Left-Right Symmetry: Minimal Model and Radiative Neutrino Mass

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Outline

Standard Model

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

Fermion representation:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

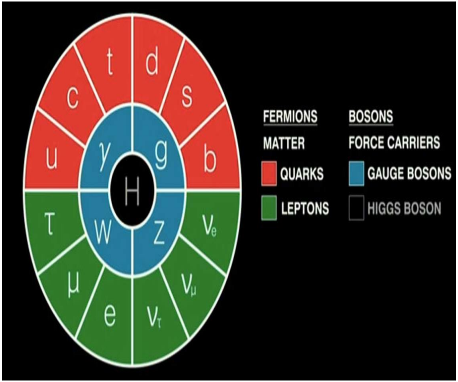
$e_R, \; u_R, \; d_R$

Higgs representation:

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Note: **No** ν_R **and** $m_\nu = 0$

Parity is maximally broken.



Motivation

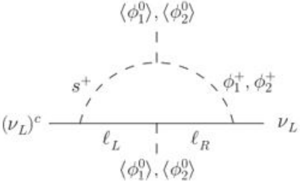
Neutrino Oscillation: Neutrino of different flavor mix in same manner as different quark flavor mix through Cabibo rotations. Thus, beam of ν can oscillate in vacuum into ν of different flavors. $\nu_{\mathsf{aL}} \leftrightarrow \nu_{\mathsf{bL}}$. This implies $\mathsf{m}_\nu \neq 0$, and requires new physics beyond SM.

Motivation for LR Symmetric Models

- Parity is explicitly broken by SM. LR symmetric model restores Parity.
- ν_R exists for $\mathrm{SU(2)_R}$ multiplet. $\mathrm{SU(2)_R}$ breaking gives heavy Majorana right handed neutrino. Thus, smallness of left-handed neutrinos is naturally realized via see-saw mechanisms.
- In SM Y (hyper charge) is arbitrary quantum number whereas in LR symmetric model Y arises more coherently from less arbitrary quantity B-L.

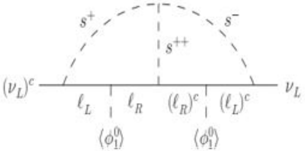
$$Y = T_R^3 + \frac{B-L}{2}$$

- Introduce ν_R . But it requires Yukawa coupling to be same order as of quark and charged leptons. But, observation shows $m_\nu << m_q$ or m_l. Introduce large Majorana mass scale Λ to suppress the neutrino mass via see-saw mechanism as $< \phi >^2 / \Lambda$.
- Radiative correction: Assumes $m_\nu = 0$ at tree level as SM and generates small mass of neutrino at 1-loop or 2-loop introducing new heavy scalar fields.



Zee Model

New particles: s^+ and second $SU(2)_L$ doublet ϕ



Zee-Babu Model

New particles: s^+ and s^{++}

Left-Right Symmetric Model

$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

Fermion representation:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_R \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_R$$

Higgs representation:

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \qquad \Delta_l = \begin{pmatrix} \frac{\Delta_L^+}{\sqrt{2}} & \Delta_L^{++} \\ \Delta_L^0 & -\frac{\Delta_L^+}{\sqrt{2}} \end{pmatrix}$$

$$\Delta_R = \begin{pmatrix} \frac{\Delta_R^+}{\sqrt{2}} & \Delta_R^{++} \\ \Delta_R^0 & -\frac{\Delta_R^+}{\sqrt{2}} \end{pmatrix}$$

Standard LR Model

$$< \phi >= \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \qquad < \Delta_L >= \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}$$

$$< \Delta_R >= \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

$$\Big\downarrow \; < \Delta_R > \neq 0$$

$$SU(2)_L \otimes U(1)_Y$$

$$\Big\downarrow \; < \phi >$$

$$U(1)_{em}$$

Neutrino Mass on Left-Right Models

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

- **Type-I plus Type II seesaw**

  $\phi \sim (2,2,0)$ $\Delta_L \sim (3,1,2)$ $\Delta_R \sim (1,3,2)$

  $< \Delta > \neq 0$ *breaks* $SU(2)_R \otimes U(1)_{B-L}$ *to* $U(1)_Y$, generating $M_{\upsilon,R}$. $M_{\upsilon,L}$ is generated via type-II see saw.

  [Ref: Mohapatra and Senjanovic; Lazarides, Shafi and Senjanovic; Scjecter, Valle]

- **Type III seesaw**

  Addition of fermionic triplet: $\rho_L \sim (3,1,0)$ $\rho_R \sim (1,3,0)$

  In addition, Higgs doublet is needed. $\chi_L \sim (2,1,1)$ $\chi_R \sim (1,2,1)$

  [Ref: P. Fileviez Perez, 2009]

- **Radiative seesaw**

  $\phi \sim (2,2,0)$ $\chi_L \sim (2,1,1)$ $\chi_R \sim (1,2,1)$

  And $\eta^+ \sim (1,1,2)$ can generate neutrino masses

Radiative ν_R Mass generation: Particle Spectrum

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

Fermion representation

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \sim (2,1,1/3) \qquad \psi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \sim (2,1,-1)$$

$$Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R \sim (1,2,1/3) \qquad \psi_R = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_R \sim (1,2,-1)$$

Higgs representation

$$\eta^+ \sim (1,1,2) \qquad \chi_L \sim (2,1,1)$$

$$\phi \sim (2,2,0) \qquad \chi_R \sim (1,2,1)$$

$$\chi_L \sim (2,1,1) \oplus \chi_R \sim (1,2,1) \oplus \phi \sim (2,2,0)$$

Particle Higgs content

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \qquad \tilde{\phi} = \tau_2 \phi^* \tau_2 = \begin{pmatrix} \phi_1^{0^*} & -\phi_1^+ \\ -\phi_2^- & \phi_2^{0^*} \end{pmatrix}$$

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix} \qquad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}$$

$$\eta^+$$

Electric charge is given by: $Q = T_L^3 + T_R^3 + \frac{B-L}{2}$

Under L-R symmetry:

$$\psi_L \leftrightarrow \psi_R \qquad \chi_L \leftrightarrow \chi_R \qquad \phi \leftrightarrow \phi^\dagger \qquad \eta^+ \leftrightarrow \eta^+ \qquad W_L^\pm \leftrightarrow W_R^\pm$$

Interaction of scalar η^+ with fermions

$$\mathcal{L}_Y \supset f_{ab}\,[\,(\psi^i_{aL} C\, \psi^j_{bL})\,\epsilon_{ij}\,\eta^+ + (\psi^i_{aR} C\, \psi^j_{bR})\,\epsilon_{ij}\,\eta^+\,] + \text{h. c.}$$

where (ab) represents generation and $f_{ab} = -f_{ba}$

(ij) are SU(2) indices

C is the charge conjugation matrix

$$\mathcal{L}_Y \sim 2[\,f_{e\mu}\,(\overline{\upsilon^c_e}\,\mu_L\eta^+ - \overline{\upsilon^c_\mu}\,e_L\eta^+\,) + \boxed{[f_{e\tau}\,(\overline{\upsilon^c_e}\,\tau_L\eta^+} - \overline{\upsilon^c_\tau}\,e_L\eta^+\,) +$$

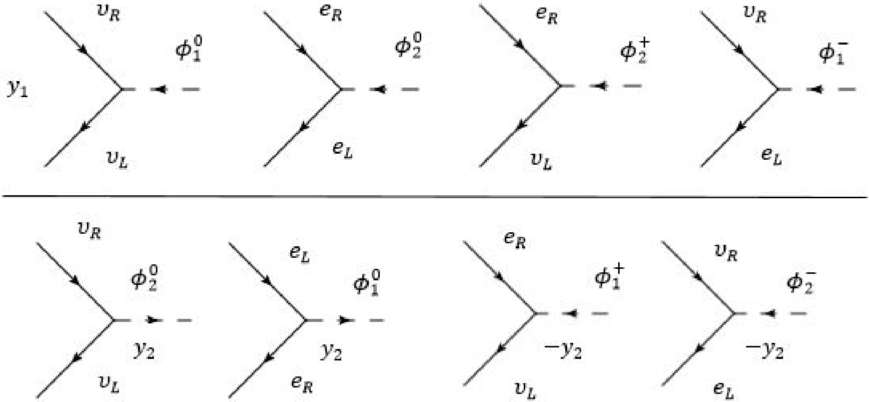
$$f_{\mu\tau}\,(\overline{\upsilon^c_\mu}\,\tau_L\eta^+ - \overline{\upsilon^c_\tau}\,\mu_L\eta^+\,)] + L \leftrightarrow \mathrm{R} + \text{h.c.}$$

Interaction of scalar ϕ with ferminos

$$\mathcal{L}_Y \supset y_1 \overline{\psi_L}\phi\,\psi_R + y_2 \overline{\psi_L}\,\tilde{\phi}\psi_R + \text{h.c.}$$

$$= y_1\left(\bar{\upsilon}_L\phi_1^0\,\upsilon_R + \bar{\upsilon}_L\phi_2^+\,e_R + \bar{e}_L\phi_1^-\,\upsilon_R + \bar{e}_L\phi_2^0\,e_R\right) +$$

$$y_2\left(\overline{\upsilon_L}\phi_2^{0^*}\,\upsilon_R - \overline{\upsilon_L}\phi_1^+\,e_R - \overline{e_L}\phi_2^-\,\upsilon_R + \overline{e_L}\,\phi_1^{0^*}\,e_R\right) + \ldots\ldots + \text{h.c.}$$



Self Interaction of Higgs particles by Higgs potential

$$\mathcal{L}_{\text{Higgs}} = Tr[(D_\mu\phi)^\dagger (D_\mu\phi)] + \left|D_\mu \chi_L\right|^2 + \left|D_\mu \chi_R\right|^2 + V(\phi, \chi_L, \chi_R, \eta)$$

$$V(\phi, \chi_L, \chi_R, \eta) = V(\phi) + V(\chi_L, \chi_R) + V(\eta) + V(cross - terms)$$

$$\begin{aligned} V(\phi) = & -\mu_1^2 Tr(\phi^\dagger\phi) - \mu_2^2 \left[Tr(\tilde{\phi}\phi^\dagger) + Tr\left(\tilde{\phi}^\dagger \phi\right)\right] + \lambda_1 [Tr(\phi^\dagger\phi)]^2 \\ & + \lambda_2 \left\{ \left[Tr\left(\tilde{\phi}^\dagger \phi\right)\right]^2 + [Tr(\tilde{\phi}\phi^\dagger)]^2 \right\} + \lambda_3 Tr(\tilde{\phi}\phi^\dagger) Tr\left(\tilde{\phi}^\dagger \phi\right) \\ & + \lambda_3 Tr(\phi^\dagger\phi)[Tr(\tilde{\phi}\phi^\dagger) + Tr(\tilde{\phi}\phi^\dagger)] \end{aligned}$$

Under transformation: $\chi_L \to U_L \chi_L$, $\phi \longrightarrow U_L \phi U_R^\dagger$

$$\begin{aligned} V(\chi_L, \chi_R) = & -\mu_3^2 \left(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R\right) + \rho_1 \left[(\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2 \right] \\ & + \rho_2 \ \chi_L^\dagger \chi_L \chi_R^\dagger \chi_R \end{aligned}$$

$$V(\eta) = -\mu_4^2 |\eta|^2 + \rho_3 |\eta|^4$$

Contd.

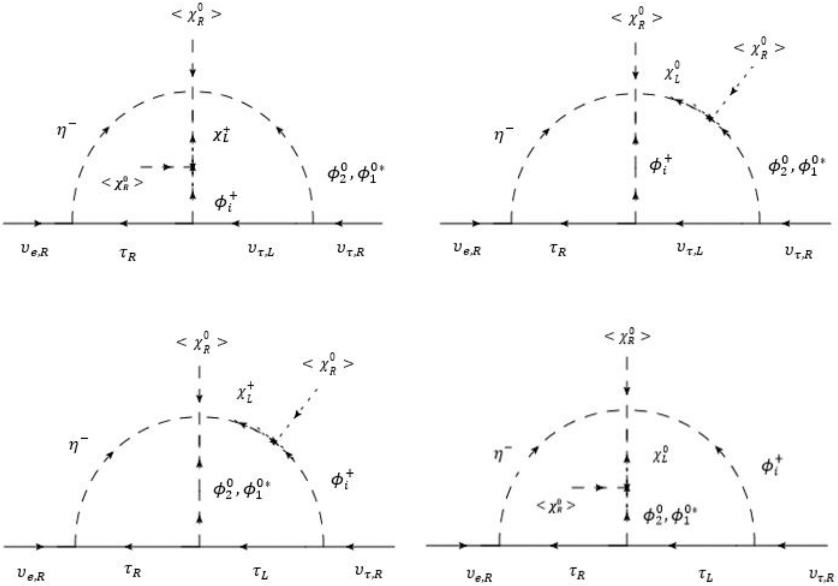
$$
\begin{aligned}
&V(cross-terms) \\
&= \alpha_1 |\eta|^2 Tr(\phi^\dagger \phi) + \alpha_2 |\eta|^2 \left[Tr(\tilde{\phi}\phi^\dagger) + Tr\left(\tilde{\phi}^\dagger \phi\right)\right] \\
&+ \alpha_3 |\eta|^2 \left(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R\right) + \boxed{\alpha_4 \left[\chi_L^\dagger \phi\, \chi_R + \chi_R^\dagger \phi^\dagger \chi_L\right]} \\
&+ \boxed{\alpha_5 \left[\chi_L^\dagger \tilde{\phi}\, \chi_R + \chi_R^\dagger \tilde{\phi}^\dagger \chi_L\right]} + \alpha_6 \left(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R\right) Tr(\phi^\dagger \phi) \\
&+ \alpha_7 \left(Tr(\tilde{\phi}\phi^\dagger)(\chi_L^\dagger \chi_L) + Tr\left(\tilde{\phi}^\dagger \phi\right)(\chi_R^\dagger \chi_R)\right) \\
&+ \alpha_7^* \left[Tr\left(\tilde{\phi}^\dagger \phi\right).(\chi_L^\dagger \chi_L) + Tr(\tilde{\phi}\phi^\dagger).(\chi_R^\dagger \chi_R)\right] + \alpha_8 [\chi_L^\dagger \phi\, \phi^\dagger \chi_L \\
&+ \chi_R^\dagger \phi^\dagger \phi\; \chi_R] + \boxed{\alpha_9 \left[\chi_L^T i\, \tau_2 \phi\, \chi_R \eta^- + \chi_R^T i\, \tau_2 \phi^\dagger \chi_L \eta^-\right] + h.c.}
\end{aligned}
$$

Contd.

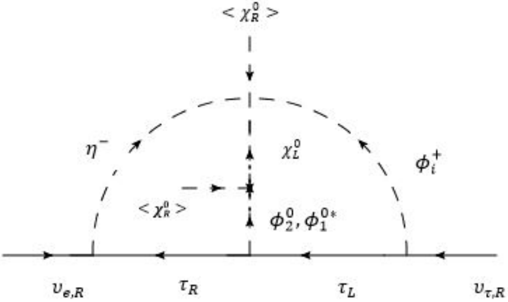
Taking only the relevant expressions from the cross-terms,

$$
\begin{aligned}
V(\,cross - terms) & \\
&= \alpha_4 \left[{\chi_L}^{0*} \phi_2^0 \chi_R^0 + {\chi_L}^{0} \phi_2^{0^*} \chi_R^{0*} \right] + \alpha_5 \left[{\chi_L}^{0} \phi_1^0 \chi_R^{0*} + {\chi_L}^{0*} \phi_1^{0^*} \chi_R^0 \right] \\
&+ \alpha_9 \left[- {\chi_L}^{0} \phi_1^{+} \chi_R^0 \eta^- - {\chi_L}^{0} \phi_2^{+} \chi_R^0 \eta^- - {\chi_L}^{+} \phi_1^{0*} \chi_R^0 \eta^- + {\chi_L}^{+} \phi_2^{0} \chi_R^0 \eta^- \right] \\
&+ \alpha_9^* \left[- \eta^+ {\chi_L}^{0*} \phi_1^{-} \chi_R^{0*} - \eta^+ {\chi_L}^{0*} \phi_2^{-} \chi_R^{0*} - {\chi_L}^{-} \phi_1^{0} \chi_R^{0*} \eta^- + {\chi_L}^{-} \phi_2^{0*} \chi_R^{0*} \eta^- \right]
\end{aligned}
$$

Loop Diagrams



Loop Diagram



$$M_{\upsilon,R} \sim \frac{(f\, y_i^\dagger y_j + y_j^T y_i^* f^T)}{(16\,\pi^2)^2} \frac{\mu_k\ \alpha_9 \upsilon_R^2}{M^2}$$

$$M^2 \approx M_\eta^2 + M_\phi^2$$

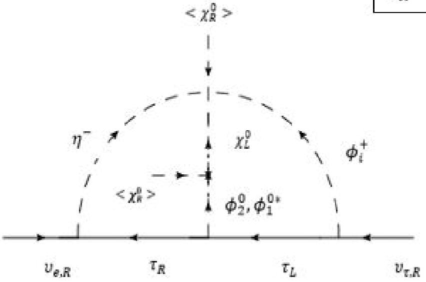
$$\mu_k \sim \upsilon_R$$

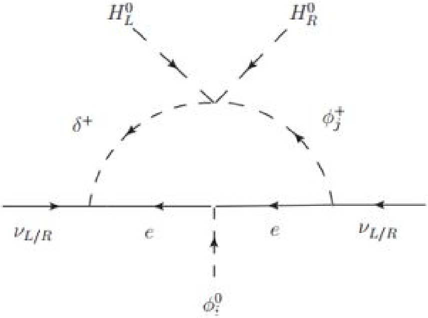
$$y_i \sim (\, O(10^{-2}) - 1)$$

$$M_{\upsilon,R} = (10^{-8} \sim 10^{-4})\ \upsilon_R$$

Comparison with previous Model

$$\boxed{\begin{array}{l} f, y \sim 1 \\ v_R \sim 10^7 \end{array}}$$





$$M_{\upsilon,R} \sim \frac{f\, y^2}{(16\,\pi^2)^2}\, v_R$$

$$M_{\upsilon,R} \sim 1 \text{ TeV}$$

$$M_{\upsilon,R} \sim 0.4 \text{ GeV}$$

Ref: Perez, Murgui, Ohmer

Mass Matrices

After Symmetry breaking masses reads,

Charged Lepton: $M_{l^+} = y_1 \, k' + y_2 \, k^*$

Dirac Mass: $M_{\upsilon,D} = y_1 \, k + y_2 \, k'^*$

Where k and k' are vacuum expectation value for ϕ_1^0 andϕ_2^0.

In the limit k' → 0,

$$M_{l^+} \approx y_2 \, k^*$$

$$M_{\upsilon,D} \approx y_1 \, k$$

After performing loop calculation, neutrino mass matrix reads:

$$\begin{pmatrix} \upsilon & \upsilon^c \end{pmatrix} \begin{pmatrix} 0 & M_{\upsilon,D} \\ (M_{\upsilon,D})^T & M_{\upsilon,R} \end{pmatrix} \begin{pmatrix} \upsilon \\ \upsilon^c \end{pmatrix}$$

$$M_\upsilon^{light} \approx - \, (M_{\upsilon,D}) \, (M_{\upsilon,R})^{-1} \, (M_{\upsilon,D})^T$$

Prediction of the Model

$$M_{\upsilon,R} \sim \frac{f\, y^2}{(16\,\pi^2)^2}\,\frac{\mu_k\;\alpha_9\;\upsilon_R^2}{M^2}$$

$$M_{\upsilon,L} \sim \frac{f\, y^2}{(16\,\pi^2)^2}\,\frac{\mu_k\;\alpha_9\upsilon_L^2}{M^2}$$

$$\frac{M_{\upsilon,L}}{M_{\upsilon,R}} \sim \frac{\upsilon_L^2}{\upsilon_R^2} \quad \Rightarrow \quad M_{\upsilon,R} \sim M_{\upsilon,L}\,\frac{\upsilon_R^2}{\upsilon_L^2} \leq (10^{-10}GeV)\,\frac{\upsilon_R^2}{\upsilon_L^2}$$

$$\upsilon_L \sim (1-100)\;GeV$$

$$\boxed{\upsilon_R \geq (10^5{\sim}10^7)\sqrt{\frac{M_{\upsilon,R}}{GeV}}}$$

Note: 1) $M_{W_R} \sim \upsilon_R$, *if* W_R *is discovered at LHC* $\Rightarrow$ υ_R *is small* $\Rightarrow$ $M_{\upsilon R}$ *has to be small*

2) No Δ^{++}

Summary

- Triplet is bigger representation than doublet. Thus, LR Symmetric Model with Higgs doublet is studied.
- η^+ is added as doublet by itself cannot generate RH ν Majorana mass.
- $\mathrm{m}_{\nu \mathrm{R}} << \mathrm{m}_{\mathrm{wR}}$ due to 2-loop suppression. However, this is still large enough to realize see-saw.