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Probing collectivity and shape evolution in ^{62}Zn via Coulomb excitation

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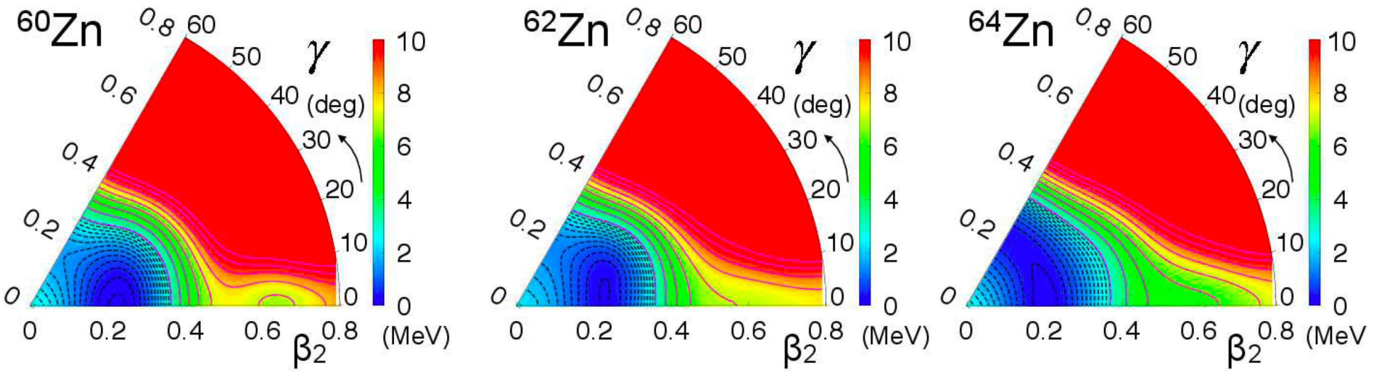
Motivation:
Shape coexistence

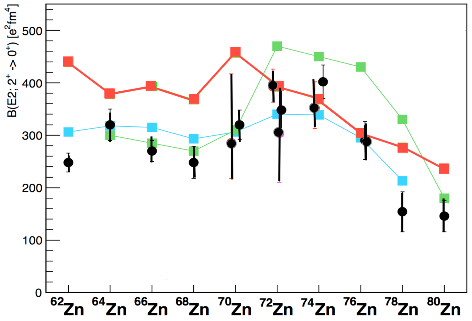
- Shape coexistence in **Z=N** regions – expected also in the vicinity of **^{60}Zn**
- BMF - **ground state deformation -** *β* **~0.2-0.3** - evolution from **prolate** in **^{60}Zn** to more **triaxially deformed/ gamma soft** for **^{62}Zn and ^{64}Zn**
- **Superdeformation in ^{60,62}Zn** – BMF calculations, experiment – high spin region
- Strongly pronounced **SD prolate minimum at** *β* **~0.6 in ^{60}Zn** - smears out and practically vanishes with the increasing number of neutrons above N=30
- **Deformation in ^{62}Zn** - a key test for understanding the evolution of deformed configurations in this isotopic chain – **properties of the SD structure studied with radioactive beam**

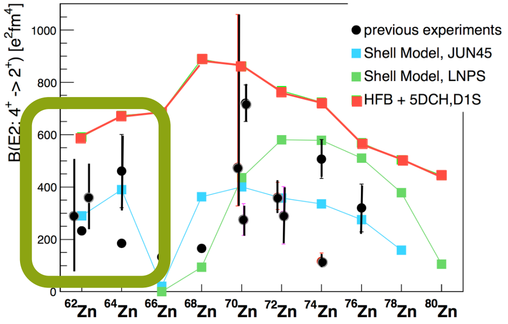


K. Hadyńska-Klęk et al., Phys. Rev. Lett. 117, 062501 (2016)

Motivation:
Collectivity of transitions in the yrast band

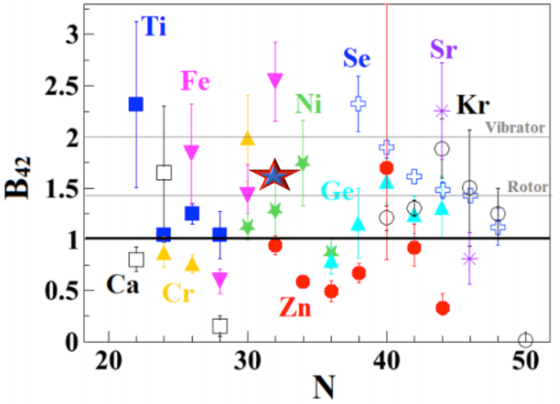
- **Zn isotopic chain** - significant disagreements exist between the results of measurements performed using different methods and between different theoretical approaches





One precise measurement of the B(E2; $4^+_1 \rightarrow 2^+_1$)
Verification needed

- **Collectivity of $4^+_1 \rightarrow 2^+_1$** transition can be resolved only by another independent high-precision measurement in order to verify the low **B_{42}**=B(E2; $4^+_1 \rightarrow 2^+_1$)/B(E2; $2^+_1 \rightarrow 0^+_1$)) ratio, observed for many Zn isotopes – **BMF calculations: B_{42}=1.6** (!)★



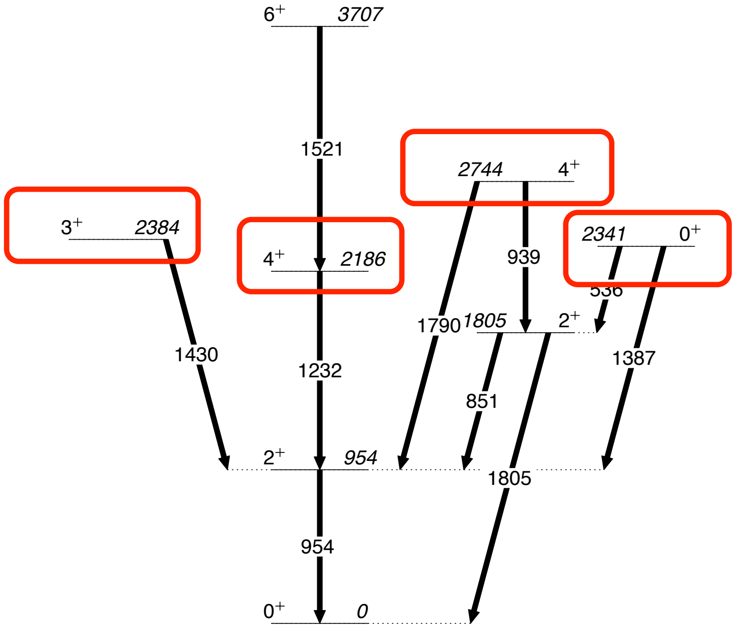
Influence of the shapes:
- Is this nucleus vibrational type, gamma-soft, traixial?
- COULEX will provide the model independent answer

Motivation: Astrophysics

- The observation of **X-ray bursts** is interpreted as thermonuclear explosions in the atmosphere of a neutron star in a close binary system.
- As temperature and density at the surface of the neutron star increase, the CNO cycles breakout into the ***rp* process**
- Sensitivity studies highlight the key reactions for understanding these bursts
- **^{61}Ga(*p*, γ)^{62}Ge** reaction – particularly important but currently completely unfeasible
- **No levels and no BE2 values are known**
- Alternative solution - to measure the microscopic nuclear properties of the mirror nucleus: **^{62}Zn**, and to extract relevant information about excited levels in the ^{62}Ge through **isospin symmetry formalisms**

Motivation:
Astrophysics

- Excitation of states in ^{62}Zn **above 2 MeV** will help to understand the structure of ^{62}Ge
- States of interest **in ^{62}Ge** – above proton separation energy**: 2053(145) keV** – very low - “exotic” case
- The **quadrupole deformation in ^{62}Zn** is expected to strongly influence **the mirror energy difference** – to be used to predict where these states are placed in **^{62}Ge**
- **The triaxial degree of freedom** has a strong influence **on the binding energies, hence masses and Q values**
- If triaxial deformation is established in *p*-rich nuclei taking part of the *rp* process, this would change dramatically **the recent mass systematics** – masses are unmeasured
- **Shapes in ^{62}Ge** – unknown – measurement of shapes in ^{62}Zn – the only way to study states in inaccessible ^{62}Ge
- **Possible unknown 2$^+$ states above 2 MeV?**



Motivation: Astrophysics

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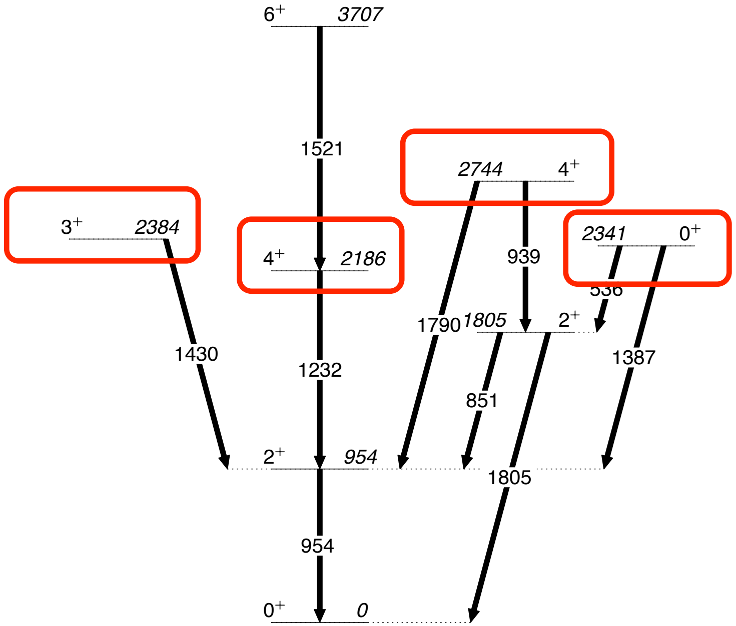

In the present experiment we will be able to excite states up to 3.7 MeV in ^{62}Zn

Goals of the proposed experiment

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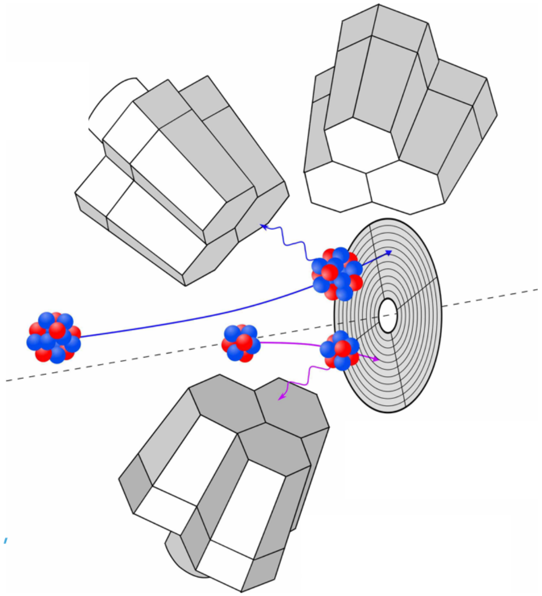
- To populate states up to the 6_1^+ one in ground state band, as well as non-yrast 0_2^+, 2_2^+, 4_2^+ and 3_1^+ states.
- To extract a complete set of matrix elements in order to describe the low-lying structure of ^{62}Zn
- To determine $Q(2_1^+)$ and $Q(2_2^+)$
- To determine quadrupole deformation of the 0_1^+ and 0_2^+ states by applying the Quadrupole Sum Rules method – direct measurement of the deformation
- To measure the collectivity of $4_1^+ \rightarrow 2_1^+$ transition and provide an additional point to resolve discrepancies in the reported results for the Zn isotopic chain
- To provide the experimental input in the discussion of the role of nuclear shapes in the astrophysical *rp*-process

Experiment

- “Safe” Coulomb excitation measurement
- **^{62}Zn from ISOLDE (ZrO_2 target):**
  - E_{beam} = 310 MeV (5 MeV/A)
  - $I_{beam} = 2.3 \times 10^7$ pps
    - 2 μA of proton beam current
    - 2% transmission efficiency to the MINIBALL beam line,
    - $I_{beam} \sim 9 \times 10^5$ pps
- **^{196}Pt target, 4 mg/cm^2**
- **MINIBALL spectrometer:**
  - 8 clusters,
  - 8% efficiency for 1.3 MeV γ-rays
- **DSSSD detector: 16° to 53°**
  - Center of Mass system: ($20° < \theta_{CM} < 137°$)
  - detection of either projectile or recoil



Courtesy of L. Gaffney

Sensitivity to the spectroscopic quadrupole moments $Q(2_1^+)$ and $Q(2_2^+)$

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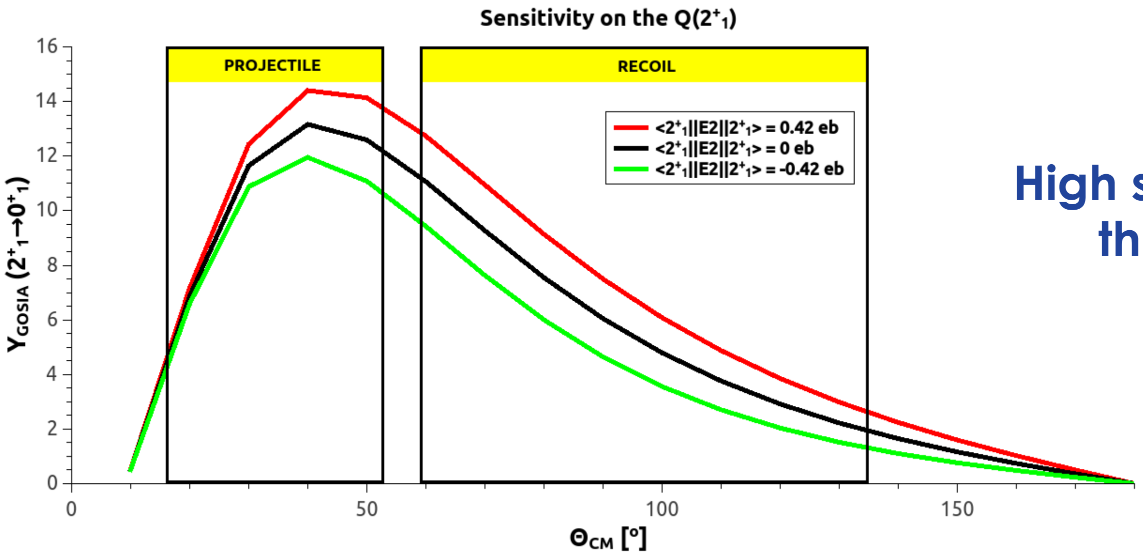
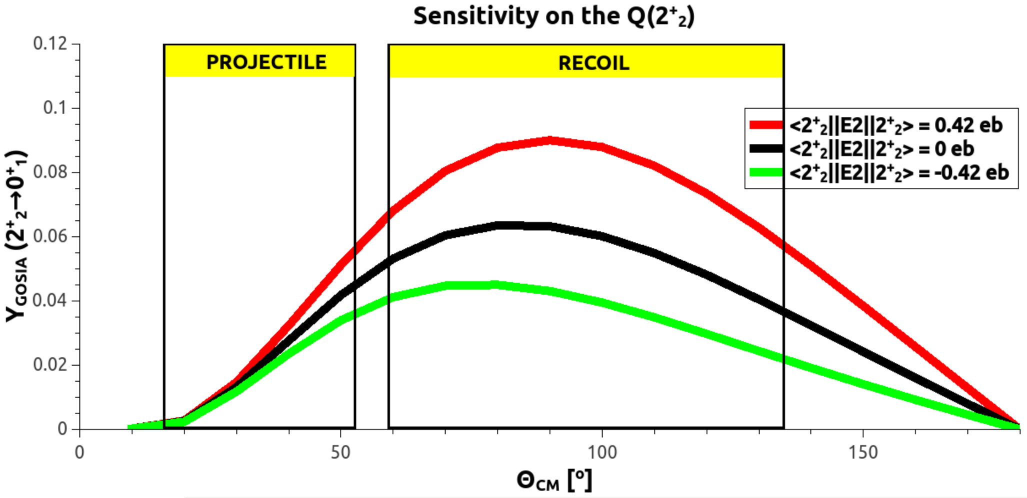
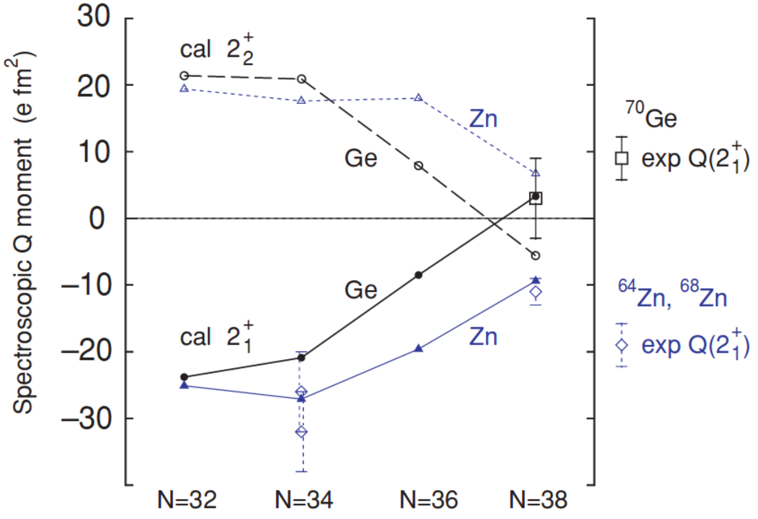


High statistics will allow to subdivide the data into angular ranges

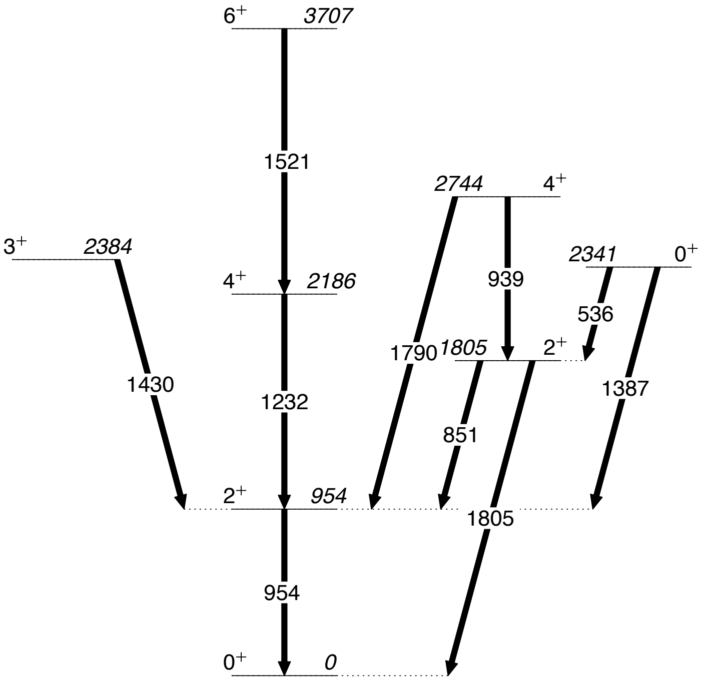




Expected rate

Expected yields calculated with the compute code GOSIA following the Coulomb excitation of **^{62}Zn beam**, (310 MeV energy, average ISOLDE beam intensity of $I_{beam} = 2.3 \times 10^7$ pps) incident on 4 mg/cm^2 **^{196}Pt target**

Transition	Energy [keV]	Number of counts / shift
$2_1^+ \rightarrow 0_1^+$	954	$3 \cdot 10^4$
$4_1^+ \rightarrow 2_1^+$	1232	$1.3 \cdot 10^3$
$6_1^+ \rightarrow 4_1^+$	1521	20
$2_2^+ \rightarrow 2_1^+$	851	600
$2_2^+ \rightarrow 0_1^+$	1805	300
$0_2^+ \rightarrow 2_1^+$	1348	40
$3_1^+ \rightarrow 2_1^+$	1430	180
$4_2^+ \rightarrow 2_1^+$	1790	40
$4_2^+ \rightarrow 2_2^+$	939	30



Matrix elements from BMF theory:

- Quadrupole moments of 2^+ states
- $2_1^+ \rightarrow 4_2^+$
- $2_1^+ \rightarrow 0_2^+$
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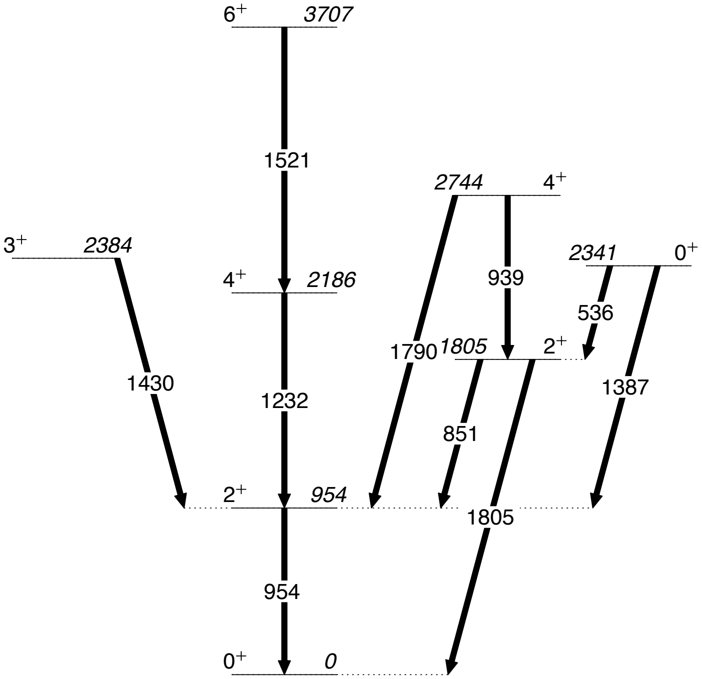

Deformation of the 0_1^+

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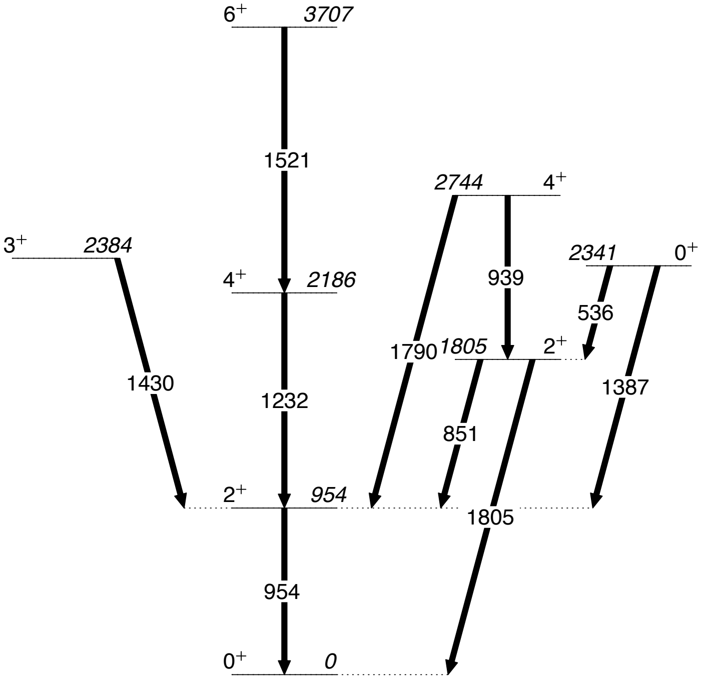

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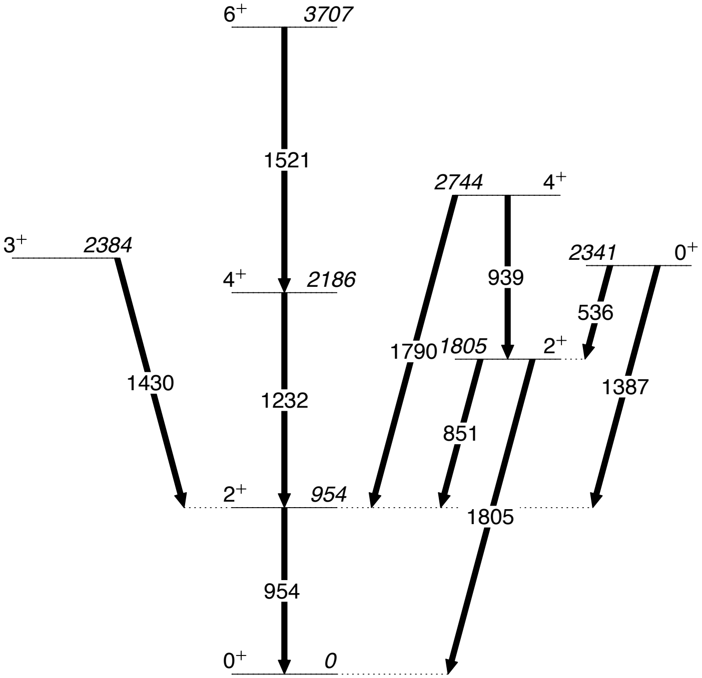

Precise measurement of 4_1^+

Summary of the beam time request

- **12 shifts for ^{62}Zn beam**
  - ✓ E_{beam} = 310 MeV (5 MeV/A)
  - ✓ $I_{beam} = 2.3 \times 10^{7}$ pps
- **3 additional shifts** to optimise the production and purification of the beam

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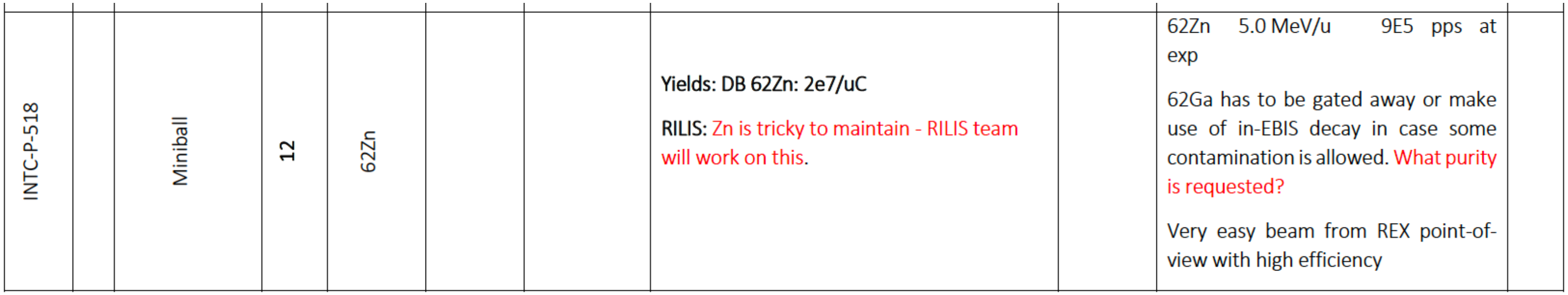
Thank you

Beam purity

INTC June 2017: Technical Advisory Committee summary file

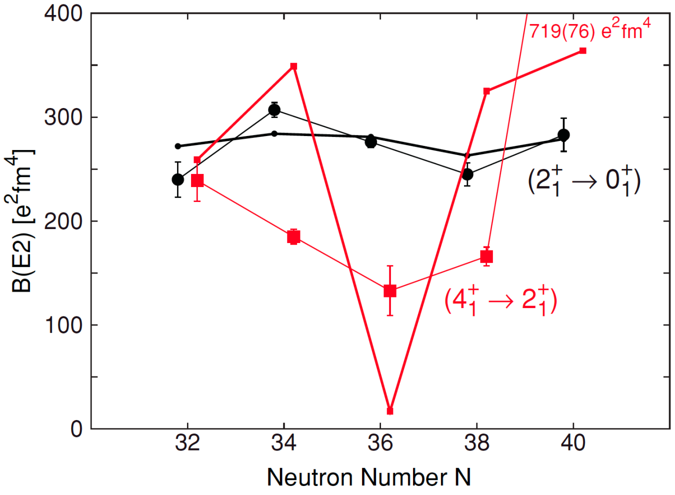


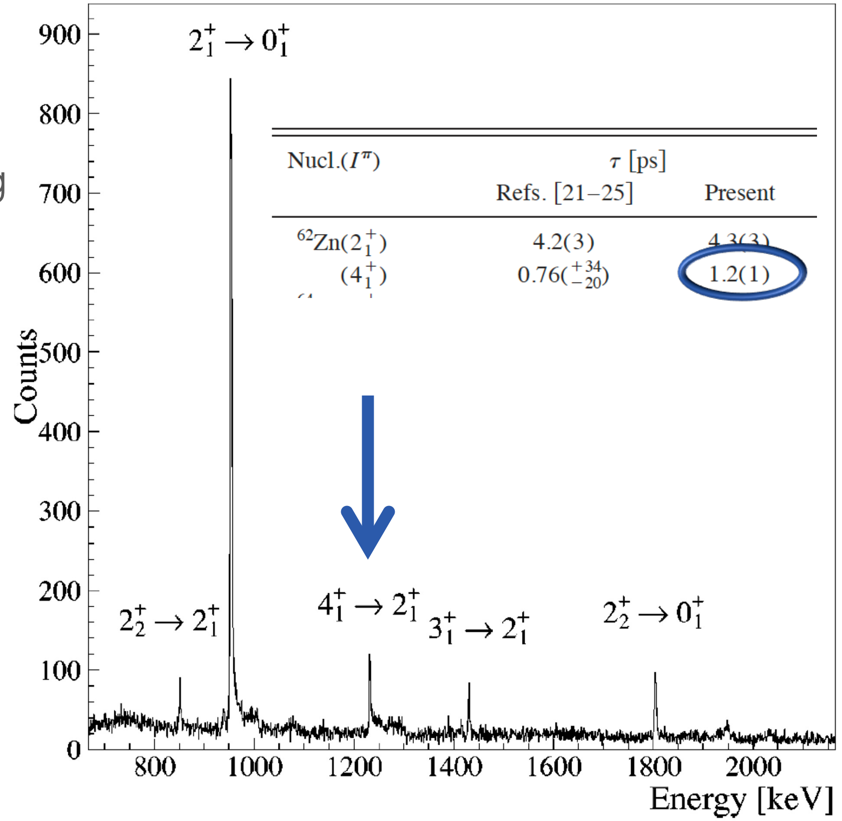
INTC-P-518		Miniball	12	62Zn			Yields: DB 62Zn: 2e7/uC RILIS: Zn is tricky to maintain - RILIS team will work on this.		62Zn 5.0 MeV/u 9E5 pps at exp 62Ga has to be gated away or make use of in-EBIS decay in case some contamination is allowed. What purity is requested? Very easy beam from REX point-of-view with high efficiency	

- $T_{1/2}$ (^{62}Zn) = 9 h
- Long-lived contamination: ^{62}Ga ($T_{1/2}$ = 116 ms)
- No lines from ^{62}Ga will overlap with ^{62}Zn
- No normalisation to the Rutherford scattering is needed
- Superallowed GT decay of ^{62}Ga to the ground state of ^{62}Zn – no chance to confuse the states populated in the beta decy and in the COULEX reactions
- The only limitation – the rate in the CD detector (MAX ~$5*10^6$ on a detector)
- We can tolerate up to 30% of the contamination (but we would prefer less)

4_1^+ state in ^{62}Zn

- ^{12}C(^{58}Ni,2α)^{62}Zn* reaction, g-factor experiment
- "In this reaction the first 2_1^+ state is predominantly populated whereas higher lying states are only weakly excited."
- Precise **B(E2; $4^+_1 \rightarrow 2^+_1$) = 240(20) e^{2}fm^4**
- Verification needed?



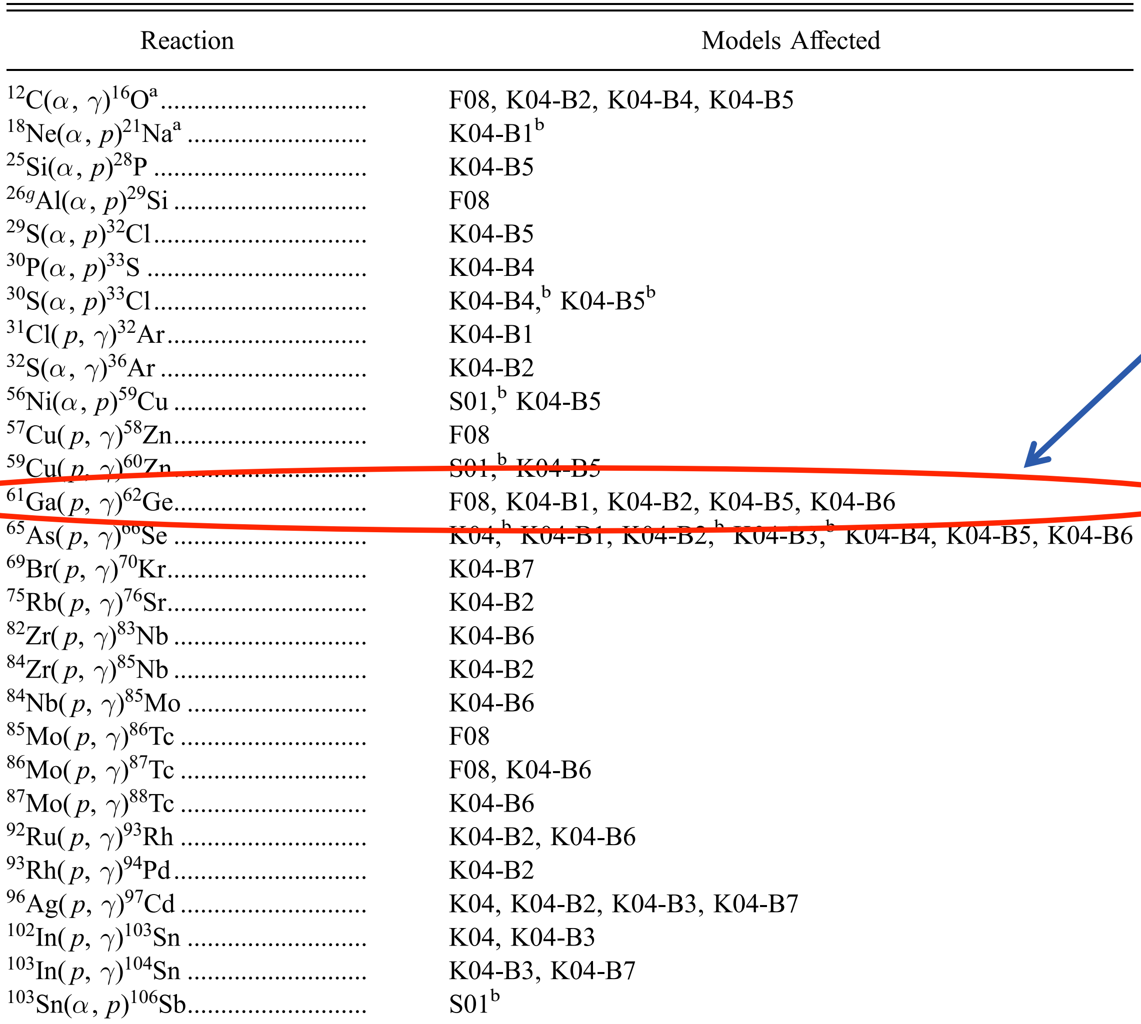


Nucl.(I^π)	τ [ps] Refs. [21–25]	Present
^{62}Zn(2_1^+)	4.2(3)	4.3(3)
(4_1^+)	$0.76(^{+34}_{-20})$	1.2(1)

FIG. 4. γ rays observed with the Ge detector in coincidence with the α-particle peak in the particle spectrum [Fig. 3(a)]. All γ-ray lines identified belong to the de-excitation of ^{62}Zn states populated in the α-transfer reaction.

O. Kenn et al, Phys. Rev. C 65, 034308 (2002)
K. Moschner et al., Phys. Rev. C 82, 014301 (2010)

$^{61}\mathrm{Ga}(p,\gamma)^{62}\mathrm{Ge}$ reaction

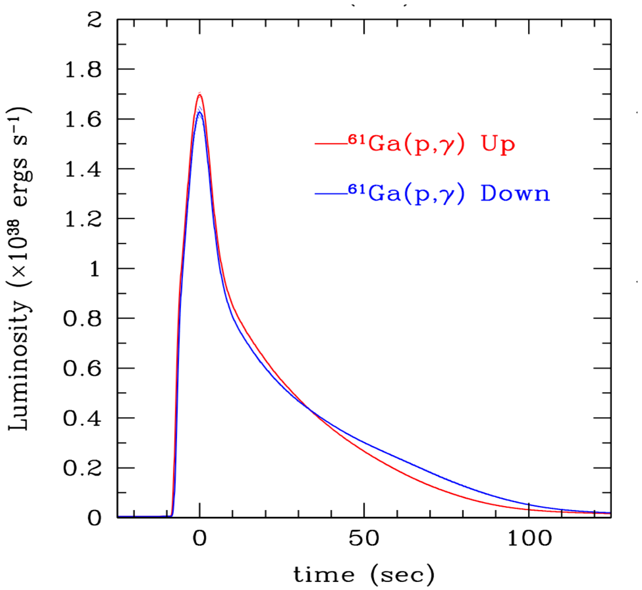


Reaction	Models Affected
$^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$[a]	F08, K04-B2, K04-B4, K04-B5
$^{18}\mathrm{Ne}(\alpha,p)^{21}\mathrm{Na}$[a]	K04-B1[b]
$^{25}\mathrm{Si}(\alpha,p)^{28}\mathrm{P}$	K04-B5
$^{26g}\mathrm{Al}(\alpha,p)^{29}\mathrm{Si}$	F08
$^{29}\mathrm{S}(\alpha,p)^{32}\mathrm{Cl}$	K04-B5
$^{30}\mathrm{P}(\alpha,p)^{33}\mathrm{S}$	K04-B4
$^{30}\mathrm{S}(\alpha,p)^{33}\mathrm{Cl}$	K04-B4,[b] K04-B5[b]
$^{31}\mathrm{Cl}(p,\gamma)^{32}\mathrm{Ar}$	K04-B1
$^{32}\mathrm{S}(\alpha,\gamma)^{36}\mathrm{Ar}$	K04-B2
$^{56}\mathrm{Ni}(\alpha,p)^{59}\mathrm{Cu}$	S01,[b] K04-B5
$^{57}\mathrm{Cu}(p,\gamma)^{58}\mathrm{Zn}$	F08
$^{59}\mathrm{Cu}(p,\gamma)^{60}\mathrm{Zn}$	S01,[b] K04-B5
$^{61}\mathrm{Ga}(p,\gamma)^{62}\mathrm{Ge}$	F08, K04-B1, K04-B2, K04-B5, K04-B6
$^{65}\mathrm{As}(p,\gamma)^{66}\mathrm{Se}$	K04,[b] K04-B1, K04-B2,[b] K04-B3,[b] K04-B4, K04-B5, K04-B6
$^{69}\mathrm{Br}(p,\gamma)^{70}\mathrm{Kr}$	K04-B7
$^{75}\mathrm{Rb}(p,\gamma)^{76}\mathrm{Sr}$	K04-B2
$^{82}\mathrm{Zr}(p,\gamma)^{83}\mathrm{Nb}$	K04-B6
$^{84}\mathrm{Zr}(p,\gamma)^{85}\mathrm{Nb}$	K04-B2
$^{84}\mathrm{Nb}(p,\gamma)^{85}\mathrm{Mo}$	K04-B6
$^{85}\mathrm{Mo}(p,\gamma)^{86}\mathrm{Tc}$	F08
$^{86}\mathrm{Mo}(p,\gamma)^{87}\mathrm{Tc}$	F08, K04-B6
$^{87}\mathrm{Mo}(p,\gamma)^{88}\mathrm{Tc}$	K04-B6
$^{92}\mathrm{Ru}(p,\gamma)^{93}\mathrm{Rh}$	K04-B2, K04-B6
$^{93}\mathrm{Rh}(p,\gamma)^{94}\mathrm{Pd}$	K04-B2
$^{96}\mathrm{Ag}(p,\gamma)^{97}\mathrm{Cd}$	K04, K04-B2, K04-B3, K04-B7
$^{102}\mathrm{In}(p,\gamma)^{103}\mathrm{Sn}$	K04, K04-B3
$^{103}\mathrm{In}(p,\gamma)^{104}\mathrm{Sn}$	K04-B3, K04-B7
$^{103}\mathrm{Sn}(\alpha,p)^{106}\mathrm{Sb}$	S01[b]

High importance in many models

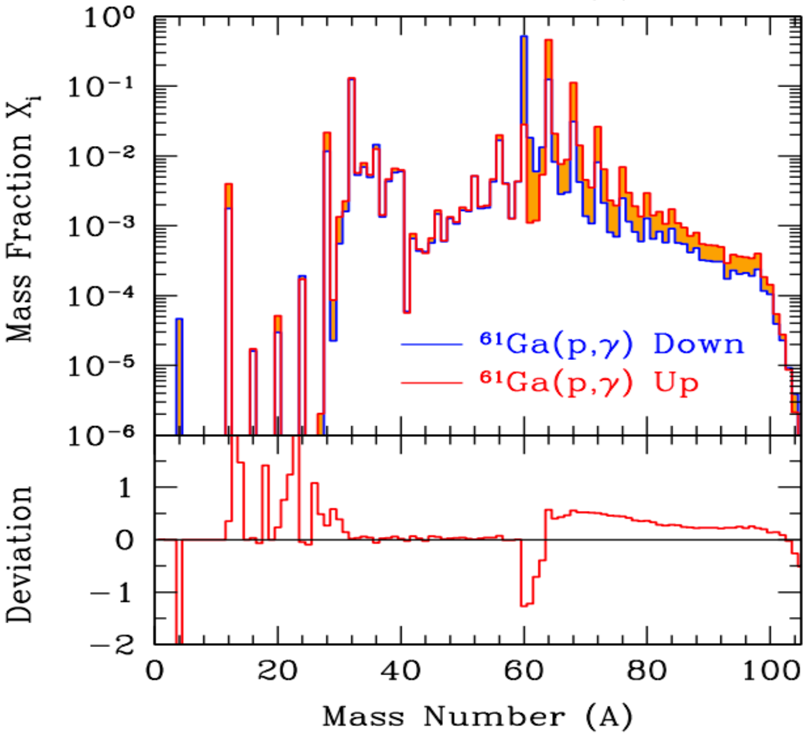
A. Parikh, J. Jose, F. Moreno and C. Iliadis, Astrophys. J. Suppl. Ser. **178**, 110 (2008).

$^{61}Ga(p,\gamma)^{62}Ge$ reaction



Effect on the X-ray burst light curve from varying the $^{61}Ga(p,\gamma)^{62}Ge$ reaction rate within its associated uncertainties.

(precise data from the satellite missions)



Effect on the final abundances from varying the $^{61}Ga(p,\gamma)^{62}Ge$ reaction rate within its associated uncertainties.

R.H. Cyburt *et al.*, Astrophys. J. **830**, 2 (2016).

How do the deformation influence the *rp* process?

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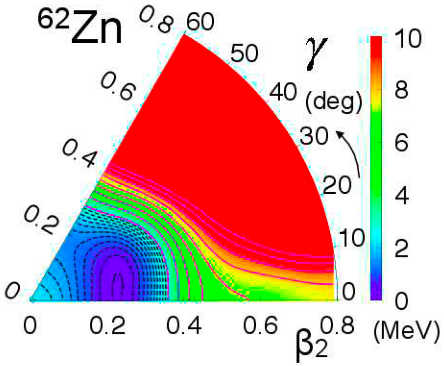
University of Surrey

- **Beta deformation,** β , has an impact on the calculated Mirror Energy Differences (MED) – even the minor changes influence the level energies in the mirror nucleus
- **Triaxial deformation** influences the nuclear binding energy (the mass) – hence the reaction Q-values.
- Most of the masses are **unmeasured** and, therefore, based on systematics
- The systematics would be wrong if **a region of triaxial deformation** is discovered along the *rp*-process and, hence, the calculated reaction rates are inaccurate.



Determination of deformation in ^{62}Zn is the only way to determine the deformation in ^{62}Ge

What is known about ^{62}Zn?

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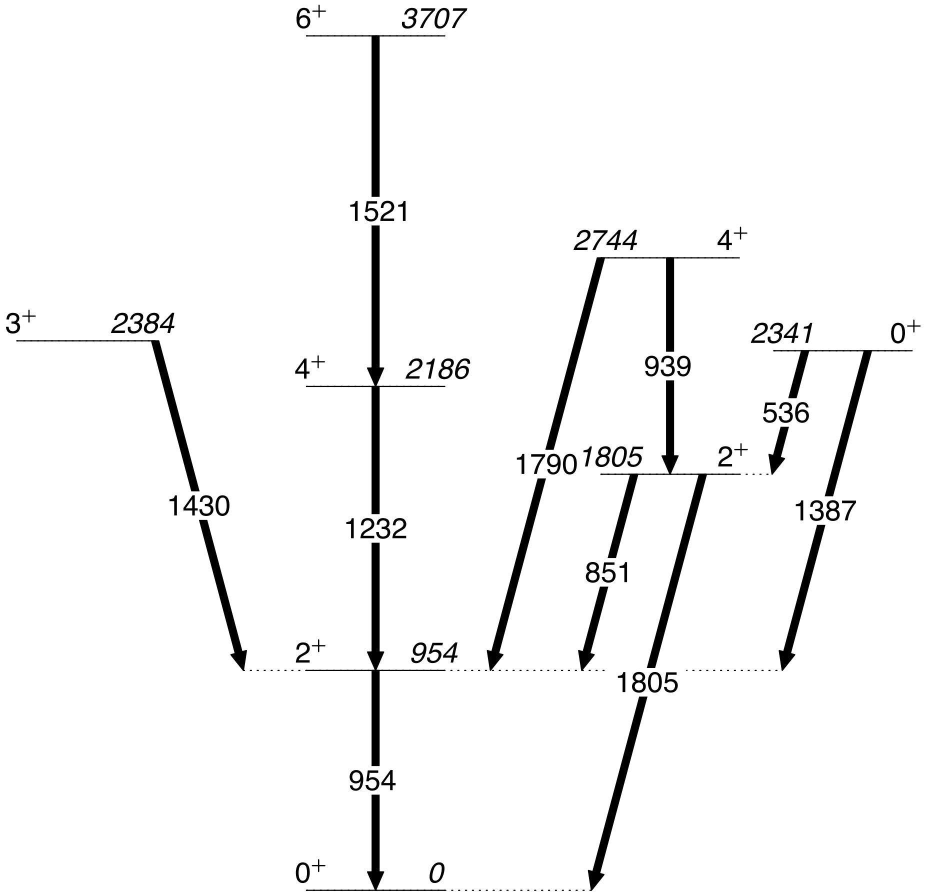


States expected to be populated in the recent experiment

6 lifetimes – in [ps]

2_1^+	4.23 (23)
4_1^+	0.76 (35)
6_1^+	0.36 (6)
2_2^+	3.79 (62)
4_2^+	3.4 (4)
3_1^+	2.5 (1.6)

4 branching ratios

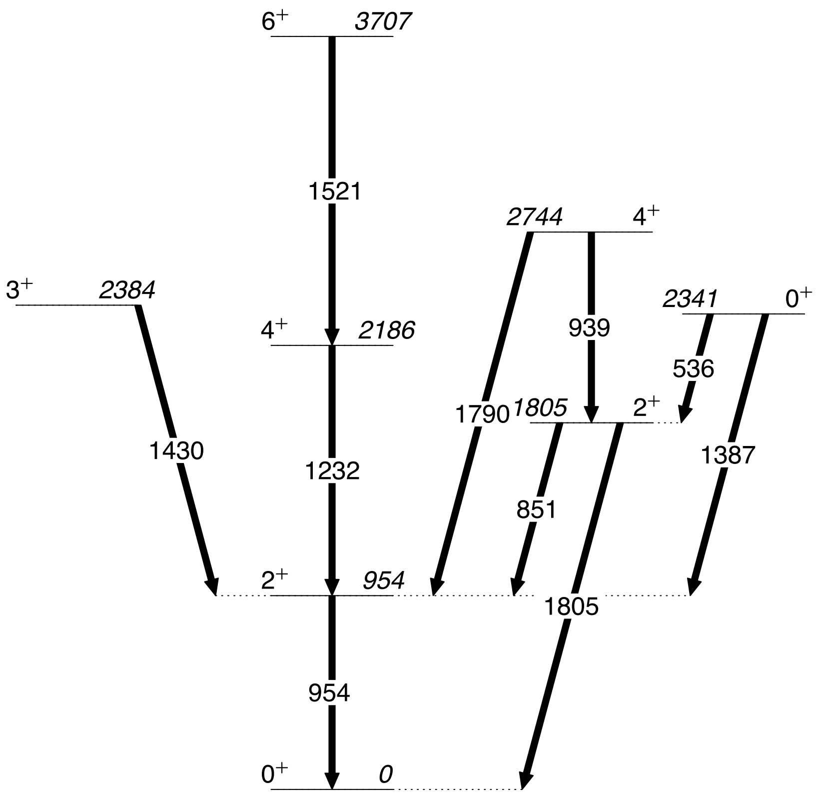
$2_2^+ \rightarrow 0_1^+ / 2_2^+ \rightarrow 2_1^+$	0.7	0.68
$3_1^+ \rightarrow 2_1^+ / 3_1^+ \rightarrow 2_2^+$	0.64	0.08
$4_2^+ \rightarrow 2_2^+ / 4_2^+ \rightarrow 4_1^+$	0.79	0.11
$4_2^+ \rightarrow 2_1^+ / 4_2^+ \rightarrow 4_1^+$	0.02	0.01

5 δ(M1/E2) mixing ratios

$2_2^+ \rightarrow 2_1^+$	-3.6 (1.0)
$4_2^+ \rightarrow 4_1^+$	-0.35 (3)
$3_1^+ \rightarrow 2_1^+$	-1.9 (5)
$3_1^+ \rightarrow 2_2^+$	-0.5 (2)
$3_1^+ \rightarrow 4_2^+$	-0.35 (3)

BMF
Quadrupole moments of 2+ and 4+ states
$2_1^+ \rightarrow 4_2^+$
$2_1^+ \rightarrow 0_2^+$
$4_1^+ \rightarrow 2_2^+$
$2_2^+ \rightarrow 0_2^+$

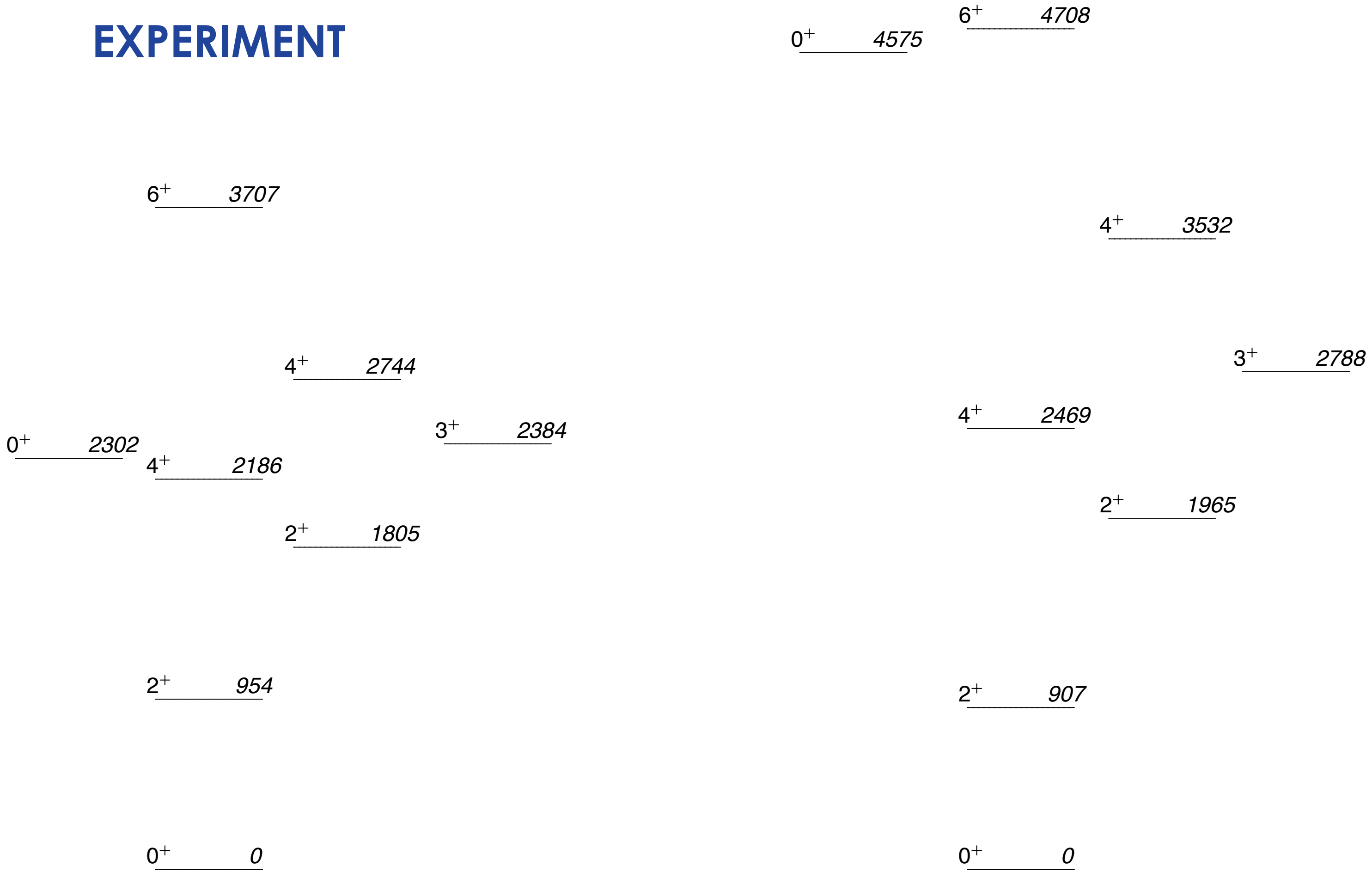
Can any states be missing in ^{62}Zn?



States expected to be populated in the recent experiment

- ^{62}Zn – a vibrational type nucleus – many 2^+ states
- 2^+ states that are known which cannot be populated here: 2804 and 2884 keV
- Are there other low-lying 2^+ states? (above 2 MeV)
- COULEX – a perfect method to populate them directly – if they are there we will see them

Beyond Mean Field calculations for ^{62}Zn



EXPERIMENT

6^+ *3707*

4^+ *2744*

3^+ *2384*

0^+ *2302*

4^+ *2186*

2^+ *1805*

2^+ *954*

0^+ *0*

BMF

6^+ *4708*

0^+ *4575*

4^+ *3532*

3^+ *2788*

4^+ *2469*

2^+ *1965*

2^+ *907*

0^+ *0*

Quadrupole Sum Rules

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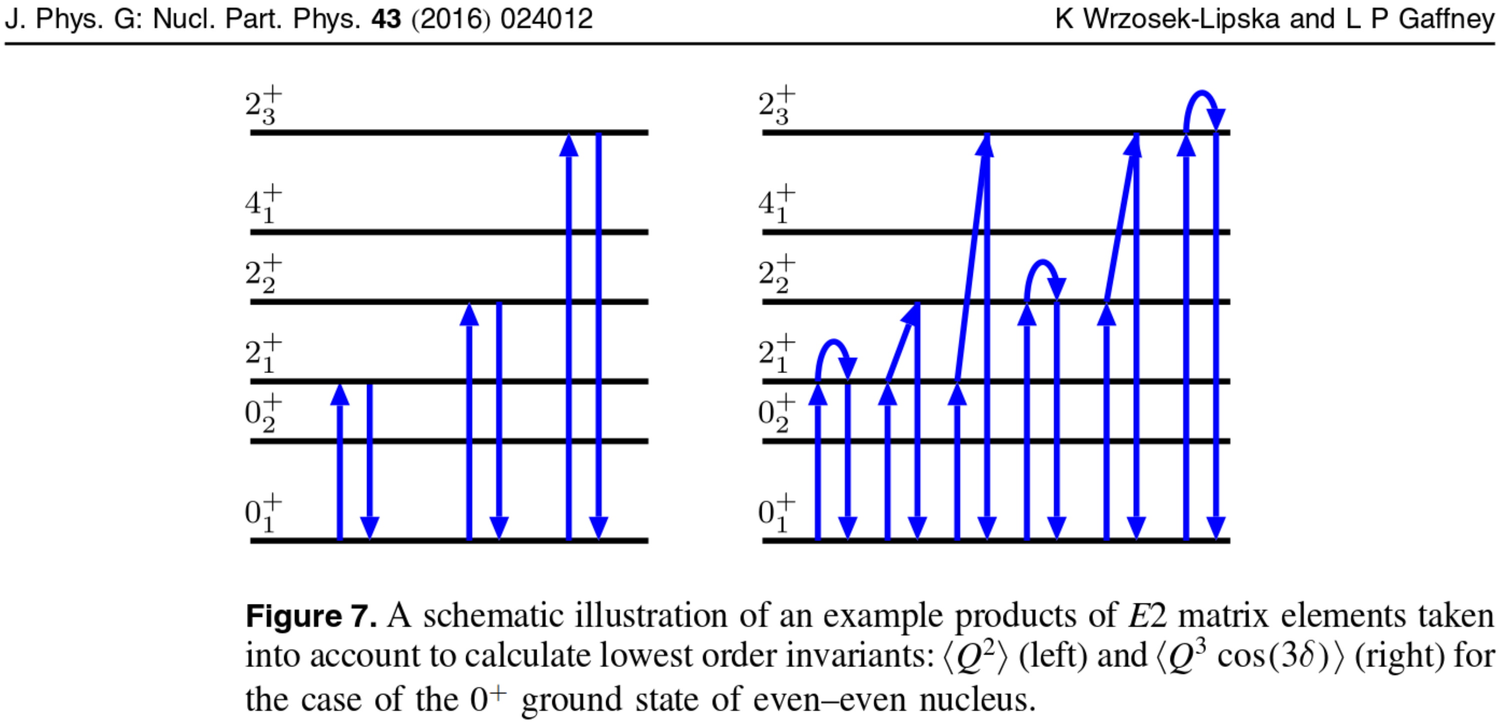

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Figure 7. A schematic illustration of an example products of $E2$ matrix elements taken into account to calculate lowest order invariants: $\langle Q^2\rangle$ (left) and $\langle Q^3\cos(3\delta)\rangle$ (right) for the case of the 0^+ ground state of even–even nucleus.

$$\frac{1}{\sqrt{5}}\langle Q^2\rangle = \frac{1}{\sqrt{2I_i+1}}\sum_t \langle i\|E2\|t\rangle\langle t\|E2\|f\rangle \begin{Bmatrix}2 & 2 & 0\\ I_i & I_f & I_t\end{Bmatrix}$$

$$\bar{\beta} = \sqrt{\langle\beta^2\rangle} = \sqrt{\frac{\langle Q^2\rangle}{q_0^2}}$$

$$\langle Q^3\cos(3\delta)\rangle = \mp\frac{\sqrt{35}}{\sqrt{2}}\frac{1}{\sqrt{2I_i+1}}\sum_{tu}\langle s\|E2\|u\rangle\langle u\|E2\|t\rangle\langle t\|E2\|s\rangle\begin{Bmatrix}2 & 2 & 2\\ I_s & I_t & I_u\end{Bmatrix}$$

$$\bar{\gamma} = \frac{1}{3}\, arc\langle cos(3\gamma)\rangle$$