Neutrino coherence and decoherence

Evgeny Akhmedov

Max-Planck Institute für Kernphysik, Heidelberg

MAX-PLANCK-INSTITUT
FÜR KERNPHYSIK

Coherence in QM

Coherence in quantum mechanics: Amplitudes corresponding to different intermediate states are summed.

Lack of coherence: Probabilities are summed.

Neutrino physics: An excellent playground for studying QM!

Examples:

- Neutrino oscillations –
  - Coherence of different mass eigenstates
  - Accel. expts. – Coherence of neutrino production at different points along the decay pipe: summation of probabilities

    $$P_{\alpha\beta}(E,L) \xrightarrow{?} \int_0^{l_p} dx P_{ab}(E,L-x) \qquad\qquad (x \in [0,l_p])$$

    or summation of amplitudes?
    (Hernandez & Smirnov, 2011; EA, Hernandez & Smirnov, 2012)
- Coherent neutrino nucleus scattering

- Coherence and decoherence in neutrino oscillations

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(Hint: the answer is mostly negative. One avenue is still being tested – work in progress).

When are neutrino oscillations observable?

Keyword: *Coherence*

Neutrino flavour eigenstates ν_e, ν_μ and ν_τ are coherent superpositions of mass eigenstates ν_1, ν_2 and ν_3 $\Rightarrow$ oscillations are only observable if

- neutrino production and detection are coherent
- coherence is not (irreversibly) lost during neutrino propagation.

Possible decoherence at production (detection): If by accurate E and p measurements one can tell (through $E=\sqrt{p^2+m^2}$) which mass eigenstate is emitted, the coherence is lost and oscillations disappear!

Full analogy with electron interference in double slit experiments: if one can establish which slit the detected electron has passed through, the interference fringes are washed out.

$\diamondsuit$ Decoherence is equivalent to averaging neutrino oscillations out.

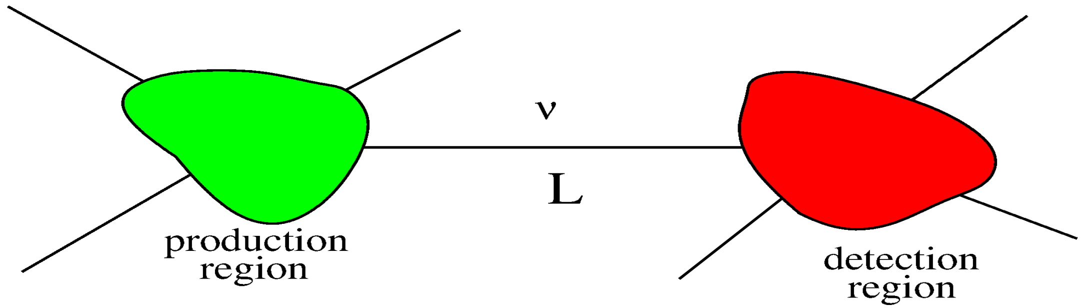
Oscillations: coherence of different ν_i

Usual assumption: the produced and detected neutrinos are flavour eigenstates

$$\diamondsuit \qquad |\nu_{\alpha L}\rangle = \sum_i U^*_{\alpha i}\,|\nu_{iL}\rangle \qquad\qquad (\alpha = e\,,\ \mu\,,\ \tau,\ \ i = 1\,,\ 2\,,\ 3)$$

$$\Updownarrow$$

oscillations



Intrinsic QM neutrino energy and momentum uncertainties (σ_E and σ_p) related to space-time localization of the production and detection processes play a crucial role.

Coherence vs. decoherence at ν production

E and p differences of neutrino mass eigenstates composing a flavour state:

$$\Delta E \equiv \Delta E_{ik} = \sqrt{p_i^2 + m_i^2} - \sqrt{p_k^2 + m_k^2}, \qquad \Delta p = p_i - p_k\,.$$

Production coherence condition (barring some cancellations): neutrino energy and momentum uncertainties must be sufficiently large to accommodate differing E_i and p_i:

$$\Delta E \ll \sigma_E, \qquad \Delta p \ll \sigma_p\,.$$

How are the oscillations destroyed when σ_E and σ_p are too small? Small σ_p means large uncertainty of the coordinate of neutrino production point. When it becomes larger than l_{osc} oscillations get washed out (Kayser 1981).

Configuration - space picture

Oscillation phase acquired over the distance x and time t:

$$\phi_{osc} = \Delta E \cdot t - \Delta p \cdot x\,.$$

Fluctuation of ϕ_{osc} due to uncertainty in 4-coordinate of neutrino production:

$$\delta\phi_{osc} = \Delta E \cdot \delta t - \Delta p \cdot \delta x\,,$$

δt and δx limited by the duration of the neutrino production process σ_t and its spatial extension σ_X: $\delta t \lesssim \sigma_t$, $|\delta x| \lesssim \sigma_X$.

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For oscillations to be observable $\delta\phi_{osc}$ must be small – otherwise oscillations will be washed out upon averaging over $(t_P,\, x_P)$ $\Rightarrow$

$$|\Delta E \cdot \delta t - \Delta p \cdot \delta x| \ll 1$$

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Barring accidental cancellations: $\Delta E \cdot \delta t \ll 1$, $\Delta p \cdot \delta x \ll 1$. From

$$\delta t \lesssim \sigma_t \sim \sigma_E^{-1}, \qquad \delta x \lesssim \sigma_X \sim \sigma_p^{-1} \qquad \Rightarrow$$

$$\diamondsuit \quad \Delta E \ll \sigma_E, \qquad \Delta p \ll \sigma_p \,.$$

Different neutrino mass eigenstates are produced (detected) coherently and hence neutrino oscillations may be observable only if the oscillation phase acquired over the space-time extension of the production (detection) region is much smaller than unity.

Propagation decoherence

Another source of decoherence: wave packet separation due to the difference of group velocities Δv of different mass eigenstates.

If coherence is lost: Flavour transition can still occur, but in a non-oscillatory way. E.g. for $\pi \to \mu\nu_i$ decay with a subsequent detection of ν_i with the emission of e:

$$P \;\propto\; \sum_i P_{\rm prod}(\mu\,\nu_i) P_{\rm det}(e\,\nu_i) \;\propto\; \sum_i |U_{\mu i}|^2 |U_{ei}|^2$$

– the same result as for averaged oscillations.

Wave packet separation

Wave packets representing different mass eigenstate components have different group velocities v_{gi} $\Rightarrow$ after time $t_{\rm coh}$ (coherence time) they separate $\Rightarrow$ Neutrinos stop oscillating! (Only averaged effect observable).

Coherence time and length:

$$\Delta v \cdot t_{\rm coh} \simeq \sigma_x\,; \qquad l_{\rm coh} \simeq v t_{\rm coh}$$

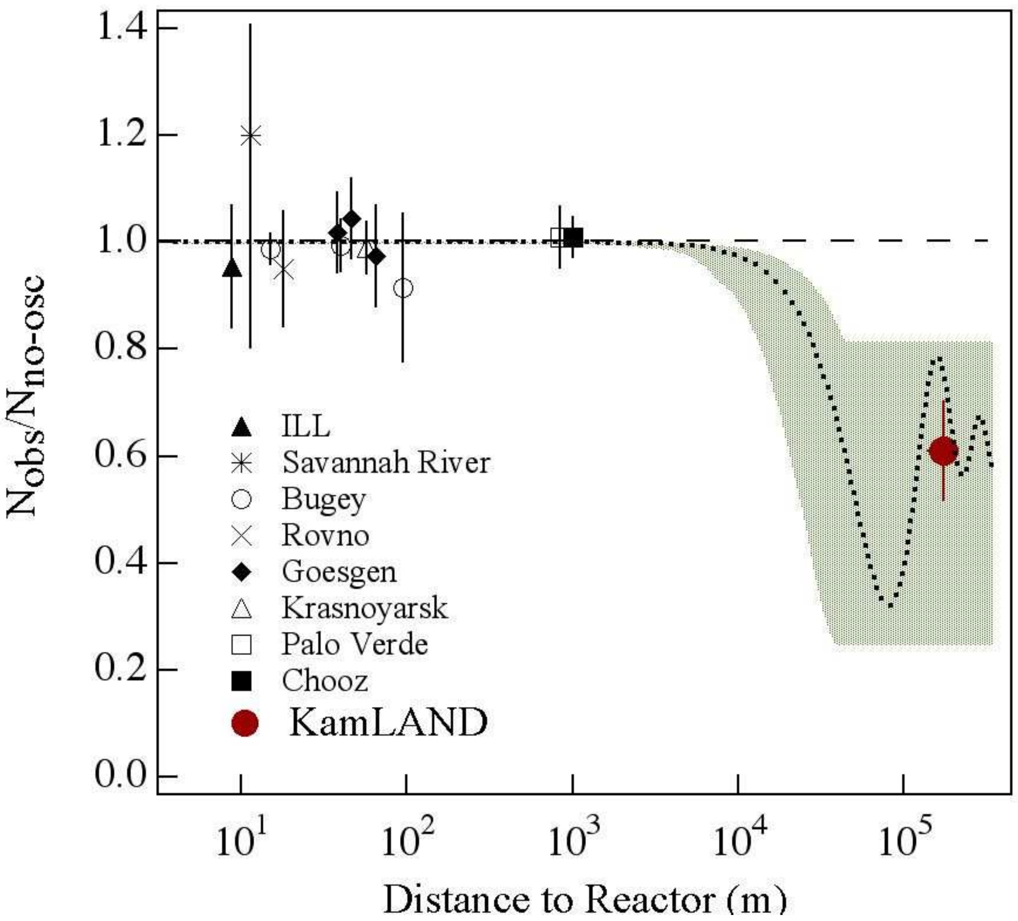
$$\Delta v = \frac{p_i}{E_i} - \frac{p_k}{E_k} \simeq \frac{\Delta m^2}{2E^2}$$

$$\boxed{l_{\rm coh} \simeq \frac{v}{\Delta v}\sigma_x = \frac{2E^2}{\Delta m^2}\, v\sigma_x}$$

The standard formula for $P_{\rm osc}$ is obtained when the decoherence effects are negligible.

A manifestation of neutrino coherence –

Non-observation of neutrino oscillations at short distances.



Expected: 365.2 ± 23.7

Background: 17.8 ± 7.3

Observed: 258

A manifestation of neutrino coherence

Even non-observation of neutrino oscillations at distances $L \ll l_{\rm osc}$ is a consequence of and an evidence for coherence of neutrino emission and detection! Two-flavour example (e.g. for ν_e emission and detection):

$$A_{\rm prod/det}(\nu_1) \sim \cos\theta\,, \qquad A_{\rm prod/det}(\nu_2) \sim \sin\theta \qquad \Rightarrow$$

$$A(\nu_e \to \nu_e) = \sum_{i=1,2} A_{\rm prod}(\nu_i) A_{\rm det}(\nu_i) = \cos^2\theta + e^{-i\Delta\phi}\sin^2\theta$$

Phase difference $\Delta\phi$ vanishes at short L $\quad\Rightarrow$

$$P(\nu_e \to \nu_e) = (\cos^2\theta + \sin^2\theta)^2 = 1$$

If ν_1 and ν_2 were emitted and absorbed incoherently $\quad\Rightarrow\quad$ one would have to sum probabilities rather than amplitudes:

$$P(\nu_e \to \nu_e) \sim \sum_{i=1,2} |A_{\rm prod}(\nu_i)|^2 |A_{\rm det}(\nu_i)|^2 \sim \cos^4\theta + \sin^4\theta \ < \ 1$$

Are coherence constraints compatible?

Observability conditions for ν oscillations:

- Coherence of ν production and detection
- Coherence of ν propagation

Both conditions put upper limits on neutrino mass squared differences Δm^2 :

$$(1) \quad \Delta E_{jk} \sim \frac{\Delta m_{jk}^2}{2E} \ll \sigma_E; \qquad\qquad (2) \quad \frac{\Delta m_{jk}^2}{2E^2} L \ll \sigma_x \simeq v_g/\sigma_E$$

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Are they compatible? – Yes, if LHS $\ll$ RHS $\Rightarrow$

$$\boxed{2\pi \frac{L}{l_{\rm osc}} \ll \frac{v_g}{\Delta v_g} \; (\gg 1)}$$

– fulfilled in most cases of practical interest

Coherence and Lorentz invariance

Consider coherent production conditions $\Delta E \ll \sigma_E$, $\Delta p \ll \sigma_p$ for ν born in $\pi^\pm$ decays in the rest frame of ν_2 (EA, 1703.08169):

$$\frac{|\Delta E'|}{\sigma'_E} \simeq \frac{\Delta m^2}{2m_2}\frac{\gamma_u}{\Gamma_\pi} \simeq \frac{\Delta m^2}{2E\Gamma_\pi}\gamma_u^2\,, \qquad \frac{|\Delta p'|}{\sigma'_{p\text{min}}} \simeq \frac{\Delta m^2}{2m_2}v_{g2}\frac{\gamma_u}{\Gamma_\pi} \simeq \frac{\Delta m^2}{2E\Gamma_\pi}\gamma_u^2$$

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Lorentz factor $\gamma_u = E/m_2 \gg 1 \quad \Rightarrow \quad$ the conditions $\Delta E' \ll \sigma'_E$, $\Delta p' \ll \sigma'_p$ can be violated for small enough m_2. Moreover, for non-rel. neutrinos quite generally $\Delta E \sim \bar{E} \gtrsim \sigma_E$!

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Resolution: the conditions $\Delta E \ll \sigma_E$, $\Delta p \ll \sigma_p$ are <u>not</u> Lorentz invarint. They follow form the Lorentz-inv. coherent production condition

$$|\Delta E \cdot \delta t - \Delta p \cdot \delta x| \ll 1$$

only assuming that the two terms on the LHS do not (approximately) cancel each other and are separately small.

In reality: Lorentz transformations with $u = -v_{g2} \simeq -1$ give

$$\delta t' = \gamma_u(\delta t + u\delta x) \simeq \gamma_u(\delta t - \delta x)\,, \qquad \delta x' = \gamma_u(\delta x + u\delta t) \simeq \gamma_u(\delta x - \delta t)\,,$$

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$$\delta\phi'_{osc} = \Delta E'\cdot\delta t' - \Delta p'\cdot\delta x' \simeq \Delta E'\cdot(\delta t' + \delta x') \simeq 0\,.$$

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– no enhancement of $\delta\phi'_{osc}$ actually occurs!
More accurate calculation (taking into account the small deviation of $u = -v_{g2}$ from -1):

$$\delta\phi'_{osc} = \delta\phi_{osc} \ll 1.$$

Conditions $\Delta E \ll \sigma_E$, $\Delta p \ll \sigma_p$ are valid only in the frames where the neutrino source is at rest or is slowly moving. Should be used with caution! Cannot be automatically extrapolated from one Lorentz frame to another.

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Sterile neutrinos: hints from SBL accelerator experiments (LSND, MiniBooNE), reactor neutrino anomaly, keV sterile neutrinos, pulsar kicks, leptogenesis via ν oscillations, SN r-process nucleosynthesis, unconventional contributions to $2\beta0\nu$ decay ...

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Production/detection coherence has to be re-checked – important implications for some neutrino experiments!

Oscillation probability in WP approach

Neutrino emission and detection times are not measured (or not accurately measured) in most experiments $\Rightarrow$ integration over T:

$$P(\nu_\alpha \to \nu_\beta; L) = \int dT\, P(\nu_\alpha \to \nu_\beta; T, L) = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\, e^{-i\frac{\Delta m_{ik}^2}{2P}L}\, \tilde{I}_{ik}$$

$$\boxed{\begin{aligned}\tilde{I}_{ik} = N \int \frac{dq}{2\pi} f_i^S(q - \Delta E_{ik}/2v + P) f_i^{D*}(q - \Delta E_{ik}/2v + P) \\ \times f_k^{S*}(q + \Delta E_{ik}/2v + P) f_k^D(q + \Delta E_{ik}/2v + P)\, e^{i\frac{\Delta v}{v}qL}\end{aligned}}$$

Here: $v \equiv \frac{v_i + v_k}{2}$, $\Delta v \equiv v_k - v_i$, $q \equiv p - P$, $N \equiv 1/[2E_i(P)2E_k(P)v]$

- For $(\Delta v/v)\sigma_p L \ll 1$ (i.e. $L \ll l_{\rm coh} = (v/\Delta v)\sigma_x$) $\tilde{I}_{ik}$ is approximately independent of L; in the opposite case $\tilde{I}_{ik}$ is strongly suppressed
- $\tilde{I}_{ik}$ is also strongly suppressed unless $\Delta E_{ik}/v \ll \sigma_p$, i.e. $\Delta E_{ik} \ll \sigma_E$ – coherent production/detection condition

Coherent neutrino scattering

Coherent elastic neutrino-nucleus scattering

NC – mediated neutrino-nucleus scattering:

$$\nu + A \to \nu + A$$

Incoherent scattering – Probabilities of scattering on individual nucleons add:

$\diamondsuit$ $\sigma \propto (\#$ of scatterers$)$

Coherent scattering on nucleus as a whole – Amplitudes of scattering on individual nucleons add

$\diamondsuit$ $\sigma \propto (\#$ of scatterers$)^2$

Significant increase of the cross sections (but requires small momentum transfer, $q \lesssim R^{-1}$)

(D.Z. Freedman, 1974)

COHERENT experiment

Neutrino energies: $E_\nu \sim 16 - 53~\mathrm{MeV}$. Nuclear recoil energy: keV - scale.

# of events expected (SM): 173 $\pm$ 48

# of events detected: 134 $\pm$ 22

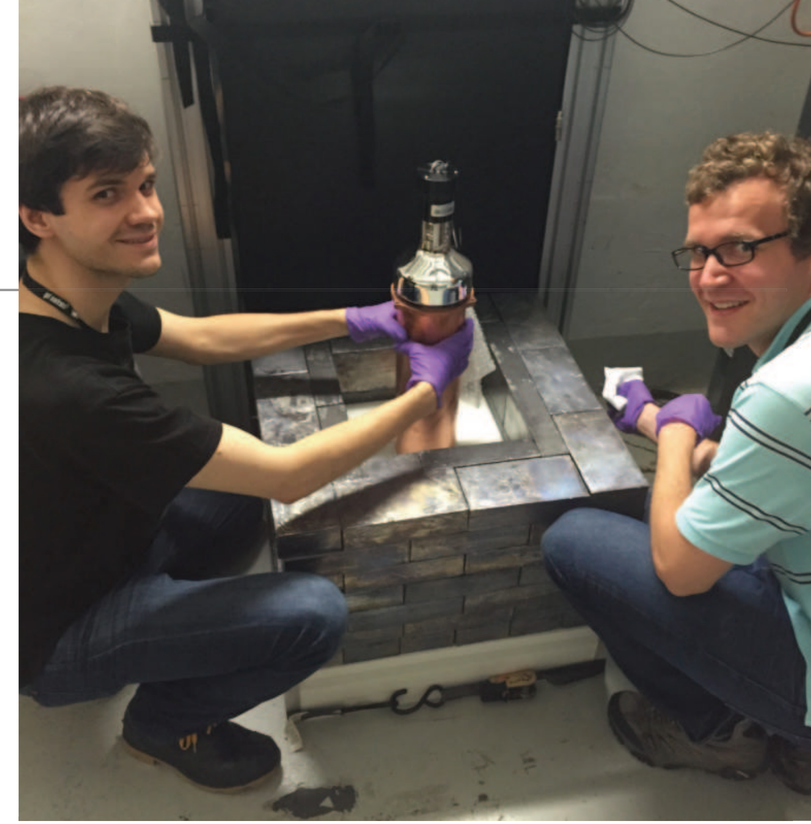
"We report a 6.7 sigma significance for an excess of events, that agrees with the standard model prediction to within 1 sigma"
$\sim 2 \times 10^{23}$ POT; $\sigma \sim 10^{-38}$ cm^2.

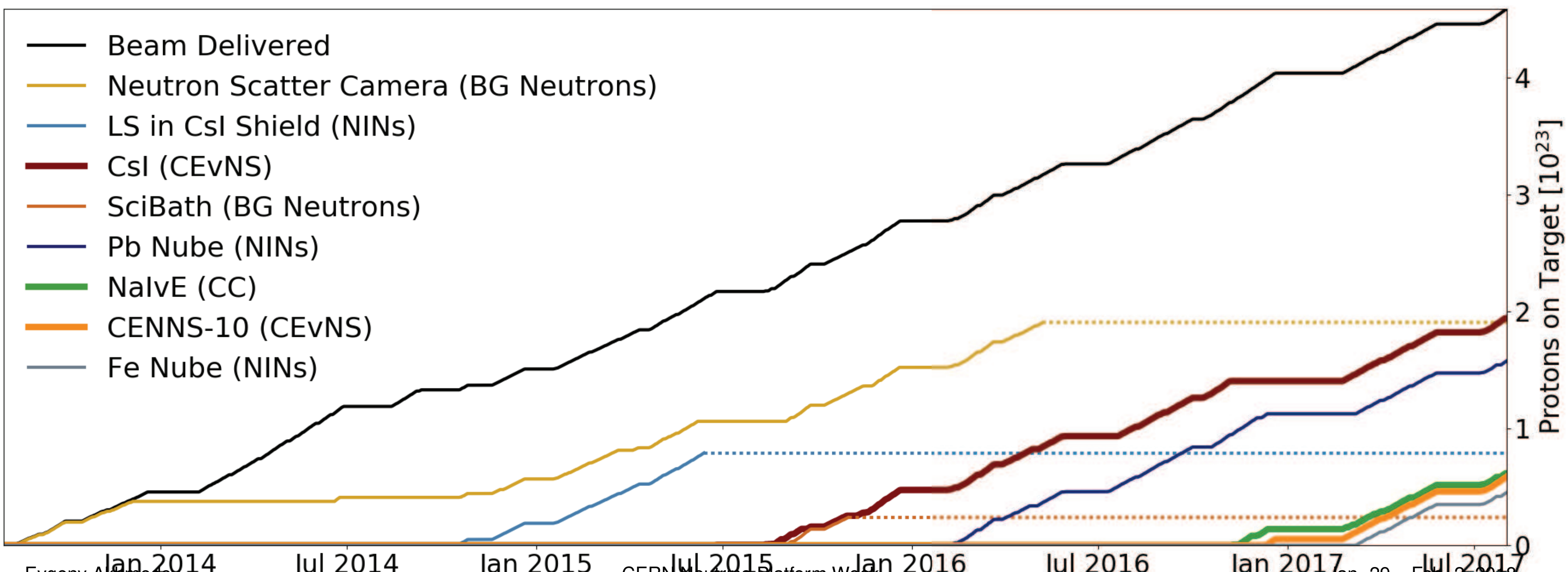
D. Akimov et al., Science 10.1126/science.aao0990 (2017).

A hand-held neutrino detector

- 14.6 kg low-background CsI[Na] detector deployed to a basement location of the SNS in the summer of 2015

- ~ $2\text{x}10^{23}$ POT delivered and recorded since CsI began taking data





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NC-induced neutrino-nucleus scattering: flavour blind.

$$\left[\frac{d\sigma_{\nu A}}{d\Omega}\right]_{\rm coh} \simeq \frac{G_F^2}{16\pi^2}[Z(4\sin^2\theta_W-1)+N]^2 E_\nu^2\,(1+\cos\theta)|F(\vec{q}^{\,2})|^2$$

$F(\vec{q}^{\,2})$ is nuclear formfactor:

$$F_{N(Z)}(\vec{q}^{\,2}) = \frac{1}{N(Z)}\int d^3x\rho_{N(Z)}(\vec{x})e^{i\vec{q}\vec{x}}, \qquad \vec{q}=\vec{k}-\vec{k}'.$$

For $q \ll R^{-1} \quad \Rightarrow \quad F(\vec{q}^{\,2})=1, \qquad \left[d\sigma_{\nu A}/d\Omega\right]_{\rm coh} \propto N^2$.

$$R \simeq 1.2\text{ fm}\,A^{1/3}; \quad A\sim 130 \quad \Rightarrow \quad R^{-1}\sim 30\text{ MeV}.$$

Recoil energy of the nucleus:

$$E_{rec} = \frac{\vec{q}^{\,2}}{2(M_A+2E_\nu)} \simeq \frac{\vec{q}^{\,2}}{2M_A}, \qquad E_{rec}^{max} = \frac{2E_\nu^2}{M_A+2E_\nu} \simeq \frac{2E_\nu^2}{M_A}.$$

Can one do better?

Coherent neutrino scattering on atoms:

- Advantages – larger number of particles (larger σ)
- CC scattering on electrons contributes – sensitivity to neutrino oscillations!
- Disadvantage: smaller q required $\Rightarrow$ much smaller recoil energies ($\sim 10^{-5}$ eV), extremely difficult to detect.

Can one have (at least in principle) macroscopic coherence?

Elastic ν scattering on macroscopic bodies

Simple estimates: for coherent scattering on a lamp of matter of linear size $\sim 1\ \mathrm{cm}$ one needs $q \lesssim q_0 \sim 10^{-5}\ \mathrm{eV}$. Gain: large number of particles in the coherent volume $1/q_0^3$.

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If typical $E_\nu \gg q_0 \sim 10^{-5}\ \mathrm{eV}$ (e.g., MeV-scale), small q means nearly forward ν scattering:

$$\vec{q}^{\,2} = 2E_\nu^2(1 - \cos\theta)$$

$\Rightarrow$ by limiting $q^2 < q_0^2$ we limit the solid angle; $\sigma \propto G_F^2 E_\nu^2 \to G_F^2 q_0^2$. But: we gain by the number of particles N in the coherent volume (get an extra factor $\propto 1/q_0^3$).

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$$\sigma_{tot} \propto N^{4/3}\,, \quad \text{not} \quad N^2.$$

Still for $N \sim N_A \simeq 6 \times 10^{23}$ a significant enhancement!

Elastic ν scattering on macroscopic bodies

The problem: detection.

Momentum transfers $q \lesssim q_0 \sim 10^{-5}$ eV to achieve a (1 cm)^3 - scale coherence would mean, for a 1 g target,

$$E_{rec} \simeq \frac{q_0^2}{2M_{tot}} \sim 10^{-43}\,\mathrm{eV}\,!$$

Leaving aside other problems, measuring such small E_{rec} would require energy resolution δE at least of the same order.
But: By time-energy uncertainty relation this would require the measurement time

$$\delta t \sim (\delta E)^{-1} \sim 10^{27}\,\mathrm{sec}$$

– 10 orders of maginutude larger than t_U !

$\Rightarrow$ New ideas are necessary.

Elastic neutrino scattering on crystals

Experiments of J. Weber in the 1980s: torsion balance expts.; sapfire crystal. Sources: solar neutrinos; reactor neutrinos; radioactive source.

The idea: if the recoil is given to the crystal as a whole (like in Mössbauer experiments), individual atoms (or nuclei) do not experience any recoil and therefore are not tagged. Coherence may occur at macroscopic level even for much larger momentum transfer – q as large as $T_{Debye} \sim 10\ \mathrm{keV}$ (rather than $10^{-5}\ \mathrm{eV}$) should work!

Recoil-free fraction

$$f \simeq \exp\left\{-\frac{E_R}{T_D}\left(\frac{3}{2}+\frac{\pi^2 T^2}{T_D^2}\right)\right\}$$

can be close to 1 for $E_R \lesssim T_D$.

Positive results claimed, in agreement with the proposed theoretical model. Force exerted on the crystal: $\sim 10^{-5}\ \mathrm{dyn}$.

Criticised from several viewpoints

- Ho, 1986: Approach excluded by expts. on neutron scattering on crystals
- Bertsch & Austin, 1986: Excluded by expts. on γ-ray scattering on crystals
- Franson & Jacobs, 1992: more sensitive torsion balance experiments with neutrinos – no signal observed
- Criticisms of Weber's theoretical model:
  - Casella, 1986
  - Butler, 1987
  - Smith, 1987
  - Aharonov, Avignone, Casher & Nussinov, 1987

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Now,

$$|F(\vec{q})|^2 \; = \sum_{i,j} e^{i\vec{q}(\vec{r}_i - \vec{r}_j)}.$$

In general, for $q \max\{|\vec{r}_i - \vec{r}_j|\} \simeq qL \ll 1$ one has $|F(\vec{q})|^2 \; \simeq \; \sum_{i,j} 1 = N^2$; in the opposite case $qL \gg 1$ only diagonal terms in the sum contribute, $|F(\vec{q})|^2 \; = \; N$.

Crystals are a special case. $|\vec{q}|$ need not be very small! For

$$\vec{q}(\vec{r}_i - \vec{r}_j) = 2\pi n_{ij}$$

– constructive interference, $d\sigma \propto N^2$. $\Leftrightarrow$ Bragg condition:

$$2d \sin\theta = n\lambda$$

(d is interplanar distance, $\lambda = 2\pi/k$). But: since $\lambda \ll d$, Bragg maxima are of high order $\Rightarrow$ very narrow solid angles around Bragg directions. When integrated over solid angle lead to the usual $\propto N$ dependence.

Need something else. Can we exploit recoil momentum rather than recoil energy?

One possibility: Photon emission accompanying neutrino scattering

$$\nu + A \to \nu + A + \gamma$$

Photon momentum can be of the order of neutrino momentum transfer. For $q \sim (1\ \mathrm{cm})^{-1}$ – photons in radiowave diapason. We lose at least a factor of α, but can we gain due to increased coherence?

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Another possibility – bremsstrahlung on free electrons, $\nu + e \to \nu + e + \gamma$. First considered by Lee and Sirlin (1964) and then by many other people. In all but one paper – also not in connection with macroscopic coherence.

An exception: implications for detection of relic neutrinos.

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A Detector for the Cosmic Neutrino Background

Abraham Loeb and Glenn D. Starkman
Institute for Advanced Study, Princeton, NJ 08540

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Can cosmic neutrinos be detected by bremsstrahlung from a metal?

Safi Bahcall
Department of Physics, Stanford University, Stanford, California 94305

Andrew Gould
Institute for Advanced Study, Princeton, New Jersey 08540

(Received 23 July 1990)

We examine the proposal of Loeb and Starkman to detect cosmic-background neutrinos by coherent bremsstrahlung from electrons in a neutral metal. We show that the positive ions in the metal exert a restoring force which suppresses the radiation by $\sim 10^{-20}$.

I. INTRODUCTION

Loeb and Starkman have recently proposed a method for detecting cosmic-background neutrinos.[1] The method relies on the coherent interaction of neutrinos with bulk matter to get an enhanced cross section for neutrinos bouncing off a slab of metal. The neutrinos accelerate conduction electrons which then radiate low-

many particles acting as a lump. In the soft-photon limit, the probability distribution for radiation is obtained by multiplying the classical radiation distribution (2.1) by the probability that the neutrino will scatter.

The probability for a neutrino to scatter off an extended object, a slab in this case, is found by summing the scattering amplitudes off individual charges times the appropriate phase shift for each scatterer:

Neutrino scattering pushes free electrons inside the metal target. For electron displacement Δz: surface charge density of positive ions $\sigma_c \simeq \Delta z n_e e \quad \Rightarrow$ restoring force per unit mass $\omega_p^2 \Delta z$ ($\omega_p = [4\pi n_e e^2/m_e]^{1/2} \sim 10 \text{ eV}$ is plasma frequency). Coherent scattering gets strongly suppressed:

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A well-known example: Rayleigh scattering *vs* Thomson scattering. For photon scattering on neutral atoms

$$\sigma_R(\omega) \sim \left[\frac{\omega^2}{(\omega^2 - \omega_0^2)}\right]^2 \sigma_T$$

(ω_0 a characteristic atomic frequency, $\sigma_T = (8\pi/3)(\alpha/m_e)^2$ – Thomson γe cross section). For $\omega \ll \omega_0$ – suppressed as $(\omega/\omega_0)^4$.

Backup slides

Oscillation probability in WP approach

Neutrino emission and detection times are not measured (or not accurately measured) in most experiments $\Rightarrow$ integration over T:

$$P(\nu_\alpha \to \nu_\beta; L) = \int dT\, P(\nu_\alpha \to \nu_\beta; T, L) = \sum_{i,k} U_{\alpha i}^* U_{\beta i} U_{\alpha k} U_{\beta k}^*\, e^{-i\frac{\Delta m_{ik}^2}{2\bar{P}}L}\, \tilde{I}_{ik}$$

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Here: $v \equiv \frac{v_i + v_k}{2}\,,\quad \Delta v \equiv v_k - v_i\,,\quad q \equiv p - P\,,\quad N \equiv 1/[2E_i(P)2E_k(P)v]$

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- $\tilde{I}_{ik}$ is also strongly suppressed unless $\Delta E_{ik}/v \ll \sigma_p$, i.e. $\Delta E_{ik} \ll \sigma_E$ – coherent production/detection condition

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They may still be realized if relatively heavy sterile neutrinos exist