Double copy & Color-kinematics on curved backgrounds

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Basic question:

As a field, what is “amplitudes” really about?

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If one believes in this:

- New structures should persist on general perturbative backgrounds.

Many reasons to be interested in curved backgrounds:

- GW physics
- Cosmology/holography
- Laser physics/strong field QED

Also: *very* little known relative to flat backgrounds

E.g., 'amplitudes' (boundary correlation functions) in AdS

@ tree-level only 3- and 4-points

[Freedman-Mathur-Matusis-Rastelli, Maldacena, Maldacena-Pimentel] [Raju]

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The structures: double copy & color-kinematics duality

The backgrounds: plane waves

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- Gravity: $\mathrm{d}s^2 = 2\mathrm{d}u\,\mathrm{d}v - \mathrm{d}x_a\,\mathrm{d}x^a - x^a\,x^b H_{ab}(u)\,\mathrm{d}u^2$, with $H_a^a = 0$.

  [Baldwin-Jeffrey, Ehlers-Kundt, Brinkmann, Einstein-Rosen]

Scattering on plane waves

Sandwich waves ($\dot{A}_a$, H_{ab} compactly supported):

- Asymptotically flat regions
- Unitary evolution
- No particle creation (in quadratic theory)

`[Gibbons, Garriga-Verdaguer,`

`TA-Casali-Mason-Nekovar]`

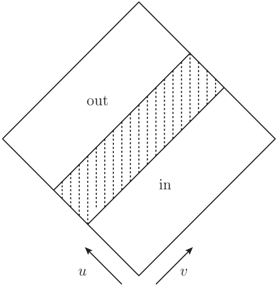

Figure: Sandwich wave

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What's the problem?

- No d-dimensional momentum conservation – integrals always left over due to wave profile
- *Memory effect*: 'out' region fields know they've passed through curvature [Zhang-Duval-Gibbons-Horvathy, TA-Casali-Mason-Nekovar]
- *Tails*: gluon/graviton perturbations do not obey Huygens' Principle [Günther-Wünsch, Mason, Harte]

Just to give you an idea...

Gluon perturbation on a gauge theory PW:

$$a^{\mathsf{a}}_{\mu} = \mathsf{T}^{\mathsf{a}} \left(\epsilon_{\mu} + \delta^{u}_{\mu} \frac{e}{k_{+}} \epsilon_{a} A^{a}(u) \right) \mathrm{e}^{\mathrm{i}\phi}$$

where e is charge under A, and

$$\phi := k_{+} v + \left(\vec{k}_{a} + e A_{a}(u) \right) x^{a} + \frac{1}{2\, k_{+}} \int^{u} \mathrm{d}s \, \left(\vec{k} + e A(s) \right)^{2}$$

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Example: 3-points

$$A_3 \sim \delta^{d-1}\left(\sum_{r=1}^{3} k_r\right) \int \mathrm{d}u\, \mathrm{e}^{\mathrm{i}\,(f_1+f_2+f_3)}\, \mathcal{I}_3\,,$$

$f_r(u)$ related to scalar free fields

$\mathcal{I}_3$ is the interesting (kinematic) part!

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$$F = -\frac{\epsilon_1 \cdot \epsilon_3}{k_{2+}}(k_{1+}\,\vec{k}_2 \cdot \epsilon_2 - k_{2+}\,\vec{k}_1 \cdot \epsilon_2) + \text{cyclic}$$

$$C = \frac{\epsilon_1 \cdot \epsilon_3}{k_{2+}}\,\epsilon_2^a\,A_a\,(e_2 k_{1+} - e_1 k_{2+}) + \text{cyclic}$$

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Gravity PW background: $\mathcal{M}_3 = F^2 - \mathrm{i}k_{1+}k_{2+}k_{3+}\,\sigma^{ab}\,\mathcal{C}_a\,\mathcal{C}_b$,

σ^{ab} background deformation tensor, $\mathcal{C}_a = \frac{\epsilon_{3\,a}}{k_{3+}}\,\epsilon_1 \cdot \epsilon_2 + \text{cyclic}$

Double copy entails 'squaring' plus replacement rules for kinematics & background:

1. Reverse charges wrt background to form $\tilde{\mathcal{A}}_3 = F - C$.
2. Take $|\mathcal{A}_3|^2 := \mathcal{A}_3\tilde{\mathcal{A}}_3 = F^2 - C^2$.
3. Replace $\vec{k}_a \to \vec{k}_i E^i_a$, and

$$e_r e_s\, A^a A^b \to \begin{cases} \mathrm{i} k_{r+}\, \sigma^{ab} & \text{if } r = s \\ \mathrm{i}(k_{r+} + k_{s+})\, \sigma^{ab} & \text{otherwise} \end{cases}$$

$$\boxed{\mathcal{M}_3 = \rho\left(|\mathcal{A}_3|^2\right)}$$

Some observations...

- Double copy persists despite tails of perturbations & amplitudes
- Suggests n-point conjecture:

$$\mathcal{M}_n = \rho\left(\sum_{\alpha,\beta\in S_{n-3}} \mathcal{A}_n\, S^A[\alpha|\beta]\, \tilde{\mathcal{A}}_n\right)$$

- This is *not* classical double copy [Luna-Monteiro-Nicholson-O'Connell-White,...] ! (PW gauge and gravity backgrounds not related by classical double copy [Ilderton])
- These 3-point amplitudes can be computed with ambitwistor strings [TA-Casali-Mason-Nekovar]

Color-Kinematics @ 4-points

Flat background: $\mathcal{A}_4 = \frac{c_s\, n_s}{s} + \frac{c_t\, n_t}{t} + \frac{c_u\, n_u}{u}$ [Bern-Carrasco-Johansson]

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PW gauge background: situation very different!

$$\mathcal{A}_4 = c_s \int \mathrm{d}\mu[s]\, n_s + c_t \int \mathrm{d}\mu[t]\, n_t + c_u \int \mathrm{d}\mu[u]\, n_u$$

$\int \mathrm{d}\mu[s] \sim$ momentum conservation and s^{-1}

Clearly, $n_s - n_t + n_u \neq 0$

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Has the correct flat background limit
Non-trivially constrains representations at higher points

Outlook

Just scratching the surface...

- Perturbative gauge theory/gravity on PW backgrounds
- Does double copy persist at higher points?
- Relation between color-kinematics and double copy?
- Derivation from string theory?