

# *Neutrinos and Symmetries*

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CP<sup>3</sup> Origins  
  
Cosmology & Particle Physics

**SDU** 

# *Flavor symmetry: reasons from experiment*

- observation of three generations of elementary particles

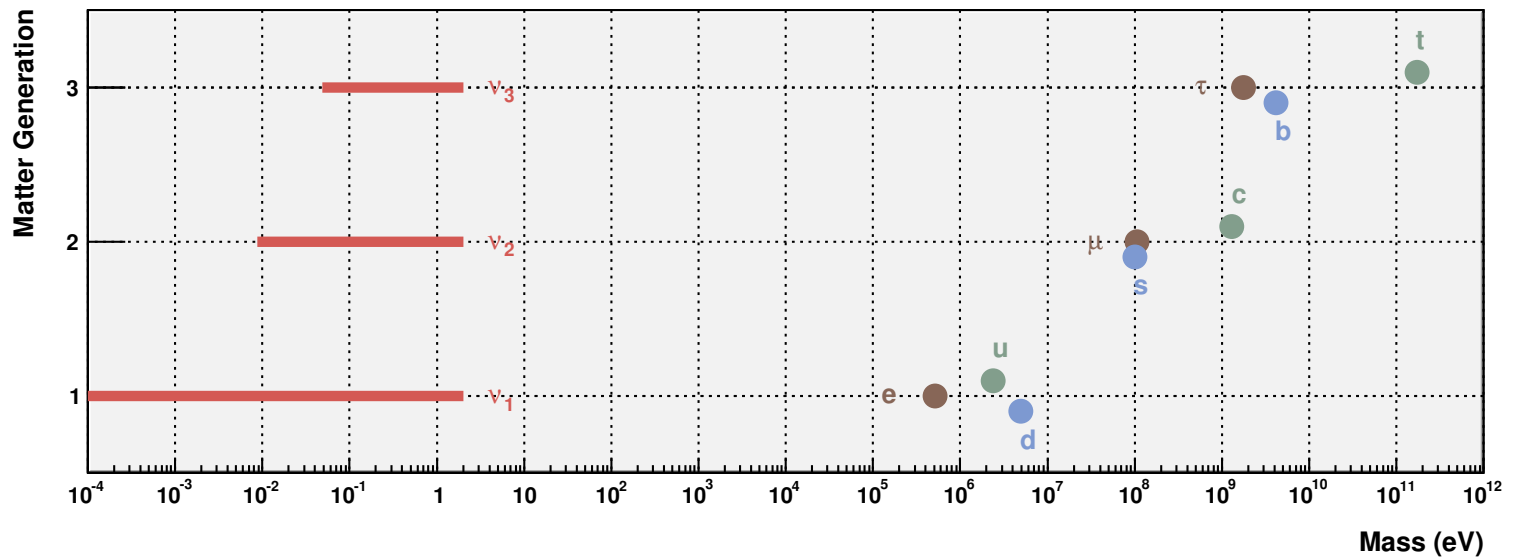
<b>Quarks</b>	$u$ up	$c$ charm	$t$ top
	$d$ down	$s$ strange	$b$ bottom
<b>Leptons</b>	$\nu_e$ electron neutrino	$\nu_\mu$ muon neutrino	$\nu_\tau$ tau neutrino
	$e$ electron	$\mu$ muon	$\tau$ tau
	I	II	III

**Three Families of Matter**

non-abelian flavor symmetry as reason for three generations

# Flavor symmetry: reasons from experiment

- observation of three generations of elementary particles
- strong mass hierarchy among charged fermions



log scale

Froggatt-Nielsen symmetry  $U(1)_{FN}$  parametrizes mass hierarchy

# *Flavor symmetry: reasons from experiment*

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- observation of three generations of elementary particles
- strong mass hierarchy among charged fermions
- solar (much) smaller than atmospheric mass splitting

$$\frac{\Delta m_{21}^2}{\Delta m_{31}^2} \approx 0.03 \quad [\text{NO}]$$

*(NuFIT ('18))*

$SO(2)$  can explain partially degenerate neutrino masses

*(Hernandez/Smirnov ('13))*

# Flavor symmetry: reasons from experiment

- observation of three generations of elementary particles
- strong mass hierarchy among charged fermions
- solar (much) smaller than atmospheric mass splitting
- very different mixing among quarks and leptons

$$\|V_{CKM}\| = \begin{pmatrix} 0.97 & 0.23 & 3.6 \cdot 10^{-3} \\ 0.22 & 0.97 & 0.041 \\ 8.8 \cdot 10^{-3} & 0.040 & 0.999 \end{pmatrix} \quad \text{[NO]}$$

*(PDG ('17 update))*

$$\|U_{PMNS}\| = \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.33 & 0.60 & 0.73 \\ 0.46 & 0.58 & 0.67 \end{pmatrix}$$

*(NuFIT ('18))*

different (residual) flavor symmetries for quarks and leptons

# ***Flavor symmetry: reasons from experiment***

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- observation of three generations of elementary particles
- strong mass hierarchy among charged fermions
- solar (much) smaller than atmospheric mass splitting
- very different mixing among quarks and leptons
- $\theta_{13} \ll \theta_{12}, \theta_{23}$  in the lepton sector

$$\sin^2 \theta_{13} = 0.02206, \quad \sin^2 \theta_{12} = 0.307, \quad \sin^2 \theta_{23} = 0.538 \quad [\text{NO}]$$

*(NuFIT ('18))*

certain values of  $\theta_{ij}$  from discrete flavor symmetries  
well-known: tri-bi-maximal mixing *(Harrison et al. ('02), Xing ('02))*

# *Flavor symmetry: reasons from experiment*

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- observation of three generations of elementary particles
- strong mass hierarchy among charged fermions
- solar (much) smaller than atmospheric mass splitting
- very different mixing among quarks and leptons
- $\theta_{13} \ll \theta_{12}, \theta_{23}$  in the lepton sector
- non-observation of e.g. charged lepton flavor violation

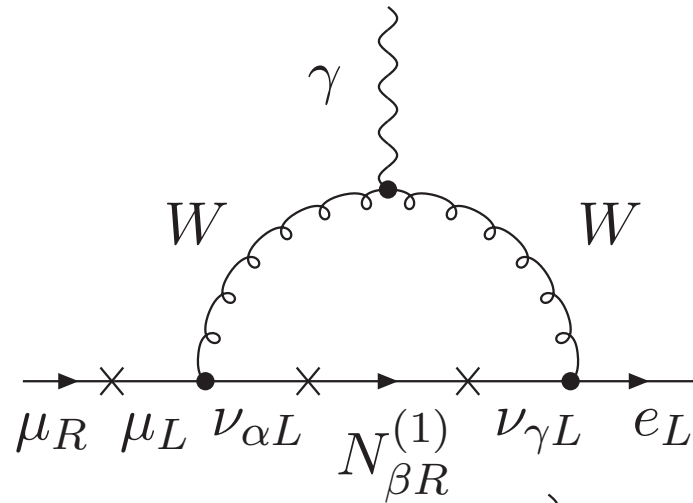
$$\text{BR} (\mu^+ \rightarrow e^+ \gamma) < 2.55 \times 10^{-13}$$

*(MEG ('16))*

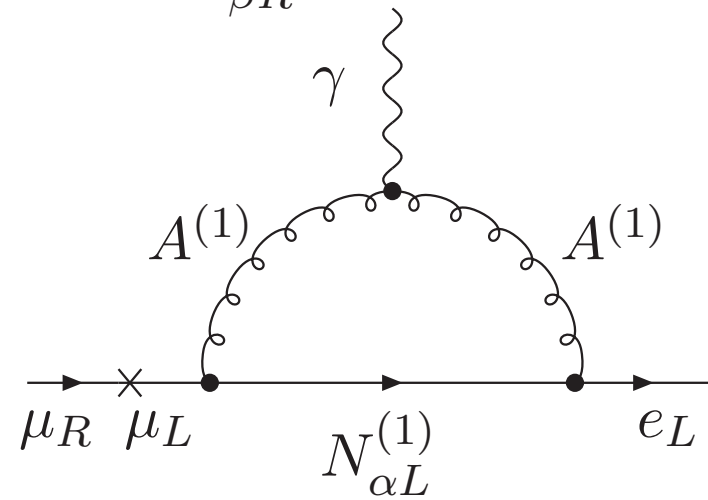
protection of flavor sector in BSM theories needed

# Flavor symmetry: reasons from experiment

KK excitation in loop



Exotic gauge boson in loop

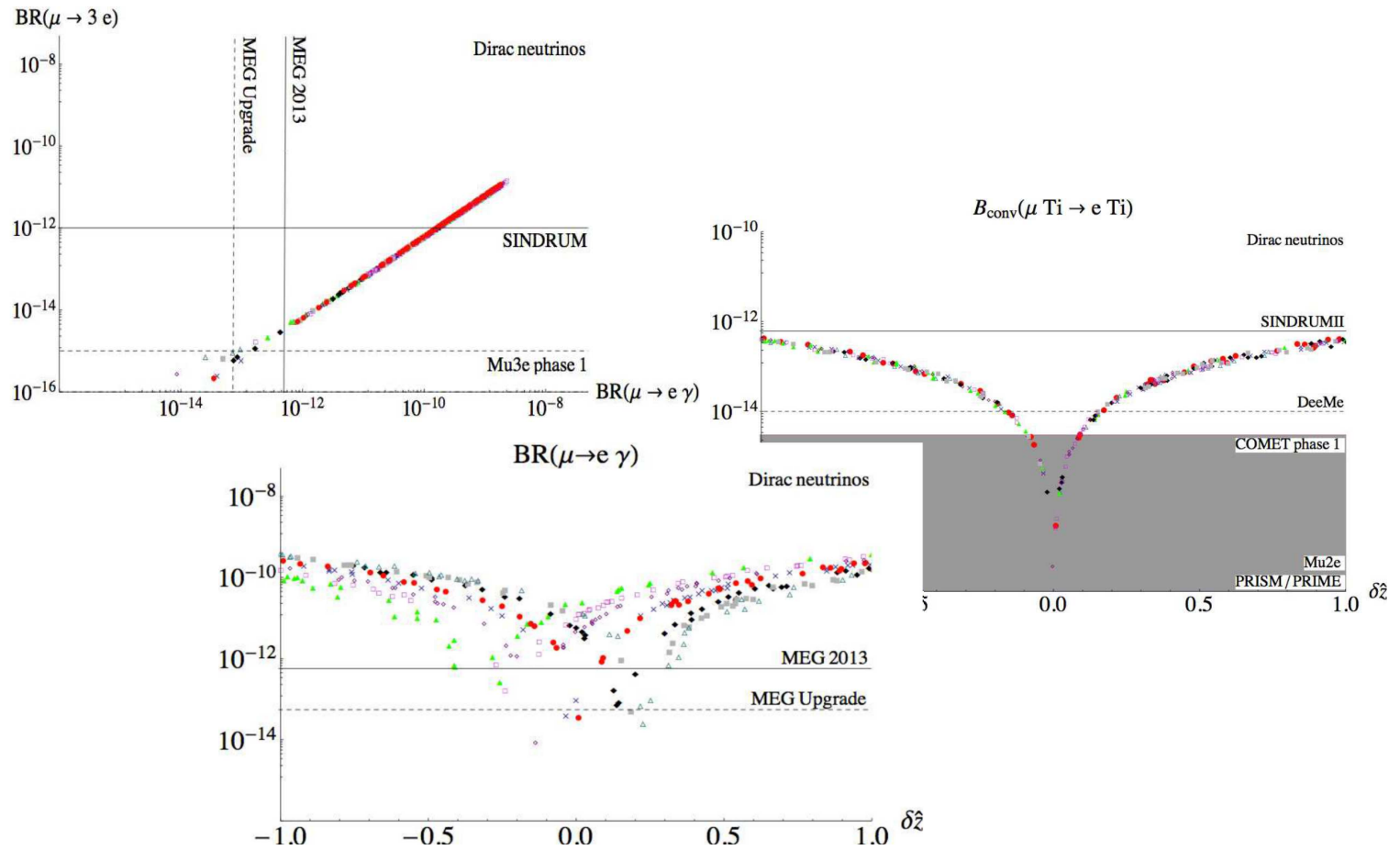




# Flavor symmetry: reasons from experiment

$\delta\hat{z}$  parametrizes boundary kinetic terms

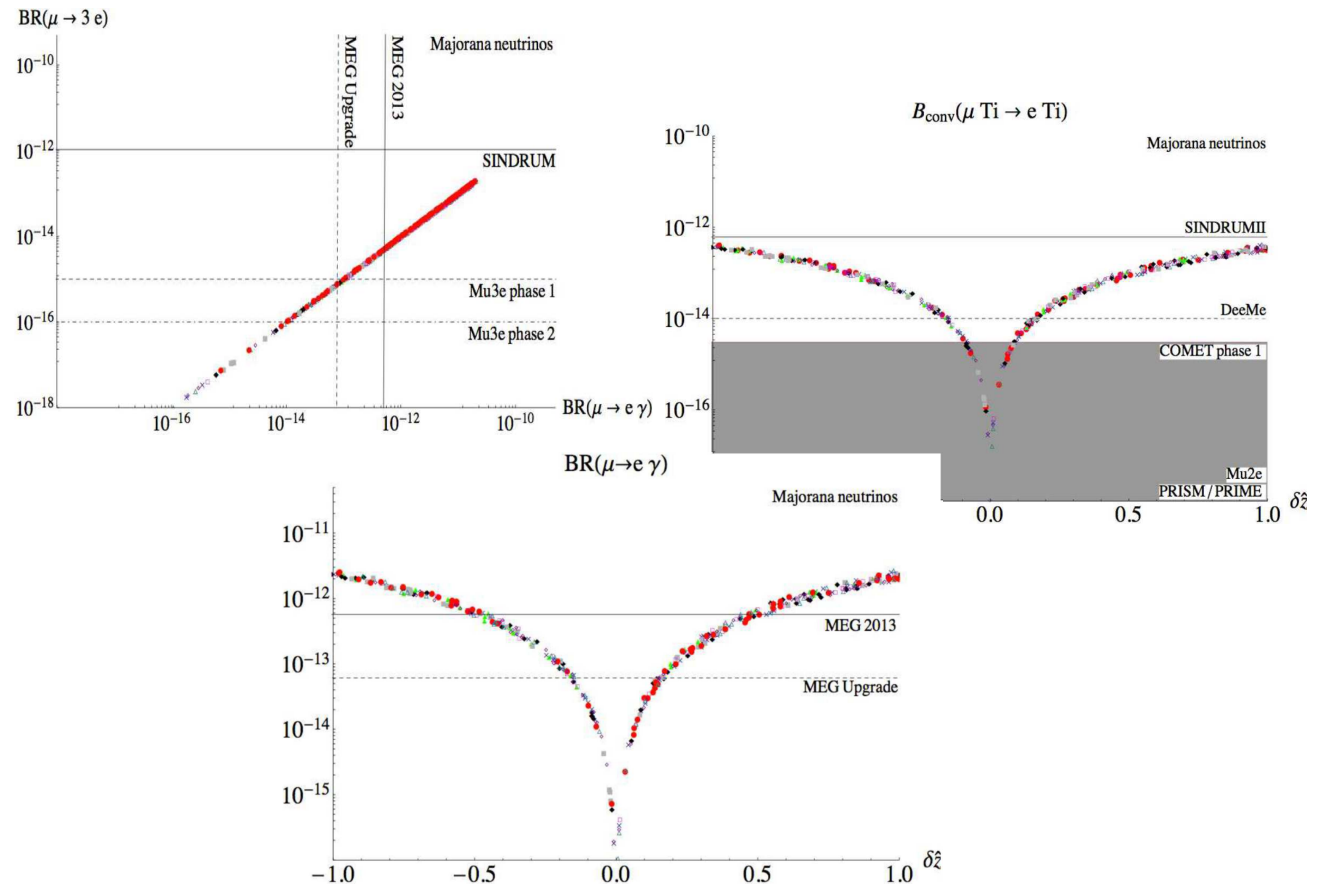
(H/Serone ('11))



# Flavor symmetry: reasons from experiment

$\delta\hat{z}$  parametrizes boundary kinetic terms

(H/Serone ('11))



# *Flavor symmetry: guesses from experiment*

---

- relation between strong quark mass hierarchy and small quark mixing?

$$|V_{us}| = |V_{cd}| = \sqrt{\frac{m_d}{m_s}}$$

*(Gatto/Sartori/Tonin ('68))*

# *Flavor symmetry: guesses from experiment*

---

- relation between strong quark mass hierarchy and small quark mixing?
- relation between origin of neutrino masses and large lepton mixing? Possibly mass hierarchy?  
Majorana nature of neutrinos

# *Flavor symmetry: guesses from experiment*

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- relation between strong quark mass hierarchy and small quark mixing?
- relation between origin of neutrino masses and large lepton mixing? Possibly mass hierarchy?
- maximal atmospheric mixing,  $\theta_{23} = \frac{\pi}{4}$ ?

$$\sin^2 \theta_{23} = 0.538_{-0.069}^{+0.033} \text{ [NO]}$$

*(NuFIT ('18))*

discrete flavor symmetries usually predict certain value of  $\theta_{23}$

# *Flavor symmetry: guesses from experiment*

- relation between strong quark mass hierarchy and small quark mixing?
- relation between origin of neutrino masses and large lepton mixing? Possibly mass hierarchy?
- maximal atmospheric mixing,  $\theta_{23} = \frac{\pi}{4}$ ?
- maximal CP violation in neutrino oscillations,  $\delta = 90^\circ$  or  $\delta = 270^\circ$ ?

$$\delta = (234_{-31}^{+43})^\circ \quad [\text{NO}]$$

*(NuFIT ('18))*

flavor and CP symmetries combined predict non-trivial  $\delta$   
 $\mu - \tau$  exchange symmetry which predicts  $\theta_{23}$  and  $\delta$  maximal

*(Harrison/Scott ('02,'04), Grimus/Lavoura ('03))*

# Flavor symmetry: guesses from experiment

- relation between strong quark mass hierarchy and small quark mixing?
- relation between origin of neutrino masses and large lepton mixing? Possibly mass hierarchy?
- maximal atmospheric mixing,  $\theta_{23} = \frac{\pi}{4}$ ?
- maximal CP violation in neutrino oscillations?
- correlation among lepton mixing angles?

e.g. *(Di Iura/H/Meloni ('15), Li/Ding ('15), Ballett/Pascoli/Turner ('15))*

flavor and CP symmetries combined can lead to sum rules

$$\sin^2 \theta_{12} = \frac{\sin^2 \varphi}{1 - \sin^2 \theta_{13}} \approx \frac{0.276}{1 - \sin^2 \theta_{13}} \gtrsim 0.276$$

$$\sin^2 \theta_{23} \approx \frac{1}{2} \left( 1 \pm (1 - \sqrt{5}) \sin \theta_{13} \right) \approx 0.5 \mp 0.618 \sin \theta_{13} \approx \begin{cases} 0.409 \\ 0.591 \end{cases}$$

# *Flavor symmetry: guesses from experiment*

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- maximal atmospheric mixing,  $\theta_{23} = \frac{\pi}{4}$ ?
- maximal CP violation in neutrino oscillations?
- correlation among lepton mixing angles?

e.g.

complementarity between mixing angles has been proposed

$$\theta_{13} + \theta_{12} \approx 45^\circ \quad \text{or} \quad \theta_{23}$$

*(Zhang/Ma ('12), Zheng/Ma ('12))*



# *Flavor symmetry: guesses from experiment*

- relation between strong quark mass hierarchy and small quark mixing?
- relation between origin of neutrino masses and large lepton mixing? Possibly mass hierarchy?
- maximal atmospheric mixing,  $\theta_{23} = \frac{\pi}{4}$ ?
- maximal CP violation in neutrino oscillations?
- correlation among lepton mixing angles?
- complementarity between lepton and quark mixing angles?

$$\theta_{12}^q + \theta_{12} \approx 45^\circ \quad \text{and} \quad \theta_{23}^q + \theta_{23} \approx 45^\circ$$

*(Minakata/Smirnov ('04), Raidal ('04), ...)*

points towards (grand) unified theory where charged leptons have mixing like down-type quarks and neutrinos a certain mixing pattern e.g. *Antusch/King/Mohapatra ('05)*

# *Flavor symmetry: guesses from experiment*

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- complementarity between lepton and quark mixing angles?
- correlation between low and high energy CP violation?

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can we correlate low energy CP phases  $\delta, \alpha, \beta$  and baryon asymmetry of the Universe?

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yes, if leptogenesis is responsible for baryon asymmetry

# *Flavor symmetry: guesses from experiment*

---

- correlation between low and high energy CP violation?  
can we correlate low energy CP phases  $\delta$ ,  $\alpha$ ,  $\beta$  and baryon asymmetry of the Universe?  
yes, if leptogenesis is responsible for baryon asymmetry  
several interesting studies in the literature  
e.g.

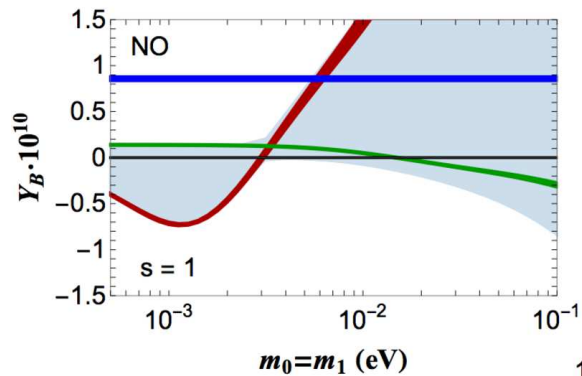
$$|\sin \delta \sin \theta_{13}| \gtrsim 0.11$$

if only  $\delta$  non-trivial and flavor effects relevant

*(Pascoli/Petcov/Riotto ('06))*

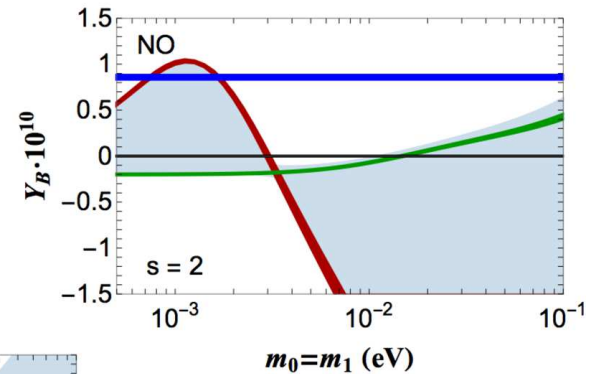
# Flavor symmetry: guesses from experiment

- correlation between low and high energy CP violation?  
unflavored leptogenesis in scenario with flavor and CP symmetries

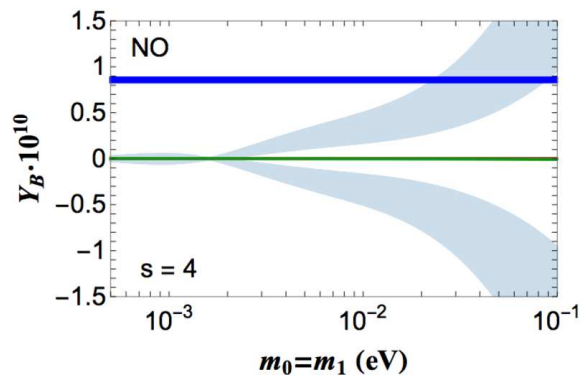


$\sin\alpha < 0$

only  $\sin\delta$   
non-zero



$\sin\alpha > 0$



(H/Molinaro ('16))

# *Flavor symmetry: future experiment*

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- (nearly) massless lightest neutrino?  
protection of lightest neutrino mass from symmetry,  
e.g. Majorana neutrinos and  $Z_N$  symmetry with  $N > 2$

*(Joshi/Patel ('14))*

# *Flavor symmetry: future experiment*

---

- (nearly) massless lightest neutrino?  
“economic” approach (Occam’s razor):  
type I seesaw with only two right-handed neutrinos

e.g. *Harigaya/Ibe/Yanagida ('12)*



# *Flavor symmetry: future experiment*

---

- (nearly) massless lightest neutrino?
- quasi-degenerate neutrino masses?  
underlying symmetry  $[S]O(3)$

*(Hernandez/Smirnov ('13), Alonso et al. ('13))*

# Flavor symmetry: future experiment

- (nearly) massless lightest neutrino?
- quasi-degenerate neutrino masses?
- correlations among lepton mixing angles and CP phase  $\delta$ ?  
e.g.  
certain mixing for neutrinos and certain correction from charged leptons

$$U_{PMNS} = U_e^\dagger \Psi U_\nu(\theta_{12}^\nu) \quad \text{with} \quad U_e = R_{23}(\theta_{23}^e) R_{12}(\theta_{12}^e) \quad \text{and} \quad \Psi \text{ phases}$$

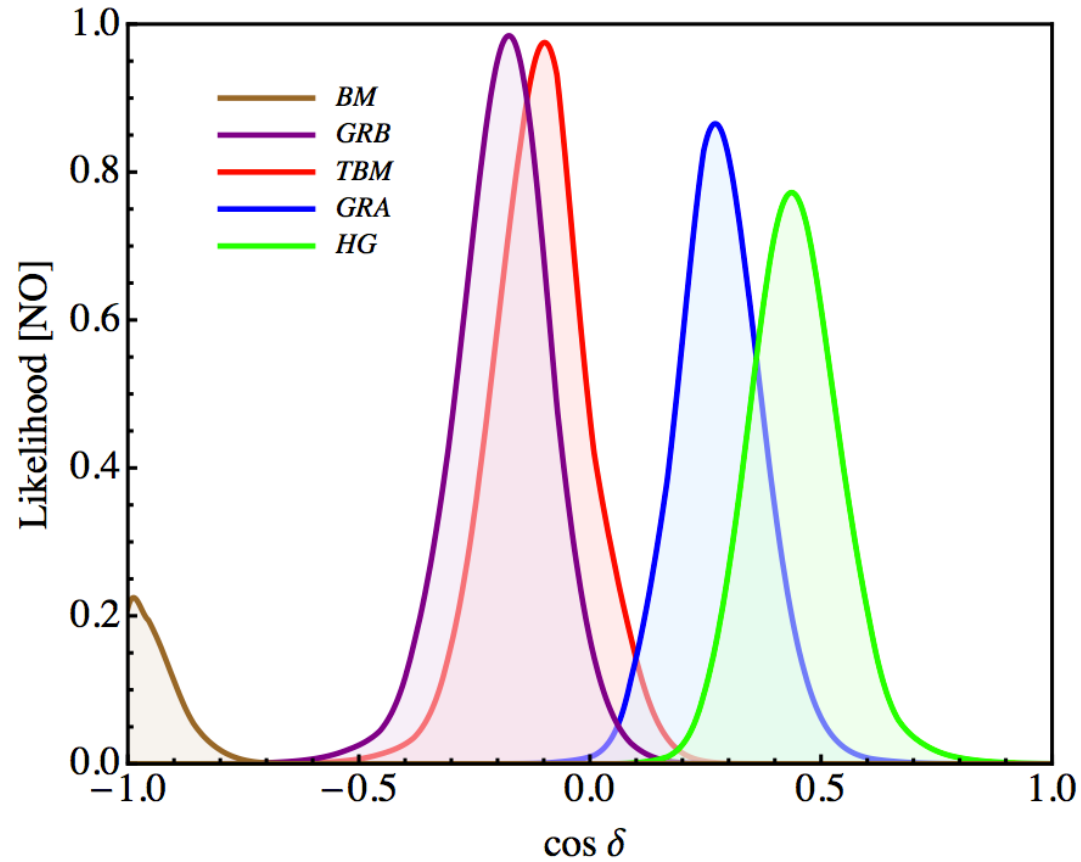
which leads to

$$\cos \delta = \frac{\tan \theta_{23} \sin^2 \theta_{12} + \frac{\sin^2 \theta_{13} \cos^2 \theta_{12}}{\tan \theta_{23}} - \sin^2 \theta_{12}^\nu \left( \tan \theta_{23} + \frac{\sin^2 \theta_{13}}{\tan \theta_{23}} \right)}{\sin 2\theta_{12} \sin \theta_{13}}$$

*(Petcov ('14), Ballett et al. ('14), Girardi et al. ('14))*

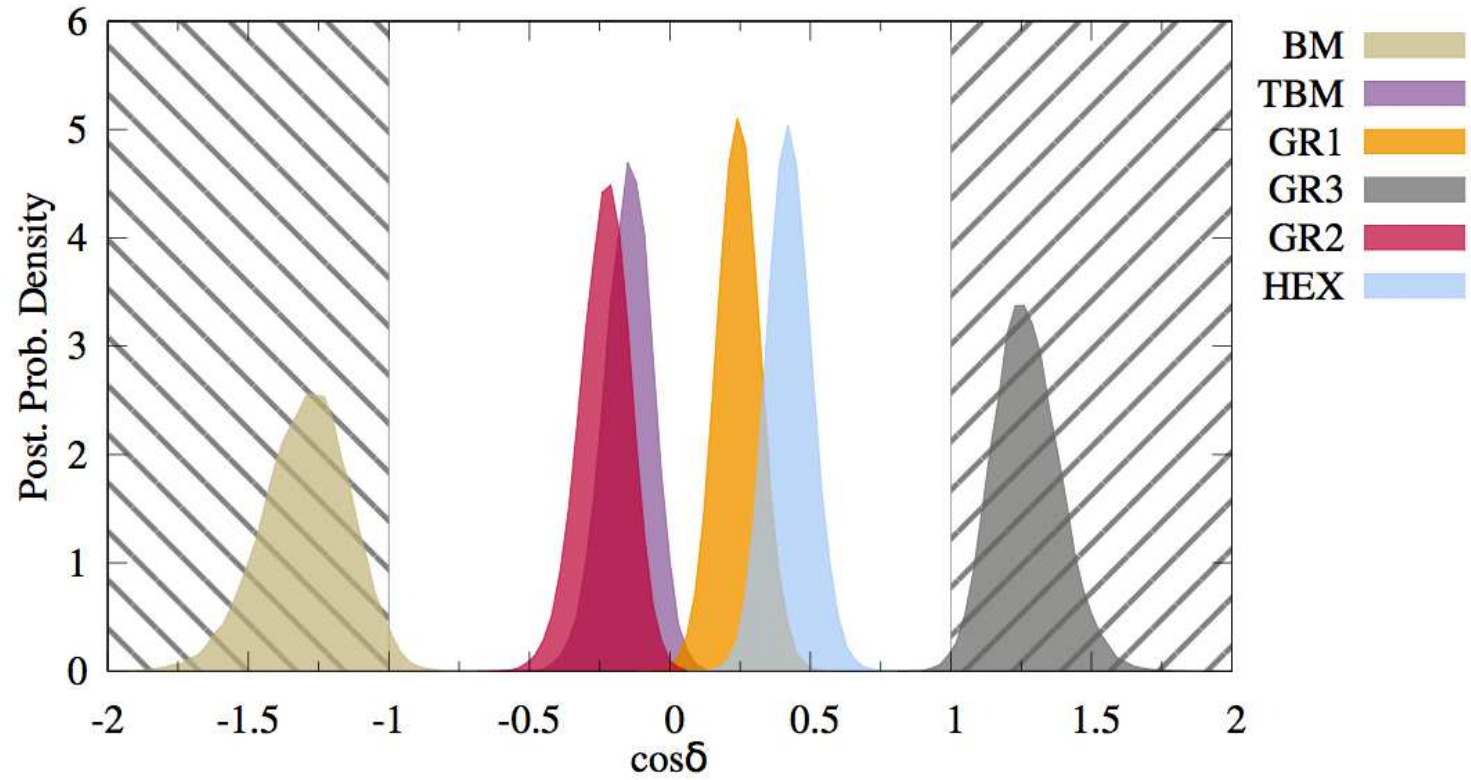
# Flavor symmetry: future experiment

(Girardi et al. ('14))



# Flavor symmetry: future experiment

(Ballett et al. ('14))



# *Flavor symmetry: future experiment*

- correlations among lepton mixing angles and CP phase  $\delta$ ?  
flavor and CP symmetries combined predict mixing patterns with only one real free parameter  
example:  
adjusting mixing angles to experimental data leads to

$$|\sin \delta| \gtrsim 0.71$$

and both Majorana phases  $\alpha, \beta$  depend on CP symmetry  $X(s)$  only

$$|\sin \alpha| = |\sin \beta| = |\sin 6\phi_s| \quad \text{with} \quad \phi_s = \frac{\pi s}{n} \quad \text{and} \quad s = 0, \dots, n-1$$

*(H/Meroni/Molinaro ('14))*

# *Flavor symmetry: future experiment*

*(H/Meroni/Molinaro ('14))*

$s$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$\sin \delta$	$\sin \alpha = \sin \beta$
$s = 1$	0.0220	0.318	0.579	0.936	$-1/\sqrt{2}$
	0.0220	0.318	0.421	-0.936	$-1/\sqrt{2}$
$s = 2$	0.0216	0.319	0.645	-0.739	1
$s = 4$	0.0220	0.318	0.5	$\mp 1$	0

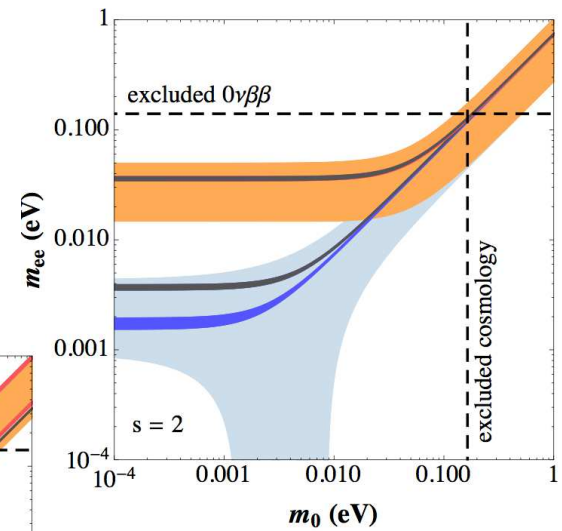
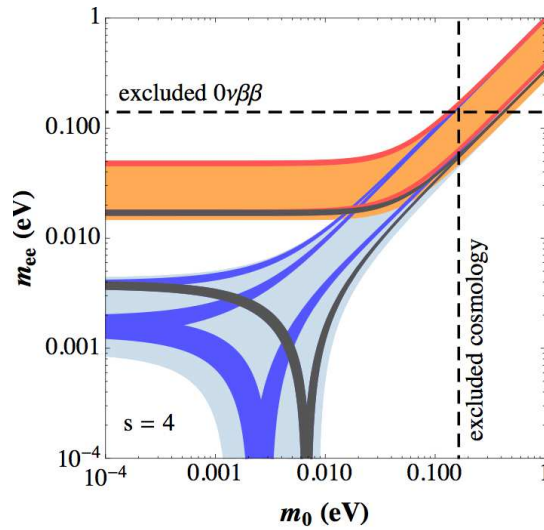
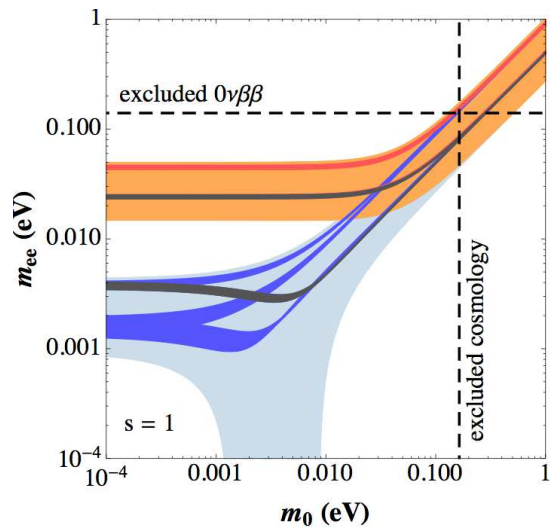
# *Flavor symmetry: future experiment*

---

- (nearly) massless lightest neutrino?
- quasi-degenerate neutrino masses?
- correlations among lepton mixing angles and CP phase  $\delta$ ?
- Majorana phases from neutrinoless double beta decay?  
e.g. flavor and CP symmetries combined can predict them

# Flavor symmetry: future experiment

(H/Molinaro ('16))





# *Flavor symmetry: future experiment*

---

- (nearly) massless lightest neutrino?
- quasi-degenerate neutrino masses?
- correlations among lepton mixing angles and CP phase  $\delta$ ?
- Majorana phases from neutrinoless double beta decay?

caveats:

only one combination of phases can be determined,  
maybe unlucky to suffer from cancellation,  
there might well be more than one mechanism

# *Flavor symmetry: future experiment*

---

- (nearly) massless lightest neutrino?
- quasi-degenerate neutrino masses?
- correlations among lepton mixing angles and CP phase  $\delta$ ?
- Majorana phases from neutrinoless double beta decay?
- lepton number violation at LHC?

# *Flavor symmetry: future experiment*

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- (nearly) massless lightest neutrino?
- quasi-degenerate neutrino masses?
- correlations among lepton mixing angles and CP phase  $\delta$ ?
- Majorana phases from neutrinoless double beta decay?
- lepton number violation at LHC?
- possibility to rule out classes of theories?

# ***Flavor symmetry: reasons from theory***

---

- symmetries are an efficient tool for reducing number of free parameters

# *Flavor symmetry: reasons from theory*

---

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- symmetries have proven very useful to describe gauge interactions

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- possibility to combine gauge and flavor symmetry in more fundamental theory?

# *Flavor symmetry: reasons from theory*

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- symmetries are an efficient tool for reducing number of free parameters
- symmetries have proven very useful to describe gauge interactions
- symmetries can protect certain features
- possibility to combine gauge and flavor symmetry in more fundamental theory?
- in BSM theories new particles also carry flavor  
e.g. supersymmetric theories, extra-dimensional theories with fermions in the bulk, extended dark sector



# *Flavor symmetry: scope?*

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- understanding of features of leptons and/or quarks?  
lepton mixing seems to prefer unification of generations in 3  
quarks seem to prefer to decouple third generation:  $2 + 1$

# *Flavor symmetry: scope?*

---

- understanding of features of leptons and/or quarks?  
refining questions:  
are leptons and quarks governed by the same flavor symmetry?  
if not, what about grand unification?

# *Flavor symmetry: scope?*

---

- understanding of features of leptons and/or quarks?
- understanding of masses and/or mixing patterns?  
often uncorrelated for leptons  
charged lepton mass hierarchy from Froggatt-Nielsen  
symmetry  
lepton mixing from discrete non-abelian symmetry

# *Flavor symmetry: scope?*

---

- understanding of features of leptons and/or quarks?
- understanding of masses and/or mixing patterns?
- what shall the symmetry describe at all?  
current best fit values for lepton mixing angles?  
or “simple” mixing pattern at leading order like  
tri-bi-maximal mixing?

# *Flavor symmetry: scope?*

---

- understanding of features of leptons and/or quarks?
- understanding of masses and/or mixing patterns?
- what shall the symmetry describe at all?  
actually, this question entails further ones like:  
at which scale shall the symmetry predict masses and  
mixing parameters?  
what corrects “simple” mixing pattern?  
and how model-dependent are such corrections?

# *Flavor symmetry: choice*

---

- non-abelian vs abelian  
 $SU(3)$ , discrete subgroups of  $SU(3)$  vs  $Z_N, U(1)$

# *Flavor symmetry: choice*

---

- non-abelian vs abelian  
or in other words: mixing vs masses  
or in other words: predicted values vs orders of magnitude

# *Flavor symmetry: choice*

---

- non-abelian vs abelian
- continuous vs discrete  
 $SU(3)$  vs discrete subgroups of  $SU(3)$



# *Flavor symmetry: choice*

---

- non-abelian vs abelian
- continuous vs discrete  
or in other words: a few vs many symmetries  
or in other words: gross structures vs predicted values

# *Flavor symmetry: choice*

---

- non-abelian vs abelian
- continuous vs discrete
  - if continuous, should it be gauged?
  - if discrete, what about gauging it?

# *Flavor symmetry: choice*

---

- non-abelian vs abelian
- continuous vs discrete
- choice of residual symmetries  
are there any?

# *Features of models: choice*

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- framework
  - supersymmetric or not
  - with extra dimension or not/composite theories
  - grand unified theories
  - string theory

# *Features of models: choice*

---

- framework
- flavor symmetry breaking
  - spontaneous
    - with which type of fields?
    - at which scale?
    - to residual symmetries?
  - soft (arbitrary?)
  - explicit (arbitrary?)

# *Features of models: choice*

---

- framework
- flavor symmetry breaking
- neutrino masses
  - Dirac vs Majorana
  - tree level vs loop level

these choices also influence choice of flavor symmetry

# *Challenges*

---

- uniqueness of flavor symmetry?
  - many symmetries lead to very similar results
  - how to find the “correct” symmetry?

# Challenges

---

- uniqueness of flavor symmetry?
  - many symmetries lead to very similar results
  - how to find the “correct” symmetry?
    - be “economical” and use smallest symmetry ?  
there are very many models with  $A_4$  or  $S_4$  as flavor symmetry



# Challenges

---

- uniqueness of flavor symmetry?
  - many symmetries lead to very similar results
  - how to find the “correct” symmetry?
    - be “economical” and use smallest symmetry ?
    - use further input on symmetries from theory?
      - gauge (discrete/continuous) flavor symmetry  
e.g. *Araki et al. ('08)*
      - try to derive flavor symmetry  
e.g. *Altarelli/Feruglio ('06)*

# Challenges

- uniqueness of flavor symmetry?
  - many symmetries lead to very similar results
  - how to find the “correct” symmetry?
    - be “economical” and use smallest symmetry ?
    - use further input on symmetries from theory?
    - shall we rely on further data (in BSM theory)?
      - suppression of charged lepton flavor violation  
e.g. *Antusch/King/Malinsky, Feruglio et al. ('08)*
      - include Dark Matter, when choosing flavor symmetry  
e.g. *Hirsch et al. ('10)*

# *Challenges*

---

- uniqueness of flavor symmetry?
- corrections
  - do always exist, are always model-dependent
  - enter usually in several places:  
corrections to vacuum alignment, mixing of symmetry breaking sectors, ...
  - might be highly relevant and/or need to be strongly suppressed
  - arise from RG running and can be large

# *Challenges*

---

- uniqueness of flavor symmetry?
- corrections
- “technical” problems, like vacuum alignment

# *Arguments against flavor symmetry*

---

- fermion masses are stable under quantum corrections

# *Arguments against flavor symmetry*

---

- fermion masses are stable under quantum corrections
- no clear pattern visible:  
is  $\theta_{13}$  really much smaller than  $\theta_{12,23}$  in the lepton sector?  
(semi-)anarchy for neutrinos OK

*(Buchmüller/Domcke/Schmitz ('11), de Gouvea/Murayama ('12), Ding/Morisi/Valle ('12), Altarelli et al. ('12), Bergström/Meloni/Merlo ('14), Brdar/König/Kopp ('15), ...)*

# *Arguments against flavor symmetry*

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- fermion masses are stable under quantum corrections
- no clear pattern visible
- we only have few data:  
fermion masses, mixing angles, two CP phases

# *Arguments against flavor symmetry*

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- fermion masses are stable under quantum corrections
- no clear pattern visible
- we only have few data
- interpretation of data is too difficult:  
hard to decide what indicates symmetry, what not



# *Arguments against flavor symmetry*

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- fermion masses are stable under quantum corrections
- no clear pattern visible
- we only have few data
- interpretation of data is too difficult
- CP phase in quark sector is large, no good explanation yet  
one reason: all quark mixing angles are small  
for recent attempt see [Li/Lu/Ding \('17\)](#)

# *Arguments against flavor symmetry*

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- fermion masses are stable under quantum corrections
- no clear pattern visible
- we only have few data
- interpretation of data is too difficult
- CP phase in quark sector is large, no good explanation yet
- models are sophisticated, yet incomplete

# *What happens, if something new shows up?*

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- confirmation of light sterile neutrino?  
e.g. *Merle/Morisi/Winter ('14)*
- NSIs found?  
e.g. *Wang/Zhou ('18)*
- charged lepton flavor violation detected?  
for  $h \rightarrow \mu\tau$  e.g. *Heeck et al. ('14)*
- a new particle directly detected?

# *In the end something beyond flavor symmetry*

---

- can one get correct lepton mixing from RG flow?

*(Casas/Espinosa/Ibarra/Navarro ('99), Chankowski/Krolikowski/Pokorski ('00), Feruglio ('16))*

- could asymptotically safe theories be the solution?

first steps, see *Pelaggi et al. ('17)*

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- could asymptotically safe theories be the solution?

first steps, see *Pelaggi et al. ('17)*

I hope this presentation triggers some discussion.

Thank you for your attention.