Zeros of the $W_L Z_L \to W_L Z_L$ amplitude : vector resonances at the LHC

Madeleine Adrien
(University of Graz, Austria)
(Work in collaboration with J. Portolés [IFIC, Spain])

18th of April, 2018

Outline for Section 1

1. Introduction
2. Pions case : Study of $\pi^- \pi^0 \longrightarrow \pi^- \pi^0$
3. Electroweak case : Study of $W_L Z_L \longrightarrow W_L Z_L$
4. Conclusion

Introduction

CMS and ATLAS : search for vector resonances at $E \sim 1$ TeV.

The Legendre zeros method helps to search for high-energy resonances using low-energy effective theories.

2 steps :

- Verification of the method with $\pi^- \pi^0 \to \pi^- \pi^0$ and the $\rho(770)$.
- Search for vector resonances in $W_L Z_L \to W_L Z_L$

Outline for Section 2

1. Introduction
2. Pions case : Study of $\pi^- \pi^0 \to \pi^- \pi^0$
3. Electroweak case : Study of $W_L Z_L \to W_L Z_L$
4. Conclusion

The variables

$$\pi^-(p_1)\,\pi^0(p_2) \rightarrow \pi^-(p_3)\,\pi^0(p_4)$$

Mandelstam variables :

$$s = (p_1 + p_2)^2 \quad \text{and} \quad t = (p_1 - p_3)^2 = -\frac{1}{2}(s - 4M_\pi^2)(1 - z)$$

where $z \equiv \cos\theta$.

Uncovering resonances : General principle

- Use the ChPT amplitude of $\pi^-\pi^0 \to \pi^-\pi^0$:

$$F(s,z) \equiv A(\pi^-\pi^0 \to \pi^-\pi^0)$$

- Chiral Perturbation Theory (ChPT) is only valid at low energies:

$$\mathrm{E} \ll \Lambda_{\mathrm{ChPT}} = 4\pi f_\pi \simeq 1.2\ \mathrm{GeV}$$

- Problem : $\mathrm{E}_{\mathrm{max\ ChPT}} \simeq 500$ MeV and $M_\rho \simeq 770$ MeV !

Uncovering resonances : General principle

- Problem : $E_{\text{max ChPT}} \simeq 500$ MeV and $M_\rho \simeq 770$ MeV !

  Solution : Using the smoothness of the zeros of $F(s,z)$:

$$F(s,z) = 0\,, \quad \text{with } F(s,z) \text{ obtained using ChPT}$$

$$\Longleftrightarrow \qquad z = z_0(s)$$

$\longrightarrow$ Provides a relation characteristic of the zeros of the amplitude even at $E \gtrsim 500$ MeV. [M.R. Pennington, J. Portolés, 1995 ; A. Filipuzzi, J. Portolés, P. Ruiz-Femenia, 2012, 2013]

Decomposition of the amplitude $F(s,z)$

Isospin defined amplitudes decomposition of the $F(s,z)$:

$$F(s,z) = \frac{1}{2}\left[\, F^1(s,z) + F^2(s,z) \,\right]$$

Legendre polynomials decomposition of the $F^{\mathrm{I}}(s,z)$ (s-channel) :

$$F^{\mathrm{I}}(s,z) = 32\,\pi \sum_{\ell=0}^{\infty} (2\,\ell + 1)\, f_\ell^{\mathrm{I}}(s)\, P_\ell(z)$$

To ensure unitarity (with $\delta_\ell^{\mathrm{I}}(s)$ the phase shift of the $f_\ell^{\mathrm{I}}(s)$) :

$$f_\ell^{\mathrm{I}}(s) = \frac{1}{\sigma}\, e^{\,i\,\delta_\ell^{\mathrm{I}}(s)}\, \sin \delta_\ell^{\mathrm{I}}(s)$$

Obtaining the resonance mass M_R

- From the previous decompositions and $z_0(s)$, we obtain:

$$\mathrm{Re}\left(z_0\left(M_R^2\right)\right) \simeq 0$$

- Using it on $\pi^-\pi^0 \to \pi^-\pi^0$ at $O(p^4)$ in ChPT, we obtain :

$$M_\rho \simeq 0.75 \text{ GeV}$$

$\longrightarrow$ Method validated!

A unitarization procedure

Similarly, it can be proved that: (unitarization procedure)

$$\tan \delta_1^1 = \frac{-\frac{1}{2}\sin 2\,\delta_0^2}{3\,(\mathrm{Re}\,\mathbf{z_0(s)}) + \sin^2 \delta_0^2}$$

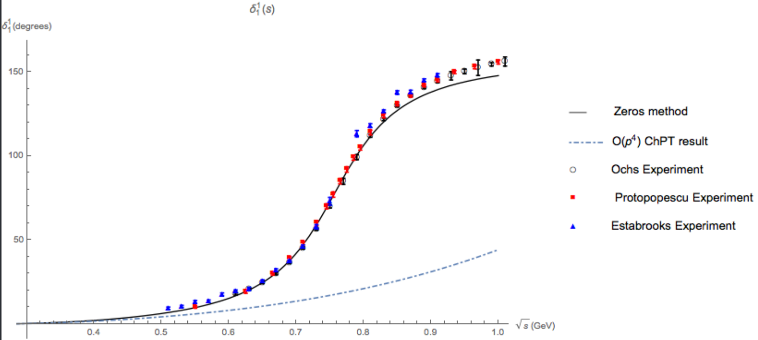
And that:

$$\delta_1^1(M_\rho^2) \to \frac{\pi}{2} + n\pi\,, \quad n \in \mathbb{Z}$$

A unitarization procedure : plotting $\delta_1^1(s)$

We see:

- Very good agreement with the experiment up to $E \simeq 0.8$ GeV
- $M_\rho \simeq 0.77$ GeV $\longrightarrow$ Method (further) validated .



[S.D. Protopopescu, et al., 1973 ; P. Estabrooks, et al., 1974 ; W. Ochs, 1973]

Outline for Section 3

1. Introduction
2. Pions case : Study of $\pi^-\pi^0 \longrightarrow \pi^-\pi^0$
3. Electroweak case : Study of $W_L Z_L \longrightarrow W_L Z_L$
4. Conclusion

Tools for computing the amplitude

- The equivalence theorem : [J.M. Cornwall, et al., 1974 ; B.W. Lee, et al., 1977 ; A. Dobado, et al., 1994]

  Links $A\left(W_L Z_L \to W_L Z_L\right)$ and $A^{(4)}(\pi^- \pi^0 \to \pi^- \pi^0)$

- The Electroweak Chiral Effective Theory (EChET) : [A.C. Longhitano, 1980, 1981 ; T. Appelquist, et al., 1993]
  - Involves the Goldstone bosons π^a
  - Similar in structure to Chiral Perturbation Theory

$\Rightarrow$ We can compute :

$$A^{(4)}(\pi^- \pi^0 \to \pi^- \pi^0) \simeq A\left(W_L Z_L \to W_L Z_L\right)$$

The validity range

- For the use of the equivalence theorem and the EChET,
- by analogy with the chiral case, [A. Filipuzzi, J. Portolés, P. Ruiz-Femenia, 2012]

our study is valid at energies :

$$M_{W,Z} \simeq 0.1\ \mathrm{TeV} \ll E \leq 2\ \mathrm{TeV}.$$

$\Rightarrow$ We can investigate the resonances between $[0.5, 2\,]$ TeV.

Legendre zeros method applied to $W_L Z_L \to W_L Z_L$

EChET is a theory similar to ChPT.

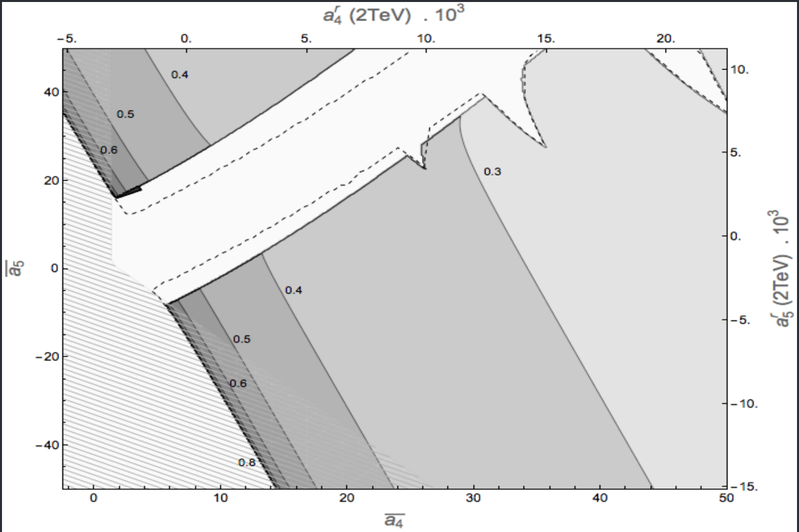
$\Rightarrow$ *We can expect the same behaviour and observing a vector resonance at $E \sim 1$ TeV.*

$\Rightarrow$ Outline of the study :

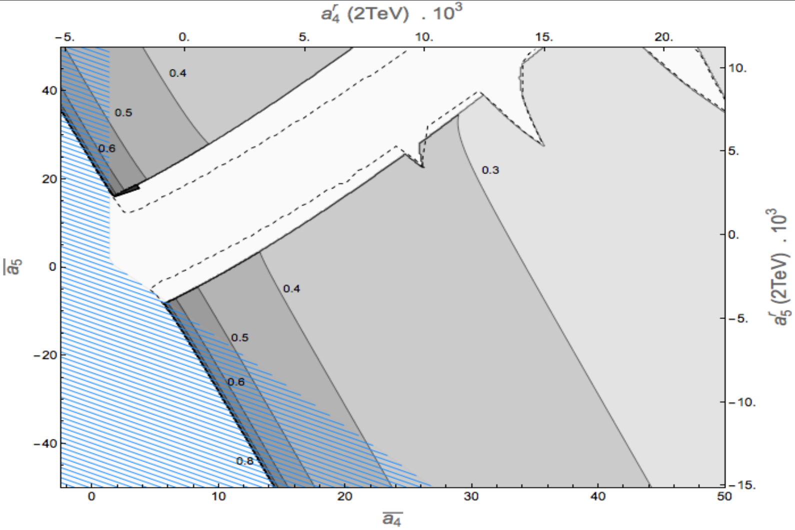
- Search for zeros of $A\left(W_L Z_L \to W_L Z_L\right) \quad \Rightarrow \quad z = z_0(s)$
- Search for the minimum resonance mass M_R for each $(\bar{a}_4, \bar{a}_5)$ with $\mathrm{Re}\left(z_0\left(M_R^2\right)\right) = 0$.
- Condition to check the validity of the hypotheses :

$$|z_0(M_R^2)| = |\mathrm{Im}\left(z_0(M_R^2)\right)| = \left| \frac{f_0^2(M_R^2)}{3\, f_1^1(M_R^2)} \right| \leq \lambda$$

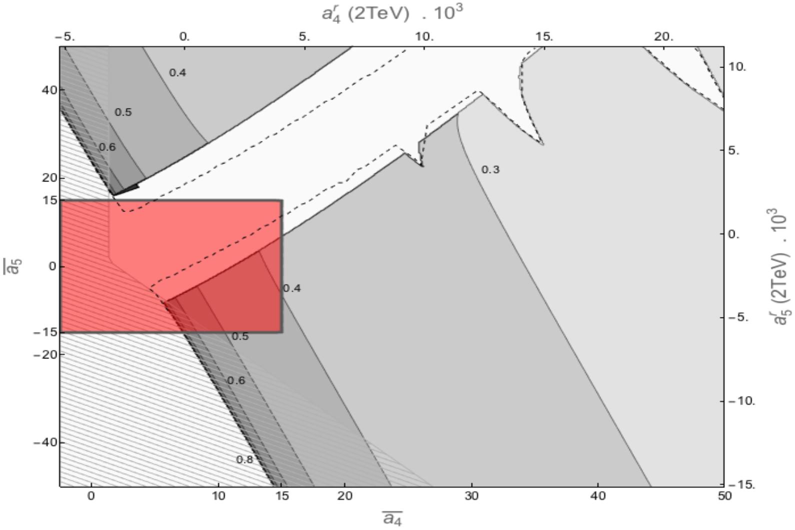
Resonance masses in the $(\overline{a}_4, \overline{a}_5)$ plane



Positivity conditions on the scattering amplitude



Natural values for the LECs : $\bar{a}_4$ and $\bar{a}_5$ $\in$ [-15, 15]



Outline for Section 4

Conclusion

The results :

- Most of the natural values of the LECs do not permit any vector resonance between [0.5, 2] TeV.
- Very few $(\bar{a}_4, \bar{a}_5)$ allow for vector resonances with $M_R \in [0.5, 0.8]$ TeV on this graph.

$\Rightarrow$ The results (compatible with other methods and experiments) :

exclude $M_R \in [0.5, 2]$ TeV for most natural values of the $(\bar{a}_4, \bar{a}_5)$.

Pushing the study further

- Increase the precision of the study going to higher orders.
- Search for resonances of higher spins $J > 1$.

Back-Up

Back-up slide : Comparison with other techniques and the experimental results

- *Inverse Amplitude Method* : [R.L. Delgado, et al., 2017]
  Finds masses $M_R \in [1.5, 2.5]$ TeV for natural LECs.
  $\Rightarrow$ Partially compatible with the main part of our current results
- *Resonance Chiral Electroweak theory* : [A. Pich, et al., 2014]
  Finds the minimum bound of 1.5 TeV $\leq M_R$.
  $\Rightarrow$ Compatible
- *Experimental data from* : [A. Hinzmann, 2013 ; The CMS Collaboration, 2017 ; The ATLAS Collaboration, 2017]
  - *CMS* : Find the minimum bound of 1.59 TeV $\leq M_R$. (C.L.= 95 %)
  - *ATLAS* : Find the minimum bound of 3 TeV $\leq M_R$. (C.L.= 95 %)

  $\Rightarrow$ Compatible

Back-up slide : Positivity conditions (1)

D-wave scattering lengths (Froissart - Gribov representation) :

[M.R. Pennington, J. Portoles, 1995]

$$a_2^{\mathrm{I}} = \frac{16}{15\,\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^3}\ \mathrm{Im}\, F^{t\,\mathrm{I}}(s', 4m_\pi^2) \qquad \mathrm{for} \quad \mathrm{I} = 0,\ 2$$

And we know that $F^{s\,\mathrm{I}} \geq 0 \quad \mathrm{so} \quad \mathrm{Im}\, F^{s\,\mathrm{I}} \geq 0.$

For $\mathrm{I} = 0$, $\quad F^{t\,0} = \frac{1}{3}\, F^{s\,0} + F^{s\,1} + \frac{5}{3}\, F^{s\,2}$

For $\mathrm{I} = 2$, $\quad F^{t\,2} = \frac{1}{3}\, F^{s\,0} - \frac{1}{2}\, F^{s\,1} + \frac{1}{6}\, F^{s\,2}$

Back-up slide : Positivity conditions (2)

So when we can take the combinations :

$$a_2^0 - a_2^2 \sim \mathrm{Im}\,[\,F^{t0} - F^{t2}\,] \sim \tfrac{3}{2}\,\mathrm{Im}\,[\,F^{s1} + F^{s2}\,]$$

$$a_2^0 + 2a_2^2 \sim \mathrm{Im}\,[\,F^{t0} + 2F^{t2}\,] \sim \mathrm{Im}\,[\,F^{s0} + 2F^{s2}\,]$$

As $\mathrm{Im}\,F^{s\mathrm{I}} \geq 0\,,\qquad a_2^0 - a_2^2 \geq 0$ and $a_2^0 + 2a_2^2 \geq 0\,.$

Back-up slide : Positivity conditions (3)

The D-wave scattering lengths can be expressed as functions of $\bar{\ell}_1$ and $\bar{\ell}_2$:

$$a_2^0 = \frac{1}{1440\,\pi^3 f_\pi^4} \left(\bar{\ell}_1 + 4\bar{\ell}_2 - \frac{53}{8} \right)$$

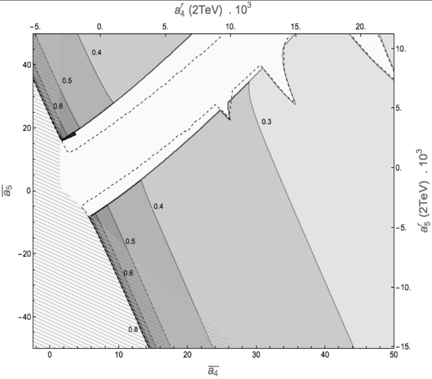
$$a_2^2 = \frac{1}{1440\,\pi^3 f_\pi^4} \left(\bar{\ell}_1 + \bar{\ell}_2 - \frac{103}{40} \right)$$

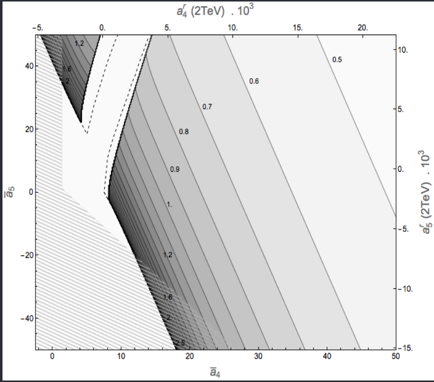
So the previous positivity conditions on the a_2^{I} translate into :

$$\bar{\ell}_1 + 2\bar{\ell}_2 \geq \frac{157}{40} \qquad \text{and} \qquad \bar{\ell}_2 \geq \frac{27}{20}$$

Back-up slide : Importance of the SM Higgs.

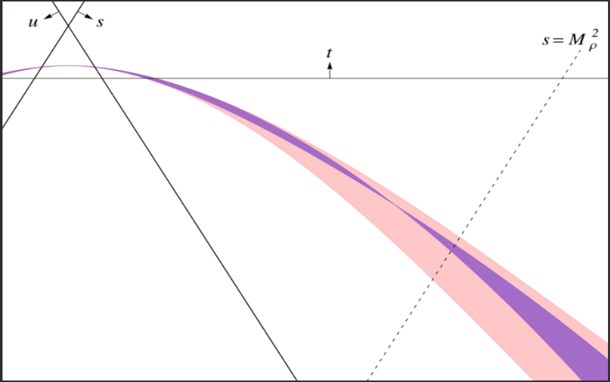
Resonance masses in the $(\overline{a}_4, \overline{a}_5)$ plane in the cases with the SM Higgs (left), and without it (right).





Back-up slide : Zero contours of $\pi^-\pi^0 \to \pi^-\pi^0$ at $O(p^4)$ and $O(p^6)$.

Connection between Weinberg's projection of the Adler zero and our Legendre zero in the Mandelstam plane.



[A. Filipuzzi, J. Portoles and P. Ruiz-Femenía, 2012]