Condensed Matter Physics

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1. An electron moves in a solid, consisting of $N = 10^{23}$ atoms arranged on a perfect lattice. In the nearly-free electron model, the energy spectrum has gaps when the momentum takes special values on the edge of the Brillouin zones.

What determines the edges of the Brillouin zones?

- The size of the potential $V(\mathbf{x})$ that the electron feels.
- The size and shape of the lattice.
- Both of these.

What determines the size of the gaps in the energy spectrum?

- The size of the potential $V(\mathbf{x})$ that the electron feels.
- The size and shape of the lattice.
- Both of these.

How many quantum states are there in the first Brillouin zone?

- 1
- 10^{23}
- 2×10^{23}
- ∞

2. In the tight-binding model, an electron sitting on the n^{th} atom has quantum state $|n\rangle$. Its dynamics are described by the Hamiltonian

$$H = E_0 \sum_n |n\rangle\langle n| - t \sum_n \Big(|n\rangle\langle n+1| + |n+1\rangle\langle n| \Big)$$

Use the general state $|\psi\rangle = \sum_m \psi_m |m\rangle$ to show that the Schrödinger equation $H|\psi\rangle = E|\psi\rangle$ can be written as

$$E_0\psi_n - t(\psi_{n+1} + \psi_{n-1}) = E\psi_n$$

Show that this is solved by $\psi_n = e^{ikna}$, with the energy E depending on the momentum k by

$$E(k) = E_0 - 2t\cos(ka)$$