ABJM flux-tube and scattering amplitudes Andrei Belitsky ASU

with Benjamin Basso

Gauge/string duality

Gauge/string duality is a gift for non-perturbative studies of gauge theories: D dimensional space-time physics is mapped into dynamics of the 2-dimensional string world-sheet!

ψ^A

 $\frac{1}{2}$

√ *N* = 4 super Yang-Mills with 't Hooft coupling $g_{\text{YM}}^2 N$ (4D) **Type-IIB superstring on AdS₅ x S**⁵ ! !Y $\frac{1}{2}$ -Mills with 't Hooft coupling $\; g^2_{\rm YM}N \;$ (4D)

 $\frac{1}{\text{Type-IIB}}$ superstring on AdS₅ **Change** $\mathsf{X} \mathsf{S}^5$

 \blacktriangledown *N* = 6 Chern-Simons with matter with 't Hooft coupling $\ N/k$ (3D ABJM) \hfill Type-IIA superstring on AdS₄ x CP³

Aharony, Bergman, Jefferis, Maldacena '08

While D-dimensional physics on the QFT side cannot be integrable per se, the 2D world-sheet theory can (and better be integrable). $\frac{1}{2}$ $\overline{\mathbf{e}}$ $\frac{1}{\sqrt{2}}$ $\frac{1}{2}$ ($\frac{1}{2}$) $\frac{1}{2}$ $\overline{}$ \cdot

 \mathbb{R}^n are light-like separated when belong to the same separated when \mathbb{R}^n separated when \mathbb{R}^n

 \mathbb{R}^n are light-like separated when belong to the same separated when s But we need a dictionary … let us review a well-studied example first

2D map for amplitudes

Alday, Maldacena '07

Planar gluon amplitudes (in *N* = 4 SYM) correspond to scattering of open strings with disk world-sheet topology (ending on D branes in the bulk $z \to \infty$).

Under T-duality (from coordinate *x* to momentum *y* space)

$$
dy_{\mu} = \frac{4x_{\mu}}{z^2} \qquad r = \frac{1}{z}
$$

the $AdS₅$ goes into itself

$$
ds^2 = \frac{dy_\mu^2 + dr^2}{r^2}
$$

and the boundary conditions change to

$$
\Delta y_\mu = 2\pi p_\mu
$$

On the boundary $(r=0)$ it becomes a holonomy of the gauge field p_6 on a piece-wise light-like contour.

Drummond, Henn, Korchemsky, Sokatchev '07 Brandhuber, Heslop, Travaglini '08

Amplitude/Wilson loop duality

In fact, the AdS₅ x S⁵ super sigma-model is self T-dual under generalization involving Grassmann coordinates as well!

AB, Korchemsky, Sokatchev '11

■ The asymptotic states of (regularized) *N* = 4 SYM Fields of superconnection(s) living on the contour

$$
A_\mu, \; \psi^A, \; \bar{\psi}_A, \; \phi^{AB}
$$

$$
A_{\mu}, \; \psi^A, \; \bar{\psi}_A, \; \phi^{AB} \qquad A_{\mu}, \; \psi^A, \; \bar{\psi}_A, \; \phi^{AB}
$$

 $A_\mu, \ \psi^A, \ \bar{\psi}_A, \ \phi^{AB}$
These source excitations propagating on 2D world-sheet

n
≈ ∂f excitations avenagating on t _{IS} *κ*οπαποπο propagamig on El \blacktriangledown The physics of excitations propagating on the color flux sourced by two fast SU(N) charged particles

N=4 flux tube <u>.</u> and the twice 0

Extracted from the large spin limit of the $N = 4$ spectral X X ^{AB, Gorsky, Korchemsky '06} problem for single-trace operators $X \rightarrow \mathcal{A}$

$$
D^s_+ X(0) \stackrel{s \to \infty}{\to} P \exp\left(ig \int_{[0,\infty)} dx_\mu A_\mu(x)\right) X(0)
$$

Multi-particle state $|\psi\rangle$ are adjoint fields (X = $A_\mu,~\psi^A,~\bar{\psi}_A,~\phi^{AB}$) inserted along the Wilson line $B₁$ ϵ \mathbf{S} **Supersymmetry and** \mathbf{S}

$$
\gamma(\lbrace s_i \rbrace) = \ln(s_1 + \cdots + s_N) \Gamma_{\text{cusp}}(g) + E_N(\psi) + \dots
$$

flux-tube vacuum $\frac{1}{2}$

 \sim $\frac{1}{2}$ $\frac{1}{\sqrt{2}}$ \mathbb{R} = iCF \mathbb{R} α segment α , α $E_N(\psi) = E(u_1) + E(u_2) + \cdots + E(u_N)$ 3!

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ϵ α˙ β˙ $\frac{1}{2}$ $\frac{1}{2}$

 \mathcal{L}_{out} to be different from zero the integration over the position of the position of the fields should should should . The product of two quantum fields separated by a light-like interval is not well defined by a light-like interval is not well defined by a light-like interval is not well defined by a light-like interval is not well defi

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Γεγονότα

Pentagon paradigm δ m

 \blacktriangledown Tesselate a polygon into squares with light-like lines: $\mathcal{O}(\mathcal{O}(\log n))$ for $\mathcal{O}(\log n)$ from the fermion equations of motion equations of motions of motions of motions of $\mathcal{O}(\log n)$

 Γ on the selection $(2, 2)$ A() = ∂ ψA() α o(g), B(tk) α For n-side polygon, (n-3) middle squares (matches the number of external kinematical (geometrical) data)

^m

(This is equivalent to multichannel OPE for correlators)

 \blacktriangleright Two squares overlap in a pentagon, (n-4) of them

$$
= \sum_{\psi_i} \left[e^{-\tau_i E(\psi_i) + i\sigma_i p(\psi_i) + im_i \phi(\psi_i)} \right]
$$

$$
\times P(0|\psi_1) P(\psi_i|\psi_2) P(\psi_2|\psi_3) P(\psi_3|0)
$$

 T The product of two quantum fields separated by a light-like interval is not well defined by a light-like interval is not well defined by a light-like interval is not well defined by a light-like interval is not well.

 \mathcal{B} The product of two graduated by a light-like interval is not well defined by a light-like interval is not well defined by a light-like interval is not well defined by a light-like interval is not well. The contra

To calculate any amplitude at any value of the coupling on needs
↑ m $\frac{1}{2}$ \mathbf{v} false of the coupling on needs

- $\mathbf{F}_{\mathbf{c}}$, the different from zero the integration over the position of the fields should show $\mathbf{f}_{\mathbf{c}}$ \blacktriangleright The spectrum of flux-tube excitations (E, p) ⟨δQWn⟩ ∼ ig Of $\overline{}$ ^{X(} ϕ
- produce a pole 1/ε. ! Employ the regularization of N = 4 SYM by dimensional reduction with D = 4 − 2ε dimensions on the segment [x1, x2] \blacktriangledown Pentagon transitions between all states

$$
P(\psi_i|\psi_j) = \langle \psi_j|\widehat{\mathcal{P}}|\psi_i\rangle
$$

is is the dynamical input! Geometry enters trivially through (τ, σ, ϕ) and \hat{p} is the dynamical input Geometry enters trivially through (τ, σ, ϕ) This is the dynamical input! Geometry enters trivially through (τ,σ,ϕ)

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 $\sum_{n=1}^{\infty}$

 $\frac{1}{2}$

tr #

tr #

tr #

 \mathbb{R}^n are light-like separated when belong to the same separated when \mathbb{R}^n

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 \mathbb{R}^n are light-like separated when belong to the same separated when \mathbb{R}^n

is is the dynamical input! Geometry enters trivially through showleds should # ψ^A be dynamical input Coometry ontore trivially through This is the dynamical input! Geometry enters trivially through

Single-particle square transition (orthogonality condition) \overline{y}

$$
\langle \psi_v | \psi_u \rangle \sim \delta(u - v)
$$

 \blacktriangledown Single-particle pentagon transition \boldsymbol{u}

$$
P(u|v) = \langle \psi_v | T | \psi_u \rangle = \frac{\Gamma(2s)\Gamma(iu - iv)}{\Gamma(s + iu)\Gamma(s - iv)}
$$
\nconformal transformation

\n

V Obeys a "bootstrap" equation (not a Watson equation!)

$$
\frac{P(u|v)}{P(v|u)} = S(u,v)
$$

transit \mathbb{R}^n

$$
S(u,v)=\frac{\Gamma(iu-iv)\Gamma(s-iu)\Gamma(s+iv)}{\Gamma(iv-iu)\Gamma(s-iv)\Gamma(s+iu)}
$$

 M_{m} Consistency with Watson equation? Move excitations from one side of the polygon to anoth Kinematics one has to go through the cuts which open up only at finite coupling. transformations. The analytic continuation in rapidity γ is non-perturbative in its origin since to reach the mirror
kinematics one has to go through the cuts which open up only at finite coupling transionmations. The analytic continuation in rapidity y is non-perturbative in its origin since to reach the
kinematics one has to go through the cuts which open up only at finite coupling. ✔ Consistency with Watson equation? Move excitations from one side of the polygon to another with mirror

To define the corresponding contribution to we have to introduce a regularization.

ε Γ(2 − ε)

$$
E(u^\gamma) = ip(u) \qquad p(u^\gamma) = iE(u) \qquad \qquad \text{as,}
$$

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P. 17/2011 - p. 17/2012 - p

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V The flux-tube scattering matrix
$$
\psi^{\sigma_2 \geq \sigma_1} e^{2iu\sigma_1}e^{2iv\sigma_2} + S(u,v)e^{2iv\sigma_1}e^{2iu\sigma_2}
$$

Pentagon bootstrap @ any coupling λ 9 \overline{a} \blacksquare α tatnon \emptyset **IO** $\frac{1}{2}$ ζ

V Watson-like equation: **The contour Cn, the contour Cn, the fermionic EOM operator is inserted over the contour Cn, the fermionic EQM operator is inserted over the fermionic EQM operator is inserted over the fermionic c**

Basso, Sever, Vieira '13

Mirror equation: The connection B(t) is integrated over the fermionic EOM operator is inserted over the fermionic EOM operator is inserted over the fermionic EOM operator is inserted over the fermionic EOM operator is in

ψ^A

Consistency condition yields the well-known Watson equation:

Solution to bootstrap equations and connection and contour Contour Contour Contour Contour Contour Contour Conto

The most uncertain part of the

 $\frac{1}{2}$

$$
P(u|v) = w(u,v) \left[\frac{S(u,v)}{S(u^{\gamma},v)} \right]^{1/2}
$$
 solutions! Huge ambiguities! Data
availability (from hexagon bootstrap) was crucial for success!

 AB' 13-14 . Basso, Sever, Vielra, Caetano, Cordova
AB '13-14 en
Basso, Sever, Vieira, Caetano, Cordova '13-15

> bootstrap) was crucial for s availability (from hexagon bootstrap) was crucial for success!

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/
Dixon, Drummond, Henn, von Hippel, McLeod, Pennington '11-'16** $\mathcal{F}_{\mathcal{A}}$ to be different from zero the integration over the position of the position of the fields showledge should show $\mathcal{F}_{\mathcal{A}}$ $\mathcal{R}^{\mathcal{R}}(x)$ are light-like separated when belong to the same separated when belong to the same segment Bourjaily, Caron-Huot, Trnka '13

Verification \bigcirc t 0

3!

 \blacktriangledown Bootstrap program explicitly verified against explicit calculations to four loops order by Dixon et al. (in fact the latter were indispensable for fixing ambiguities in solution of functional equations!)

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- The OPE series can be resummed to recover the exact kinematics 3! aa
Martasumr 0 od to recover the exact kiner \blacktriangledown The OPE series can be resummed to recover the exact kinematics
- \blacktriangledown At strong coupling it reproduces TBA and provides results beyond the minimal area approximation $\mathcal{L}(\mathcal{X})$ is the sequence of \mathcal{X}

To define the corresponding contribution to we have to introduce a regularization.

ABJM S-matrices and pentagons <u>y</u> $S-m$ 0 atricae ar " 1 $\overline{\mathcal{L}}$ n enta \mathcal{S} on P. \mathbf{d}

∕ Twist-1— Twist-1 excitations (square of *N* = 4 SYM)

$$
S_{FF}(u, v) = S_{FF}^{N=4}(u, v) \times S_{F\bar{F}}^{N=4}(u, v)
$$

\n
$$
S_{\Psi\Psi}(u, v) = S_{\Psi\Psi}^{N=4}(u, v) \times S_{\Psi\bar{\Psi}}^{N=4}(u, v)
$$

\n
$$
P_{\Psi}
$$

$$
P_{F|F}(u|v) = P_{F|F}^{N=4}(u|v) \times P_{F|\bar{F}}^{N=4}(u|v)
$$

\n
$$
P_{\Psi|\Psi}(u|v) = P_{\Psi|\Psi}^{N=4}(u|v) \times P_{\Psi|\bar{\Psi}}^{N=4}(u|v)
$$

\n
$$
P_{\Psi|\Psi}(u|v) = P_{\Psi|\Psi}^{N=4}(u|v) \times P_{\Psi|\bar{\Psi}}^{N=4}(u|v)
$$

obey both the fundamental and mirror axioms (as a consequence of their validity in *N* = 4 SYM)

To define the corresponding contribution to we have to introduce a regularization.

 $\frac{m}{n}$ = 4 SYM)
wist-1/2 excitations (square root of $N = 4$ SYM) \blacktriangledown Twist-1/2 — Twist-1/2 excitations (square root of $N = 4$ SYM) $\frac{B}{B}$ ($\frac{B}{B}$ are light-like separated when belong to the same separated when $\frac{B}{B}$

 σ (1) σ $(N=4$ k integration over the fields should show σ (1) in σ (1) $S_{Z|Z}(u|v) \times S_{\bar{Z}|Z}(u|v) = S_{\phi|\phi}^{N=4}(u|v)$ $P_{Z|Z}(u|v) \times P_{\bar{Z}|Z}(u|v) = P_{\phi|\phi}(v)$ $S_{Z|Z}(u|v)/S_{\bar{Z}|Z}(u|v) = S^{SU(2)}(u-v)$ $S_{\pi}(\nu|x) = S^{SU(2)}(\nu)$ $- v$) $S_{Z|Z}(u|v)/S_{\bar{Z}|Z}(u|v) = S^{\rm SU(2)}(u-v)$

Solved up to an unknown function $f(u,v)$:

110)
1110)
1111

$$
v)
$$

\n
$$
P_{Z|F}(u|v) = P_{\bar{Z}|F}(u|v) = P_{\phi|F}^{N=4}(u|v)
$$

\n
$$
P_{Z|\Psi}(u|v) = P_{\bar{Z}|\Psi}(u|v) = P_{\phi|\Psi}^{N=4}(u|v)
$$

$$
P_{Z|Z}(u|v) \times P_{\bar{Z}|Z}(u|v) = P_{\phi|\phi}(u|v)
$$

$$
P_{Z|Z}(u|v)^2 = f(u,v) \times P_{\phi|\phi}^{N=4}(u|v), \qquad P_{\bar{Z}|Z}(u|v)^2 = \frac{1}{f(u,v)} \times P_{\phi|\phi}^{N=4}(u|v)
$$

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ABJM NPWL **Supersymmetry anomaly Supersymmetry anomaly** 2.26

A Wilson loop (there are other options as well) for ABJM theory

$$
W_n = \frac{1}{2N} \left\langle \text{tr}_{\text{SU}_k(N)} \exp \left(\int_{C_n} dy_\mu A^\mu \right) + \text{tr}_{\text{SU}_{-k}(N)} \exp \left(\int_{C_n} dy_\mu \widehat{A}^\mu \right) \right\rangle
$$

Any number of edges **3. The connection B(t) is integrated over the connection B(t) is integrated over the contour Cn, the fermionic EOM operator Cn, the fermionic EOM operator Cn, the fermionic EOM operator Contour Contour Contour Cn of Conto**

enn, Plefka, Wiegandt '10 Cn %&' Bianchi, Gilbert, Leoni, Penati '11 $^{\prime}$ $^{\prime$

Only even powers in 't Hooft coupling *N*/*k*

Computed at two loops and found to be given by one-loop BDS ansatz of *N*=4 SYM (i.e., they obey ! The connection B(t) is integrated over the contour Cn, the fermionic EOM operator is inserted on the segment [x1, x2] anomalous conformal Ward Identities of dual conformal boost!) for for *n*=4 and by BDS along with N=4 remainder for *n*=6 anomalous comonnal ward
N=4 remainder for *n*=6 \sim Only even powers in 't Hooft coupling N/k
 \sim Connected of the large and familia beginned to get a fermion contract in the fermionic entry of N , the fermionic contour is inserted of the fermionic contour is inse $der for n=6$) = ∂ ψA() $k = \frac{1}{2}$ $\overline{}$

Properly subtracted *n*=6 NPWL $\frac{1}{\sqrt{2}}$

on the segment $\mathcal{L}(\mathcal{L})$ and the sequence of the sequence of the sequence of the sequence of the sequence

$$
W_6^R = \exp\left(\Gamma_{\text{cusp}}(N/k)[2\zeta_2 - \ln(1-u_2)\ln\frac{u_1u_2}{(1-u_2)u_3} - \ln u_1\ln u_3 - \sum_{i=1}^3 \text{Li}_2(1-u_i)]\right)
$$

Within pentagon OPE, the leading twist contribution comes from an exchange of a gluon and spinon-antispinon pair

$$
I_F = \int \frac{du}{2\pi} \mu_F(u)e^{-\tau E_F(u) + i\sigma p_F(u)}
$$

\n
$$
= -(N/k)^2 e^{-\tau} \frac{\sigma}{\sinh \sigma}
$$

\n
$$
I_{Z\bar{Z}} = \int \frac{du_1 du_2}{(2\pi)^2} \frac{4\mu_Z(u_1)\mu_{\bar{Z}}(u_2)}{[(u_1 - u_2)^2 + 4] |P_{Z|\bar{Z}}(u_1|u_2)|^2} e^{-\tau(E_Z(u_1) + E_Z(u_2)) + i\sigma(p_Z(u_1) + p_Z(u_2))}
$$

\n
$$
= -(N/k)^2 e^{-\tau} \left[2 \cosh \sigma \log (1 + e^{-2\sigma}) - \frac{\sigma e^{-2\sigma}}{\sinh \sigma} \right]
$$

The applicability of pentagon OPE to null polygonal Wilson loops is not surprising at all. \blacktriangledown Note that spurious poles cancel only in the sum of the two reproduces perturbative calculation! L
The that spurious poles cancel only in the sum of the two reproduces perturbative calculation! Note that spurious poles cancel only in the sum of the two reproduces perturbative calculation! Workshop on Recent Developments in Supersymmetric Gauge Theory, 26 April 2011 - p. 17/20 Workshop on Recent Developments in Supersymmetric Gauge Theory, 26 April 2011 - p. 17/20

Scattering amplitudes ing amplitudes

N = 4 SYM ABJM

The connection B(t) is integrated over the connection B(t) is integrated over the connection B(t) is inserted over the contour Cn, the fermionic EOM operator is inserted over the fermionic EOM operator is inserted over th

Packed in to a single CPT self-conjugate superfield $N = 3$ superfields

$$
\Phi = F + \theta^A \psi_A + \frac{1}{2} \phi_{AB} + \frac{1}{3!} \varepsilon_{ABCD} \theta^A \theta^B \theta^C \bar{\psi}^D
$$

$$
+ \frac{1}{4!} \varepsilon_{ABCD} \theta^A \theta^B \theta^C \theta^D \bar{F}
$$

V Nomenclature: integrated over the connection by dimensional reduction \sim 2.8 \sim

h

Two $N = 3$ superfields (only U(3) of SU(4) is manifest)

$$
\Phi = F + \theta^A \psi_A + \frac{1}{2} \phi_{AB} + \frac{1}{3!} \varepsilon_{ABCD} \theta^A \theta^B \theta^C \bar{\psi}^D
$$
\n
$$
\Phi = Y^4 + \theta^a \psi_a + \frac{1}{2} \varepsilon_{abc} \theta^a \theta^a Y^c + \frac{1}{6} \varepsilon_{abc} \theta^a \theta^b \theta^c \psi_4
$$
\n
$$
+ \frac{1}{4!} \varepsilon_{ABCD} \theta^A \theta^B \theta^C \theta^D \bar{F}
$$
\n
$$
\bar{\Psi} = \bar{\psi}^4 + \theta^a \bar{Y}_a + \frac{1}{2} \varepsilon_{abc} \theta^a \theta^b \bar{\psi}^c + \frac{1}{6} \varepsilon_{abc} \theta^a \theta^b \theta^c \bar{Y}_4
$$
\nAgarwal, Beisert, McLoughlin's

\nAgarwal, Beisert, McLoughlin's

Burneyer are light-like separated when belong to the same segment of the same separated when belong to the same segment Agarwal, Beisert, McLoughlin '09

Flux-tube for ABJM amplitudes \cup hef 9 r ABJM ar .U amplitudes po in correction corrections to the fermion of motion of motion equations of motion equations of motion equations of motion exterministic motion and the fermion exterministic motion and the fermion exterministic motion ext

√ Tree level: contribution from leading-twist spinons
◆ Tree level: contribution from leading-twist spinons

only two independent components: The contour C

$$
\mathcal{Z}_3 = e^{-\tau/2} \left[\chi_1^3 \frac{2e^{\sigma/2}}{1 + e^{2\sigma}} + \chi_4^3 \frac{2e^{3\sigma/2}}{1 + e^{2\sigma}} \right]
$$

$$
= e^{-\tau/2} \int \frac{du}{2\pi} e^{i\sigma u} \left[\chi_1^3 \mu_Z^{(0)}(u) + \chi_4^3 \mu_{\bar{Z}}^{(0)}(u) \right]
$$

 $\overline{z_2}$ v "doubling" of spinon measures: $\mathcal{O}(\mathcal{O}_\mathcal{A})$ for a fermion to fermion the fermion equations of motions of motions of motions of motions of motions of $\mathcal{O}(\mathcal{A})$

$$
\mu_Z^{(0)}(u) = \Gamma\left(\frac{3}{4} + \frac{i}{2}u\right)\Gamma\left(\frac{1}{4} - \frac{i}{2}u\right), \qquad \mu_{\bar{Z}}^{(0)}(u) = \Gamma\left(\frac{3}{4} - \frac{i}{2}u\right)\Gamma\left(\frac{1}{4} + \frac{i}{2}u\right)
$$

To define the corresponding contribution to we have to introduce a regularization.

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 $square:$ $Square:$ $\boldsymbol{\mathsf{v}}$ squaring to $\boldsymbol{N} = 4$ scalar measure:
 $\boldsymbol{\mathsf{v}}$ \cdot scalar measure: fermion \mathbf{r}

$$
\mu_Z^{(0)}(u)\mu_{\bar Z}^{(0)}(u)=\mu_\phi^{(0)N=4}(u)
$$

V Two loops: contribution from leading-twist spinons

■ Two loops: contribution from leading-twist spinons . Due to light-cone singularities, the Wilson loop Wilson loop Wilson loop Wilson loop Wilson loop Wilson loop W

$$
+ \left(\frac{N}{k}\right)^2 e^{-\tau/2} \int \frac{du}{2\pi} e^{i\sigma u} \left[\chi_1^3 \mu_Z^{(0)}(u) + \chi_4^3 \mu_Z^{(0)}(u) \right] \left[-\tau E_Z^{(2)}(u) + i\sigma p_Z^{(2)}(u) + \delta \mu_Z^{(2)}(u) \right]
$$

here

$$
E_Z^{(2)}(u) = \frac{1}{2} E_\phi^{(2)N=4}(u), \qquad p_Z^{(2)}(u) = \frac{1}{2} p_\phi^{(2)N=4}(u)
$$

\n
$$
\delta \mu_Z^{(2)}(u) = \frac{1}{2} \delta \mu_\phi^{(2)N=4}(u) - \pi^2 \text{sech}^2(\pi u) - 2\zeta_2
$$

\n
$$
f(u, u) = 1 + 2\left(\frac{N}{k}\right)^2 \left(\pi^2 \text{sech}^2(\pi u) + 2\zeta_2\right) + \dots
$$

\nNot enough data (and imagination) to uncover $f(u, v)$!

! Due to light-cone singularities, the Wilson loop Wⁿ is not well defined in D = 4 dimensions!

OPE resummation

Small fermions (i.e., fermions living on the second sheet of the $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ Riemann rapidity surface) act as supersymmetry generators (as their Hiemann rapidity surface) act as supersymmetry generators (as their
momentum goes to zero) and do not induce suppression in 't Hooft coupling! momentum goes to zero) and do not induce suppression
coupling! in 't Hooft

. One-loop corrections to $\mathcal{O}(\mathcal{C})$ from the fermion equations of motion equations of motions of motions of motions of \mathcal{C}

 \blacktriangledown In fact the entire OPE series can be resummed: \blacktriangle \blacktriangle

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$$
= e^{-\tau/2} \sum_{n=0}^{\infty} e^{-2n\tau} \int \frac{du}{2\pi} e^{i\sigma u} \left[\frac{(iu+3/2)_{2n}}{(2n)!} \mu_{Z/\bar{Z}}(u) + e^{-\tau} \frac{(iu+3/2)_{2n+1}}{(2n+1)!} \mu_{\bar{Z}/Z}(u) \right]
$$

$$
\times \left\{ 1 + \left(\frac{N}{k} \right)^2 \left[-\tau E_Z^{(2)}(u) + \delta \mu_Z^{(2)}(u) + i\sigma p_Z^{(2)}(u) + i\sigma \sum_{j=1}^n p_{\psi_s}^{(2)}(u - i(1/2+j)) - \sum_{j=1}^n \frac{\pi \tanh(\pi u)}{u - i(1/2+j)} + \sum_{j > k=1}^n \frac{2}{(u - i(1/2+j))(u - i(1/2+k))} \right] \right\}
$$

v Complete agreement with available two-loop data!

Somplete agreement with available two-loop data!

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- of both Wilson loops and scattering an V Natural (democratic) description of both Wilson loops and scattering amplitudes. $\ddot{\theta}$ ering $f(x)$

Supersymmetry anomaly

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► Odd and even order loop amplitudes require separate analyses. Bootstrap equations for pure spinon sector were not solved to all orders. Currently, it is not even clear which of them are more fundamental. $\frac{311}{f_{1}}$ $\frac{a}{c}$ there will be interest that the state of the
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der loop amplitudes require separate analys 0 Nc n order loop amplitudes require separate analyses.
■ ignorestigations of motions of m $\frac{d}{dt}$ **Moder and even order loop amplitudes require separate analyses.**
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- Wanted: \triangledown Wanted: \triangledown is integrated over the contour Cn, the fermionic EOM operator is inserted over the fermionic EOM operator is inserted over the fermionic EOM operator is inserted over the fermionic EOM operator is in P The product of two guantum fields separated by a light-like interval is not well defined by a light-like interval is not well defined by a light-like interval is not well defined by a light-like interval is not w Id : the Id matrix Id :

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- **THES-IOC** $\sum h$ is a sequent $\sum_{n=1}^{\infty}$ MABCD **is integrated over the fermionic EOM operator is instrumented over the fermionic EOM operator is instrumented to a state of motion equations of motions of motions of motions of motions of motions of motions of** \checkmark

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! Due to light-cone singularities, the Wilson loop Wⁿ is not well defined in D = 4 dimensions!

- hat is the unifying object accounting for both Wilson loops and scattering amplitudes? When addressed
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the language of flux-tube physics, it seems to suggest that it does indeed exist. V What is the unifying object accounting for both Wilson loops and scattering amplitudes? When addressed in
the language of flux-tube physics, it seems to suggest that it does indeed exist. and scattering amplitudes? When addressed i in What is the unifying object accounting for both Wilson loops and scattering amplitudes? When addressed in the language of hux-tube physics, it seems to suggest that it does indeed exist. hat is the unifying object accounting for both Wilson loops and scattering amplitudes? When addresse \sim To define the corresponding contribution to we have to the corresponding contribution to introduce a regularization.
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! Due to light-cone singularities, the Wilson loop Wⁿ is not well defined in D = 4 dimensions!

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