Contribution of QCD Condensates to the OPE of Green Functions of Chiral Currents

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Introduction

- The amplitudes of physical processes can be computed using LSZ reduction formula from the Green functions, the vacuum expectation values of the time ordered products of composite operators $\mathcal{O}$ (the group and Lorentz indices are suppresed):

$$\Pi(p,q) = \int \mathrm{d}^4x\,\mathrm{d}^4y\, e^{-i(p\cdot x+q\cdot y)} \left\langle \mathrm{T}\,\mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(0)\right\rangle .$$

- The operators $\mathcal{O}$ stand for any of the chiral:
  - vector $V^a_\mu(x) = \overline{q}(x)\gamma_\mu T^a q(x)$ or axial $A^a_\mu(x) = \overline{q}(x)\gamma_\mu\gamma_5 T^a q(x)$ currents,
  - scalar $S^a(x) = \overline{q}(x)T^a q(x)$ or pseudoscalar $P^a(x) = i\overline{q}(x)\gamma_5 T^a q(x)$ densities.
- There are 15 nontrivial three-point Green functions:
  - Set I: $\langle ASP\rangle$, $\langle VSS\rangle$, $\langle VPP\rangle$, $\langle VVA\rangle$, $\langle AAA\rangle$, $\langle AAV\rangle$, $\langle VVV\rangle$.
  - Set II: $\langle SSS\rangle$, $\langle SPP\rangle$, $\langle VVP\rangle$, $\langle AAP\rangle$, $\langle VAS\rangle$, $\langle VVS\rangle$, $\langle AAS\rangle$, $\langle VAP\rangle$.

OPE and QCD condensates

- OPE: at large external momenta, the Green function can be written down as a sum of Wilson coefficients proportional to vacuum avarages of composite gauge-invariant local operators (QCD condensates), made of quark and gluon fields:

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(y)\mathcal{O}_3(0)\rangle = C_0 + C_1\langle\overline{q}q\rangle + C_2\langle G_{\mu\nu}G^{\mu\nu}\rangle + C_3\langle\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle + C_4\langle\overline{q}q\rangle^2 + \dots$$

- The first term corresponds to the perturbative contribution and the subsequent ones stand for the quark, gluon, quark-gluon, four-quark condensates.
- The Wilson coefficients C_i contain informations about short-distance physics, i.e. the dynamics above some scale μ, and are calculable in perturbative QCD by means of Feynman diagrams.

Odd-intrinsic parity sector of QCD: $\langle VVA\rangle$ (example)

- Important phenomenological object, connection with the decay of axial resonance $f_1(1285)$, see for example [1].
- The Ward identities restrict the general decomposition of the tensor part of the $\langle VVA\rangle$ correlator into four terms:

$$\left[\Pi_{VVA}(p,q;r)\right]^{abc}_{\mu\nu\rho} = d^{abc}\left(w_L\varepsilon_{\mu\nu(p)(q)}r_\rho + \sum_{i=1}^{3} w_T^{(i)}\Pi^{(i)}_{\mu\nu\rho}\right).$$

  - Longitudinal formfactor w_L is fixed entirely by the chiral anomaly.
  - Transversal tensors $\Pi^{(i)}_{\mu\nu\rho}$ can be found in [2].
- Contribution of resonances at NLO are given by [3]

$$\mathcal{L}^{(6)}_{\mathrm{R}\chi\mathrm{T}} = \sum_X\sum_i \kappa_i^X \widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta}\varepsilon^{\mu\nu\alpha\beta}.$$

  - X stands for the channels with one resonance (V, A, S, P), two resonances (VV, AA, SA, SV, VA, PA, PV) and three resonances (VVP, VAS, AAP).
  - 67 operators and 67 coupling constants κ_i^X (many unknown).
- How to determine the couplings for $\langle VVA\rangle$?
  - Calculate resonance contributions to $w_T^{(i)}$ and construct the formfactor w_T:

$$w_T(Q^2) = -16\pi^2\left[w_T^{(1)}(-Q^2,0,-Q^2) + w_T^{(3)}(-Q^2,0,-Q^2)\right].$$

  - We already know the result for OPE of $\langle V^\star VA\rangle$, where one of the momenta is soft [4]:

$$w_T(Q^2) = \frac{N_c}{Q^2} + \frac{128\pi^3\alpha_s\chi\langle\overline{q}q\rangle^2}{9Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right).$$

  - After expanding the resonance contribution up to $\mathcal{O}(\frac{1}{Q^8})$, we can obtain constraints for respective couplings.

Results: Coupling constants

- For example [5]: $\kappa_5^{VA} = -0.086$, which can be compared with the value obtained from the decay of the axial resonance $f_1(1285)$, which gives $\kappa_5^{VA} = -0.062 \pm 0.030$.
- We are also able to predict the value for the deviation parameter, which describes by how much is the Brodsky-Lepage behaviour of the pion transition formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\gamma\gamma}$ violated.
  - We found $\delta_{\mathrm{BL}} = -1.342$.
  - Then, we have taken data sets for the measured pion transition formfactor, obtained by the BABAR [6], BELLE [7] and CLEO [8] collaborations, and compared them with the formula for the pion transition formfactor for various values of the deviation parameter.
- A disagreement between our prediction and experiments has been found.

Results: Pion transition formfactor

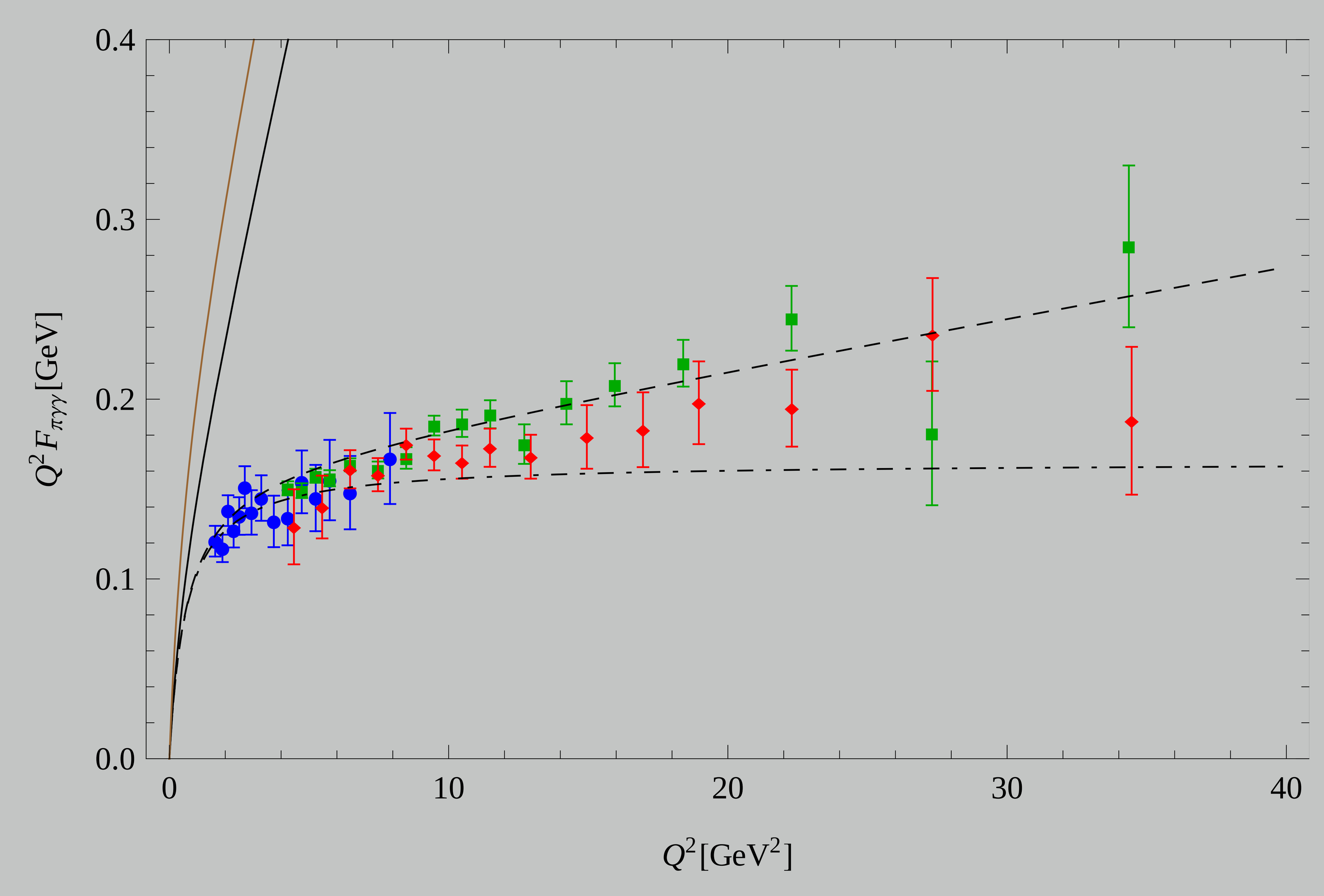

Figure 1: A plot of BABAR (green), BELLE (red) and CLEO (blue) data fitted with the formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\gamma\gamma}(0,-Q^2;0)$ using the modified Brodsky-Lepage condition. The full black line represents our fit with $\delta_{\mathrm{BL}} = -1.342$, and the full brown line is a fit using the LMD formfactor. The dashed line stands for $\delta_{\mathrm{BL}} = -0.055$ and the dot-dashed line for $\delta_{\mathrm{BL}} = 0$.

OPE for all momenta large: Propagation of QCD condensates

- We tried to solve this inconsistency by calculating OPE with all three momenta large, instead of using the OPE for only two large momenta.
  - We also included higher-order contributions of QCD condensates and used them once again to express the couplings of the NLO resonance Lagrangian.
- Propagation of nonlocal condensates!
  - The Fock-Schwinger gauge, $(x-x_0)^\mu\mathcal{A}^a_\mu(x) = 0$, allows us to obtain expansion of the nonlocal QCD condensates in terms of local ones.
  - Nonlocal quark and quark-gluon condensates propagate as local quark, quark-gluon and four-quark condensates.
  - "*This effect has been one of the main source of errors in the existing QCD spectral sum rules literature.*" [9]

$$\langle\overline{q}(x)q(y)\rangle \sim \langle\overline{q}q\rangle + \langle\overline{q}Gq\rangle\,F^{\langle\overline{q}q\rangle}(x,y) + \langle\overline{q}q\rangle^2\,G^{\langle\overline{q}q\rangle}(x,y)\,,$$
$$\langle\overline{q}(x)\mathcal{A}_\mu(y)q(z)\rangle \sim \langle\overline{q}Gq\rangle\,F_\mu^{\langle\overline{q}\mathcal{A}q\rangle}(x,y,z) + \langle\overline{q}q\rangle^2\,G_\mu^{\langle\overline{q}\mathcal{A}q\rangle}(x,y,z)\,.$$

- The functions F and G are highly nontrivial [5].
  - Our results are in the most general form.
  - So far in the literature, one usually takes one of the coordinates as zero, so the formulas were not applicable for three-point Green functions.
  - Contributions of quark, gluon, quark-gluon and four-quark condensates have been obtained for all existing three-point correlators.

Conclusion

- We calculated OPE of all three-point Green functions of chiral currents for all momenta large.
  - We expressed these correlators at large energies in terms of QCD condensates.
  - We also tried to match the OPE with RχT, however, it is still unclear how to deal with logarithmic terms for which one would need infinite tower of resonances to get rid of them.
- Our work is still ongoing and the final results should be available soon in [5].

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