Quantum Tomography Finds Unexpected Vortex-like Spin Distribution in Z Boson Production

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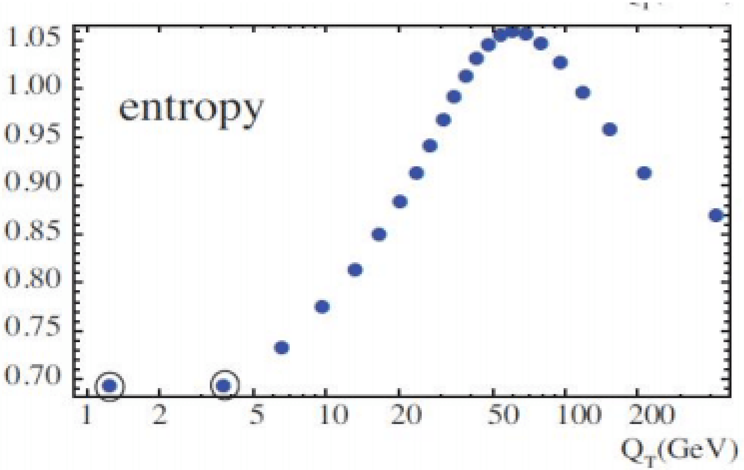
September 30, 2017

Overview

A New Resonance!

Using **quantum tomography** (QT), *1707.01638: Ralston, Tapia Takaki, JM*
we find evidence of a new resonance-like structure associated with inclusive Z production.



QT extracts maximum polarization info from data

→ excellent exploratory tool

Quantum Tomography

Inclusive Z production:

$$pp \to Z + X \to \ell^+ \ell^- + X$$

Consider the cross section for the inclusive production of two final-state leptons of spin s, s' from particle intermediates:

$$d\sigma \sim \sum_{s,s'} |\sum_J \mathcal{M}(\chi_J \to f)|^2 d\Pi_{LIPS},$$

where $\sum_J \mathcal{M}(\chi_J \to f) = \sum_J < f|T|\chi_J > \delta^4(\sum p_i)$, T is a transfer matrix, and $d\Pi_{LIPS}$ is the Lorentz-invariant phase space.

Quantum Tomography

Plugging $\sum_J \mathcal{M}(\chi_J \to f) = \sum_J < f|T|\chi_J > \delta^4(\sum p_i)$ into the expression for $d\sigma$, we find ...

$$d\sigma \sim \sum_{s,s'} [(\sum_J < f_{s,s'}|T|\chi_J >) \cdot (\sum_K < \chi_K|T^\dagger|f_{s,s'} >)] \cdot d\Pi_{LIPS}$$

$$= tr[(\sum_{s,s'} |f_{s,s'} >< f_{s,s'}|) \cdot (\sum_{J,K} T|\chi_J >< \chi_K|T^\dagger)] \cdot d\Pi_{LIPS}$$

T, $T^\dagger$ in the second set of parens ensures the overlap between the intermediate and final states is taken at 'equal time'.
$\sum_{J,K}$ accounts for any interference between intermediate states.

Quantum Tomography

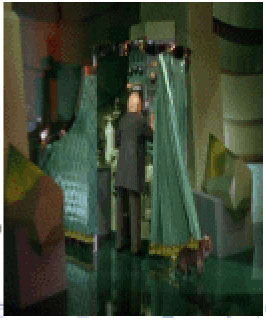
We can *define* $d\sigma$ in terms of the density matrices ρ_f, ρ_χ:

$$d\sigma \sim tr[(\sum_{s,s'} |f_{s,s'} >< f_{s,s'}|) \cdot (\sum_{J,K} T|\chi_J >< \chi_K|T^\dagger)] \cdot d\Pi_{LIPS}$$

$$= tr(\rho_{lep} \cdot \rho_\chi) \cdot d\Pi_{LIPS}$$

Note: ρ_{lep}, ρ_χ are OBSERVABLE since $d\sigma$ is observable.
QT deals only with ρ_{lep}, ρ_χ (i.e. only with observable dof).

"Less theory is better theory."
-Man behind the curtain



Quantum Tomography

Making use of the identity $d\Pi \sim d\Omega \cdot d^4 q$, where q is the sum of the final-state four-momenta ...
We find for given q,

$$\frac{d\sigma}{d\Omega}|_q \sim tr(\rho_{lep} \cdot \rho_\chi)$$

$$\sim \frac{1}{4\pi} + \sum_M \rho_M Y_M(\theta, \phi),$$

--lepton matrix is a 'probe'--

where ρ_M are the components of ρ_χ, Y_M are real-valued spherical harmonics ($\ell = 1, 2; m = -\ell, ...\ell$), and θ, ϕ are defined with respect to rest frame of Z in a frame to be explained.
Hence ρ_M can be reconstructed from orthogonality relations of the form:

$$\rho_M \sim \int d\Omega (d\sigma / d\Omega) \cdot Y_M$$

Quantum Tomography

Let's pause to consider what we've accomplished.

Takeaway #1

$d\sigma/d\Omega$ has good transformation properties. ρ_M transform like spin-1 or spin-2 (depending on Y_M).

Takeaway #2

We can reconstruct ρ_χ, which is 3×3 Hermitian and has $8 = 9 - 1$ real params. Subtracting 3 rotational freedoms leaves 5 invariants.

Quantum tomography

ρ_X has 5 invariants.

We can construct invariants of invariants.

Our favorite for present purposes... **entanglement entropy**:

$$\Sigma = -tr(\rho_X log \rho_X)$$

$$\Sigma_{min} = 0 \leftrightarrow \textit{pure state}$$
$$\Sigma_{max} = logN = log3 \sim 1.09 \leftrightarrow \textit{unpolarized}$$

Quantum Tomography Applied to ATLAS Z Study

ATLAS 1606.0089

$$Z/\gamma^* \to e^+e^-, \quad Z/\gamma^* \to \mu^+\mu^-,$$

where $\sqrt{s}=8\,TeV, 80<\sqrt{Q^2}<100\,GeV, |y|<3.5$

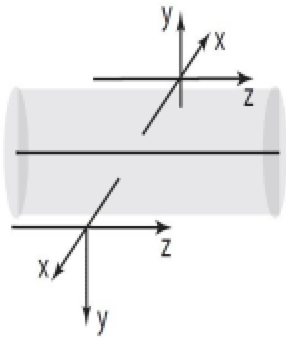
$d\sigma/d\Omega$ expanded in angular coeffs A_j

Our study:

We take $A_j \to \rho_M$ and populate ρ_X.

We compute evals, evecs, entropy of ρ_X.

QT Applied to ATLAS Z Study

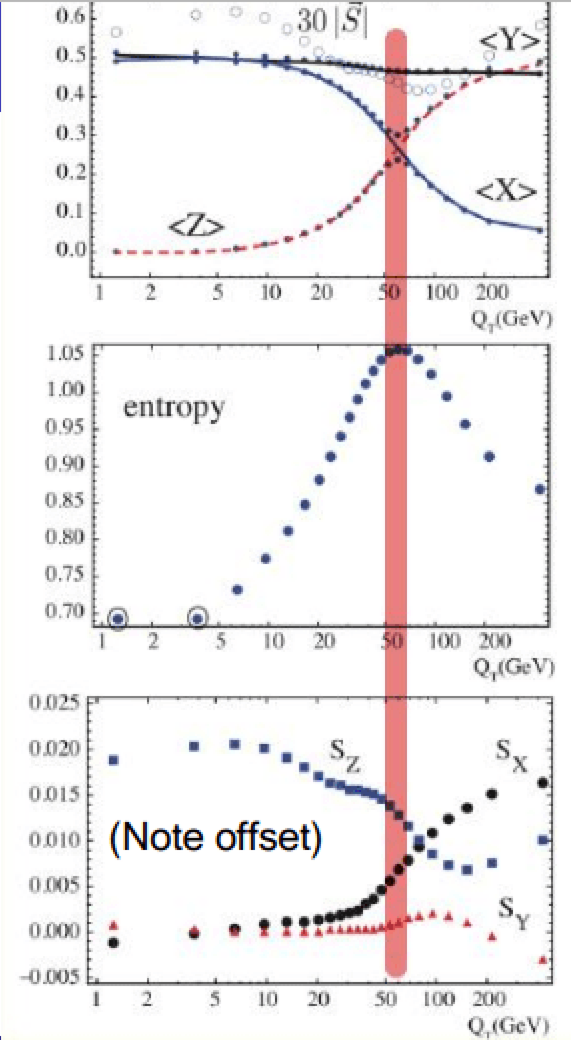


Collins Soper frame

Evals of polz density matrix plus $30|\vec{S}|$ and $< X >, < Y >, < Z >$, where $< Y > = < Y|\rho_X|Y >$, etc.

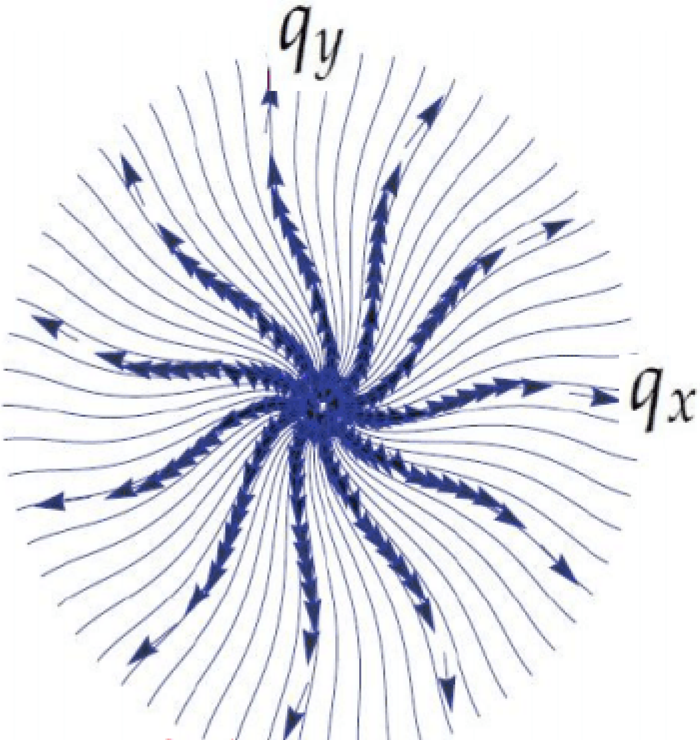
Entanglement entropy

Spin parameters $\quad S_j \sim \rho_M$

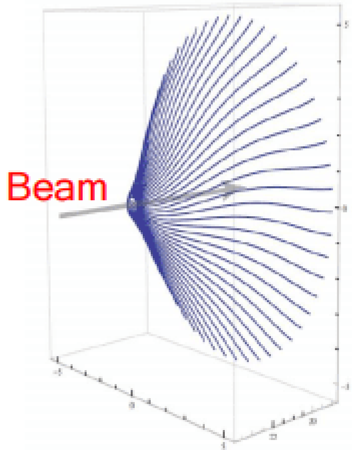


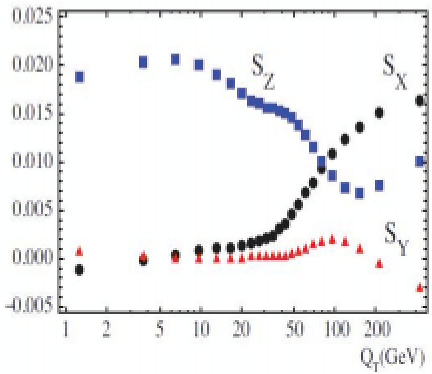
Quantum Tomography Applied to ATLAS Z Study

Looking down beam ...



$S_y \neq 0 \to P - violation$





Quantum Tomography Applied to ATLAS Z Study

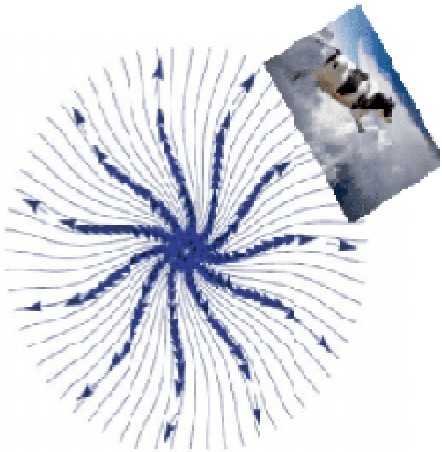
What does it *mean*?

Resonance-like structure associated w $\rho_3, \rho_4 \neq 0 \leftrightarrow T - odd \leftrightarrow NNLO$
ATLAS 1606.0089 finds Lam-Tung violation bigger than NNLO prediction

We understand why eigenvectors turn corner $\hat{X} \to \hat{Z}$

We don't understand near degeneracy of eigenvalues

Important point: speculation aside ...
QT finds what's there.



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Backup

Properties of Density Matrices

When $|\psi>$ is the only non-zero eigenvector of a 'density matrix' $\rho\,(=|\psi><\psi|)$, we say ρ is a 'pure state'. Special case !
Define $\psi_j =< j|\psi>$.
Then for some operator A,

$$< A >=< \psi|A|\psi >= \sum_{ij} \psi_i^* A_{ij} \psi_j = \sum_{ij} A_{ij} \psi_j \psi_i^*$$

$$= \sum_i [\sum_j A_{ij} \rho_{ji}] = \sum_i (A\rho)_{ii} = tr(A\rho).$$

Properties of Density Matrices

More generally, ρ is a linear combination (viz. an ensemble) of pure states,

$$\rho = \sum_j c_j |\rho_j >< \rho_j|.$$

The $|\rho_j >$ are eigenvectors and the c_j eigenvalues of ρ.
We require $\sum_j c_j = 1$ and $c_j \geq 0$. This allows us to interpret the c_j as probabilities.
We then have

$$< A >= tr(A\rho) = \sum_j c_j tr(A|\rho_j >< \rho_j|) = \sum_j c_j < \rho_j|A|\rho_j > .$$

Properties of Density Matrices

Takeaway #1

ρ is an $N \times N$ Hermitian matrix describing a QM system. ρ has N eigenvectors $|\rho_j>$ and N eigenvalues c_j. Pure state j is defined by $|\rho_j><\rho_j|$ and weighted within ρ by c_j.

Takeaway #2

$<A>= tr(A\rho)$ samples ρ. Hence, with enough observations, ρ is completely observable.