Amplitudes for precision collider physics

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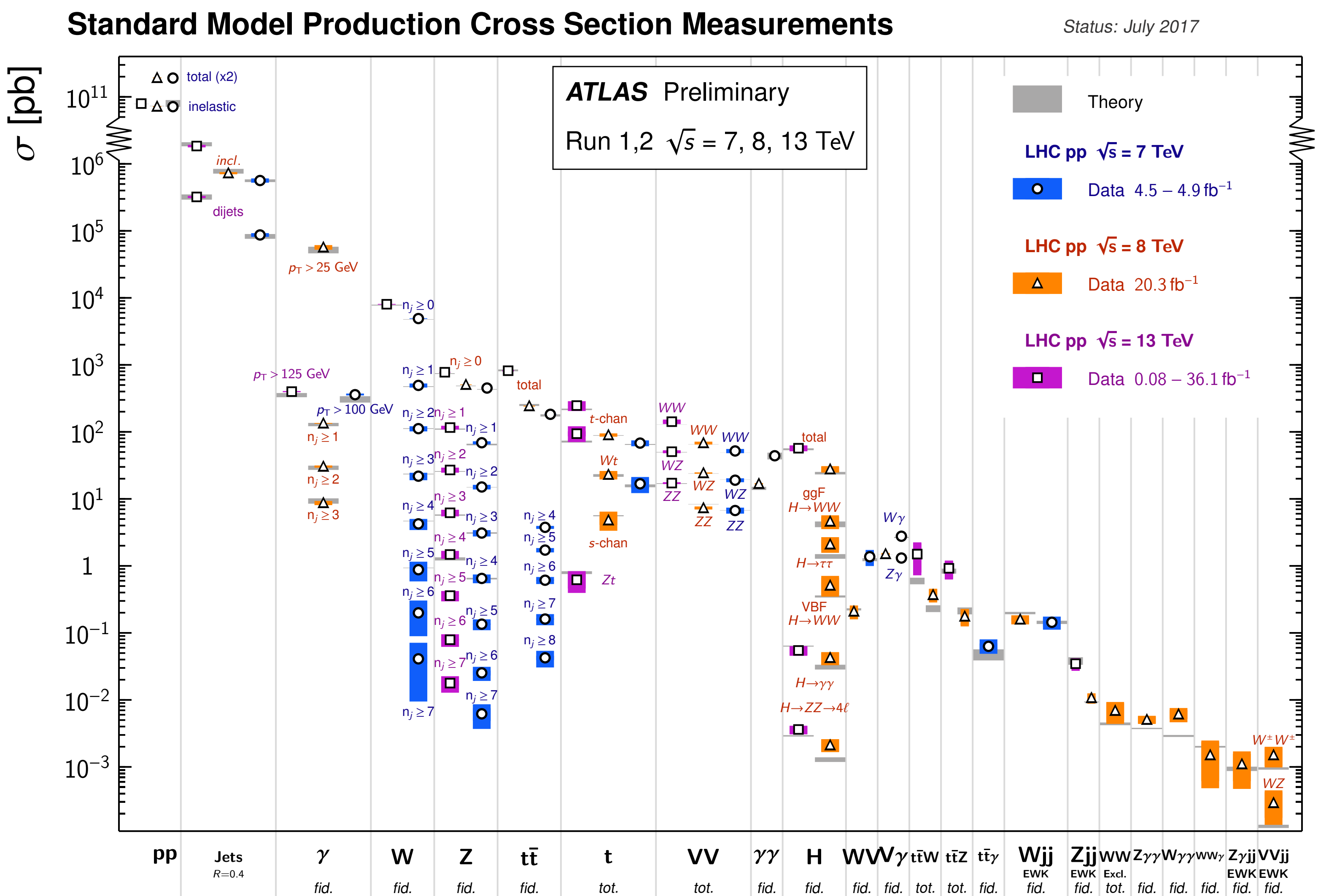
Benchmark processes at LHC



Benchmark processes

Low multiplicity

- Large cross sections
  - Precise multi-differential measurements
- Enable precision studies
  - Masses, couplings
  - Parton distributions
- Indirect tests of Standard model

High multiplicity

- New physics signatures
  - New particles, new interactions, dark matter,…
  - Observe only decay products
- Standard Model backgrounds
  - Combine theory and data-driven methods

Require precise Standard Model predictions

Higher order corrections

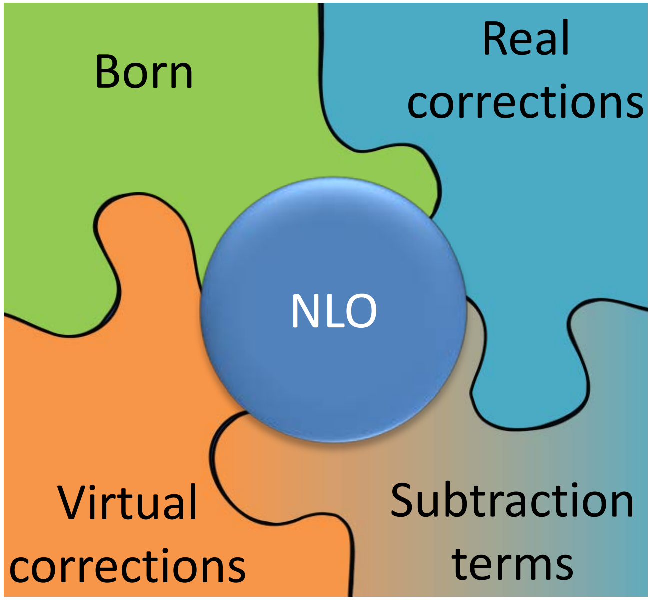
- Amplitudes for all relevant subprocesses
  - QCD and/or electroweak corrections
  - Virtual loop corrections and real radiation
- Fast and reliable numerical evaluation
  - Amplitude coefficients and master integrals
  - Generalized polylogarithms and beyond
- Phase space integration
  - Handle infrared divergences in subprocesses
  - Reproduce experimental observable definition
- Interplay of fixed order and resummation

Next-to-leading order

- Why NLO?
  - reduce scale uncertainty of LO theory prediction
  - reliable normalization and shape
  - accounts for effects of extra radiation
  - jet algorithm dependence
- Typical observations
  - sizable NLO corrections
  - corrections not constant, but kinematics-dependent
  - remaining uncertainty at NLO typically 10-20%
- NLO QCD and EW automated
  - Integral basis: maximally four-point functions
  - Methods: generalized unitarity, integrand reduction,…

NLO automation

- One-loop amplitudes
  - BlackHat (Z. Bern, L. Dixon, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre)
  - GoSam (G.Cullen, N.Greiner, G.Heinrich, G Luisoni, P. Mastrolia, G.Ossola, F.Tramontano)
  - RECOLA (S. Actis, A. Denner, L. Hofer, J.N. Lang, A. Scharf, S. Uccirati)
  - OpenLoops (F. Cascioli, P. Maierhöfer, S. Pozzorini)
  - NJet (S. Badger, B. Biedermann, P. Uwer, V. Yundin)
  - MadLoop/aMC@NLO (R. Frederix et al.)
  - CutTools (G. Ossola, C. Papadopoulos, R. Pittau)
- Real radiation, subtraction terms phase space (infrastructure)
  - From event generator programs
  - Well-defined interfaces (Binoth Les Houches accord)



Tools for NLO calculations

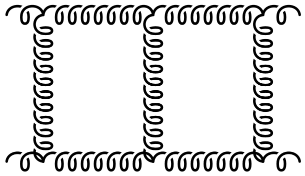
- MCFM (J. Campbell, K. Ellis, C. Williams) , VBFNLO (D. Zeppenfeld et al.)
  - Extensive libraries of NLO QCD processes
- MG5_aMC@NLO (F. Maltoni, S. Frixione et al.)
  - Full event generation with automation of one-loop amplitudes
  - Matching to parton shower (MC@NLO method)
- SHERPA (S. Höche, F. Krauss et al.)
  - Interfaces to one-loop codes (OpenLoops, BlackHat, Njet, GoSam)
  - Matching to parton shower (MC@NLO, POWHEG methods)
  - Matching of NLO multiplicities (MENLOPS)
- HERWIG (S. Gieseke, S. Plätzer, P. Richardson et al.)
  - Full event generation with one-loop from GoSam or VBFNLO
  - Matching to parton shower (MC@NLO method)

Next-to-next-to-leading order

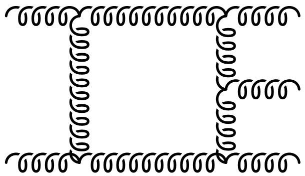
- NNLO predictions
  - expected to have a per-cent level accuracy
  - yielding first reliable estimate of theoretical uncertainty
- For processes measured to few per cent accuracy
    - jet production
    - vector boson (+jet) production
    - vector boson pair prodcution
    - top quark pair production
- For processes with potentially large perturbative corrections
  - New channels and/or phase space regions open up
    - Higgs or vector boson pair production

NNLO calculations

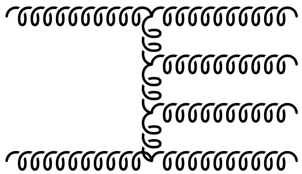
- Require three principal ingredients
  - two-loop matrix elements
    - explicit infrared poles from loop integral



  - one-loop matrix elements
    - explicit infrared poles from loop integral
    - and implicit poles from single real emission



  - tree-level matrix elements
    - implicit poles from double real emission



- Infrared poles cancel in the sum
- Combine contributions into parton-level generator

Two-loop QCD amplitudes: analytical

- Methods for analytic expressions
  - IBP reduction, differential equations for masters
  - Results in GHPL (tdhpl, GiNaC) (E. Remiddi, TG; J. Vollinga, S. Weinzierl)
- Jets in 2→2 processes
  - Di-jet production (C. Anastasiou, E.W.N. Glover, C. Oleari, M. Tejeida-Yeomans; Z. Bern, L. Dixon, A. De Freitas)
  - Vector-boson-plus-jet production (L. Garland, E.W.N. Glover, A. Koukoutsakis, E. Remiddi, L. Tancredi, E. Weihs, TG)
  - Higgs-boson-plus-jet production (E.W.N. Glover, M. Jaquier, A. Koukoutsakis, TG)
- Vector boson pairs
  - Vector-boson-plus-photon (L. Tancredi, TG)
  - Pair of massive vector bosons: W^+W^-, Z^0Z^0, $W^\pm Z^0$ (F. Caola, J. Henn, K. Melnikov, A&V. Smirnov; A. von Manteuffel, L. Tancredi, TG)

Two-loop QCD amplitudes: numerical

- Top quark pair production
  (P. Bärnreuther, M. Czakon, P. Fieldler; L. Chen, M. Czakon, R. Poncelet)
  - IBP reduction
  - Numerical solution (Runge-Kutta) of DE
  - Expansions around kinematical limits
- Heavy quark loop in HH and H+jet production
  (S. Borowka, N. Greiner, G. Heinrich, S. Jones, G. Luisoni, M. Kerner, J. Schlenk, T. Zirke)
  - IBP reduction to finite basis
  - Master integrals using sector decomposition
- Results
  - Grid tables in kinematical variables

Real radiation at NNLO: factorization

- Single unresolved radiation at one loop
  - One-loop correction to collinear splitting factors
    (Z. Bern, V. Del Duca, W. Kilgore, C. Schmidt)
  - One-loop correction to soft eikonal factor (S. Catani, M. Grazzini)
- Double unresolved radiation factors at tree level
  (J. Campbell, E.W.N. Glover; S. Catani, M. Grazzini; V. Del Duca, A. Frizzo, F. Maltoni)
  - Double soft
  - Soft/Collinear
  - Triple collinear
  - Double single collinear
- Require method to extract singular contributions

Real radiation at NNLO: methods

- Sector decompostion
  (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello)
  - Partition of phase space into sectors of non-overlapping singularities
  - Laurent expansion of integrand

$$(1-z)^{-1-\epsilon} = -\frac{1}{\epsilon}\,\delta(1-z) + \sum_n \frac{(-\epsilon)^n}{n!}\left(\frac{\ln^n(1-z)}{1-z}\right)_+$$

  - Numerical sector integrals
- Sector-improved subtraction schemes
  (M. Czakon; R. Boughezal, K. Melinkov, F. Petriello)
  - Residue subtraction: universal singular factors
  - Integration of subtraction terms using sector decomposition

Real radiation at NNLO: methods

- q_T-subtraction (S. Catani, M. Grazzini)
  - Production of colourless final states at hadron colliders
  - Universal behaviour for small transverse momentum from resummation
  - Cut off real radiation phase space at small transverse momentum

$$d\sigma^F_{NNLO} = \mathcal{H}^F_{NNLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\mathrm{jet}}_{NLO} - d\sigma^{CT}_{NLO} \right]$$

- N-jettiness subtraction

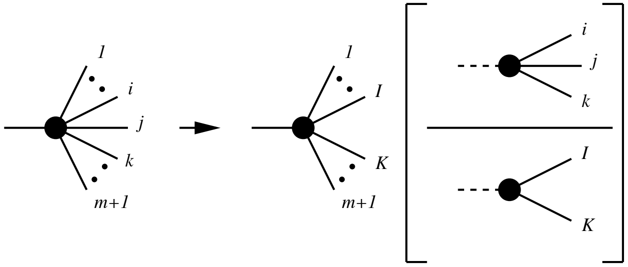
(R. Boughezal, X. Liu, F. Petriello; J. Gaunt, M. Stahlhofen, F. Tackmann, J. Walsh)

  - Use N-jettiness variable as cut-off for N-jet final state

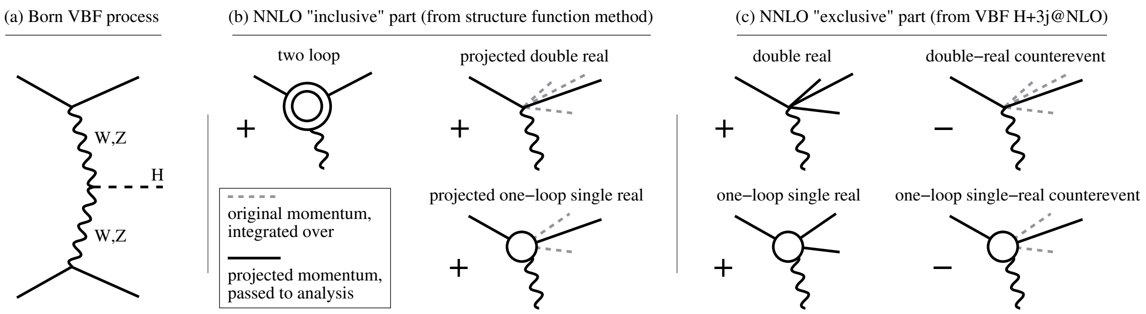
$$\tau_N = \frac{2}{Q^2} \sum_k \min \{ q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \dots q_N \cdot p_k \}$$

Real radiation at NNLO: methods

- Antenna subtraction
  (J. Currie, A. Daleo, A. Gehrmann-De Ridder, E.W.N. Glover, D. Maitre, TG)
  - Subtraction terms constructed from antenna functions
  - Antenna function contains all emission between two partons
  - Integration of antenna functions
    - IBP for phase space integrals
    - Differential equations



- Projection-to-Born (M. Cacciari, F. Dreyer, A. Karlberg, G. Salam, G. Zanderighi)



NNLO calculations: results

- Sector decomposition
  - pp → H, pp → V, including decays
    (C. Anastasiou, K. Melnikov, F. Petriello; S. Bühler, F. Herzog, A. Lazopoulos, R. Müller)
- Sector-improved residues
  - pp → tt (M. Czakon, P. Fiedler, A. Mitov)
  - pp → H+j (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
  - pp → t+j (M. Brucherseifer, F. Caola, K. Melnikov)
- q_T subtraction
  - pp → H, pp → V, pp →γγ, pp → VH
    (S. Catani, L. Cieri, D. de Florian, G. Ferrera M. Grazzini, F. Tramontano)
  - pp → VV (M. Grazzini et al.)
- N-Jettiness subtraction
  - pp → H+j (R. Boughezal, C. Focke, W. Giele, X. Liu, F. Petriello)
  - pp → W+j (R. Boughezal, C. Focke, X. Liu, F. Petriello)
  - pp → Z+j (R. Boughezal, J. Campbell, K. Ellis, C. Focke, W. Giele, X. Liu, F. Petriello)
  - pp → γ+j (J. Campbell, K. Ellis, C. Williams)

NNLO calculations: results

- Antenna subtraction (NNLOJET)
  - pp → H+j (J. Cruz-Martinez, X. Chen, E.W.N. Glover, M. Jaquier, TG)
  - pp → Z+j (A. Gehrmann-De Ridder, E.W.N. Glover, A. Huss, T. Morgan, TG)
  - pp → W+j (A. Gehrmann-De Ridder, E.W.N. Glover, A. Huss, D. Walker, TG)
  - pp → 2j (J. Currie, A. Gehrmann-De Ridder, E.W.N. Glover, A. Huss, J. Pires, TG)
  - pp → H+2j (VBF) (J. Cruz-Martinez, E.W.N. Glover, A.Huss, TG)
  - ep → 2j (J. Currie, A. Huss, J. Niehues, TG)
- Projection-to-Born
  - pp → H+2j (VBF) (M. Cacciari, F. Dreyer, A. Karlberg, G. Salam, G. Zanderighi)

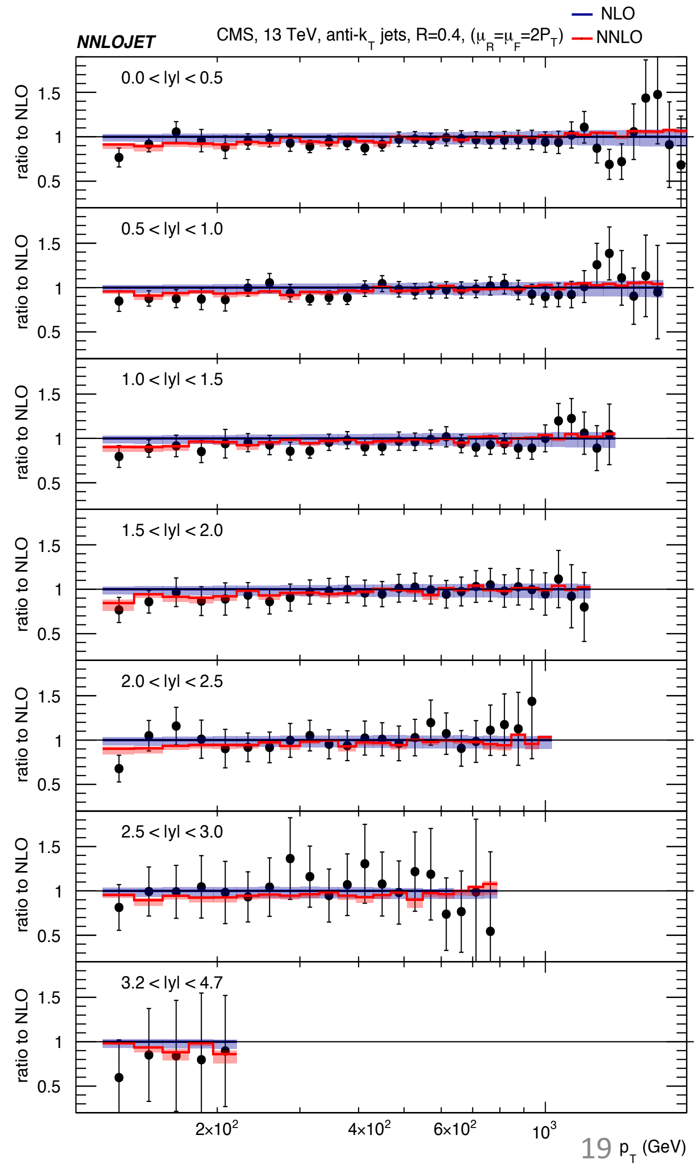
NNLOJET code

- NNLO parton level event generator
  - Based on antenna subtraction technique
- Infrastructure
  - Process management
  - Phase space, histogram routines
  - Validation and testing
  - Parallel computing support
- Processes implemented at NNLO
  - Z+(0,1)jet, W+(0,1)jet, H+(0,1)jet
  - DIS-2j, LHC-2j, VBF H+2jet
  - Typical runtimes: 60’000-250’000 core-hours

NNLOJET project:
X. Chen, J. Cruz-Martinez, J, Currie,
R. Gauld, A.Gehrmann-De Ridder,
E.W.N. Glover, M. Höfer, A. Huss,
T. Morgan, J. Niehues, J. Pires,
M. Sutton, D. Walker, TG

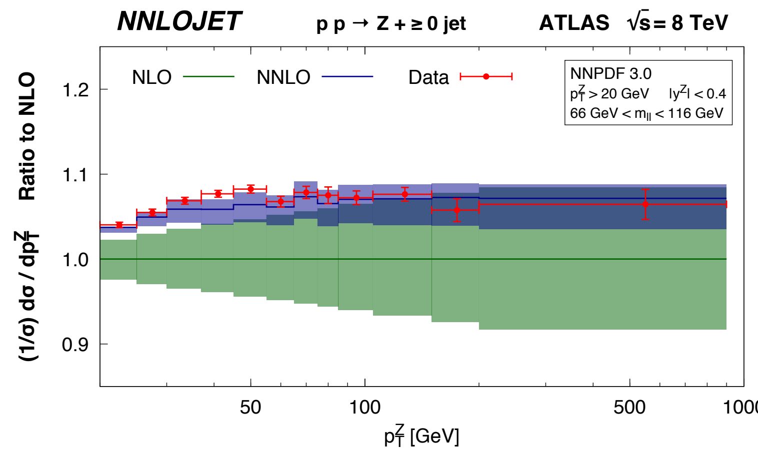
Jet cross sections

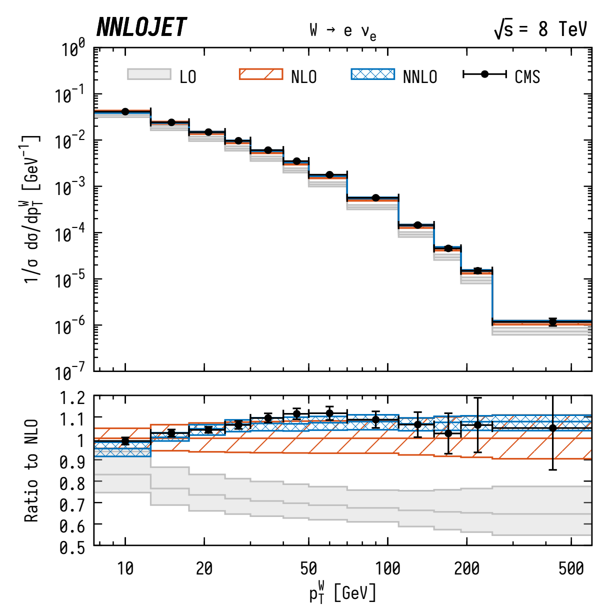
- Inclusive jet and di-jet production
  - Multiple-differential measurements
  - Constraints on parton distributions
  - Determination of α_s
- NNLO predictions
  - Per-cent level accuracy
  - Reduced spread among scale choices
  - Improved description of dependence transverse momentum and rapidity
  - Enable precision phenomenology



Transverse momentum distributions

- Z-boson p_T distribution
  - Measured: p_T= 1...1000 GeV
  - Constrains gluon distribution
    (R.Boughezal, A. Guffanti, F. Petriello, M. Ubiali)
  - NNLO for fiducial cross section improves description of data in shape and normalization
- W-boson p_T distribution
  - Important ingredient to W-mass determination
  - NNLO required to match accuracy of experimental data





Transverse momentum resummation

- Large logarithmic terms $\log^n(p_T/M)$
  - Factorization of cross section

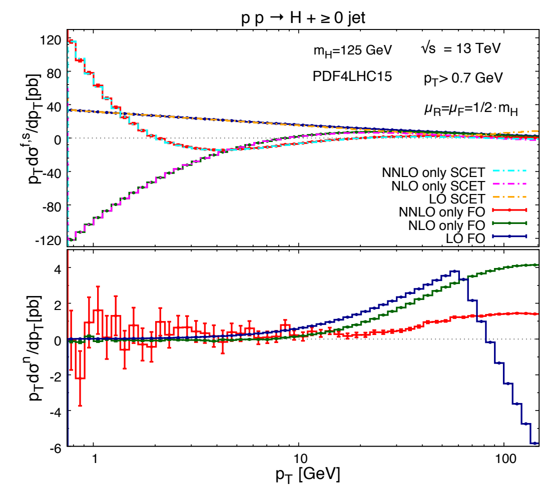
$$\frac{d\sigma}{dp_T^2} \sim \sigma_0 \, \mathcal{H} \otimes \mathcal{S} \otimes \mathcal{B}_1 \otimes \mathcal{B}_2$$

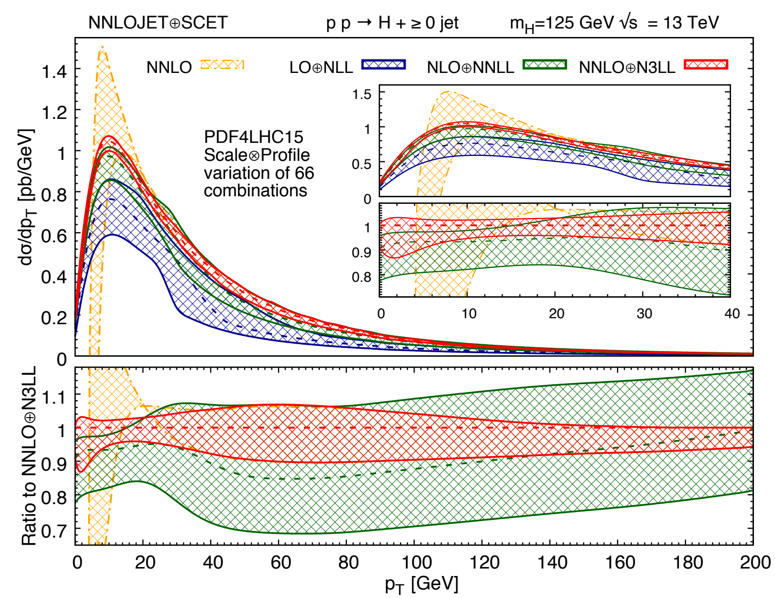
  - Hard, soft and beam functions fulfill evolution equations
    - Anomalous dimensions: cusp, quark/gluon, soft, hard, rapidity
    - Boundary conditions: form factors, TMD parton distributions
- Multi-loop corrections
  - Three-loop anomalous dimensions (S. Moch, J. Vermaseren, A. Vogt; Y.Li, H.X.Zhu; A. Vladimirov)
  - Three-loop form factors (P. Baikov, K. Chetyrkin, A.Smirnov, V. Smirnov, M. Steinhauser; E.W.N. Glover, T. Huber, N. Ikizlerli, C. Studerus, TG)
  - Two-loop soft function (Y.Li, A. von Manteuffel, R. Schabinger, H.X. Zhu)
  - Two-loop TMD parton distributions (T. Lübbert, L.L. Yang, TG; M. Echevarria, I. Scimemi, A. Vladimirov)

Transverse momentum resummation

- Predict low-p_T behaviour at fixed order
  - NLL → LO, NNLL → NLO, N3LL → NNLO
- Enables matching of fixed order and resummation
  - Precise predictions for full p_T range
- Inclusive Higgs boson p_T distribution at N3LL⊕NNLO

(X.Chen, E.W.N.Glover, A.Huss, Y.Li, D. Neill, M.Schulze, I.Stewart, H.X.Zhu, TG)

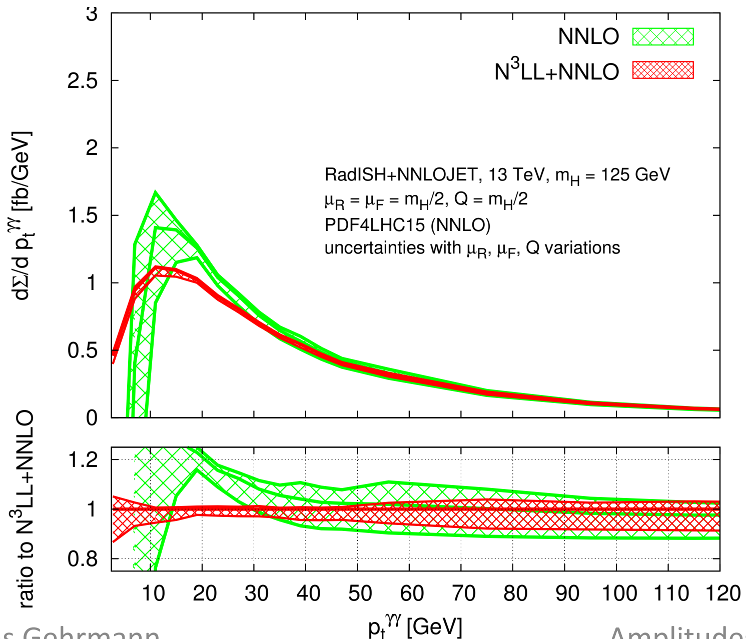




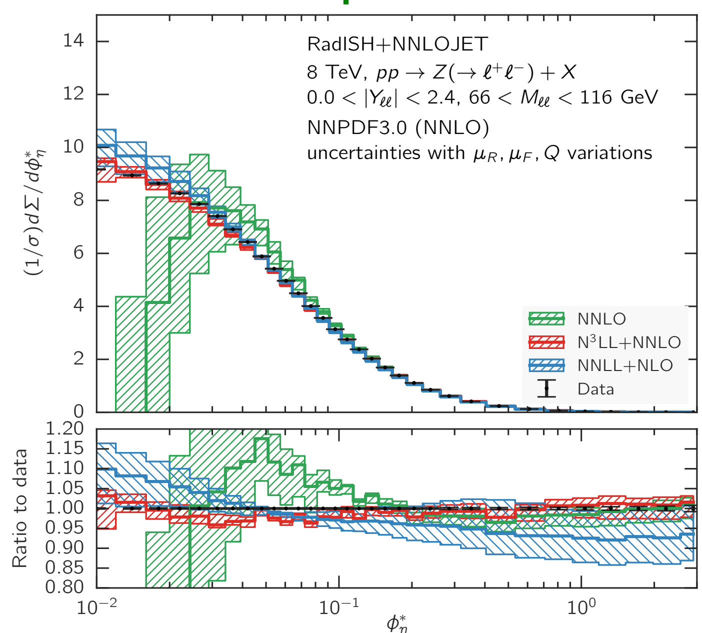
Transverse momentum resummation

- Formulate resummation as Monte Carlo
  - N3LL: RadISH code (W.Bizon, P.F.Monni, E.Re, L.Rottoli, P.Torrielli)
  - Describes fiducial cross sections
- N3LL multiplicatively matched to NNLO (W.Bizon, X. Chen, A.Gehrmann-De Ridder, E.W.N.Glover, A. Huss, P.F.Monni, E.Re, L.Rottoli, P.Torrielli, TG)

Fiducial cross section for H→γγ



Drell-Yan ϕ* distribution

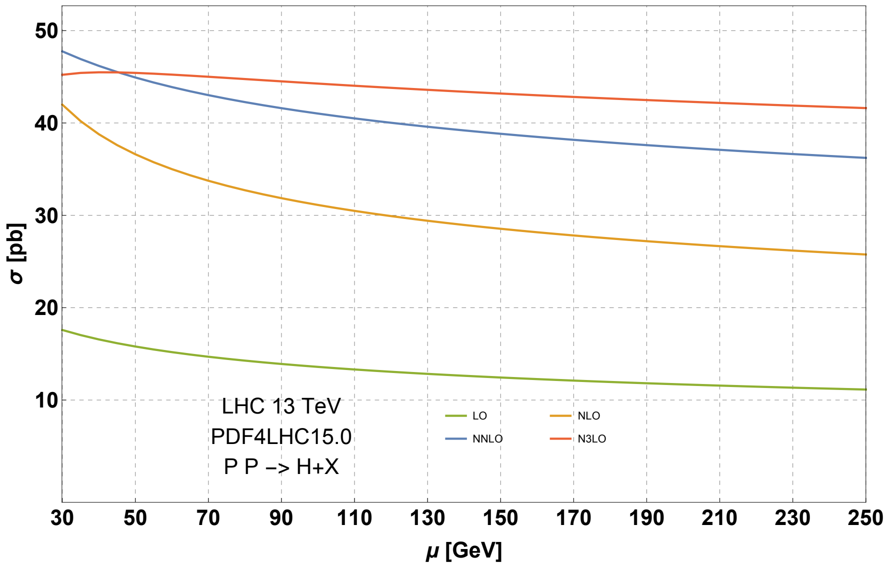


The precision frontier

- NNLO results for many processes in the last years
  - Have exhausted stockpile of two-loop $2 \longrightarrow 2$ amplitudes
- Aiming for NNLO at higher multiplicities: $2 \longrightarrow 3,4,\ldots$
  - Need two-loop amplitude reduction, master integrals ($\longrightarrow$many talks at Amplitudes2018)
  - Automation of real radiation subtraction
  - Computational efficiency and dissemination of results
- N3LO for benchmark reactions
  - Fully inclusive cross sections
  - Low-multiplicity $2 \longrightarrow 1$ processes

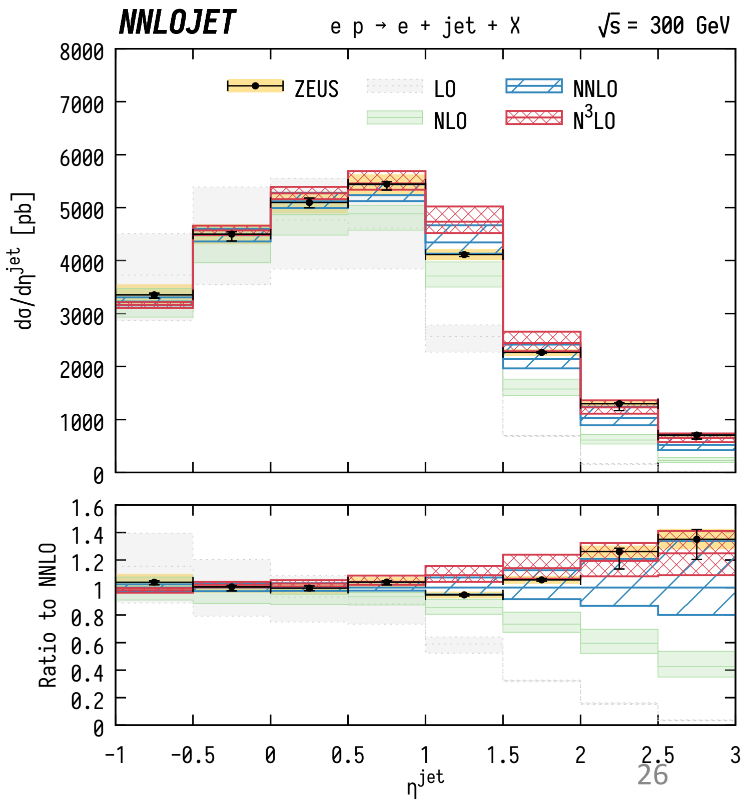
Higgs production in gluon fusion at N3LO

- N3LO contributions to Higgs production: cut integrals
  - Three-loop form factor
  - Two-loop single real
  - One-loop double real
  - Tree-level triple real
- Coefficient function
  - From high-order expansion around threshold (C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger)
  - Exact result, yielding GHPL and an elliptic integral (B. Mistlberger)



Differential distributions at N3LO

- Real radiation subtraction: potential methods
  - Always require NNLO calculation with one extra jet
  - q_T subtraction or N-jettiness subtraction: require
    - N3LL resummation and three-loop beam functions
  - Projection-to-Born (P2B): require
    - Inclusive coefficient function for full Born kinematics at N3LO
- P2B method at N3LO
  - 1-Jet production in deep inelastic scattering (J. Currie, E.W.N.Glover, A.Huss, J.Niehues, A.Vogt, TG)
  - Born process: inclusive three-loop DIS coefficient functions (S.Moch, J.Vermaseren, A.Vogt)



Where do we stand?

- Witnessed an NLO revolution
  - Previously unthinkable NLO QCD+EW multi-particle calculations now feasible due to technological breakthroughs
  - High-level of automation
  - Standarization of interfaces: combine different codes (providers)
  - Interface to experiment (codes, ntuples, histograms,..)?
- Substantial progress on NNLO calculations
  - Several different methods available
  - Close interplay with resummation
  - Calculations on process-by-process basis
  - Codes typically require HPC infrastructure
  - Preparing for precision phenomenology

Future Directions

- NNLO calculations
  - Desperately need two-loop amplitudes for
    - Higher multiplicities
    - Multiple mass scales
  - Gain better understanding of underlying structures
  - Automation: two-loop amplitudes and real radiation subtraction
  - Performance and efficiency of evaluations
- Beyond NNLO
  - N3LO for benchmark processes: inclusive, differential, fiducial