Exercises
(Ann Nelson's Cosmology lectures)

Physical Constants, Units

$$c = 3.00 \times 10^8 \text{m/s}, \quad \hbar = 6.58 \times 10^{-22} \text{MeV} - \text{s}, \quad \hbar c = 197 \text{ MeV} - \text{fm}, \quad 1 \text{ fm} = 10^{-15} \text{m}$$

$$G_N = 6.674 \times 10^{-8} \text{cm}^3 \text{g}^{-1} \text{s}^{-2} = \frac{\hbar c^5}{(1.2 \times 10^{19} \text{GeV})^2} , \quad k_B = 8.6 \times 10^{-5} \text{eV K}^{-1}$$

$$1 \text{ keV} = 10^3 \text{eV}, \quad 1 \text{ MeV} = 10^6 \text{eV}, \quad 1 \text{ GeV} = 10^9 \text{eV}, \quad 1 \text{ TeV} = 10^{12} \text{eV}, \quad 1 \text{ eV} = 1.6 \times 10^{-19} \text{J}$$

$$1 \text{ pc} = 3.3 \text{ Light years}, \quad 1 \text{ Light year} = 9.5 \times 10^{15} \text{m}, \quad 1 \text{ Mpc} = 10^6 \text{pc}$$

$$m_e = 0.510 \text{ MeV/c}^2, \quad m_p = 938 \text{ MeV/c}^2, \quad m_\mu = 106 \text{ MeV/c}^2$$

Lorentz transformation: boost in z direction with velocity βc, $\gamma = 1/\sqrt{1-\beta^2}$

$$E' = \gamma(E - \beta c p_z), \quad p'_z = \gamma(p_z - \beta E/c), \quad p'_x = p_x, \quad p'_y = p_y, \quad E^2 - |\vec{p}|^2 c^2 = E'^2 - |\vec{p}'|^2 c^2$$

Reference: See the excellent online cosmology notes at
http://people.virginia.edu/~dmw8f/astr5630/Topic16/Lecture_16.html

Set 1

[**1**] (multiple choice question) Dimensional analysis is a very useful tool in physics. The trick is to begin by identifying the dimensionful physical constants which are relevant for a problem, determine what units they are measured in (e.g. units of length, units of time, units of energy, mass....). These can include fundamental constants and such as the speed of light and particular constants such as a combination of masses or charges. Then dimensional analysis is the analysis of how any given physical quantity can depend on the dimensionful constants. For instance in the hydrogen atom, the important physical quantities are the reduced mass $\mu = m_e m_p/(m_e + m_p) \approx m_e$, the fine structure constant (dimensionless), Planck's constant: $\hbar$, and the speed of light, c. Use dimensional analysis to find out how if the electron in a hydrogen atom were to be replaced by a muon, which is exactly like an electron except that it is 200 times heavier.

1a) how would the expectation value of the radius change?
A: radius would be approximately 200 times larger
B: radius would not change
C: radius would decrease by factor of about 200
D: radius would decrease by factor of about 200^2

1b) how would the binding energy of the ground state change?
A: Binding energy would be about 200 times larger
B: Binding energy would be about 200^2 times larger
C: Binding energy would be the same
D: Binding energy would be about 200 times smaller

[**2**] (open ended question) As far as we know, Planck's constant: $\hbar$, the speed of light, c, and Newton's constant G_N are fundamental constants of nature. Use dimensional analysis to construct from these constants: a) a mass (the Planck mass) b) an energy (the Planck energy), c) a time (the Planck time), and d) a distance (the Planck length). Express the Planck mass in grams, the Planck energy in GeV, the Planck time in seconds, and the Planck length in centimeters. The tiny sizes of the Planck length and Planck time and huge sizes of the Planck mass and Planck energy (relative to typical particle physics sizes) are striking. (Open ended question) Explain why and in what situations particle physicists can justify ignoring gravity for non-cosmological situations.

[**3**] The Friedman- Robertson-Walker metric for flat space is, In units where the speed of light is set to 1:

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$$

where $a(t)$ is known as the scale factor and s is the interval. When s^2 is negative $|s|$ is known as the proper time (time elapsed according to a watch being carried along a space-time trajectory) and when positive (as is the case for $dt = 0$) $|s|$ is known as the proper distance. Note that all observers agree on the interval–it is an observer-independent quantity, also called a relativistic invariant. We can imagine space time points as being specified by "co-moving coordinates" which are embedded in space time while the metric which gives physical intervals is dynamic. According to this metric the physical volume of any region of space which is specified by co-moving coordinates changes with time as a^3. By convention we use units with $c = 1$ and take $a = 1$ today. Consider some source of energy and momentum which fills space which has equation of state $\epsilon = -wp$ where ϵ is the energy density, w is a constant known as the equation of state parameter, and p is the pressure. For example, cold noninteracting particles have $w = 0$, radiation has $w = 1/3$, and a cosmological constant has $w = -1$ (negative pressure for positive energy density). Recall the thermodynamic relation which expresses the conservation of energy: $dE = -pdV$, where E is the total energy in some volume V. Assuming this thermodynamic relation applies (it does) to expanding space, find out how E, the total energy in some coming volume, and ϵ, the energy density, depend on the scale factor a, as a function of w.

[4] (open ended question) In thinking about the results of your previous question, discuss in what sense energy is conserved in General relativity and in what sense it is not conserved.

End of set 1.