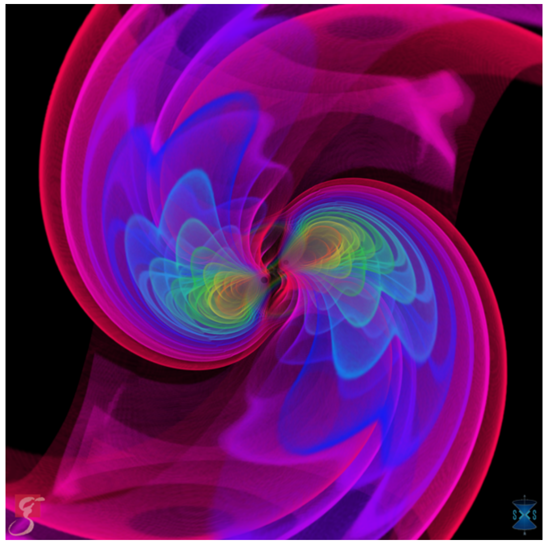
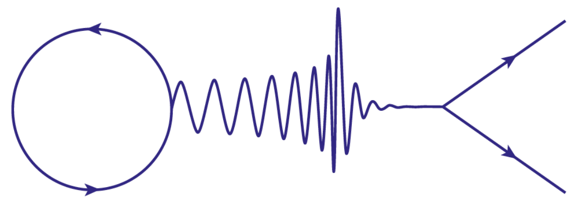
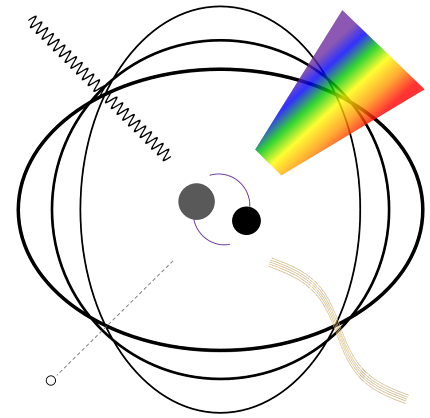
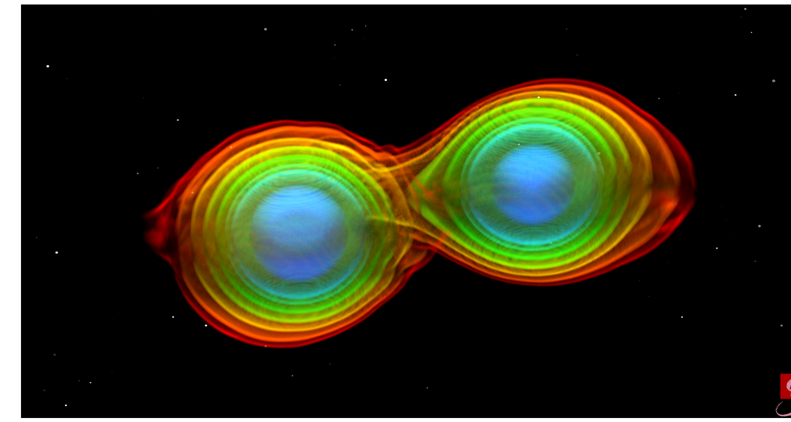

The New Era of Precision Gravitational-Wave (astro)Physics

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Max Planck Institute for Gravitational Physics

(Albert Einstein Institute)

Department of Physics, University of Maryland

"Amplitudes 2018", SLAC, Stanford University

June 18, 2017

Outline

- The science from GW experiments stems on our ability to make precise predictions: brief review of theoretical groundwork to identify and interpret the signals.

- GWs detected from binary black holes & binary neutron stars: astrophysical and fundamental physics implications. Tests of GR.

- Effective-one-body theory: it can re-sum and re-organize perturbative results to improve accuracy and include strong-field effects close to merger; it can model semi-analytically merger-ringdown.

- The bright future of GW astrophysics comes with new theoretical challenges and opportunities.

- Can gravitational waveforms be obtained more efficiently with modern scattering amplitude techniques?

Solving two-body problem in General Relativity (including radiation)

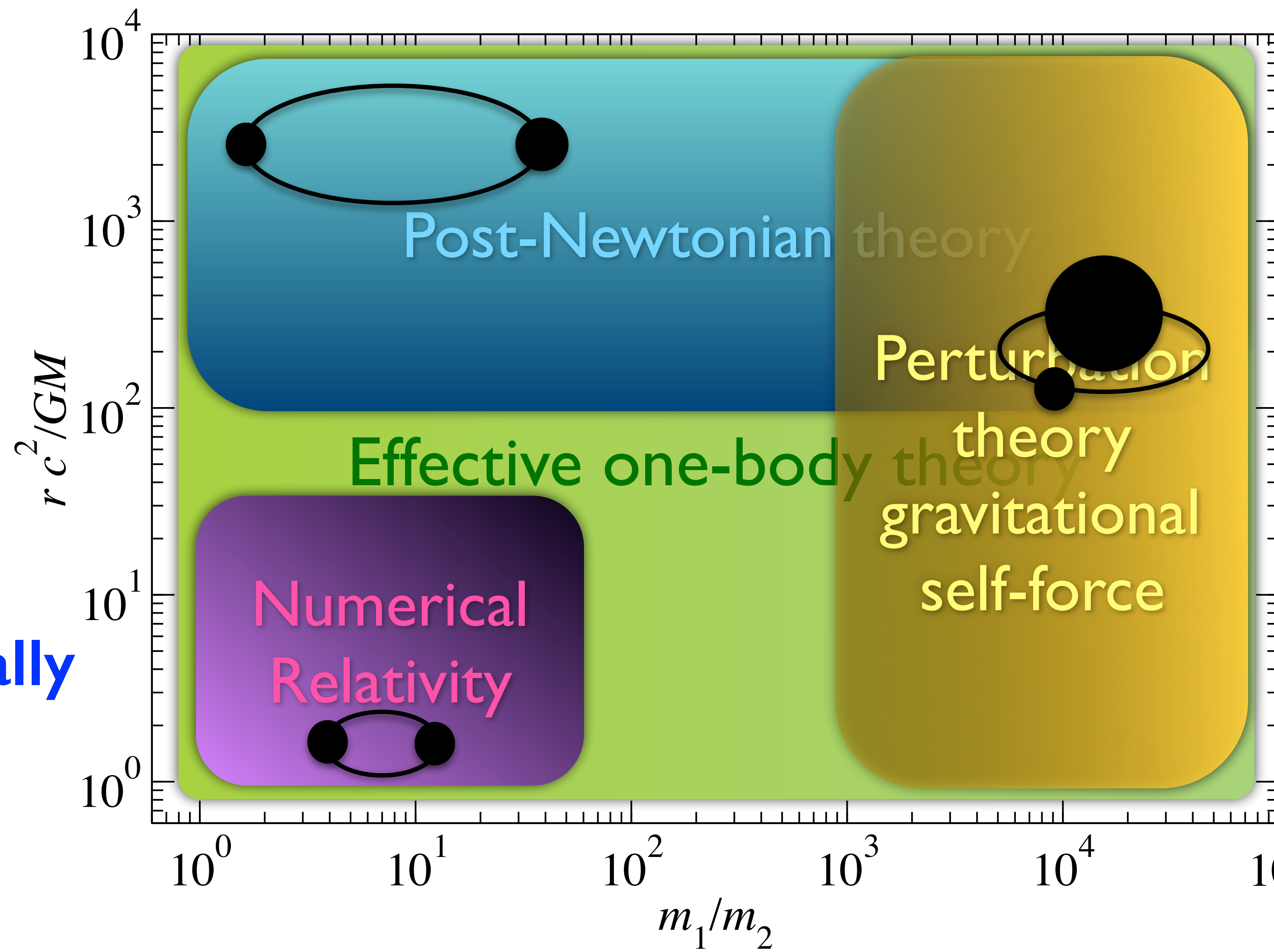
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **GR** is **non-linear theory**.

- Einstein's field equations can be solved:

  - **approximately,** but **analytically** (**fast** way)
  - **exactly**, but **numerically** on supercomputers (**slow** way)

$v^2/c^2 \sim GM/rc^2$



(AB & Sathyaprakash 14)

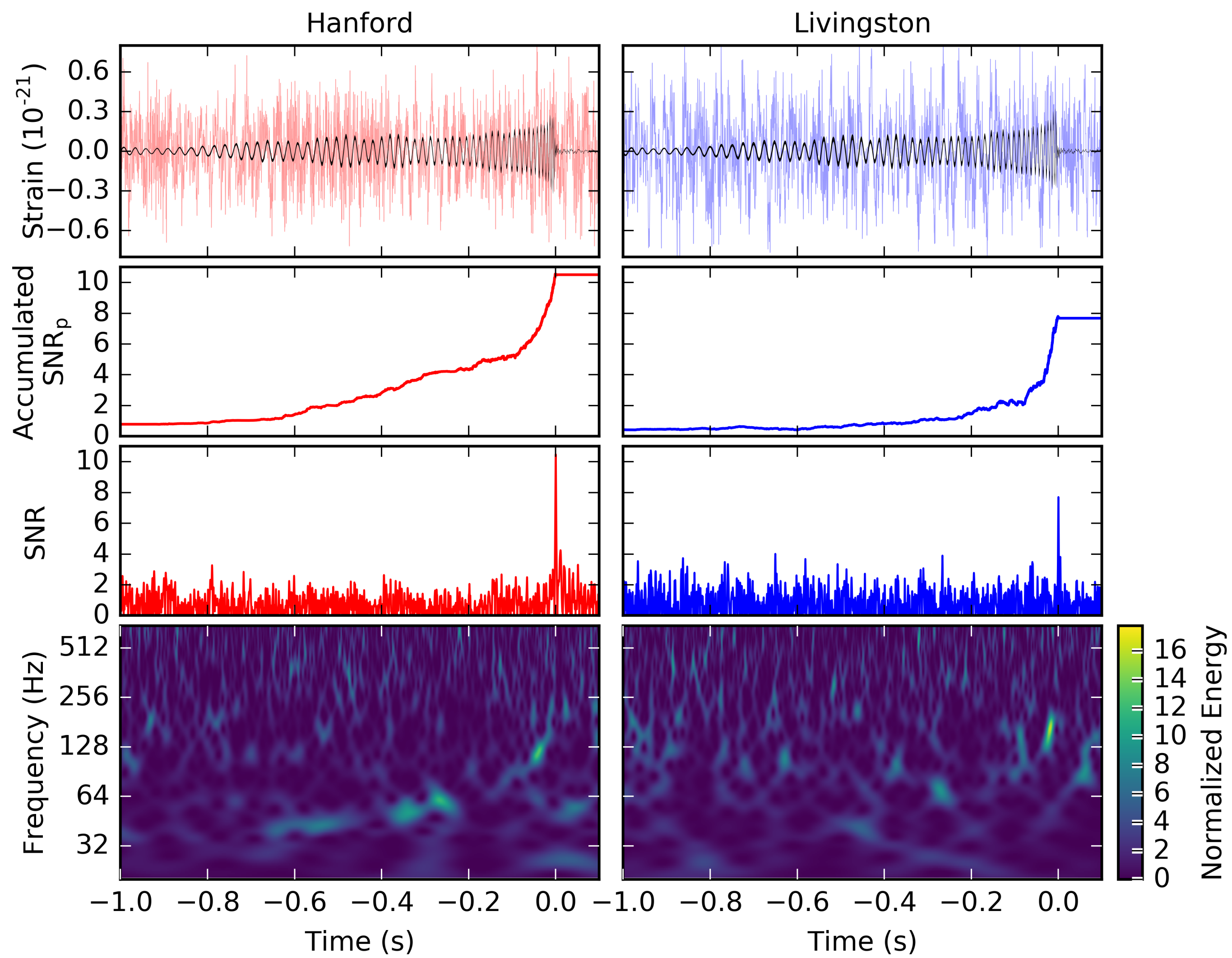
- **Synergy** between **analytical** and **numerical relativity** is **crucial.**

Solving two-body problem in General Relativity (including radiation)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

(Abbott et al. PRL 116 (2016) 241103)

- **GW151226**: SNR=13, 55 cycles (from 35 Hz), **1 sec.**



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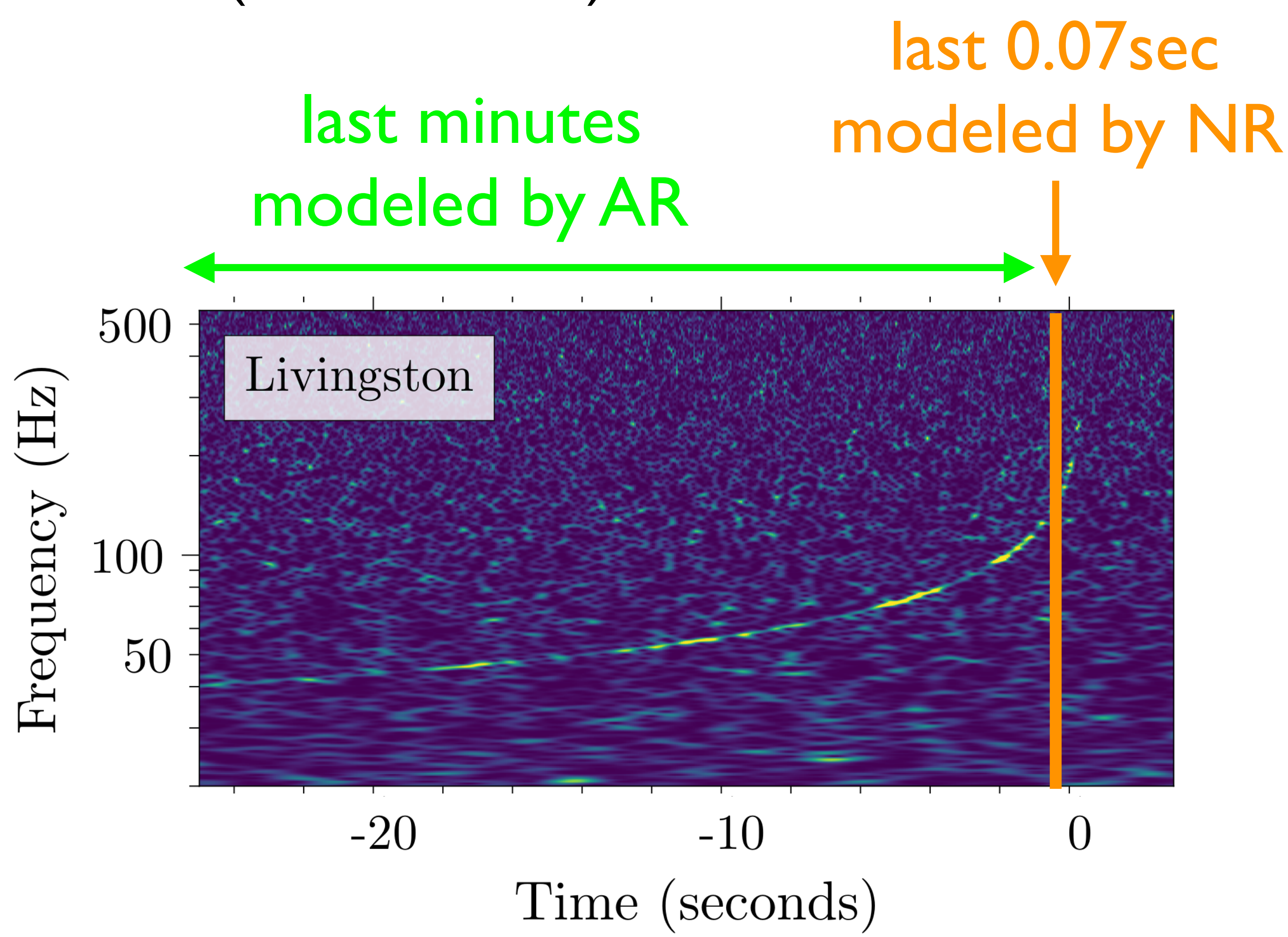
- GR is non-linear theory.

- Einstein's field equations can be solved:

  - approximately, but analytically (fast way)
  - exactly, but numerically on supercomputers (slow way)

(Abbott et al. PRL 119 (2017) 161101)

- GW170817: SNR=32, 3000 cycles (from 30 Hz), one minute.

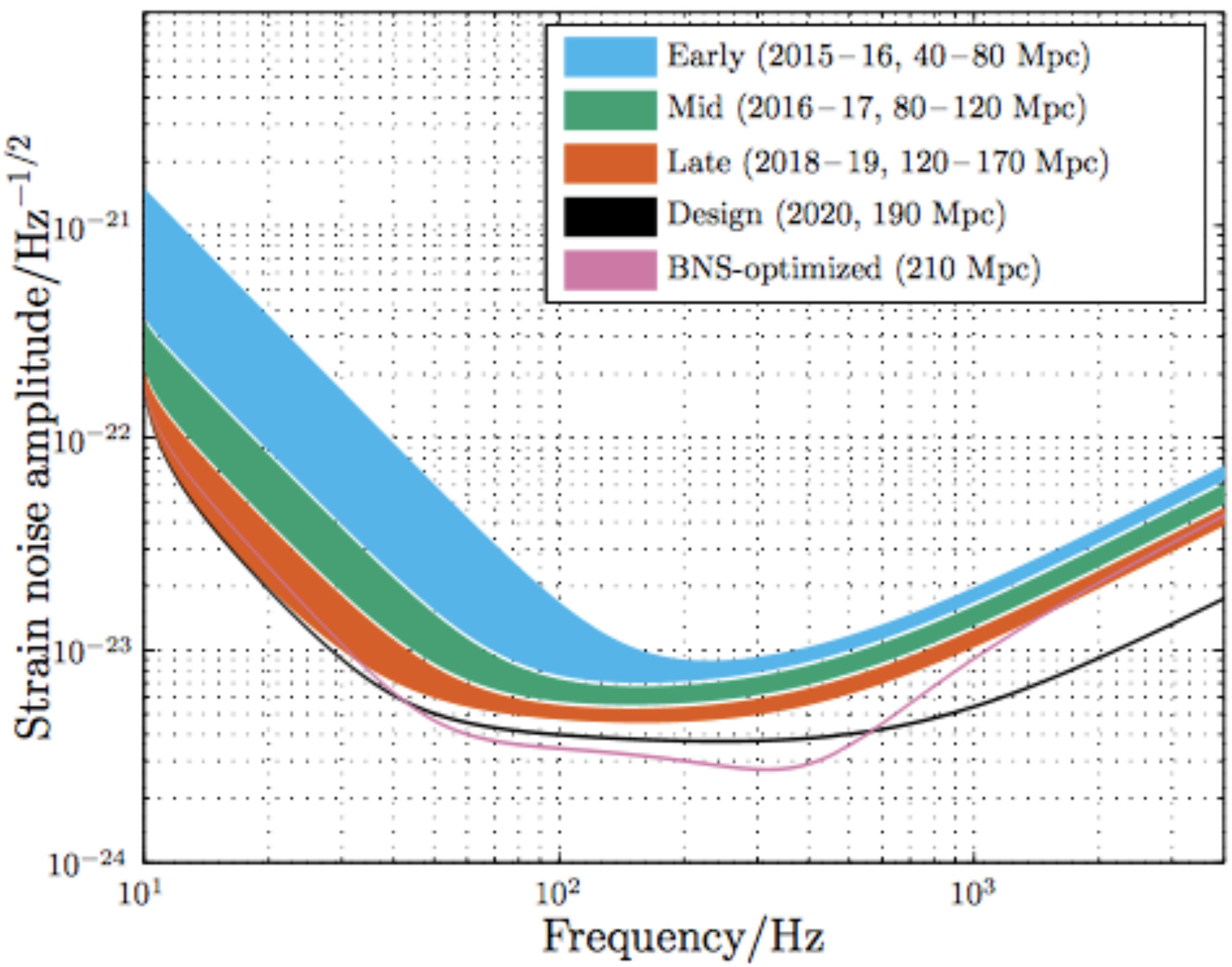


- Synergy between analytical and numerical relativity is crucial.

Advanced detectors' roadmap and rates

O3 is planned to start in February 2019

Advanced LIGO



(Aasi et al. Living Rev. Rel. 21, 2018)

BNS rate: 320 - 4740 /(Gpc)^3/yr

BBH rate: 12 - 213 /(Gpc)^3/yr

$$R = \frac{\Lambda}{< VT >}$$

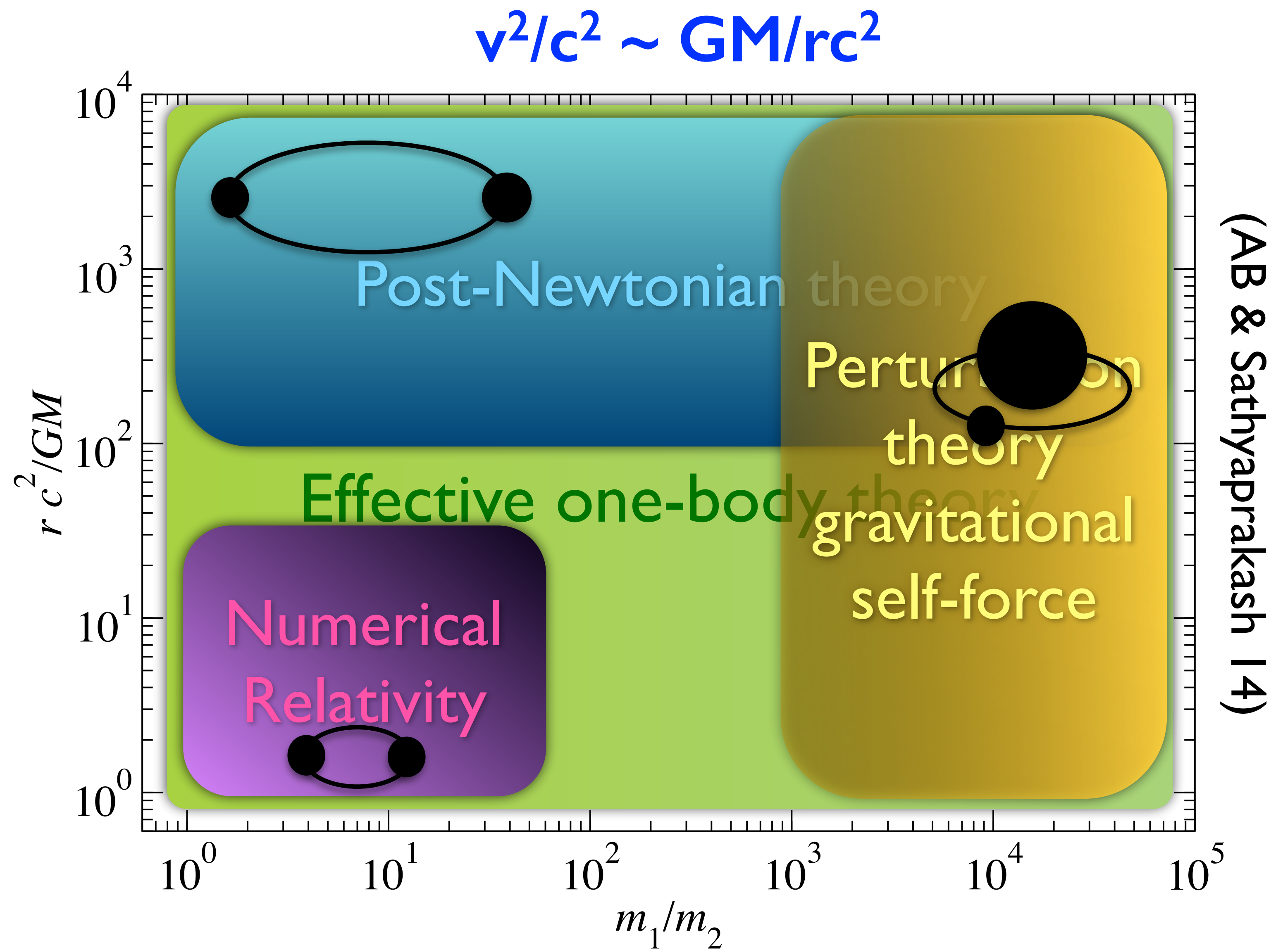
Detection rates @ design sensitivity:

- **Binary neutron stars: 4 - 80 per year**
- **Binary black holes: tens to hundreds per year**

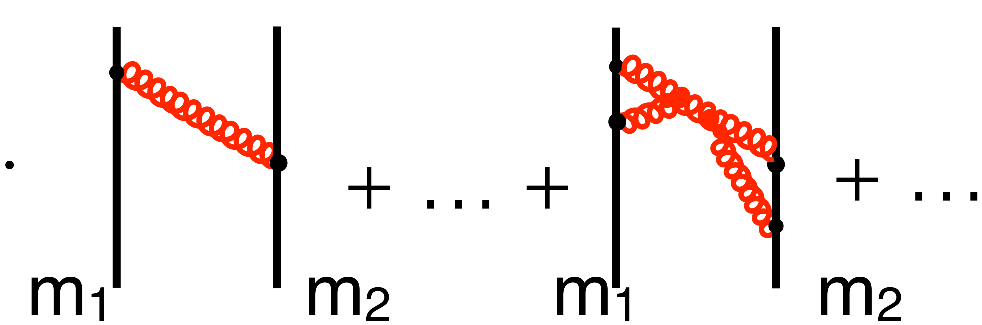
Solving two-body problem in General Relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **GR** is **non-linear theory**. Complexity similar to QCD.
- Einstein's field equations can be solved:
  - **approximately,** but **analytically** (**fast** way)
  - **exactly**, but **numerically** on supercomputers (**slow** way)

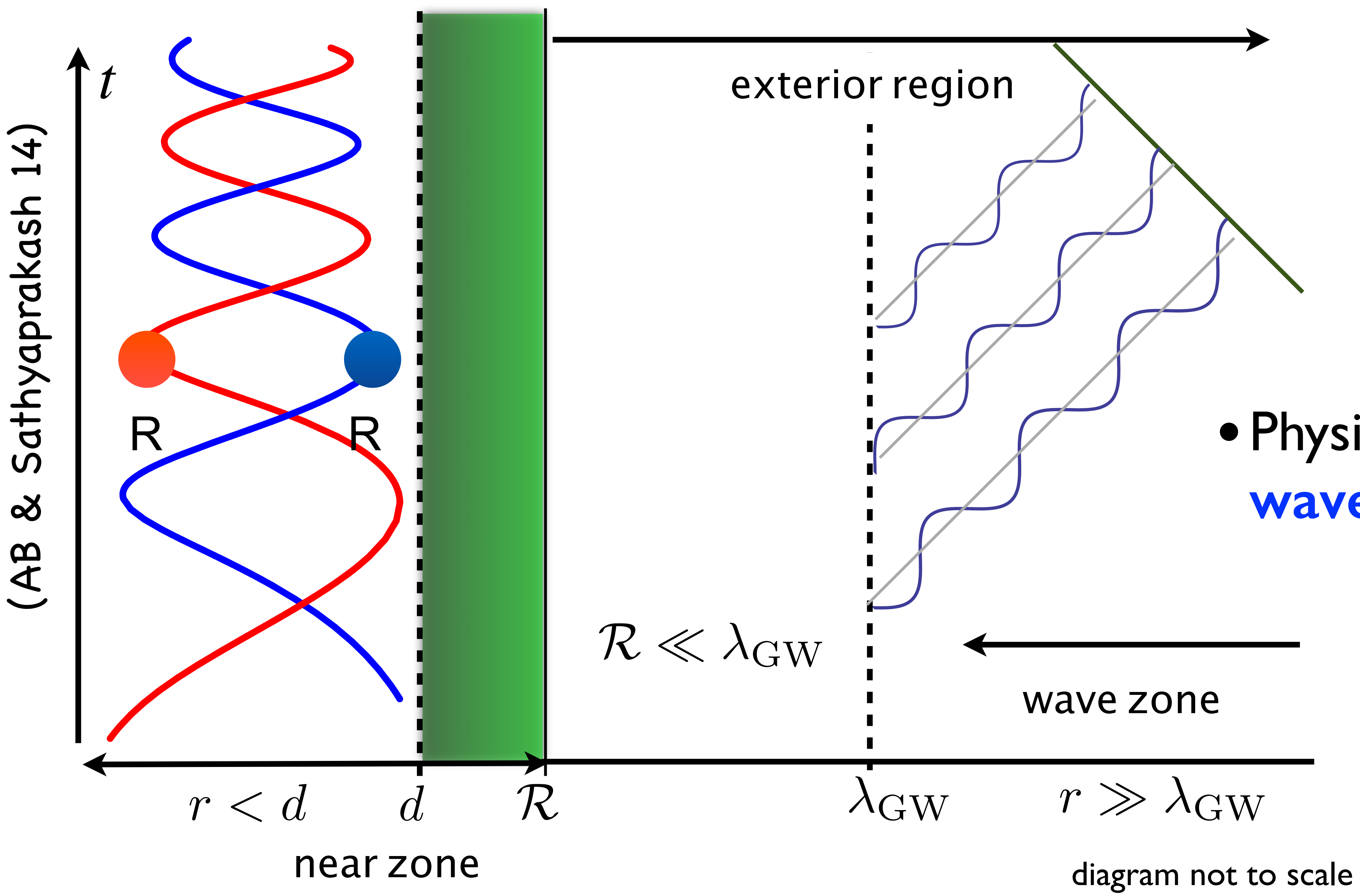


- **Analytical methods**: post-Newtonian/post-Minkowskian/small mass-ratio expansion, effective-one-body theory
  - **effective field-theory**, **dimensional regularization**, etc.
  - **diagrammatic** approach to organize expansions



Post-Newtonian/post-Minkowskian formalism/effective field theory

diverse scales: R, d, $\lambda_{\rm GW}$



- Physical **observables are waveforms** at null infinity

- **Multi-chart** approach to describe motion of **strong self-gravity bodies**, such as NS & BH.
- **Radiation-reaction** problem.

- **Matched asymptotic** expansions.
- **Generation** problem.

Equations of motion/Hamiltonian in post-Newtonian theory

- **First introduced in 1917** *(Droste & Lorentz 1917, … Einstein, Infeld & Hoffmann 1938)*

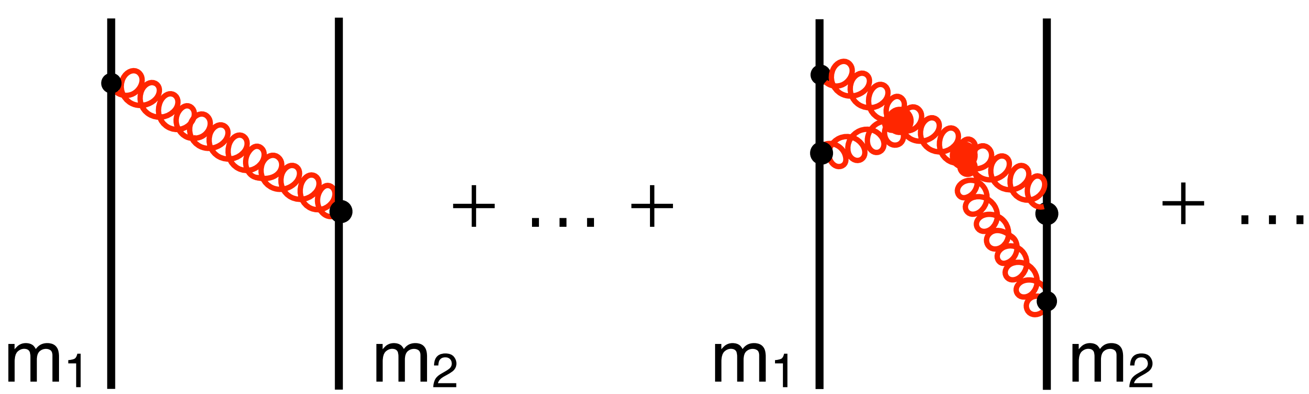
(Blanchet, Damour, Iyer, Faye, Bernard, Bohe', AB, Marsat; Jaranowski, Schaefer, Steinhoff; Will, Wiseman; Flanagan, Hinderer, Vines; Goldberger, Porto, Rothstein; Kol, Levi, Smolkin; Foffa, Sturani; …)

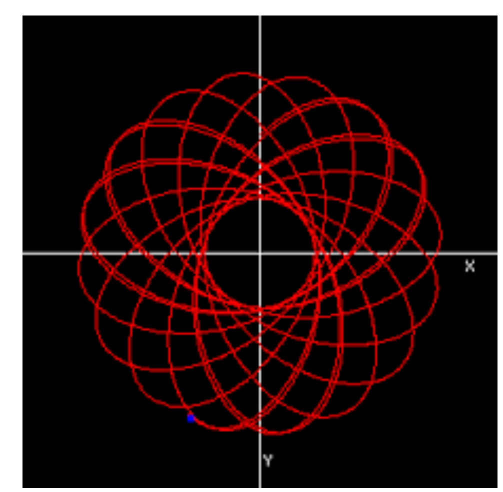
$$\widehat{H}_{\rm N}(\mathbf{r},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{r}$$

Small parameter is **$v/c << 1$, $v^2/c^2 \sim GM/rc^2$**

$$\widehat{H}_{\rm 1PN}(\mathbf{r},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\left\{(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\right\}\frac{1}{r} + \frac{1}{2r^2}$$

$$\widehat{H}_{\rm 2PN}(\mathbf{r},\mathbf{p}) = \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\left\{\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 \quad + \ldots\right.$$





- Compact object is **point-like body endowed** with time-dependent **multipole moments** (skeletonization).

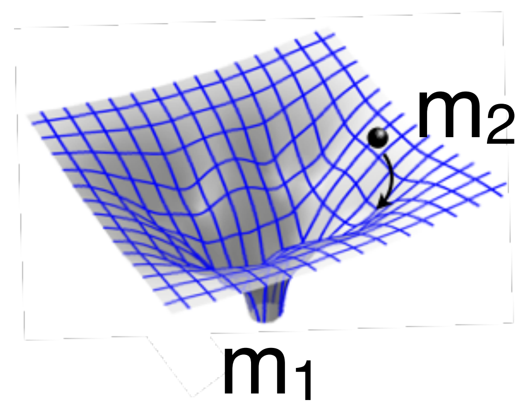
Small mass-ratio expansion/gravitational self-force formalism

- **First works in 50-70s** *(Regge & Wheeler 56, Zerilli 70, Teukolsky 72)*

Small parameter is $m_{2}/m_{1} \ll 1$, $v^2/c^2 \sim GM/rc^2 \sim 1$, $M = m_1 + m_2$

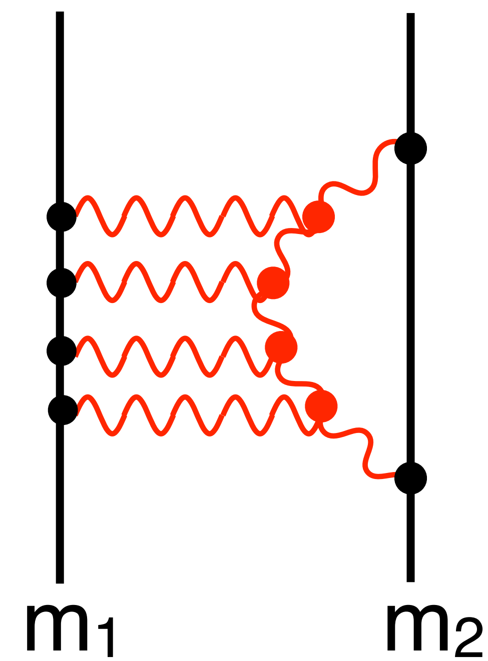
Equation of gravitational perturbations in black-hole spacetime:

$$\frac{\partial^2\Psi}{\partial t^2} - \frac{\partial^2\Psi}{\partial r_\star^2} + V_{\ell m}\Psi = S_{\ell m}$$



Green functions in Schwarzschild/Kerr spacetimes.

(Fujita, Poisson, Sasaki, Shibata, Khanna, Hughes, Bernuzzi, Harms, Nagar…)



- Accurate modeling of **relativistic dynamics of large mass-ratio** inspirals **requires** to include **back-reaction effects** due to interaction of small object with its own gravitational perturbation field.

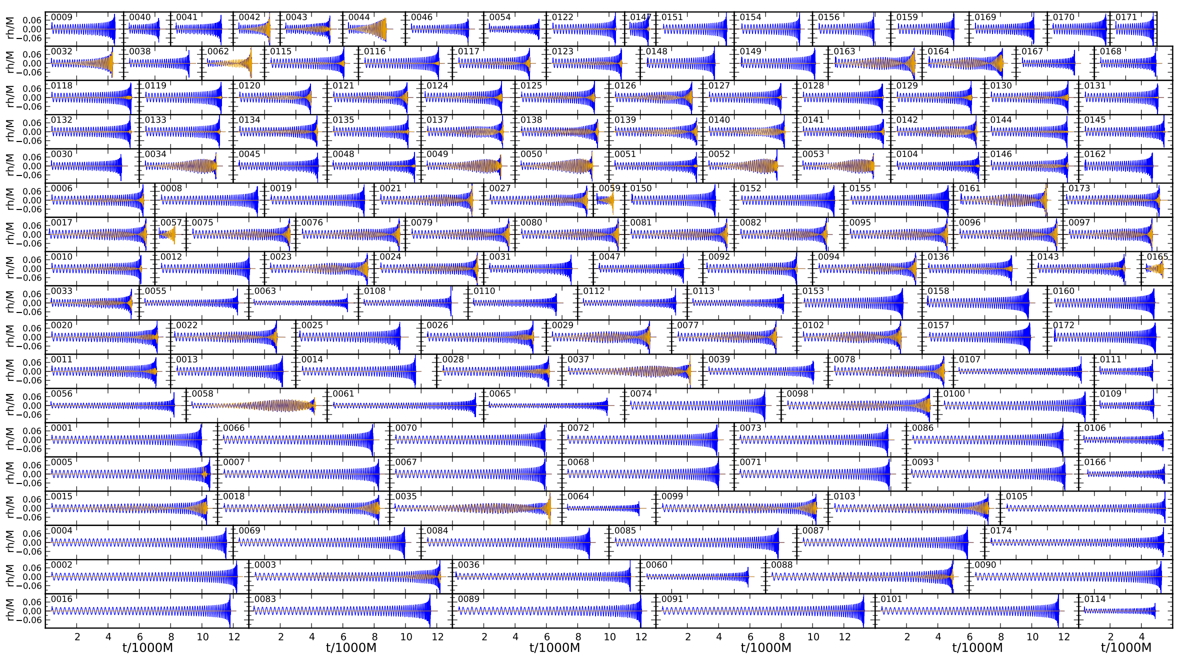
(Deitweiler, Whiting, Mino, Poisson, Quinn, Wald, Sasaki, Tanaka, Barack, Ori, Pound, van de Meent, …)

Numerical Relativity: binary black holes

- **Breakthrough** in 2005 *(Pretorius 05, Campanelli et al. 06, Baker et al. 06)*

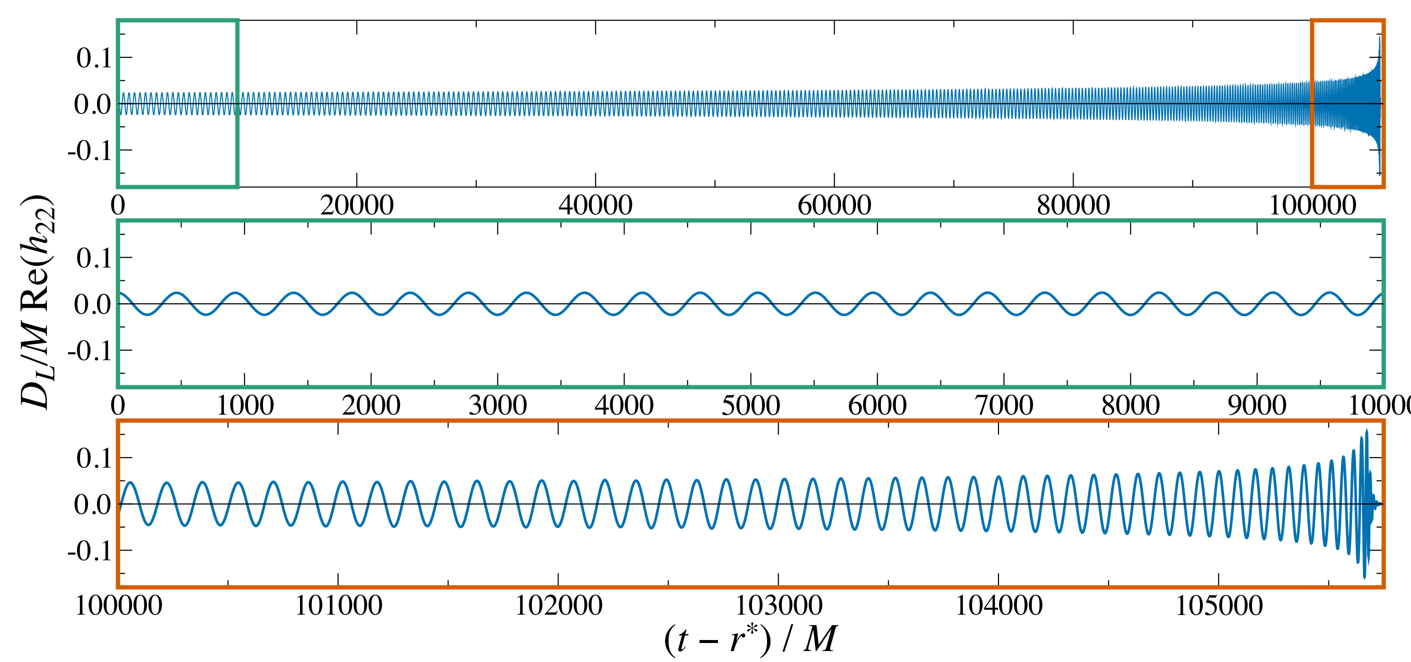
(Kidder, Pfeiffer, Scheel, Lindblom, Szilagyi; Bruegmann; Hannam, Husa, Tichy; Laguna, Shoemaker; …)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$



- **376 GW cycles**, zero spins & mass-ratio 7 (8 months, few millions CPU-h)

(Szilagyi, Blackman, AB, Taracchini et al. 15)



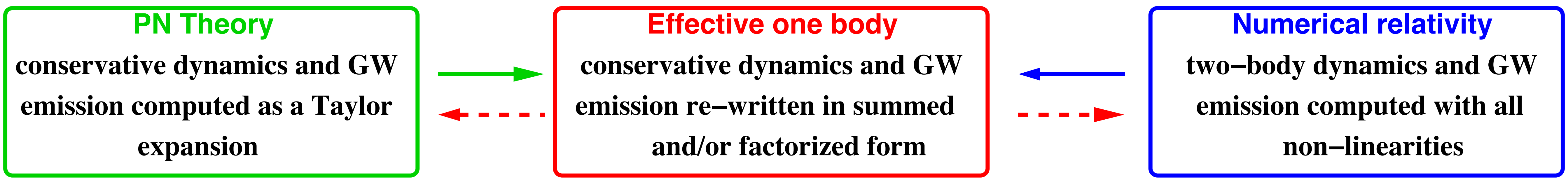
- **Simulating eXtreme Spacetimes (SXS)** collaboration *(Mroue et al. 13)*

- **Numerical-Relativity & Analytical-Relativity** collaboration *(Hinder et al. 13)*

The effective-one-body (EOB) approach

- **EOB** approach **introduced before NR** breakthrough

(AB, Pan, Taracchini, Barausse, Bohe', Cotesta, Shao, Hinderer, Steinhoff, Vines; Damour, Nagar, Bernuzzi, Bini, Balmelli, Messina; Iyer, Sathyaprakash; Jaranowski, Schaefer)



- **EOB** model uses best information available in PN theory, but **resums PN terms** in suitable way to describe accurately dynamics and radiation during inspiral and plunge (going beyond quasi-circular adiabatic motion).

- **EOB** assumes **comparable-mass** description is **smooth deformation of test-particle limit**. It employs non-perturbative ingredients and **models analytically merger-ringdown** signal.

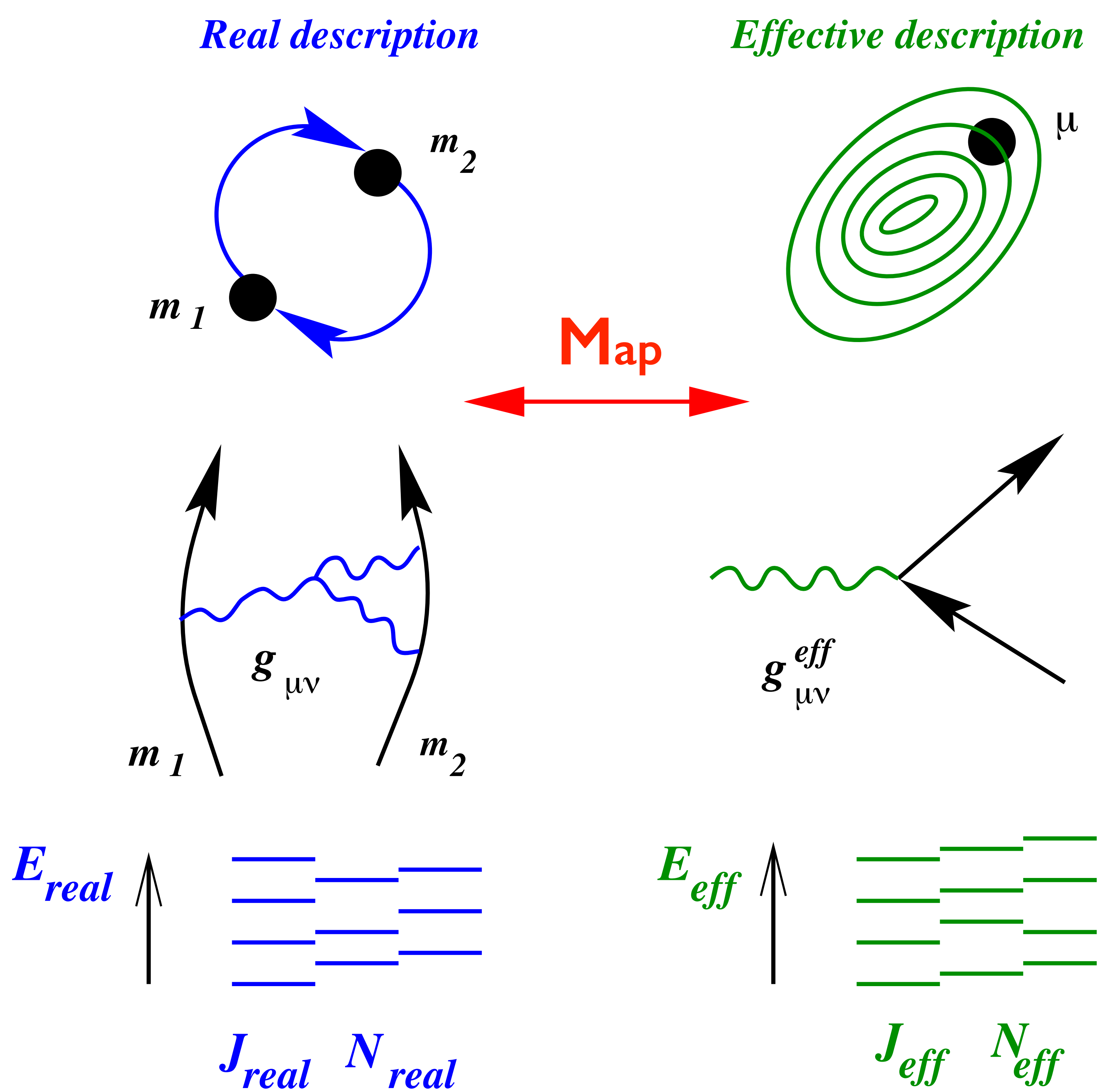
The effective-one-body approach in a nutshell

$$\nu = \frac{\mu}{M} \qquad 0 \leq \nu \leq 1/4$$

$$\mu = \frac{m_1 m_2}{M} \qquad M = m_1 + m_2$$

- **Two-body dynamics is mapped** into dynamics of **one-effective body** moving **in deformed black-hole spacetime**, deformation being the mass ratio.

- Some key **ideas** of EOB model were **inspired by quantum field theory** when describing energy of comparable-mass charged bodies.



(AB & Damour 1998)

Energy for comparable-mass bodies

- Classical gravity: *(AB & Damour 98)*

$$E_{\rm real}^2 = m_1^2 + m_2^2 + 2m_1 m_2 \left(\frac{E_{\rm eff}}{\mu}\right)$$

- Quantum electrodynamics: *(Brezin, Itzykson & Zinn-Justin 1970)*

$$E_{\rm real}^2 = m_1^2 + m_2^2 + 2m_1 m_2 \frac{1}{\sqrt{1 + Z^2\,\alpha^2/(n-\epsilon_j)^2}}$$

- Considering scattering states:

$$\varphi(s) \equiv \frac{s - m_1^2 - m_2^2}{2m_2 m_2} = \frac{-(p_1+p_2)^2 - m_1^2 - m_2^2}{2m_2 m_2} = -\frac{p_1\cdot p_2}{m_1 m_2}$$

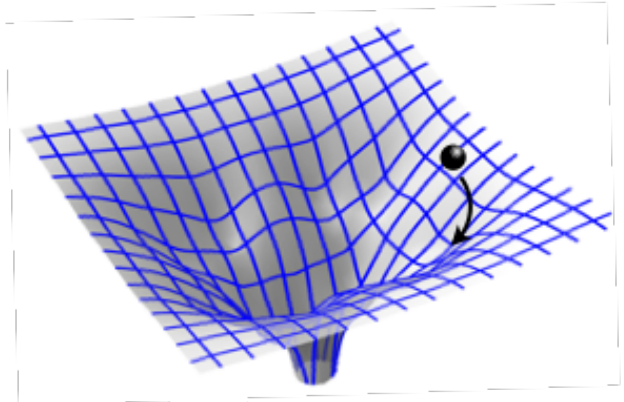
EOB Hamiltonian: resummed conservative dynamics (@2PN)

- Real Hamiltonian → • Effective Hamiltonian

$$H_{\rm real}^{\rm PN} = H_{\rm Newt} + H_{\rm 1PN} + H_{\rm 2PN} + \cdots$$

$$H_{\rm eff}^{\nu} = \mu\sqrt{A_\nu(r)\left[1+\frac{\mathbf{p}^2}{\mu^2}+\left(\frac{1}{B_\nu(r)}-1\right)\frac{p_r^2}{\mu^2}\right]}$$

$$ds^2_{\rm eff} = -A_\nu(r)dt^2 + B_\nu(r)dr^2 + r^2 d\Omega^2$$



(credit: Hinderer)

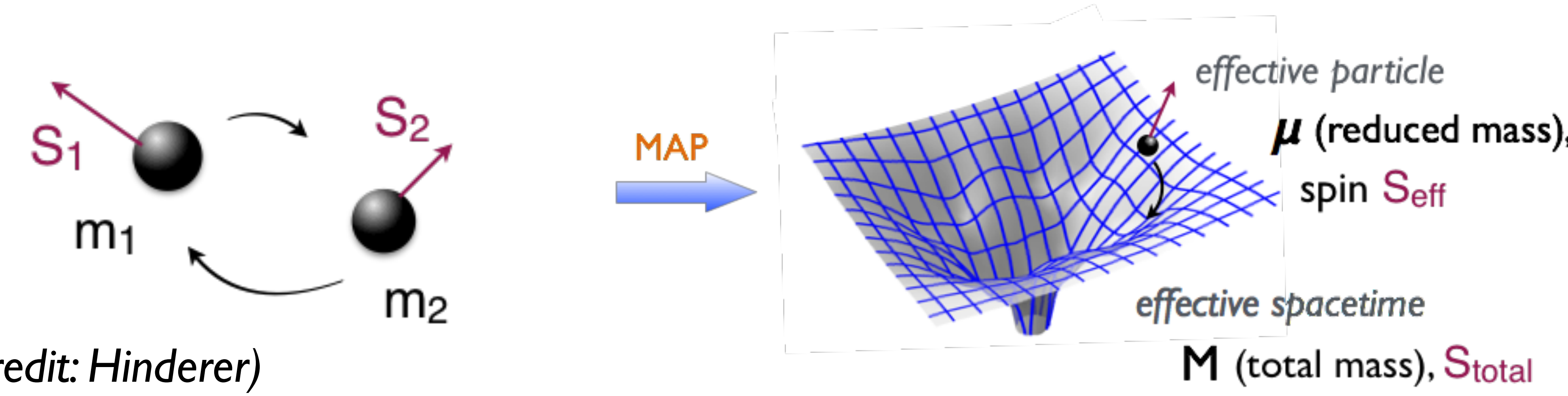
- EOB Hamiltonian: $H_{\rm real}^{\rm EOB} = M\sqrt{1+2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu}-1\right)}$

- Dynamics condensed in $A_\nu(r)$ and $B_\nu(r)$

- $A_\nu(r)$, which encodes the energetics of circular orbits, is quite simple:

$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3}-\frac{41}{32}\pi^2\right)\frac{M^4\nu}{r^4} + \frac{a_5(\nu)+a_5^{\log}(\nu)\log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots$$

EOB resummed spin dynamics & waveforms



(credit: Hinderer)

$$H_{\rm real}^{\rm EOB} = M\sqrt{1+2\nu\left(\frac{{H_{\rm eff}}^{\nu}}{\mu}-1\right)}$$

- ${H_{\rm eff}}^{\nu}$ with spins, two EOB resummations:

(Barausse, Racine & AB 09; Barausse & AB 10, 11)

(Damour 01, Damour, Jaranowski & Schäfer 08; Damour & Nagar 14)

- EOB equations of motion *(AB et al. 00, 05; Damour et al. 09)*:

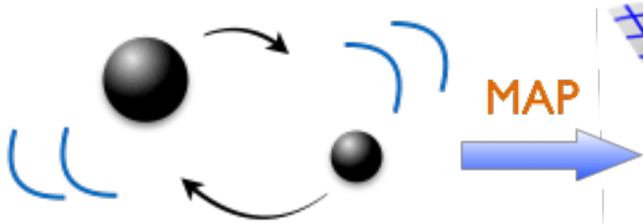
$$\dot{\mathbf{r}} = \frac{\partial H_{\rm real}^{\rm EOB}}{\partial \mathbf{p}} \qquad F \propto \frac{dE}{dt}, \quad \frac{dE}{dt} \propto \sum_{\ell m} |h_{\ell m}|^2$$

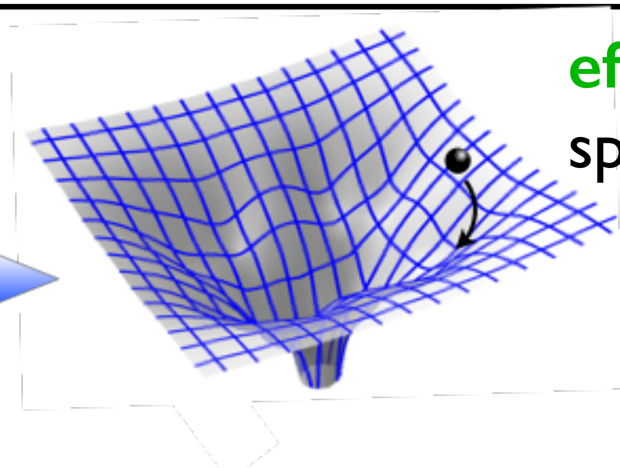
$$\dot{\mathbf{p}} = -\frac{\partial H_{\rm real}^{\rm EOB}}{\partial \mathbf{r}} + \mathbf{F} \qquad \dot{\mathbf{S}} = \{\mathbf{S}, H_{\rm real}^{\rm EOB}\}$$

- EOB waveforms *(AB et al. 00; Damour et al. 09; Pan, AB et al. 11)*:

$$h_{\ell m}^{\rm insp-plunge} = h_{\ell m}^{\rm Newt}\, e^{-im\Phi}\, S_{\rm eff}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, (\rho_{\ell m})^{\ell}\, h_{\ell m}^{\rm NQC}$$

EOB inspiral-merger-ringdown analytic waveform





(credit: Hinderer)

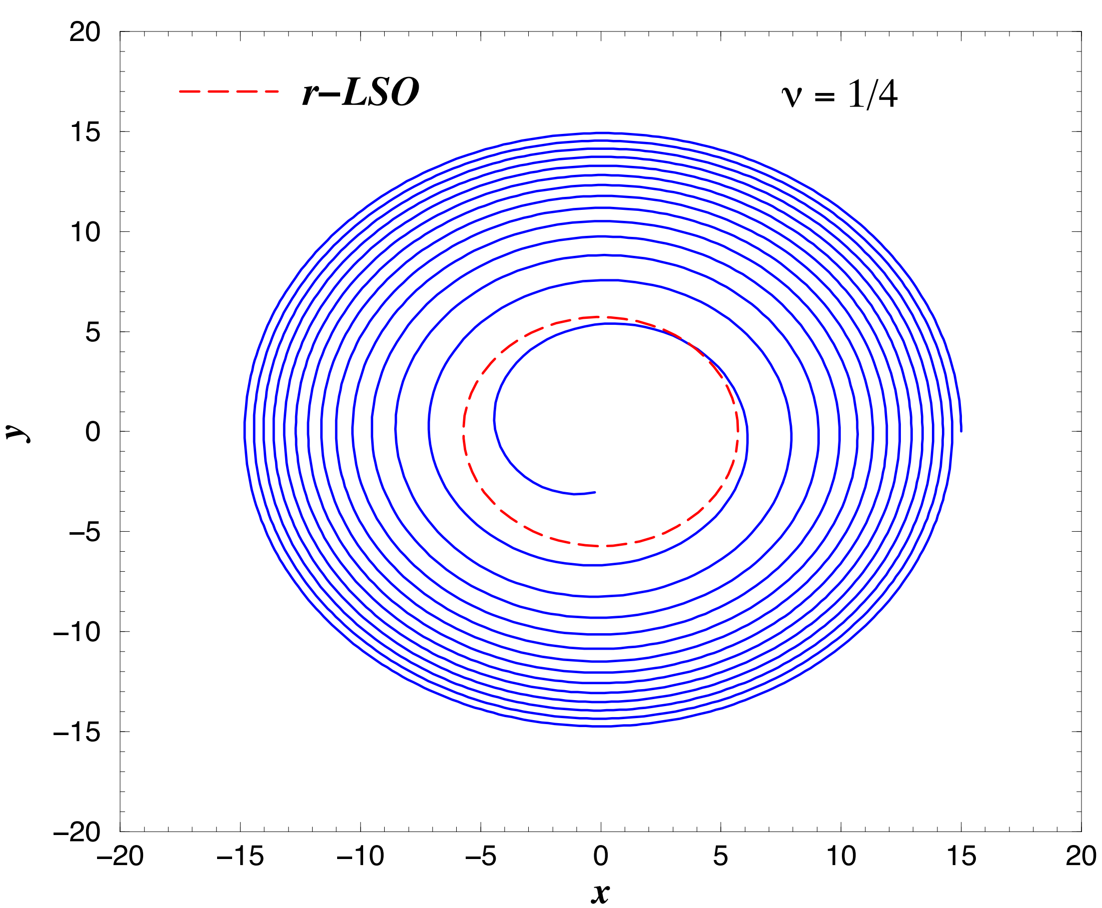
effective one-body Hamiltonian (deformed-Kerr spacetime, deformation being mass ratio)

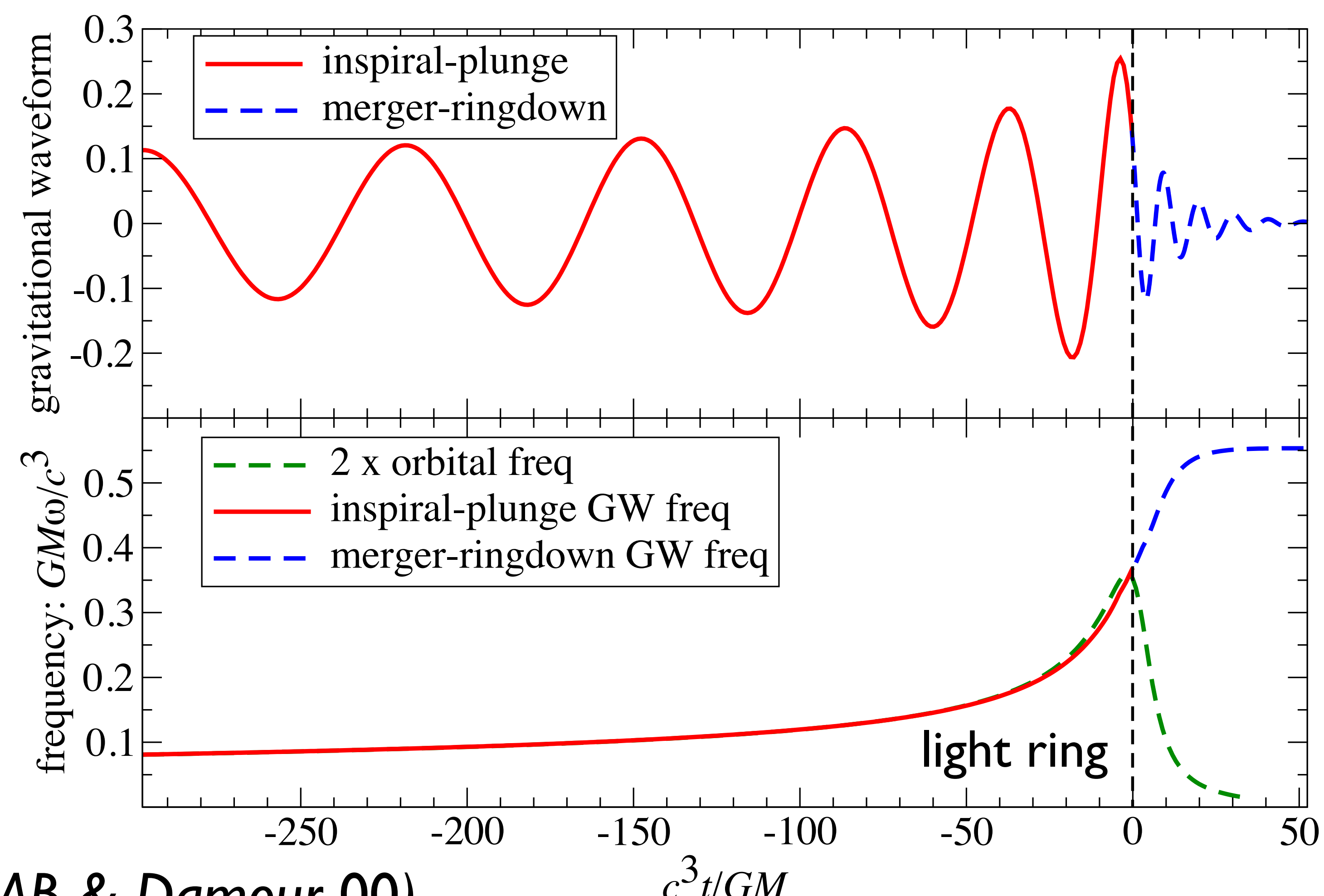
$$H_{\rm real}^2 = m_1^2 + m_2^2 + 2 m_1 m_2 \left(\frac{H_{\rm eff}}{\mu} \right)$$

real two-body Hamiltonian

(AB & Damour 98)

- By solving Hamilton equations with appropriate, resummed radiation-reaction force, we obtain orbital motion and gravitational waveform.





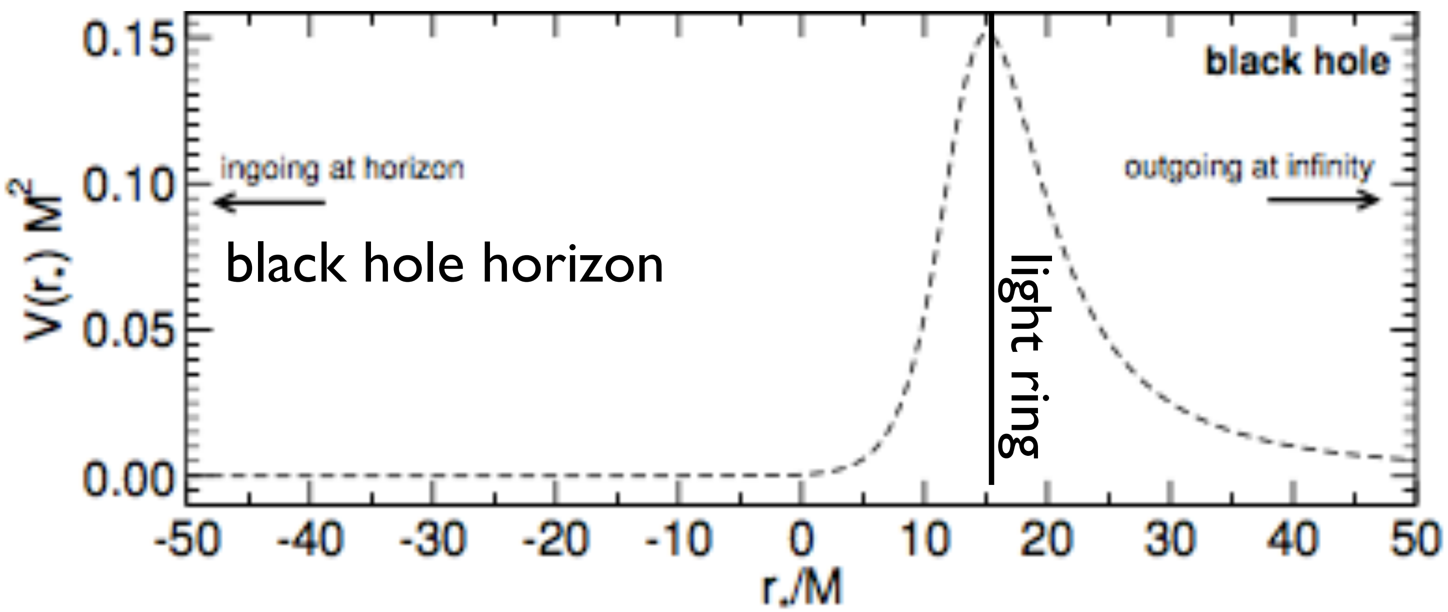
(AB & Damour 00)

On the simplicity of merger signal in small-mass ratio limit

- Equation of gravitational perturbations in black-hole spacetime

(Regge & Wheeler 56, Zerilli 70, Teukolsky 72)

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial r_\star^2} + V_{\ell m} \Psi = S_{\ell m}$$



- Peak of black-hole potential close to "light ring".

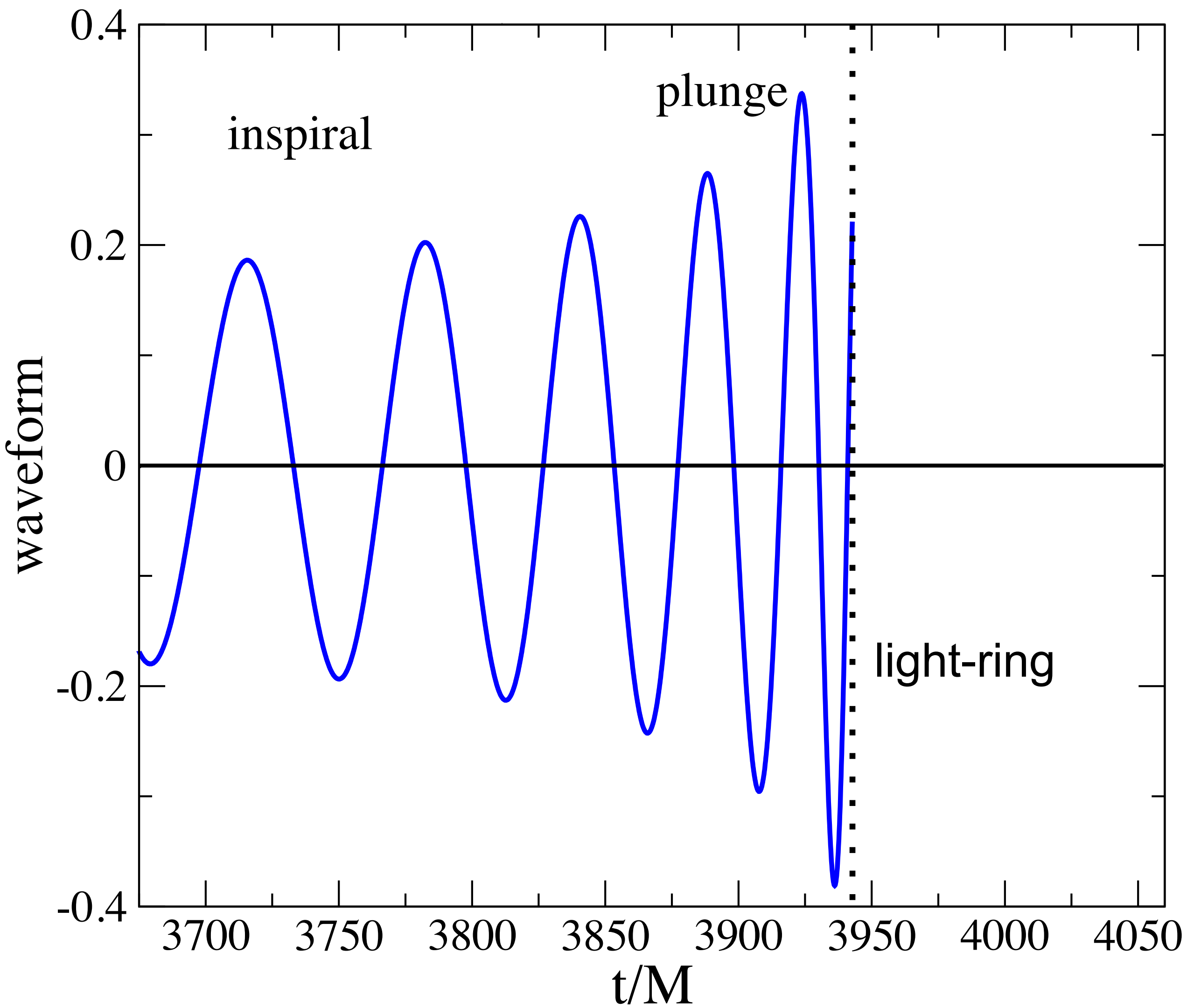
- Once particle is inside potential, direct gravitational radiation from its motion is strongly filtered by potential barrier (high-pass filter).

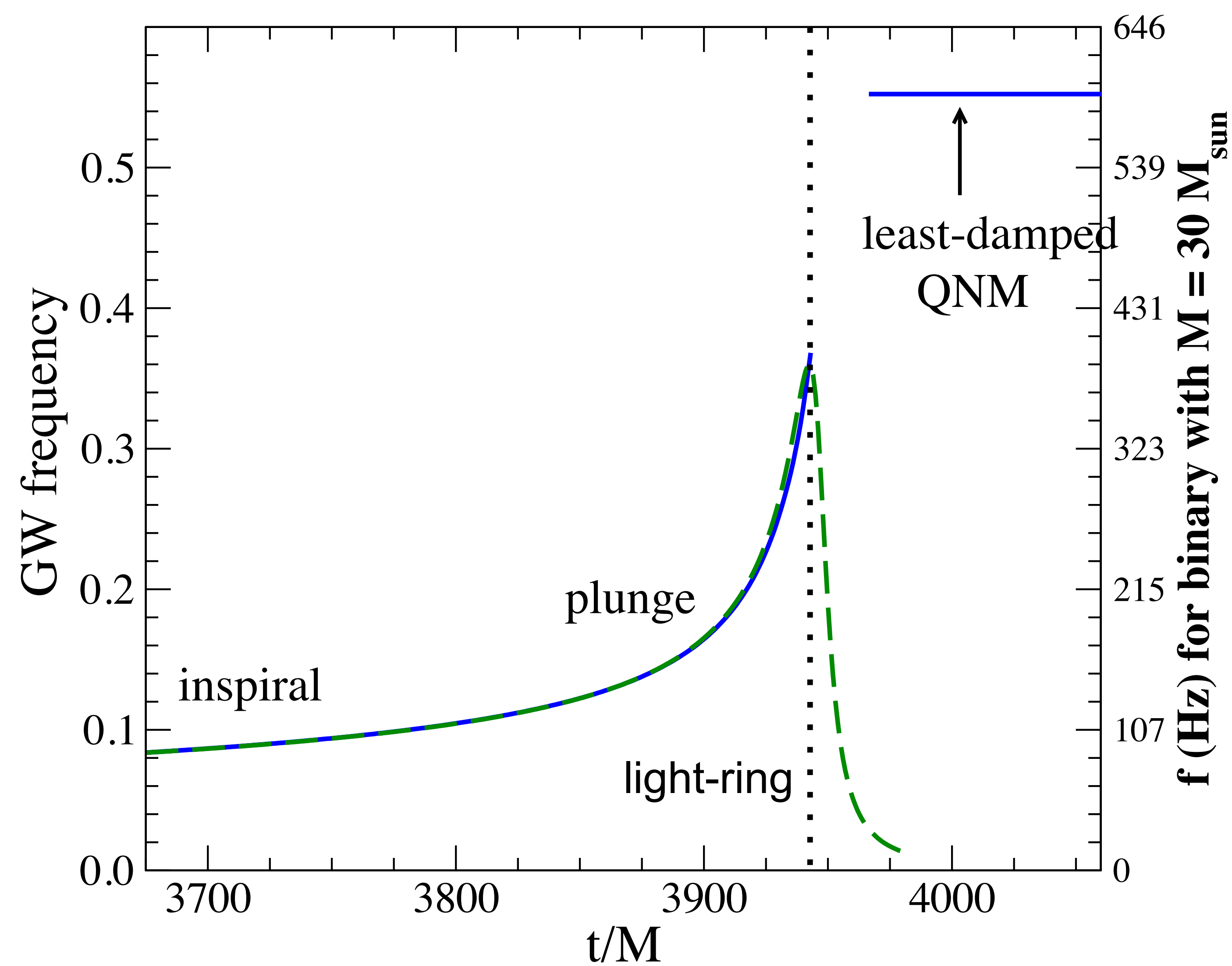
- Only black-hole spacetime vibrations (quasi-normal modes) leaks out black-hole potential.

(Goebel 1972, Davis et al. 1972, Ferrari & Mashhoon 1984)

On the full effective-one-body waveforms

- Evolve **two-body dynamics up to light ring** (or photon orbit) and then …



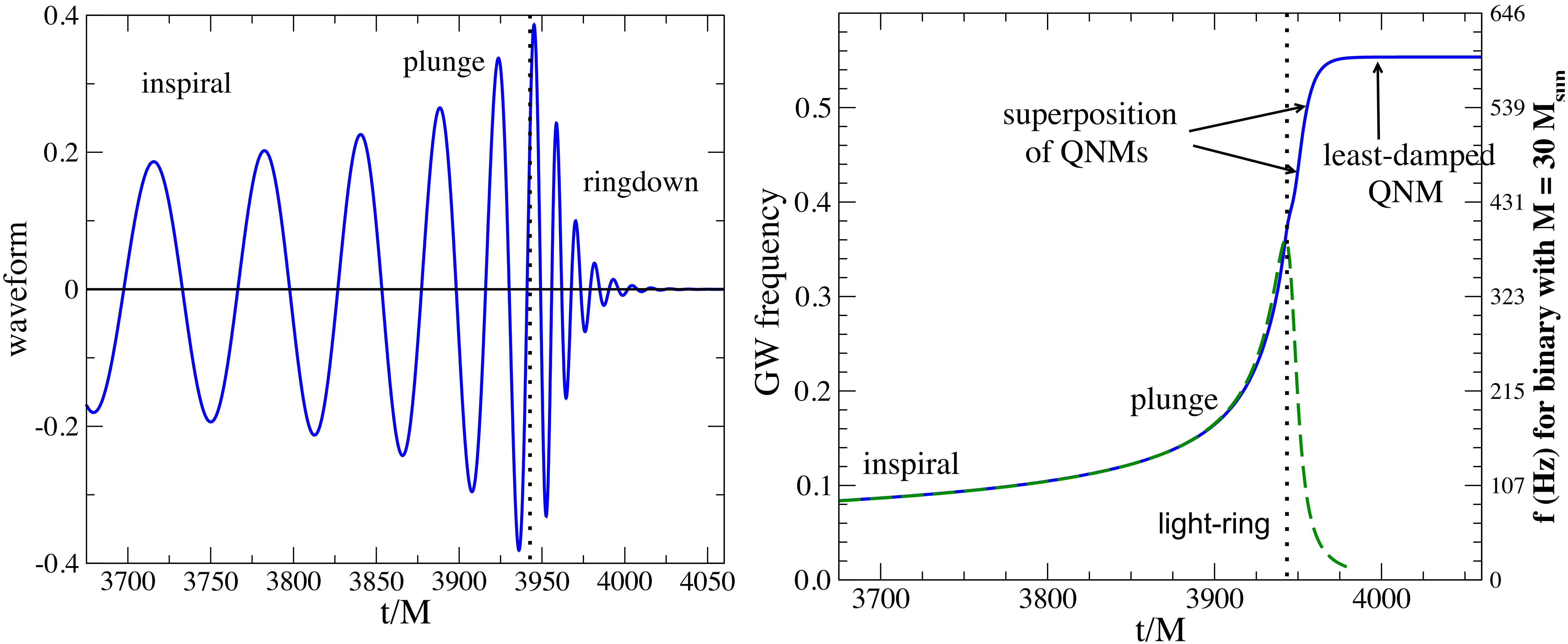


- **Quasi-normal modes** excited at **light-ring crossing**

(Goebel 1972, Davis et al. 1972, Ferrari et al. 1984, Damour et al. 07, Barausse et al. 11, Price et al. 15)

On the full effective-one-body waveforms (contd.)

… attach **superposition** of **quasi-normal modes** of **remnant** black hole.



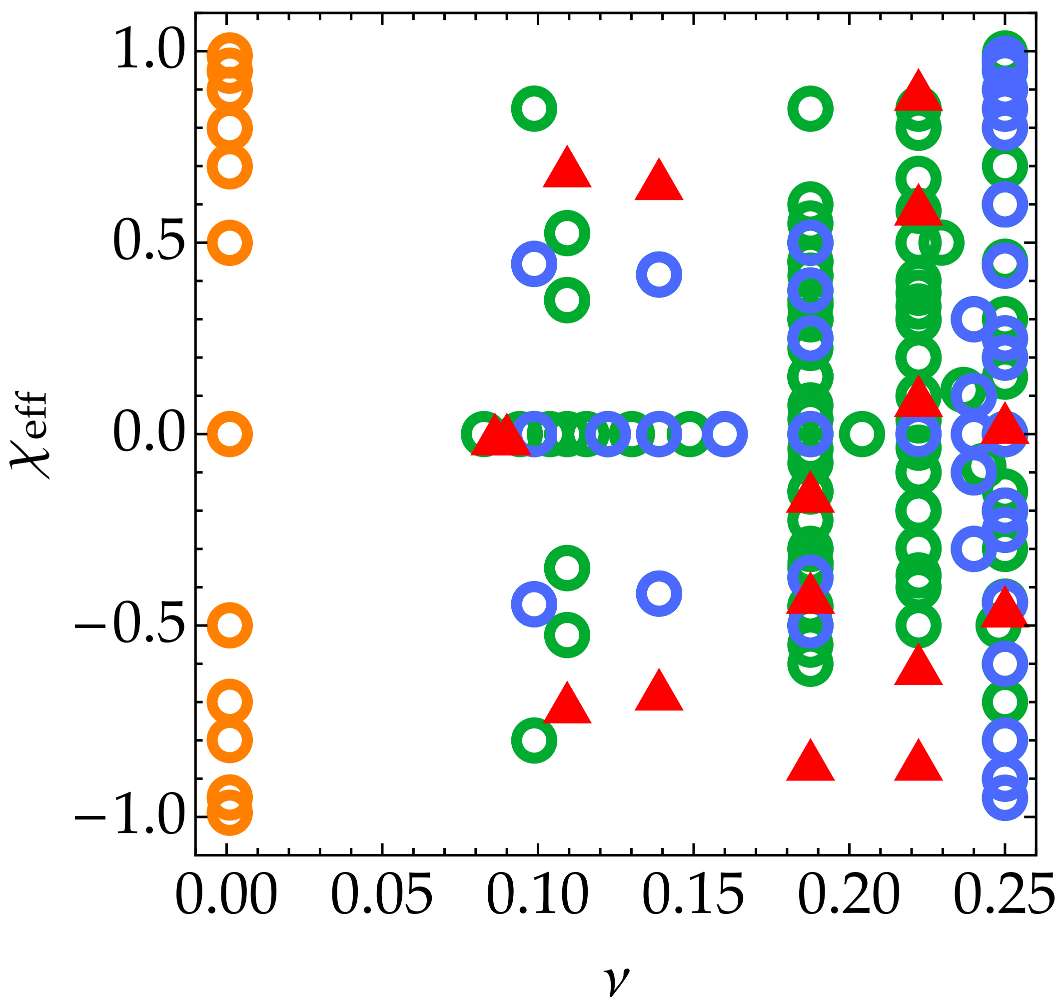
$$h_{22}^{\mathsf{EOB}}(t) = h_{22}^{\mathrm{insp-plunge}}\,\theta(t_{\mathrm{match}} - t) + h_{22}^{\mathrm{merger-RD}}\theta(t - t_{\mathrm{match}})$$

$$h_{22}^{\mathrm{merger-RD}}(t) = \sum_{n=0}^{N-1} A_n\, e^{-i\sigma_{22n}(t - t_{\mathrm{match}})} \qquad \sigma_{22n} \equiv \omega_{22n} - i/\tau_{22n}$$

Waveforms combining analytical & numerical relativity

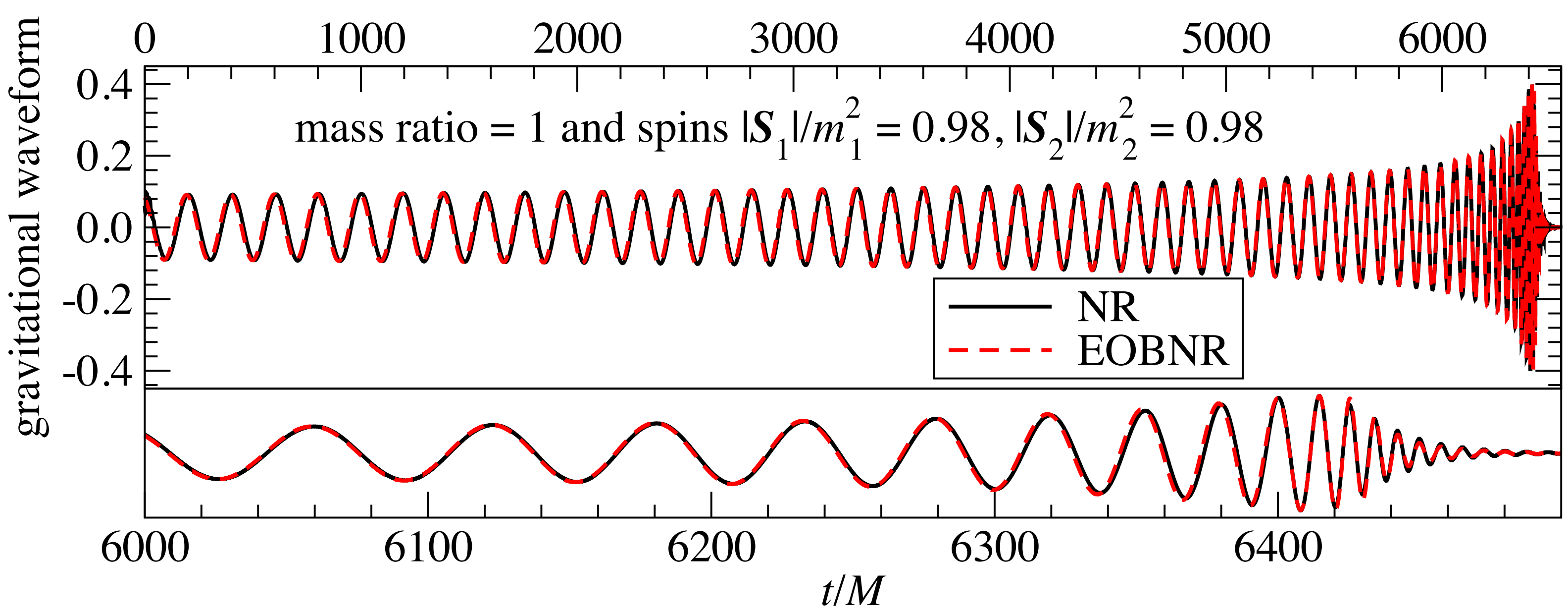
- **Effective-one-body** (EOB) theory & NR (EOBNR)

141 SXS simulations



(Pan, AB et al. 13, Taracchini, AB, Pan, Hinderer & SXS 14, Puerrer 15)

(Bohe', Shao, Taracchini, AB & SXS 16, Babak et al. 16)

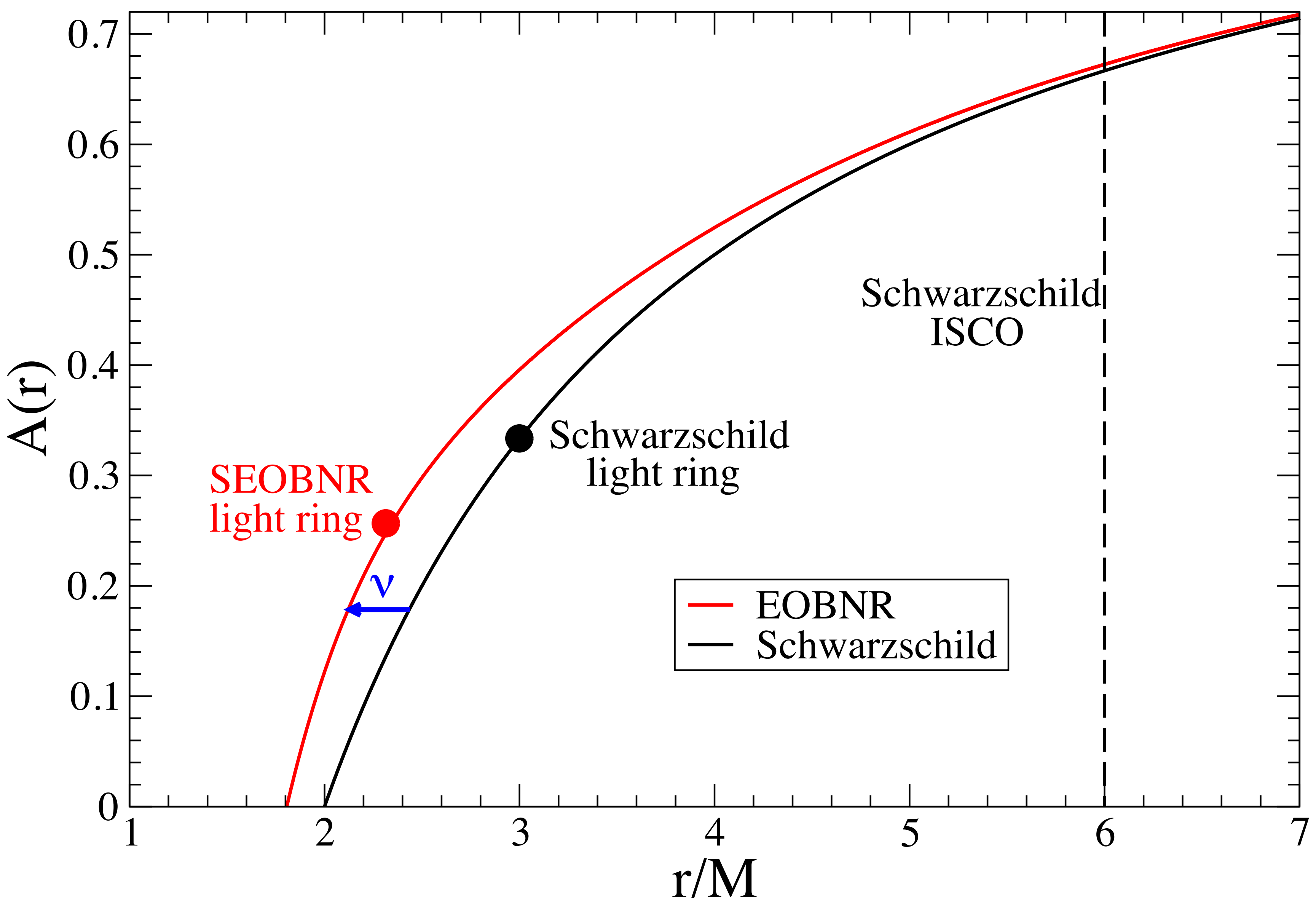


- **Inspiral-merger-ringdown phenomenological** waveforms fitting EOB & NR (IMRPhenom) *(Khan et al. 16, Hannam et al 16)*

Strong-field effects in binary black holes included in EOB

Finite mass-ratio effects make **gravitational** interaction **less attractive**

(Taracchini, AB, Pan, Hinderer & SXS 14)

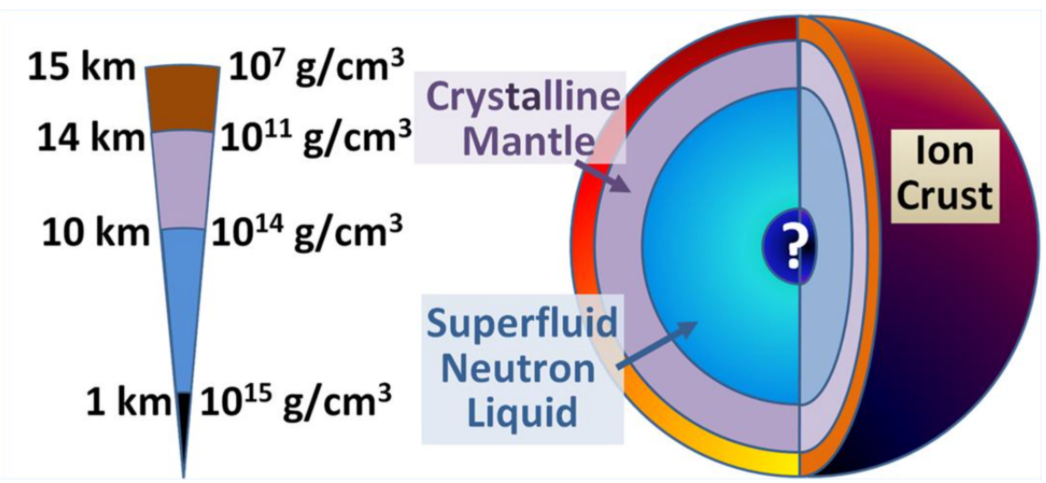


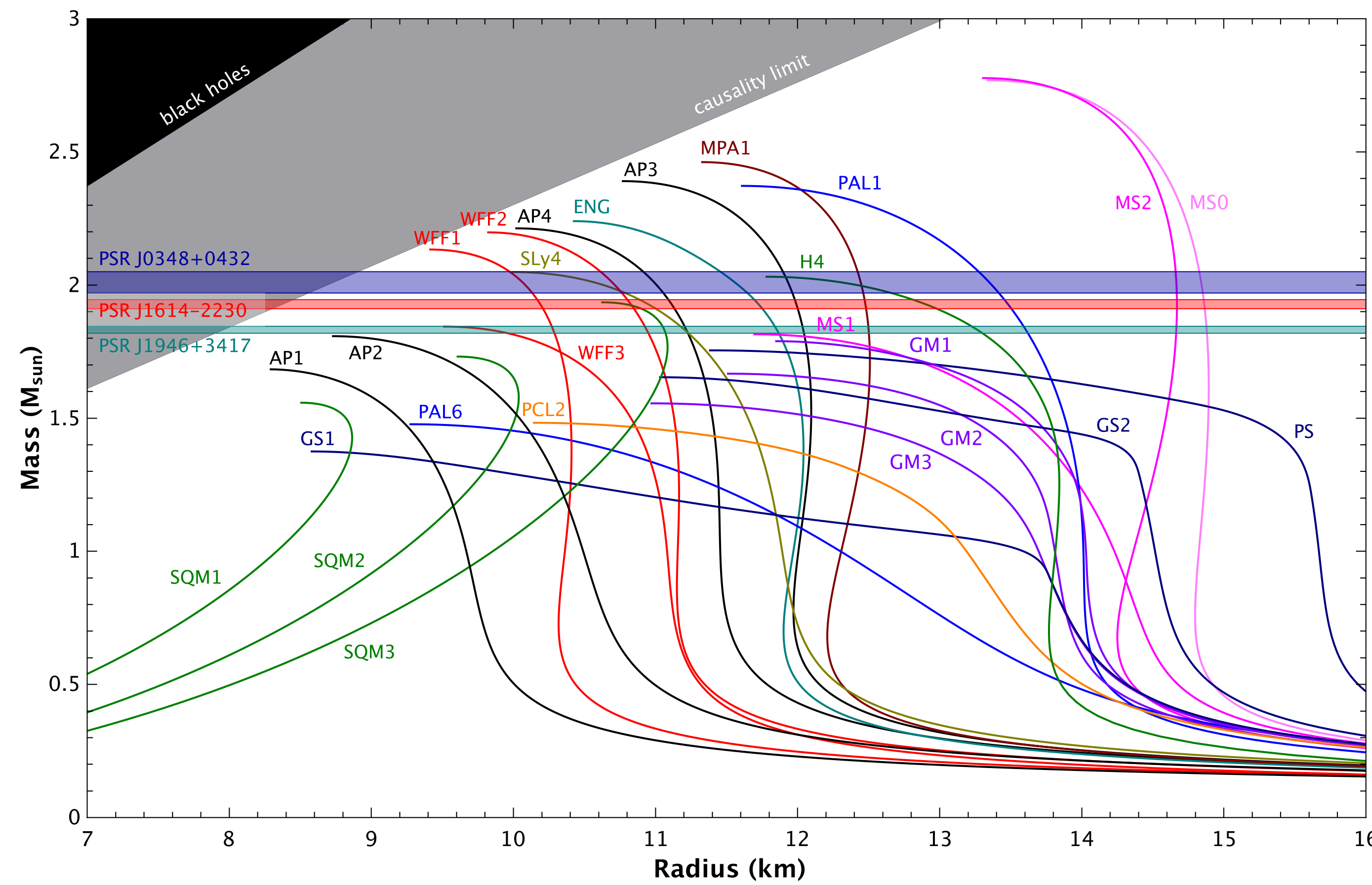
$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\frac{M^4\nu}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu)\log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots$$

Probing equation of state of neutron stars

Neutron Star:

- mass: 1-3 Msun
- radius: 9-15 km
- core density > 10^{14}g/cm^3

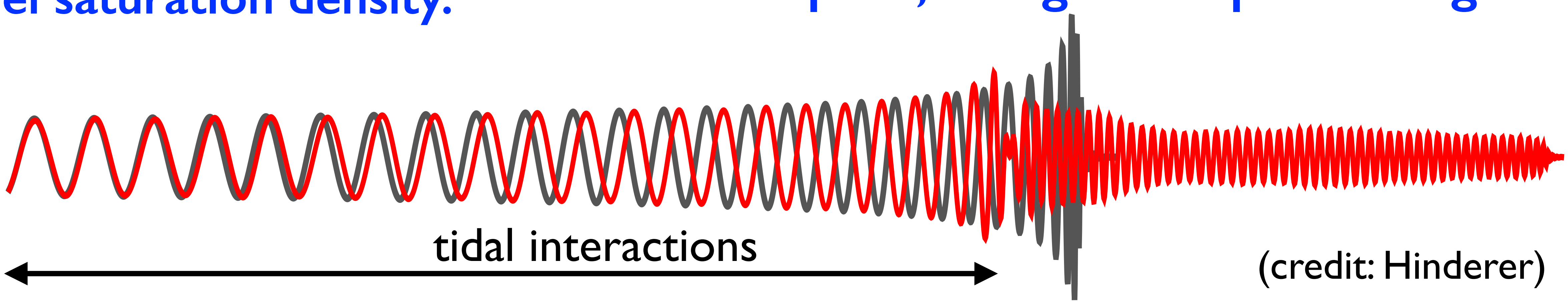




(Antoniadis et al. 16)

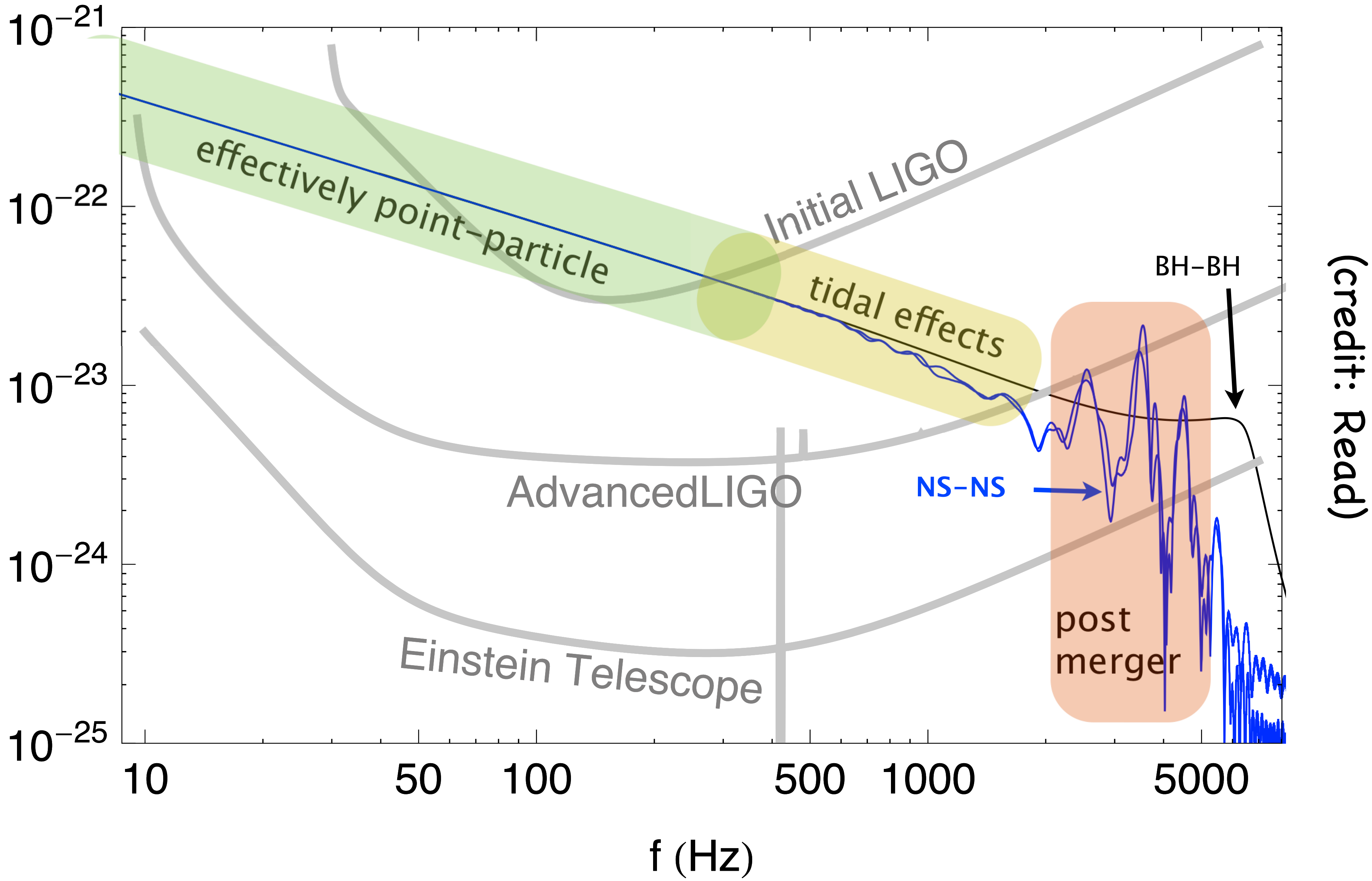
- **Terrestrial experiments** test EOS at **densities smaller** than **nuclei saturation density.**

- **NS equation of state** (EOS) affects gravitational **waveform** during **late inspiral, merger** and **post-merger**.



(credit: Hinderer)

Probing equation of state of neutron stars



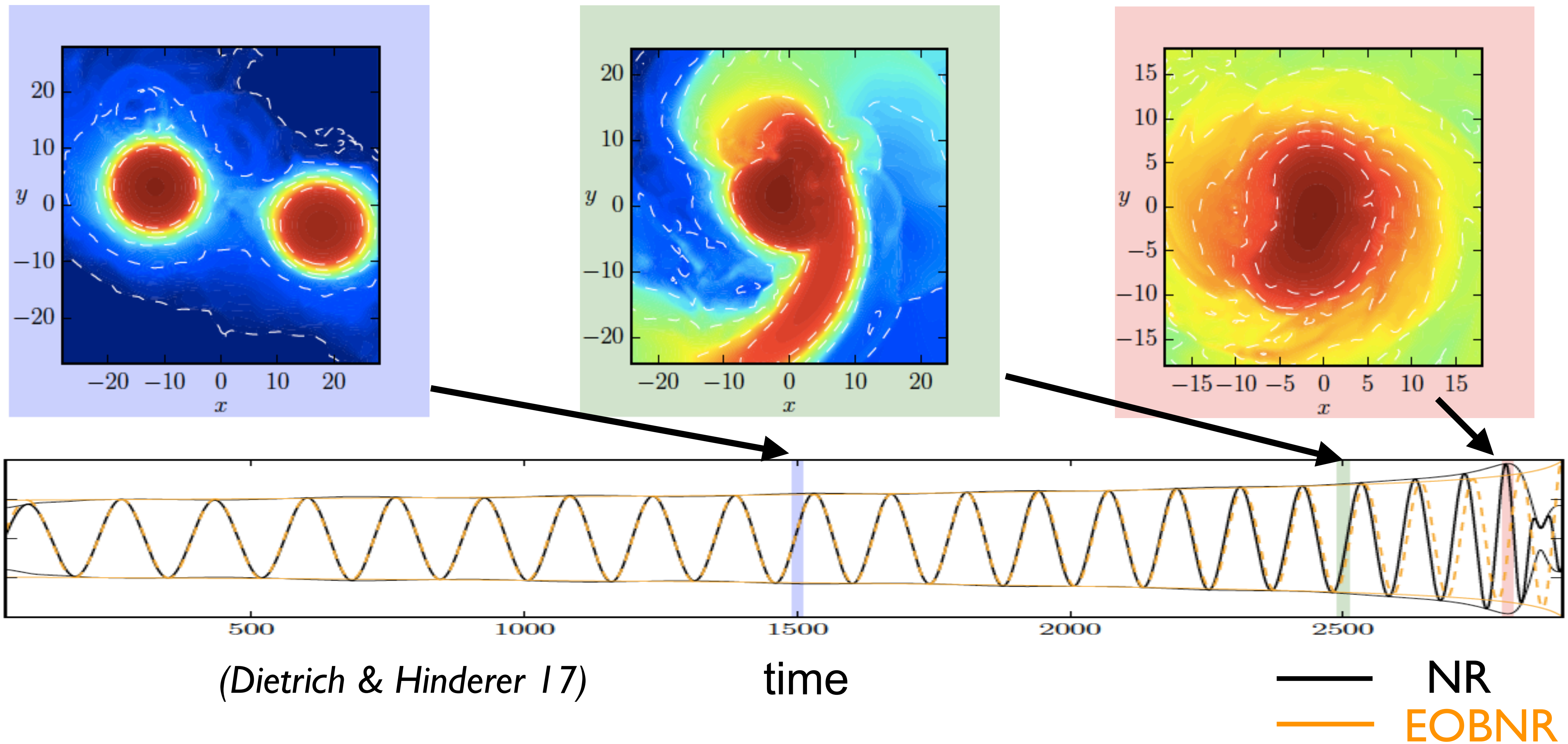
- **Tidal effects imprinted** on gravitational waveform during inspiral through **parameter** λ.

- λ **measures** star's **quadrupole deformation** in response to companion **perturbing tidal field:**

$$Q_{ij} = -\lambda\,\mathcal{E}_{ij}$$

State-of-art waveform models for binary neutron stars

- Synergy between **analytical** and **numerical work** is **crucial.**

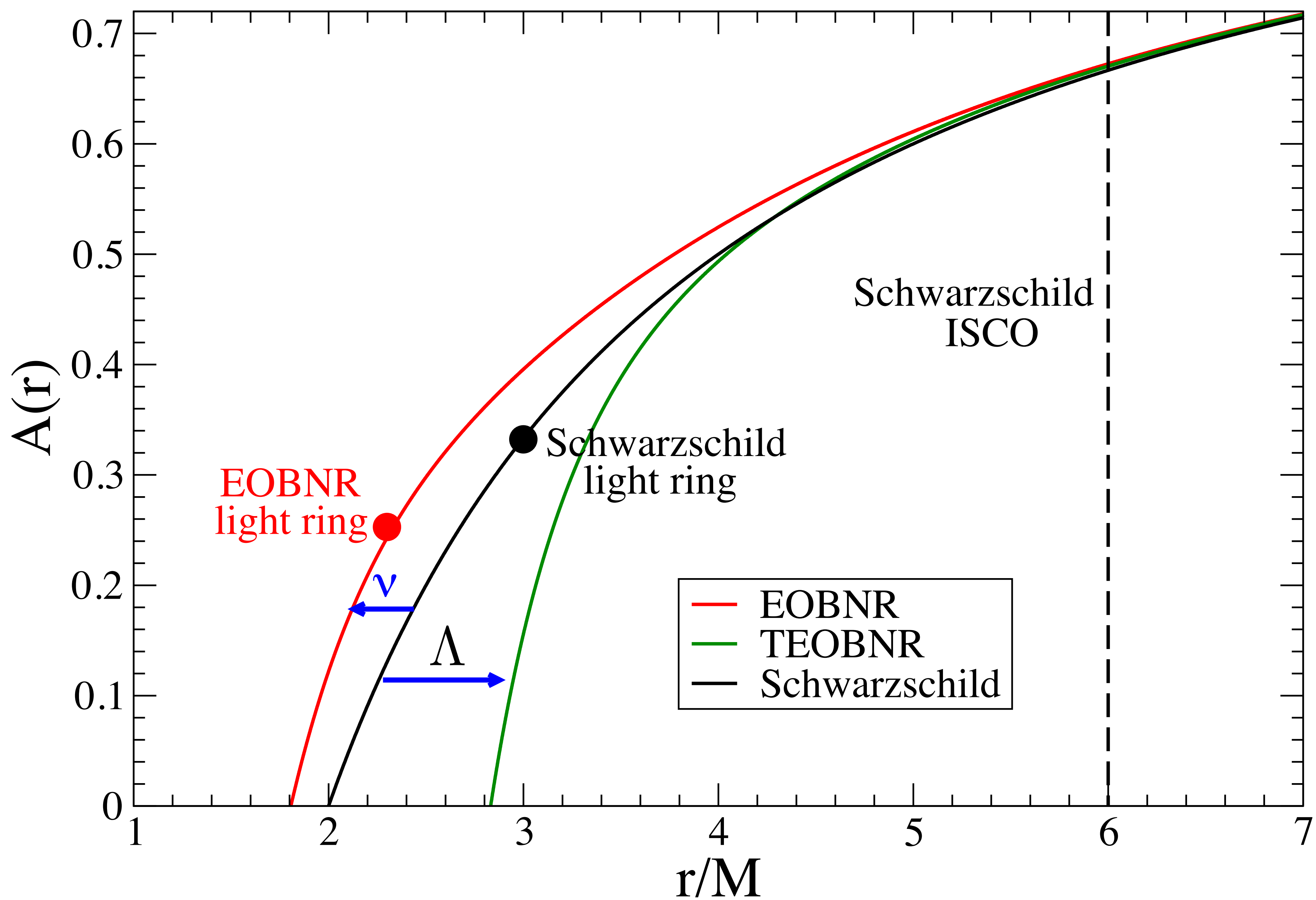


(Dietrich & Hinderer 17)

(Damour 1983, Flanagan & Hinderer 08, Binnington & Poisson 09, Vines et al. 11, Damour & Nagar 09, 12, Bernuzzi et al. 15, Hinderer et al. 16, Steinhoff et al. 16, Dietrich et al. 17-18, Nagar et al. 18)

Strong-field effects in presence of matter in EOB theory

$$A(r) = A_\nu(r) + A_{\text{tides}}(r)$$



(Hinderer et al. 2016, Steinhoff et al. 2016, see also Bernuzzi et al. 15)

Tides make **gravitational** interaction **more attractive**

PN templates for compact-object binary inspirals

$$\tilde{h}(f) = \mathcal{A}_{\rm SPA}(f)\, e^{i\psi_{\rm SPA}(f)}$$

$$\mathcal{M} = \nu^{3/5}\, M$$

0PN

$$\psi_{\rm SPA}(f) = 2\pi f t_c - \Phi_c - \pi/4 + \frac{3}{128}(\pi\mathcal{M}f)^{-5/3}\,\{1+$$

$$-\frac{5\hat{\alpha}^2}{336\omega_{\rm BD}}\,\nu^{2/5}\,(\pi\mathcal{M}f)^{-2/3} - \frac{128}{3}\,\frac{\pi^2 D\,\mathcal{M}}{\lambda_g^2\,(1+z)}\,(\pi\mathcal{M}f)^{2/3}$$

$$+\left(\frac{3715}{756}+\frac{55}{9}\nu\right)\nu^{-2/5}\,(\pi\mathcal{M}f)^{2/3} - 16\pi\,\nu^{-3/5}\,(\pi\mathcal{M}f) + 4\beta\,\nu^{-3/5}\,(\pi\mathcal{M}f)$$

$$+\left(\frac{15293365}{508032}+\frac{27145}{504}\nu+\frac{3085}{72}\nu^2\right)\nu^{-4/5}\,(\pi\mathcal{M}f)^{4/3} - 10\sigma\,\nu^{-4/5}\,(\pi\mathcal{M}f)^{4/3}$$

$$\cdots - \frac{39}{2}\,\nu^{-2}\,\tilde{\Lambda}\,(\pi\mathcal{M}f)^{10/3}\Big\}$$

-1PN: dipole radiation

1PN: graviton with non zero mass

1PN

1.5PN

1.5PN: spin-orbit

2PN

2PN: spin-spin

5PN: tidal

$$\tilde{\Lambda} = \frac{16}{13}\,\frac{(m_1+12m_2)m_1^4\Lambda_1 + (m_2+12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

$$\Lambda = \frac{\lambda}{m_{\rm NS}^5} = \frac{2}{3}k_2\left(\frac{R_{\rm NS}c^2}{Gm_{\rm NS}}\right)^5$$

k_2: Depends on EOS & compactness

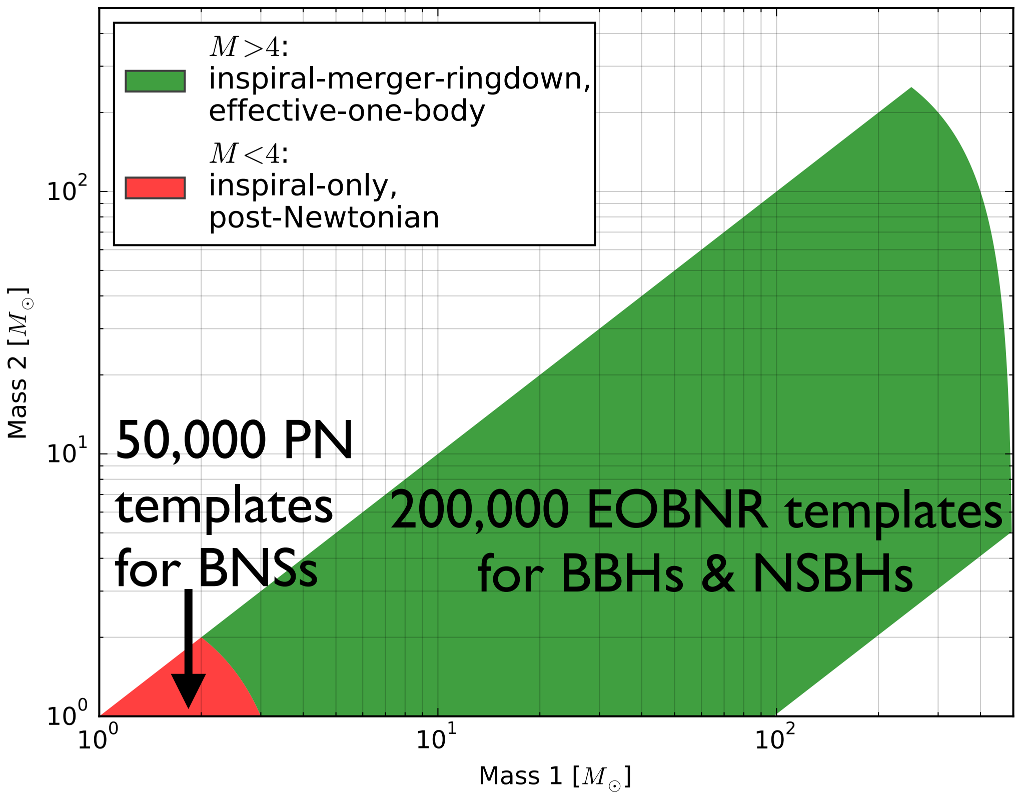
$\left(\frac{R_{\rm NS}c^2}{Gm_{\rm NS}}\right)^5$: it can be large

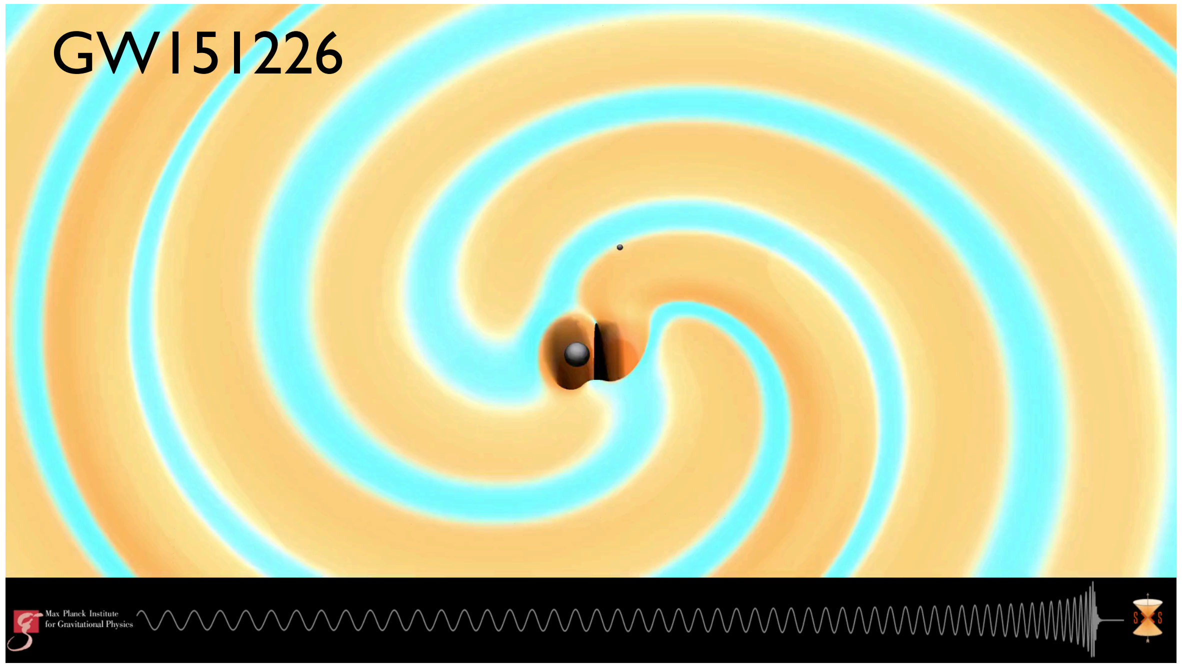
Template bank for modeled search & possible systematics

- **Matched filtering** employed

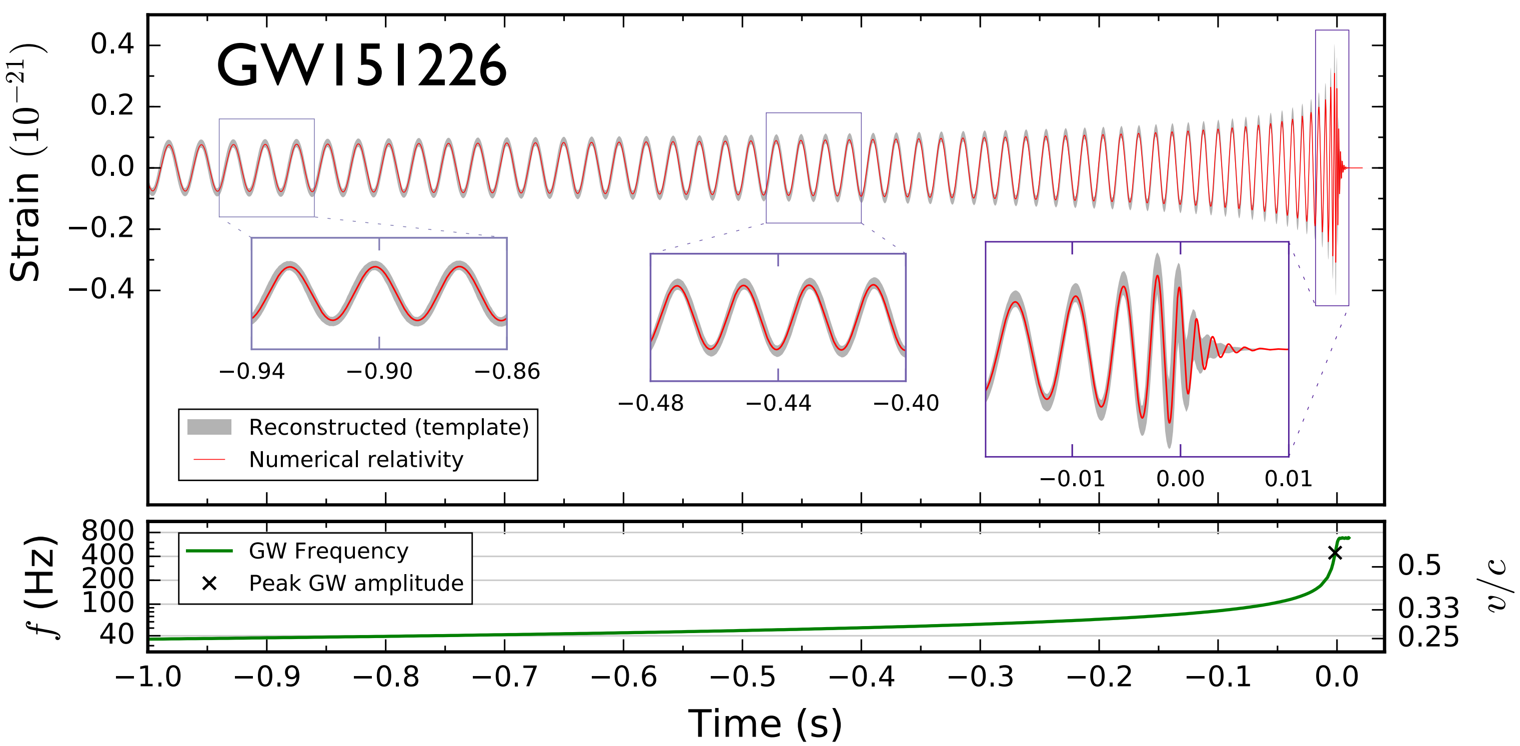
(visualization credit: Dietrich, Haas @AEI)
(Ossokine, AB & SXS project)

(Dal Canton & Harry 17)





(Abbott et al. PRL 116 (2016) 241103)

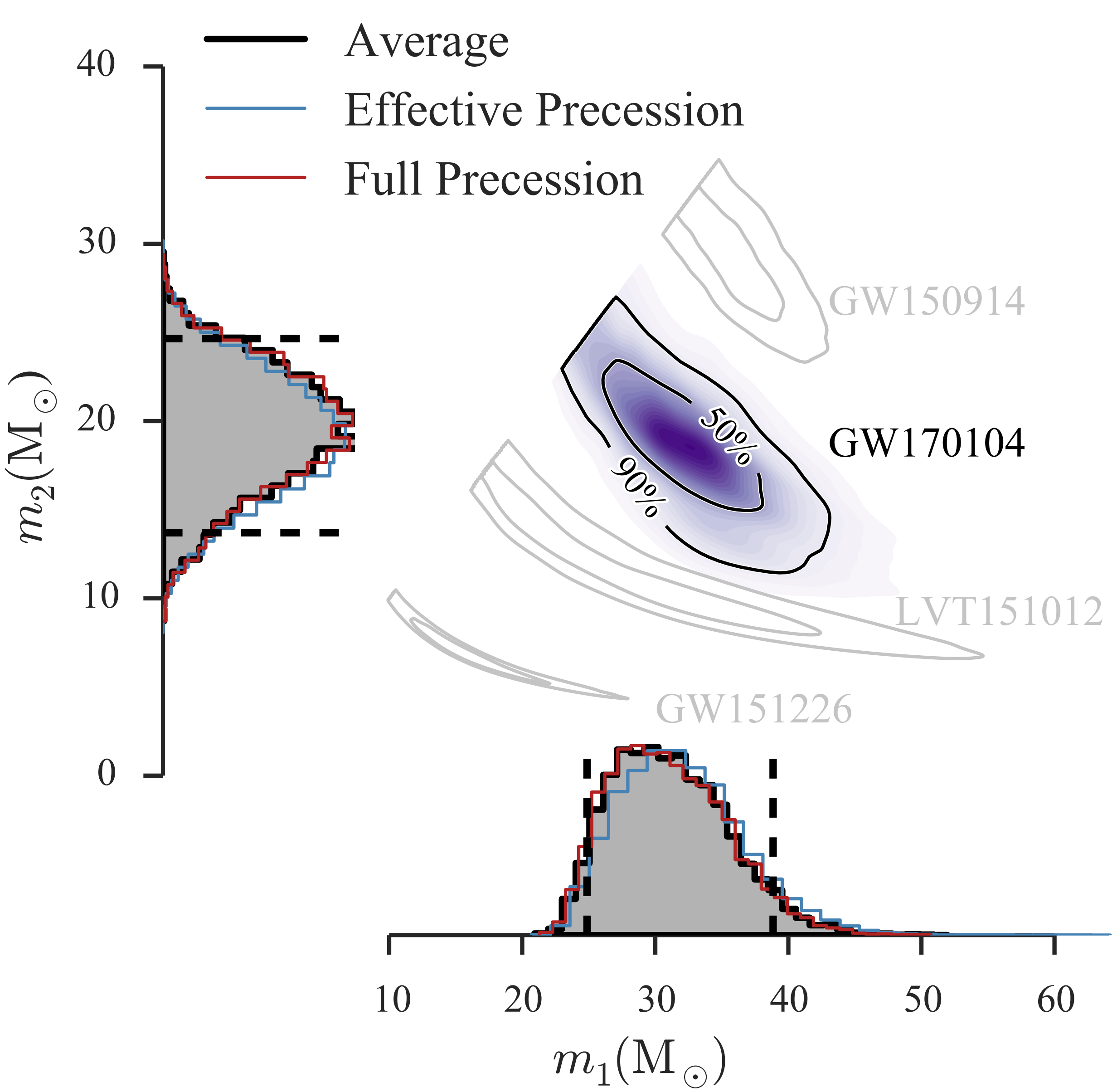


- **Systematics** due to modeling **are smaller than statistical** errors for GW events observed in **O1 & O2 runs**.

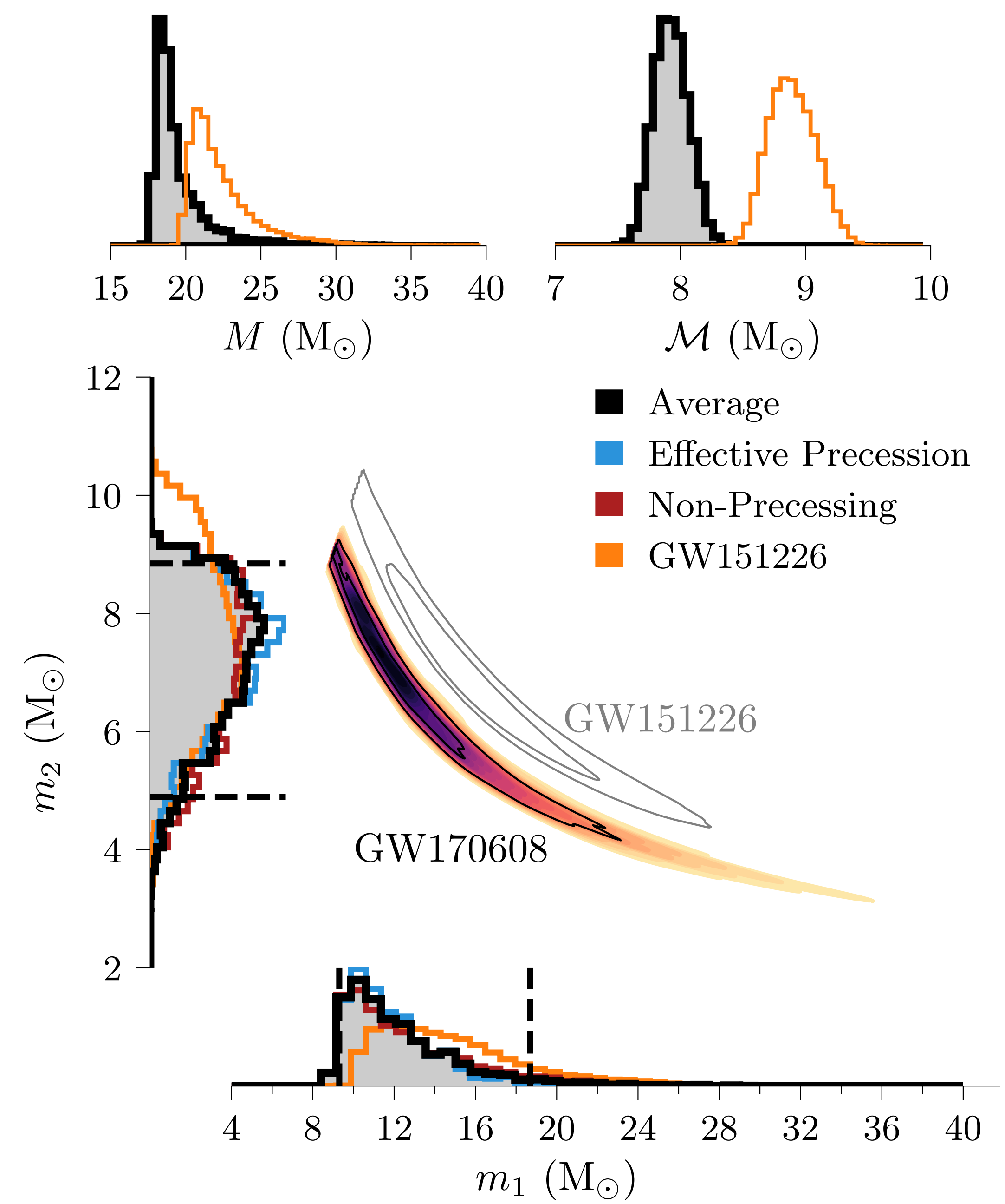
(see also Abbott et al. CQG 34 (2017) 104002)

Unveiling binary black-hole properties: masses

(Abbott et al. PRL 118 (2017) 221101)



(Abbott et al. ApJ 851 (2017) L35)



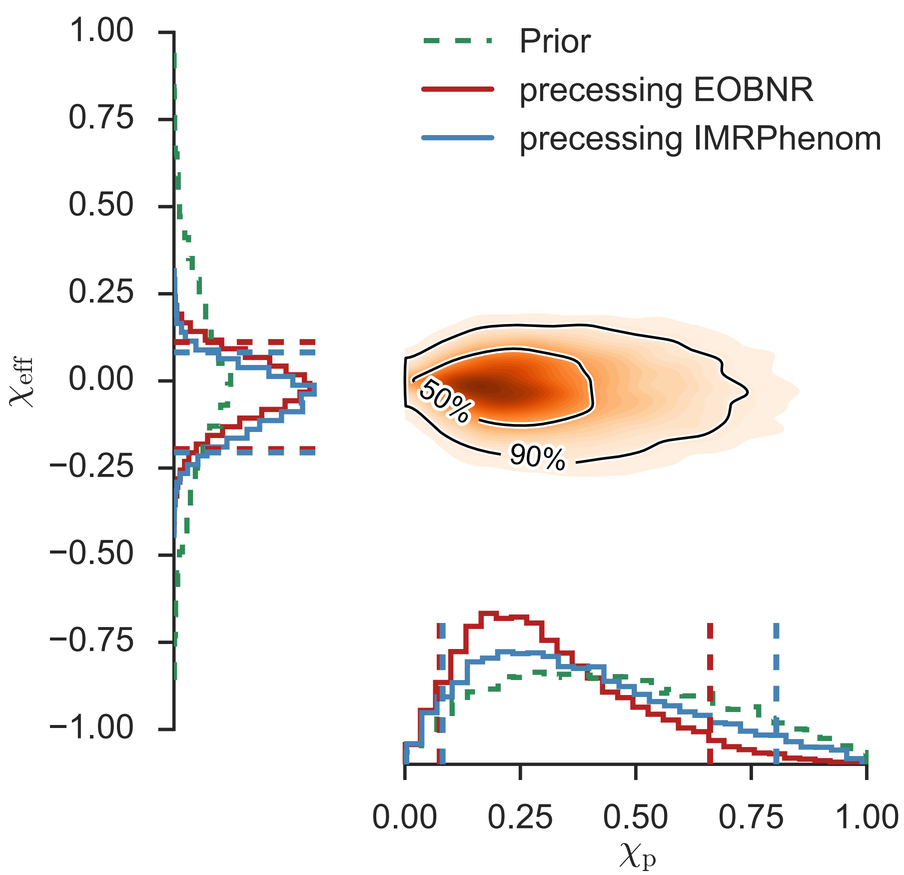
- **Chirp mass** is **best measured**. **Individual masses** can be **better measured** if **merger is observed**, because total mass is measured at merger.

Unveiling binary black-hole properties: spins

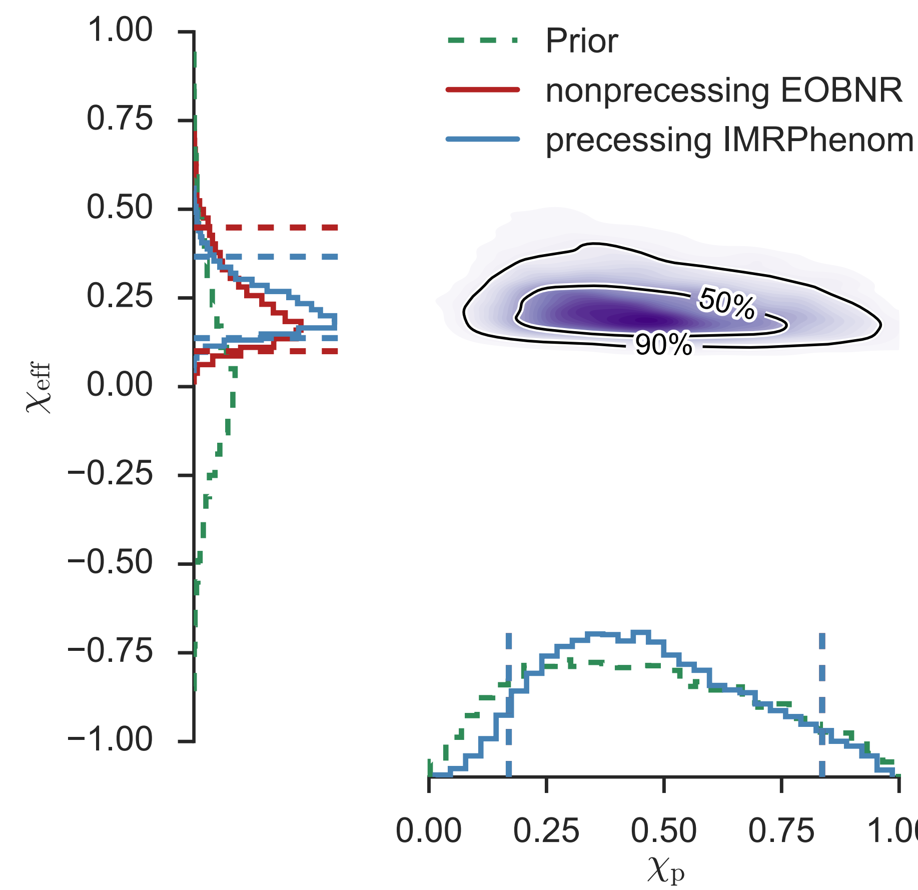
(measurements @ 25Hz)

(Abbott et al. PRX 6 (2016) 041014)

GW150914



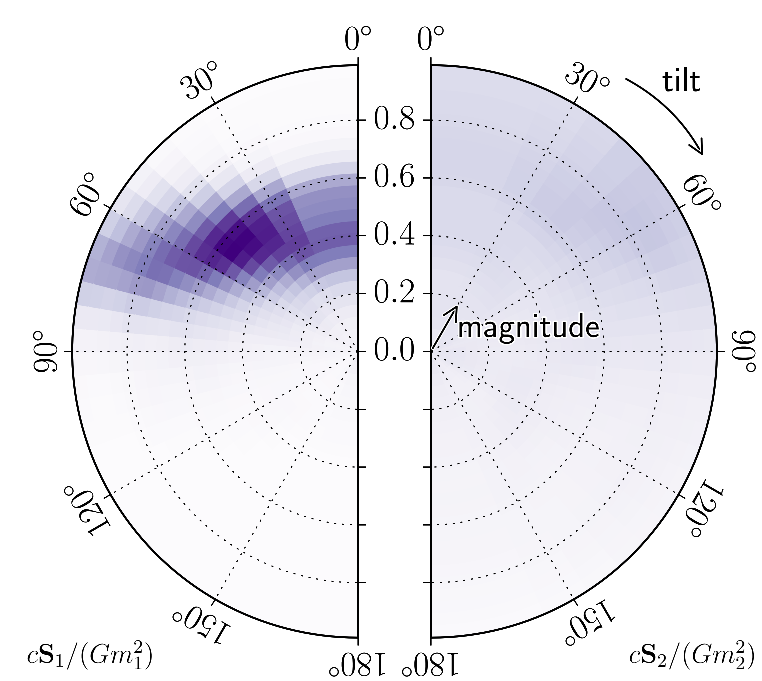
GW151226



(Abbott et al. PRL 116 (2016) 241103)

$$\chi_{\rm eff} = \frac{c}{GM}\left(\frac{\mathbf{S}_1}{m_1} + \frac{\mathbf{S}_2}{m_2}\right)\cdot\frac{\mathbf{L}}{|\mathbf{L}|}$$

(constant through 2PN order)

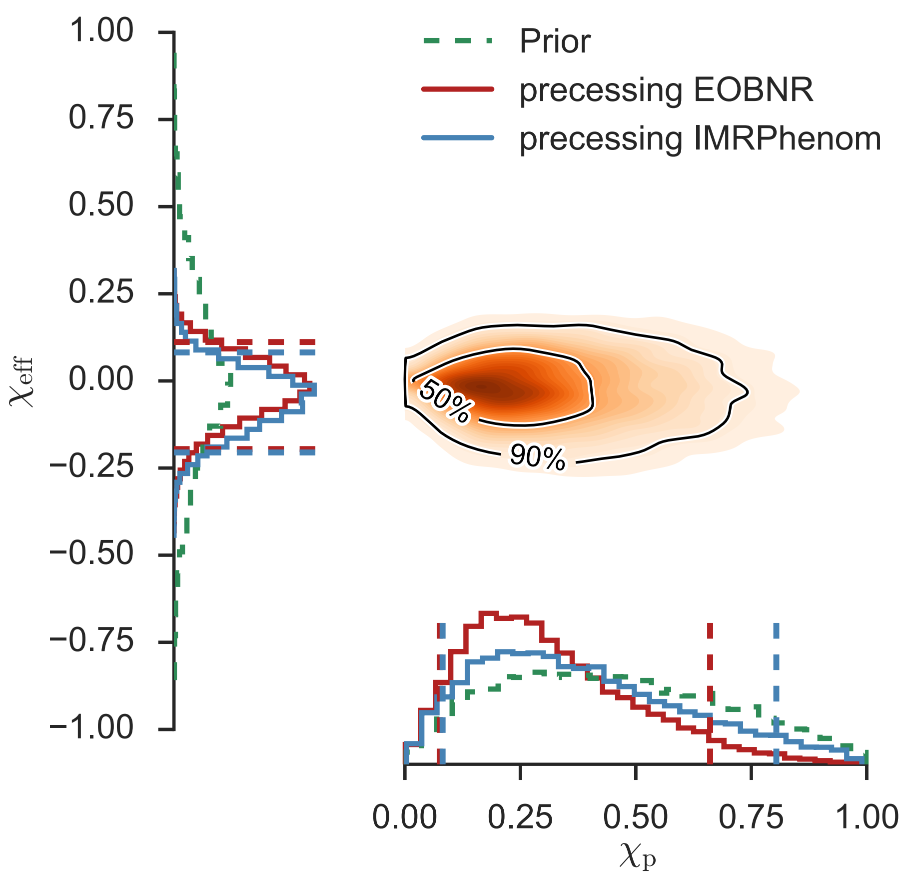


- BHs' **spins not maximal**, and for GW151226 **one BH's spin larger than 0.2** at 99% confidence.
- **Spins < 0.7**. **No information** about **precession**.

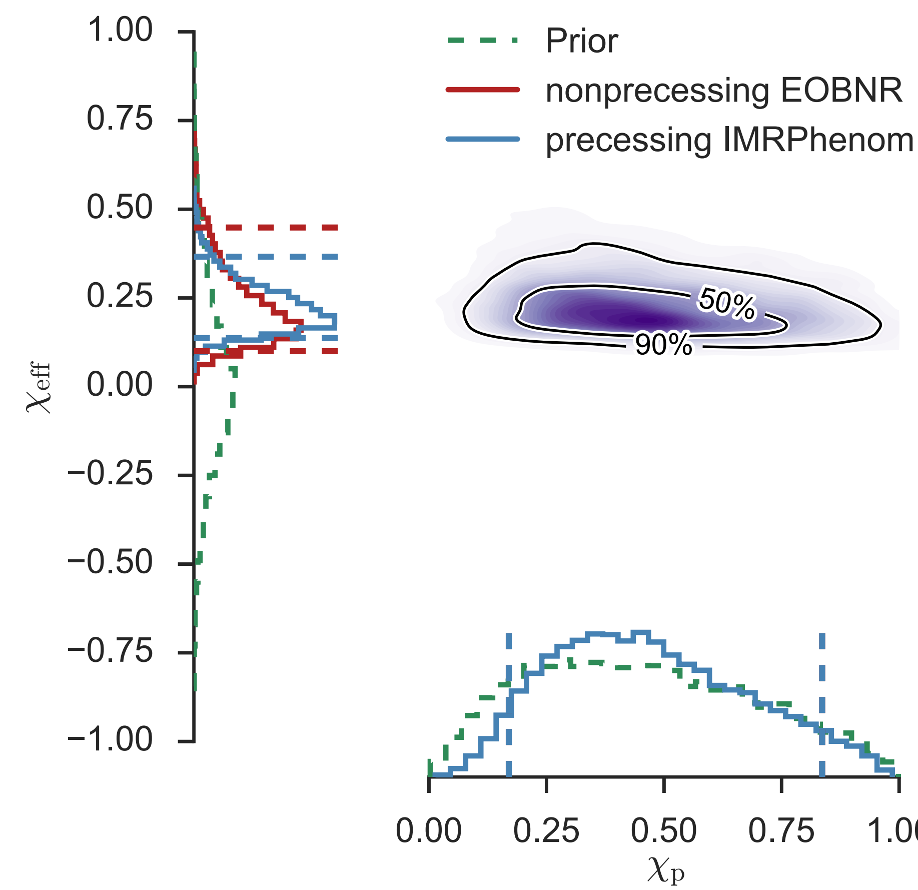
Unveiling binary black-hole properties: spins

(measurements @ 25Hz)

GW150914



GW151226

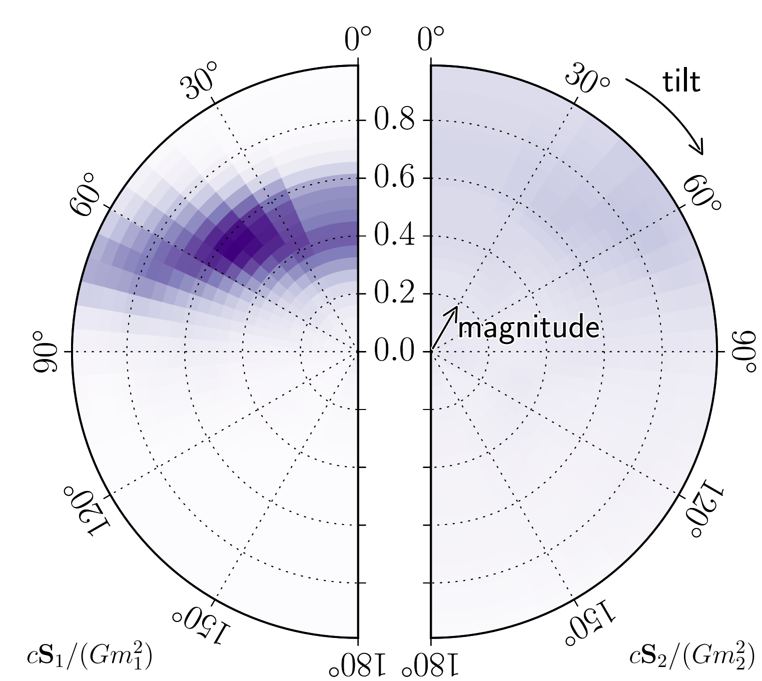


(Abbott et al. PRX 6 (2016) 041014)

(Abbott et al. PRL 116 (2016) 241103)

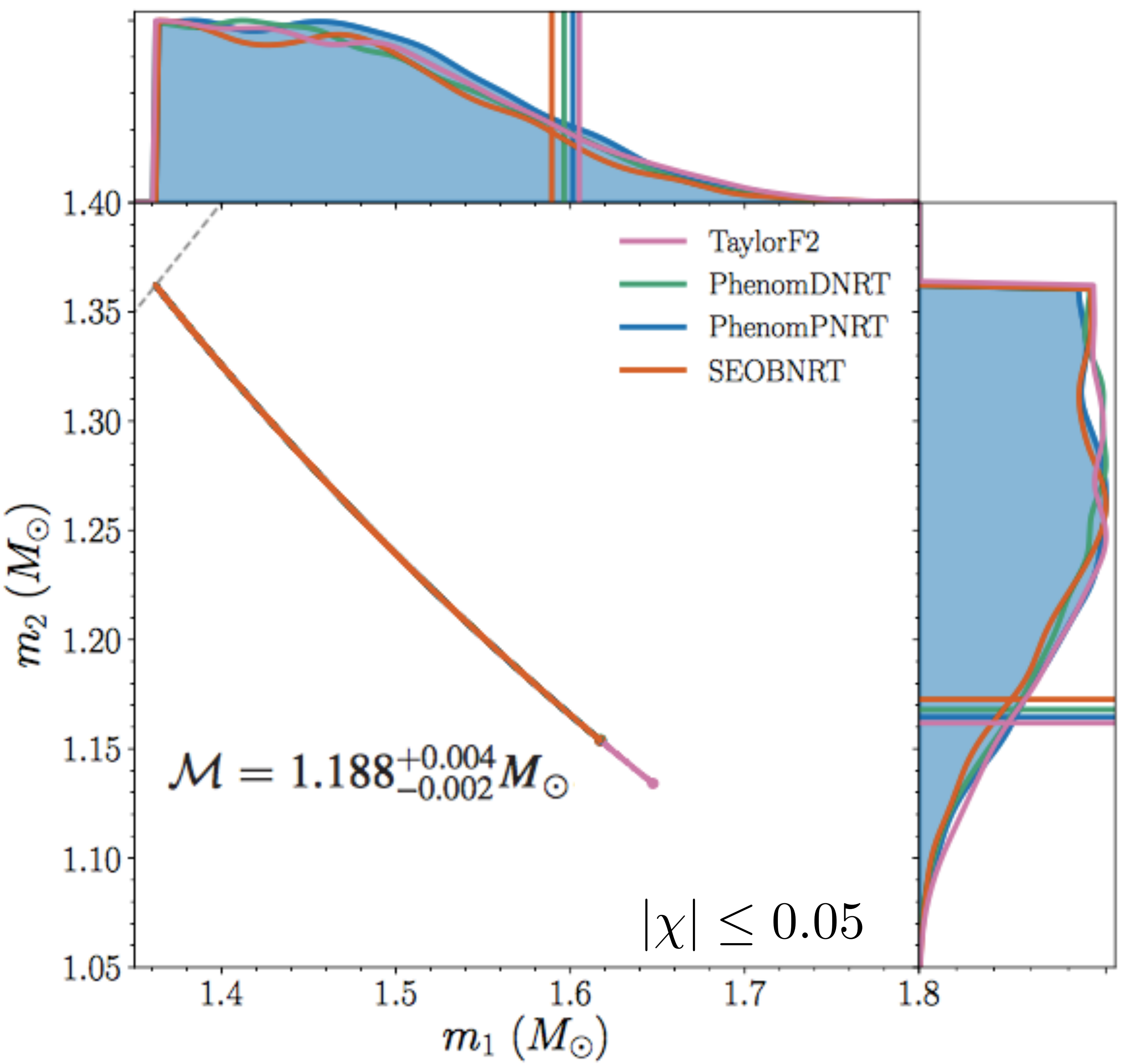
$$\chi_p = \frac{c}{B_1 G m_1^2} \max(B_1 S_{1\perp}, B_2 S_{2\perp})$$

- BHs' **spins not maximal**, and for GW151226 **one BH's spin larger than 0.2** at 99% confidence.
- **Spins < 0.7**. **No information** about **precession**.



Unveiling binary neutron star properties: masses

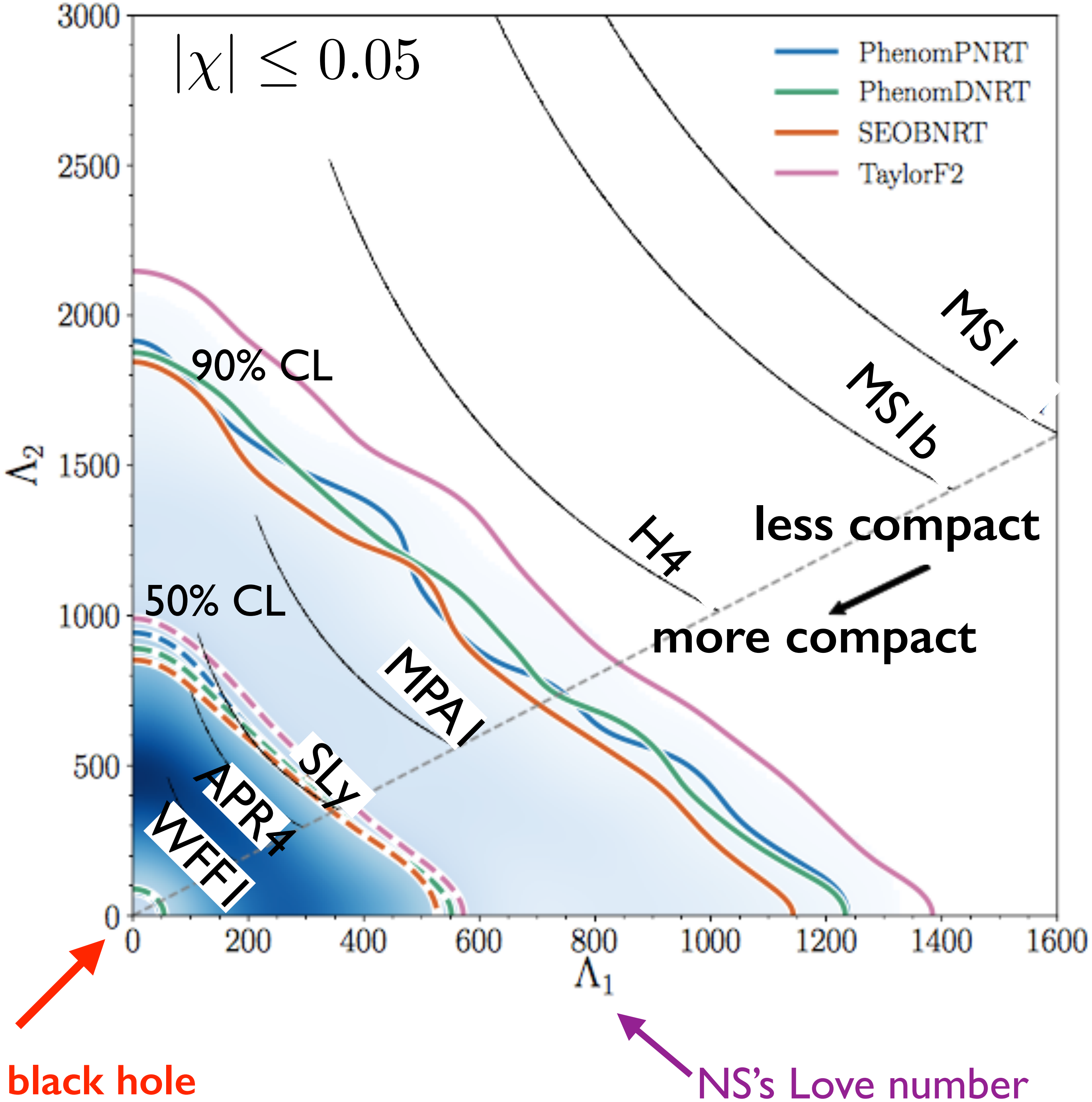
(Abbott et al. arXiv:1805.11579)



- **Degeneracy** between **masses and spins** limit their measured accuracy.
- **Fastest-spinning** neutron star has (dimensionless) **spin ~0.4.**
- Observation of **binary pulsars** in our galaxy indicate spins are **not larger than ~0.04**.
- Current **measurements** of NS masses **dominated by statistical** error.

Constraining NS equation of states with GW170817

(Abbott et al. arXiv:1805.11579)



Depends on EOS & compactness

$$\Lambda = \frac{\lambda}{m_{\rm NS}^5} = \frac{2}{3} k_2 \left(\frac{R_{\rm NS} c^2}{G m_{\rm NS}}\right)^5$$

- **Effective tidal deformability** enters **GW phase at 5PN order:**

$$\tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12 m_2) m_1^4 \Lambda_1 + (m_2 + 12 m_1) m_2^4 \Lambda_2}{(m_1 + m_2)^5}$$

$$\tilde{\Lambda} : 300^{+500}_{-190} \quad @ \text{ 90\% CL}$$

- Current **measurements** of tidal effects **dominated by statistical** error.

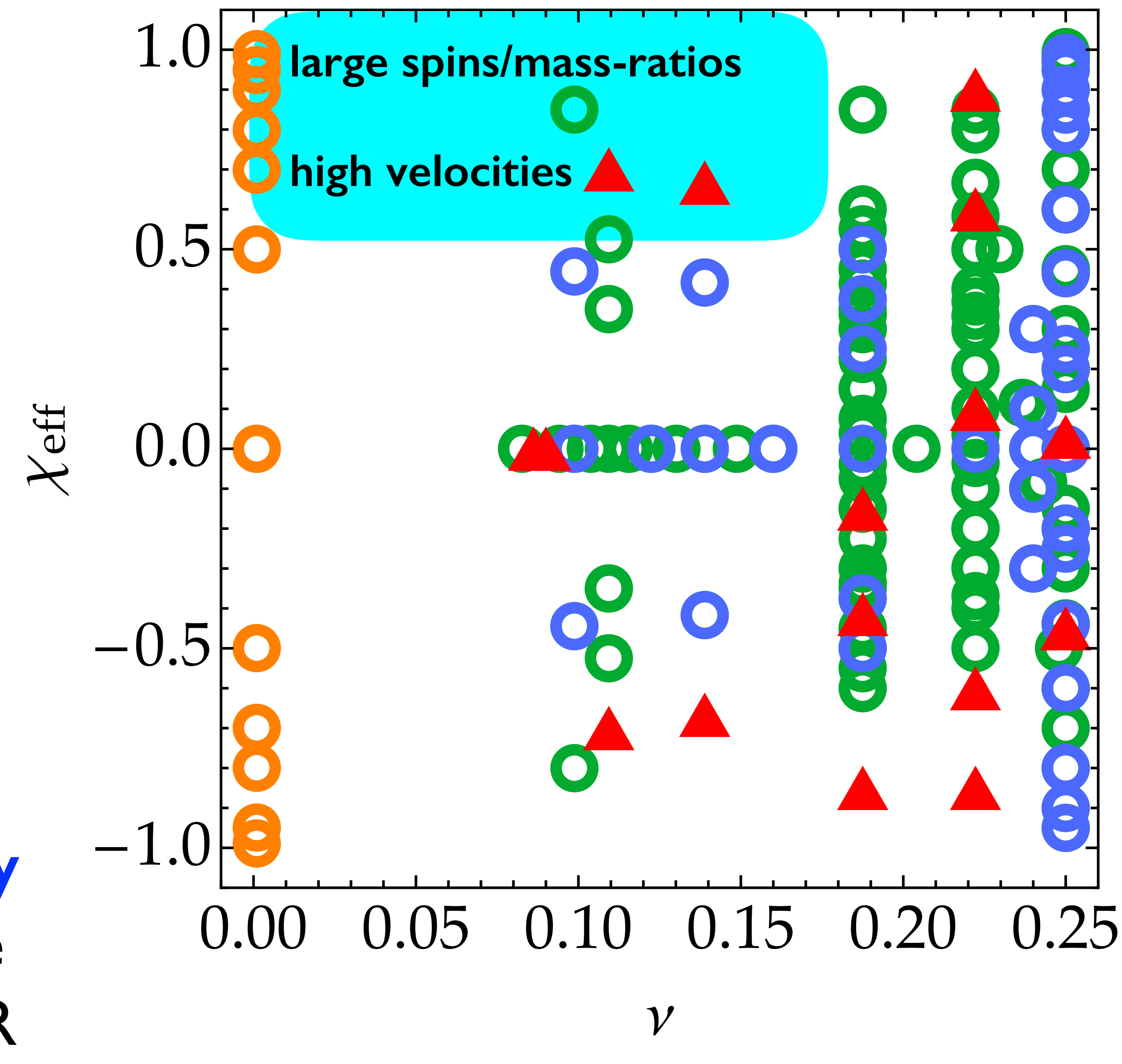
Extending waveform modeling in all binary parameter space

- Difficult to run **NR simulations** for **large mass ratios** (> 4) **& large spins** (> 0.8), with **large number** of GW **cycles** (> 50).

- For **large mass ratios and spins** crucial to **combine PN, GSF** and **NR** results in **EOB** framework.
  (Damour 09, Barausse, AB et al. 12, Le Tiec , … AB 12, Bini et al. 12-16, Antonelli et al. in prep)

- Perform **EOB internal consistency checks** to control **systematics** due to limited length and number of NR simulations.
  (Pan, AB et al. 14, Bohe' …AB et al. 16)

141 NR waveforms



(Bohe' …AB et al. 16)

- Compare with **other waveform** models. *(Khan et al. 16, Nagar et al. 15-18)*

Need more efficient ways to solve two-body problem, analytically

- In test-body limit, spinning EOB Hamiltonian includes **linear terms in spin of test body at all PN orders**.

(Barausse et al. 10, Barausse & AB 11, 12; Vines et al. 15)

$$(\sigma + \sigma^2)\left(1 + \frac{v^2}{c^2} + \cdots\right)$$

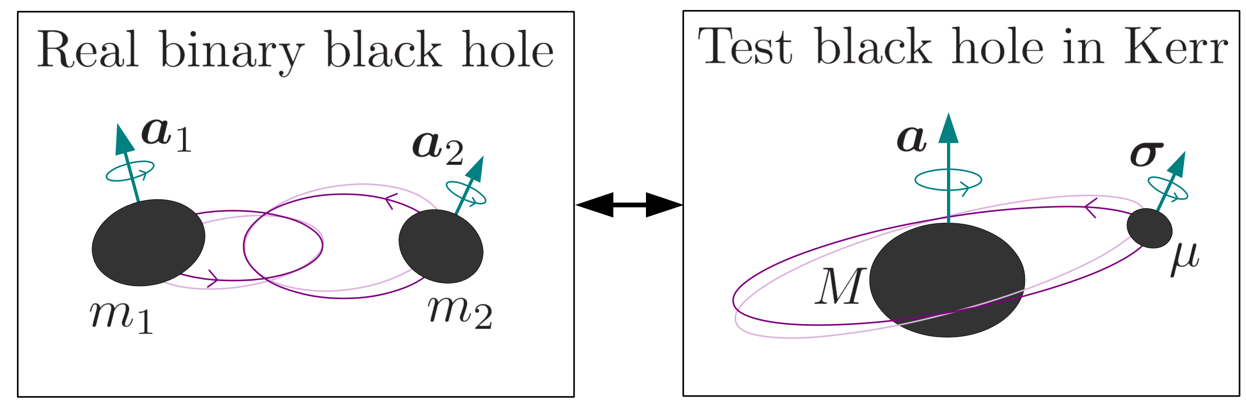
- Is EOB **mapping unique** at all orders?

$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu} - 1\right)}$$

Using **unbound orbits** and **scattering angle** as adiabatic invariant, **at 1PM: mapping unique** & 2-body relativistic motion equivalent to 1-body motion in Kerr. *(Damour 16, Bini et al. 17-18, Vines 17)*

$$\frac{Gm}{rc^2}\left(1 + \frac{v^2}{c^2} + \cdots\right)$$

$GM/rc^2 << v^2/c^2 \sim 1$



exact mapping at the leading PN orders

- Results at **leading PN order** but **all orders in spin.**

(Vines & Steinhoff 16; Vines & Harte 16, Siemonsen, Steinhoff & Vines17)

$$\frac{v^2}{c^2}\left(S_i + S_i^2 + \cdots\right)$$

On quantum-field theory methods in classical gravity & GWs

- **Modern scattering amplitude** methods of quantum fields/particles:
  - Bern-Carrasco-Johansson (BCJ) duality/ double copy *(Bern et al. 10, Monteiro et al. 15, Bjerrum-Bohr et al. 15, Luna et al. 16, 17, Goldberger & Ridgway 17, Goldberger, Li & Prabhu 17)*
  - Britto-Cachazo-Feng-Witten (BCFW) on-shell recursion relations/unitary methods *(Britto et al. 04, 05, Bern et al. 1994, 1995, Neil & Rothstein 13)*
  - Higher spin fields *(Vaidya 16; Guevara 17)*

- Can those techniques **be *really* more efficient** in solving two-body problem, including radiation?

- Those methods naturally **allow to obtain PM results** ($GM/rc^2 << v^2/c^2 \sim 1$). How to go **beyond perturbative** calculations? We are interested in strong-field regime ($GM/rc^2 \sim 1$).

- How to **reconstruct classical dynamics** from **quantum scattering amplitudes?** *(Damour 17, Bjerrum-Bohr et al.18)*

- How to **efficiently compute gravitational waveforms?**

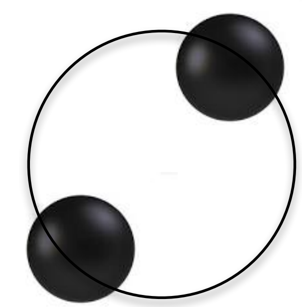
Gravitational waveforms built from conservative & dissipative dynamics

- GW from time-dependent **quadrupole moment**: $h_{ij} \sim \frac{G}{c^4}\frac{\ddot{Q}_{ij}}{R}$

$$h = \nu \left(\frac{GM}{c^2\,R}\right)\frac{v^2}{c^2}\cos 2\Phi$$

$$\frac{v}{c} = \left(\frac{GM\omega}{c^3}\right)^{1/3}$$

$$\nu = \mu/M$$



- Center-of-mass energy: $E(\omega)$

$$E(v) = -\frac{\mu}{2}v^2 + \cdots$$

- GW luminosity: $\mathcal{L}_{\rm GW}(\omega) \equiv F(\omega)$

$$F(v) = \frac{32}{5}\nu^2\frac{c^5}{G}\left(\frac{v}{c}\right)^{10} + \cdots$$

- Balance equation: $\frac{dE(\omega)}{dt} = -F(\omega)$ $\rightarrow$ $\dot{\omega}(t) = -\frac{F(\omega)}{dE(\omega)/d\omega}$

- Gravitational-wave **phase**: $\Phi_{\rm GW}(t) = 2\Phi(t) = \frac{1}{\pi}\int^t \omega(t')dt'$

PN versus PM expansion for conservative two-body dynamics

$$\mathcal{L} = -Mc^2 + \underbrace{\frac{\mu v^2}{2} + \frac{GM\mu}{r}} + \frac{1}{c^2}[\ldots] + \frac{1}{c^4}[\ldots] + \ldots$$

$$E(v) = -\frac{\mu}{2}v^2 + \cdots$$

non-spinning compact objects

		0PN	1PN	2PN	3PN	4PN	5PN	...
0PM:	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	...
1PM:		$1/r$	v^2/r	v^4/r	v^6/r	v^8/r	v^{10}/r	...
2PM:			$1/r^2$	v^2/r^2	v^4/r^2	v^6/r^2	v^8/r^2	...
3PM:				$1/r^3$	v^2/r^3	v^4/r^3	v^6/r^3	...
4PM:					$1/r^4$	v^2/r^4	v^4/r^4	...
...						...	...	

(credit: Justin Vines)

$$1 \to Mc^2, \qquad v^2 \to \frac{v^2}{c^2}, \qquad \frac{1}{r} \to \frac{GM}{rc^2}.$$

current known PN results — current known PM results — overlap between PN & PM results — unknown

- **PM results** *(Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)*

PN versus PM expansion for conservative two-body dynamics

spinning compact objects

linear in spins		0PN		1PN	$\frac{3}{2}$PN	2PN	$\frac{5}{2}$PN	3PN	$\frac{7}{2}$PN	4PN	$\frac{9}{2}$PN
spin[1]: 1PM:	$Gm\cdot$				va/r^2		v^3a/r^2		v^5a/r^2		v^7a/r^2
2PM:	$(Gm)^2\cdot$						va/r^3		v^3a/r^3		av^5/r^3
3PM:	$(Gm)^3\cdot$								va/r^4		v^3a/r^4
...											...

(credit: Justin Vines)

current known PN results — current known PM results — overlap between PN & PM results — unknown

- **PM results** *(Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)*

PN versus PM expansion for conservative two-body dynamics

spinning compact objects

non-linear in spins			0PN	1PN	$\frac{3}{2}$PN	2PN	$\frac{5}{2}$PN	3PN	$\frac{7}{2}$PN	4PN	$\frac{9}{2}$PN	5PN
spin^3:	1PM:	$Gm\,\cdot$							va^3/r^4		v^3a^3/r^4	
	2PM:	$(Gm)^2\,\cdot$									va^3/r^5	
	$\cdots$											
spin^4:	1PM:	$Gm\,\cdot$								a^4/r^5		v^2a^4/r^5
	2PM:	$(Gm)^2\,\cdot$										a^4/r^6
	$\cdots$											

(credit: Justin Vines)

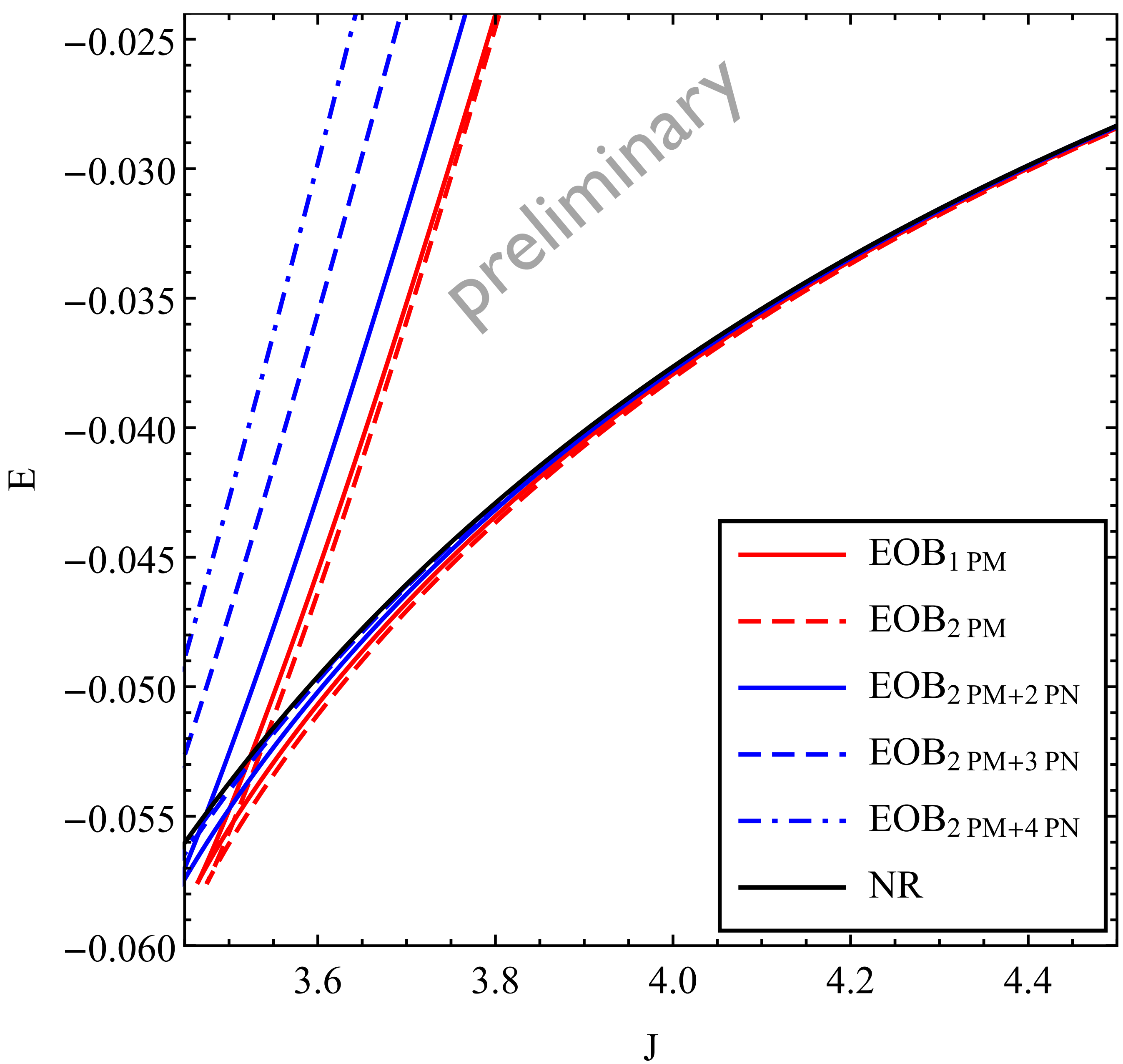
current known PN results | current known PM results | overlap between PN & PM results | unknown

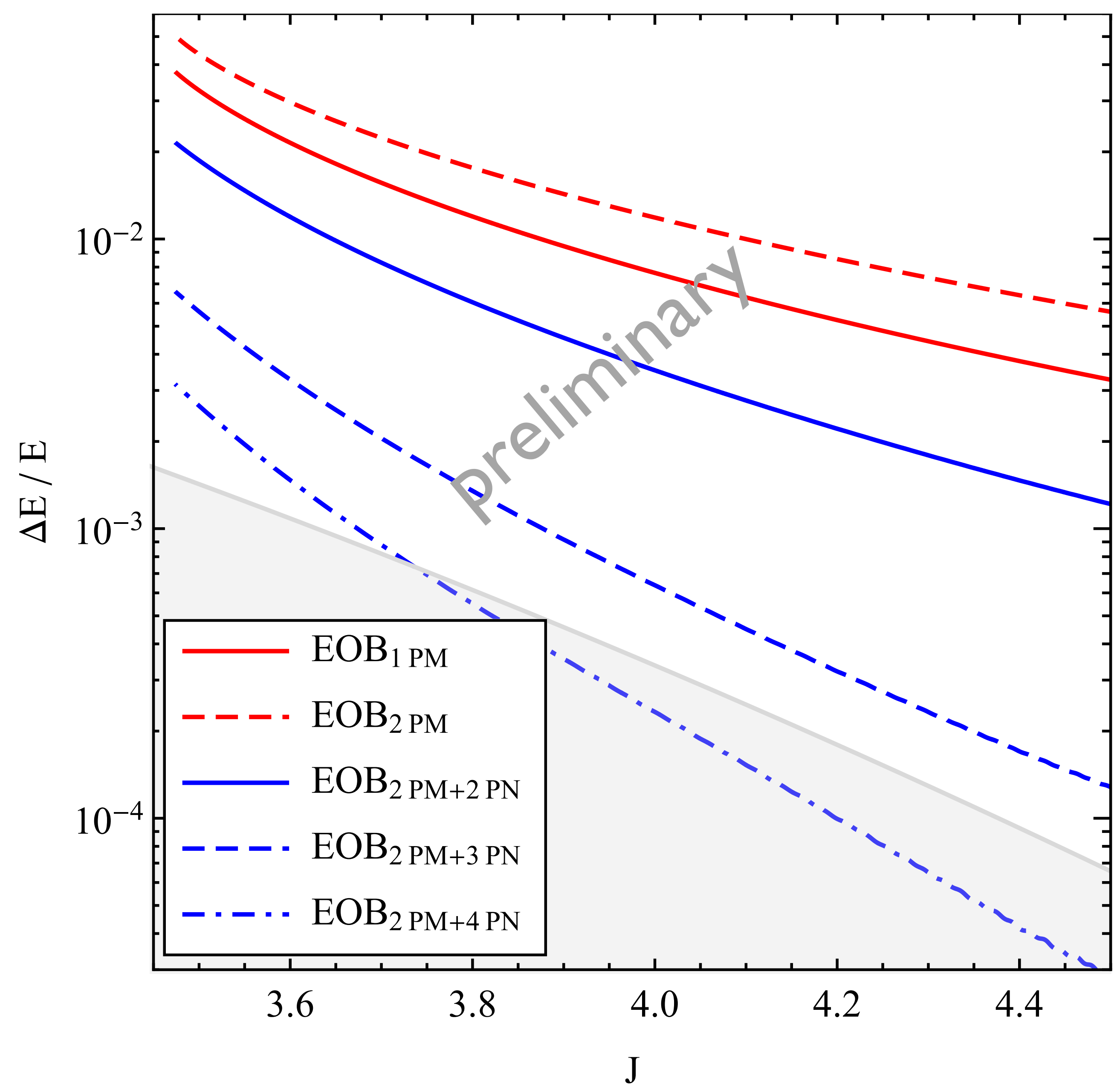
- **PM results** *(Westfahl 79, Westfahl & Goller 80, Portilla 79-80, Bel et al. 81, Ledvinka et al. 10, Damour 16-17, Guevara 17, Vines 17, Bini & Damour 17-18, Vines in prep)*

Binding energy at 2PM in EOB theory: comparison to NR

(Antonelli, van de Meent, … AB et al. in prep)

non-spinning, equal mass BBH





- Accurate **NR data** (*Ossokine & Dietrich 17*)
- Using **a 2PM EOB Hamiltonian** (*Damour 17*)
- Crucial to **complete 2PM** results **with PN** terms for **bounded orbits** to improve accuracy.
- **Important** to compute **3PM**.

~~A wish-list of precision calculations which could have "real" phenomenological impact~~

Note 1: Until we have **new results** to check against **current analytical waveform models** and against **numerical-relativity computations**, it is **not possible to understand** the **"real" phenomenological impact.**

Note 2: To have "real" phenomenological impact, we **need conservative** and **dissipative results** (i.e., also **waveforms**).

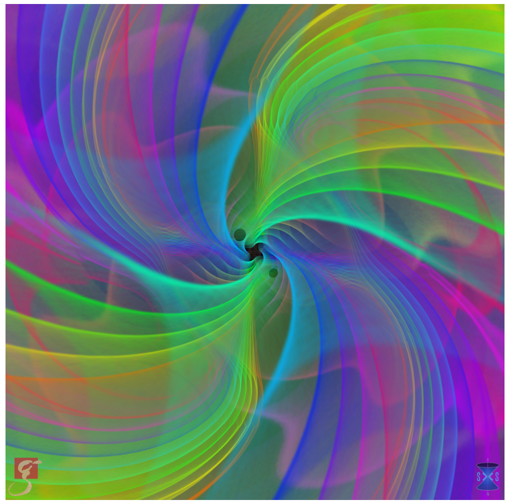
Possible problems to tackle:

- Scattering amplitudes at 2 loops [**3PM ($GM/rc^2 << v^2/c^2 \sim 1$)**] for **non-spinning** and **then spinning** particles [black holes].

- Scattering amplitudes at 1 loop [**2PM ($GM/rc^2 << v^2/c^2 \sim 1$)**] for particles endowed with multipole moments [neutron stars].

- **Non-perturbative [$GM/rc^2 \sim v^2/c^2 \sim 1$]** scattering amplitude from "condensates" [strongly curved spacetimes] ???

The new era of precision gravitational-wave astrophysics

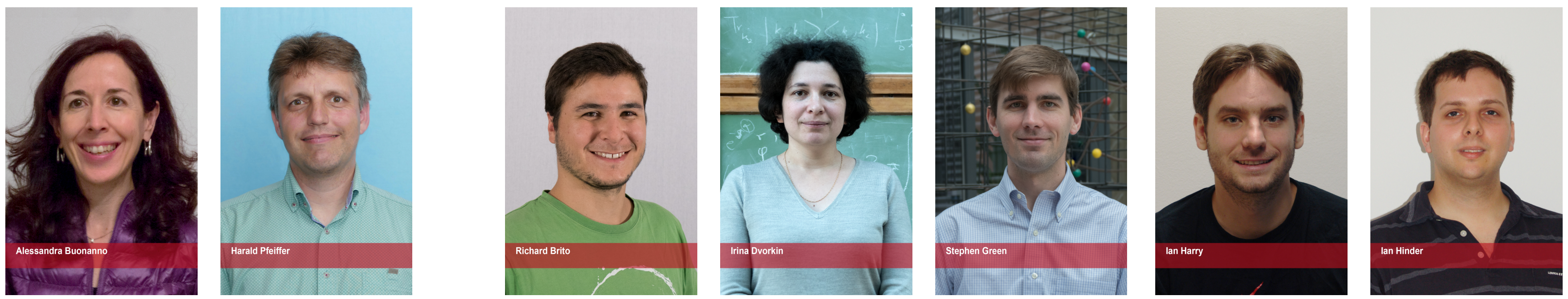


- Theoretical groundwork in **analytical and numerical relativity** has allowed us to build **faithful waveform models** to **search** for signals, **infer properties** and **test GR**.

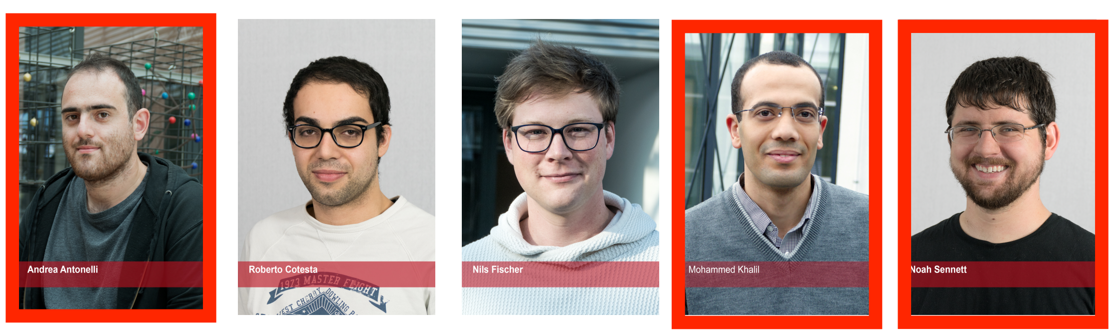
- We can now **learn about gravity** in the genuinely **highly dynamical, strong field** regime.
- We can now **unveil properties of neutron stars** unaccessible in other ways.
- **To take full advantage of discovery potential** in next years and decades **we need** to continue to make **precise theoretical predictions.**
- **Analytical relativity** work **still needed** to cover the **entire parameter space.** New **opportunities** for **theoretical physicists to contribute!**

“Astrophysical & Cosmological Relativity” Department

- **Current members**



Marsat   Steinhoff   Vines   van de Meent



Antonelli   Khalil   Sennett

- **Past members contributed to work presented**

Hinderer