The physical spectrum of theories with a Brout-Englert-Higgs effect

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with Axel Maas and René Sondenheimer

University of Graz

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[1709.07477 and 1804.04453]

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FWF

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- Not obvious that W/Z, Higgs, fermions, etc., are physical particles from theoretical p.o.v.
  - Does not matter in the SM
  - Can matter in BSM theories
- Problems can be treated using gauge-invariant perturbation theory [Maas, 1712.04721 ← Review]

Weak-Higgs sector of SM - Basics

- Consider bosonic weak-Higgs sector of SM

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} + \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right) - V\left(\phi^\dagger \phi\right)$$

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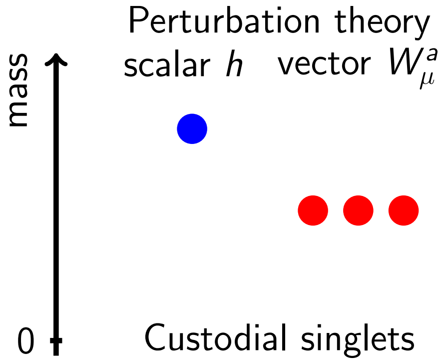
  - Masses of Higgs, W/Z, depend on vev
  - Perform PT (small fluctuations φ)

Weak-Higgs sector of SM - Spectrum



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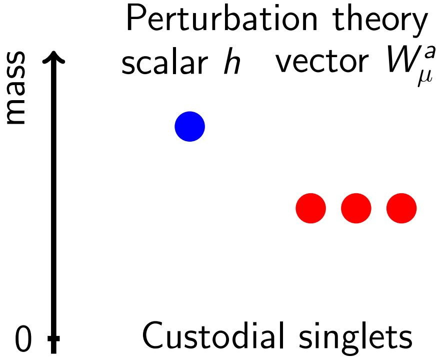

- Gauge transformation for vev: Gauge choice 'Spontaneous gauge symmetry breaking'
- There are gauges where $\langle \phi \rangle = 0 \Rightarrow$ PT not sensible
- Symmetry is not manifest (hidden)

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  $\Rightarrow$ Cannot be observable
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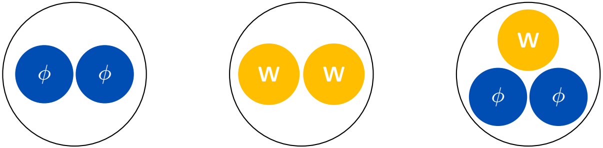
Higgs-Higgs W-Ball Higgs-Higgs-W



et cetera
...

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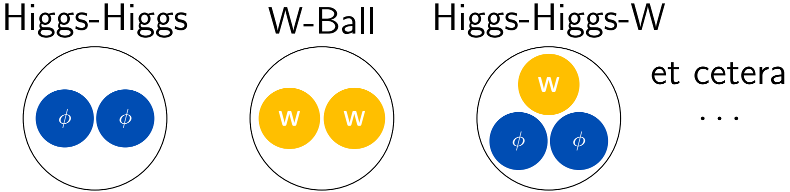

- Why does perturbation theory work so well?

- What is the mass spectrum?

Weak-Higgs sector of SM - Spectrum

- Lattice spectroscopy $\Rightarrow$ Spectrum of bound states

  [Maas, MPL **A28** (2013) / Maas and Mufti, JHEP (2014)]

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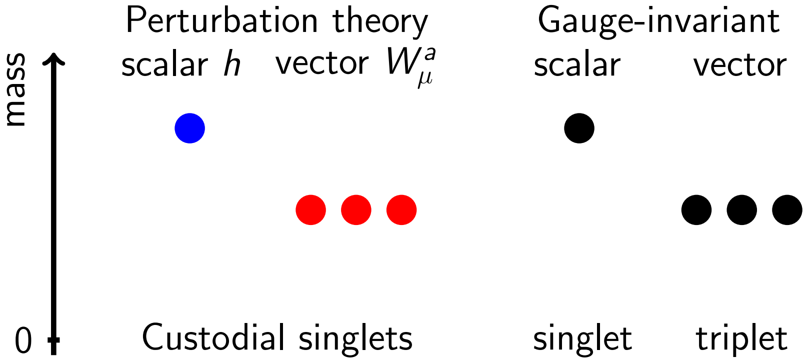

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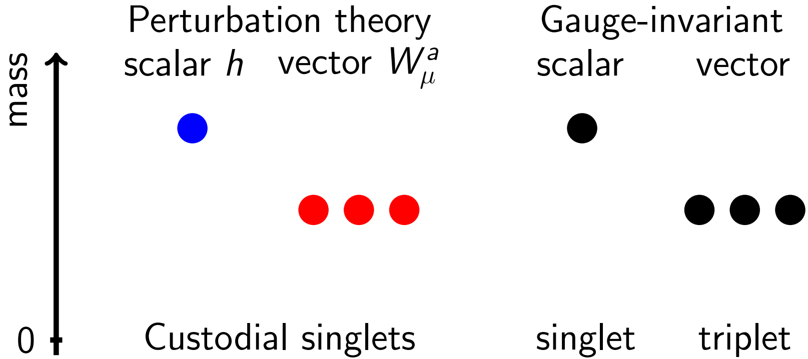

- Masses of bound states = elementary fields
- This is not a coincidence!

  [Fröhlich *et al.*, PL **B97** (1980) and NP **B190** (1981)]

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- Expand correlator in Higgs fluctuations φ (FMS mechanism)

$$\left\langle O(x) O(y)^\dagger \right\rangle \underset{h=\sqrt{2}\mathrm{Re}[n^\dagger \phi]}{\overset{\phi=\frac{v}{\sqrt{2}}n+\varphi}{=}} \frac{v^4}{4} + \frac{v^3}{2}\left\langle h(x) + h(y) \right\rangle + v^2 \left\langle h(x) h(y) \right\rangle$$
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- Exact identity
- Sum on r.h.s. is gauge-invariant but each term individually is gauge-variant

Gauge-invariant perturbation theory II

- Perform standard perturbation theory on r.h.s.:

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- Confirmed on the lattice

  [Maas, MPL **A28** (2013) / Maas and Mufti, JHEP (2014)]

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- Physical states are bound states
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- Has to be checked for BSM theories

SU(N) + fundamental scalar - Toy GUT

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  - Global symmetry: U(1)
- Perturbative construction: SU(N) $\xrightarrow{\langle\phi\rangle}$ SU($N-1$)
  - $2(N-1)+1$ massive and $N(N-2)$ massless gauge bosons
  - 1 massive real scalar field

Physical spectrum of the toy GUT

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- Focus on $N = 3$ and vector channel in the following

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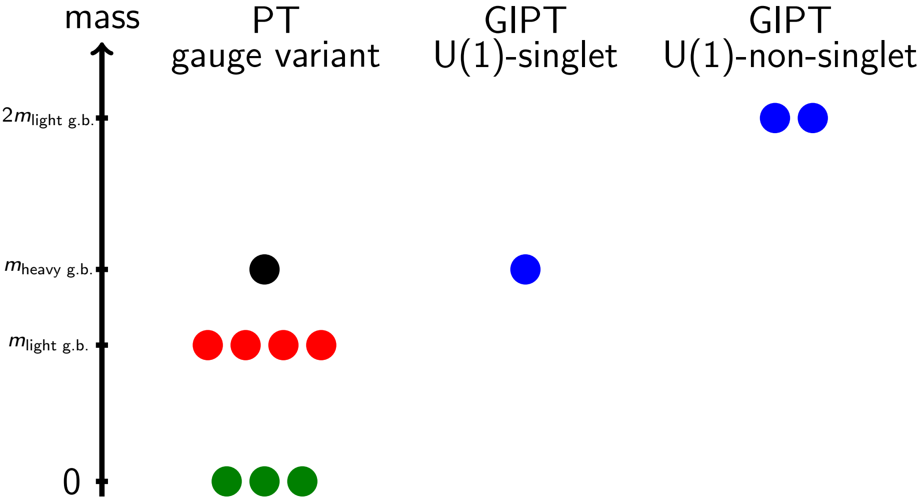
- Spectrum in the vector channel



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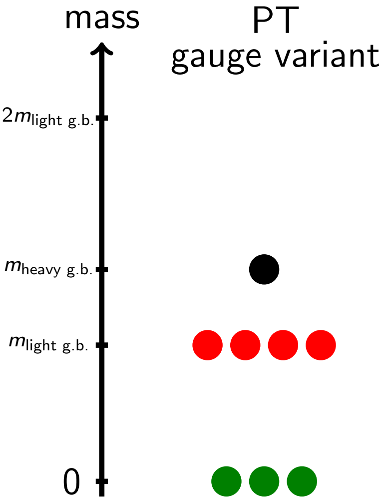

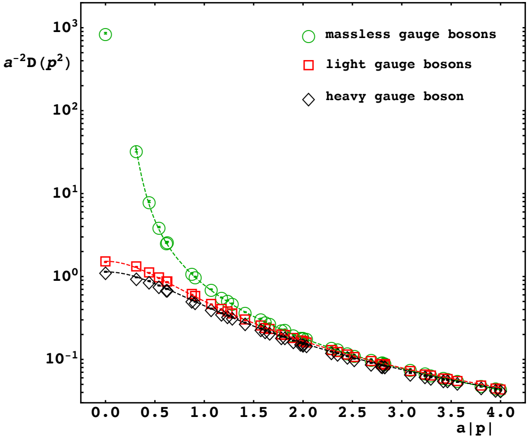

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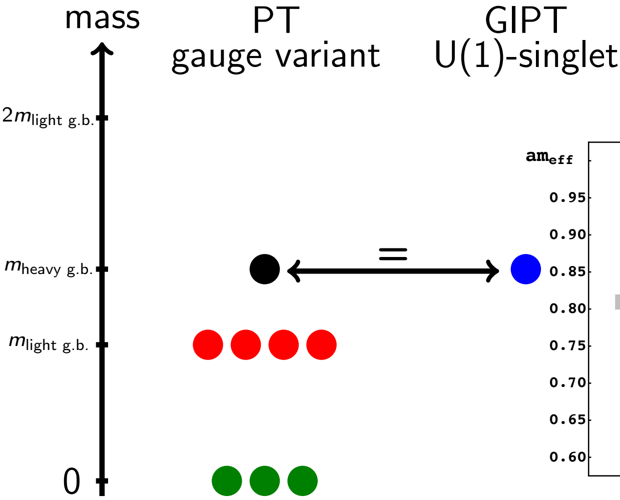

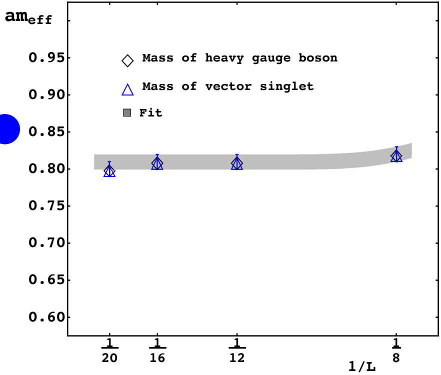

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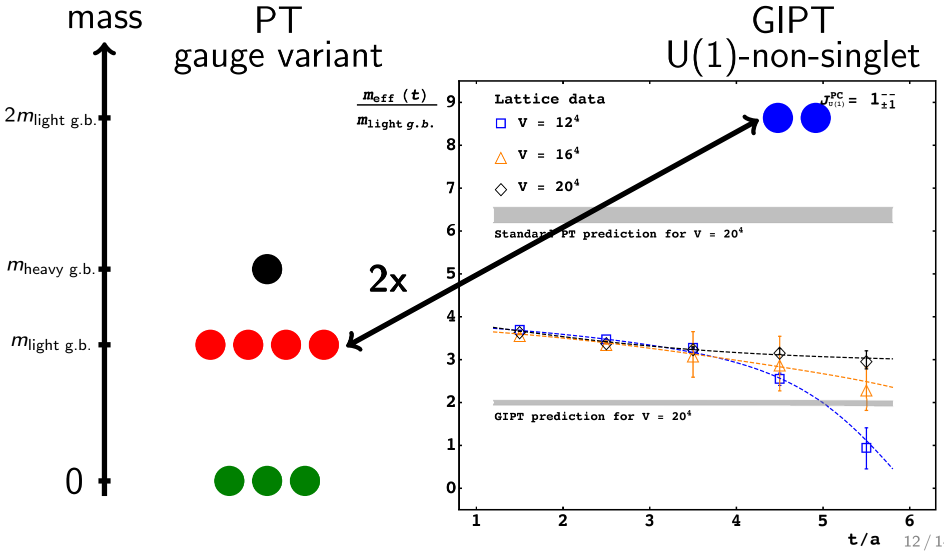

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- Conventional GUTs unlikely to reproduce low-energy spectrum according to GIPT
  - Larger custodial groups needed (?)

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  - SU(2)×U(1) with Higgs (no direct verification)
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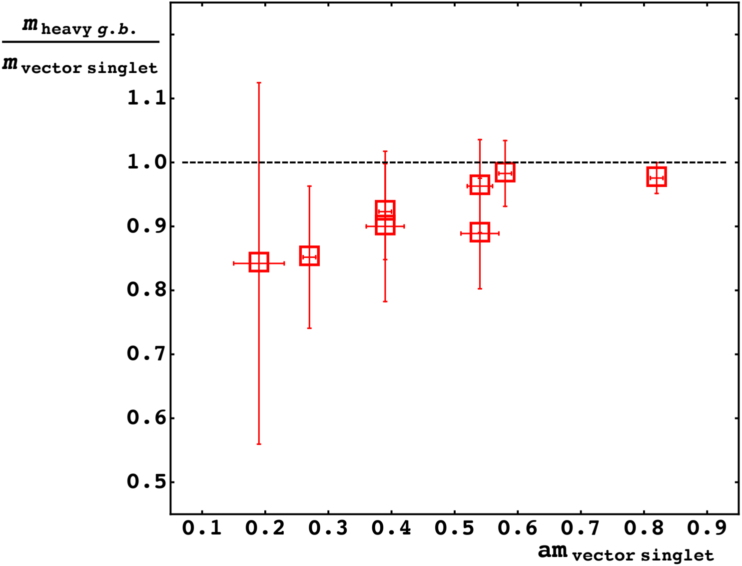
- Procedure can be used to build or rule out BSM theories (e.g. std. SU(5)-GUT construction)

Thank you!

GIPT works well!



SU(5) GUT - Bosonic sector

- Fundamental scalar φ and adjoint scalar σ
- Custodial group: $\mathrm{U}(1) \times \mathbb{Z}_2$
- GUT-scale: w

	elementary spectrum			gauge-invariant spectrum			
J^P	Field	Mass	Deg.	$(\mathrm{U}(1), \mathbb{Z}_2)$	Mass	N.-I.	Deg.
0^+	h	m_h	1	$(0,+)$	m_h	$\sim w$	1
	$\varphi_{1,\ldots,6}$	$m_{\varphi_{1,\ldots,6}}$	6	$(0,-)$	m_h	$\sim w$	1
	$\sigma^{1,\ldots,8}$	$m_{\sigma^{1,\ldots,8}}$	8	$(\pm 1,+)$	$\sim w$	$\sim w$	1
	$\sigma^{21,22,23}$	m_σ	3	$(\pm 1,-)$	$\sim w$	$\sim w$	1
	σ^{24}	M_σ	1				
1^-	A_μ	$m_\mathrm{A} = 0$	1	$(0,+)$	m_A	m_Z	1
	$W_\mu^\pm$	m_W	2	$(0,-)$	m_A	m_Z	1
	Z_μ	m_Z	1	$(\pm 1,+)$	$\sim w$	$\sim w$	1
	$A_\mu^{9,\ldots,14}$	m_L	6	$(\pm 1,-)$	$\sim w$	$\sim w$	1
	$A_\mu^{15,\ldots,20}$	M_L	6				