

# Non-geodesic corrections to mass-spin estimates for Galactic microquasars implied by QPO models

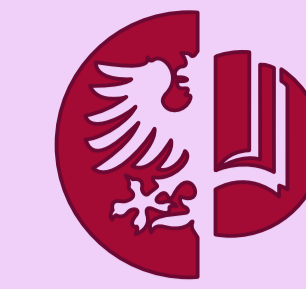
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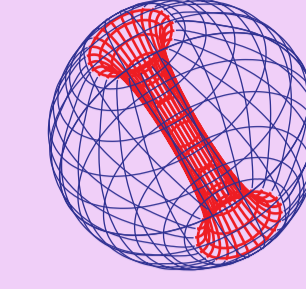
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## Abstract

We study frequencies of axisymmetric and non-axisymmetric epicyclic modes of accretion disc oscillations and explore the influence of pressure forces present in the disc. We discuss the implications for estimations of black hole spin in the three Galactic microquasars, GRS 1915+105, GRO J1655-40, and XTE J1550-564, that have been carried out based on several models of 3:2 high-frequency quasi-periodic oscillations (QPOs). Our findings show that in the particular case of 3:2 epicyclic resonance model the presence of pressure forces affects the predicted QPO frequencies only slightly when  $a < 0.9$ . On the contrary, when  $a > 0.9$ , the influence of pressure forces is non-negligible. For several models this influence can be quite significant even for low values of spin. We furthermore discuss the differences between results obtained based on approximative analytic methods and those carried out using exact numerical solutions.

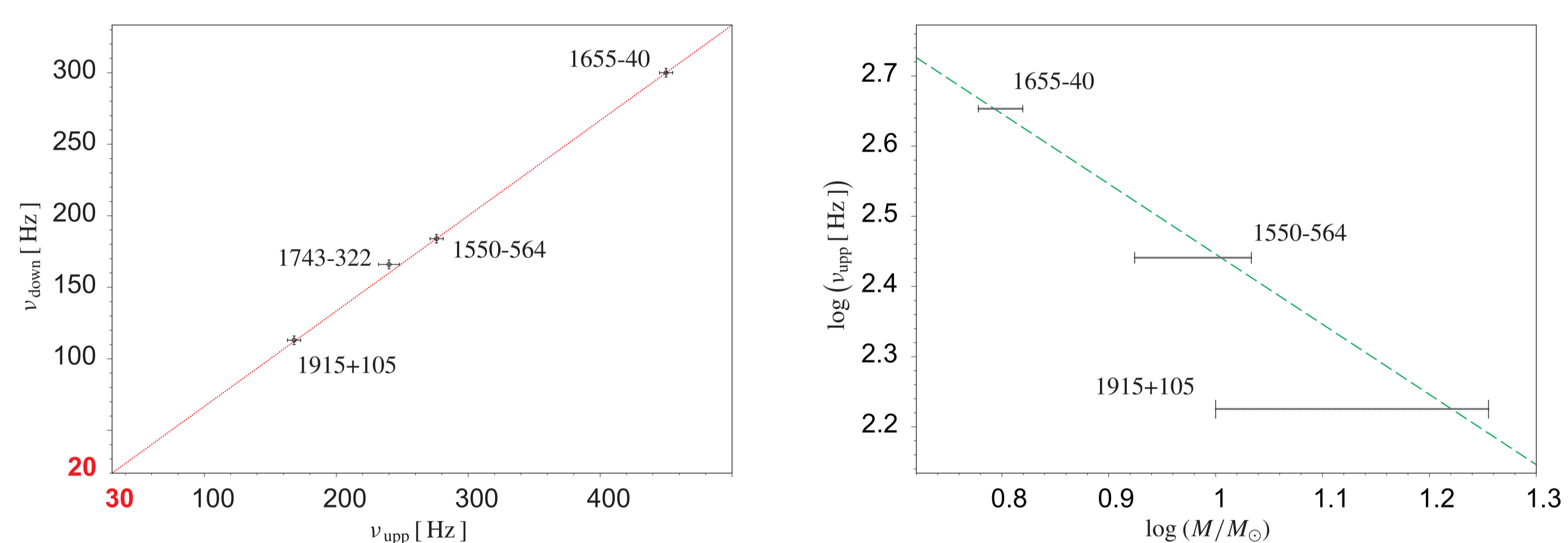
## Models of HF QPOs and Properties of Black Holes

Studying the X-ray spectra and variability provides a useful tool for putting constraints on the properties of compact objects like is the mass or spin of a black hole. One of the standard ways to measure the black hole spin is through fitting the X-ray spectral continuum or the relativistically broadened Fe K alpha lines. Within the recent years, another approach has been gaining popularity, in particular determination of their properties through the theory of twin-peak high-frequency quasi-periodic oscillations (HF QPOs). Such BH spin estimations have been carried out by several authors in the past (a list of references can be found in, e.g., Török et al., 2011, A&A). Several of the so-far obtained black hole spin estimations based on the QPO models have been carried out considering geodesic accretion flow. In the case of more general flows non-geodesic effects connected to e.g. pressure gradients, magnetic fields or other forces may have potentially significant impact on the spin predictions implied by these models. We aim to quantify such impact in the particular case of non-geodesic influence introduced by pressure forces that are present in a specific type of accretion flow modeled by a pressure-supported perfect fluid torus. We assume the set of QPO models that were discussed in a study by Török et al. (2011, A&A) who calculated BH spin values predicted by this set of models dealing with purely geodesic flow for three Galactic microquasars displaying the 3:2 twin-peak HF QPOs – GRS 1915+105, GRO J1655-40, and XTE J1550-564. We extend that work by calculating modifications to spin estimates predicted by the set of models that are introduced by the pressure forces.

We focus our attention on the so-called ‘disc-oscillation’ models that involve various oscillatory modes of accretion disc oscillations. We consider non-geodesic flow modeled by an equilibrium, slightly non-slender pressure-supported perfect fluid torus. The torus is assumed to have constant specific angular momentum distribution and orbit a rotating Kerr black hole. A detailed description of such model of torus is given in Straub & Šrámková (2009, A&A).

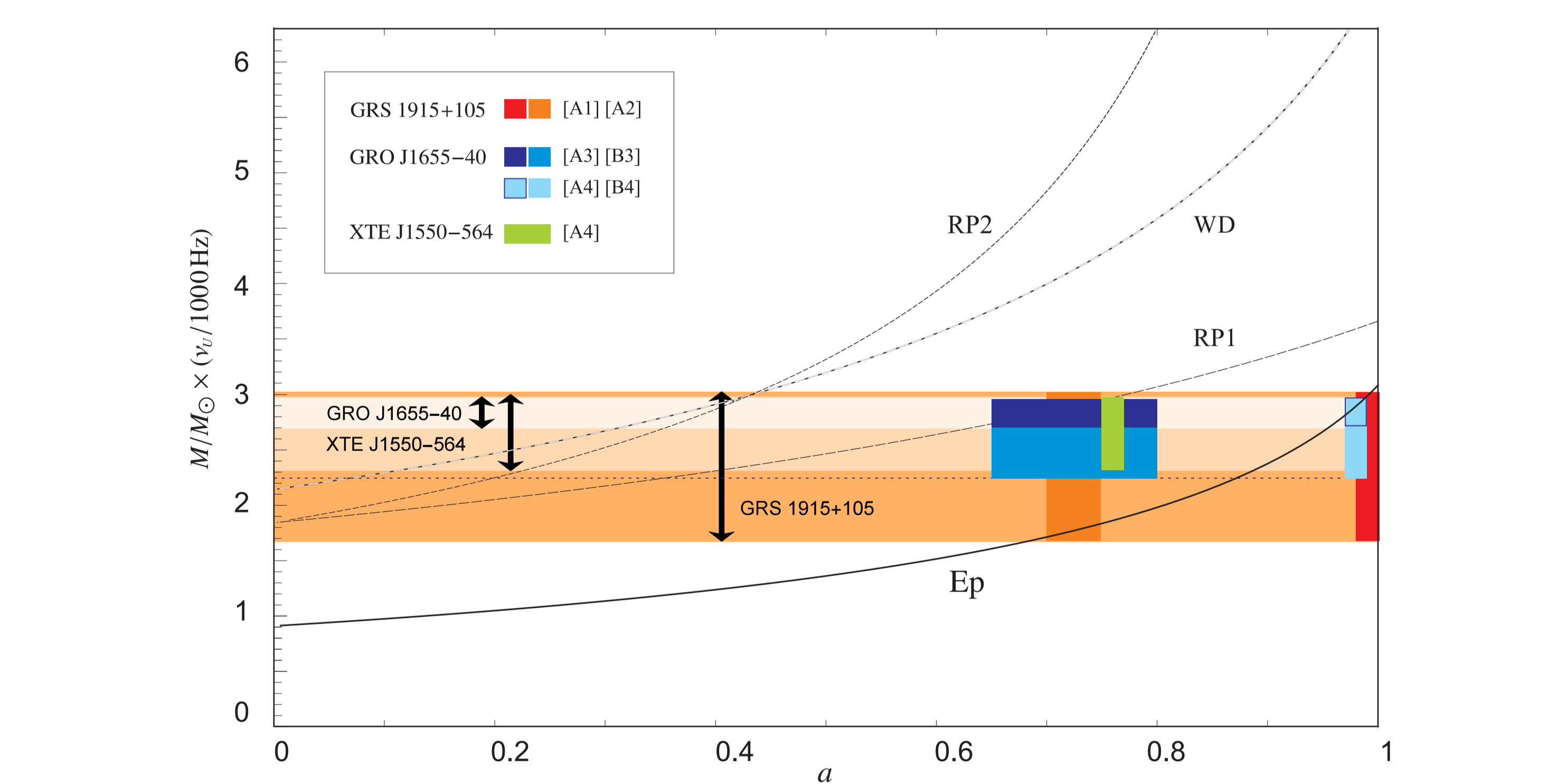
The list of the considered QPO models and their corresponding frequency relations of the lower and upper QPO modes is summarized in Table 2. It comprehends the particular ‘warped disc’ (WD) model representing concept proposed by Kato (2001, 2004, PASJ) that in general assume oscillation modes in a warped accretion disc. Then there is the ‘epicyclic resonance’ model of Abramowicz and Kluźniak that attributes the twin-peak HF QPOs to a non-linear resonance between two axisymmetric epicyclic accretion disc oscillation modes. Furthermore there are another two resonance models that we denote as the ‘RP1’ model (Bursa, 2005, Proceedings of RAGtime) and the ‘RP2’ model (Török et al., 2010, ApJ). Both of these models deal with certain combination of non-axisymmetric disc-oscillations modes.

## Figure 1



Left: The 3:2 HF QPO frequencies displayed by Galactic microquasars. Right: Inverse mass scaling of these frequencies supporting the QPO orbital origin hypothesis (McClintock & Remillard, 2003).

## Figure 2



Curves  $M(a)$  implied by the individual geodesic models. The light full horizontal range rectangles indicate the observationally determined interval of  $\nu_i \times M$  for each of the individual microquasars. The color boxes are drawn for the QPO independent mass and spin estimates given by different authors.

**Table 1.** The 3:2 QPO frequencies in three microquasars and QPO independent estimates of the central black hole mass (see, e.g., Török et al., 2005, A&A, for appropriate references and details).

Source	$\nu_L$ [Hz]	$\nu_U$ [Hz]	Mass [ $M_\odot$ ]
GRO 1655-40	300	450	6.0 – 6.6
GRS 1915+105	113	168	10.0 – 18.0
XTE 1550-564	184	276	8.4 – 10.8

**Table 2.** Frequency relations corresponding to individual QPO models ( $\beta = 0$ ) and the spins of the three microquasars implied by these models for the geodesic ( $a$ ) and non-geodesic ( $a^*$ ) case.

Model	Frequency Relations	$a \sim$	$a^* \sim$
WD	$\nu_L = 2(\nu_K - \nu_r)$ $\nu_U = 2\nu_K - \nu_r$	$< 0.44$	$< 0.44$
Ep	$\nu_L = \nu_r$ $\nu_U = \nu_\theta$	$0.68 - 0.99$	$0.62 - 1$
RP1	$\nu_L = \nu_K - \nu_r$ $\nu_U = \nu_\theta$	$< 0.78$	$0 - 1$
RP2	$\nu_L = \nu_K - \nu_r$ $\nu_U = 2\nu_K - \nu_\theta$	$< 0.44$	$< 0.44$

## Black Hole Spin Estimation

### Comparison of Spin Estimates Obtained by Different Methods

Several groups of authors applied different spectral fitting methods to measure the black hole spin in the three assumed sources (see Figure 2 for illustration). It is apparent that the spin predictions carried out by different authors are somewhat inconsistent. Comparing the spin measurements obtained by the spectral methods to those predicted by theoretical QPO models may help to shed some light on the present puzzling situation. We present such comparison within a mass-spin diagram displayed in Figure 2. In the Figure 2, the intervals of mass and spin based on QPO independent estimates are illustrated by the several colored boxes. The individual curves in the Figure then correspond to spin values predicted by the several QPO models given in Table 1.

### Spin Estimates Implied by the Non-Geodesic QPO Models

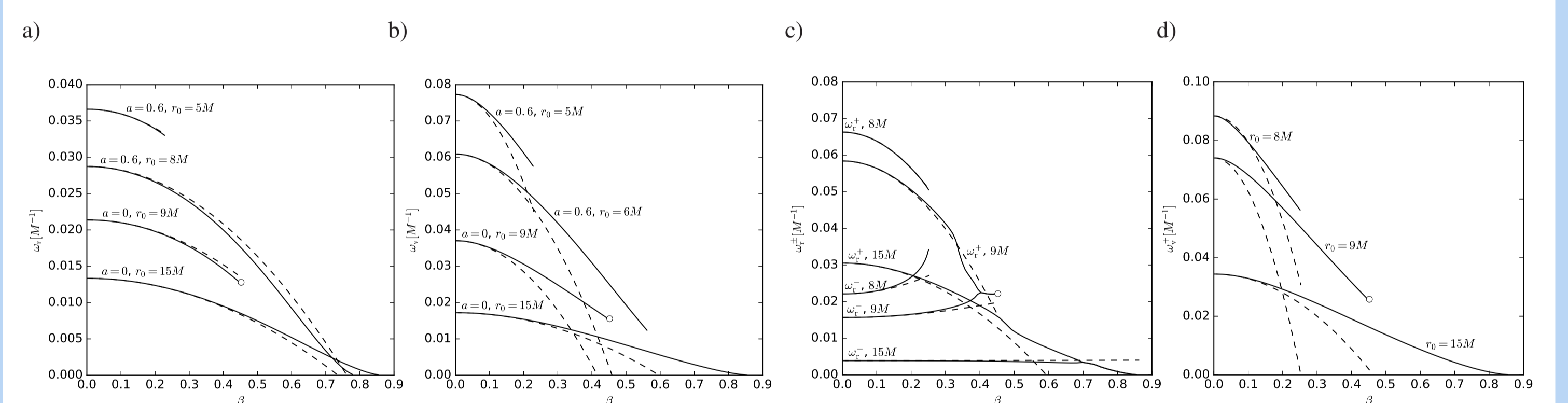
The relevant properties of the three microquasars are summarized in Table 1. We assume a slightly non-geodesic flow with non-negligible pressure gradients. In this flow the radial and vertical epicyclic oscillations of the fluid are naturally modified by the pressure forces. Such modifications were explored by Straub & Šrámková (2009, A&A) who calculated explicit formulae for the pressure corrections to epicyclic frequencies in a slightly non-slender constant specific angular momentum torus orbiting a Kerr black hole. Here we use their formulae, and, from the 3:2 observed QPO frequencies and estimated ranges of mass of the three microquasars, we infer the spin predicted by the QPO models listed in Table 2. The obtained intervals of spin for the individual models are presented in the right columns of this Table for the geodesic and non-geodesic case.

## Epicyclic Oscillations of Thick relativistic Disks

In our previous analytical work, considering epicyclic oscillations of pressure-supported perfect fluid tori orbiting Kerr black holes, non-geodesic (pressure) effects on the epicyclic modes properties were examined. Using a perturbation method we derived fully general relativistic formulae for eigenfunctions and eigenfrequencies of the radial and vertical epicyclic modes of a slightly non-slender, constant specific angular momentum torus up to second-order accuracy with respect to the torus thickness. The behaviour of the axisymmetric and lowest-order non-axisymmetric epicyclic modes was investigated. For an arbitrary black hole spin we found that, in comparison with the (axisymmetric) epicyclic frequencies of free test particles, non-slender tori receive negative pressure corrections and thus exhibit lower frequencies.

In present numerical study we examine epicyclic oscillations of thick relativistic tori with constant specific angular momentum distribution using the finite element numerical method. We have compared frequencies of the axisymmetric and non-axisymmetric modes with the analytic formulae obtained by Straub & Šrámková (2009, A&A) and Fragile et al. (2016, MNRAS). We have found excellent agreement in the case of axisymmetric radial epicyclic modes. In the case of the axisymmetric vertical epicyclic modes and non-axisymmetric modes in general, the analytic approximation agrees with numerical results only for tori of moderate thicknesses. These findings are illustrated in Figure 3.

## Figure 3

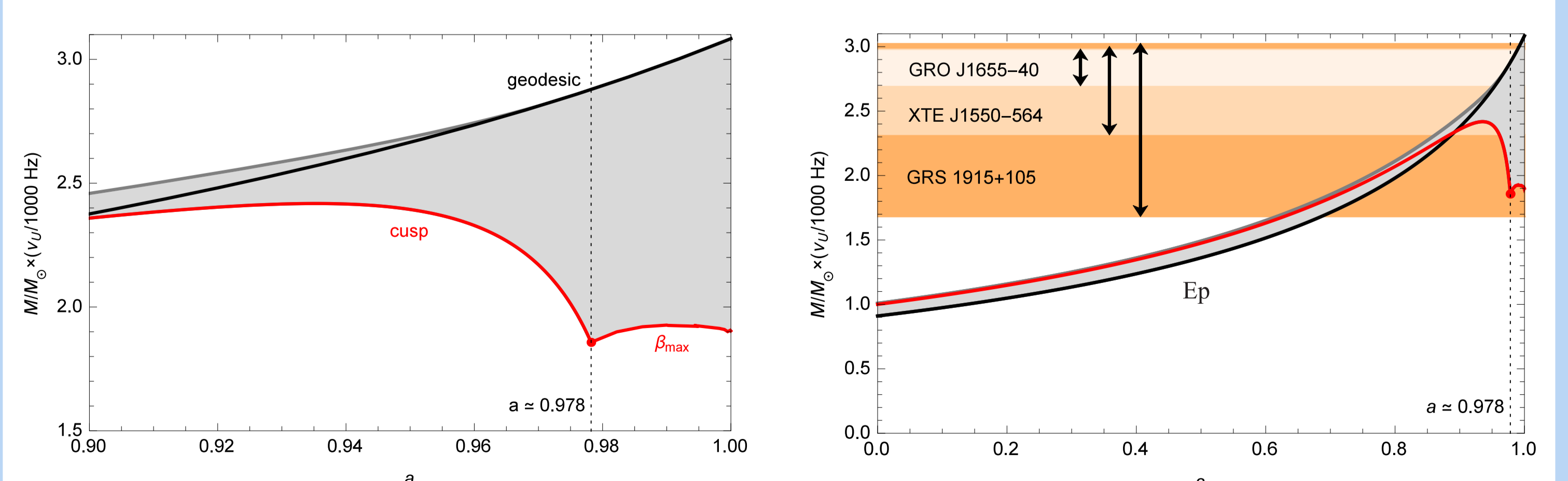


Eigenfrequencies of the axisymmetric ( $m = 0$ ) and non-axisymmetric ( $m = 1$ ) epicyclic modes in relativistic tori surrounding Kerr black hole. The dashed lines show the analytical approximations. The solid lines denote the numerical solutions. a) Radial mode ( $m = 0$ ). b) Vertical mode ( $m = 0$ ). c) Radial mode ( $m = 1$ ). d) Vertical mode ( $m = 1$ ).

## Implications for Epicyclic Resonance Model of QPOs

We conclude that the assumed non-geodesic effects shift the lower limit of the spin, implied for the three microquasars by the epicyclic model and independently measured masses, from  $a \sim 0.7$  to a  $a \sim 0.6$ . Furthermore, their consideration provides highly testable predictions on the QPO frequencies (see Figure 4). Individual sources with a moderate spin ( $a < 0.9$ ) should exhibit a smaller spread of the measured 3:2 QPO frequencies than sources with a near-extreme spin ( $a \sim 1$ ). This should be further examined using the large amount of high-resolution data expected to become available with the next generation of X-ray instruments, such as the LAD (Large Area Detector) previously proposed for Large Observatory for X-ray Timing (LOFT) or detectors considered for other future X-ray missions (The enhanced X-ray Timing and Polarimetry mission – eXTP).

## Figure 4



The non-geodesic corrections to the frequencies predicted by the epicyclic resonance model ( $m = 0$ ) are small for all but not high values of the spin  $a$ . Left: The  $M(a)$  relation implied by the epicyclic resonance model. The thick black curve corresponds to the geodesic case, i.e. it is the same curve as shown in Figure 2. The gray shaded region corresponds to the consideration of non-geodesic effects. Right: Impact on the estimation of the BH spin.

## Acknowledgements

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