factors

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- $A_i^{P\ell_1\ell_2}(p^2, \ldots)$ are the new scalar form-factors
- The general amplitude then becomes:

$$A(P(p)\to\gamma(k)\ell_1(p_1)\overline{\ell}_2(p_2)) = \sum_i A_i^{P\ell_1\ell_2}(p^2,\ldots)\ \overline{u}(p_1,s_1)\ L_i^{\mu}(p,q,k)\ v(p_2,s_2)\ \epsilon_{\mu}^{*}(k)$$

Hadronic Form Factors

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  - Kinematical invariants i.e. p^2, q^2, $p \cdot k$, etc.
  - 8 linearly independent form-factors
  - In this talk, we will only consider 4 of the 8 form-factors

Hadronic Form Factors

- The scalar functions $A_i^{P\ell_1\ell_2}(p^2,\ldots)$ depend on
  - Kinematical invariants i.e. p^2, q^2, $p\cdot k$, etc.
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- Upon inspecting the four-fermion Lagrangian one finds that the $A_i^{P\ell_1\ell_2}(p^2,\ldots)$ functions should depend on P(p) to γ (k) form-factors:

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\langle\gamma(k)|\overline{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle &= i\sqrt{4\pi\alpha}\ f_A^P\left[k_\mu\left(p\cdot\epsilon^*\right)-\epsilon^*_\mu\left(p\cdot k\right)\right],\\
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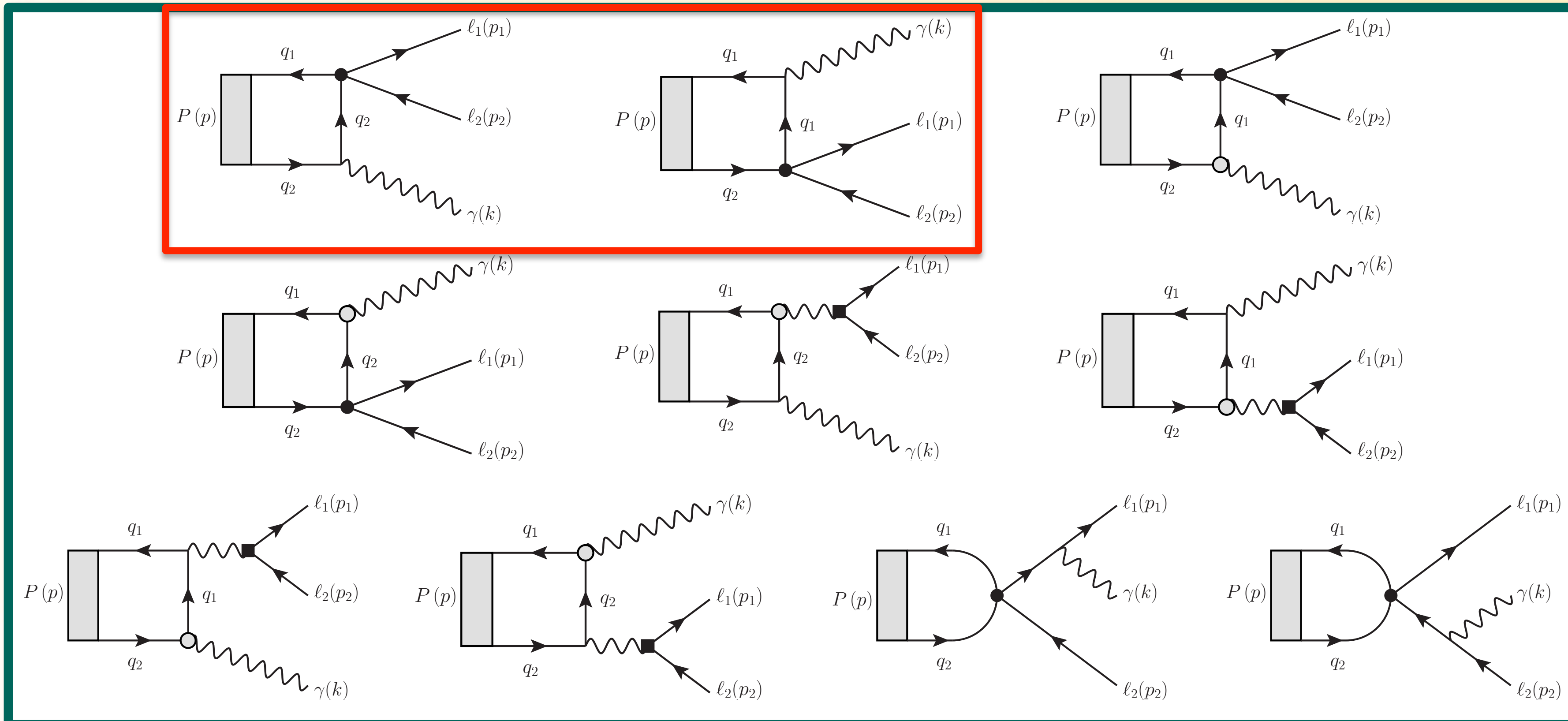
- Let's look at the vector operator contributions using $f_V^P(Q^2)$

Feynman diagrams B,D,K to $\gamma\ell_1\ell_2$



Vector operator contributions for $q_1q_2\ell_1\ell_2$ operators

Vector operator contributions for B_q to $\gamma \ell_1 \ell_2$ ($bq\ell_1\ell_2$ operators)

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A_1^{B_q\ell_1\ell_2} &= \frac{m_{B_q} y}{2\Lambda^2}\sqrt{4\pi\alpha} f_V^{B_q}[m_{12}^2]\left(C_{VR}^{qb\ell_1\ell_2} - C_{VL}^{qb\ell_1\ell_2}\right)\\
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- A_3-A_6 are zero in this case
- Now we can use a vector form factor parameterization from any model as an input for our work.

Vector operator contributions for B_q to $\gamma\ell_1\ell_2$ ($bq\ell_1\ell_2$ operators)

- We use the following parameterization*:

	Parameter	F_V
$B^0_{d,s} \to \gamma$	$\beta\,(\mathrm{GeV}^{-1})$	0.28
	$\Delta\,(\mathrm{GeV})$	0.04

$$F_V^{(B_q)}(E) = \beta \frac{f_{B_q}}{\Delta + E_\gamma}$$

*F. Kruger and D. Melikhov, Phys. Rev. D 67, 034002 (2003)

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- Such constraints could be found by B-factories such as LHCb or Belle 2.

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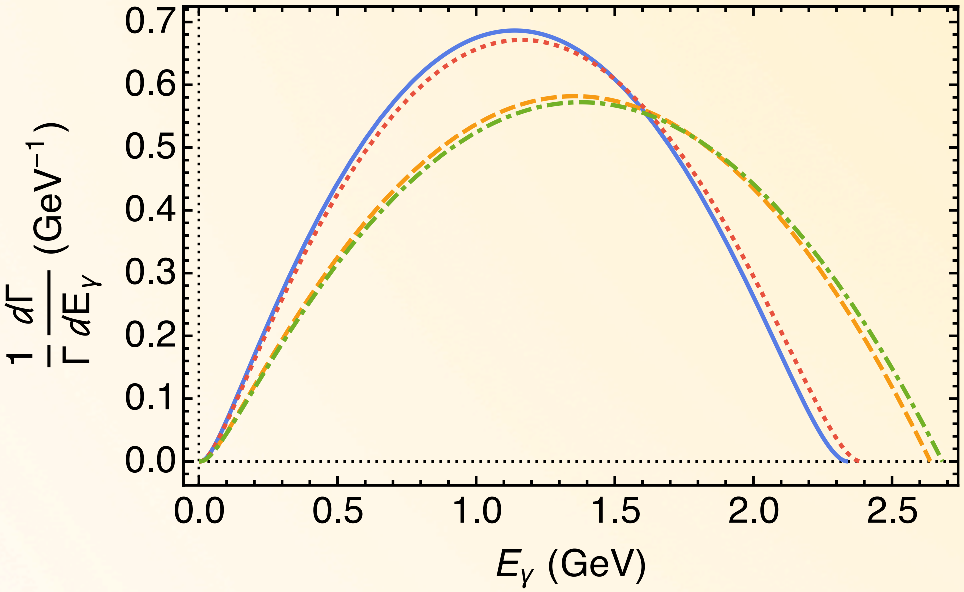
Differential decay rates B_q to $\gamma \ell_1 \ell_2$ ($bq\ell_1\ell_2$ operators)

While we cannot constrain the Wilson coefficients without experimental data, we can look at the differential decay rates:



$B_d \to \gamma\,\mu\,\tau$ or $\gamma\, e\, \tau$ (solid blue curve), $B_d \to \gamma\, e\, \mu$ (short-dashed gold curve), $B_s \to \gamma\,\mu\,\tau$ or $\gamma\, e\, \tau$ (dotted red curve), $B_s \to \gamma\, e\, \mu$ (dot-dashed green curve)

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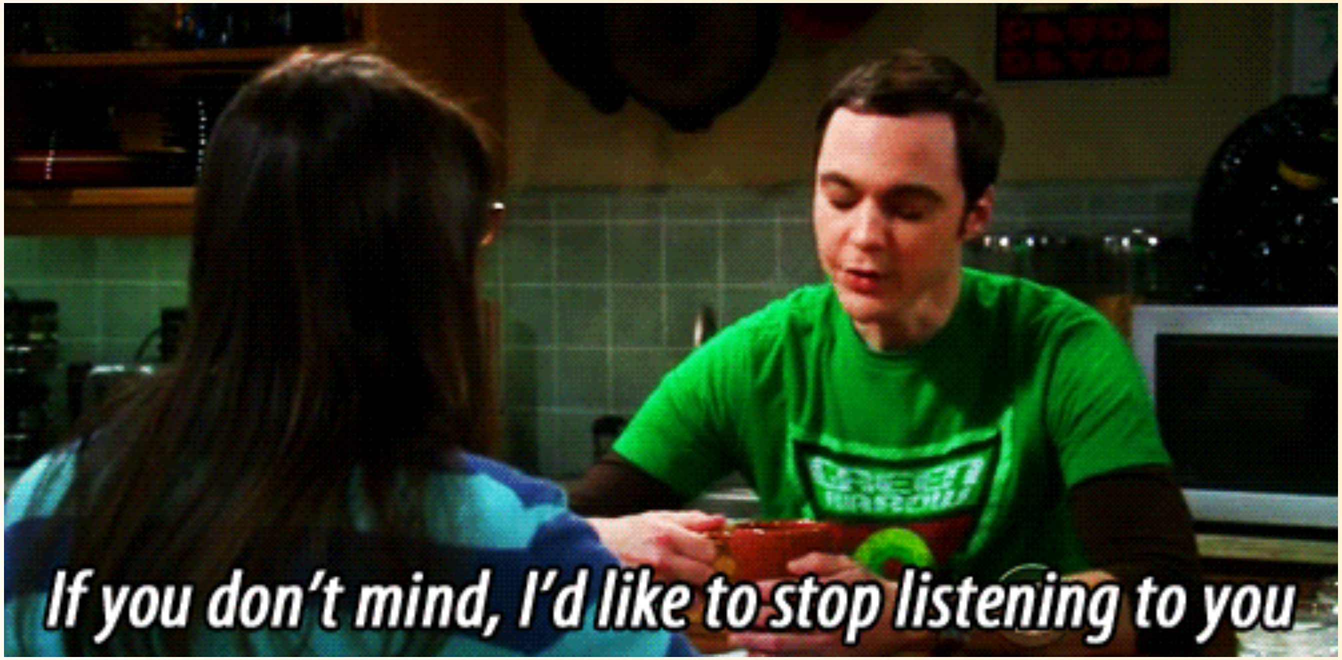
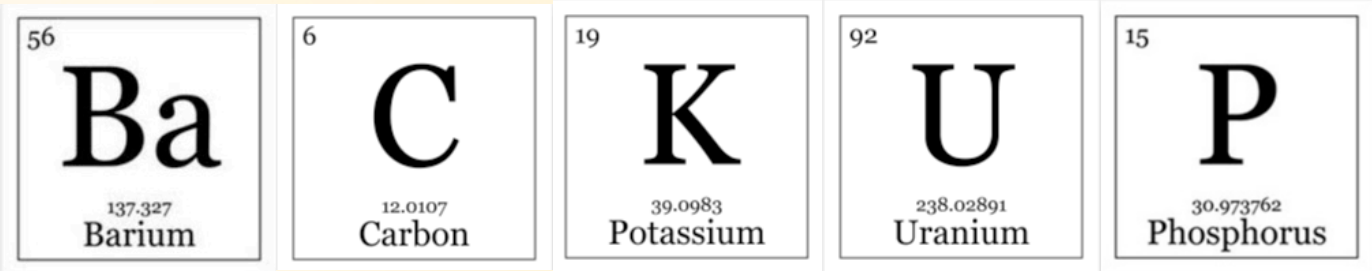
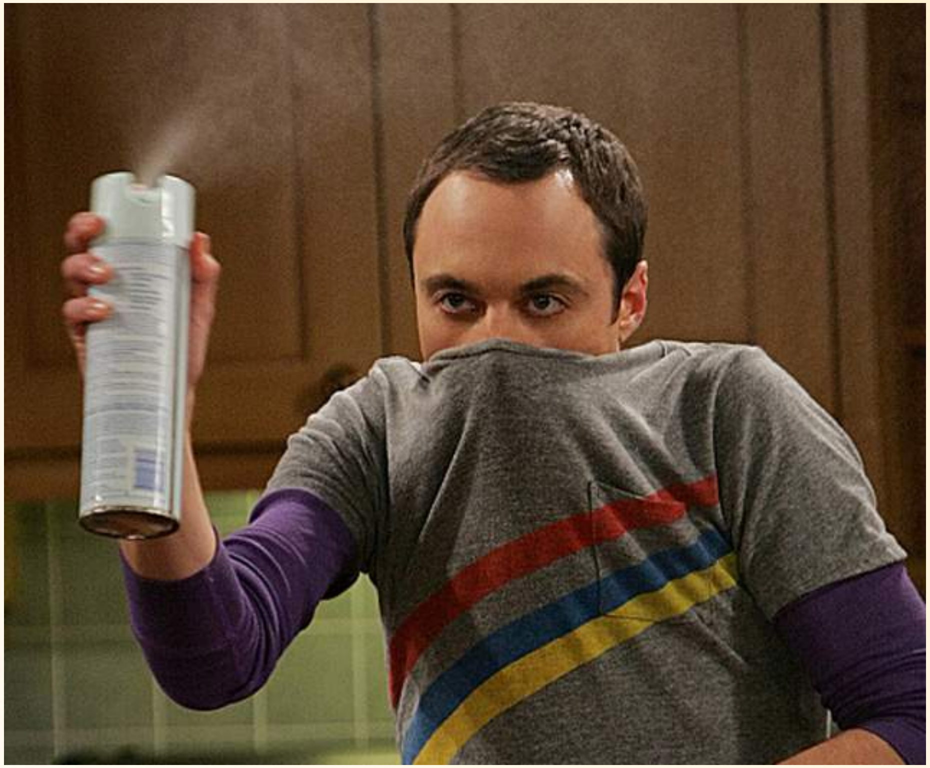
- Calculating three-body decays using form-factors provides a model independent result.
- RLFV decays can provide access to operators that cannot otherwise be seen via two-body decays.


If you don't mind, I'd like to stop listening to you


56
Ba
137.327
Barium
6
C
12.0107
Carbon
19
K
39.0983
Potassium
92
U
238.02891
Uranium
15
P
30.973762
Phosphorus



Form Factors

- There is no scalar or pseudo-scalar operator contribution to the P(p) to γ (k) form-factors.
- Proven by taking the divergence of the vector or axial current

$$
\begin{aligned}
\langle \gamma(k) | \overline{q}_1 \gamma_5 q_2 | P(p) \rangle &= -\frac{1}{m_{q_1} + m_{q_2}} p^\mu \langle \gamma(k) | \overline{q}_1 \gamma_\mu \gamma_5 q_2 | P(p) \rangle \\
&= -\frac{1}{m_{q_1} + m_{q_2}} p^\mu \sqrt{4\pi\alpha}\ f_V^P \epsilon^{*\nu} \epsilon_{\mu\nu\alpha\beta} p^\alpha k^\beta \\
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$$
\begin{aligned}
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