Confinamento de Quarks:
uma pergunta de um milhão de dólares

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Color confinement: Why care?

Why are there no free color charged particles (quarks or gluons), but only objects built out of them (hadrons), like mesons and baryons?

How does this phenomenon emerge from QCD?

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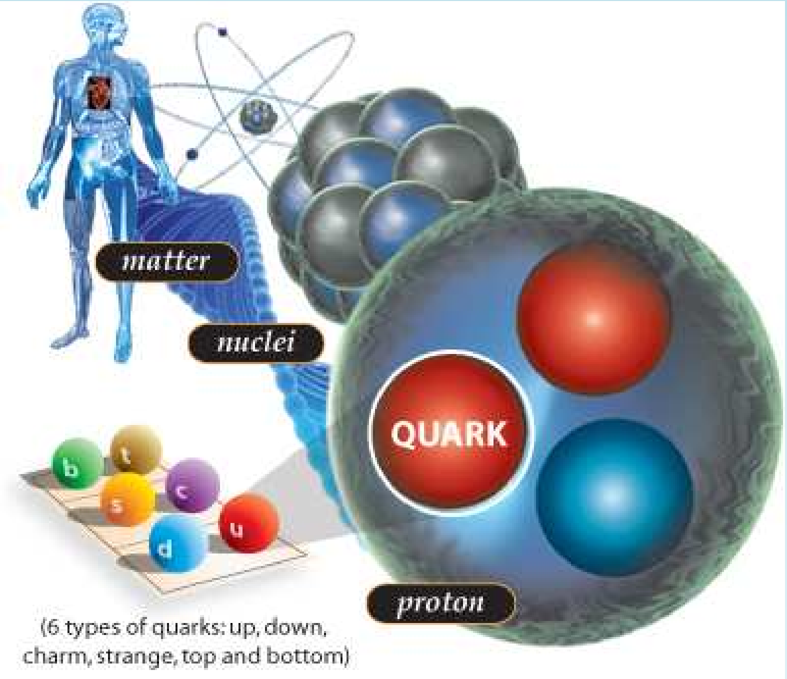
$\Rightarrow$ Millennium Prize Problems by the Clay Mathematics Institute (US$1,000,000): Yang-Mills Existence and Mass Gap: Prove that for any compact simple gauge group G, a non-trivial quantum Yang-Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$.

Quarks: in the Heart of Matter



Interior of the atom: nucleus, made up of nucleons, made up of quarks

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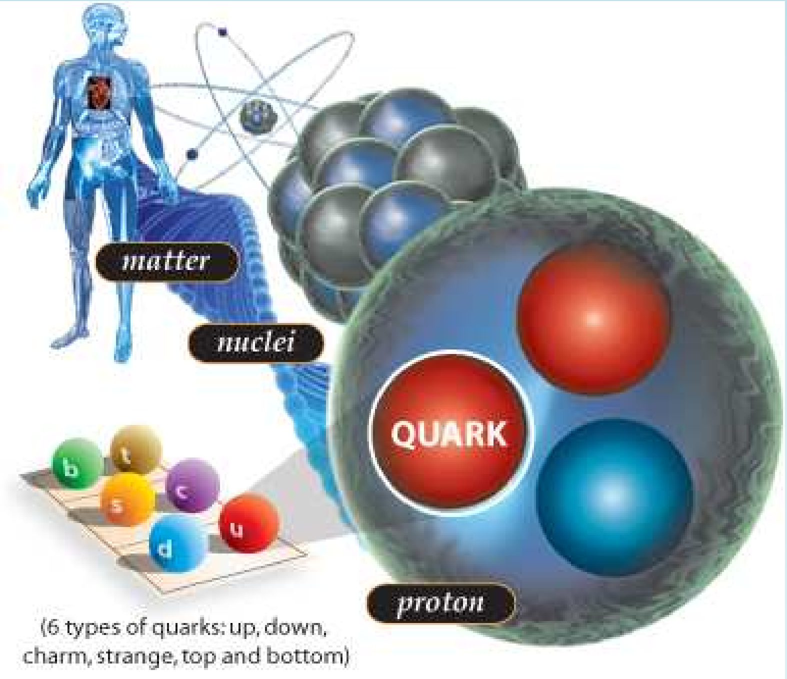

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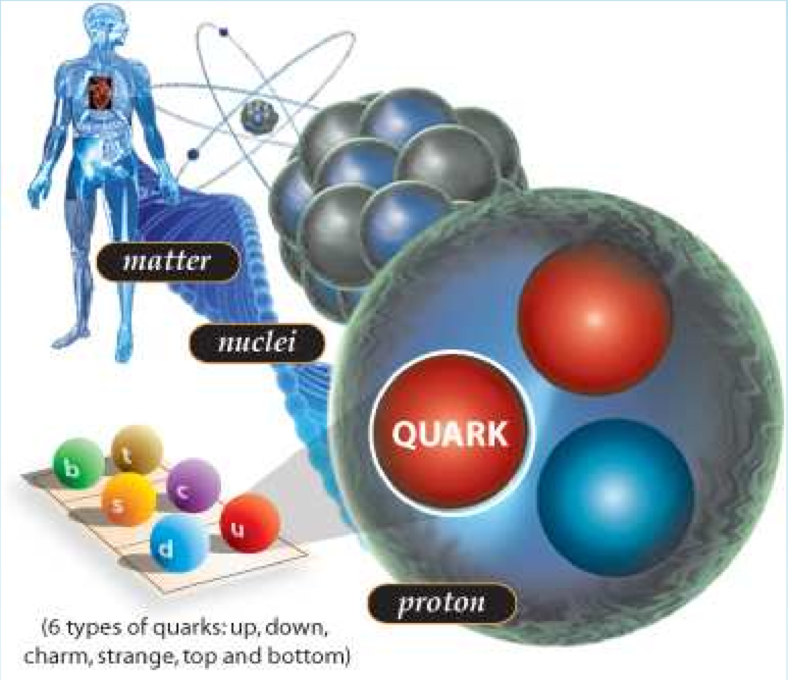

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Hadrons (e.g. protons, neutrons, mesons) are made up of quarks

Quarks: Bizarre Properties

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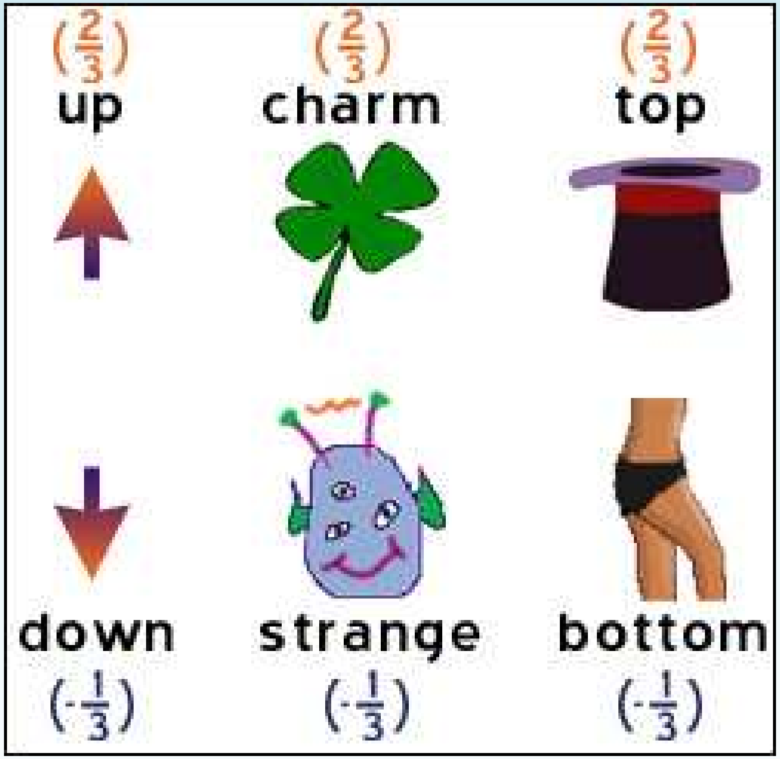
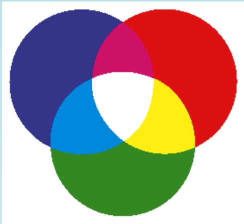


Quantum Chromodynamics

Strong interaction btw. protons e neutrons is residue of interaction btw. their quarks. Nucleons are made up of 3 quarks of different colors



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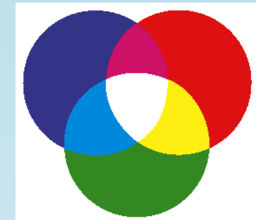
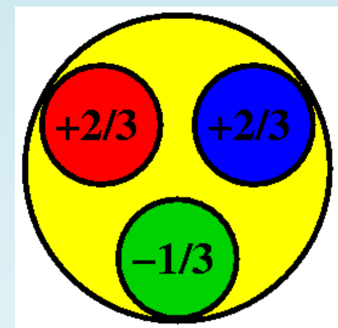


The proton is a color-neutral bound state of quarks interacting through the exchange of (massless) gluons



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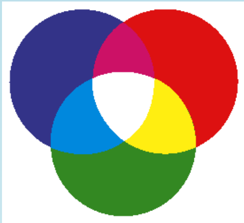

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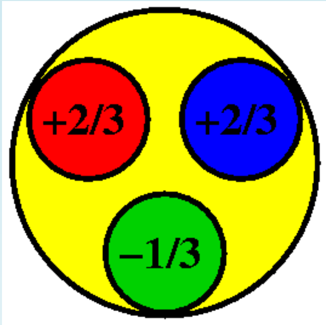

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Protons and Neutrons:

- 99% of the mass of the bound state comes from the interaction!

  ⇒ we are not star dust, we are (virtual) gluons!

Free the Quarks!

- Why was a fractionary electric charge never observed?
- Answer: quarks are confined inside hadrons!

Confinement: it would take infinite energy to separate the quarks that constitute a hadron

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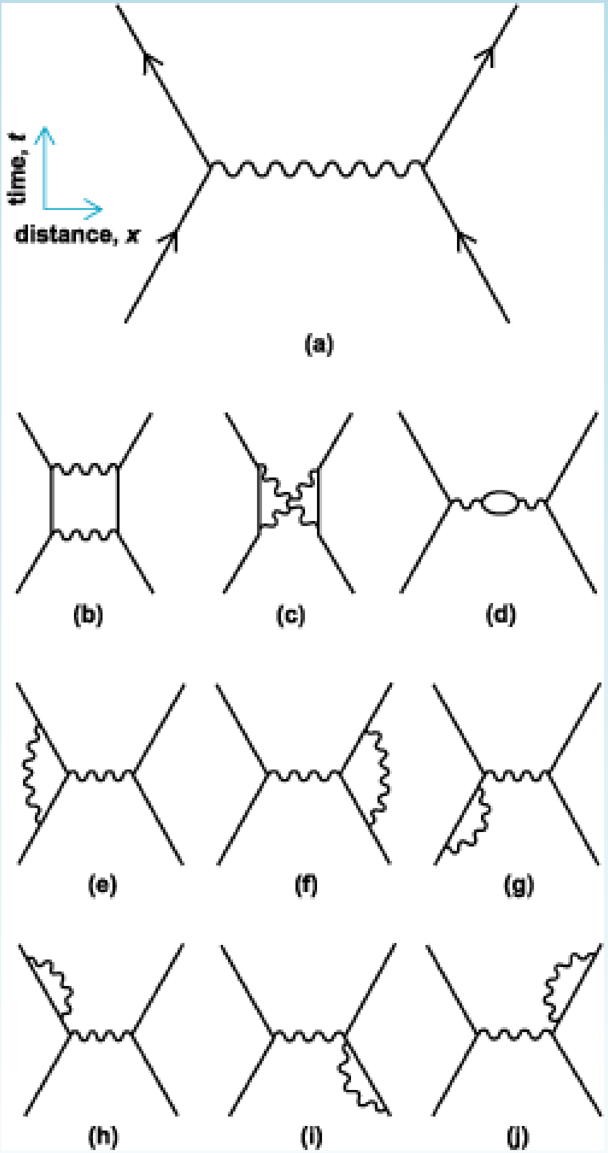
Quarks have flavor, and color, but no freedom 

(Usual) Quantum Field Theory



QED Lagrangian:

$$\mathcal{L} \;=\; \bar{\psi}\,(i\,\gamma^{\mu} D_{\mu} - m)\,\psi \;-\; \frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu}$$

where

$$D_{\mu} \equiv \partial_{\mu} - i\,e\,A_{\mu}\,, \qquad F_{\mu\nu} \equiv \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

Perturbative calculation: Feynman diagrams for electron scattering; it is possible to infer the redefinition of m and e to obtain finite results

Quantum Chromodynamics (QCD)

QCD Lagrangian is just like the one of QED:

quarks (spin-1/2 fermions)	$\Leftrightarrow$	electrons
gluons (vector bosons) / color charge		photons / electric charge

But: gauge symmetry is $SU(3)$ (non-Abelian) instead of $U(1)$

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$$\mathcal{L} \;=\; -\frac{1}{4}\, F^{a}_{\mu\nu}\, F^{\mu\nu}_{a} \;+\; \sum_{f=1}^{6} \bar{\psi}_{f,i} \left(i\,\gamma^{\mu} D^{ij}_{\mu} - m_f\, \delta_{ij} \right) \psi_{f,j}$$

where [$a = 1, \ldots, 8$; $i = 1, \ldots, 3$; T^{a}_{ij} = $SU(3)$ generators]

$$F^{a}{}_{\mu\nu} \;\equiv\; \partial_{\mu} A^{a}{}_{\nu} - \partial_{\nu} A^{a}{}_{\mu} + g_0\, f_{abc}\, A^{b}{}_{\mu}\, A^{c}{}_{\nu}$$

$$D_{\mu} \;\equiv\; \partial_{\mu} - i\, g_0\, A^{a}{}_{\mu}\, T_{a}$$

Note: g_0, m_f are bare parameters.

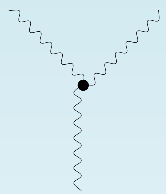
Gluons Have Color

Note: $F^a_{\mu\nu} \sim g_0 \, f^{abc} \, A^b_\mu \, A^c_\nu$

$\Rightarrow$ QCD Lagrangian contains terms with three and four gauge fields in addition to quadratic terms (propagators)

$\mathcal{L}_{\bar{\psi}\psi A} \;=\; g_0 \, \bar{\psi} \, \gamma^\mu A_\mu \, \psi \Rightarrow$ quark-quark-gluon vertex

$\mathcal{L}_{AAA} \;=\; g_0 \, f^{abc} \, A^\mu_a \, A^\nu_b \, \partial_\mu A^c_\nu \Rightarrow$ three-gluon vertex



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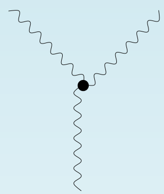
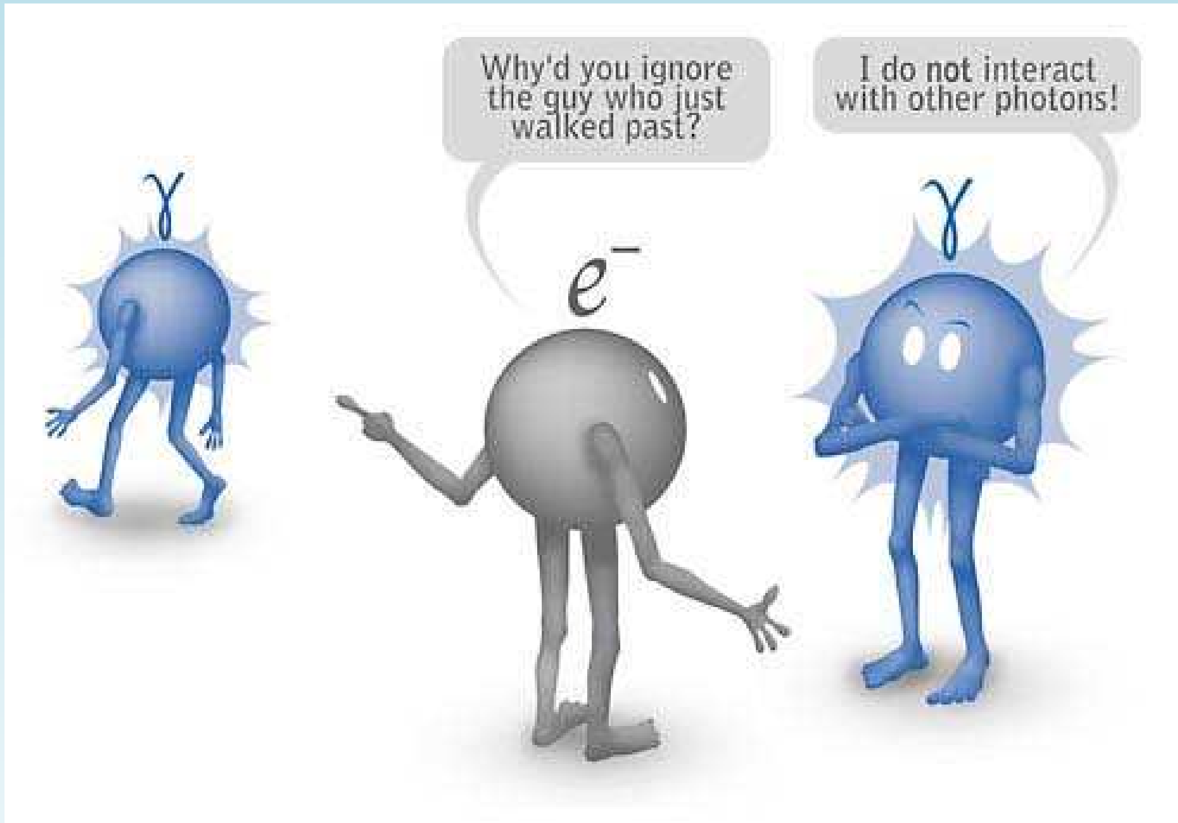


$\Rightarrow$ gluons interact with each other (have color charge), determining the peculiar properties and the nonperturbative nature of low-energy QCD

$\Rightarrow$ Running coupling $\alpha_s(p)$ instead of $\alpha \approx 1/137$

Photons vs. Gluons



Photons do not interact directly with one another

⇒ lightsabers (Star Wars) could not possibly work...

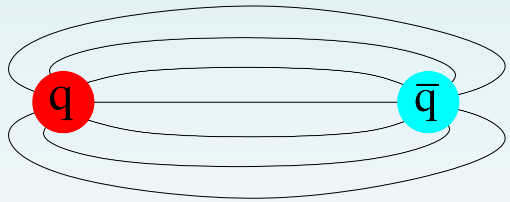
QCD vs. QED

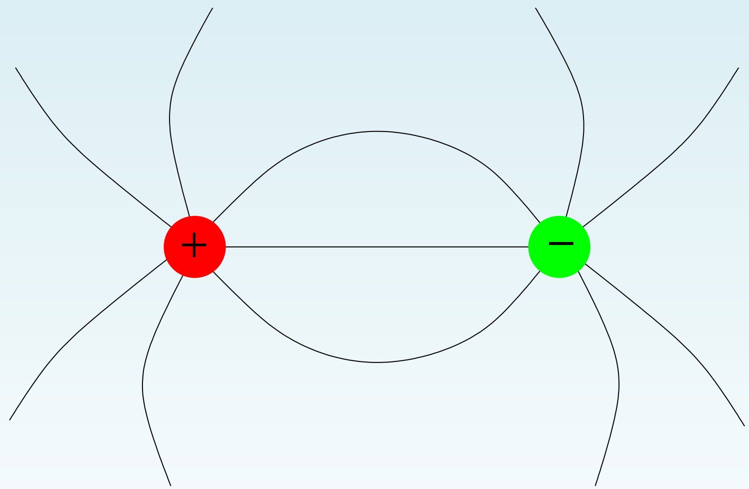
QCD (strong force) vs. QED (EM force)

quarks, gluons | electrons, photons

$SU(3)$ (3 "colors") | $U(1)$

m_q, $\alpha_s(p)$ | m_e, $\alpha \approx 1/137$





Confinement vs. Aymptotic Freedom (I)

- At high energies: deep inelastic scattering of electrons reveals proton made of partons: pointlike and free. In this limit $\alpha_s(p) \ll 1$ (asymptotic freedom) and QCD is perturbative

$$\alpha_s(p) = \frac{4\pi}{\beta_0 \log\left(p^2/\Lambda^2\right)} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\log\left(\log\left(p^2/\Lambda^2\right)\right)}{\log\left(p^2/\Lambda^2\right)} + \ldots\right]$$

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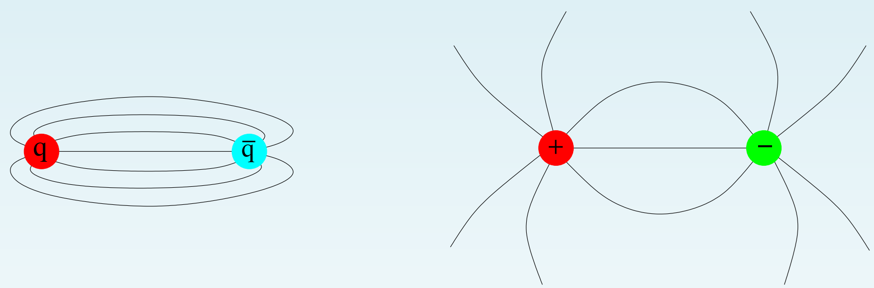
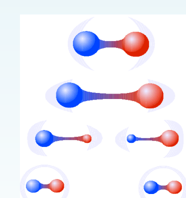
- At low energies: interaction gets stronger, $\alpha_s \approx 1$ and confinement occurs. Color field may form flux tubes



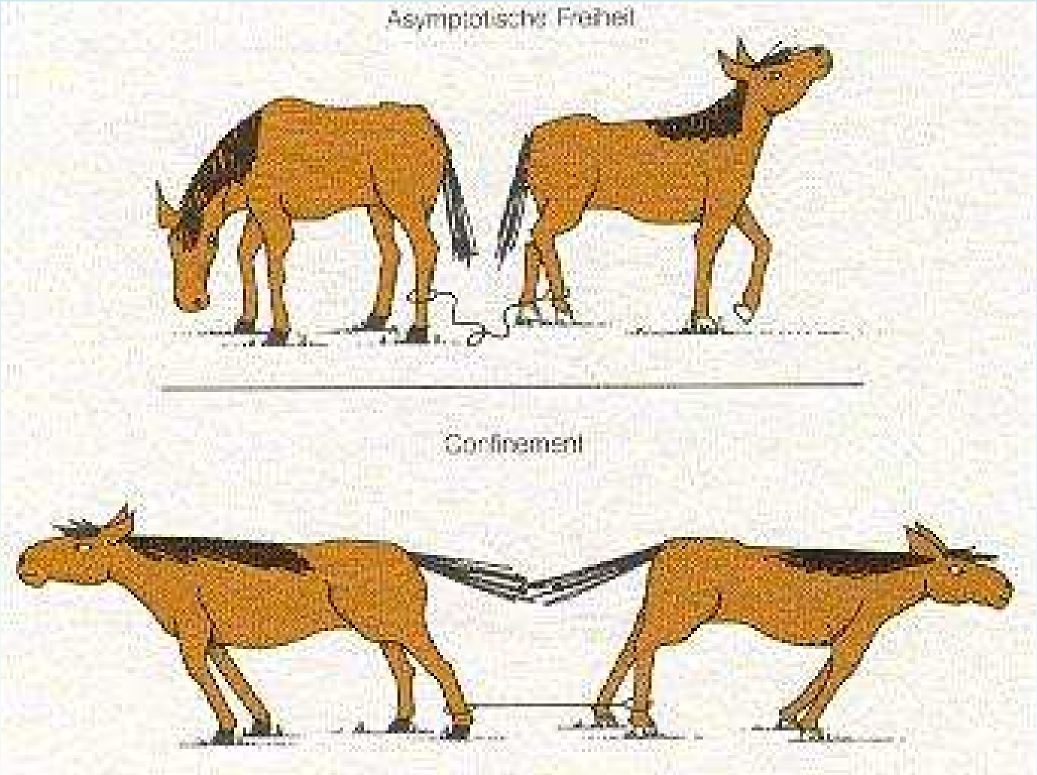
linear increase of inter-quark potential $\rightarrow$ string tension

At large distances $\rightarrow$ string breaks



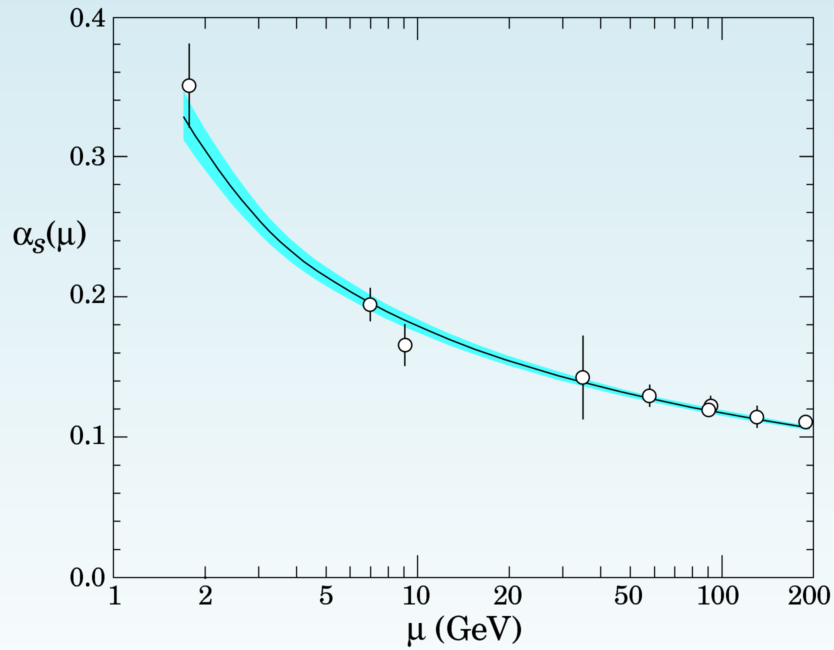
Confinement vs. Aymptotic Freedom (II)

At high energies (small distances) quarks behave as free particles, but at large distances the strength of the interaction becomes constant and an infinite energy would be necessary to separate two quarks ⇒ confinement



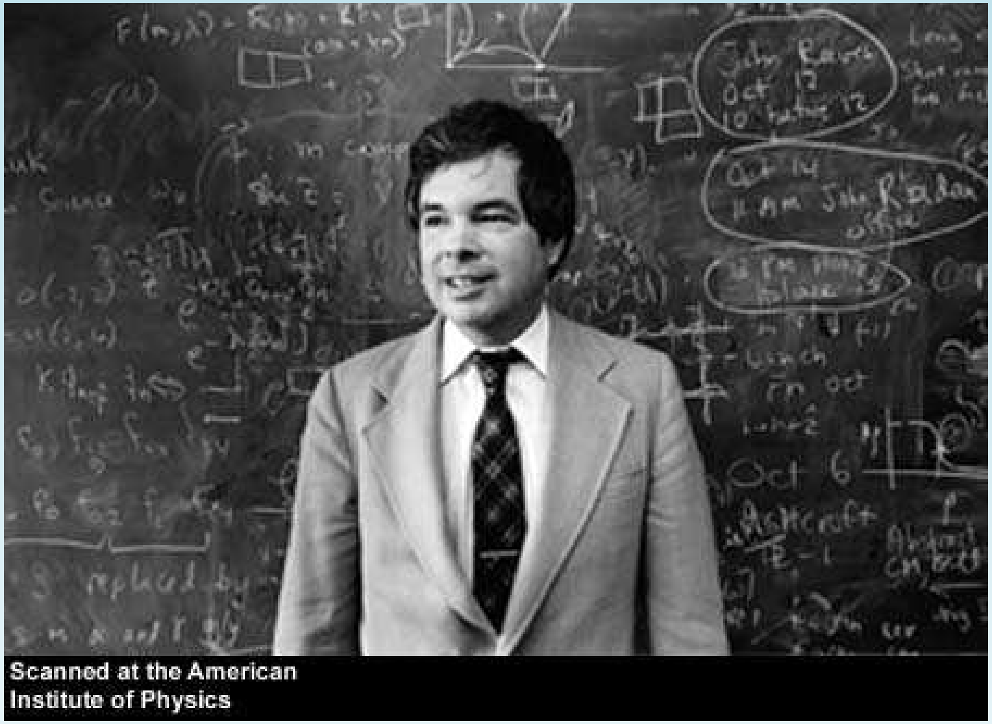
How do we perform calculations?

The strength of the interaction α_s increases for larger r (smaller p) and vice-versa (asymptotic freedom).
Perturbation theory breaks down in the limit of "small" energies



QCD on a Lattice (I)

Kenneth Geddes Wilson (June 8, 1936 – June 15, 2013)



Lattice used by Wilson in 1974 as a trick to prove confinement in (strong-coupling) QCD

[*Confinement of quarks*, Phys. Rev. D 10, 2445 (1974)]

QCD on a Lattice (II)

As recalled by Wilson

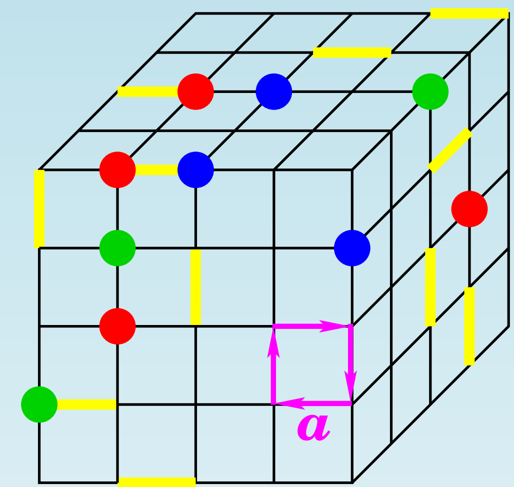
> *[...] Unfortunately, I found myself lacking the detailed knowledge and skills required to conduct research using renormalized non-Abelian gauge theories. What was I to do, especially as I was eager to jump into this research with as little delay as possible? [...] from my previous work in statistical mechanics I knew a lot about working with lattice theories...*
> *[...] I decided I might find it easier to work with a lattice version of QCD. . .*

The Origins of Lattice Gauge Theory, hep-lat/0412043 (Lattice 2004)

Lattice QCD Ingredients

Three ingredients

1. Quantization by path integrals $\Rightarrow$ sum over configurations with “weights” $e^{i\,S/\hbar}$
2. Euclidean formulation (analytic continuation to imaginary time) $\Rightarrow$ weight becomes $e^{-S/\hbar}$
3. Discrete space-time $\Rightarrow$ UV cut at momenta $p \lesssim 1/a$ $\Rightarrow$ regularization



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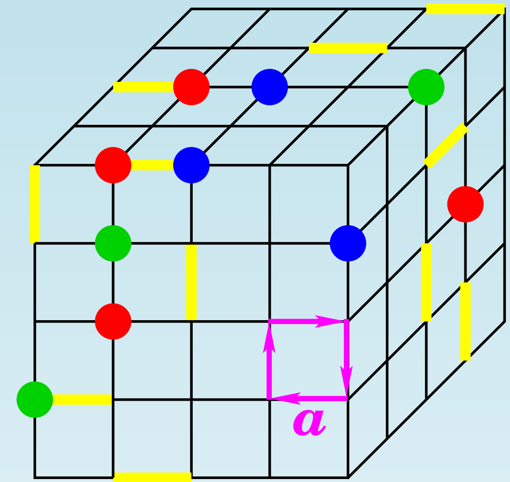

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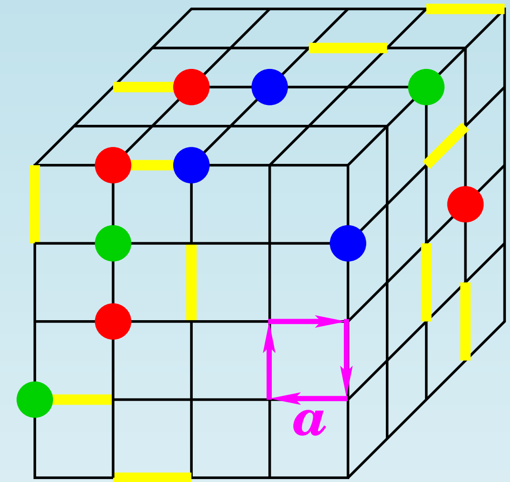

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The Wilson action

- is written for the gauge links $U_{x,\mu} \;\equiv\; e^{i g_0 a A^b_\mu(x) T_b}$
- reduces to the usual action for $a \to 0$
- is gauge-invariant

The Lattice Action

The Wilson action (1974)

$$S \;=\; -\frac{\beta}{3}\sum_{\Box} \mathrm{ReTr}\, U_{\Box}\,, \quad U_{x,\mu} \;\equiv\; e^{i g_0 a A_\mu^b(x) T_b}\,, \quad \beta \;=\; 6/{g_0}^2$$

- written in terms of oriented plaquettes formed by the link variables $U_{x,\mu}$, which are group elements
- under gauge transformations: $U_{x,\mu} \to g(x)\, U_{x,\mu}\, g^\dagger(x+\mu)$, where $g \in SU(3)$ $\Rightarrow$ closed loops are gauge-invariant quantities
- integration volume is finite: no need for gauge-fixing

At small β (i.e. strong coupling) we can perform an expansion analogous to the high-temperature expansion in statistical mechanics. At lowest order, the only surviving terms are represented by diagrams with “double” or “partner” links, i.e. the same link should appear in both orientations, since $\int dU\, U_{x,\mu} = 0$

Confinement and Area Law

Considering a rectangular loop with sides R and T (the Wilson loop) as our observable, the leading contribution to the observable's expectation value is obtained by "tiling" its inside with plaquettes, yielding the area law

$$< W(R,T) > \sim \ \beta^{RT}$$

But this observable is related to the interquark potential for a static quark-antiquark pair

$$< W(R,T) > = \ e^{-V(R)T}$$

We thus have $V(R) \sim \sigma R$, demonstrating confinement at strong coupling (small β)!

Problem: the physical limit is at large β...

Numerical Lattice QCD (I)

The approach had a “marvelous side effect”, as Michael Creutz calls it

> *By discreetly making the system discrete, it becomes sufficiently well defined to be placed on a computer. This was fairly straightforward, and came at the same time that computers were growing rapidly in power. Indeed, numerical simulations and computer capabilities have continued to grow together, making these efforts the mainstay of lattice gauge theory.*

The Early days of lattice gauge theory,
AIP Conf. Proc. 690, 52 (2003)

Numerical Lattice QCD (II)

We are left with a (classical) Statistical-Mechanics model with the partition function

$$Z = \int \mathcal{D}U \, e^{-S_g} \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \; e^{-\int d^4x \, \overline{\psi}(x) \, K \, \psi(x)} = \int \mathcal{D}U \, e^{-S_g} \, \det K(U)$$

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All we need is to evaluate expectation values

$$< \mathcal{O} > = \int \mathcal{D}U \, \mathcal{O}(U) \, P(U)$$

with the weight

$$P(U) \;=\; \frac{e^{-S_g(U)} \;\det K(U)}{Z}$$

$\Rightarrow$ analogous to thermodynamic averages in Statistical Mechanics

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Very complicated (high-dimensional) integral to compute!

May perform Monte Carlo simulations, as in statistical mechanics!

Monte Carlo Simulations

Stochastic systems are simulated on the computer using a random number generator

⇒ theoretical approach, with experimental aspects:

- data, errors
- "measurements" in time

Lattice QCD Simulations

(Classical) Statistical-Mechanics model — which may be studied by Monte Carlo simulations — with the partition function

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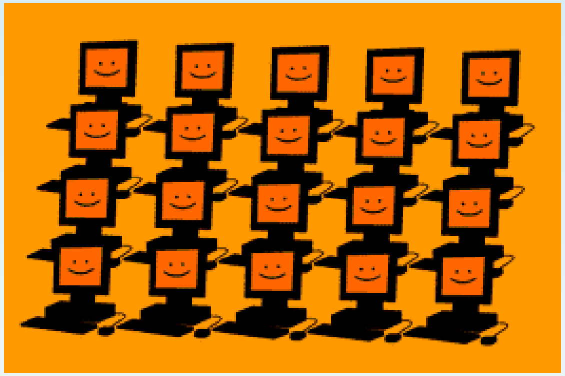
Monte Carlo methods

- pure gauge (quenched):
  Metropolis / Heat Bath $+$ Overrelaxation
- gauge $+$ dynamic quarks (full QCD):
  Hybrid Monte Carlo (HMC)



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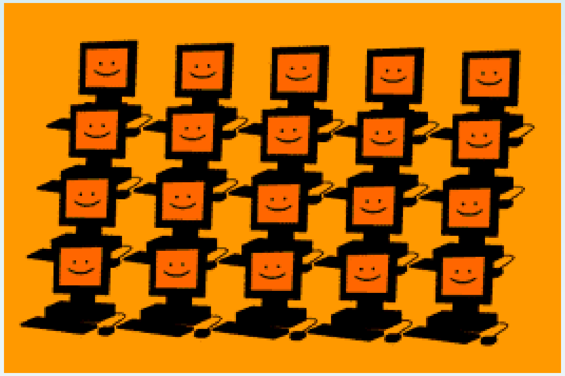

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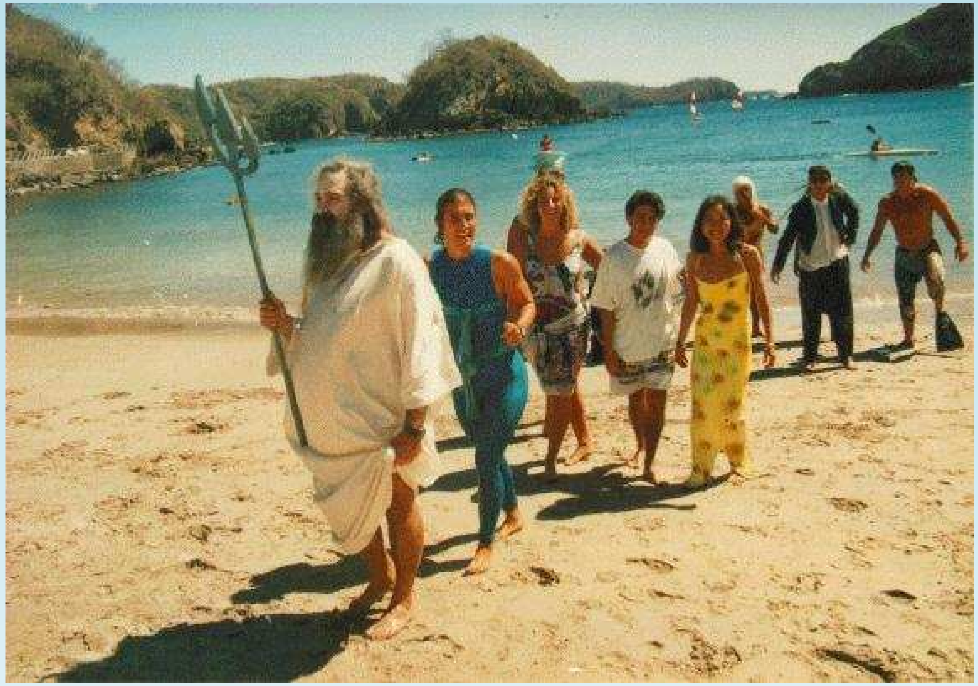
Note: $m = m_{\rm latt}/a$; as $a \to 0$ correlation length $\xi_{\rm latt} = 1/m_{\rm latt} \to \infty$

$\Rightarrow$ Continuum limit corresponds to critical point of the lattice theory

Life on the Lattice



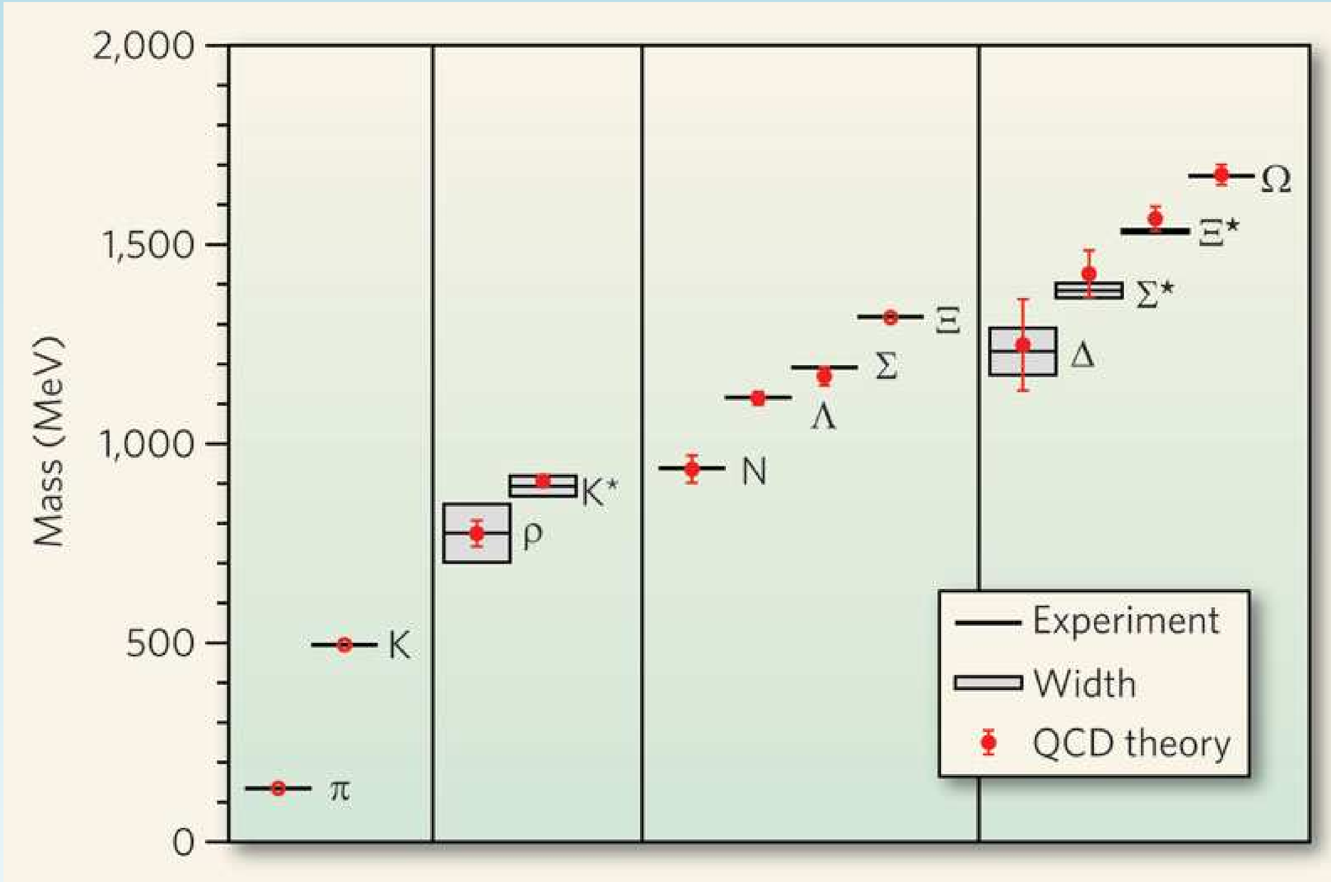
Lattice links: `http://latticeguy.net/lattice.html`

Blog: `http://latticeqcd.blogspot.com.br/`

Yearly meetings: Lattice Conference (latest was Lattice 2017 in Granada, Spain)

Papers: `arXiv/hep-lat`

Lattice QCD Results: Mass Spectrum

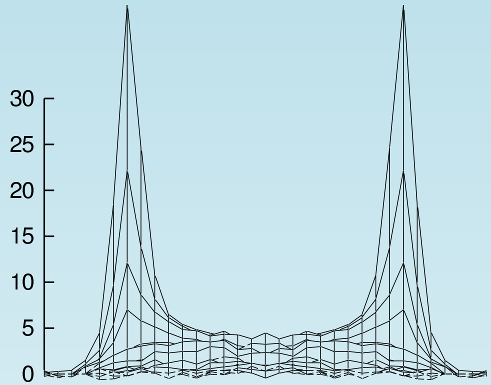


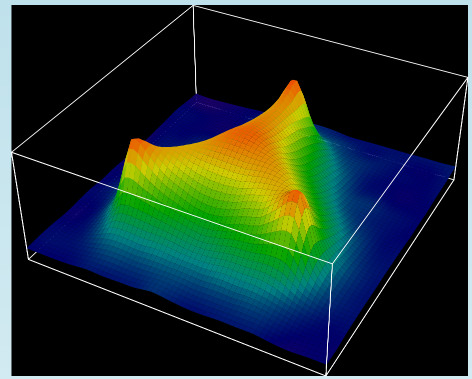
Light hadron masses obtained by S. Dürr et al. (Science, 2008) vs. experimental values. Note: π, K, Ξ as inputs.

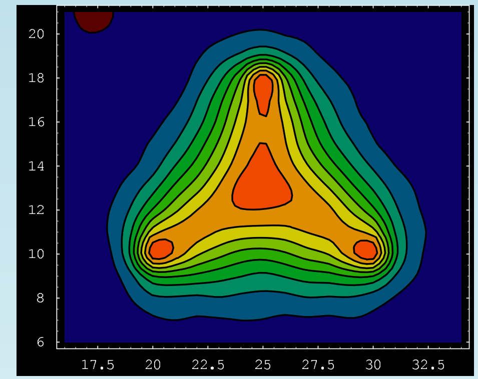
Press: (November 2008): United Press, Scientific American, Nature (F. Wilczek)

Lattice QCD Results (II)

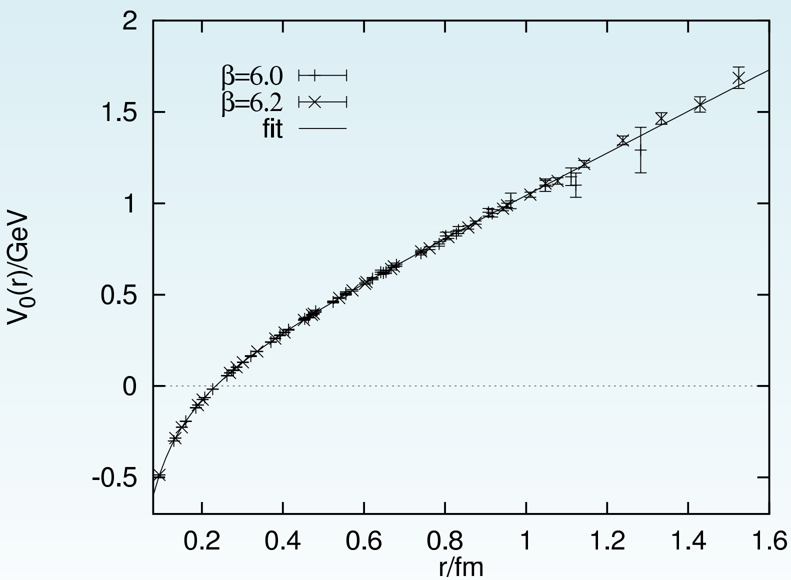
May observe formation of flux tubes

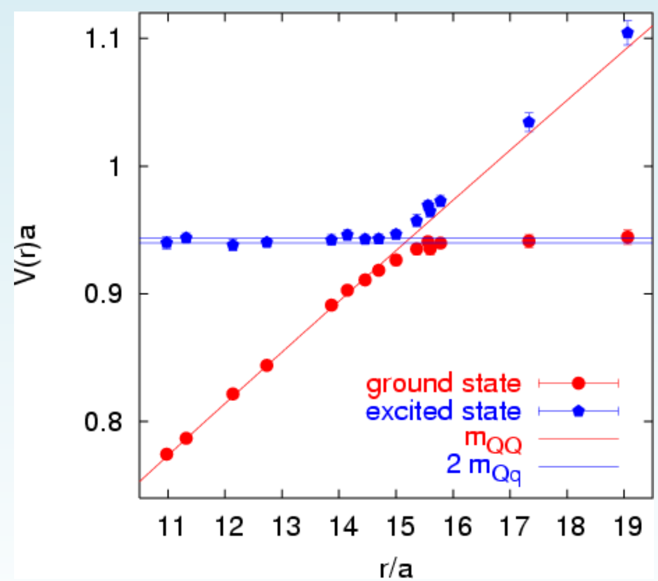






Linear Growth of potential between quarks, string breaking





Confinement: the Elephant in the Room

Do we understand confinement?

$\Rightarrow$ we know what it looks like,
but do we know what it is?

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Millenium Prize Problems (Clay Mathematics Institute, USA/UK)

Yang-Mills and Mass Gap: Experiment and computer simulations suggest the existence of a mass gap in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Pathways to Confinement

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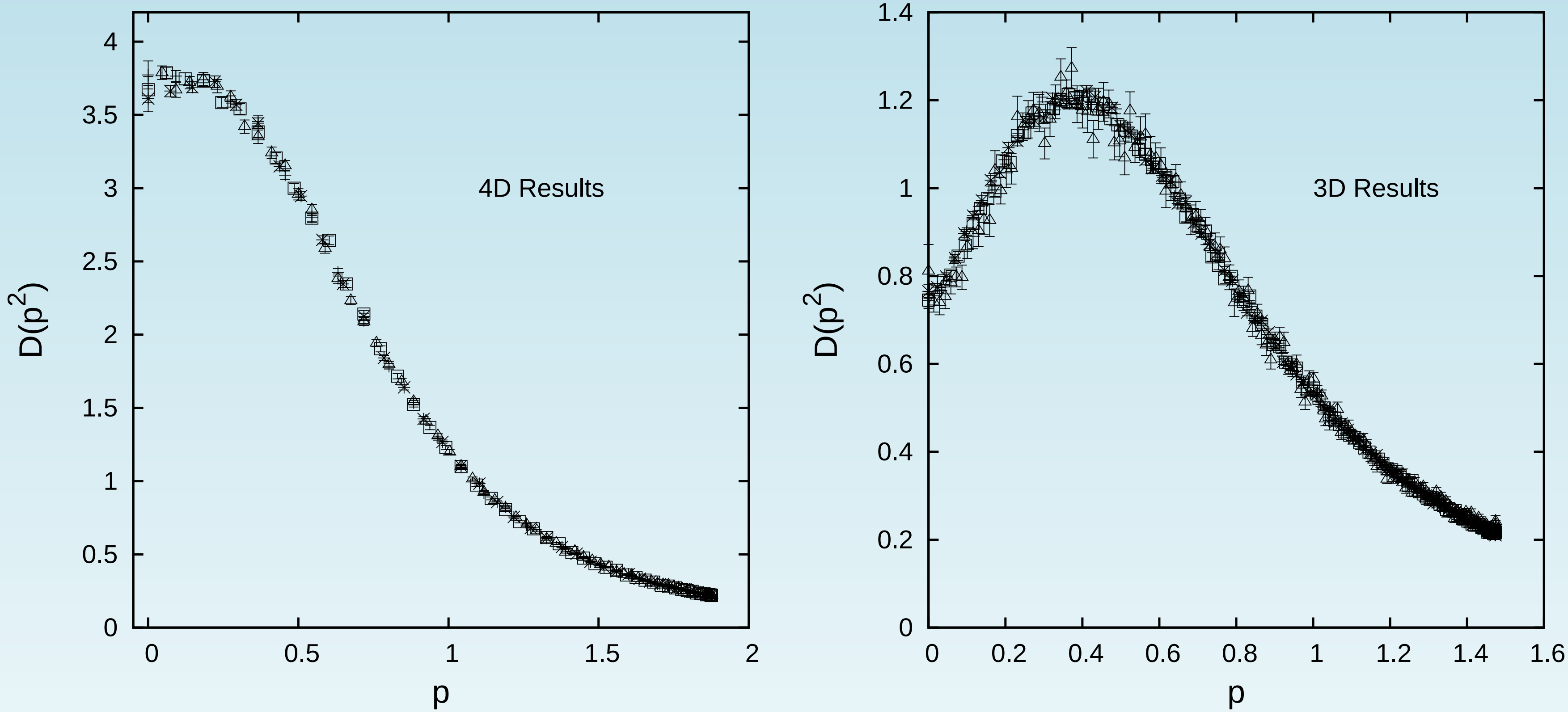
- Gribov-Zwanziger confinement scenario based on suppressed gluon propagator and enhanced ghost propagator in the infrared

Gluon Propagator at "Infinite" Volume

Attilio Cucchieri & T.M. (2008)



Gluon propagator $D(k)$ as a function of the lattice momenta k (both in physical units) for the pure-$SU(2)$ case in $d = 4$ (left), considering volumes of up to 128^4 (lattice extent ~ 27 fm) and $d = 3$ (right), considering volumes of up to 320^3 (lattice extent ~ 85 fm).

GZ Scenario: Confinement by Ghost

Formulated for Landau gauge, predicts gluon propagator

$$D^{ab}_{\mu\nu}(p) = \sum_x e^{-2i\pi k\cdot x} \langle A^a_\mu(x)\, A^b_\nu(0)\rangle = \delta^{ab} \left(g_{\mu\nu} - \frac{p_\mu\, p_\nu}{p^2} \right) D(p^2)$$

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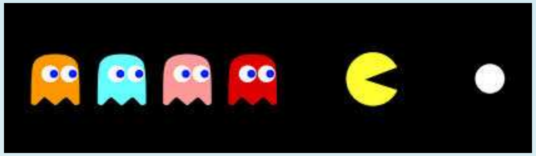

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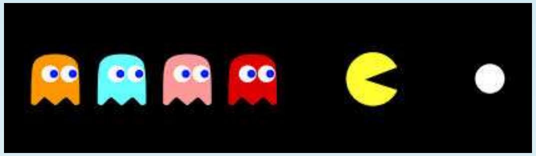

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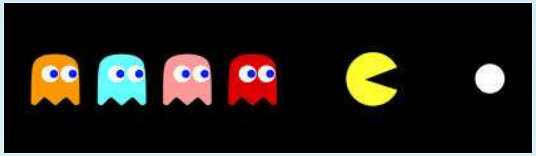

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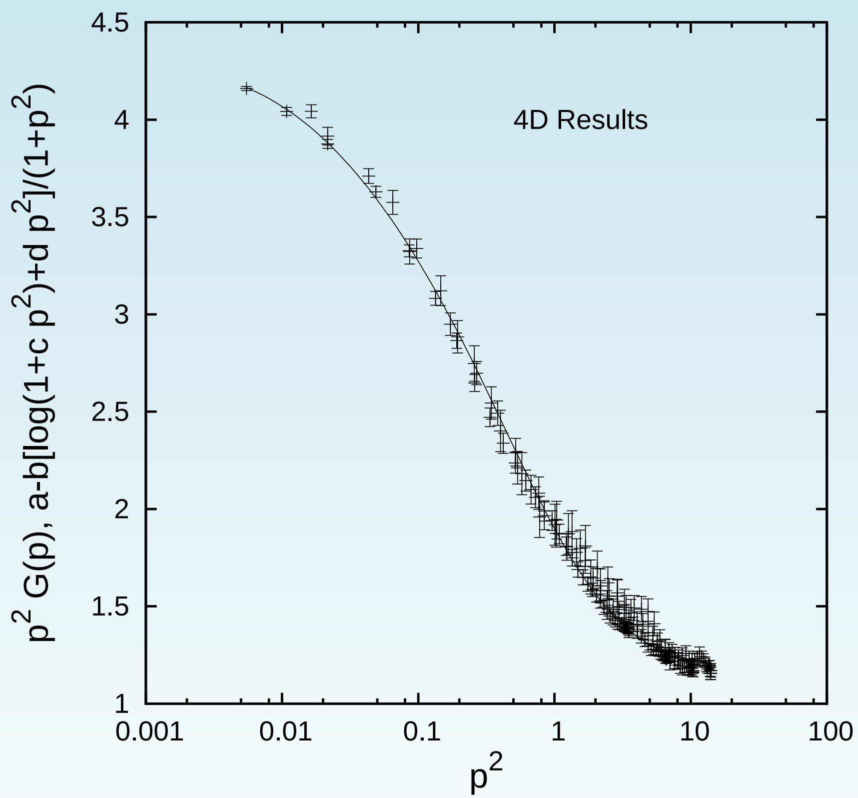
- In turn, $G(p)$ should be IR enhanced, introducing long-range effects, which are related to the color-confinement mechanism

Ghost Propagator Results

Fit of the ghost dressing function $p^2G(p^2)$ as a function of p^2 (in GeV) for the 4d case ($\beta = 2.2$ with volume 80^4). We find that $p^2G(p^2)$ is best fitted by the form $p^2G(p^2) = a - b[\log(1 + cp^2) + dp^2]/(1 + p^2)$, with



$a = 4.32(2)$,

$b = 0.38(1)\, GeV^2$,

$c = 80(10)\, GeV^{-2}$,

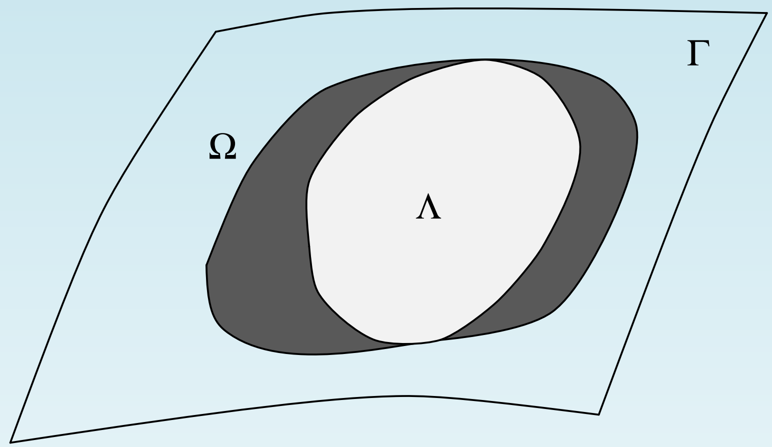
$d = 8.2(3)\, GeV^{-2}$.

In IR limit $p^2G(p^2) \sim a$.

Attilio Cucchieri & T.M. (2008)

The Infinite-Volume Limit

As the infinite-volume limit is approached, the sampled configurations (inside Ω = region for which $\mathcal{M}$ is positive semi-definite) are closer and closer to the first Gribov horizon $\partial\Omega$



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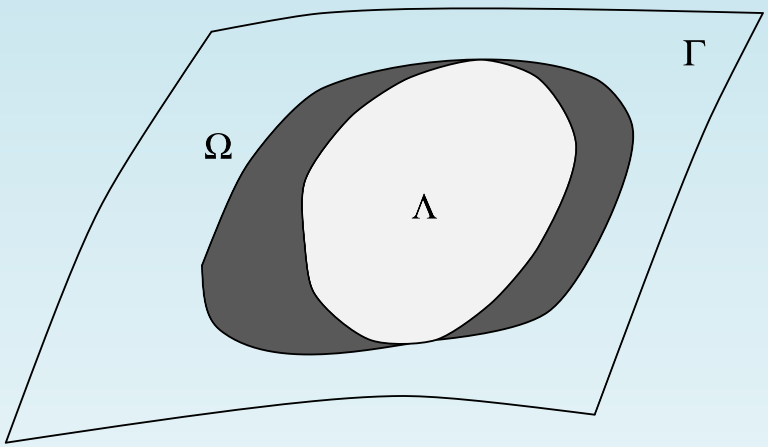

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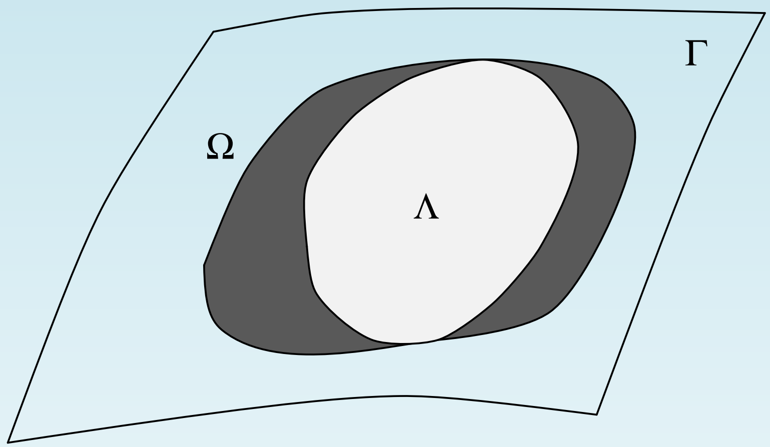

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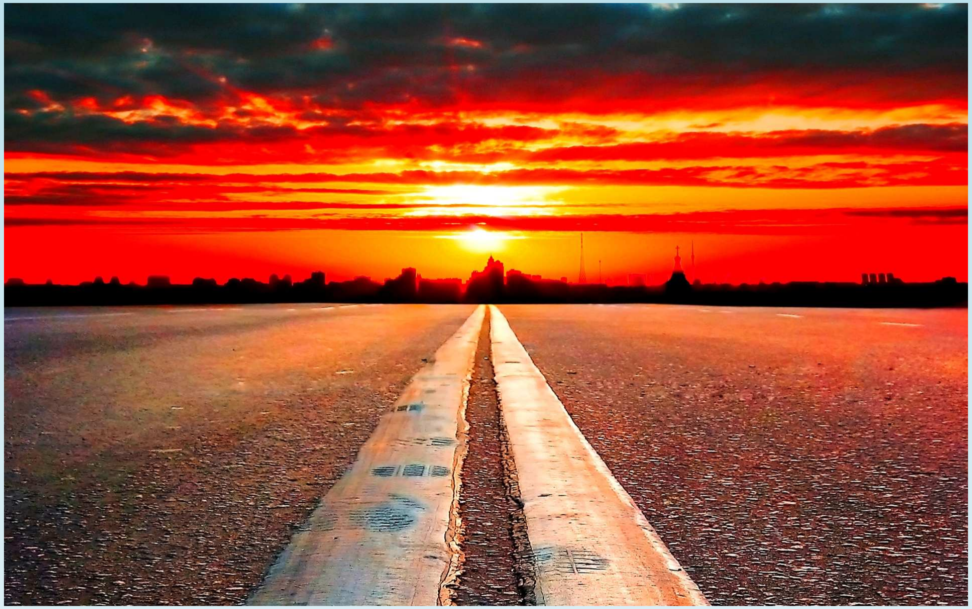
Lattice simulation produces thermalized gauge configurations, but we can also “visit” nearby configs and extract info from them!

Reaching (and Crossing!) the Horizon



How many roads have I wondered?
None, and each my own
Behind me the bridges have crumbled
No question of return

Nowhere to go but the horizon
where, then, will I call my home?

The Same Song, Susheela Raman

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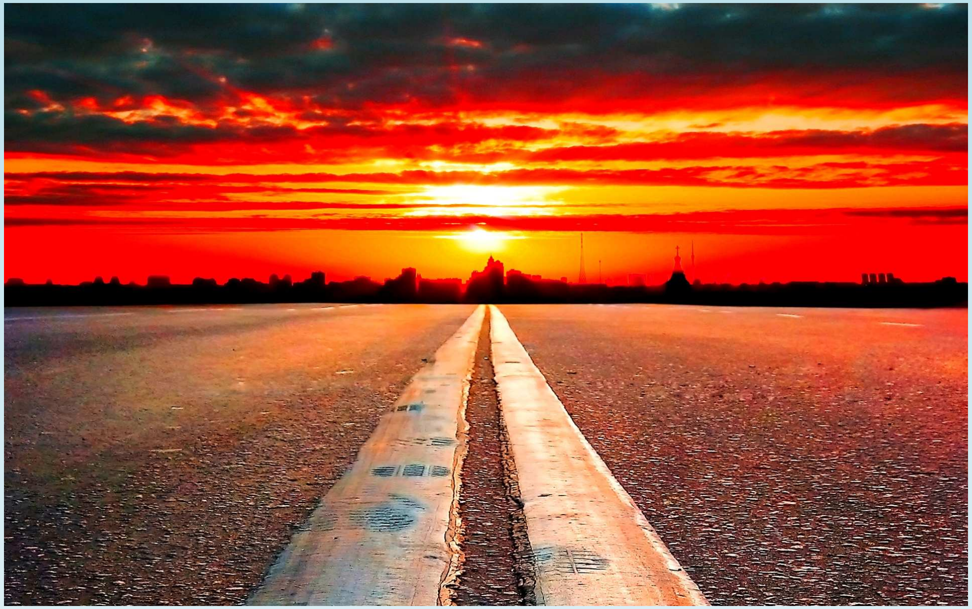

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— They say that communism is just over the horizon. What's a horizon?
— A horizon is an imaginary line which continues to recede as you approach it.

Russian joke from Khrushchev's time

Relating λ_{min} and Geometry

Using properties of Ω and the concavity of the minimum function, one can show (A. Cucchieri, TM, PRD 2013)

$$\lambda_{min}\left[\,\mathcal{M}[A]\,\right] \;\geq\; [1-\rho(A)]\,p^2_{min}$$

Here $1-\rho(A)\leq 1$ measures the distance of a configuration $A\in\Omega$ from the boundary $\partial\Omega$ (in such a way that $\rho^{-1}A\equiv A'\in\partial\Omega$). This result applies to any Gribov copy belonging to Ω

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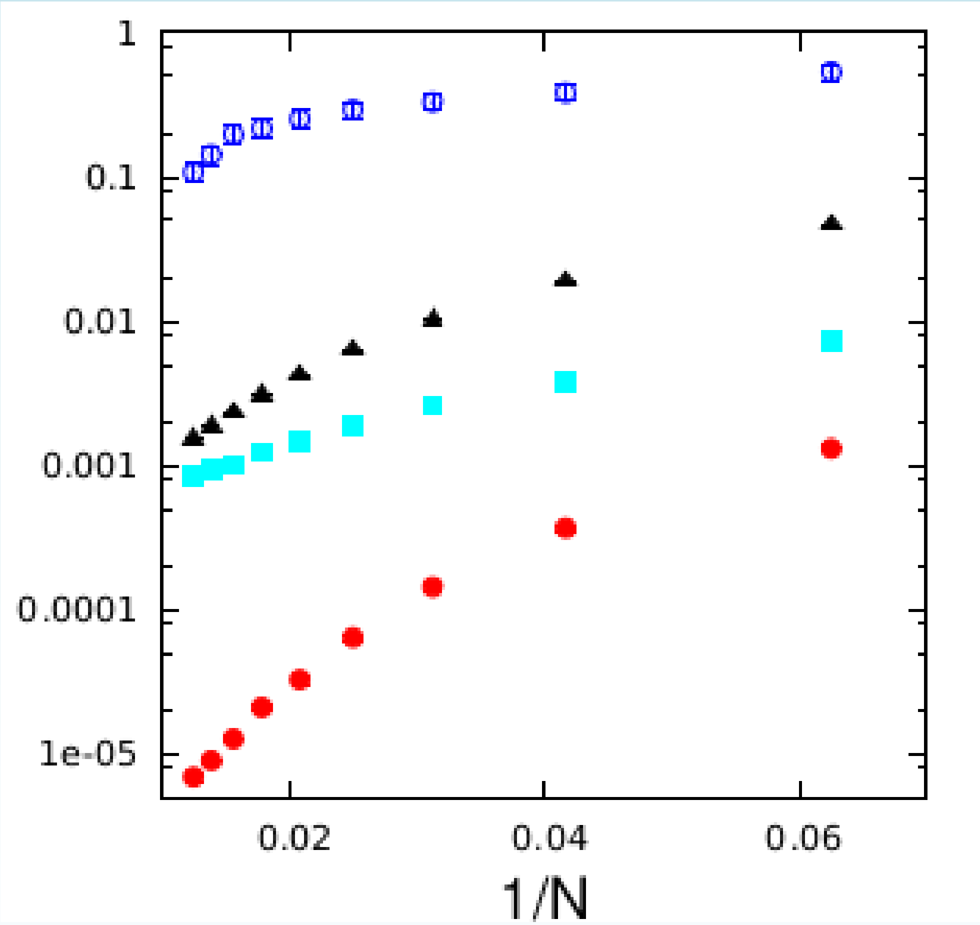
In the Abelian case one has $\mathcal{M}=-\partial^2$ and $\lambda_{min}=p^2_{min}$
$\Longrightarrow$ non-Abelian effects are included in the $(1-\rho)$ factor

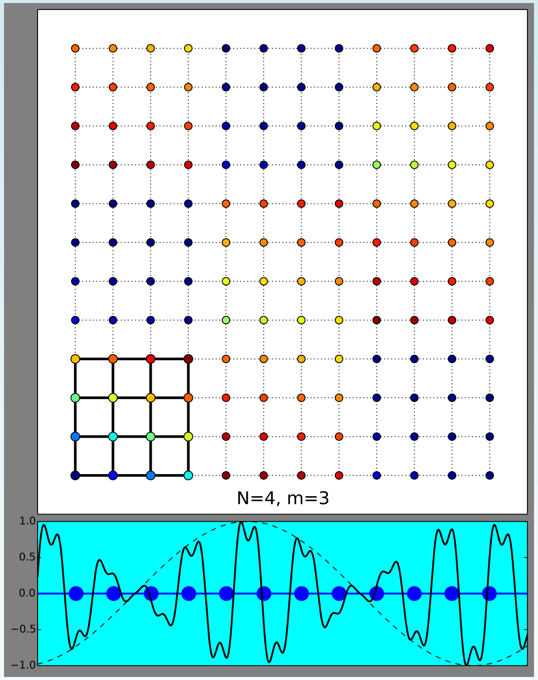
How far from Equality? Very far...

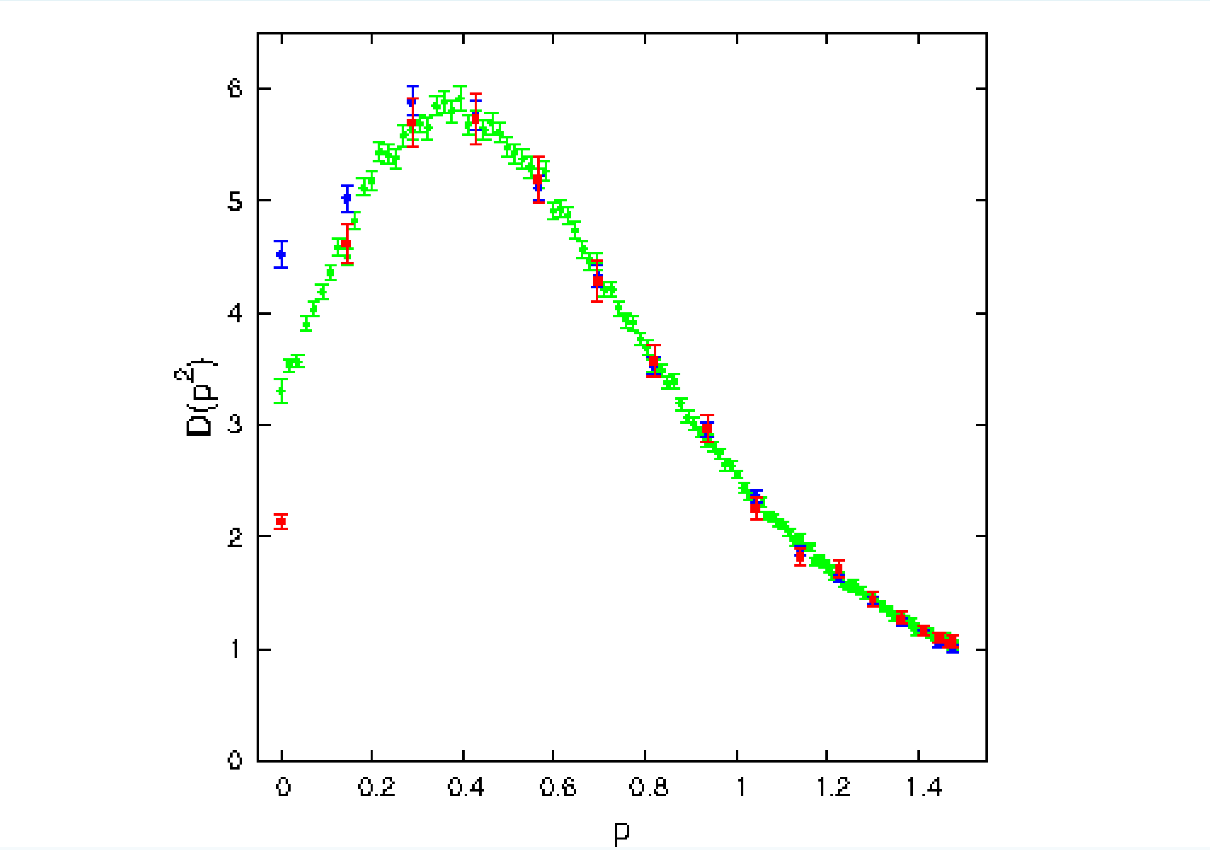
Using $A' = \widetilde{\tau}\, A \equiv A(\tau_{n-1} + \tau_n)/2 \in \partial\Omega$ and $\rho = 1/\widetilde{\tau} < 1$: plot inverse of the lower bound for $G(p)$, $1/G(p_{min})$, λ_{min} and the quantity $(1-\rho)\, p^2_{min}$ as functions of the inverse lattice size $1/N$.



Large Lattices via Bloch's Theorem

Perform thermalization step on small lattice, then replicate it and use Bloch's theorem from condensed-matter physics to obtain gauge-fixing step for much larger lattice (A. Cucchieri, TM, PRL 2017)





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We've come a long way... in discarding things we thought we knew about confinement

Conclusion

Still lots to be understood inside a Proton!!