Diagrammatic Approach to the Two-Body Dynamics & PostNewtonian Corrections

Pierpaolo Mastrolia

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based on: **Foffa, PM, Sturani, Sturm, PRD95 (2017) 10, 104009**

in collaboration with: **- Foffa, Sturani, Sturm, Cristofoli, Torres-Bobadilla**

Physics and Astronomy Department *Galileo Galilei* University of Padova - Italy

Outline

- Effective Field Theory General Relativity Feynman Rules
- Post-Newtonian expansion
- \bullet Amplitudes @ 4PN O(G^5)
- Feynman Integrals in Dimensional Regularisation
	- Untegration-by-parts identities
	- Master Integrals
	- Dimensional Recurrence Relations
	- Differential Equations
- **C**Results
- \bullet Toward the Potential @ 5PN O(G^6)
- Conclusions/Outlook

Motivations

PDescribing the Two-body dynamics at high precision

 \bullet Providing an independent calculation of the 4PN-O($G \wedge 5$) Lagrangian

 \bullet Providing a new calculation of the 5PN-O($G^{\wedge}6$) Lagrangian

Broadening the application range of Multi-loop Amplitudes/High-Energy Computational Tools

- **>> A. Buonanno**
- **>> C. Cheung**
- **>> D. O'Connell**
- **>> S. Caron-Huot**

Length scales in a Binary System ary : **1. Tength scales in a Binary Sy** S vsto *M*[*d*] *n* ngth scales in a Binary System In general, *n* MIs obey a system of 1st ODE 2. EFT

$$
\boxed{r_\star << r} << \lambda_{GW}
$$

 $r_{\star} << r$ $<< \lambda_{GW}$ conservative system \cdots $\left\{ r_{\star} << r \right\} << \lambda_{GW}$ Conservative system ::

$$
G_N \frac{m}{r} \sim v^2 \ll 1
$$
expansion parameter

Effective Field Theory t_{rel} $\mathbf{E} \cdot \mathbf{E} = \mathbf{E} \cdot \mathbf{E} \cdot \mathbf{E}$ ective

Goldberger, Ross, Rothstein-Hilbert on

$$
\sum_{m_{\alpha}} \text{Einstein-Hilbert}
$$
\n
$$
S_{tot}[x,h] = S_{ij} \delta_{\theta} \left[\frac{1}{2} \delta_{\theta} \frac{1}{2} \delta_{\
$$

$Fevn$ 2*k*² nan Rules **Feynman Rules Propagators** and P

Feynman Ru r<mark>j kl</mark> *Spp* = *m^a d^a x*˙ *^u ^ax*˙ *^v ^bguv*(*xa*) (4.42) *Spp* = *m^a* Z *d^a e* щ ¹ *^e*(2+*cd*)

 \overline{p} and \overline{p} and \overline{p}

Interactions

fields, we have a divergent term that can be interpreted as the emission of a tensor of a tensor of a tensor of

$$
\Lambda^{-1} = \left(32\pi G_N\right)^{\frac{1}{2}}
$$

r (2.5) *r (2.5) <i>r* (2.5) *r* (2.5) *r* (2.5) *r* (2.5) *r* (2.5) *r*

✓

Newton Potential

Post-Newtonian Corrections C fi ²*k*² (2.2)

Post-Newtonian expansion

n-th order correction

$$
\mathcal{O}(G_N^k, v^{2m}), \qquad n = k + 2m - 1
$$
\nFind

$$
S_{eff}[x_1, x_2] \supset -i \int_{t_1, t_2} \left(\frac{-im_1 V_{\phi}(t_1)}{m_p} \right) \left(\frac{-im_2 V_{\phi}(t_2)}{m_p} \right) \langle \phi(t_1, x_1) \phi(t_2, x_2) \rangle
$$

=
$$
\frac{32\pi G m_1 m_2}{8} \int_{t_1, t_2, k} \frac{1}{k^2 - \partial_{t_1} \partial_{t_2}} V_{\phi}(t_1) V_{\phi}(t_2) e^{ik.(x_1(t_1) - x_2(t_2))} \delta(t_1 - t_2)
$$

4-PN (595 diagrams)

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1 of 2 12/11/09 12:00 12:00 12:00 12:00 12:00 12:00 12:00 12:00 12:00 12:00 12:00 12:00 12:00 12:00 12:00 12:0

4-PN (595 diagrams)

courtesy of Foffa & Sturani

Amplitudes @ 4PN - O(G^5)

Foffa, Sturani, Sturm, & **P.M.**

50 Amplitudes @ 4-loops

I*Irom* **Amplitudes to Lagrangian** Finalge to Lagrangian **Lagrangian** iphtudes to Lag

In FET. GR Diagrams vs 2-noint OFT Diagrams propagate, any gravity-amplitude of order *G*` *^N* can be mapped into an (` 1)-loop 2-point **EFT-GR Diagrams** *vs* 2-point QFT Diagrams \mathbb{I} in graphic general, since the sources (since the sources (black lines) and do not static and do not so not

Dimensionally Regulated Integrals

Graph Topology & Integrals

N = # scalar products (of types *qⁱ · p^j* and *qⁱ · q^j*)

 $\frac{1}{2}$ + 1) $\frac{1}{2}$ *n* = # reducible scalar products (expressed in terms of denominators); $N =$ **#** scalar products (of types $q_i \cdot p_j$ and $q_i \cdot q_j$) $N = \ell(e-1) + \frac{\ell(\ell+1)}{2}$ $\frac{1}{2}$ $v = \ell(e-1) + \frac{2}{2}$

n = # denominators :: *Dⁱ* (*i* = 1*,...,n*);

 $n = #$ reducible scalar products (expressed in terms of denominators); *n* = # reducible scalar products (expressed in terms of denominators);

 $m = \texttt{\#}$ **irreducible** scalar products $N = N - n$:: S_i $(i = 1, \ldots, n)$ $m = # \text{imoducible scalars node.}$ $m = #$ irreducible scalar products $= N - n :: S_i \quad (i = 1, ..., m)$

Graph Topology & Integrals

n = # denominators :: *Dⁱ* (*i* = 1*,...,n*);

$$
e = # \text{ legs} :: p_i, \quad (i = 1, \ldots, e);
$$

$$
\ell = # \text{ loops} :: q_i \quad (i = 1, \ldots, \ell);
$$

$$
n = # \text{ denominators} :: D_i \quad (i = 1, \ldots, n);
$$

 $\frac{1}{2}$ + 1)

 $\frac{1}{2}$

n = # reducible scalar products (expressed in terms of denominators); $N =$ # scalar products (of types $q_i \cdot p_j$ and $q_i \cdot q_j$) $N = \ell(e-1) + \frac{\ell(\ell+1)}{2}$ $v = \ell(e-1) + \frac{1}{2}$

m = # irreducible scalar products = *N n* :: *Sⁱ* (*i* = 1*,...,m*)

 $n = #$ reducible scalar products (expressed in terms of denominators); *n* and λ reducible scalar products (expressed in terms of denominators);

 $m = \texttt{\#}$ **irreducible** scalar products $N = N - n$:: S_i ($i = 1, \ldots, n$) $m = #$ **irreducible** scalar products $N = n$... $S = (i-1, ..., n)$ $m = #$ irreducible scalar products $= N - n :: S_i \quad (i = 1, \ldots, m)$

Graph Topology & Integrals Simps! *N* Crank Tanalagy 8. Integr *e* = # legs :: *pi*, (*i* = 1*,...,e*); ` = # loops :: *qⁱ* (*i* = 1*,...,* `); 1. IBP

$$
\ell = \# \text{ loops} :: p_i, \quad (i = 1, ..., e);
$$
\n
$$
\ell = \# \text{ loops} :: q_i \quad (i = 1, ..., e);
$$
\n
$$
n = \# \text{ denominators} :: \overline{(D_i \quad (i = 1, ..., n))}
$$

1.1 Integration-by-parts Identities (IBPs)

*Dx*¹

¹ *··· ^Dxⁿ*

$$
N = # \text{ scalar products (of types } q_i \cdot p_j \text{ and } q_i \cdot q_j) \qquad N = \ell(e-1) + \frac{\ell(\ell+1)}{2}
$$

 α = α (expressed in terms of denominators); $p = #$ reducible scalar products (expressed in terms of denominators); $n = #$ reducible scalar products (expressed in terms of denominators); *n* = # reducible scalar products (expressed in terms of denominators);

$$
n =
$$
reducible scalar products (expressed in terms of denominators);
\n $m =$ # irreducible scalar products = $N - n$:: $(S_i \quad (i = 1, ..., m))$

Associated Integrals :: $\ddot{}$ $\frac{1}{2}$ ated Integrals :: Associated Integrations

n,m(x*,* y) ⌘

Associated Integrals ::

\n
$$
F_{n,m}^{[d]}(\mathbf{x}, \mathbf{y}) \equiv \int_{q_1 \dots q_\ell} f_{n,m}(\mathbf{x}, \mathbf{y}) , \qquad \int_{q_1 \dots q_\ell} \equiv \int \frac{d^d q_1}{(2\pi)^d} \cdots \frac{d^d q_\ell}{(2\pi)^d}
$$
\n
$$
f_{n,m}(\mathbf{x}, \mathbf{y}) = \frac{S_1^{y_1} \cdots S_m^{y_m}}{D_1^{x_1} \cdots D_n^{x_n}}
$$

Associated Integrals ::

Integration-by-parts Identities (IBPs) 8(*n, m*), *N*IBP = # of IBP relations = `(` + *g* 1)

Tkachov; Chetyrkin, Tkachov; Laporta;

= 0 *, v* = *q*1*,...,q*`*, p*1*,...,pe*1*.* (1.4)

= 0 *, v* = *q*1*,...,q*`*, p*1*,...,pe*1*.* (1.4)

$$
\int_{q_1\ldots q_\ell} \frac{\partial}{\partial q_i^\mu} \Big(v^\mu\; f_{n,m}(\mathbf{x},\mathbf{y}) \Big) = 0 \;, \qquad v = q_1,\ldots,q_\ell,\; p_1,\ldots,p_{e-1}.
$$

 $\forall (n, m), N_{\text{IBP}} = \# \text{ of IBP relations } = \ell(\ell + e - 1)$ \mathbb{R}^{n} , \mathbb{R}^{I} = \mathbb{R}^{I} of IBP relations = $\ell(\ell + e^{-1})$

1.1 Integration-by-parts Identities (IBPs)

8(*n, m*), *N*IBP = # of IBP relations = `(` + *e* 1)

 $\frac{1}{2}$ the same topology (or subtopologies) y^j = *{y*1*,...,y^j ±* 1*,...yn}* (1.7) Relations between integrals associated to the same topology (or subtopologies) Relations between integrals associated to the same topology (or subtopologies) \overline{a} \overline{a}

*q*1*...q*`

such that

Relations between integrals associated to the same topology (or subtopologies)

$$
c_0 F_{n,m}^{[d]}(\mathbf{x}, \mathbf{y}) + \sum_{i,j} c_{i,j} F_{n,m}^{[d]}(\mathbf{x_i}, \mathbf{y_j}) = 0,
$$

$$
\mathbf{x_i} = \{x_1, \dots, x_i \pm 1, \dots, x_n\}
$$

@*q*

µ

v^µ fn,m(x*,* y)

$$
\mathbf{y_j} = \{y_1, \ldots, y_j \pm 1, \ldots y_n\}
$$

public codes :: AIR; Reduze2; FIRE; LiteRed; *IIC COOC*
'ate cod *q*1*...q*` $\frac{1}{2}$, $\frac{1}{2}$, Independent set of integrals *M*[*d*] $\frac{d}{dx}$ Z **private codes :: ... many authors ... Sturm ...**

mi(¯x*,* ¯y) *,* (1.8)

Master Integrals (MIs) y^j = *{y*1*,...,y^j ±* 1*,...yn}* (1.7) 1.2 Master Integrals (MIs) 1.2 matrix Integrals (A) *i* , y^j = *{y*1*,...,y^j ±* 1*,...yn}* (1.7) 1.2 Master Integrals (MIs) $1.2 M_{\odot}$ Independent set of integrals *M*[*d*] 1.2 Master Integrals (MIs) *i Master Integrals i* Master Integrals *(MIs)* y^j = *{y*1*,...,y^j ±* 1*,...yn}* (1.7) 1.2 Master Integrals (MIs) y^j = *{y*1*,...,y^j ±* 1*,...yn}* (1.7) $\mathbf{A} \mathbf{A} \mathbf{A}$ α *y* α *is* (*MIS)* <u>Igrals</u> (MIS)

 $\text{Independent set of integrals } M_i^{[d]},$ Independent set of integral $\frac{i}{\sqrt{2}}$ \blacksquare Independent set of integrals $M_i^{[d]},$ $\lceil d \rceil$

$$
M_i^{[d]} \equiv \int_{q_1...q_\ell} m_i(\mathbf{\bar{x}}, \mathbf{\bar{y}}) ,
$$

mi(¯x*,* ¯y) *,* (1.8)

k

ⁱ ,

 $\overline{\text{with a definite}}$ with a definite set of powers $\bar{\mathbf{x}}, \bar{\mathbf{y}}$ such that with a definite set of powers \bar{x} , \bar{y} such with a definite set of powers $\bar{\mathbf{x}}$, $\bar{\mathbf{x}}$ $\frac{1}{2}$ with a definite set of powers $\bar{\mathbf{x}}, \bar{\mathbf{y}}$ such that *M*[*d*] *i* ⌘ \vec{r} a such that such that

with a definite set of powers ¯x*,* ¯y

with a definite set of powers $\mathcal{L}_\mathcal{A}$

with a definite set of powers \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , \mathcal{A} , $\mathcal{A$

$$
F_{n,m}^{[d]}(\mathbf{x},\mathbf{y}) \stackrel{\text{IBP}}{=} \sum_{k} c_k M_k^{[d]}, \qquad \forall (n,m)
$$

*q*1*...q*`

with a definite set of powers ¯x*,* ¯y

Z

k

Z

They form a *basis* for the integrals of the corresponding topology. tey form a *basis* for the integrals of the corresponding topology. They form a *basis* for the integrals of the corresponding topology.

Two special cases Two special cases Two special cases ${\bf SES}$ of integrals generated from the master integrals generated from the master integrals generated from the master integrals \sim

• Polynomial insertion: Two types of integrals generated from the master integrands *•* Polynomial insertion: T types of integrals generated from the master integrals generated from the master interaction of \mathbf{r} Two types of integrals generated from the **master integrands** *•* Polynomial insertion: Two types of integrals generated from the master integrands

such that

M[*d*]

@

^k =

pµ i

pµ

pµ i

@*p^µ*

@

@*p^µ*

@*p^µ*

M[*d*] *^k* =

pµ

Z

@*p^µ*

pµ

@

pµ i

mi(¯x*,* ¯y) = *F*[*d*]

mi(¯x*,* ¯y) = *F*[*d*]

 $\ddot{\mathbf{z}}$ • Polynomial insertion: *•* Polynomial insertion: • Polynomial insertion:

ion:
$$
\int_{q_1 \dots q_\ell} P(q_i \cdot p_j, q_i \cdot q_j) \ m_i(\mathbf{\bar{x}}, \mathbf{\bar{y}}) = \sum_{n,m} \alpha_{n,m} \ F_{n,m}^{[d]}(\mathbf{x}, \mathbf{y}) \stackrel{\text{IBP}}{=} \sum_i c_i \ M_i^{[d]}
$$

• External-leg derivatives: p_i^L • External-leg derivatives:

• Polynomial insertion:

pµ

• External-leg derivatives:

pµ i

• External-leg derivatives:

• External-leg derivatives:

pµ

@*p^µ*

@

@*p^µ*

@*p^µ*

M[*d*] *^k* =

M[*d*] *^k* =

• External-leg derivatives:
$$
p_i^{\mu} \frac{\partial}{\partial p_j^{\mu}} M_k^{[d]} = \int_{q_1...q_\ell} p_i^{\mu} \frac{\partial}{\partial p_j^{\mu}} m_k(\bar{\mathbf{x}}, \bar{\mathbf{y}}) = \sum_{n,m} \beta_{n,m} F_{n,m}^{[d]}(\mathbf{x}, \mathbf{y}) \stackrel{\text{IBP}}{=} \sum_i c_i M_i^{[d]}
$$

cⁱ M[*d*]

cⁱ M[*d*]

@

pµ

n,m(x*,* ^y) IBP

n,m F[*d*]

ⁱ (1.11)

ⁱ (1.11)

 \sim

mi(¯x*,* ¯y) = *F*[*d*]

n,m(x*,* ^y) IBP

cⁱ M[*d*]

 $\overline{}$

n,m(x*,* ^y) IBP

cⁱ M[*d*]

ⁱ (1.11)

ⁱ (1.11)

 $\overline{}$

mi(¯x*,* ¯y) *,* (1.8)

^k , 8(*n, m*) (1.9)

ⁱ (1.10)

cⁱ M[*d*]

mi(¯x*,* ¯y) = *F*[*d*]

Z

 $\overline{}$

^mk(¯x*,* ¯y) = ^X

 $-$

@

@*p^µ*

n,m(x*,* ^y) IBP

^k =

n,m(x*,* ^y) IBP

M[*d*]

Dimensional Recurrence Relations for MIs

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F[*d*]

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í.

*q*2

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Tarasov; Baikov; Lee; Bern, Dixon, Kosower Gluza, Kajda, Kosower

 $\overline{}$ \vert $\overline{}$ $\overline{}$ \vert $\overline{}$ $\overline{}$ \vert

fn,m(x*,* y) *,* (1.14)

Gram determinant *.*

t
$$
P(q_i \cdot p_j, q_i \cdot q_j) = \mathbf{G}(q_i, p_j) = \begin{vmatrix} q_1^2 & \cdots & (q_1 \cdot p_{e-1}) & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ (p_{e-1} \cdot q_1) & \cdots & p_{e-1}^2 & \cdots & p_{e-1}^2 \end{vmatrix}
$$

Ĩ

¹ *...* (*q*¹ *· pe*1)

Z

. .

G(*qi, p^j*) = egrals G(*qi, p^j*) = y.
D $\overline{}$ Dimension-shifted integrals

$$
F_{n,m}^{[d]}(\mathbf{x},\mathbf{y})\equiv\int_{q_1...q_\ell}f_{n,m}(\mathbf{x},\mathbf{y})
$$

G-insertion:

Dimensional Recurrence Relations for MIs G(*qi, p^j*) = Ī Ī \overline{P} *.* \overline{a} $\overline{}$ (113) IVI IVIIS

Gram determinant
\n*P(q_i · p_j, q_i · q_j)* = *G(q_i, p_j)* =
$$
\begin{pmatrix}\nq_1^2 & \cdots & (q_1 \cdot p_{e-1}) \\
\vdots & \ddots & \vdots \\
(p_{e-1} \cdot q_1) & \cdots & p_{e-1}^2\n\end{pmatrix}
$$
\n**Dimension-shifted integrals**
\n*F*_{n,m}^[d](**x**, **y**) $\equiv \int_{q_1 \dots q_\ell} f_{n,m}(\mathbf{x}, \mathbf{y}) \Rightarrow \int_{q_1 \dots q_\ell} \mathbf{G} f_{n,m}(\mathbf{x}, \mathbf{y}) = \Omega(d, p_i) F_{n,m}^{[d+2]}(\mathbf{x}, \mathbf{y})$

F[*d*]

Tarasov; Baikov; Lee; Bern, Dixon, Kosower Gluza, Kajda, Kosower

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Dimensional Recurrence Relations for MIs G(*qi, p^j*) = Ī Ī \overline{P} *.* \overline{a} $\overline{}$ (113) IVI IVIIS

Gram determinant
\n*P(q_i · p_j, q_i · q_j)* = *G(q_i, p_j)* =
$$
\begin{pmatrix}\nq_1^2 & \cdots & (q_1 \cdot p_{e-1}) \\
\vdots & \ddots & \vdots \\
(p_{e-1} \cdot q_1) & \cdots & p_{e-1}^2\n\end{pmatrix}
$$
\n**Dimension-shifted integrals**
\n
$$
\overline{F_{n,m}^{[d]}(x,y)} \equiv \int_{q_1...q_\ell} f_{n,m}(x,y) \implies \int_{q_1...q_\ell} G_{n,m}(x,y) = \Omega(d, p_i) \overline{F_{n,m}^{[d+2]}(x,y)}
$$

Tarasov; Baikov; Lee; Bern, Dixon, Kosower Gluza, Kajda, Kosower

 $\overline{}$ $\overline{}$ $\overline{}$ $\overline{}$ \vert $\overline{}$ $\overline{}$ \vert

n $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ *F*[*d*] *n,m*(x*,* y) ⌘ *inte fn,m*(x*,* y) *,* (1.14) **G-insertion** generates shifted dim. Integr **G-insertion** generates shifted dim. integrals**: d --> d+2**

Dimensional Recurrence Relations for MIs G(*qi, p^j*) = Ī Ī \overline{P} *.* \overline{a} $\overline{}$ (113) IVI IVIIS \blacksquare 1 **....** (*q*1 *r c***₁) ***e*₁ i, I *.* $$ $\overline{1}$ $\overline{}$ $\overline{}$

Z

*q*1*...q*`

Tarasov; Baikov; Lee; Bern, Dixon, Kosower Gluza, Kajda, Kosower

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Gram determinant iniano

erminant
$$
P(q_i \cdot p_j, q_i \cdot q_j) = \mathbf{G}(q_i, p_j) = \begin{vmatrix} q_1^2 & \dots & (q_1 \cdot p_{e-1}) \\ \vdots & \ddots & \vdots \\ (p_{e-1} \cdot q_1) & \dots & p_{e-1}^2 \end{vmatrix}
$$
 $\begin{array}{c} \text{Tarasov; Baikov; Lee;} \\ \text{Gluza, Kajda, Kosower} \end{array}$

G(*qi, p^j*) = egrals G(*qi, p^j*) = r
D \overline{a} Dimension-shifted integrals *n*_{ttegral}

$$
F_{n,m}^{[d]}(\mathbf{x},\mathbf{y})\equiv \int_{q_1\ldots q_\ell} f_{n,m}(\mathbf{x},\mathbf{y}) \qquad \Rightarrow \int_{q_1\ldots q_\ell} \mathbf{G}\,\, f_{n,m}(\mathbf{x},\mathbf{y}) = \Omega(d,p_i)\,\, F_{n,m}^{[d+2]}(\mathbf{x},\mathbf{y})
$$

which can be seen as a Dimensional records as a Dimensional recurrence relationship as a Dimensional recurrence

*q*1*...q*` In the case of Master integrals

In the case of Master integrals

)

^k = ⌦(*d, pi*)

*q*1*...q*`

*q*1*...q*`

ntegrals
$$
M_k^{[d+2]} = \Omega(d, p_i)^{-1} \int_{q_1 \dots q_\ell} \mathbf{G} \ m_k(\mathbf{\bar{x}}, \mathbf{\bar{y}}) \stackrel{\text{IBP}}{=} \sum_i c_{k,i} \ M_i^{[d]}
$$

Dimensional Recurrence Relations for MIs G(*qi, p^j*) = Ī Ī \overline{P} *.* \overline{a} $\overline{}$ (113) IVI IVIIS \blacksquare 1 **....** (*q*1 *r c***₁) ***e*₁ i, I *.* $$ $\overline{1}$ $\overline{}$ $\overline{}$ \overline{a} \mathbf{I} $\overline{}$ *x x e <i>n* \blacksquare $\overline{}$ \blacksquare with Gram determinant Gram determinant C *d***₁** *r***_e₁** *c***₁** *<i>d*₁ *a*₂ $\overline{}$ 1.4 Dimension *F*[*d*] *n,m*(x*,* y) ⌘ *q*1*...q*` *fn,m*(x*,* y) *,* (1.14)

Tarasov; Baikov; Lee; Bern, Dixon, Kosower

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Gram determinant iniano $\boldsymbol{\mathsf{Gram}}$ determinant determinant)

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M[*d*+2]

erminant
$$
P(q_i \cdot p_j, q_i \cdot q_j) = \mathbf{G}(q_i, p_j) = \begin{vmatrix} q_1^2 & \dots & (q_1 \cdot p_{e-1}) \\ \vdots & \ddots & \vdots \\ (p_{e-1} \cdot q_1) & \dots & p_{e-1}^2 \end{vmatrix}
$$
 $\begin{array}{c} \text{Tarasov; Baikov; Lee;} \\ \text{Gluza, Kajda, Kosower} \end{array}$

G(*qi, p^j*) = egrals G(*qi, p^j*) = r
D \overline{a} Dimension-shifted integrals *n*_{ttegral} $\frac{1}{2}$ $\frac{1}{2}$ egrals \mathbf{D} : $\frac{1}{2}$ *M* $\frac{1}{2}$ $\frac{1}{2}$

1.5 Di↵erential Equations

$$
F_{n,m}^{[d]}(\mathbf{x},\mathbf{y})\equiv \int_{q_1\ldots q_\ell} f_{n,m}(\mathbf{x},\mathbf{y}) \qquad \Rightarrow \int_{q_1\ldots q_\ell} \mathbf{G}\,\, f_{n,m}(\mathbf{x},\mathbf{y})=\Omega(d,p_i)\,\,F_{n,m}^{[d+2]}(\mathbf{x},\mathbf{y})
$$

In the case of Master integrals\n
$$
M_k^{[d+2]} = \Omega(d, p_i)^{-1} \int_{q_1 \dots q_\ell} \mathbf{G} \ m_k(\bar{\mathbf{x}}, \bar{\mathbf{y}}) \stackrel{\text{IBP}}{=} \sum_i c_{k,i} \ M_i^{[d]}
$$
\nwhich can be seen as a
Dimensional recurrence relation

which can be seen as a Dimensional recurrence relation \mathcal{L} In general, n MIs obey a system of Dimension In general, n MIs obey a system of Dimensional recurrence relations of \bf{u} In general, *n* MIs obey a system of Dimensional recurrence relations In general, *n* MIs obey a system of Dimensional recurrence relations In general, *n* MIs obey a system of Dimensional recurrence relations In general, *n* MIs obey a system of Dimensional recurrence relations *M*[*d*] *n*

$$
\mathbf{M}^{[d]} = \begin{pmatrix} M_1^{[d]} \\ \vdots \\ M_n^{[d]} \end{pmatrix} \qquad \mathbf{M}^{[d+2]} = \mathbb{C}(d) \mathbf{M}^{[d]}
$$

Pifferential Equations for MIs $\overline{}$ • Differential ∂ M uations f or M_{s} ∂m² where, for simplicity, we assume that is just one propagator of mass measurement of mass μ where α simplicity, we assume that is just one propagator of α $\frac{1}{2}$ for M_{IS} Z G *fn,m*(x*,* y) = ⌦(*d, pi*) *F*[*d*+2] **Ellual Equation**

 $p^2\frac{\partial}{\partial}$ ∂p^2 $\left\{p \quad \} = \frac{1}{2}$ 2 p_μ ∂ ∂p_μ $\bigg\{\left\langle p\right\rangle =\frac{1}{2}p_{\mu}\frac{\partial}{\partial p}\left\{p-\left\langle p\right\rangle -p\right\}$ ∂p² $\begin{pmatrix} 1 \\ -n \end{pmatrix} = \frac{1}{2} n_{\theta} \frac{\partial}{\partial n_{\theta}} \left\{ n_{\theta} \right\}$ \boldsymbol{p} pµ ∂ ∂p^µ \mathbb{R}^n $\left\{\frac{p}{\partial p^2}\left\{p-\left(\rule{0pt}{10pt}\right.\right.\left.\left.\rule{0pt}{10pt}\right.\right. p\right\}=\frac{1}{2}p.$ $\overline{}$ $\frac{1}{2}$ $\frac{1}{2}$ *q*1*...q*` \mathcal{C}_P can be seen as \mathcal{C}_P and \mathcal{C}_P

bern, Dixon, Roso
Kotikov; Remiddi; where, for simplicity, we assume the simplicity, we assume the simplicity of mass media of mass media of mass m
∂p2 and the simplicity of mass media of • Differentiation with respect to the squared momentum Bern, Dixon, Kosower
*K*otikov; Remiddi; Argeri, Bonciani, Ferroglia, Remiddi, **P.M**.

 \mathbb{R} Henn; Henn, Smirno
Lee; Papadopoulos; • 4-point case.

Henn; Henn, Smirnov; $\frac{4}{3}$ P2
P \$ ^p¹ % Argeri, diVita, Mirabella, Schlenk, Schubert, Tancredi, **P.M**. Argeri, diVita, Mirabella, Schle
diVita, Schubert, Yundin, **P.M**. Primo, Tancredi … ... Zeng \ldots , \ldots , \ldots *c*_{*c*} *i d*_{*i*} *c*_{*c*} *i d*_{*i*} *d*_{*i*} *i* which can be seen as a Dimensional recurrence relationship as a Dimensional recurrence relationship and the seeing of the se

 \mathbf{R}

$$
P^2 \frac{\partial}{\partial P^2} \left\{ \sum_{p_2}^{p_1} \bigodot -p_3 \right\} = \left[A \left(p_{1,\mu} \frac{\partial}{\partial p_{1,\mu}} + p_{2,\mu} \frac{\partial}{\partial p_{2,\mu}} \right) + B \left(p_{1,\mu} \frac{\partial}{\partial p_{2,\mu}} + p_{2,\mu} \frac{\partial}{\partial p_{1,\mu}} \right) \right] \left\{ \sum_{p_2}^{p_1} \bigodot -p_3 \right\}
$$

$$
P = p_1 + p_2,
$$

^G *^mk*(¯x*,* ¯y) IBP

In general, *n* MIs obey a system of Dimensional recurrence relations

$$
P^2\frac{\partial}{\partial P^2}\left\{\sum_{p_2}^{p_3}\middle(\sum_{p_4}^{p_3}\right) = \left[C\left(p_{1,\mu}\frac{\partial}{\partial p_{1,\mu}}-p_{3,\mu}\frac{\partial}{\partial p_{3,\mu}}\right)+Dp_{2,\mu}\frac{\partial}{\partial p_{2,\mu}}+E(p_{1,\mu}+p_{3,\mu})\left(\frac{\partial}{\partial p_{3,\mu}}-\frac{\partial}{\partial p_{1,\mu}}+\frac{\partial}{\partial p_{2,\mu}}\right)\right]\left\{\sum_{p_2}^{p_3}\middle(\sum_{p_4}^{p_4}\right\}\right\}
$$

 α general, $n \n$ χ asystem of 1st ODE $\ddot{}$ $\ddot{}$ $\frac{1}{2}$ ral, *n* MIs obey a $etom$ of 1 a system of 1st ODE In general, n MIs obey a system of 1st ODE [−] [∂]

$$
\partial_z \mathbf{M}^{[d]} = \mathbb{A}(d,z) \; \mathbf{M}^{[d]}
$$

back to EFT-GR @ 4PN - O(G^5)

7 Master Integrals We performed the calculation with the calculation with the dimensional regularization scheme, and the contribution to the Lagrangian of each graph was given as Laurent series in *d* =3+ ", being *d* $\frac{1}{\sqrt{2}}$ *N* actor Intornale \blacksquare in the analogy between such diagrams in the EFT gravitation \blacksquare functions in massless gauge theory, we addressed their calculation by means of multi-loop

b

IBP Reduction (i. *in-house* code + ii. Reduze2) $\frac{1}{2}$. In thouse court \pm ii. Neuazez We performed the calculation with the dimensional regularization with the dimensional regularization scheme, an r keauction (i. *in-nous*e code + II. Keauze*z)*

50 EFT Integrals ==> 29 Topologies ==> 7 MIs limit, the sum of the fifty terms is found to be finite at *d* = 3. Our result may indicate the **EFT INTEGRAIS ==> 29 TOPOIOGIES ==> / MIS**

 \bigcap $\mathbf{\bar{x}} = (1, \ldots, 1)$, $\mathbf{\bar{y}} \neq (0, \ldots, 0)$ problem. Further investigation on such discrepancy will be subject of future work. that can be found in the literature, which adopted different approaches to the two-body

\cup $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ **7 Master Integrals**

5 easy MIs

*d/*2*a^b*

*s*1+2"

*M*1*,*3 *M*1*,*4 *M*2*,*2 *M*3*,*6 $\Gamma(\frac{5}{2}\alpha)$ $\mathcal{M}_{1,1} = (4\pi)^{-2d} s^{2d-6} \frac{1}{\Gamma(1-\Gamma)}$ $\Gamma(r \approx l)\Gamma(d+1)5$ $-c_{2d-2d-5}$ (3 – 20)1 ($\frac{1}{2}$ – 1) $\frac{1}{2}$ $\frac{2}{3}$ ² ⁼ *,* (3.2) $\frac{dS}{dt}$ states states states states states states in $\frac{dS}{dt}$ $f(x)$ $M_{0,1} = (4\pi)^{-2d} s^{2d-5} \frac{\Gamma(5-2d)\Gamma(\frac{d}{2}-1)^5}{\Gamma(\frac{5}{2}d-5)}$ $\Gamma\left(\frac{5}{2}d - 5\right)$ $\overline{)}$ $\sum_{a=1}^{8} a^2 = \frac{1}{2}$ $\sum(\frac{d}{2}-\frac{1}{2})\Gamma(\frac{d}{2}-1)$ \int_0^6 \mathcal{V} \searrow \overline{a} ◆ $\Gamma(d-2)\Gamma(2d)$ *^d*=3+" = (4⇡) 62" *s*1+2" *e*2"*^E* $\int_{\mathcal{L}} | \Gamma(\frac{d}{2} - 1)^5$ "² $\int_{-\infty}^{\infty} s^{2a-1}$ $\int \frac{5}{2}d-5$ \overline{a} ⁺ *^O*("3) *,* (A.2) $M_{1,1} = (4\pi)^{-2d} s^{2d-6} \frac{\Gamma(4-\frac{3}{2}d)\Gamma(2-\frac{d}{2})}{\Gamma(d-2)\Gamma(2d)}$ $\int \Gamma\left(\frac{d}{2}-1\right)^6$ $\frac{2^{n} \sqrt{2^{n}+2^{n}+2^{n}+3^{n}+3^{n}+4}}{\Gamma(d-2)\Gamma(2d-4)}$ $(4\pi$ $-2d_{\alpha}2d$ $\frac{15a-2d\pi\sqrt{\frac{a}{2}}-1}{\Gamma(\frac{5}{2}d-5)}$ $\frac{1}{\sqrt{2}}$ ⁺ *^O*("3) $M_{\text{max}} = (4\pi)^{-2d}e^{2d-6}\Gamma(4-\frac{3}{2}d)\Gamma(2-\frac{d}{2})$ $\sum_{\Gamma/d}$ ⁶ $\left(\frac{3}{2}d\right)\Gamma\left(2-\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-1\right)^{6}$ $f(a^2-2) f(2a^2-4)$ \int_{A} $(4\pi)^{-2d}e^{2d-5}\Gamma(5-2d)\Gamma(\frac{d}{2}-1)$ \overline{a} $\frac{(d-2d)\Gamma(\frac{d}{2}-1)^5}{\Gamma(\frac{5}{2}-1)}$ $\frac{1(\frac{1}{2}a-3)}{2}$ *,* (A.4) $M_{1,1} = (4\pi)^{-2d} s^{2d-6} \frac{\Gamma(4-\frac{3}{2}d)\Gamma(2-\frac{d}{2})}{\Gamma(d-3)\Gamma(2d)}$ $\Gamma(d-2)$ $\frac{(2)}{2\Gamma(2d-4)}$ $\Gamma(5-2d)\Gamma(\frac{d}{2}-1)$ *,* (A.4) $M_{0,1} = (4\pi)^{-2d} s^{2d-5} \frac{\Gamma(5d-5)}{\Gamma(\frac{5}{2}d-5)}$ $\begin{array}{ccc} \n\Gamma(\frac{3}{2}d-5) & & & \rightarrow & \longrightarrow & \longrightarrow \n\end{array}$ ² *d* 3 $=$ $(4\pi)^{-2d} s^{2d-6} \frac{\Gamma(4)}{s^2}$ $\frac{-\frac{3}{2}d\Gamma(2-\frac{3}{2})}{\Gamma(d-2)\Gamma}$ $-\frac{2}{2}$) 1 $\frac{a}{2}$ 4 $\frac{-1}{2}$ ा अन्य अन्तर्गत अस्ति। अस्ति अस्ति अस्ति ।

The following master integrals are known in closed analytical form, exact in *d*:

$$
\mathcal{N}_{\mathcal{A}} = \mathcal{N}_{\mathcal{A}} \mathcal{L}_{\mathcal{A}} \mathcal
$$

 $\overline{}$

 $\overline{}$

^d=3+" = (4⇡)

$$
\mathcal{M}_{1,3} = (4\pi)^{-2d} s^{2d-6} \frac{\Gamma(6-2d)\Gamma(3-d)\Gamma(2-\frac{d}{2})\Gamma(\frac{d}{2}-1)^6 \Gamma(2d-5)}{\Gamma(5-\frac{3}{2}d)\Gamma(d-2)\Gamma(\frac{3}{2}d-3)\Gamma(\frac{5}{2}d-6)}
$$

$$
\mathcal{M}_{1,4} = (4\pi)^{-2d} s^{2d-6} \frac{\Gamma(6-2d)\Gamma(2-\frac{d}{2})^2 \Gamma(\frac{d}{2}-1)^6 \Gamma(\frac{3}{2}d-4)}{\Gamma(4-d)\Gamma(d-2)^2 \Gamma(\frac{5}{2}d-6)}
$$

 \bullet M_{2,2} drops out in the d --> 3 limit

a

b

 \bullet M_{2,2} drops out in the d --> 3 limit

a

b

The set *A^I* contains diagrams with a simpler internal structure, and they have been **M3,6 non-trivial**

$$ *a***2** \overline{AB} \overline{BA} \overline{BA} (14 ⁵*d*)(63872 ⁴⁰¹⁶²*^d* + 8403*d*² ⁵⁸⁵*d*3) always appears multiplied by a positive power of \mathbb{R}^2 . The Laurent expansion in \mathbb{R}^2 *d* $\overline{}$ $\$ **Numerical Solution of Dim. Rec. Rel.** SUMMERTIME Lee, Mingulov

3(*^d* 6)2(3*^d* 16)(3*^d* 14)*s*⁴ *,* (3.5) *^a*² = 80(*^d* 3)3(2*^d* 7)(5*^d* 26)(5*^d* 24)(5*^d* 22)(5*^d* 18)(5*^d* 16) ⇥ $-0.000030401300124000302431031400030$ $+0.00008751790270929812451430800595930715306389454769730508$ $+0.00083640896480242565453996588706281367341758130837556548495\,\varepsilon$ $+O\left(\varepsilon^2\right)$ $\frac{119949575742549538723187}{\varepsilon^2}$ **a**3740685543350373855084153798514138/ ε (5*d* 16)(5*d* 14)(7*d* 32) +0.00008751790270929812451430800595930715306389454769730505664
+0.00008*640006400849565459006595950506894955941550190095556540405* $\mathcal{M}_{3,6} = s^{2\varepsilon-2} \left[\begin{array}{c} 0.00002005074659118034216631402981859119949575742549538723187/\varepsilon^2 \end{array} \right.$ $-0.00009840138812460833249783740685543350373855084153798514138$ $\{\varepsilon^2\}$ (ε^2)

Experimental Mathematics for M3,6

ü Numerical Reconstruction

ü from SUMMERTIME

In[1]:= **nM36 = H**

0.00002005074659118034216631402981859119949575742549538723187 ê e^2 - 0.00009840138812460833249783740685543350373855084153798514138 ê e + 0.00008751790270929812451430800595930715306389454769730505664 + 0.00083640896480242565453996588706281367341758130837556548495 * e $+ \epsilon$ ^ 2 $*$ **Help** ϵ , 2 **L;** $\ln[2]$:= pref = $(4 * Pi)$ ^ $(-4 - 2 * \epsilon) * Exp [2 * \epsilon * EulerGamma]$ Out[2]= $e^{2\,\gamma\,\epsilon}\,(4\,\pi)^{-2\,\epsilon-4}$ $In[3]:$ **npref** = $N[Series[prefix {e, 0, 2}]$, 50] // Chop $Out[3] = 0.000040101493182360684332628059637182398991514850990774 0.00015670128306685598066304675407368460848558683208520 \epsilon +$ $0.00030616431167705224971803922217880567378514178260532 $\epsilon^2+O(\epsilon^3)$$ In[4]:= **nBexp = nM36 ê npref** Out $[4]$ = 0.500 ϵ^2 -0.5000 ϵ - $3.58876648328794339088189620833849370269526252470 + O(\epsilon^1)$

ü 50 digits

 $test = N[Coefficient[nBexp, \epsilon, 0], 50]$

 $-3.58876648328794339088189620833849370269526252470$

Experimental Mathematics for M3,6

ü Numerical Reconstruction

ü from SUMMERTIME

In[1]:= **nM36 = H**

 $-3.58876648328794339088189620833849370269526252470$

 $$ *a***2** \overline{AB} \overline{BA} \overline{BA} (14 ⁵*d*)(63872 ⁴⁰¹⁶²*^d* + 8403*d*² ⁵⁸⁵*d*3) *d* $\overline{}$ $\$ **Numerical Solution of Dim. Rec. Rel.** SUMMERTIME Lee, Mingulov

3(*^d* 6)2(3*^d* 16)(3*^d* 14)*s*⁴ *,* (3.5) *^a*² = 80(*^d* 3)3(2*^d* 7)(5*^d* 26)(5*^d* 24)(5*^d* 22)(5*^d* 18)(5*^d* 16) ⇥ $-0.000030401300124000302431031400030$ $+0.00008751790270929812451430800595930715306389454769730508$ $+0.00083640896480242565453996588706281367341758130837556548495\,\varepsilon$ $+O\left(\varepsilon^2\right)$ 3(*^d* 6)2(*^d* 4)2(3*^d* 16)(3*^d* 14)*s*⁶ *,* (3.7) $\frac{119949575742549538723187}{\varepsilon^2}$ **a**3740685543350373855084153798514138/ ε (5*d* 16)(5*d* 14)(7*d* 32) +0.00008751790270929812451430800595930715306389454769730505664
+0.00008*640006400849565459006595950506894955941550190095556540405* $\mathcal{M}_{3,6} = s^{2\varepsilon-2} \left[\begin{array}{c} 0.00002005074659118034216631402981859119949575742549538723187/\varepsilon^2 \end{array} \right]$ $-0.00009840138812460833249783740685543350373855084153798514138/\varepsilon$ $\{\varepsilon^2\}$ (ε^2) 0*.*00002005074659118034216631402981859119949575742549538723187*/*"² +0*.*00008751790270929812451430800595930715306389454769730505664

Anary treating ansatz and the experimental m \overline{z} *experimental mat* z :: experimental mathematics :: \mathbf{B} and the matics :: **Analytic ansatz :: experimental mathematics ::**

$$
= s^{2\varepsilon - 2} (4\pi)^{-4 - 2\varepsilon} e^{2\varepsilon \gamma_E} \frac{1}{2} \left[\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} - 8 + \frac{\pi^2}{12} - \varepsilon \left(18 - \pi^2 \left(\frac{13}{4} - 2 \log 2 \right) - \frac{77}{3} \zeta_3 \right) + \mathcal{O} \left(\varepsilon^2 \right) \right] \tag{2.2}
$$

9(*^d* 6)2(*^d* 5)(*^d* 4)3(3*^d* 16)2(3*^d* 14)2*s*⁷ *,* (3.9) T expansion have been obtained from the " expansion have been obtained from the high precision \mathcal{L} **Damour, Jaranowski (analytical)**

*a*⁵ = 20(*d* 3)(2*d* 7)(2*d* 5)(5*d* 26)(5*d* 24) ⇥

 $$ *a***2** \overline{AB} \overline{BA} \overline{BA} (14 ⁵*d*)(63872 ⁴⁰¹⁶²*^d* + 8403*d*² ⁵⁸⁵*d*3) *d* $\overline{}$ $\$ **Numerical Solution of Dim. Rec. Rel.** SUMMERTIME Lee, Mingulov

3(*^d* 6)2(3*^d* 16)(3*^d* 14)*s*⁴ *,* (3.5) *^a*² = 80(*^d* 3)3(2*^d* 7)(5*^d* 26)(5*^d* 24)(5*^d* 22)(5*^d* 18)(5*^d* 16) ⇥ $-0.000030401300124000302431031400030$ $+0.00008751790270929812451430800595930715306389454769730508$ $+0.00083640896480242565453996588706281367341758130837556548495\,\varepsilon$ $+O\left(\varepsilon^2\right)$ 3(*^d* 6)2(*^d* 4)2(3*^d* 16)(3*^d* 14)*s*⁶ *,* (3.7) $\frac{119949575742549538723187}{\varepsilon^2}$ **a**3740685543350373855084153798514138/ ε (5*d* 16)(5*d* 14)(7*d* 32) +0.00008751790270929812451430800595930715306389454769730505664
+0.00008*640006400849565459006595950506894955941550190095556540405* $\mathcal{M}_{3,6} = s^{2\varepsilon-2} \left[\begin{array}{c} 0.00002005074659118034216631402981859119949575742549538723187/\varepsilon^2 \end{array} \right]$ $-0.00009840138812460833249783740685543350373855084153798514138/\varepsilon$ $\{\varepsilon^2\}$ (ε^2) 0*.*00002005074659118034216631402981859119949575742549538723187*/*"² +0*.*00008751790270929812451430800595930715306389454769730505664

Anary treating ansatz and the experimental m \overline{z} *experimental mat* **z** :: experimental mathematics :: and the matics :: **Analytic ansatz** :: experimental mathematics ::

$$
= s^{2\varepsilon - 2} (4\pi)^{-4 - 2\varepsilon} e^{2\varepsilon \gamma_E} \frac{1}{2} \left[\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon} - 8 + \frac{\pi^2}{12} - \varepsilon \left(18 - \pi^2 \left(\frac{13}{4} - 2 \log 2 \right) - \frac{77}{3} \zeta_3 \right) + \mathcal{O} \left(\varepsilon^2 \right) \right]
$$
\nConfirmed by Danour, Jaranowski (analytical)

 S^2 important impact (1972736 ¹⁶⁶⁶⁴¹⁸*^d* + 527297*d*² ⁷⁴⁰⁷⁰*d*³ + 3897*d*4) important impact the PSLQ algorithm \sim

(Impact on) **The 4PN O(G^5) Lagrangian** (Impact on) The 4PN O(G^5) Lagrangian <u>ran</u> Foffa, Stur

In this appendix we collect the collect the contributions to the L agrangian coming from all the ampli-**P** Individual terms

Foffa, Sturani, Sturm, & **P.M.** Sturm, $\&$ P.M. **Damour, Jaranowski**

1 **a**, Sturani, Sturr 1, & **P.M. , Owski**
● G57 → G57 →

$$
0={\cal L}_9={\cal L}_{12}={\cal L}_{13}={\cal L}_{22}={\cal L}_{26}={\cal L}_{27}={\cal L}_{31}={\cal L}_{36}={\cal L}_{46}={\cal L}_{47}\,,
$$

$$
\frac{1}{2}\frac{G_N^5 m_1^3 m_2^3}{r^5} = \mathcal{L}_1 = \mathcal{L}_3 = 4\mathcal{L}_5 = 3\mathcal{L}_{14} = \frac{\mathcal{L}_{19}}{8} = \frac{3\mathcal{L}_{20}}{2} = \frac{3\mathcal{L}_{21}}{4} = \frac{\mathcal{L}_{23}}{4} = \frac{\mathcal{L}_{24}}{4} = \frac{3\mathcal{L}_{25}}{2},
$$

$$
\frac{1}{2}\frac{G_N^5 m_1^4 m_2^2}{r^5} = \mathcal{L}_2 = 3\mathcal{L}_4 = \frac{3\mathcal{L}_8}{2} = \frac{3\mathcal{L}_{10}}{2} = \frac{3\mathcal{L}_{11}}{2} = \frac{\mathcal{L}_{15}}{4} = \frac{3\mathcal{L}_{16}}{4} = \frac{3\mathcal{L}_{17}}{4} = \frac{\mathcal{L}_{18}}{4},
$$

$$
\frac{1}{120}\frac{G_N^5 m_1^5 m_2}{r^5} = \mathcal{L}_6 = \frac{\mathcal{L}_7}{20} = \frac{3\mathcal{L}_{30}}{20} = -\frac{3\mathcal{L}_{35}}{56} = \frac{\mathcal{L}_{39}}{24} = \frac{\mathcal{L}_{45}}{12},
$$

$$
\mathcal{L}_{28} = \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[\frac{428}{75} + \frac{4}{15} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{32} = \frac{G_N^5 m_1^3 m_2^3}{r^5} \left[-\frac{91}{450} + \frac{1}{15} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{33} = \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[\frac{13}{5} - \frac{2}{3} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{34} = \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[\frac{13}{5} - \frac{2}{3} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{35} = \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[\frac{147}{25} + \frac{8}{15} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{40} = \frac{G_N^5 m_1^3 m_2^3}{r}
$$
\n
$$
\mathcal{L}_{41} = \frac{G_N^5 m_1^3 m_2^3}{r^5} \left[\frac{49}{18} + \frac{1}{3} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{42} = -\frac{G_N^5}{r}
$$
\n
$$
\mathcal{L}_{43} = -\frac{G_N^5 m_1^3 m_2^3}{r^5} \left[\frac{53}{150} + \frac{2}{15} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{44} = -\frac{G_N^5 m_1^5 m_2^5}{r}
$$
\n
$$
\mathcal{L}_{48} = \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[\frac{578}{75} + \frac{8}{5} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{49} = \frac{G_N^5 m_1^4 m_2^2}{r}
$$

$$
\mathcal{L}_{29} = \frac{G_N^5 m_1^3 m_2^3}{r^5} \left[-\frac{409}{450} + \frac{1}{5} \mathcal{P} \right], \qquad \mathcal{L}_{50} = \frac{G_N^5 m_1^3 m_2^3}{r^5}
$$
\n
$$
\mathcal{L}_{33} = \frac{G_N^5 m_1^3 m_2^3}{r^5} \left(16 - \pi^2 \right),
$$
\n
$$
\mathcal{L}_{37} = -\frac{G_N^5 m_1^4 m_2^2}{r^5} \left[17 + 2 \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{40} = \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[-\frac{39}{25} + \frac{4}{15} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{42} = -\frac{G_N^5 m_1^3 m_2^3}{r^5} \left[\frac{97}{225} + \frac{1}{15} \mathcal{P} \right], \qquad \mathcal{P} \equiv \frac{1}{\varepsilon}
$$
\n
$$
\mathcal{L}_{44} = -\frac{G_N^5 m_1^3 m_2^3}{r^5} \left[\frac{37}{75} + \frac{2}{5} \mathcal{P} \right], \qquad \mathcal{P} \equiv \frac{1}{\varepsilon}
$$
\n
$$
\mathcal{L}_{49} = \frac{G_N^5 m_1^3 m_2^3}{r^5} \left(32 - 3\pi^2 \right),
$$

$$
\mathcal{L}_{50} = \frac{G_N^5 m_1^3 m_2^3}{r^5} \left(4\pi^2 - \frac{124}{3} \right)
$$

$$
\mathcal{P} \equiv \frac{1}{\varepsilon} - 5 \log \frac{r}{L_0}
$$

binaries, *Living Reviews in Relativity* 5 (2002), no. 3.

A³³ = = − i (8⇡*G^N*)

[1] L. Blanchet, *Gravitational radiation from post-newtonian sources and inspiralling compact*

binaries, *Living Reviews in Relativity* 5 (2002), no. 3.

$$
L=\sqrt{4\pi {\rm e}^{\gamma_E}}L_0
$$

 $\Lambda^{-2} \equiv 32\pi G_N L^{d-3}$

[1] L. Blanchet, *Gravitational radiation from post-newtonian sources and inspiralling compact*

always cancels out in the expression of physical observables.

length scale which keeps the correct dimensions of \mathbb{R}^n

In this framework, a Kaluza-Klein (KK) parametrization of the metric [32, 33] is usually

(Impact on) **The 4PN O(G^5) Lagrangian B** Results for a line $rac{1}{2}$ <u>ran</u> Foffa, Stur

In this appendix we collect the collect the contributions to the L agrangian coming from all the ampli-**P** Individual terms

Sturm, $\&$ P.M. **Damour, Jaranowski**

1 **a**, Sturani, Sturr 1, & **P.M. , Owski**
● G57 → Foffa, Sturani, Sturm, & **P.M.**

$$
0={\cal L}_9={\cal L}_{12}={\cal L}_{13}={\cal L}_{22}={\cal L}_{26}={\cal L}_{27}={\cal L}_{31}={\cal L}_{36}={\cal L}_{46}={\cal L}_{47}\,,
$$

$$
\frac{1}{2}\frac{G_N^5 m_1^3 m_2^3}{r^5} = \mathcal{L}_1 = \mathcal{L}_3 = 4\mathcal{L}_5 = 3\mathcal{L}_{14} = \frac{\mathcal{L}_{19}}{8} = \frac{3\mathcal{L}_{20}}{2} = \frac{3\mathcal{L}_{21}}{4} = \frac{\mathcal{L}_{23}}{4} = \frac{\mathcal{L}_{24}}{4} = \frac{3\mathcal{L}_{25}}{2},
$$

$$
\frac{1}{2}\frac{G_N^5 m_1^4 m_2^2}{r^5} = \mathcal{L}_2 = 3\mathcal{L}_4 = \frac{3\mathcal{L}_8}{2} = \frac{3\mathcal{L}_{10}}{2} = \frac{3\mathcal{L}_{11}}{2} = \frac{\mathcal{L}_{15}}{4} = \frac{3\mathcal{L}_{16}}{4} = \frac{3\mathcal{L}_{17}}{4} = \frac{\mathcal{L}_{18}}{4},
$$

$$
\frac{1}{120}\frac{G_N^5 m_1^5 m_2}{r^5} = \mathcal{L}_6 = \frac{\mathcal{L}_7}{20} = \frac{3\mathcal{L}_{30}}{20} = -\frac{3\mathcal{L}_{35}}{56} = \frac{\mathcal{L}_{39}}{24} = \frac{\mathcal{L}_{45}}{12},
$$

 \sim

$$
\mathcal{L}_{28} = \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[\frac{428}{75} + \frac{4}{15} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{32} = \frac{G_N^5 m_1^3 m_2^3}{r^5} \left[-\frac{91}{450} + \frac{1}{15} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{34} = \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[\frac{13}{5} - \frac{2}{3} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{38} = \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[\frac{147}{25} + \frac{8}{15} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{41} = \frac{G_N^5 m_1^3 m_2^3}{r^5} \left[\frac{49}{18} + \frac{1}{3} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{43} = -\frac{G_N^5 m_1^3 m_2^3}{r^5} \left[\frac{53}{150} + \frac{2}{15} \mathcal{P} \right],
$$
\n
$$
\mathcal{L}_{48} = \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[\frac{578}{75} + \frac{8}{5} \mathcal{P} \right],
$$

$$
\begin{aligned}\n\zeta_{29} &= \frac{G_N^5 m_1^3 m_2^3}{r^5} \left[-\frac{409}{450} + \frac{1}{5} \mathcal{P} \right], \\
\zeta_{33} &= \frac{G_N^5 m_1^3 m_2^3}{r^5} \left(16 - \pi^2 \right), \\
\zeta_{37} &= -\frac{G_N^5 m_1^4 m_2^2}{r^5} \left[17 + 2 \mathcal{P} \right], \\
\zeta_{40} &= \frac{G_N^5 m_1^4 m_2^2}{r^5} \left[-\frac{39}{25} + \frac{4}{15} \mathcal{P} \right], \\
\zeta_{42} &= -\frac{G_N^5 m_1^3 m_2^3}{r^5} \left[\frac{97}{225} + \frac{1}{15} \mathcal{P} \right], \\
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\zeta_{49} &= \frac{G_N^5 m_1^3 m_2^3}{r^5} \left(32 - 3\pi^2 \right),\n\end{aligned}
$$

 ϵ

$$
\mathcal{L}_{50} = \frac{G_N^5 m_1^3 m_2^3}{r^5} \left(4\pi^2 - \frac{124}{3} \right)
$$

$$
\frac{\frac{1}{15}\mathcal{P}}{\frac{1}{15}\mathcal{P}},
$$
\n
$$
\left(\mathcal{P}\equiv \frac{1}{\varepsilon} - 5\log\frac{r}{L_0}\right),
$$
\n
$$
L = \sqrt{4\pi e^{\gamma_E}}L_0
$$
\n
$$
\Lambda^{-2} \equiv 32\pi G_N L^{d-3}
$$

binaries, *Living Reviews in Relativity* 5 (2002), no. 3.

A³³ = = − i (8⇡*G^N*)

diverge @ d=3 \parallel

binaries, *Living Reviews in Relativity* 5 (2002), no. 3.

[1] L. Blanchet, *Gravitational radiation from post-newtonian sources and inspiralling compact*

always cancels out in the expression of physical observables.

length scale which keeps the correct dimensions of ⇤ in dimensional regularization, and

In this framework, a Kaluza-Klein (KK) parametrization of the metric [32, 33] is usually

i=1*,*2

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$$
\n
$$
\mathcal{L}_{10} = \frac{G_N^5 m_1^3 m_2^3}{r^5} \left(32 - 3\pi^2 \right),
$$
\n
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$$
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$$
\mathcal{P} \equiv \frac{1}{\varepsilon} - 5 \log \frac{r}{L_0}
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$$
L=\sqrt{4\pi {\rm e}^{\gamma_E}}L_0
$$

(Impact on) **The 4PN O(G^5) Lagrangian** L*G*5 *N* 4*P N* = 50 1 **N** ∪(∪¹′′) Lagranglan

Foffa, Sturani, Sturm, & **P.M.**

Damour, Jaranowski

Total contribution

$$
\sum_{a=1}^{50} \mathcal{L}_a = \frac{3}{8} \frac{G_N^5 m_1^5 m_2}{r^5} + \frac{31}{3} \frac{G_N^5 m_1^4 m_2^2}{r^5} + \frac{141}{8} \frac{G_N^5 m_1^3 m_2^3}{r^5}.
$$

^a=¹ ^L*^a* represents the main result of this work, and it amounts to

Towards 5PN-O(G^6)

Vacuum Diagrams for Newton Potential *r* **Pracuding Diagrams for** *r* **Vacuum Diagrams for New** *m* Diegrens for Newton Potential *n* **Newton Potential** = *i* ⇤ ⁺ *^O*(*v*2) + *^O*(*v*3) + *^O*(*v*4) (2.4) $\frac{1}{2}$ (*p k*)² *^dd^k ^eik·^r* **h** Z *eip·^r* Z <u>ent</u> $\overline{}$ (*p k*)² *^dd^k ^eik·^r k*2 = *ddp ei*(*pk*)*·^r* **Ne** $\frac{d}{dx}$ **h**

Eoffa, Sturani, Sturm, 2014.
Torres-Bobadilla & **P.M.** Foffa, Sturani, Sturm, (coming soon)

r **Newton Potential**

Amplitudes

^dd^k ¹

d $\overline{}$ *static source* (*p k*)² *k*² = 1t *ddp* es <==> non prop *ddp* \le = > pinching lines \overline{a} *d*_{pro} non propagating <==> pinching lines *ddp eip·^r* **static sources <==> non propagating <==> pinching lines**

r ⇠ *^v*² *<<* 1 (2.5) 2.1 Diagrammatica for Newton potential ostNewtonian Correct 2.1 Diagrammatica for Newton potential *r* ⇠ *^v*² *<<* 1 (2.5) **PostNewtonian Corrections**

² PN correction :: explicit calculation $\frac{1}{2}$ *d*
d^{*r*}/*d*

Z $d^d p \; e^{i p \cdot r}$ Z $\int d^dp e^{ip\cdot r}$ $\int d^d p \, e^{ip \cdot r}$ *p*2 (2.6) *p*2 (2.6) *p*2 (2.6) *p* = *^p*² (2.6)

$$
= \int d^d p \; e^{ip \cdot r} \int d^d k \; \frac{1}{(p-k)^2 \; k^2}
$$

$$
=\int d^dp\; \frac{e^{i(p-k)\cdot r}}{(p-k)^2}\int d^dk\; \frac{e^{ik\cdot r}}{k^2}
$$

 $e^{ik \cdot r}$ $\int^{a} P^2 \int^{a} \frac{R}{f}$ **k**² *ei*(*pk*)*·^r* (*^p ^k*)² ⇥ $\frac{d}{p}p \frac{e^{i p \cdot t}}{p^2}\int d^d k$ \dot{p} *^dd^k ^eik·^r* e^{ik} = Z d^dp $e^{ip\cdot r}$ $p^{\mathbf{2}}$ $\int d^d k \frac{e^{ik \cdot r}}{l^2}$ *k*2

Foffa, Stur *^p*² (2.6) Foffa, Sturani, Sturm, Torres-Bobadilla & **P.M.** (coming soon)

> (2.7) Static 1PN = Newton^2

r ⇠ *^v*² *<<* 1 (2.5) 2.1 Diagrammatica for Newton potential **PostNewtonian Corrections**

2.1 PN correction :: diagrammatic $\overline{\mathbf{n}}$ *ddd* $\frac{1}{2}$ **1PN correction :: diagrammatic**

> Z $d^d p \; e^{i p \cdot r}$ Z $\int d^dp e^{ip\cdot r}$ $\int d^d p \, e^{ip \cdot r}$ $\int d^d x e^{ip \cdot r}$

> > *eip·^r*

=

Z *^dd^k* ¹

1

^dd^k ¹

(*p k*)² *k*²

p
p2 (2.6) and produced by the product of the product

Foffa, Stur **ric** *v v v v v v (coming soon) (coming soon)* Foffa, Sturani, Sturm, Torres-Bobadilla & **P.M.** (coming soon)

r ⇠ *^v*² *<<* 1 (2.5) 2.1 Diagrammatica for Newton potential **PostNewtonian Corrections**

2.1 PN correction

Foffa, Stur **v**2 *v*2 *v*2 *v*² *v*² *v*² *(coming soon) v*² *v*² *v* **v**2 *v*2 *v*2 *v*² *v*² *v*² *v*² *v*² *(coming soon) v*² *v*² *v* Foffa, Sturani, Sturm, Torres-Bobadilla & **P.M.** (coming soon)

^ddk ei(*pk*)*·r*+*ik·^r* ¹ = $\overline{ }$ *d* Z *^ddk ei*(*pk*)*·r*+*ik·^r* ¹ (*p k*)² *k*² Z (*p k*)² *k*² *eip·^r* **static sources <==> non propagating <==> pinching lines** *ddp eip·^r*

m G^N m r **PostNewtonian Corrections** Expediance of the Correction of *Posta, Sturani, S*
Foffa, Sturani, S *m*

2.1 Diagrammatica for Newton potential 2.1 Diagrammatica for Newton potential 2.1 Diagrammatica for Newton potential **(2j+1)PN correction**

Foffa, Sturani, Sturm, Foffa, Sturani, Sturm, Torres-Bobadilla & **P.M.** (coming soon)

m G^N m r ⇠ *^v*² *<<* 1 (2.5) = *i m* ⇤ ⁺ *^O*(*v*2) + *^O*(*v*3) + *^O*(*v*4) (2.4) **PostNewtonian Corrections** Expediance of the Correction of *Posta, Sturani, S*
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Foffa, Sturani, Sturm, Foffa, Sturani, Sturm, Torres-Bobadilla & **P.M.** (coming soon)

<mark>es <==> non p</mark> *ing lines*

⇤ ⁺ *^O*(*v*2) + *^O*(*v*3) + *^O*(*v*4) (2.4) $\dot{}$ 0 **PostNewtonian Corrections**

orred r 2.1 diagrammatica for $\ddot{}$ **(2j+1)PN correction**

v
 v2 *v*2 *v*² *v* Foffa, Sturani, Sturm, Torres-Bobadilla & **P.M.** (coming soon)

*e¹·<i><i>r***₁</sup>** \ddot{p} <u>zeci ci (-1)</u> *^dd^k* ¹ **static (2j+1)-PN diagrams as product of (2j)-PN diagrams and the Newtonian term**

Amplitudes @ 4PN - O(G^5) Foffa, Sturani, Sturm, & P.M.

50 Amplitudes @ 4 loop: 25 of them…

 $\sqrt{ }$

and its Taylor expansion provides the various particle-gravity vertices of the EFT.

Foffa, Sturani, Sturm, Torres-Bobadilla & **P.M.** (coming soon)

Digital properties of the set of the UNIC(G^5) diagram + 1 Phi > \cdots **< insertion** 1 "N O(G^5) diagram + 1 Phi

Foffa, Sturani, Sturm, Torres-Bobadilla & **P.M.** (coming soon)

 $D = 1$ is the state of U or $D(G^55)$ diagram + 1 Phi > \cdots insertion $\overline{1}$ **a** \bigcirc 2(G[^]5) diagram + 1 Phi

(∇⃗)² [≡] *abcdijab,icd,j* and *ij* ⁼ (−¹)*ij* (and on the second line *ij* ⁼ *ij* , ⁼ *ijij*).

Summary ...

Multi-Loop Diagrammatic Techniques Powerful tools for General Relativity IBPs + Difference & Differential Equations

Application 1 **:: Coalescent Binaries Dynamics @ 4PN-O(G^5) Basic idea @** *Amplitudes* **:: EFT-GR Diagrams ~ 2-point QFT Diagrams 4-loop EFT-GR Diagrams mapped into 4-loop QFT Problem**

Application 2 **:: Coalescent Binaries Dynamics @ 5PN-O(G^6) Basic idea @** *Potential* **:: EFT-GR Diagrams ~ Vacuum Diagrams + factorisation 5-loop EFT-GR Diagrams mapped into 5-loop factorised Vacuum Diagrams**

... and Outlook

How about more-legs (diagrams with radiation) ?

- CR & GW-physics EFT vs HEP & Amplitudes
- Unitarity-based methods, Multi-loop Integrals and Integrands decomposition, Amplitudes-inspired dualities, BCJ/Double-copy
- UBPs for Fourier Transform Integrals
- Post-Minkowskian approximation: v-expansion *vs* complete-v dependence

Status of PN Corrections Porto

no spin spin spin

Blanchet, Damour '88 Foffa, Sturani Le Tiec, Blanchet, Whiting Jaranowski, Schaefer Jaranowski, Schaefer Damour, Jaranowski, Schaefer Bernard, Blanchet, Bohe, Faye, Marsat Foffa, Sturani, Sturm, & **PM** Damour, Jaranowski

3PN Damour, Jaranowski, Schaefer Blanchet, Faye De Andrade, Esposito-Farese, Itho, Futamase

2PN Damour, Deruelle '81-'83 Damour, Schaefer '85

1PN Einstein-Infeld-Hoffman 1938

OPN Newton 1687

NNLO Steir

Steinhoff

Porto, Rothstein Buonanno, Faye, Blanchet Damour, Jaranowski, Schaefer Steinhoff, Hergt, Schaefer

NLO

Barker, O'Connel '75

4PN

Binary coalescence: a tale made of three stories

Inspiral phase post-Newtonian approximation: *v/c*

Merger: fully non-perturbative: Nu- Perturbed merical Relativity Ring-down: Kerr Black Hole

> Spin can induce precession and change the amplitude (and phase) of the waveform due to $cos(\theta_{LN})$ factors in $h_{+,\times}$

Riccardo Sturani (IFT-UNESP/ICTP-SAIFR) GW Detection Pedra Azul - Sept 29

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Was it necessary to build a detector? The Hulse-Taylor binary pulsar

GW's first observed in the NS-NS binary system PSR B1913+16 Observation of orbital parameters $(a_p\sin\iota,\; e,\; P,\; \dot{\theta},\; \gamma,\; \dot{P})$

determination of *mp*, *m^c* (1PN physics, GR)

Energy dissipation in GW's \rightarrow $\dot{P}^{(GR)}(m_p, m_c, P, e)$ vs. $\dot{P}^{(obs)}$

$$
\frac{1}{2\pi}\phi = \int_0^T \frac{1}{P(t)} dt \simeq \frac{T}{P_0} - \frac{\dot{P_0}}{P_0^2} \frac{T^2}{2}
$$

• Test of the 1PN conservative

$$
E(v) = -\frac{1}{2} \nu M v^2 (1 + \#(\nu) v^2 + \#(\nu) v^4 + \ldots)
$$

• leading order dissipative dynamics

$$
F(v) \equiv -\frac{dE}{dt} = \frac{32}{5G_N}v^{10}\left(1+\#(\nu)v^2 + \#(\nu)v^3 + \ldots\right)
$$

Modeling the inspiral

 \ln **Spiral** $h = A \cos(\phi(t))$ $\frac{\dot{A}}{4} \ll \dot{\phi}$ Virial relation:

$$
v \equiv (G_N M \pi f_{GW})^{1/3}
$$
 $v = \frac{m_1 m_2}{(m_1 + m_2)^2}$

$$
E(v) = -\frac{1}{2}\nu Mv^2 (1 + \#(\nu)v^2 + \#(\nu)v^4 + ...)
$$

$$
P(v) = -\frac{dE}{dt} = \frac{32}{5G_N}v^{10} (1 + \#(\nu)v^2 + \#(\nu)v^3 + ...)
$$

 $E(v)(P(v))$ known up to 3(3.5)PN

 $\frac{\dot{P}_{GR}-\dot{P}_{exp}}{\dot{P}}\sim10^{-3}$

$$
\frac{1}{2\pi}\phi(\mathcal{T}) = \frac{1}{2\pi}\int_{0}^{\mathcal{T}}\omega(t)dt = -\int_{0}^{\mathcal{V}}\frac{\omega(v)dE/dv}{P(v)}dv
$$

$$
\sim \int_{0}^{\mathcal{T}}(1+\#(\nu)v^{2}+\ldots+\#(\nu)v^{6}+\ldots)\frac{dv}{v^{6}}
$$

Post-Newtonian Coefficients $A \equiv \mathbf{1} \times \mathbf{1} \oplus \mathbf{1} \times \mathbf{1} \oplus \mathbf{1} \times \mathbf{1} \oplus \mathbf{1} \times \mathbf{1}$

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Weisberg and Taylor (2004)

10 pulsars in NS-NS, still \sim 100Myr for coalescence $\frac{1}{\sqrt{2}}$

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