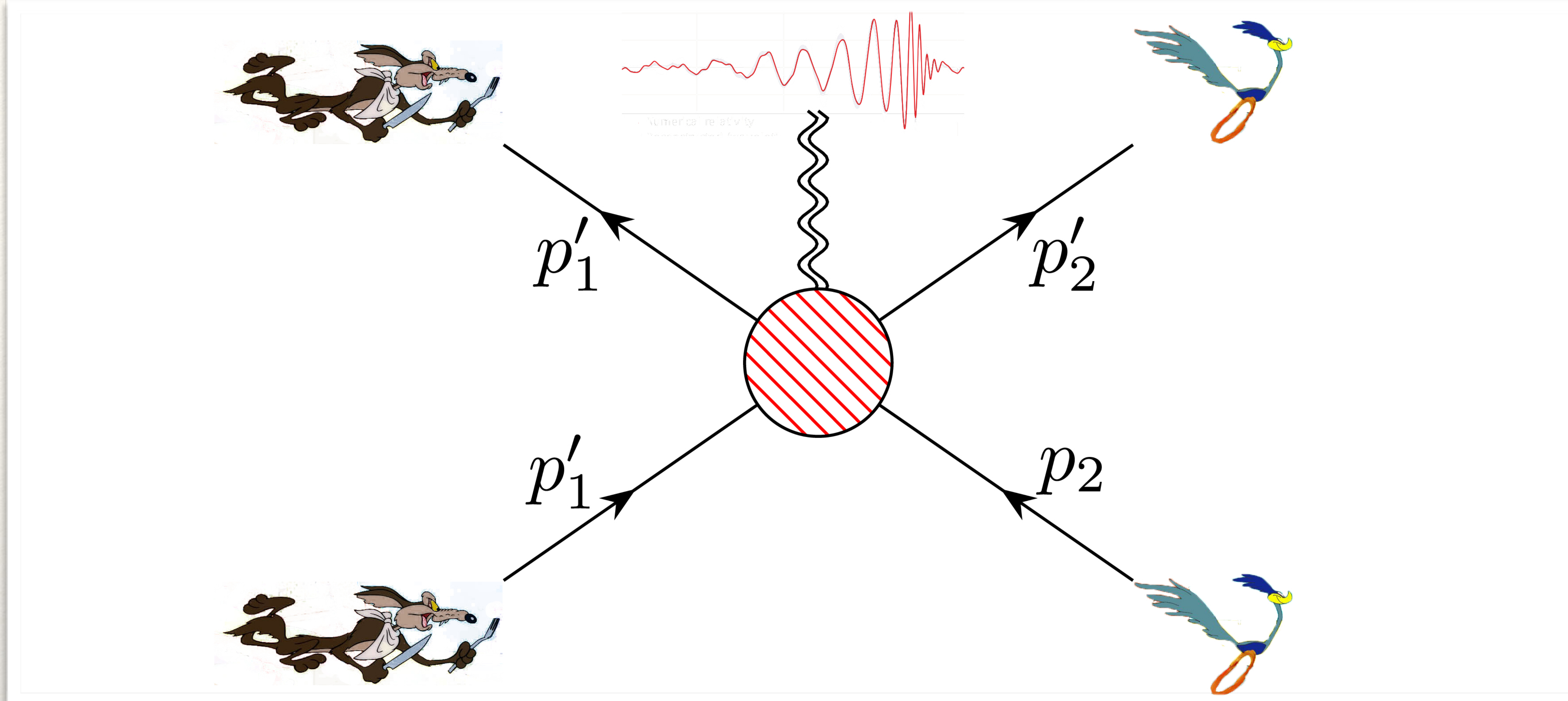
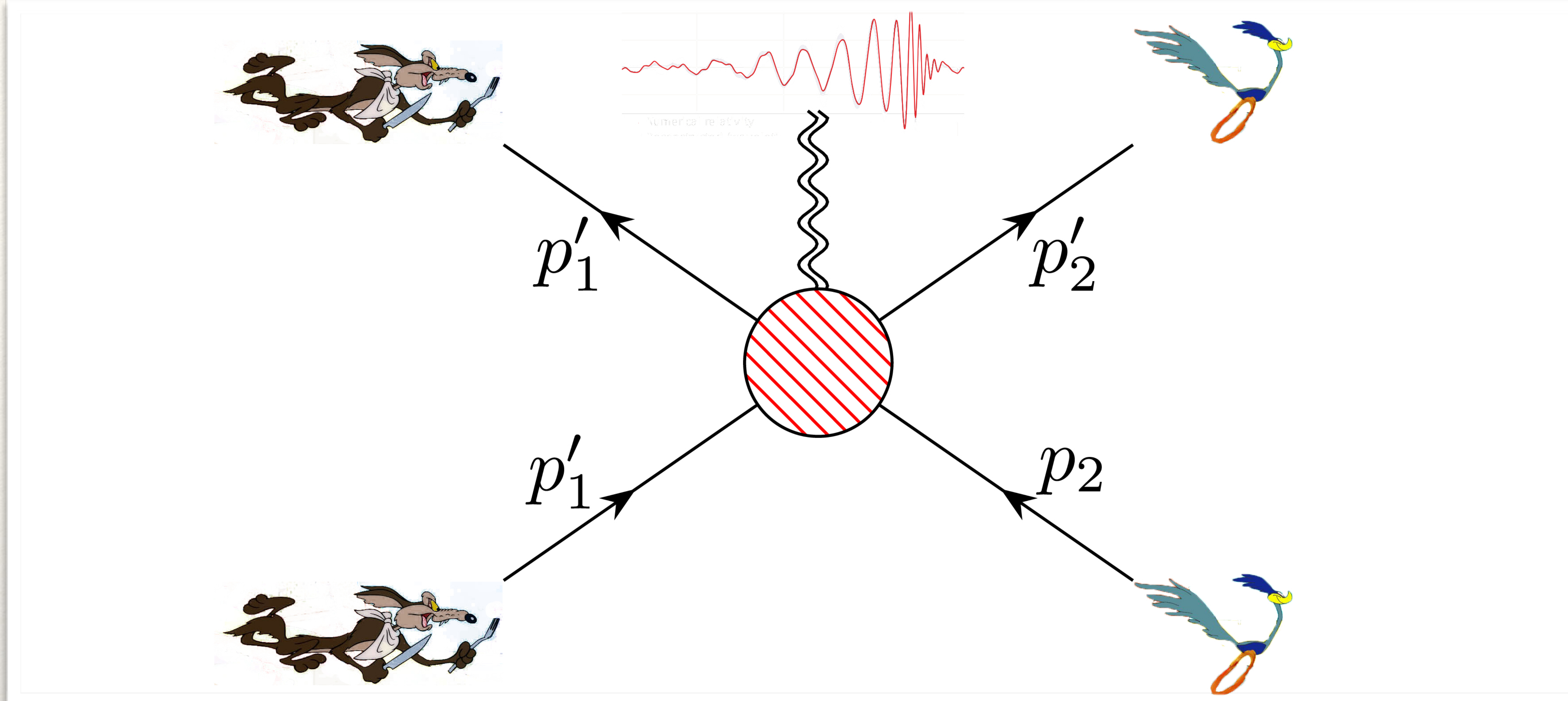


Amplitudes for Fast Classical Scattering

Donal O'Connell
University of Edinburgh



Amplitudes for Fast Classical Scattering

Andrés Luna, Isobel Nicholson, Chris White 1711.03901
David Kosower, Ben Maybee 180x.xxxxx

Motivation

- Gravitational wave physics: potential and flux terms in PN EFT
  - Get flux terms by comparing predictions for fast scattering → *Alessandra's talk*
- Goldberger & Ridgway: LO step using double copy-like rule
  - At higher order, probably need a more systematic double copy
- So want to understand classical limit of momentum flux in QFT
  - This limit is intricate

    Donoghue, Holstein, …

    Bjerrum-Bohr, Damgaard, Festuccia, Planté, Vanhove
- Make contact with Abraham-Lorentz-Dirac radiation reaction

Talk outline

1. Radiated momentum in field theory
2. Classical limits
3. Momentum deflection
4. Radiation reaction and the Abraham-Lorentz-Dirac force

Radiated Momentum

Incoming state

- For simplicity, scatter different distinguishable, stable, scalar particles
- In the far past, prepare two well-separated localised states as usual:

$$|\psi\rangle_{\text{in}} = \int Dp_1\, Dp_2\, e^{ib\cdot p_1} \phi_1(p_1)\phi_2(p_2)|p_1, p_2\rangle_{\text{in}}$$

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$$Dp_1 \equiv \frac{d^4 p_1}{(2\pi)^4}(2\pi)\delta(p_1^2 - m_1^2)$$

Integral over massive on-shell phase space

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Particles displaced by impact parameter, *b*

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Integral over massive on-shell phase space

Particles displaced by impact parameter, b

Wavefunctions: peaked at incoming momenta mu^μ.

Radiation

- Always assume particles 1 and 2 present in final state
- Expectation of radiated momentum is

$$\langle k^\mu \rangle = \sum_X \int Dp_1 Dp_2 Dk \, k^\mu \, |\langle p_1 p_2 k X | S | \psi \rangle|^2$$

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Time evolution operator

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Sum over states

Since $\langle p_1 p_2 k X | \psi \rangle = 0$, can replace $S \to T$

- An on-shell observable, in both classical and quantum theories

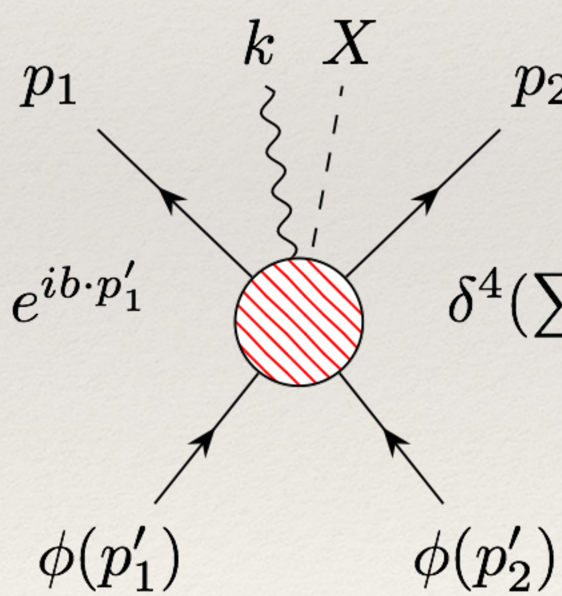
Radiation

$$\langle k^\mu \rangle = \sum_X \int Dp_1 Dp_2 Dk \, k^\mu \, |\langle p_1 p_2 k X | S | \psi \rangle|^2$$

- Insert the initial state $|\psi\rangle_{\text{in}} = \int Dp_1 \, Dp_2 \, e^{ib\cdot p_1} \phi_1(p_1) \phi_2(p_2) |p_1, p_2\rangle_{\text{in}}$

- Find:

$$\langle k^\mu \rangle = \textstyle\sum_X \int Dp_1 Dp_2 Dk \; k^\mu \left| \int Dp_1' Dp_2' \; e^{ib\cdot p_1'} \right.$$



$$\left. \delta^4(\textstyle\sum p) \right|^2$$

Classical Limit

Classical Limit

- If momentum transferred is very long wavelength compared to modes in wavefunctions, point particle approximation should be valid
- Then result should equal classical value, plus quantum corrections
  1. Take mass m very large: point particle EFT
  2. Take $\hbar$ to zero
- Also helpful to think about wavefunction

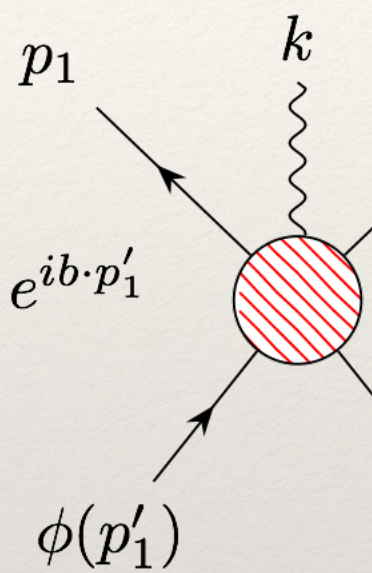
Classical Limit

- When $\hbar$ goes to zero, want particle's position *and* momentum to be sharply defined
- Appropriate wavefunction looks like $\psi(p) \sim \mathcal{N} \exp\left[\frac{(p-\bar{p})^2}{2\hbar\sigma^2}\right]$
- Encounter integrals like $\int Dp\, \psi^*(p+q)\psi(p) = \exp\left[-\frac{q^2}{4\hbar\sigma^2}\right]$
  - So exponentially suppressed unless shift $q \sim O(\hbar)$
  - Write $q = \hbar\bar{q}$, then $\bar{q} \sim O(1)$ is a wavenumber

Classical Limit

- Classical limit of radiated momentum (only photons for simplicity)

$$\langle k^\mu \rangle = \int Dp_1 Dp_2 Dk\; k^\mu \left| \int Dp_1' Dp_2'\; e^{ib\cdot p_1'} \right.$$



$$\left. \delta^4\left(\sum p\right) \right|^2$$

$$\to \int D\bar{k}\; \bar{k}^\mu\; \hbar^7 \left| \int d^4\bar{q}_1 d^4\bar{q}_2 \delta(2m_1 u_1 \cdot \bar{q}_1 + \hbar \bar{q}_1^2) \delta(2m_2 u_2 \cdot \bar{q}_2 + \hbar \bar{q}_2^2)\; e^{ib\cdot\bar{q}_1} \right.$$

$$\left. \times\, \bar{\mathcal{A}}(\bar{q}_1, \bar{q}_2)\; \delta^4(\bar{q}_1 + \bar{q}_2 - \bar{k}) \right|^2$$

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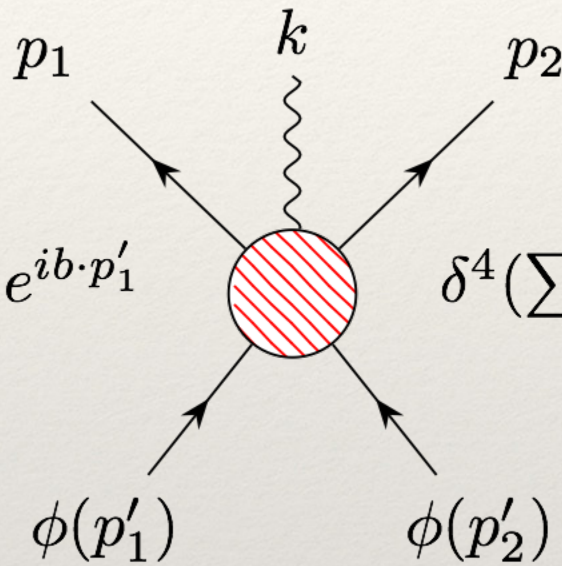

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$$\left. \times\, \bar{\mathcal{A}}(\bar{q}_1, \bar{q}_2)\; \delta^4(\bar{q}_1 + \bar{q}_2 - \bar{k}) \right|^2$$

Quite a lot of $\hbar$'s

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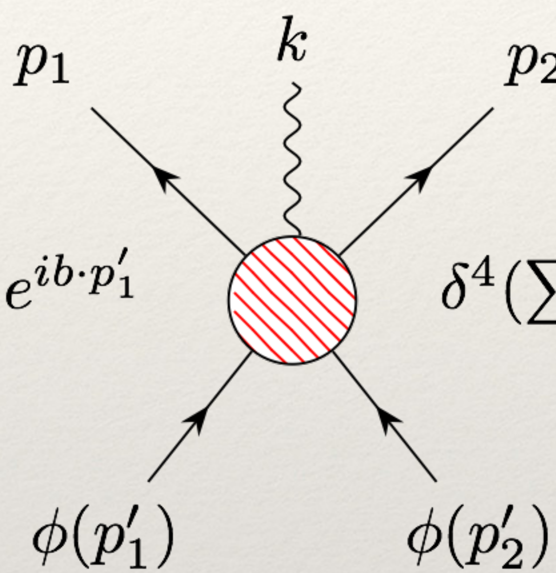

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Will see these may be relevant

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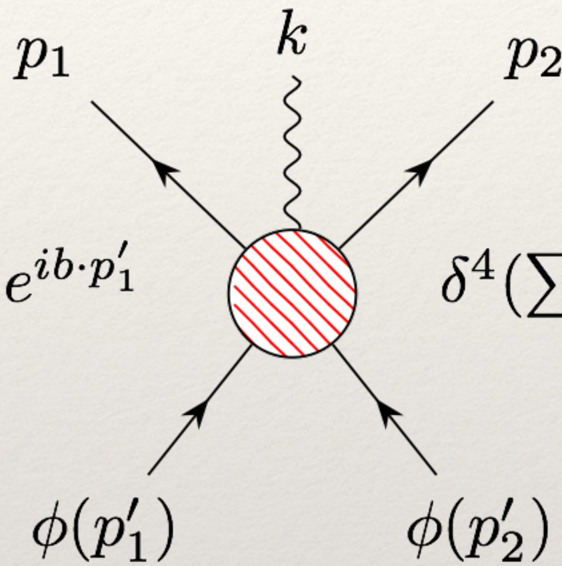

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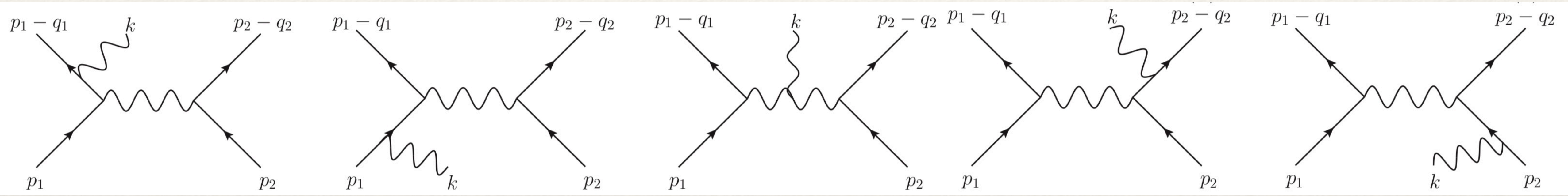
Mass expanded 5 point amplitude

Classical Limit

- Leading order in mass expansion:



Classical Limit

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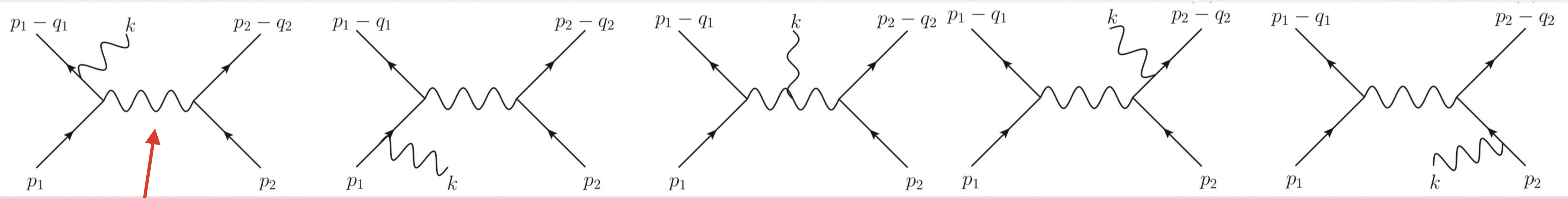
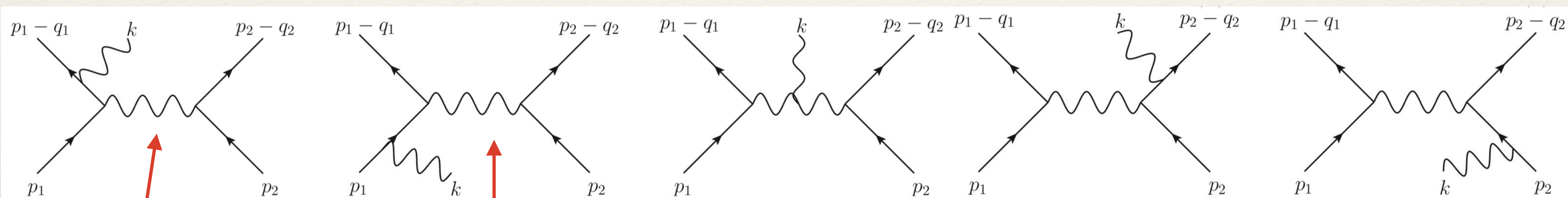


$$n_A = 8m_1^2 m_2\, u_1 \cdot u_2\, \epsilon \cdot u_1$$

$$d_A = 2m_1\, u_1 \cdot k\, q_2^2$$

Classical Limit

- Leading order in mass expansion:



$$n_B = 8m_1^2 m_2\, u_1 \cdot u_2\, \epsilon \cdot u_1$$

$$d_B = -2m_1\, u_1 \cdot k\, q_2^2$$

$$n_A = 8m_1^2 m_2\, u_1 \cdot u_2\, \epsilon \cdot u_1$$

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Classical Limit

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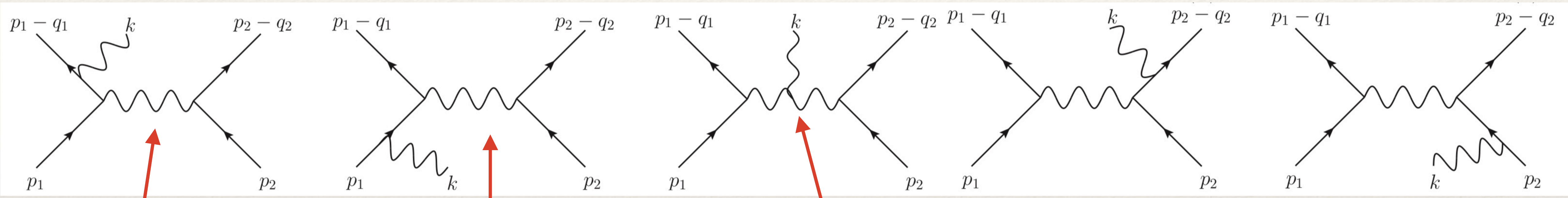

$$n_C = n_A - n_B = 0$$

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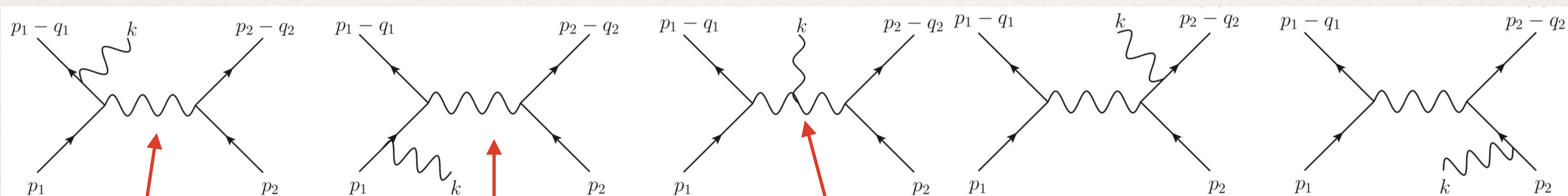

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$$\Rightarrow \bar{\mathcal{M}}_{\mathrm{LO}} = \frac{n_A^2}{d_A} + \frac{n_B^2}{d_B} + \cdots = 0$$

Classical Limit

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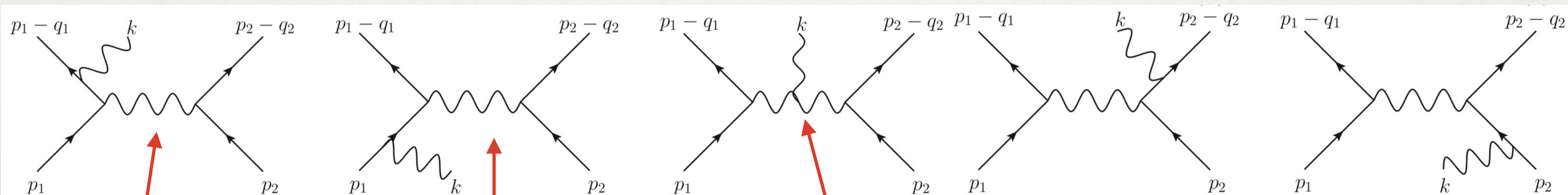

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$$\Rightarrow \bar{\mathcal{M}}_{\text{LO}} = \frac{n_A^2}{d_A} + \frac{n_B^2}{d_B} + \cdots = 0$$

- So need to go to next order in mass expansion

Classical Limit

- This is good: $\mathcal{M}_{\rm LO}$ otherwise has wrong power of $\hbar$

$$n_A = 8m_1^2 m_2\, u_1 \cdot u_2\, \epsilon \cdot u_1 \qquad \hbar^0$$

$$d_A = 2m_1\, u_1 \cdot k\, q_2^2 \qquad \hbar^{-3}$$

$$\kappa^3 = \left(\frac{32\pi G}{\hbar}\right)^{\frac{3}{2}} \qquad \hbar^{-\frac{3}{2}}$$

- Radiation has two powers of amplitude, so this is two too many $\hbar$'s
- Next to leading order: correct scaling
    - Quantum corrections in delta functions become important

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- Radiation has two powers of amplitude, so this is two too many $\hbar$'s
- Next to leading order: correct scaling ← Happens frequently
  - Quantum corrections in delta functions become important

Classical Limit

Classical Limit

- Double copy leads to

$$\bar{\mathcal{M}} = 16 m_1^2 m_2^2 \, \epsilon_\mu \epsilon_\nu \Bigg[4 \frac{P_{12}^\mu P_{12}^\nu}{q_1^2 q_2^2} + 2 \frac{u_1 \cdot u_2}{q_1^2 q_2^2} \left(Q_{12}^\mu P_{12}^\nu + Q_{12}^\nu P_{12}^\mu \right) + (u_1 \cdot u_2)^2 \left(\frac{Q_{12}^\mu Q_{12}^\nu}{q_1^2 q_2^2} - \frac{P_{12}^\mu P_{12}^\nu}{(k \cdot u_1)^2 (k \cdot u_2)^2} \right) \Bigg] .$$

Luna, Nicholson, White, DOC

Classical Limit

$$P_{12}^\mu \equiv k \cdot u_1 \, u_2^\mu - k \cdot u_2 \, u_1^\mu$$

- Double copy leads to

$$Q_{12}^\mu \equiv (q_1 - q_2)^\mu - \frac{q_1^2}{k \cdot u_1} u_1^\mu + \frac{q_2^2}{k \cdot u_2} u_2^\mu$$

$$\begin{aligned}\bar{\mathcal{M}} =& 16 m_1^2 m_2^2 \, \epsilon_\mu \epsilon_\nu \left[4 \frac{P_{12}^\mu P_{12}^\nu}{q_1^2 q_2^2} + 2 \frac{u_1 \cdot u_2}{q_1^2 q_2^2} \left(Q_{12}^\mu P_{12}^\nu + Q_{12}^\nu P_{12}^\mu \right) \right. \\ & \left. + (u_1 \cdot u_2)^2 \left(\frac{Q_{12}^\mu Q_{12}^\nu}{q_1^2 q_2^2} - \frac{P_{12}^\mu P_{12}^\nu}{(k \cdot u_1)^2 (k \cdot u_2)^2} \right) \right] .\end{aligned}$$

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- By investigating factorisation channels, easy to remove dilaton:

$$\bar{\mathcal{M}}_{\mathrm{GR}} = 16 m_1^2 m_2^2 \; \epsilon_{\mu}\epsilon_{\nu} \left[4 \frac{P_{12}^{\mu} P_{12}^{\nu}}{q_1^2 q_2^2} + 2 \frac{u_1 \cdot u_2}{q_1^2 q_2^2} \left(Q_{12}^{\mu} P_{12}^{\nu} + Q_{12}^{\nu} P_{12}^{\mu} \right) + \left((u_1 \cdot u_2)^2 - \frac{1}{2} \right) \left(\frac{Q_{12}^{\mu} Q_{12}^{\nu}}{q_1^2 q_2^2} - \frac{P_{12}^{\mu} P_{12}^{\nu}}{(k \cdot u_1)^2 (k \cdot u_2)^2} \right) \right].$$

Luna, Nicholson, White, DOC

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Luna, Nicholson, White, DOC

- Insert into radiation formula; matches direct GR calculation
- Expression with dilaton leads to Goldberger & Ridgway's result

Deflection

Momentum Deflection

- Classical limit intricate: investigate other on-shell observables
- Expectation of incoming/outgoing momentum of eg particle 1:

$$\langle p_1^\mu \rangle \equiv \langle \psi | \hat{P}_1^\mu | \psi \rangle \qquad\qquad \langle p_1'^\mu \rangle = \langle \psi | S^\dagger \hat{P}_1^\mu S | \psi \rangle$$

- Operator $\hat{P}_1^\mu$: momentum operator of quantum field of particle 1
- Change in momentum is

$$\langle \Delta p_1^\mu \rangle = \langle \psi | i[\hat{P}_1^\mu, T] | \psi \rangle + \langle \psi | T^\dagger [\hat{P}_1^\mu, T] | \psi \rangle$$

- Other approaches: review Bjerrum-Bohr, Holstein, Donoghue, Planté & Vanhove (+ recent paper with Damgaard, Festuccia)

Momentum Deflection

$$\langle \Delta p_1^\mu \rangle = \langle \psi | i[\hat{P}_1^\mu, T] | \psi \rangle + \langle \psi | T^\dagger [\hat{P}_1^\mu, T] | \psi \rangle$$

- Two types of deflection:
  - momentum exchange (classically due to interaction force)
  - momentum radiated (radiation reaction)

$$\langle \Delta p_1^\mu \rangle = \langle \psi | i[\hat{P}_1^\mu, T] | \psi \rangle + \sum_X \langle \psi | T^\dagger | p_1 p_2 X \rangle \langle p_1 p_2 X | [\hat{P}_1^\mu, T] | \psi \rangle$$

Momentum Deflection

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Momentum delta function:
initial & final momenta equal

Momentum Deflection

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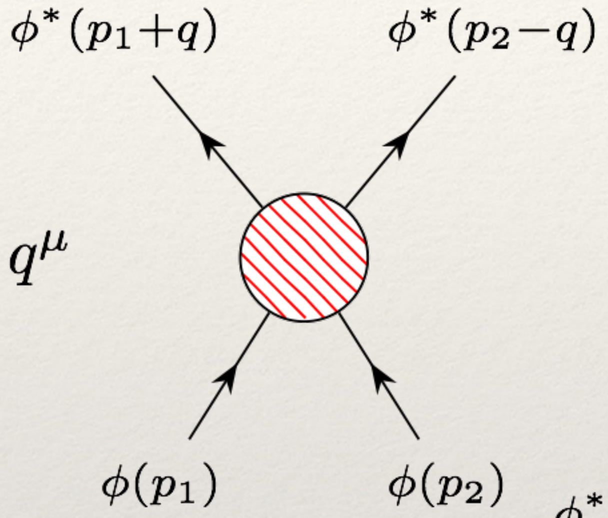
Momentum delta function: initial & final momenta equal

Intermediate states carrying momentum: radiation reaction

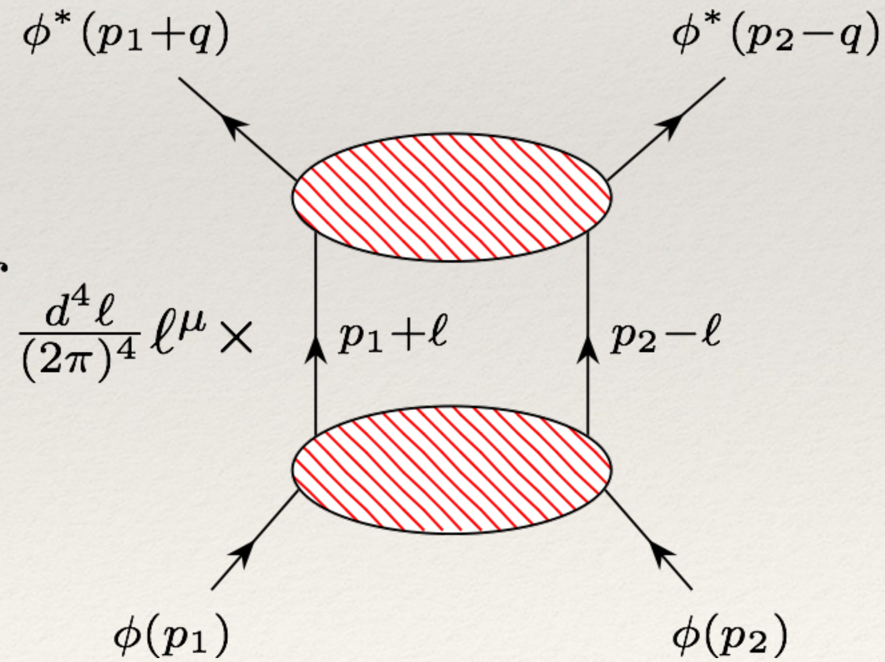
Momentum Deflection

- Complete momentum exchanged:

$$\langle \Delta p_1^\mu \rangle_{\text{exchange}} = \int_{\text{on-shell}} e^{-ib\cdot q}\; q^\mu$$



$$+\int_{\text{on shell}} e^{-ib\cdot q} \int \frac{d^4\ell}{(2\pi)^4}\,\ell^\mu \times$$



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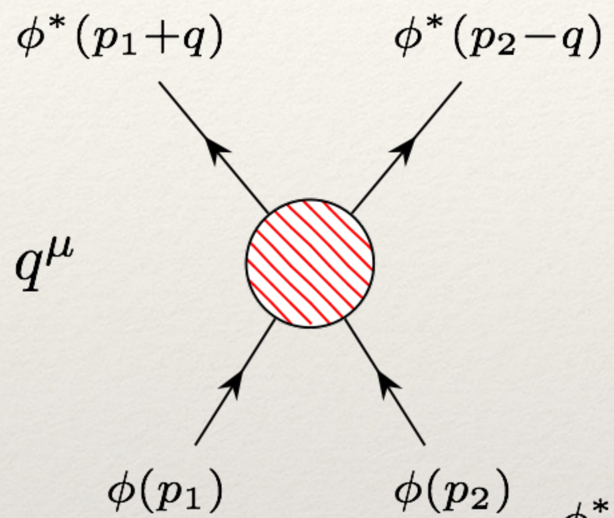

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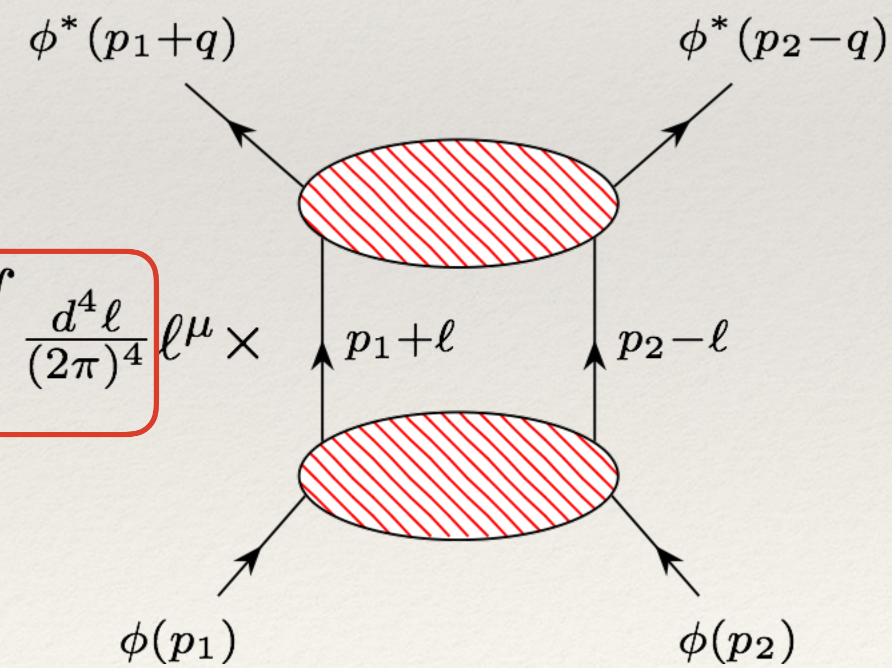
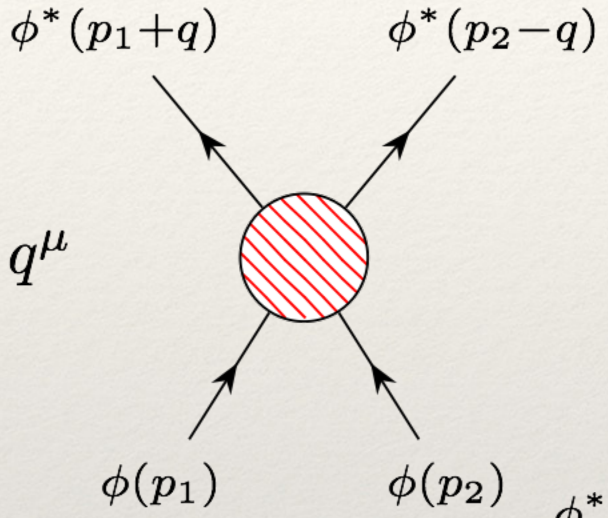


Integrate over undetermined cut momentum

Momentum Deflection

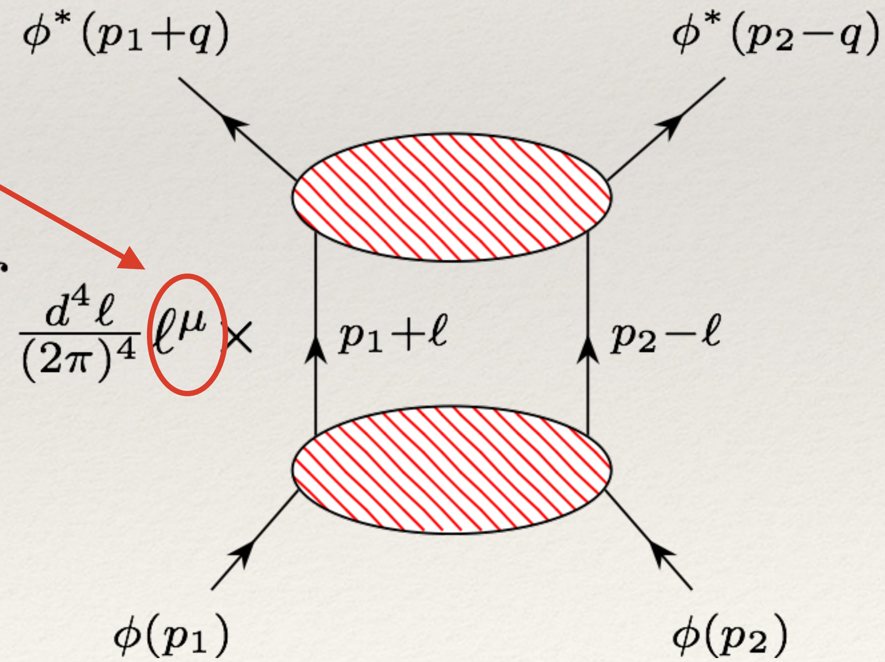
- Complete momentum exchanged:

$$\langle \Delta p_1^{\mu} \rangle_{\text{exchange}} = \int_{\text{on-shell}} e^{-ib\cdot q}\; q^{\mu}$$



Explicit factor of cut loop momentum

$$+\int_{\text{on shell}} e^{-ib\cdot q} \int \frac{d^4\ell}{(2\pi)^4}\, \ell^{\mu} \times$$



Momentum Deflection

- Take classical limit as before
- One loop agrees with direct NLO calculation using Lorentz force

- Cancellations between box, cross-box and cut term necessary to get correct power of $\hbar$

Radiation Reaction

Radiation Reaction

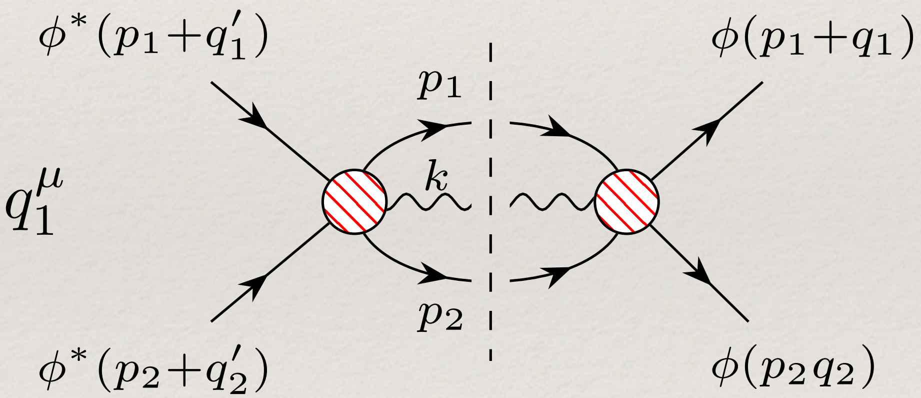
- The momentum of particle 1 lost to radiation is

$$\langle \Delta p_1^\mu \rangle_{\text{radiation}} = \sum_{X \neq \{\}} \int Dp_1 Dp_2 \langle \psi | T^\dagger | p_1 p_2 X \rangle \langle p_1 p_2 X | [\hat{P}_1^\mu, T] | \psi \rangle$$

$$= \int Dp_1 Dp_2 Dk \langle \psi | T^\dagger | p_1 p_2 k \rangle \langle p_1 p_2 k | [\hat{P}_1^\mu, T] | \psi \rangle$$

Simplest case

$$= \int_{\text{on shell}} e^{-ib\cdot(q_1 - q_1')} \, q_1^\mu$$



- Classically, this deflection is due to the Abraham-Lorentz-Dirac radiation reaction

Radiation Reaction

- The ALD force is

$$F^\mu = \frac{e^2}{6\pi m}\left[\frac{d^2p^\mu}{d\tau^2} - \frac{p^\mu}{m^2}\left(\frac{dp}{d\tau}\cdot\frac{dp}{d\tau}\right)\right]$$

- Contribution to deflection is simple

$$(\Delta p_1^\mu)_{\text{ALD}} = \int d^4q\, d^4q'\, e^{-i(q-q')\cdot b}\,\delta(q\cdot u_2)\delta(q'\cdot u_2)\delta((q-q')\cdot u_1)$$
$$\times \frac{e^6 u_1^\mu}{6\pi m_1^2}\frac{1}{q^2 q'^2}\left((u_1\cdot q)^2 + q\cdot q'(u_1\cdot u_2)^2\right)$$

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Case where $m_2 \gg m_1$ for simplicity

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Integral measure

$$(\Delta p_1^\mu)_{\mathrm{ALD}} = \boxed{\int d^4q\, d^4q'\, e^{-i(q-q')\cdot b}\, \delta(q\cdot u_2)\delta(q'\cdot u_2)\delta((q-q')\cdot u_1)}$$
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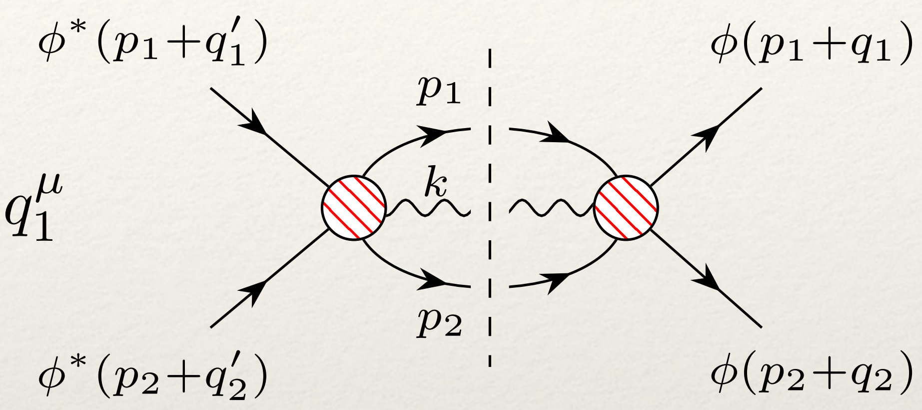
ALD kernel

Radiation Reaction

- The classical limit is

$$\langle \Delta p_1^\mu \rangle_{\text{radiation}} = \int_{\text{on shell}} e^{ib\cdot(q_1-q_1')}\, q_1^\mu$$



$$\rightarrow \int d^4q d^4q'\, \delta(q\cdot u_2)\delta(q'\cdot u_2)\delta\left((q-q')\cdot u_1\right) e^{-ib\cdot(q-q')}$$

$$\times \int Dk\, \delta\left((k-q)\cdot u_1\right)(k^\mu - q^\mu)\bar{\mathcal{A}}(q,k)\bar{\mathcal{A}}^*(q',k)\frac{1}{(4m_1m_2)^2}$$

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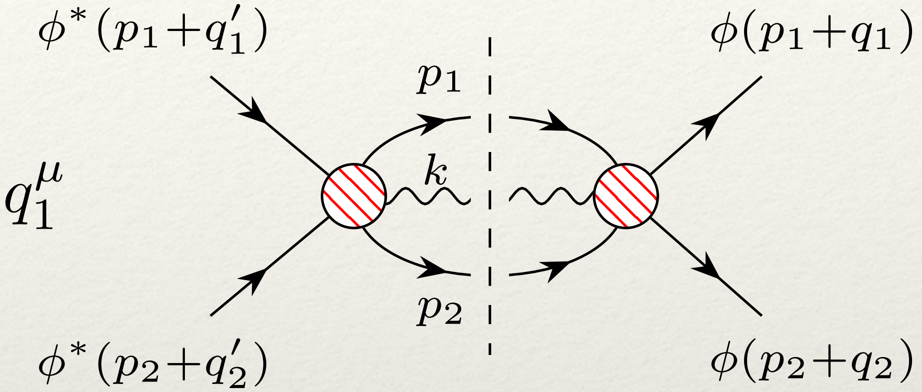

ALD measure

$$\to \int d^4q d^4q'\, \delta(q\cdot u_2)\delta(q'\cdot u_2)\delta\left((q-q')\cdot u_1\right) e^{-ib\cdot(q-q')}$$

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Radiation Reaction

- The mass-expanded amplitude isn't quite as simple as the ALD kernel

$$\bar{\mathcal{A}} = \frac{4e^3}{q^2} m_2 \left(\frac{u_1 \cdot u_2 \, q \cdot \varepsilon}{u_1 \cdot k} - \frac{k \cdot q \, u_1 \cdot u_2 \, u_1 \cdot \varepsilon}{(u_1 \cdot k)^2} + \frac{u_2 \cdot k \, u_1 \cdot \varepsilon}{u_1 \cdot k} - u_2 \cdot \varepsilon \right) + \cdots$$

- But photon phase space integral simplifies matters

$$\int Dk (k^\mu - q^\mu) \delta((k - q) \cdot u_1) \frac{|\bar{\mathcal{A}}|^2}{(4m_1 m_2)^2} \sim \frac{1}{q^2 q'^2} \left[(u_1 \cdot q)^2 + q \cdot q' (u_1 \cdot u_2)^2 \right]$$

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ALD kernel

Conclusions

- Can access many interesting classical observables with amplitudes
    - Gravitational radiation reaction, higher order momentum flux via double copy / any method of constructing GR amplitudes
- The classical limit is intricate: cancellations among various terms
- Further simplifications occur in practise
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