Palestinian Advanced Physics School

Condensed Matter Physics

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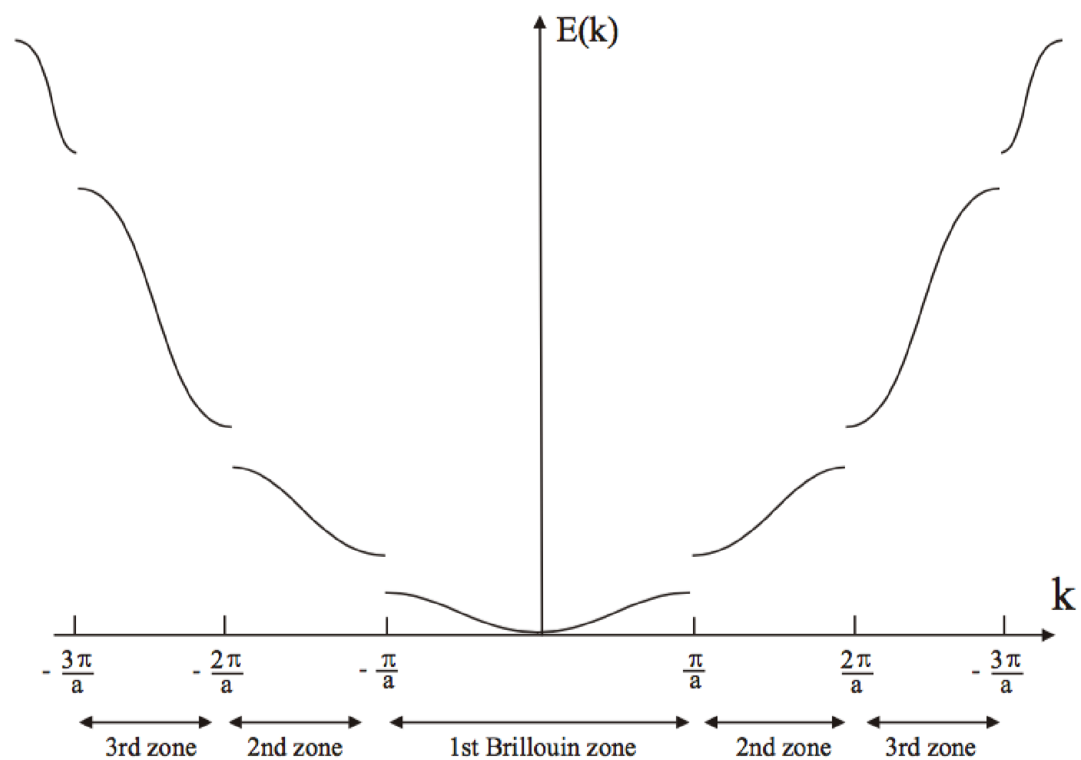
Lecture 2: Fermi Surfaces

Review of Lecture 1

In the last lecture, we looked at a single electron moving in a lattice. The energy spectrum forms a *band structure* with gaps.



In this lecture, we look at what happens when we have many electrons.

Fermi Surfaces

Let’s first think about a Fermi surface without a lattice

Consider a single electron in a box of size *L.* The energy is

$$E = \frac{\hbar^2}{2m} \sum_{i=1}^{3} k_i^2 \qquad \text{with} \qquad k_i = \frac{2\pi n_i}{L} \qquad n_i \in \mathbf{Z}$$

Make some assumptions

- Electron has two spin states: $|\uparrow\rangle$ and $|\downarrow\rangle$
- Electrons do not interact with each other

We also need a key principle of physics: the *Pauli exclusion principle*

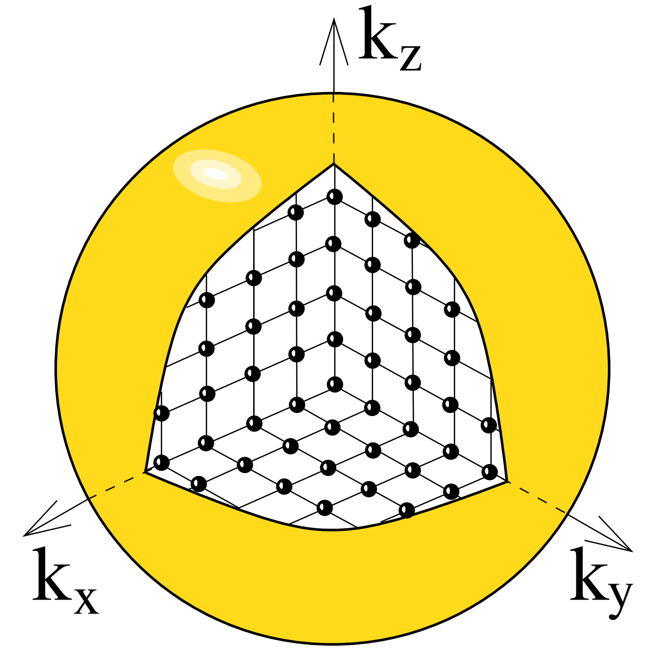
- No two electrons can occupy the same state

Fermi Surfaces

Now we throw N electrons into the box. (With $N=10^{23}$ roughly). What is the ground state?

- The first electron has $k=0$ and, say, spin $|\uparrow\rangle$
- The second electron has $k=0$ and spin $|\downarrow\rangle$
- The next electron must have non-zero momentum. We can set $k=2\pi/L$.
  - There are six states with this energy (three directions; spin up or down)
- Now we keep going. As we throw in more electrons, they have to have higher momentum and higher energy.

The electrons fill out a ball in momentum space



Fermi Surfaces

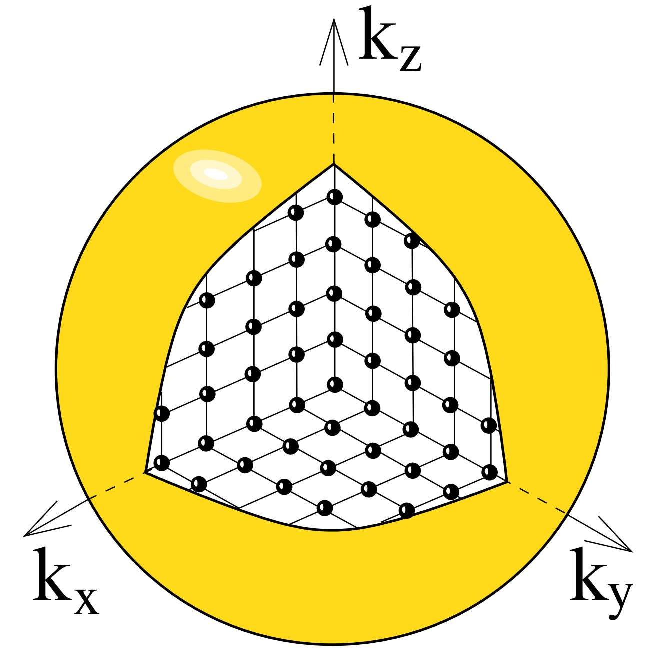
Everything in the Fermi surface is called the *Fermi-something*

- This inside is the *Fermi sea*
- The edge is the *Fermi surface*
- The electrons on the edge have *Fermi momentum*, k_F
- The energy of this last electron is the *Fermi energy* $$E_F = \frac{\hbar^2 k_F^2}{2m}$$

<u>An important fact:</u>

Only the electrons near the Fermi surface can contribute to any dynamical process.

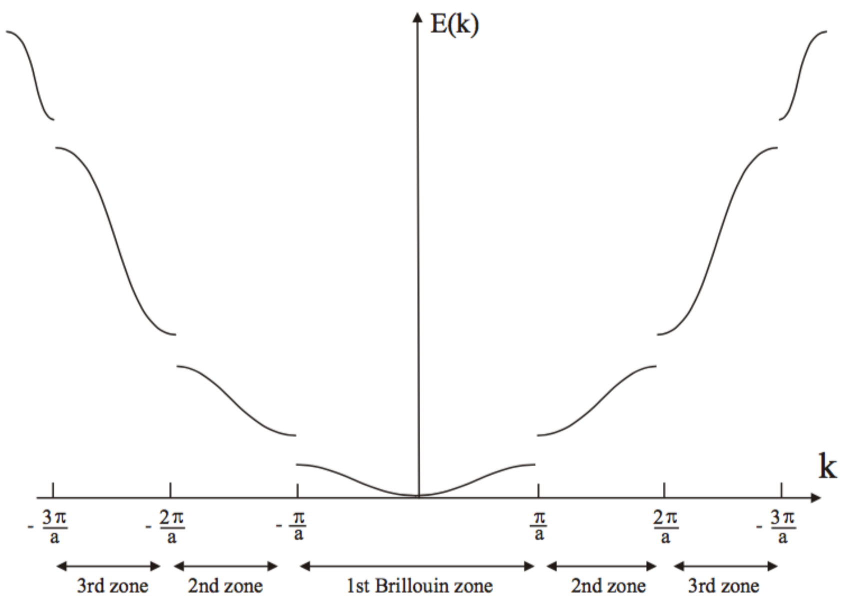
All others are trapped.



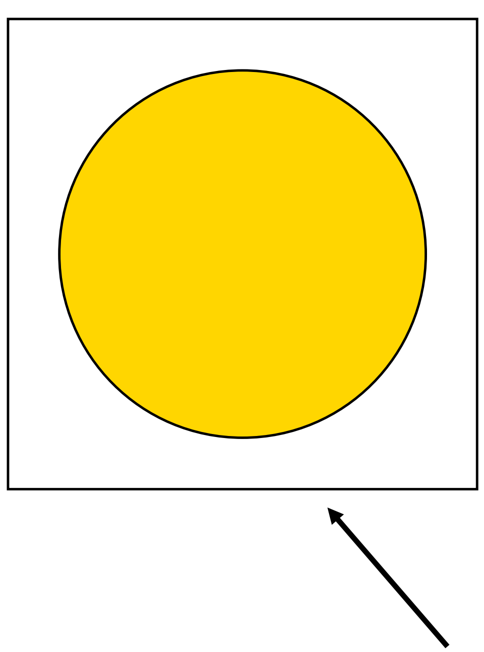
Fermi Surfaces with a Lattice

How does this change in a solid where there is a lattice?

- The energy spectrum for a single electron forms bands.
- The bands sit within Brillouin zone



Let's consider electrons moving in two-dimensions. Throw in electrons and they form a Fermi surface



edge of Brillouin zone

Fermi Surfaces with a Lattice

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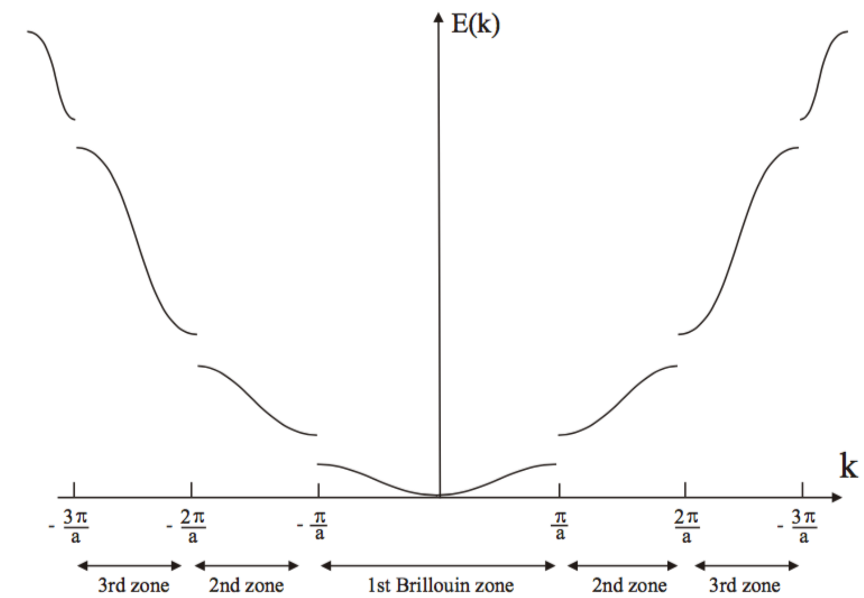

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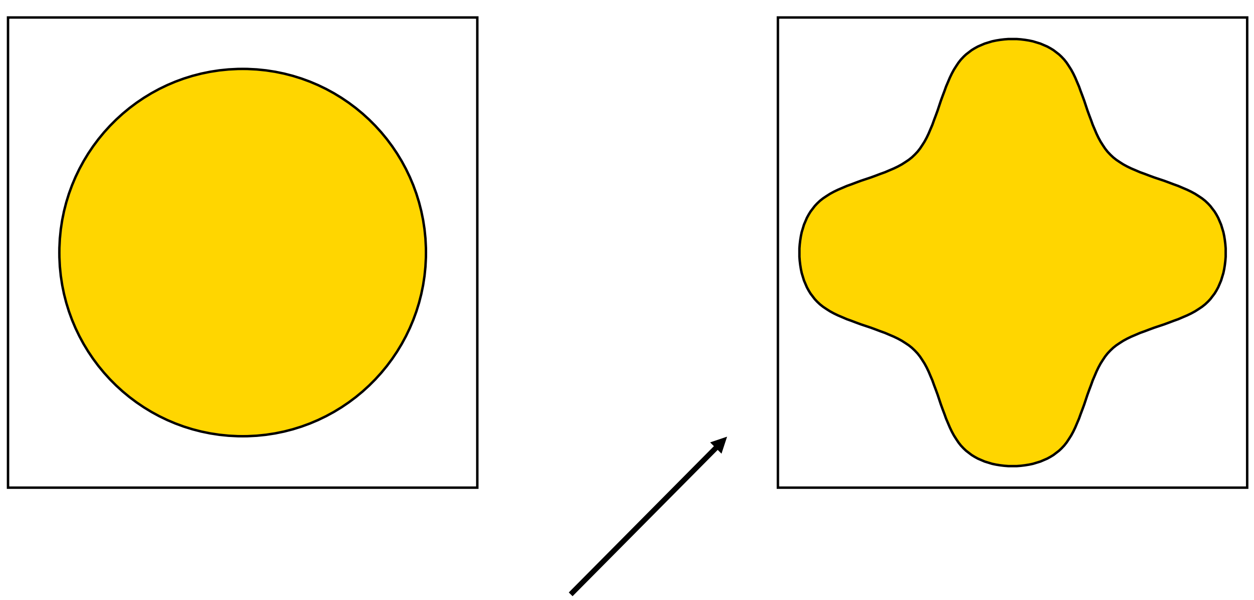

As the lattice gets stronger, the energy gets pushed down near the edge of the BZ.

Fermi Surfaces with a Lattice

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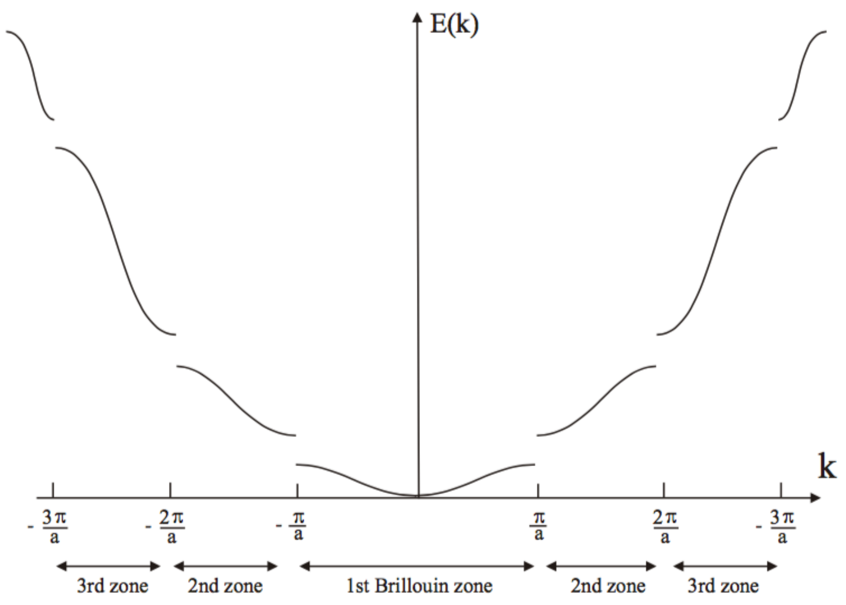

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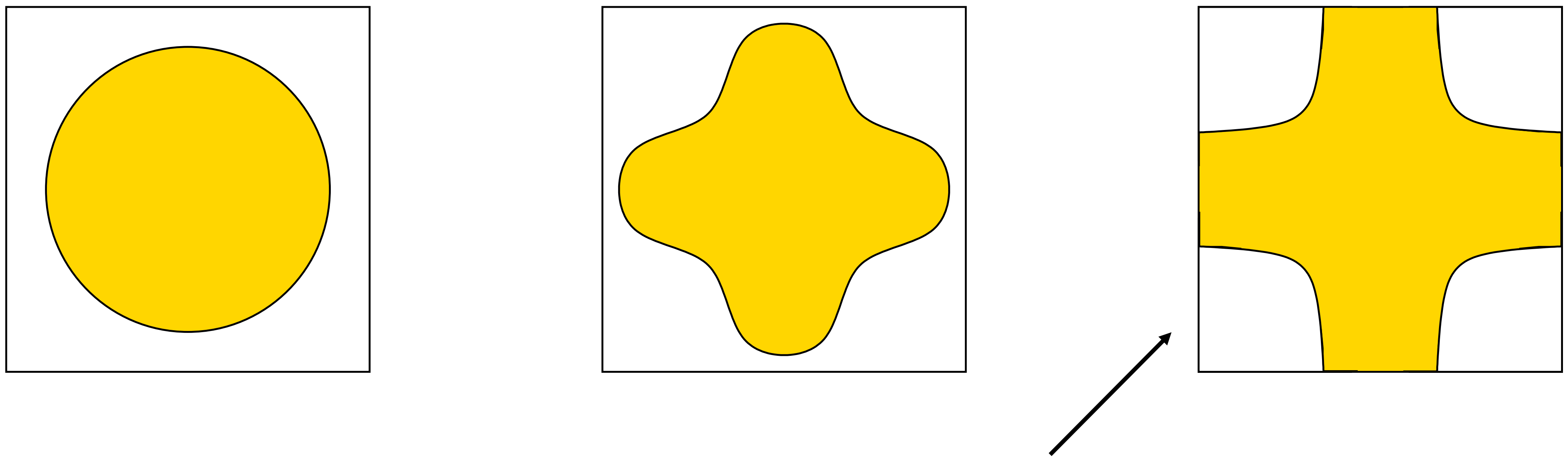
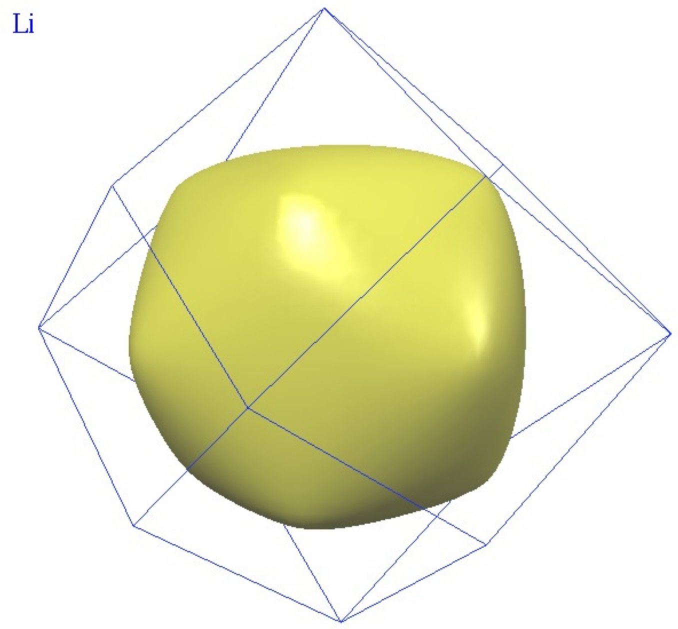
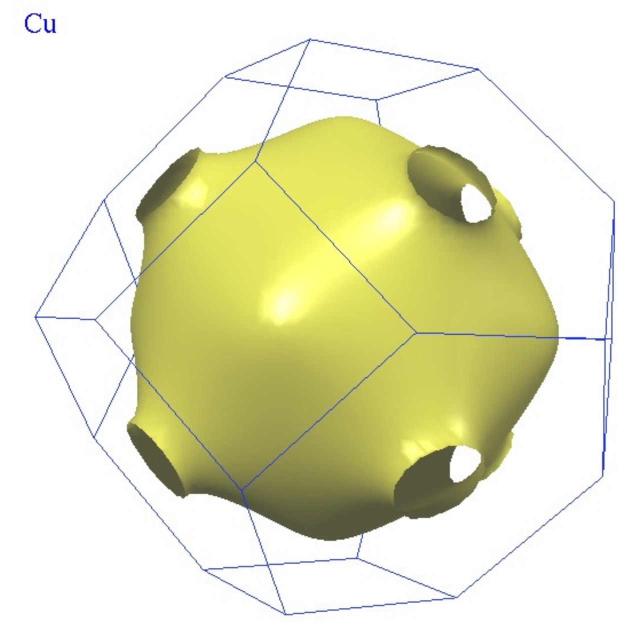


As the lattice gets very strong it can distort the Fermi surface to touch the edge of the BZ.

Real Fermi Surfaces



Lithium

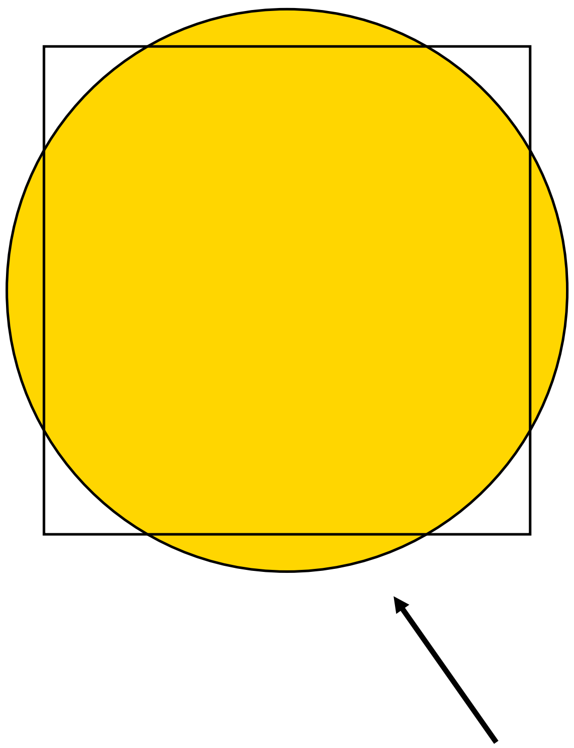


Copper

http://www.phys.ufl.edu/fermisurface/

Fermi Surfaces with a Lattice

What happens if we keep adding electrons?



With no lattice, the electrons spill over into the next Brillouin zone

Fermi Surfaces with a Lattice

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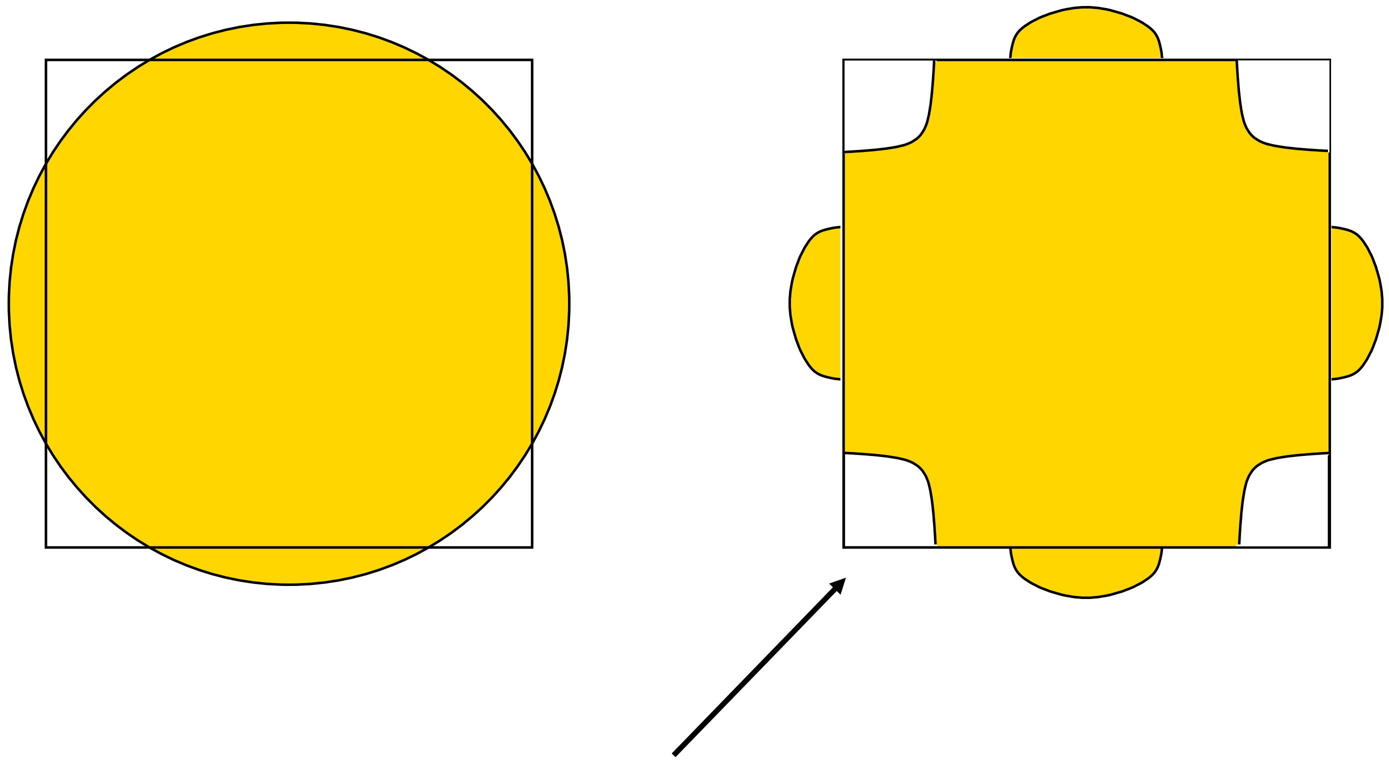
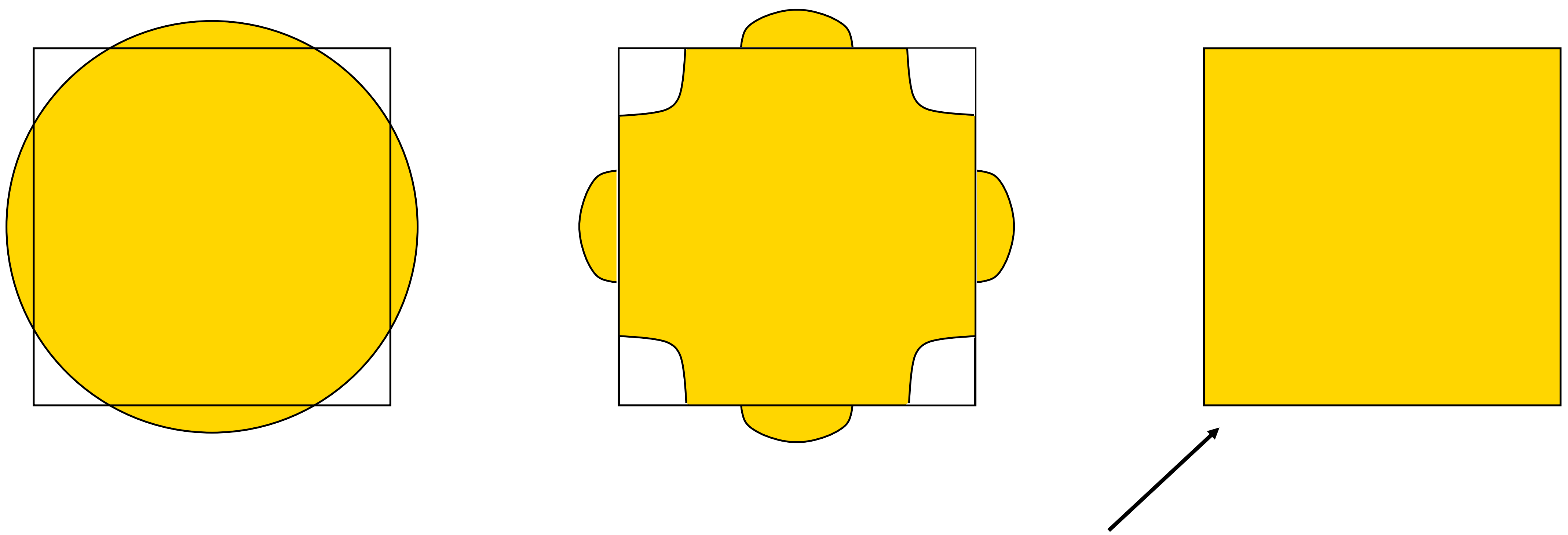


The lattice distorts the energy spectrum, creating a gap at the BZ. If the gap is small, the electrons still spill over into the second BZ

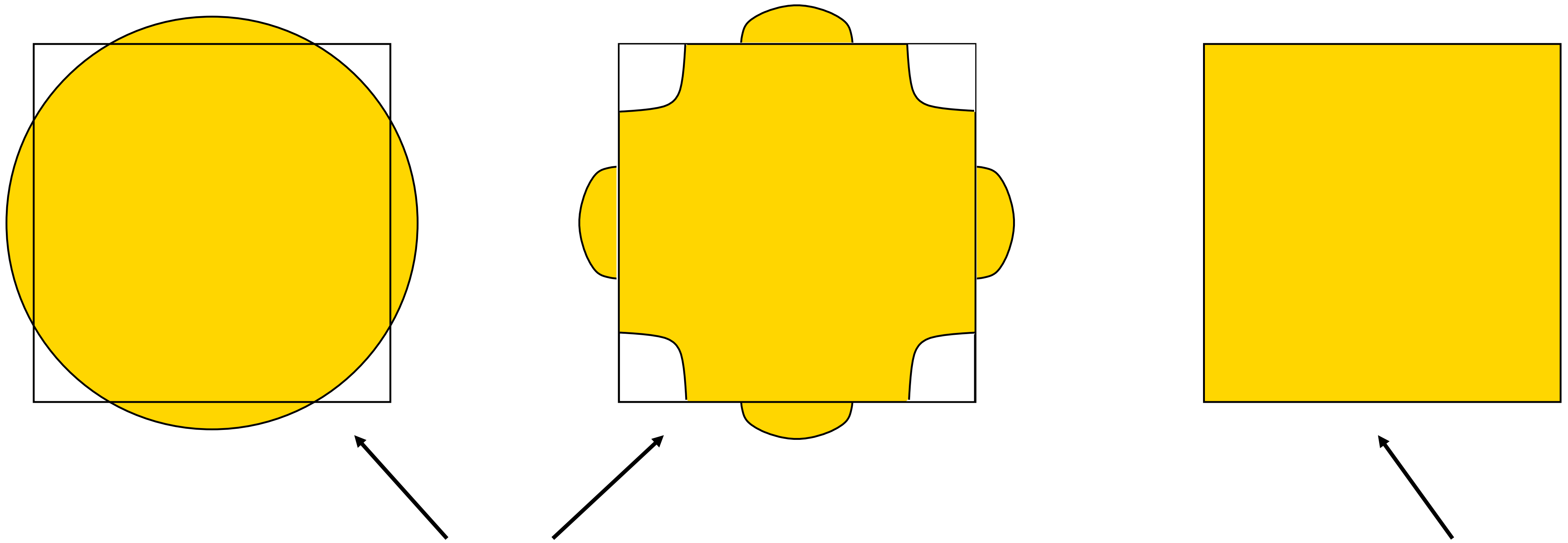
Fermi Surfaces with a Lattice

What happens if we keep adding electrons?



But if the lattice is strong, then the electrons fill up all of the first BZ and none of the second BZ.

Fermi Surfaces with a Lattice



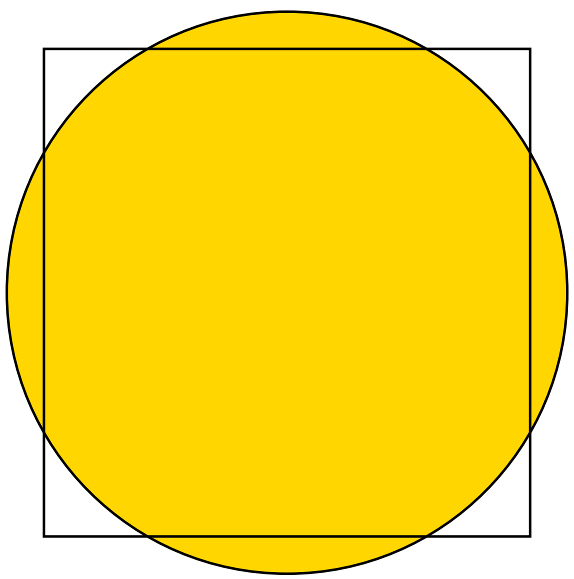
These two cases are similar. If we perturb the system a little bit, the electrons can shift into new states at very little cost of energy,

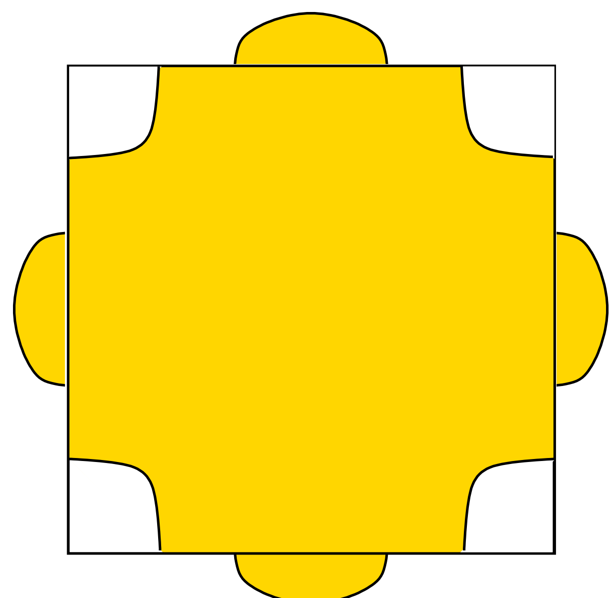
This is very different. If we perturb the system, the electrons can move. There is an energy gap to the next available state

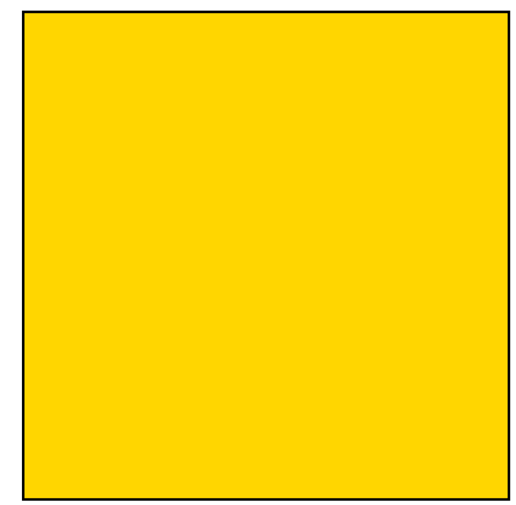
Fermi Surfaces with a Lattice

These are *metals*

This is an *insulator*







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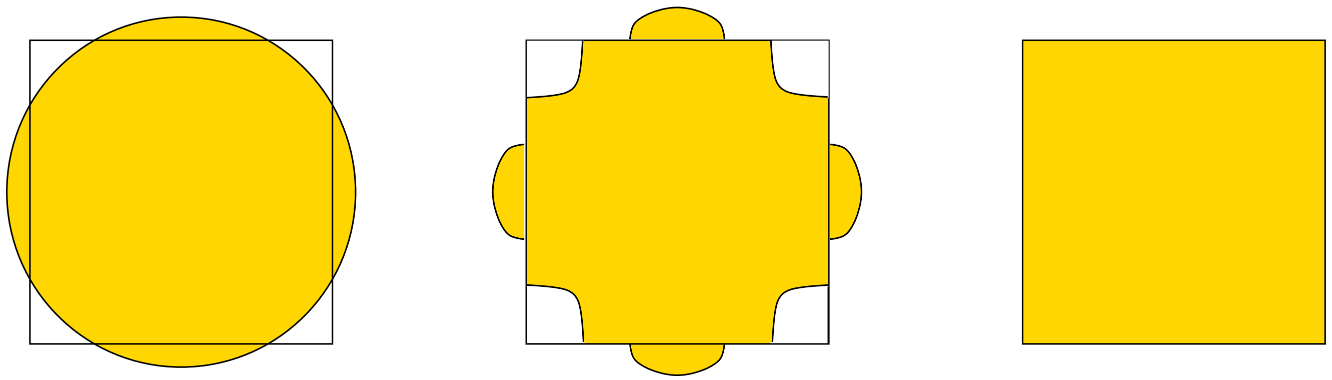
Fermi Surfaces with a Lattice

To make an insulator, we have to fill one Brillouin zone exactly, with no spillover.



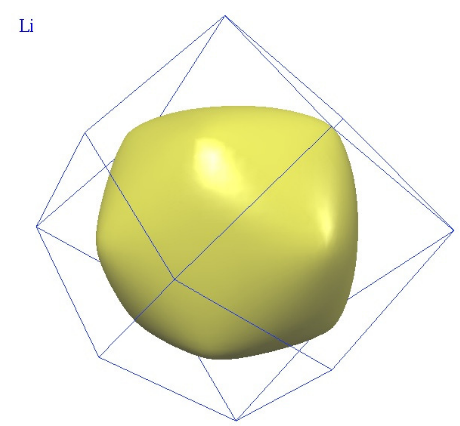
Recall:

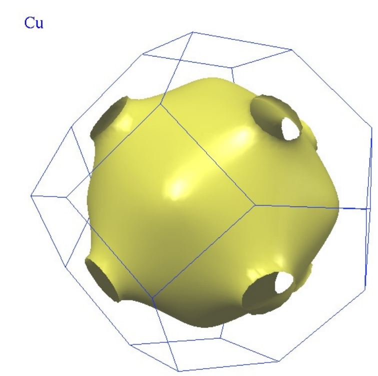
- Each Brillouin zone contains *N* states. (where *N* is the number of atoms)
- Each electron has two spin states, up and down
- So each Brillouin zone can accommodate *2N* electrons

Metals vs Insulators

- Each atom donates some number *Z* of electrons which wander through the materia; with Z an integer. This is called the valence of the atom. The number of electrons in a solid is *NZ.*

- For *Z=1*, the first BZ must be exactly half-filled. Any element with *Z=1* is a metal!
  - e.g. Lithium, Copper





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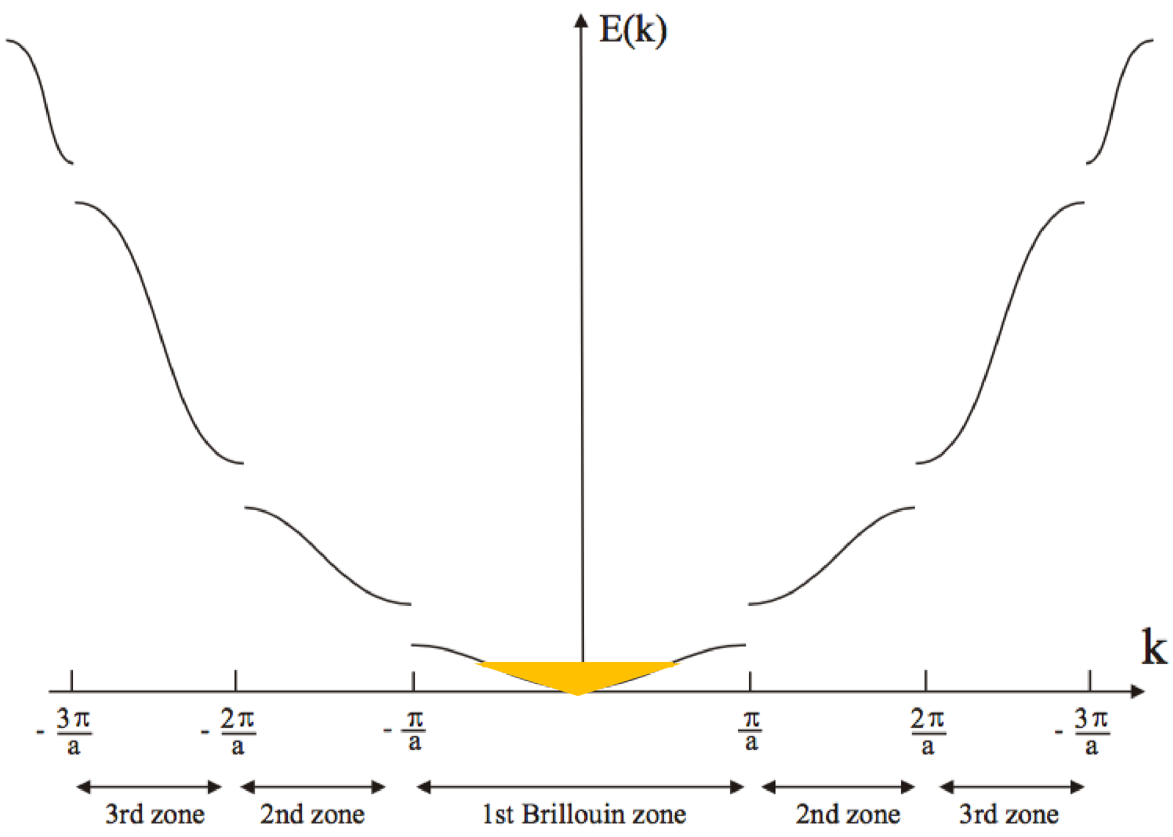
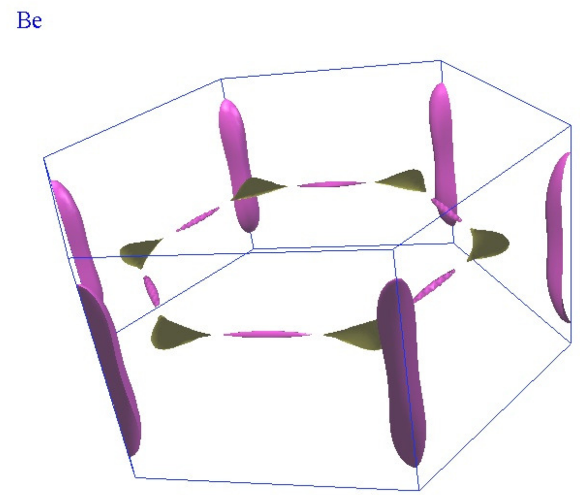


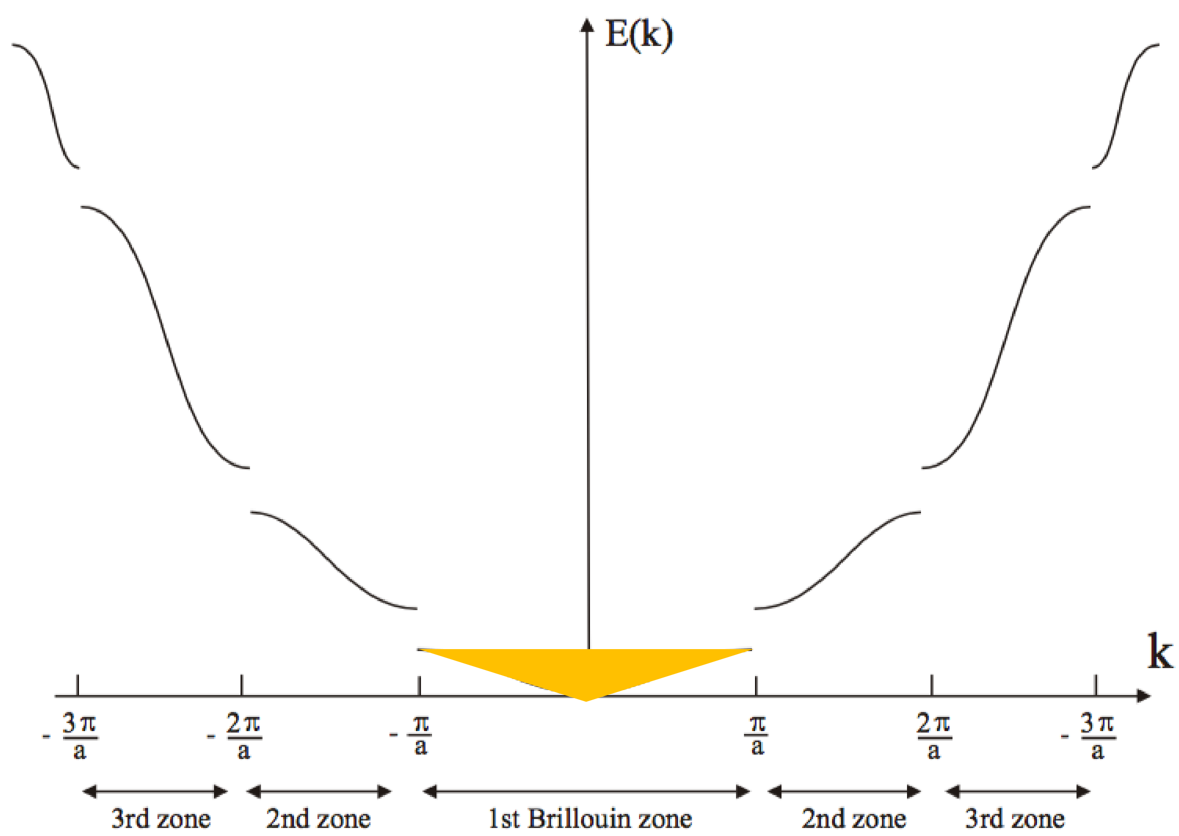
Metals vs Insulators

- For $Z=2$, with a weak lattice, there will be some spillover. Some elements with $Z=2$ are metals.
  - e.g. Beryllium



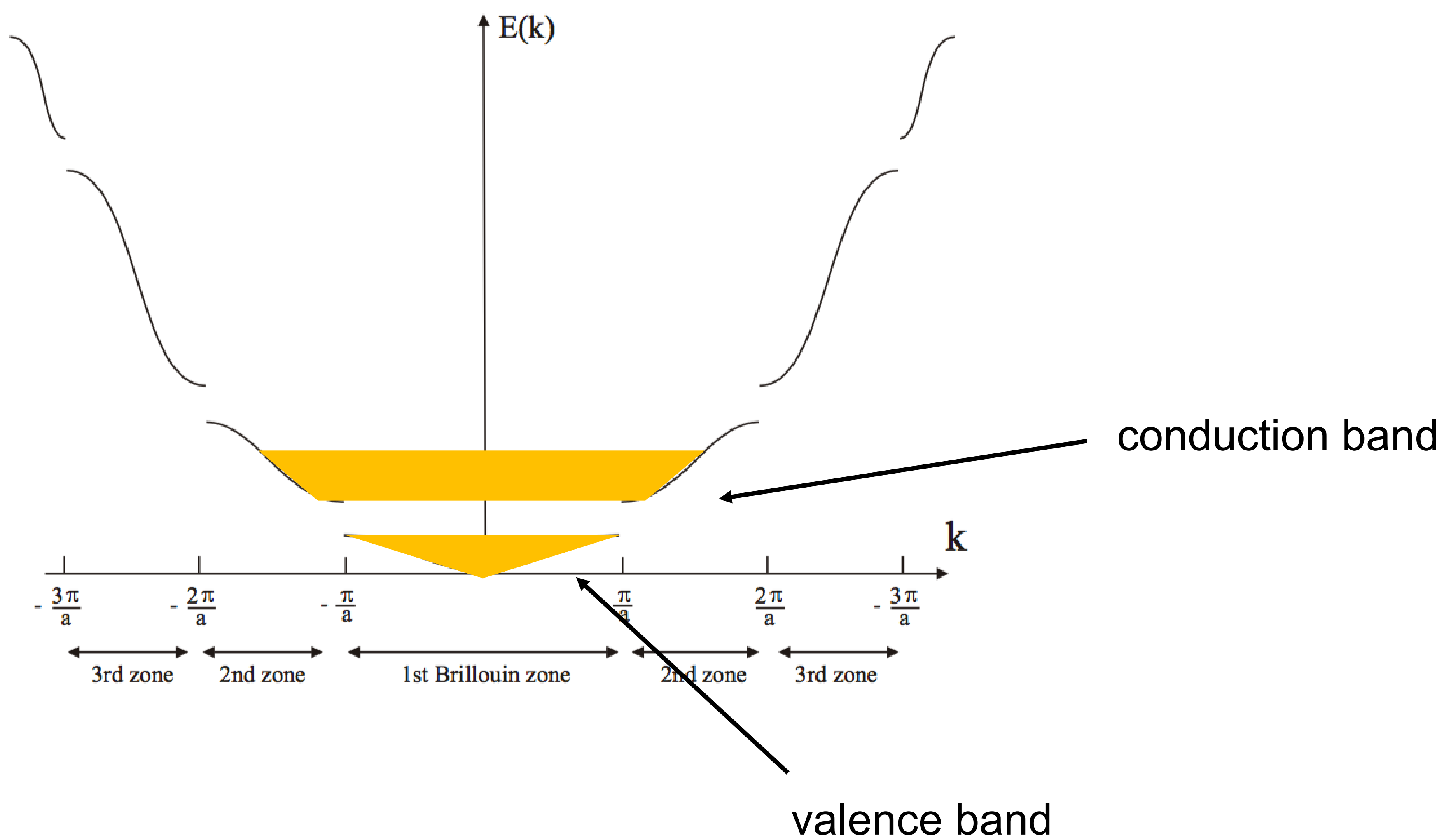
Metals vs Insulators

- For *Z=2*, with a strong lattice, the first BZ will be exactly filled with no spillover. This is an insulator.



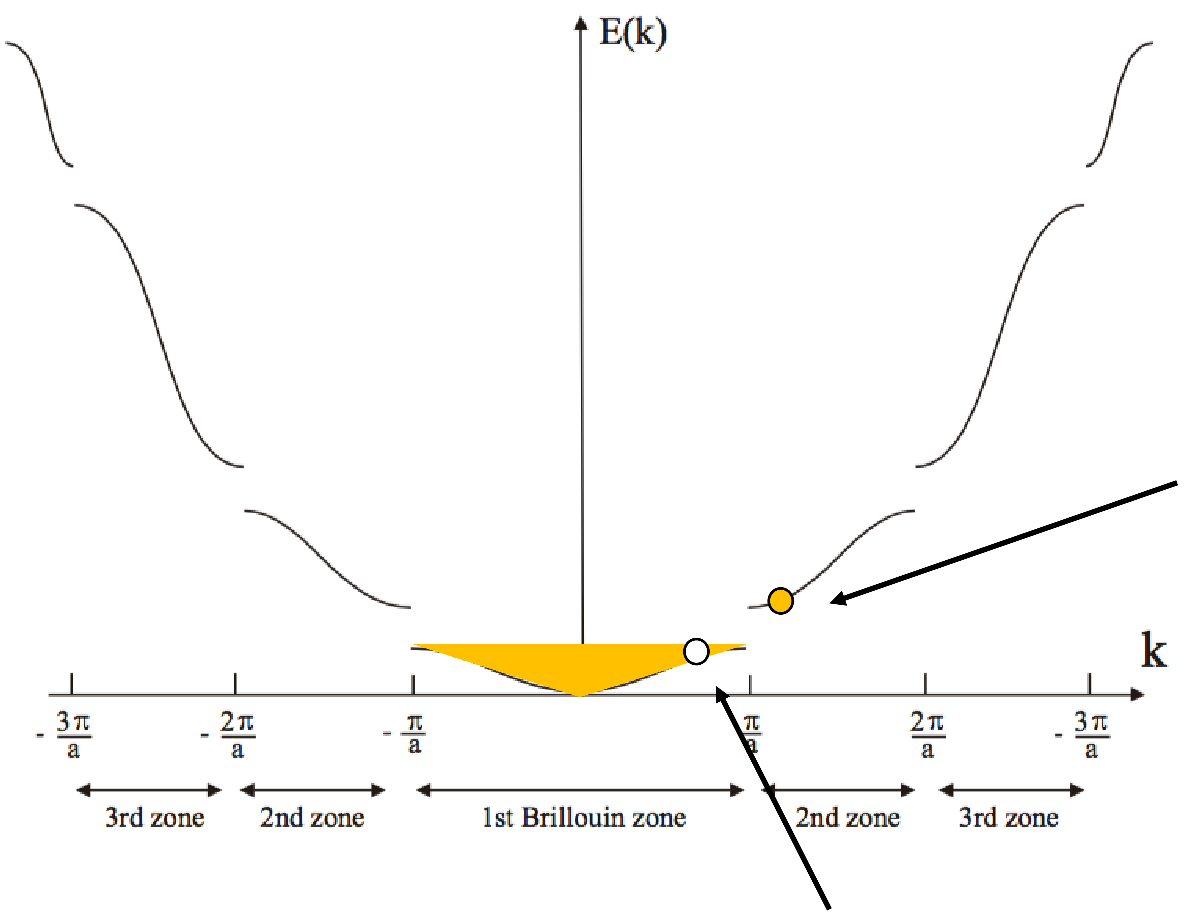
Metals vs Insulators

- For *Z=3* we get a metal again.



Electrons and Holes

Take an insulator, and inject enough energy to move an electron from the valence band to the conduction band.



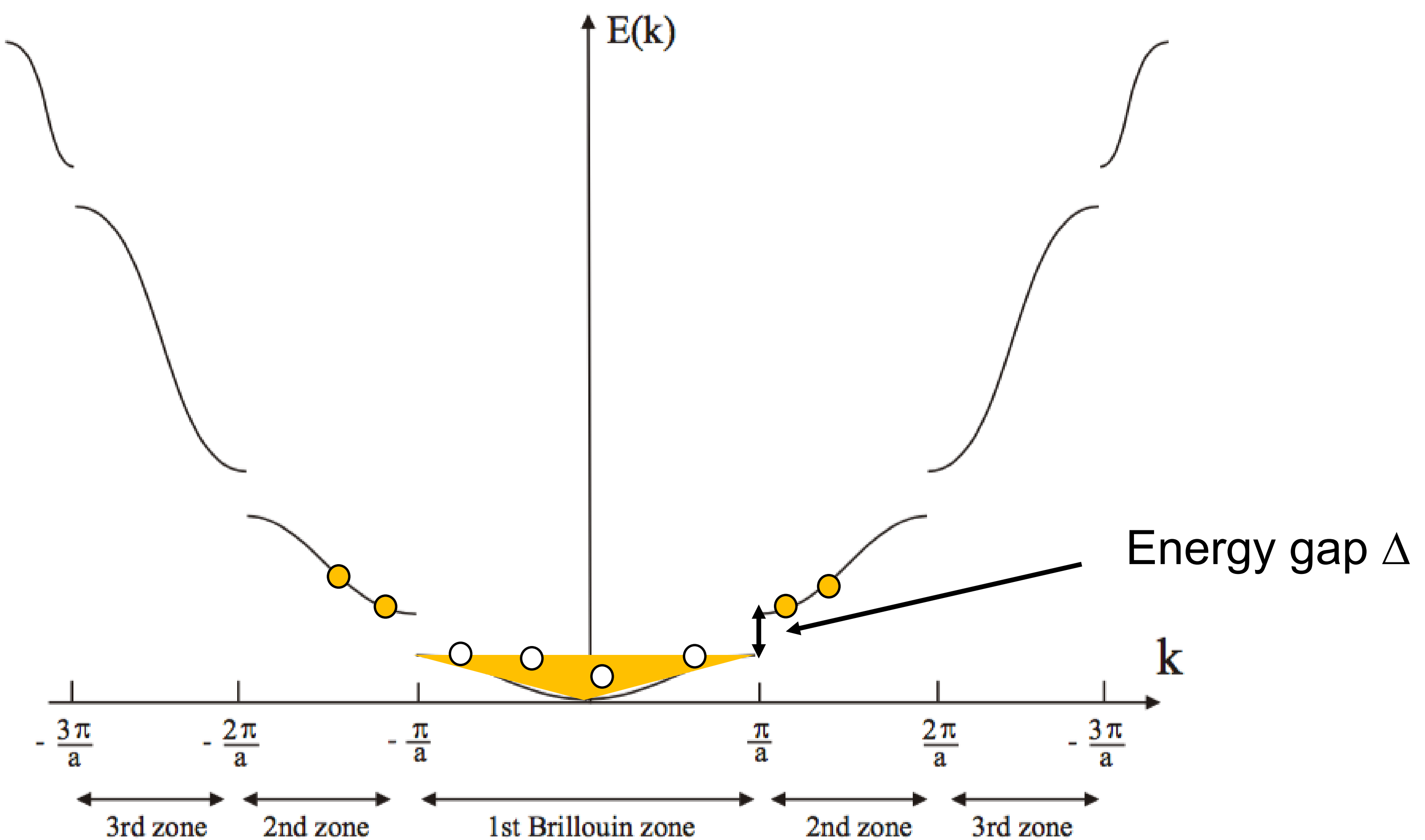
This is an electron which is free to move around

This is an absence of an electron, which is also free to move around. We call it a *hole*. It acts like a particle with positive charge.

The *hole* is like an anti-particle! (Although it is not related to special relativity.)

Semiconductors

- A semiconductor is an insulator, but with a small energy gap



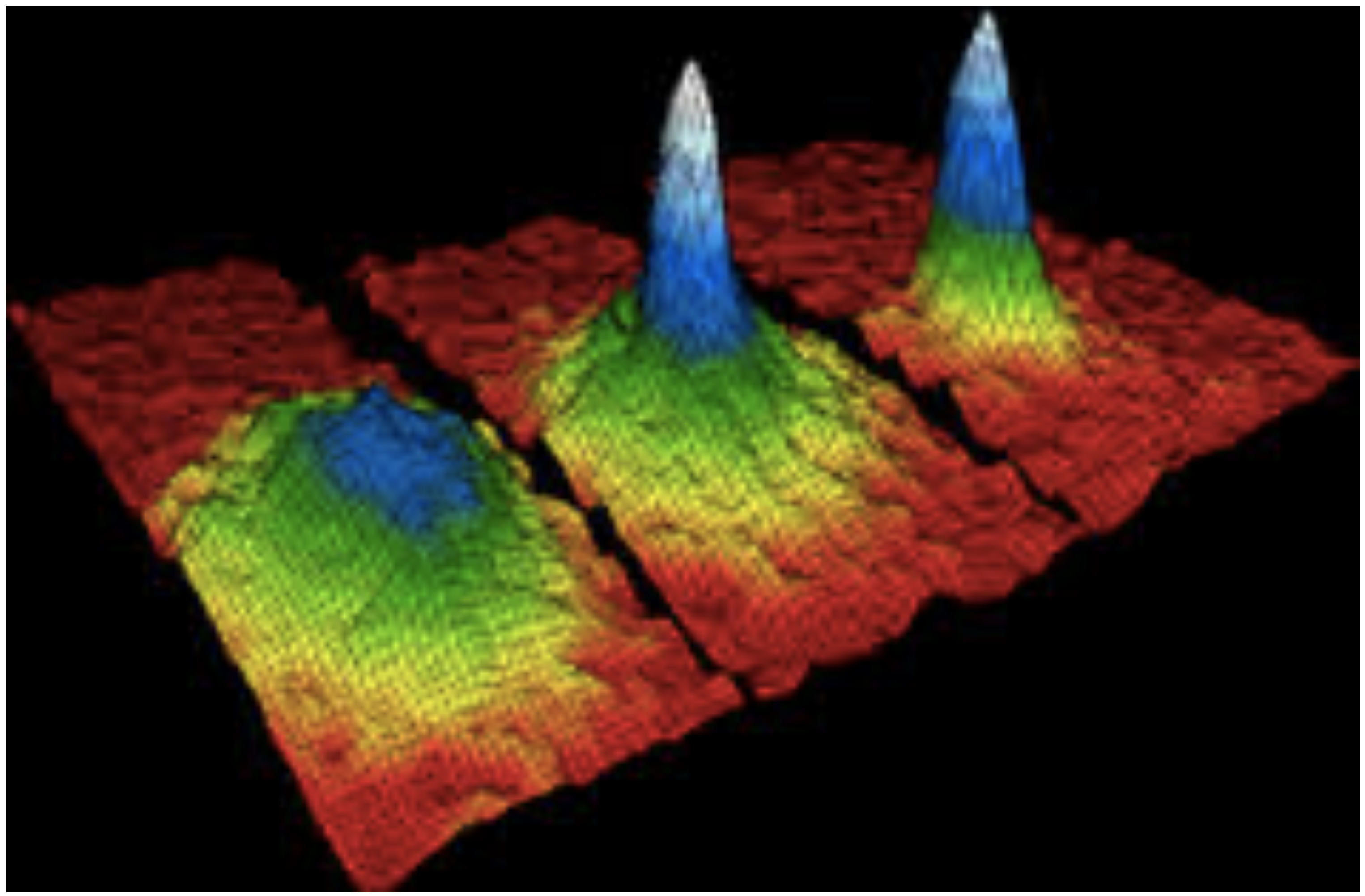
If the temperature T is comparable to the gap, $k_B T \sim \Delta$, then electrons get excited into the conduction band, leaving behind holes. This is a semi-conductor.

Summary

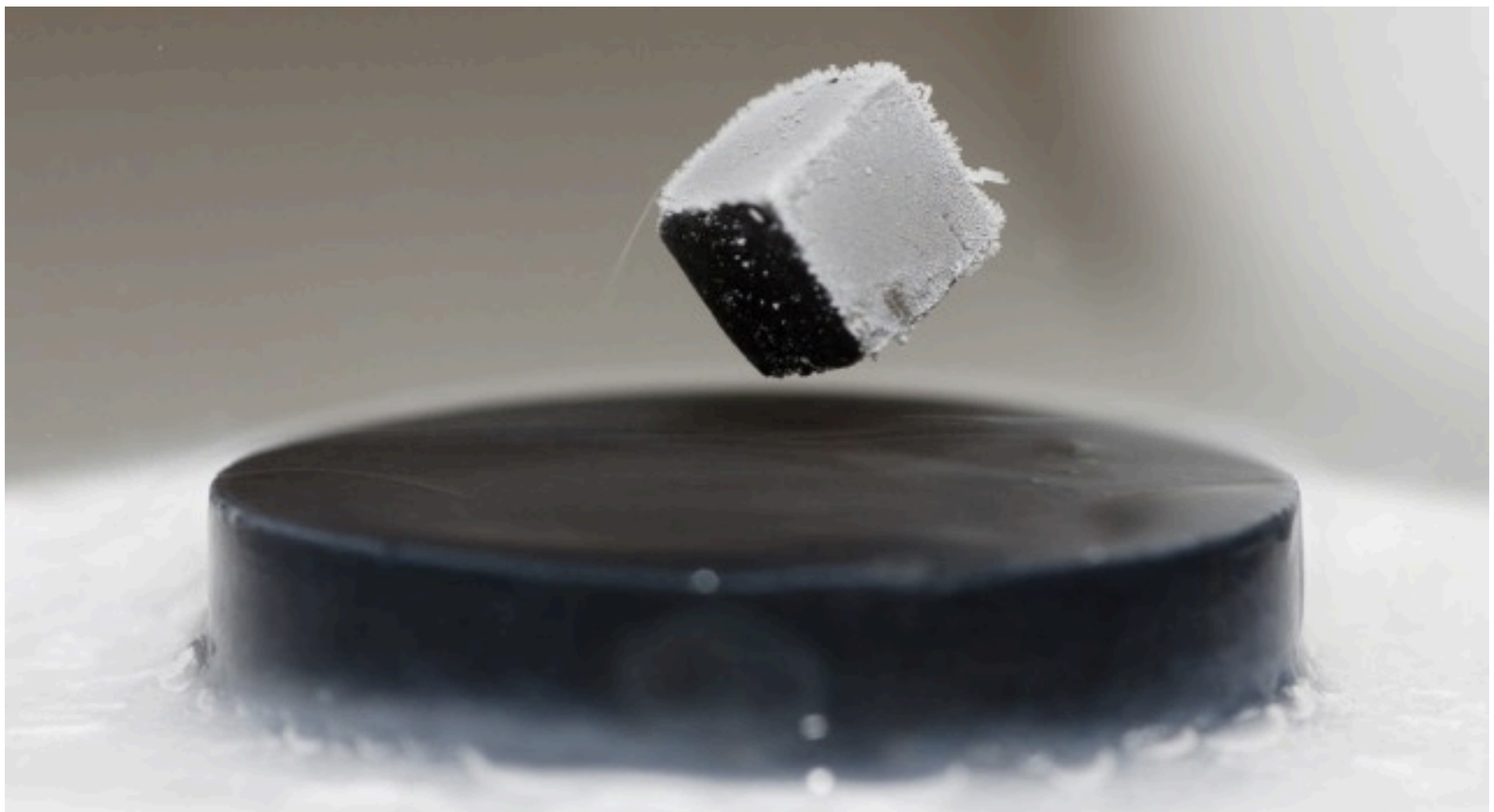
- Lattice + Quantum Mechanics gives band structure
- Pauli exclusion principle gives metals and insulators

Lots of very interesting things still to discuss, and even more still to discover

Bose-Einstein Condensates

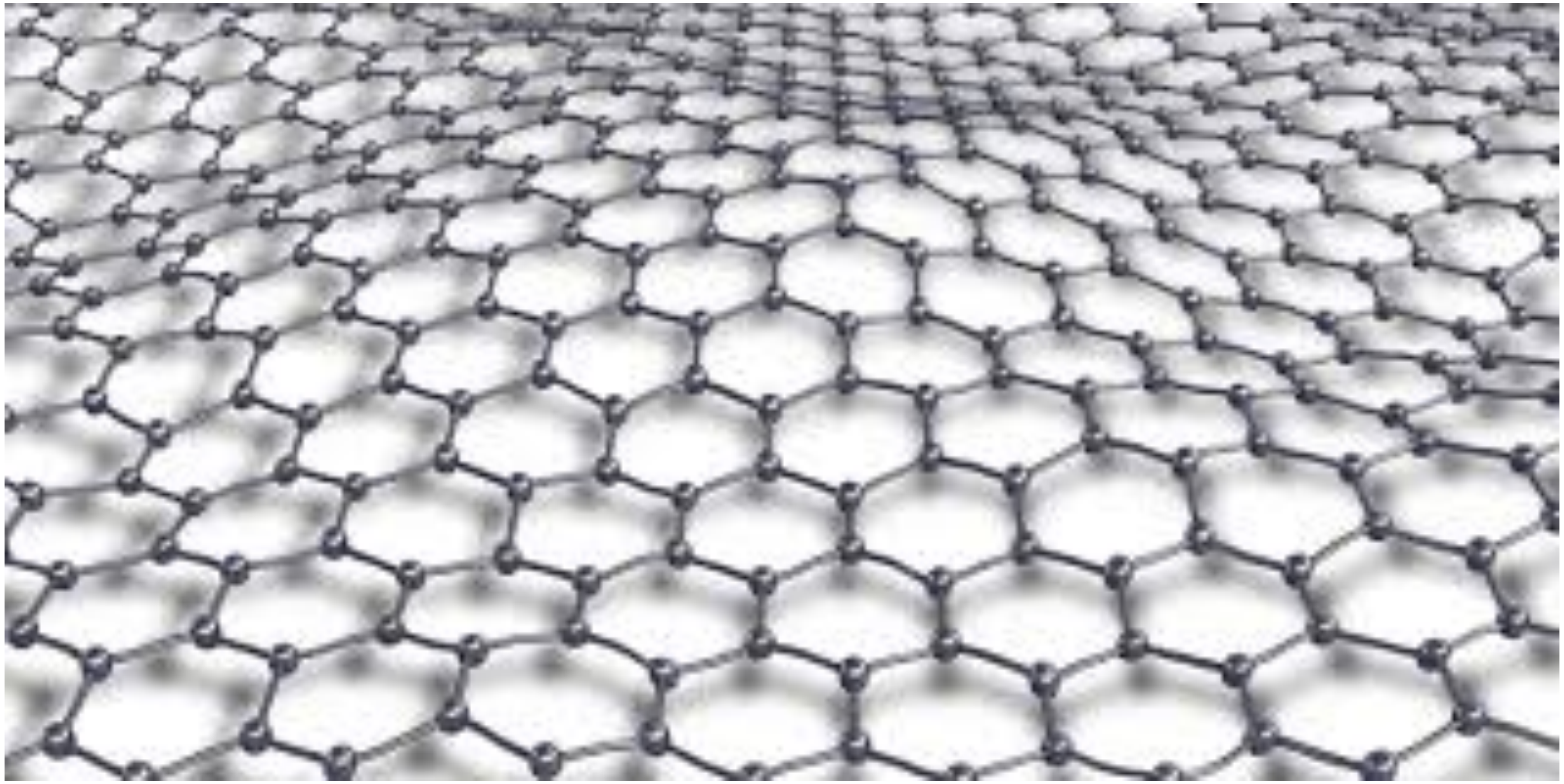


Superconductors

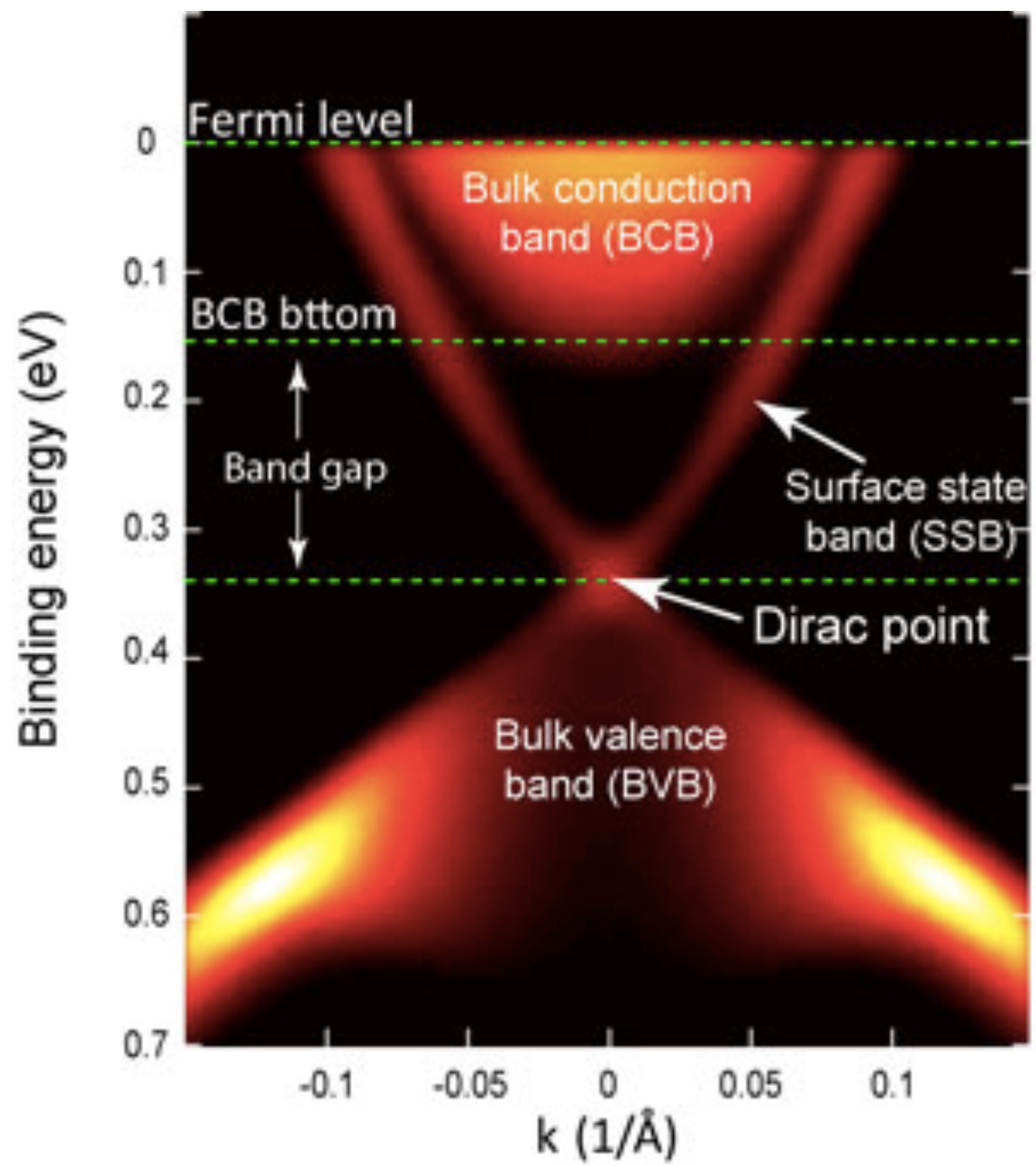


- Conventional superconductors
- High temperature superconductors

Graphene



Topological Insulators



Quantum Hall Effect