B_c excitations at LHC

Workshop on Heavy Baryons at LHCb, CERN

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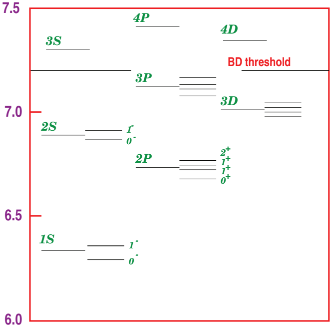
B_c family

- All excitations below the threshold decay into 1^1S_0.
- Charge parity can not be determined: $h_Q\ \chi_{1Q} \xrightarrow{\text{mixing}} 1^+\ 1^{+'}$.



state	Martin	BT
1^1S_0	6.253	6.264
1^1S_1	6.317	6.337
2^1S_0	6.867	6.856
2^1S_1	6.902	6.899
2^1P_0	6.683	6.700
$2P\ 1^+$	6.717	6.730
$2P\ 1'^+$	6.729	6.736
2^3P_2	6.743	6.747
3^1P_0	7.088	7.108
$3P\ 1^+$	7.113	7.135
$3P\ 1'^+$	7.124	7.142
3^3P_2	7.134	7.153
$3D\ 2^-$	7.001	7.009
3^5D_3	7.007	7.005
3^3D_1	7.008	7.012
$3D\ 2'^-$	7.016	7.012

Figure 1: The mass spectrum of $(\bar{b}c)$ with account for the spin-dependent splittings.

[Gouz et al.(2004)Gouz, Kiselev, Likhoded, Romanovsky, and Yushchenko, Godfrey(2004)]

How to estimate the cross section for B_c excitations?

The same method as for the ground state can used.

δ-approximation:

- the matrix element of S wave state production is proportional to the matrix element of $b\bar{c}c\bar{c}$ production timed by S wave function at origin[1],
- $\bar{b}$ and c quark moves with the same velocities,
- $\bar{b}$ and c quarks are in appropriate spin state.

Color model:

- $\bar{b}c$ pair is produced in color singlet.

Heavy quark matrix element:

- calculated within leading order of QCD.

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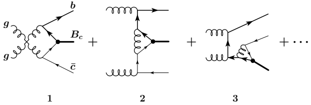
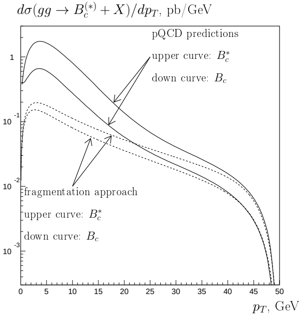
[1] For the P wave production the amplitude in proportional to the derivatives of the hard part of amplitudes over internal momentum of quark in quarkonuim timed by the derivative of P wave function at origin

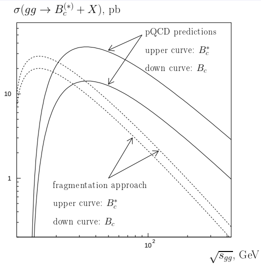
$gg \to B_c + X$

The examples of diagrams for $gg \to B_c b\bar{c}$:



p_T-distribution for $\sqrt{\hat{s}_{gg}} = 100$ GeV:





The difference is partially hiden by convolution with PDFs.

For $P_T(B_c) < 35 - 40$ GeV recombination diagrams (such as 1 and 2) dominates and

$$\boxed{\frac{\sigma(B_c^*)}{\sigma(B_c)} \sim 2.5 \div 3}$$

Fragmentation approach is valid for $P_T(B_c) > 35 - 40$ GeV:

$$\boxed{\frac{\sigma(B_c^*)}{\sigma(B_c)} \sim 1.4}$$

What will be observed first: B_c^* or B_c^P?

$$\boxed{B_c^*(1^3S_1)\xrightarrow[100\%]{\gamma}B_c(1^1S_0)}$$

$$M(B_c^*) - M(B_c) \approx 65 \text{ MeV}$$

Maximum transverse energy ω_T of γ in the lab. system:

$$\omega_T^{max} = \left(1+\frac{\Delta M}{2M_{B_c^*}}\right)\left(\sqrt{M_{B_c^*}^2+p_T^2}+p_T\right)\frac{\Delta M}{M_{B_c^*}}$$
$$\approx 0.01\left(\sqrt{M_{B_c^*}^2+p_T^2}+p_T\right)$$

B_c^* v.s. B_c^P:

- a lot of B_c^*, but high p_T is needed.
- Small amount of B_c^P, but p_T is not essential.

Seems, that B_c^P family win:

$$\frac{\sigma_{2P}(\omega_T^\gamma>0.5\text{ GeV})}{\sigma_{1S}(\omega_T^\gamma>0.5\text{ GeV}\Longleftrightarrow p_T^{Bc}>24\text{ GeV})}\sim 25\div 50$$

Table : Decays $P\to 1S+\gamma$

[Godfrey(2004), Gupta and Johnson(1996), Kiselev et al.(1995)Kiselev, Likhoded, and Tkabladze]

state	decay to 1S	Br, %	ΔM, MeV
2^3P_0	$1^3S_1+\gamma$	100	363-366
$2P1^+$	$1^3S_1+\gamma$	87	393-400
	$1^1S_0+\gamma$	13	393-400
$2P1'^+$	$1^1S_0+\gamma$	94	472-476
	$1^3S_1+\gamma$	6	472-476
2^3P_2	$1^3S_1+\gamma$	100	410-426
3^3P_0	$1^3S_1+\gamma$	2	741
$3P1^+$	$1^3S_1+\gamma$	8.5	761
	$1^1S_0+\gamma$	3.3	820
$3P1'^+$	$1^1S_0+\gamma$	22.6	825
	$1^3S_1+\gamma$	0.7	769
3^3P_2	$1^3S_1+\gamma$	18	778

What to do with the soft photon from B_c^* decay?

Only $\sim 20\%$ of $2P$-wave states decay radiating only one photon:

$$2P1^+(B_c) \xrightarrow[\sim 13\%]{\gamma} 1^1S_0(B_c)$$

$$2P1'^+(B_c) \xrightarrow[\sim 94\%]{\gamma} 1^1S_0(B_c)$$

In most cases

$$2P \xrightarrow{\gamma_1^{\text{hard}}} 1^3S_1(B_c^*) \xrightarrow{\gamma_2^{\text{soft}}} 1^1S_0(B_c)$$

To find γ_2 the high p_T of B_c^* is needed.

But, it seems that we can see P-wave excitations without the reconstruction of soft γ_2.

$$\boxed{\Delta \approx 2\frac{(M_{B_c(P)} - M_{B_c^*})(M_{B_c^*} - M_{B_c})}{M_{B_c}}}$$

width of partially reconstructed B_c^P

$$B_c^P \;:\; M_{B_c^P} = M + \Delta M_1 + \Delta M_2$$

$$B_c^* \;:\; M_{B_c^*} = M + \Delta M_2$$

$$B_c \;:\; M_{B_c} = M$$

$$B_c^P \xrightarrow[\Delta M_1]{\gamma_1(\omega_1)} B_c^* \xrightarrow[\Delta M_2]{\gamma_2(\omega_2)} B_c$$

In the B_c rest frame:

$$M^2_{B_c^P} = (M + \Delta M_1 + \Delta M_2)^2 = \\ M^2 + 2(\omega_1 + \omega_2)M + 2\omega_1\omega_2(1 - \cos\Theta)$$

$$M^2_{B_c^*} = (M + \Delta M_2)^2 = M^2 + 2\omega_2 M$$

Θ is the angle between γ_1 and γ_2 in the B_c rest frame.

$$\omega_2 = \left(1 + \frac{\Delta M_2}{2M}\right)\Delta M_2$$

$$\omega_1 = \frac{2M + \Delta M_1}{2M + 2\omega_2(1 - \cos\Theta)}\Delta M_1$$

$$\omega_1^{max} = \left(1 + \frac{\Delta M_1}{2M}\right)\Delta M_1$$

$$\omega_1^{min} = \frac{1 + \frac{\Delta M_1}{2M}}{\left(1 + \frac{\Delta M_2}{M}\right)^2}\Delta M_1$$

$\tilde{M}$ is the mass of $B_c + \gamma_1$ system:

$$\tilde{M}^2 = (M + \omega_1)^2 - \omega_1^2 = M^2 + 2\omega_1 M$$

$$\tilde{M}^2_{max} = (M + \Delta M_1)^2$$

$$\tilde{M}^2_{min} = \\ = (M + \Delta M1)^2 - 4\Delta M_1 \Delta M_2 \frac{\left(1 + \frac{\Delta M_1}{2M}\right)\left(1 + \frac{\Delta M_2}{2M}\right)}{\left(1 + \frac{\Delta M_2}{M}\right)^2}$$

$$\boxed{\tilde{M}_{max} - \tilde{M}_{min} \approx 2\frac{\Delta M_1 \Delta M_2}{M}}$$

P-wave excitations in $B_c + \gamma$ mass spectrum

Peaks for $2P$ wave excitations ("soft" photon is lost):

- shifted by $\sim$ 65 MeV
- have a width $\sim$ 10 MeV

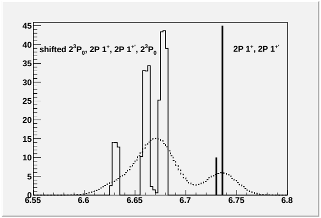
Peaks for $3P$ wave excitations ("soft" photon is lost):

- shifted by $\sim$ 65 MeV
- have a width $\sim$ 20 MeV

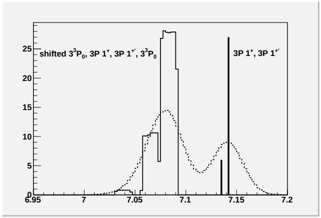
$\sigma(3P) \sim \sigma(2P)$, but only $\sim 20\%$ of $B_c(3P)$ decay electromagnetically into $1^3S_1(B_c^*)$ or $1^1S_0(B_c)$.

$\omega_T^{\min}$, GeV	B_c state	relative yield %
0.3	$B_c(2P)$	~ 5.0
	$B_c(3P)$	~ 1.0
	B_c^*	~ 0.8
0.5	$B_c(2P)$	~ 3.5
	$B_c(3P)$	~ 0.7
	B_c^*	~ 0.06
1.0	$B_c(2P)$	~ 0.9
	$B_c(3P)$	~ 0.4
	B_c^*	~ 0.005

$B_c(2P) \to B_c + \gamma[+\gamma_{\text{lost}}]$



$B_c(3P) \to B_c + \gamma[+\gamma_{\text{lost}}]$



$B_c(2S) \to B_c(B_c^*) + \pi\pi$

$$2^1S_0(B_c) \xrightarrow[\sim 50\%]{\pi^+\pi^-} 1^1S_0(B_c)$$

$$2^3S_1(B_c) \xrightarrow[\sim 40\%]{\pi^+\pi^-} 1^3S_1(B_c)$$

$$\sigma(B_c(2S))/\sigma^{\text{total}}(B_c) \sim 20 \div 30 \ \%$$

$$\sigma(2^3S_1)/\sigma(2^1S_0) \sim 2.6$$

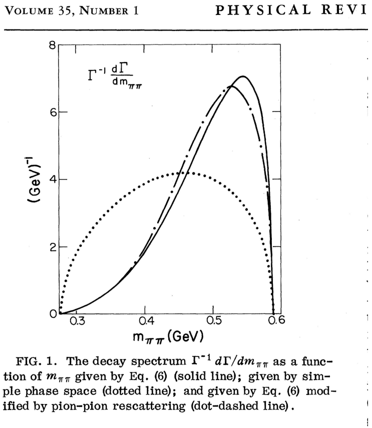
feed down from $B_c(2S) \to B_c(1S) + \pi^+\pi^-$ $\sim 10 \div 15\ \%$

For $2S$ vector, as well as pseudoscalar

$$\frac{1}{\Gamma}\frac{d\Gamma}{dm_{2\pi}} \sim \frac{|\mathbf{k}_{\pi\pi}|}{M^2}(2x^2-1)\sqrt{x^2-1}$$

where $x = m_{\pi\pi}/2m_\pi$ and $\mathbf{k}_{\pi\pi}$ is the momentum of $\pi\pi$-pair in the $B_c(2S)$ rest frame.

$$\langle m_{\pi\pi}\rangle \sim 0.5 \text{ GeV}$$



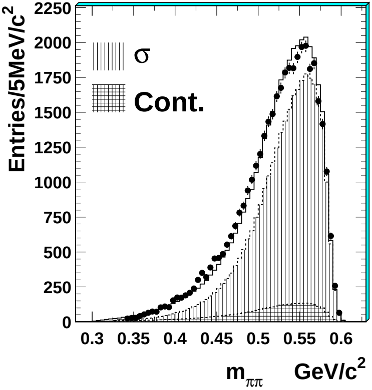
VOLUME 35, NUMBER 1 PHYSICAL REVI

FIG. 1. The decay spectrum $\Gamma^{-1}\, d\Gamma/dm_{\pi\pi}$ as a function of $m_{\pi\pi}$ given by Eq. (6) (solid line); given by simple phase space (dotted line); and given by Eq. (6) modified by pion-pion rescattering (dot-dashed line).

[Brown and Cahn(1975), Novikov and Shifman(1981), Voloshin(1975), Voloshin and Zakharov(1980)]

σ-meson in the quarkonia decays

Distribution over $m_{\pi\pi}$ for the process $\psi' \to J/\psi\pi\pi$.
The resonance σ ($f_0(500)$) has been included into the fit (BESII) [Ablikim et al.(2007)].



$f_0(500)$ or σ
$J^{PC} = 0^{++}$
$(400-550) - i(200-350)$ МэВ

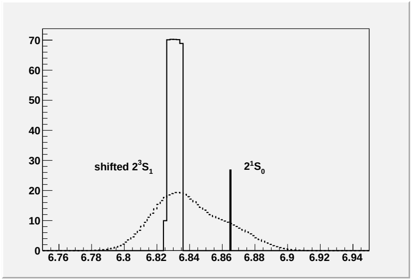
$B_c^*(2S)$ with lost photon

Peak for 2^3S_1 excitation ("soft" photon is lost):

- shifted by ~ 65 MeV $\Rightarrow$ appeared ~ 30 MeV before 2^1S_0
- have a width ~ 10 MeV

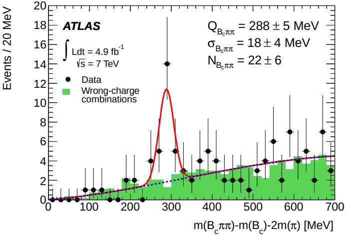
$$\boxed{\Delta \approx 2\frac{\sqrt{(M_{B_c^*(2S)}-M_{B_c^*})^2-4m_\pi^2}(M_{B_c^*}-M_{B_c})}{M_{B_c}}}$$

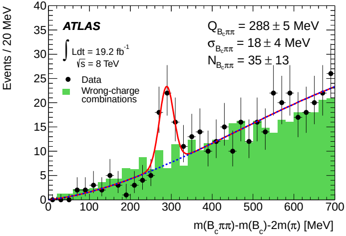
$B_c(2S) \to B_c + \pi\pi[+\gamma_{\text{lost}}]$



ATLAS results for $B_c(2S)$

ATLAS Coll. [Aad et al.(2014)]





$B_c \to J/\psi\pi$ (ATLAS)

Data	Signal events	Peak mean [MeV]
7 TeV	100 ± 23	6282 ± 8
8 TeV	227 ± 25	6277 ± 6

$$\frac{N(B_c(2S) \to B_c\pi^+\pi^-)}{N(B_c)} \sim 0.17$$

Taking into account $B_c\pi^0\pi^0$ one can obtain

$$\frac{\sigma(B_c(2S))}{\sigma(B_c^{\text{direct}}) + \sigma(B_c(2P) + \sigma(B_c(3P))} \gtrsim 0.4$$

Accounting acceptance efficiency and pion track efficiency could increase this ratio to the unexpectedly large value.

- $B_c(2S)$ yield is quite large.
- The peak mean value can be interpreted as 2^3S_1 shifted by 65 MeV due to the unreconstructed soft photon ($B_c^*(2S) \to B_c^* + \pi^+\pi^-$, $B_c^* \to B_c + \gamma_{\text{lost}}$).
- It could be supposed, that the peak have a quite large width due to additional contribution from 2^1S_0.

$B_c(2S)$ yield at LHCb

At first sight

$$R_{2S/1S} = \frac{\sigma(B_c(2S))}{\sigma(B_c(1S))} \sim \frac{R^2_{2S}(0)}{R^2_{1S}(0)}$$

and doesn't depends on kinematic region.

Several unpleasant features of theoretical model:
The predicted cross section strongly depends on choice of mass values. Within the model $M(B_c(nS)) = m_b + m_c$ and therefore quark mass values in estimation of $\sigma(B_c(2S))$ differ from ones in estimation of $\sigma(B_c(1S))$. And this leads (very rough estimation!) to

$$\frac{R_{2S/1S}[\text{LHCb}]}{R_{2S/1S}[\text{ATLAS}]} \sim 0.7 \qquad \text{(Does it have a physical meaning?)}$$

Anyway, the relative yields of $B_c(2S)$ at LHCb and at ATLAS are comparable.

Conclusions

- The missing of photon from $B_c^* \to B_c + \gamma$ shifts peaks for $2S$ and P states by 65 MeV and broadens them by $10-20$ MeV. This photon is too soft to be detected.
- The best chance to be found first in the $M_{B_c} + \gamma$ spectrum belongs to $2P$ excitations.
- If $B_c(2S) \to B_c(B_c^*) + \pi^+\pi^-$ mode is seen at ATLAS, it should be seen at LHCb.
- New results on $B_c(2S)$ production are needed from ATLAS (full data set of Run-I + Run-II). It is very useful to know the efficiency of $\pi^+\pi^-$ registration.
- New results on $B_c(2S)$ production are needed from CMS.
- The LHCb results on $B_c(2S)$ production are still awaited.

Thank for your attention!

backup slides

B_c properties

B_c family have a spectroscopy similar to $c\bar{c}$ or $b\bar{b}$ quarkonium spectroscopy and decays like B meson

1. All excitations below the threshold decay to the ground state 1^1S_0.
2. The absence of strong annihilation channels leads the very narrow ground state (practically as B-meson).
3. Spectroscopy can be investigated within the same frame work as for $c\bar{c}$ and $b\bar{b}$ quarkoniums.
4. The small total yield comparing to the $c\bar{c}$ and $b\bar{b}$ quarkonia case.
5. The small relative yield of P-wave excitations comparing to the $c\bar{c}$ and $b\bar{b}$ quarkonia case.

- The main difference in decays (comparing to B meson): the both quarks in B_c are heavy.
- The main difference in spectroscopy (comparing to $c\bar{c}$ and $b\bar{b}$ quarkonia): charge parity can not be determined.

$$h_Q \ \chi_{1Q} \xrightarrow{\text{mixing}} 1^+ \ 1^{+'}$$

$$\begin{aligned}
|2P, 1^{'+}\rangle &= 0.294|S=1\rangle + 0.956|S=0\rangle \\
|2P, 1^{+}\rangle &= 0.956|S=1\rangle - 0.294|S=0\rangle \\
|3P, 1^{'+}\rangle &= 0.371|S=1\rangle + 0.929|S=0\rangle \\
|3P, 1^{+}\rangle &= 0.929|S=1\rangle - 0.371|S=0\rangle
\end{aligned}$$

[Kiselev et al.(1995)Kiselev, Likhoded, and Tkabladze, Gershtein et al.(1995)Gershtein, Kiselev, Likhoded, and Tkal

B_c production amplitude

$$A^{SJj_z} = \int T^{Ss_z}_{b\bar{b}c\bar{c}}(p_i, k(\vec{q})) \cdot \left(\Psi^{Ll_z}_{\bar{b}c}(\vec{q})\right)^* \cdot C^{Jj_z}_{s_z l_z} \frac{d^3\vec{q}}{(2\pi)^3},$$

where $T^{Ss_z}_{b\bar{b}c\bar{c}}$ is an amplitude of the hard production of two heavy quark pairs;
$\Psi^{Ll_z}_{\bar{b}c}$ is the quarkonium wave function (color singlet, *in some calculations — color octet*);
J and j_z are the total angular momentum and its projection on z-axis in the B_c rest frame;
L and l_z are the orbital angular momentum of B_c meson and its projection on z-axis;
S and s_z are B_c spin and its projection;
$C^{Jj_z}_{s_z l_z}$ are Clebsh-Gordon coefficients;
p_i are four momenta of B_c meson, b quark and $\bar{c}$ quark;
$\vec{q}$ is three momentum of $\bar{b}$-quark in the B_c rest frame (in this frame $(0, \vec{q}) = k(\vec{q})$).
Under assumption of small dependence of $T^{Ss_z}_{b\bar{b}c\bar{c}}$ on $k(\vec{q})$

$$A \sim \int d^3q\, \Psi^*(\vec{q}) \left\{ T(p_i, \vec{q})|_{\vec{q}=0} + \vec{q}\frac{\partial}{\partial\vec{q}} T(p_i, \vec{q})|_{\vec{q}=0} + \cdots \right\}$$

and, particularly, for the S-wave states

$$A \sim R_S(0) \cdot T_{b\bar{b}c\bar{c}}(p_i)|_{\vec{q}=0},$$

where $R_S(0)$ is a value of radial wave function at origin.

Papers on B_c production

Hadronic S-wave B_c-meson prodcution:
[Chang et al.(2005)Chang, Qiao, Wang, and Wu,
Berezhnoy et al.(1995)Berezhnoy, Likhoded, and Shevlyagin,
Berezhnoy et al.(1997b)Berezhnoy, Kiselev, Likhoded, and Onishchenko,
Kolodziej et al.(1995)Kolodziej, Leike, and Ruckl,
Chang et al.(1995)Chang, Chen, Han, and Jiang, Baranov(1997a),
Baranov(1997b)].

P-wave B_c-meson:
[Berezhnoy et al.(1996)Berezhnoy, Kiselev, and Likhoded,
Berezhnoy et al.(1997a)Berezhnoy, Kiselev, and Likhoded,
Chang et al.(2004)Chang, Wang, and Wu].

Some more papers on B_c production:
[Berezhnoy(2005), Chang et al.(2006)Chang, Wang, and Wu,
Berezhnoy et al.(2011)Berezhnoy, Likhoded, and Martynov,
Chang et al.(2015)Chang, Wang, and Wu].

First theoretical research of the hadronic four heavy quark production
($gg \to Q_1\bar{Q}_1Q_2\bar{Q}_2$ and $q\bar{q} \to Q_1\bar{Q}_1Q_2\bar{Q}_2$):
[Barger et al.(1991)Barger, Stange, and Phillips]

Fragmentation in e^+e^-

It the special gluonic gauge the only diagram (1) contributes to the cross section at

$$\frac{M_{B_c}^2}{s_{e^+e^-}} \to 0$$

$$\frac{d\sigma_{B_c}}{dz} = D_{\bar{b}\to B_c}(z)\cdot \sigma_{b\bar{b}}$$

$$D_{\bar{b}\to B_c^*}(z) =$$

$$\frac{2\alpha^2|R_S(0)|^2}{81\pi m_c^3}\frac{rz(1-z)^2}{(1-(1-r)z)^6}(6-18(1-2r)z+(21-74r+68r^2)z^2$$

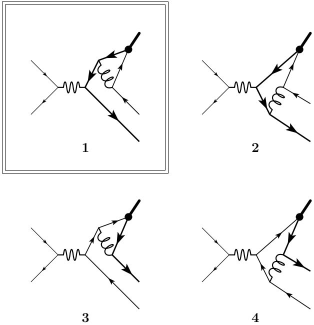
$$-2(1-r)(6-19r+18r^2)z^3+3(1-r)^2(1-2r+2r^2)z^4)$$

$$D_{\bar{b}\to B_c}(z) =$$

$$\frac{2\alpha^2|R_S(0)|^2}{27\pi m_c^3}\frac{rz(1-z)^2}{(1-(1-r)z)^6}(2-2(3-2r)z+3(3-2r+4r^2)z^2$$

$$-2(1-r)(4-r+2r^2)z^3+(1-r)^2(3-2r+2r^2)z^4)$$

$$r = \frac{m_c}{m_c+m_b}$$



[Clavelli(1982), Ji and Amiri(1987),
Amiri and Ji(1987), Chang and Chen(1992a),
Chang and Chen(1992b),
Braaten et al.(1993)Braaten, Cheung, and Yuan,
Kiselev et al.(1994)Kiselev, Likhoded, and Shevlyag

The analog of Peterson function

To describe heavy quark hadronization into heavy-light meson the Peterson FF is usually used:

$$D_{Q\to(Q\bar{q})}(z) \sim \frac{1}{z\left(1-\frac{1}{z}-\frac{\epsilon}{1-z}\right)^2}.$$

The dependence of "nonperturbative" Peterson FF is partially determined by denominator of perturbative propagator for $Q^* \to (Q\bar{q}) + X$ process:

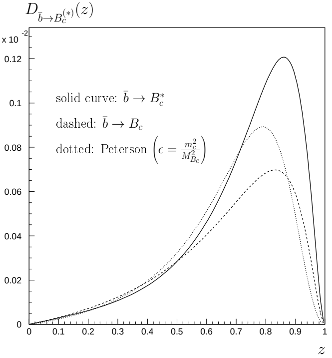
$$m_Q^2 - (P_{(Q\bar{q})} + P_{\mathrm{X}})^2.$$

$$m_Q^2 - (P_{(Q\bar{q})} + P_{\mathrm{X}})^2 \approx m_Q^2 - \frac{m^2_{(Q\bar{q})}}{z} - \frac{m_{\mathrm{X}}^2}{1-z}.$$

For $m_Q \approx m_{(Q\bar{q})}$

$$m_Q^2 - (P_{(Q\bar{q})} + P_{\mathrm{X}})^2 \sim 1 - \frac{1}{z} - \frac{m_{\mathrm{X}}^2}{m_Q^2}\frac{1}{1-z}.$$

FF for $b \to B_c^{(*)}$ obtained within pQCD vs. Peterson parametrization:



The FFs from the previous slide are used within FONLL to describe $c \to D$ (r — free parameter).

Additional contributions to B_c hadronic production

Comparable contributions to the B_c production:

- $gg \to B_c b\bar{c}$
- $\bar{b}(c)g \to B_c + \bar{c}(b)$
- Pythia gluonic showering $g \to B_c$.

Small contributions:

- $q\bar{q} \to B_c b\bar{c}$ (about 10% $gg \to B_c b\bar{c}$)
- color octet (unessential for B_c inclusive production).

Problems:

- How to avoid double counting?
- How to take into account the mass of $b(\bar{c})$-quark from the sea in matrix element?
- How to obtain reasonable p_T distribution?

Methods used for b production

GM-VFNS	General-mass variable- flavor-number scheme (GM-VFNS): The heavy quark is treated as any other massless parton, the mass is taken into account as large logarithms $\ln(p_T/m)$ in parton distribution and fragmentation functions, where they are resummed by imposing DGLAP evolution [Kniehl et al.(2012)Kniehl, Kramer, Schienbein, and Spiesberger].
FONLL	NLO (massive quark) + resummation of large logs: at $p_T < 5m_c$ NLO works without logarithm resummation (see eq. 6.1 in [Cacciari et al.(1998)Cacciari, Greco, and Nason]).
k_T fact.	LO (massive quark) + virtual initial gluons (It seems that sea c quark is not needed) [Baranov et al.(2005)Baranov, Lipatov, and Zotov].

B_c/B ratio

LHCb experimental value [Aaij et al.(2014a)]:

$$R^{\rm exp} = \frac{\sigma(B_c^+) \times \mathcal{B}(B_c^+ \to J/\psi\pi^+)}{\sigma(B^+) \times \mathcal{B}(B^+ \to J/\psi K^+)} =$$
$$= (0.683 \pm 0.018 \pm 0.009)\%$$

$\mathcal{B}(B^+ \to J/\psi K^+) \approx 0.1\%$ is known experimentally. $\mathcal{B}(B_c^+ \to J/\psi\pi^+)$ is theoretically predicted within different models. The obtained predictions contradict each other :

$$\mathcal{B}(B_c^+ \to J/\psi\pi^+) = 0.061 \div 0.29\%$$

$$r_c^{\rm theor.} \sim \frac{\sigma(B_c^+)}{\sigma(B^+)} \sim 0.1 \div 0.3\%$$

The $R^{\rm exp}$ is in agreement the largest predicted value for $\mathcal{B}(B_c^+ \to J/\psi\pi^+)$:

$$R^{\rm theor.}_{\rm optimistic} \sim 0.3 \div 0.9\%$$

However the choice of $\mathcal{B}(B_c^+ \to J/\psi\pi^+) \sim (0.06 \div 1.1)\%$ looks more reasonable, because these values obtain within the models [Ebert et al.(2003)Ebert, Faustov, and Galkin, Abd El-Hady et al.(2000)Abd El-Hady, Munoz, and Vary], which describe the the experimentally obtained ratio [Aaij et al.(2014b)]:

$$\frac{\mathcal{B}(B_c^+ \to J/\psi\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\mu\nu_\mu)} = 0.0469 \pm 0.0028 \pm 0.0046$$

. For this case

$$R^{\rm theor.}_{\rm reasonalbe} \sim 0.06 \div 0.3\%$$

It seems that the theoretical predictions underestimate the experimental values.

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