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Equivalance of 1801.10488 to 1604.08167

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Abstract: We give the one to one mapping between 1801.10488 and 1604.08167.

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Contents

1 Mapping between 1801.10488 and 1604.08167

The key results of 1801.10488, [1], are contained in equations (34), (35), (36) and (38). (37) is just a different way to write (38). Here, we show how to trivially derive these equations by using the results of 1604.08167, [2], and changing the notation. We first consider θ_{13}, then θ_{12} and Δm^2_{21} and finally Δm^2_{31} in matter.

1.1 $\theta^m_{13} = \phi$: Start from 1604.08167/eqn 2.3.5:

$$\cos 2\phi = \frac{(\lambda_c - \lambda_a)}{\Delta\lambda_{+-}} \quad \text{where} \quad \Delta\lambda_{+-} \equiv \lambda_+ - \lambda_- = \sqrt{(\lambda_c - \lambda_a)^2 + (\sin 2\theta_{13}\Delta m^2_{ee})^2}$$

$$\lambda_c - \lambda_a = \Delta m^2_{ee}\cos 2\theta_{13} - a = \Delta m^2_{ee}(\cos 2\theta_{13} - \epsilon_a) \quad \text{with } \epsilon_a \equiv a/\Delta m^2_{ee}$$

$$\text{using} \quad \lambda_a = a + s^2_{13}\Delta m^2_{ee} + s^2_{12}\Delta m^2_{21}, \quad \lambda_c = c^2_{13}\Delta m^2_{ee} + s^2_{12}\Delta m^2_{21}$$

$$\Delta\lambda_{+-} = \Delta m^2_{ee}\sqrt{(\cos 2\theta_{13} - \epsilon_a)^2 + \sin^2 2\theta_{13}} \qquad \text{used later in } \widetilde{\Delta \mathrm{m}^2}_{31}$$

$$\cos 2\phi = \frac{(\cos 2\theta_{13} - \epsilon_a)}{\sqrt{(\cos 2\theta_{13} - \epsilon_a)^2 + \sin^2 2\theta_{13}}} = \cos 2\theta^m_{13} \quad = 1801.10488/\text{eqn } (34).$$

1.2 $\theta^m_{12} = \psi$ and $\widetilde{\Delta m^2}_{21} = \Delta\lambda_{21}$: Start from 1604.08167/eqn 2.4.9:

$$\cos 2\psi = \frac{(\lambda_0 - \lambda_-)}{\Delta\lambda_{21}} \quad \text{where} \quad \Delta\lambda_{21} = \sqrt{(\lambda_0 - \lambda_-)^2 + (\cos(\phi - \theta_{13})\sin 2\theta_{12}\Delta m^2_{21})^2}$$

$$\lambda_0 - \lambda_- = \Delta m^2_{21}\cos 2\theta_{12} - (ac^2_\phi + \Delta m^2_{ee}\sin^2(\phi - \theta_{13})) = \Delta m^2_{21}(\cos 2\theta_{12} - \epsilon_\odot)$$
$$\text{with } \epsilon_\odot \equiv (ac^2_\phi + \Delta m^2_{ee}\sin^2(\phi - \theta_{13}))/\Delta m^2_{21}$$

$$\text{using} \quad \lambda_0 = \lambda_b = c^2_{12}\Delta m^2_{21}, \quad \lambda_- = \lambda_a c^2_\phi + \lambda_c s^2_\phi - 2\Delta m^2_{ee}\, s_{13}c_{13}\, s_\phi c_\phi$$

$$\Delta\lambda_{21} = \Delta m^2_{21}\sqrt{(\cos 2\theta_{12} - \epsilon_\odot)^2 + \cos^2(\phi - \theta_{13})\sin^2 2\theta_{12}} \quad = 1801.10488/\text{eqn } (36)$$

$$\cos 2\psi = \frac{(\cos 2\theta_{12} - \epsilon_\odot)}{\sqrt{(\cos 2\theta_{12} - \epsilon_\odot)^2 + \cos^2(\phi - \theta_{13})\sin^2 2\theta_{12}}} = \cos 2\theta^m_{12} \quad = 1801.10488/\text{eqn } (35)$$

1.3 $\widetilde{\Delta m^2}_{31} = \Delta\lambda_{31}$: Starting from 1604.08167/eqn 2.4.5:

$$\begin{aligned}\Delta\lambda_{31} &= \lambda_3 - \lambda_1 = \lambda_+ - \frac{1}{2}(\lambda_0 + \lambda_- - \Delta\lambda_{21}) \qquad \text{then using} \quad \lambda_\pm = \frac{1}{2}(\lambda_a + \lambda_c \pm \Delta\lambda_{+-}) \\ &= \frac{3}{4}\Delta\lambda_{+-} + \frac{1}{4}(\lambda_a + \lambda_c - 2\lambda_0) + \frac{1}{2}\Delta\lambda_{21} \\ &= \frac{3}{4}\Delta\lambda_{+-} + \frac{1}{4}(\Delta m^2_{ee} + a) + \frac{1}{2}(\Delta\lambda_{21} - \Delta m^2_{21}\cos 2\theta_{12}) = \widetilde{\Delta m^2}_{31} \quad = 1801.10488/\text{eqn } (38)\end{aligned}$$

with $\Delta\lambda_{+-}$ and $\Delta\lambda_{21}$ given above.

2 Appendix A of 1801.10488

Eqn (44) of Appendix A in 1801.10488 is equivalent to eqn 3.2.5 of 1604.08167.

References

[1] A. Ioannisian and S. Pokorski, "Three Neutrino Oscillations in Matter," arXiv:1801.10488 [hep-ph].

[2] P. B. Denton, H. Minakata and S. J. Parke, "Compact Perturbative Expressions For Neutrino Oscillations in Matter," JHEP **1606**, 051 (2016) doi:10.1007/JHEP06(2016)051 [arXiv:1604.08167 [hep-ph]].