On the Magnetic Field of Pulsars with Realistic Neutron Star Configurations

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PLAN OF THE TALK

1. MOTIVATION
   - High-B Pulsar Class
2. NEUTRON STAR MODEL
   - Einstein-Maxwell-Thomas-Fermi
   - Rotating Neutron Stars: Hartle-Thorne Perturbative Method
   - Neutron Star Structure
3. PULSAR'S MAGNETIC FIELD
   - Pulsar's Field Corrections
   - Magnetic Field with Realistic NS Configurations
   - Rotational Energy Loss for High-B Pulsars
4. RESULTS

Surface Magnetic Field

Newtonian Magnetic Field (Dipolar Approximation)

$$\dot{E}_{\rm rot} = -4\pi^2 I \frac{\dot{P}}{P^3}$$

$$P_{\rm dip} = -\frac{2}{3}\frac{\mu_\perp^2 \Omega^4}{c^3}$$

$$B \sin\chi = \sqrt{\frac{3c^3 I}{8\pi^2 R^6} P\dot{P}}$$

$(\mu_\perp = \mu \sin\chi, \mu = BR^3)$

J. E. Gunn and J. P. Ostriker, Nature **221**, 454 (1969) – J. P. Ostriker and J. E. Gunn, ApJ **157**, 1395 (1969) – A. Ferrari and R. Ruffini, ApJ **158**, L71 (1969)

$\dot{E}$ and B with Fiducial Values: $M = 1.4\, M_\odot$, $R = 10$ km, $I = 10^{45}$ g cm^2

$$\dot{E}^f_{\rm rot} = -3.95 \times 10^{46} \dot{P}/P^3 \; erg\; s^{-1} \quad , \quad B_f = 3.2 \times 10^{19} \left(P/\dot{P}\right)^{1/2} \text{G}$$

HIGH-MAGNETIC FIELD PULSAR CLASS

High-Magnetic Field Pulsars

J-Name	B_f/B_c	L_X (10^{33} erg s^{-1})	$L_X/\lvert\dot{E}^f_{\rm rot}\rvert$	P (s)	$\dot{P}$ (10^{-12})
J1846–0258	1.11	$25-28^a$, $120-170^b$	$0.0031-0.0035^a$, $0.015-0.021^b$	0.323	7.08
J1819–1458[c]	1.13	$1.8-2.4$	$6.21-8.28$	4.263	0.58
J1734–3333	1.18	$0.1-3.4$	$0.0018-0.0607$	1.169	2.28
J1814–1744	1.24	< 43	< 91.5	1.169	2.28
J1718–3718	1.67	$0.14-2.6$	$0.0875-1.625$	3.378	1.60
J1847–0130	2.13	< 34	< 200	6.707	1.27

a) in 2000, prior to the 2006 outburst
b) during the outburst in 2006
c) classified as a rotating radio transient (RRAT)

C.-Y. Ng and V. M. Kaspi, Am. Inst. Phys. Conf. Series **1379**, 60 (2011) – V. M. Kaspi et Al., ApJ **734**, 44 (2011)

CRITICAL B-FIELD FOR VACUUM POLARIZATION

$$B_c = \frac{m_e^2 c^2}{e\hbar} \approx 4.41 \times 10^{13}\ \mathrm{G}$$

High-Magnetic Field Pulsar as missing link, i.e. transition objects, between the rotation powered pulsars and the magnetars?

BUILDING NSs WITH GLOBAL CHARGE NEUTRALITY (GCN)

- Thermodynamical Equilibrium: $\rightarrow \mu_{\text{K.P.}} = \sqrt{g_{00}}\,\mu = const$
  O. Klein, Rev. Mod. Phys. **21**, 531 (1949)
- a self-gravitating system of degenerate neutrons, protons and electrons in β-equilibrium fulfill **Global Charge Neutrality** and *NOT* **Local Charge Neutrality**, the latter violating the Klein's equilibrium conditions.
  M. Rotondo, Jorge A. Rueda, R. Ruffini and S.-S. Xue, Phys. Lett. B **701**, 667 (2011)
- In presence of gravitational and Coulomb fields, global electric polarization effects at macroscopic scales occur (S. Rosseland, MNRAS **84**, 720 (1924)) $\Rightarrow$ if one turns to examine gravito-polarization effects in neutron stars, the equations describing baryonic matter (i.e. the EOS) need to be solved simultaneously in combination with the **Einstein-Maxwell**.
- The Klein thermodynamic equilibrium conditions are the generalization, to the relativistic case, of the electronic equilibrium in the **Thomas-Fermi** model of the atom.
- The total Lagrangian density of the system has to take into account gravitational, strong, weak and electromagnetic interactions and is given by

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\gamma + \mathcal{L}_{\text{int}}$$

J. Boguta and A. R. Bodmer, Nucl. Phys. A **292**, 413 (1977)

EMTF SYSTEM OF EQUATIONS

- **NS composition**: neutrons, protons and electrons
- **Interactions**: Gravitational, Electromagnetic, Strong (σ- ω-ρ model), Weak. (GR)
- **Space-time metric**: $ds^2 = \mathrm{e}^{\nu(r)}dt^2 - \mathrm{e}^{\lambda(r)}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$

$$\mathrm{e}^{-\lambda(r)}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2} = -8\pi G T_0^0$$

$$\mathrm{e}^{-\lambda(r)}\left(\frac{1}{r^2} + \frac{1}{r}\frac{d\nu}{dr}\right) - \frac{1}{r^2} = -8\pi G T_1^1$$

$$\frac{d^2V}{dr^2} + \frac{dV}{dr}\left[\frac{2}{r} - \frac{1}{2}\left(\frac{d\nu}{dr} + \frac{d\lambda}{dr}\right)\right] = -\mathrm{e}^{\lambda} e J_0^{ch}$$

$$\frac{d^2\sigma}{dr^2} + \frac{d\sigma}{dr}\left[\frac{2}{r} + \frac{1}{2}\left(\frac{d\nu}{dr} - \frac{d\lambda}{dr}\right)\right] = \mathrm{e}^{\lambda}\left[\partial_\sigma U(\sigma) + g_s n_s\right]$$

$$\frac{d^2\omega}{dr^2} + \frac{d\omega}{dr}\left[\frac{2}{r} - \frac{1}{2}\left(\frac{d\nu}{dr} + \frac{d\lambda}{dr}\right)\right] = -\mathrm{e}^{\lambda}\left[g_\omega J_0^\omega - m_\omega^2\omega\right]$$

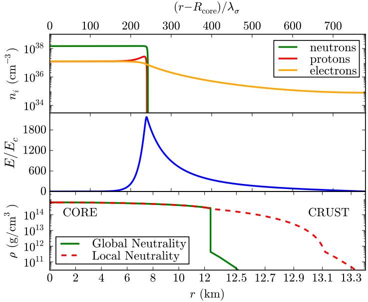
$$\frac{d^2\rho}{dr^2} + \frac{d\rho}{dr}\left[\frac{2}{r} - \frac{1}{2}\left(\frac{d\nu}{dr} + \frac{d\lambda}{dr}\right)\right] = -\mathrm{e}^{\lambda}\left[g_\rho J_0^\rho - m_\rho^2\rho\right]$$

$$E_e^F = \mathrm{e}^{\nu/2}\mu_e - eV = \text{constant}$$

$$E_p^F = \mathrm{e}^{\nu/2}\mu_p + \mathcal{V}_p = \text{constant}$$

$$E_n^F = \mathrm{e}^{\nu/2}\mu_n + \mathcal{V}_n = \text{constant}$$

GCN VS LCN NS STRUCTURE



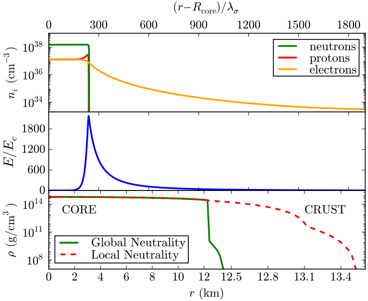


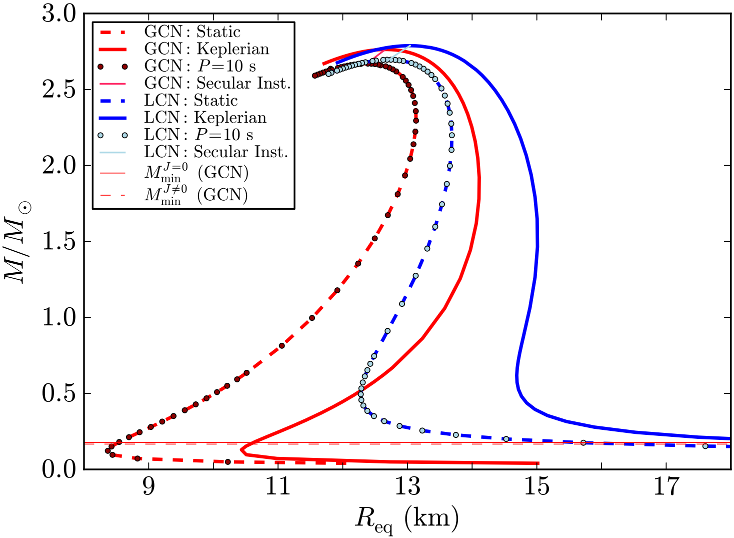
FIGURE: Upper panel: particle density profiles in the core-crust boundary interface, in units of cm^{-3}. Middle panel: electric field in the core-crust transition layer, in units of the critical field E_c. Lower panel: density profile inside a neutron star with central density $\rho(0) \sim 5\rho_{\rm nuc}$. We compare and contrast the structural differences between the solution obtained from the traditional TOV equations (locally neutral case) and the globally neutral solution presented here. $\lambda_\sigma = \hbar/(m_\sigma c) \sim 0.4$ fm, denotes the sigma-meson Compton wavelength. Left: the density at the edge of the crust is $\rho_{\rm crust} = \rho_{\rm drip} = 4.3 \times 10^{11}$ g/cm^3. Right: the density at the edge of the crust is $\rho_{\rm crust} = 10^{10}$ g/cm^3.

R. B., D. Pugliese, J. A. Rueda, R. Ruffini and S.-S. Xue, NPA **883**, 1 (2012)

Hartle Slow Rotation Approximation

- Solution obtained thorough a perturbative method, expanding the metric functions up to the second order in the angular velocity Ω.
- The structure of compact objects can be approximately described by M, J and Q.
- Slow rotation regime $\rightarrow$ perturbations owing to the rotation $<$ the known non-rotating geometry.
- Interior solution derived by solving numerically a system of ordinary differential equations for the perturbation functions.
- The exterior solution for the vacuum surrounding the star, can be written analytically in terms of M, J, and Q.
  J. B. Hartle, ApJ **150**, 1005 (1967). J. B. Hartle and K. S. Thorne, ApJ **153**, 807 (1968).
- $$ds^2 = e^{\nu}\left(1+2h\right)dt^2 - e^{\lambda}\left[1+\frac{2m}{r-2M_0}\right]dr^2 - r^2\left(1+2k\right)\left[d\theta^2+\sin^2\theta\left(d\phi-\omega dt\right)^2\right]$$
  being $\omega = \omega(r)$ the fluid angular velocity in the local inertial frame.
- $$J = \frac{1}{6}R^4\left(\frac{d\bar{\omega}}{dr}\right)_{r=R}\ ,\ \bar{\omega}(r) = \Omega - \omega(r)\ ,\ I = \frac{J}{\Omega}$$
- $$M = M_0 + \delta M\ ,\ \delta M = m_0(R) + J^2/R^3$$
- $$Q = \frac{J^2}{M_0} + \frac{8}{5}\mathcal{K}M_0^3$$

NEUTRON STAR STRUCTURE



R. B., K. Boshkayev, J. A. Rueda, and R. Ruffini, NPA **921**, 33 (2014)

Pulsar's Field Corrections

- Existence of a plasma magnetosphere instead of an electrovacuum → many competing models of the pulsar's magnetosphere: NO.

- The dependence on EOS by the neutron star structure parameters, with respect to the oversimplification lead by the use of fiducial values→ just shown how the structure parameters depend on the neutron star theory and EOS: YES.

- The effects due to the relativistic fast rotation, as measured by the fastness parameter, $\Omega R/c$ → this specific correction is expected to be important for millisecond pulsars. For the pulsar class discussed in this work, with rotation periods $P \sim 10$ s (hence, $\Omega R/c = 2\pi R/(cP) \sim 10^{-5}$), such a correction is negligible and the solution in the slow rotation regime is sufficiently accurate: NO.

- The corrections measured by the compactness parameter $GM/(c^2 R)$, introduced by the finiteness of the mass and size of the star → details in the next slides: YES.

COMPACTNESS PARAMETER CORRECTIONS

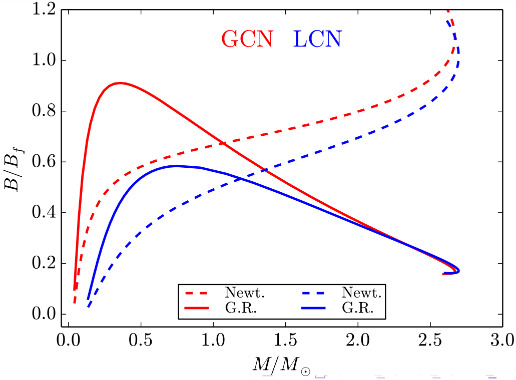
DIPOLE RADIATION POWER

- $P_{\rm dip}^{\rm G.R.} = -\frac{2}{3}\frac{\mu_\perp^2\Omega^4}{c^3}\left(\frac{f}{N^2}\right)^2$
- $f = -3/8(R/M_0)^3[\ln(N^2) + 2M_0/R(1 + M_0/R)]$; $N = \sqrt{1 - 2M_0/R}$

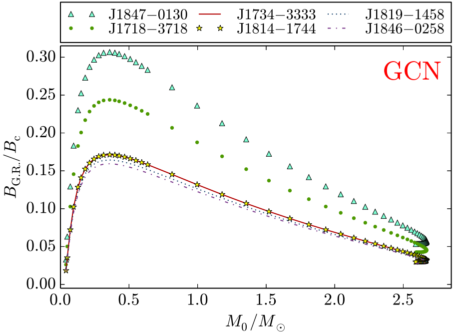
L. Rezzolla and B. J. Ahmedov, MNRAS **352**, 1161 (2004)

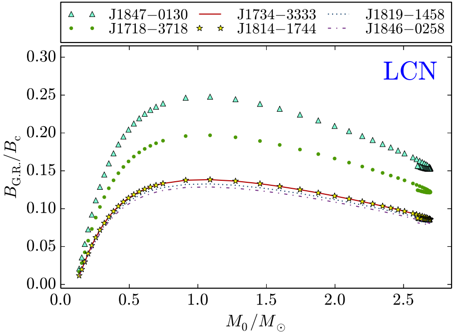
G.R. MAGNETIC FIELD

$$B\sin\chi = \frac{N^2}{f}\left(\frac{3c^3}{8\pi^2}\frac{I}{R^6}P\dot{P}\right)^{1/2}$$



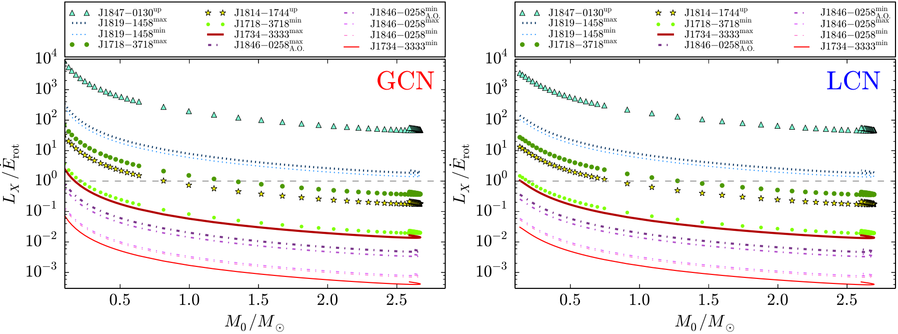
Magnetic Field for High-B Pulsars





⇒ The magnetic field of all the high-magnetic field pulsars turn to be under-critical for any values of the neutron star mass!!!

Rotational Energy to Electromagnetic Radiation Conversion Efficency



⇒ The X-ray luminosity of these pulsars can be well explained via the loss of rotational energy and therefore they fall into the family of ordinary Rotation Powered Pulsars. The only possible exceptions were found to be PSR J1847–0130 and PSR J1819–1458, which however still present observational uncertainties in the determination of their distances and/or luminosities that leave room for a possible explanation in terms of spin-down power.

R. B., J. A. Rueda and R. Ruffini, Ap. J. **799**, 23 (2015)

SUMMARY

1. The magnetic field is overestimated when fiducial parameters are adopted, independently on the use of either the Newtonian or the general relativistic radiation formula of the rotating magnetic dipole (see Fig. $B/B_{\rm f}$ vs $M/M_{\odot}$).

2. The use of the Newtonian formula for the magnetic field can overestimate the surface magnetic field of up to one order of magnitude with respect to the general relativistic one (see Fig. $B/B_{\rm f}$ vs $M/M_{\odot}$). We applied these considerations to the specific case of the high-magnetic field pulsar class, for which overcritical magnetic fields have been obtained in the literature with the use of fiducial neutron star parameters within the Newtonian rotating magnetic dipole model. We found that, instead, the magnetic field inferred for these pulsars turn to be undercritical for any values of the neutron star mass (see Fig. $B_{\rm G.R.}/B_{\rm c}$ vs $M_0/M_{\odot}$).

3. The nontrivial dependence of the inferred magnetic field on the neutron star mass, in addition to the dependence on P and $\dot{P}$, namely $B = B(I(M_0), R(M_0), P, \dot{P})$, leads to the impossibility of accommodating the pulsars in a typical $\dot{P} - P$ diagram together with a priori fixed values of the magnetic field (see Fig. $B_{\rm G.R.}/B_{\rm c}$ vs $M_0/M_{\odot}$).

4. We computed the range of neutron star masses for which the X-ray luminosity of these pulsars can be well explained via the loss of rotational energy of the neutron star and therefore falling into the family of ordinary rotation-powered pulsars (see Fig. $L_{\rm X}/\dot{E}_{\rm rot}$ vs $M_0/M_{\odot}$).